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Comparing the Cognitive Demand of Traditional and Reform Algebra 1 Textbooks

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Abstract

Research has shown that students achieved higher standardized test scores in mathematics and gained more positive attitudes towards mathematics after learning from reform curricula. Because these studies involve actual students and teachers, there are classroom variables that are involved in these findings (Silver and Stein, 1996; Stein et al., 1996). To understand how much these curricula by themselves contribute to higher test scores, I have studied the cognitive demand of tasks in two traditional and two reform curricula. This work required the creation of a scale to categorize tasks based on their level of cognitive demand. This scale relates to those by Stein, Schoenfeld, and Bloom. Based on this task analysis, I have found that more tasks in the reform curricula require higher cognitive demand than tasks in the traditional curricula. These findings confirm other results that posing tasks with higher cognitive demand to students can lead to higher student achievement.

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Chapter 1

Introduction

My motivation for this entire project is how mathematical tasks might play an important role in student achievement. I have found that many factors affect student achievement, and content is one of them. Because of the abundant textbook choices available in our nation today, I want to examine which textbooks are best for students. Many studies have shown the positive effects of reform curricula in mathematics classrooms. Students perform better on tests and have a positive attitude towards mathematics (Silver and Stein, 1996). However, is it because the teacher implemented the curriculum well? How important is the textbook? The purpose of this investigation is to compare traditional and reform Algebra 1 textbooks based on the task's level of cognitive demand, the amount of intellectual activity needed to perform a task. Because findings (Chávez-López, 2003) show that teachers use textbooks often, analyzing the written curriculum eliminates classroom variables, such as teachers' interpretations of the textbook or teachers' beliefs in the use of textbooks.

What is a curriculum? Several definitions and meanings of curriculum exist, but I will use the term curriculum to refer to the set of materials and guidelines used to plan and guide learning. The term traditional mathematics is used to describe the conventional methods used in mathematics instruction. The emphasis is on direct instruction, memorization, and skill building as opposed to understanding and thinking about mathematics. On the contrary, the term reform mathematics is used to describe approaches different from traditional mathematics and is based on the ideas published in the 1989 National Council of Teachers of Mathematics (NCTM) document concerning a new vision of mathematics education. More about the reform movement is described in the background.

Chapter 2

Background

There have been many reform movements in the history of American mathematics education. In the 1950s, there had already been concerns about the inadequacy of the mathematics curriculum, but the launch of the Russian satellite *Sputnik* in 1957 sparked the first mathematics reform movement (Herrera and Owens, 2001). This event created the perception that the United States was falling behind in the world of technology. However, *new math*, as it was called, was short-lived. Many teachers felt unprepared to implement the new curriculum, and the public believed that the *new math* had failed. Thus in the 1970s, the country moved to the *back to the basics* era. During this decade, the nation went back to emphasizing basic skills and procedures similar to the pre-Sputnik era.

By 1980, results showed no improvements in students' problem solving or even basic skills (Herrera and Owens, 2001). The National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* to emphasize that problem solving was needed in the mathematics curricula. Many publishers began to incorporate word problems into their textbooks. However, the changes were usually trivial (Schoenfeld, 2004). Finally, in 1989, the NCTM published the *Curriculum and Evaluation Standards for School Mathematics*. This document was revised and in 2000, the NCTM published the *Principles and Standards for School Mathematics*, which was later referred to as simply the *Standards* (Herrera and Owens, 2001). It was based on a constructivist view of the learning process focusing on students' confidence in mathematics, attitude towards mathematics, and higher-level thinking and problem solving in mathematics (Schoenfeld, 2004).

Proponents of this new mathematics movement argue that it is important for students to communicate and reason mathematically to solve problems. They advocate student-centered, problem solving. Studies show that when reform curricula are used in classrooms, students of those classes show more confidence in their mathematical abilities, achieve higher scores in mathematics, and even have a positive attitude towards mathematics (Silver and Stein, 1996). The researchers argue that traditional mathematics curricula do not foster students' conceptual understanding and real-world applications of mathematics. Traditional mathematics education mainly focuses on procedural knowledge. Thus the reform curricula challenge students to reason and communicate mathematics with a higher level of intellectual capability. They emphasize group work, written and verbal communication of mathematics, and deeper conceptual understanding of mathematics through the connection of ideas whereas traditional curricula emphasize procedures and arithmetic skills (Silver and Stein, 1996; Stein et al., 1996).

In 2000, the NCTM published the *Principles and Standards for School Mathematics* serving as an update to the 1989 *Standards*. In this document, the NCTM states six principles for school mathematics: equity, curriculum, teaching, learning, assessment, and technology. In learning, they emphasize that students must align "factual knowledge and procedural proficiency with conceptual knowledge to be effective learners (National Council of Teachers of Mathematics, 2000)." Thus a good balance between high- and low-level cognitive tasks can make students effective learners.

Following the reform movement, the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project in 1990-1991 selected six schools to improve the mathematics education at the school sites. At these schools, teachers had high expectations of students and posed more complex problems to encourage a deeper conceptual understanding of mathematics. Through high-level cognitive tasks, collaboration, and instruction that emphasized deeper mathematical understanding, students showed greater achievement. To qualify for a grade 9 algebra class, the percentage of students who passed the qualifying exam increased by 8% after the first year and 40% after the fourth year. Furthermore, students who qualified to take algebra were able to sustain their performance throughout the course (Silver and Stein, 1996). These successes show that high-level cognitive tasks can improve student achievement. Perhaps we can find a difference in the cognitive level of these tasks within the textbooks themselves.

2.1 The Role of the Written Curriculum

As mentioned previously, a curriculum is the set of materials and guidelines used to plan and guide learning, but the materials and guidelines that affect students' learning take on various forms. For instance, intended, written, enacted, or attained curriculum. The intended curriculum is the standards, or any guidelines to dictate what will be included in the written curriculum, or textbook. The enacted curriculum is how the teacher implements the written curriculum, and the attained curriculum is what the student actually learns and retains. Historically, the written curriculum has influenced mathematics practice in the classroom. Unlike objectives and standards that guide the curriculum, textbooks are concrete and provide the teacher with all of the information needed to implement lessons.

Textbooks are common resources in the classroom. They are frequently used by teachers and students and influence the instructional designs that teachers make for every lesson. In the 2000 National Assessment of Educational Progress, or NAEP, 72% of the eighth grade test takers reported using the textbook for mathematics problems everyday (Jones and Tarr, 2007). Another study has found that "70% of the teachers reported using their mathematics textbook in 75% or more of the lessons during two 10-day periods (Chávez-López, 2003)." These findings confirm that even though textbooks restrict the content that teachers teach, they provide many activities teachers can use and have the potential to influence how and how often the textbook is used for instruction.

From my observations of traditional textbooks, I found that many of the classrooms in which traditional textbooks were used emphasized teacher-directed instruction most of the time, which agrees with the traditional view of mathematics. Reform-based textbooks, I have found, are designed to develop conceptual understand through a variety of ways including big picture open-ended problems. If this is true of most traditional and reform textbooks, then from what we know of textbook use in mathematics instruction, examining textbooks may be helpful for understanding the connection between content and student achievement.

2.2 Classification of Tasks

Cognitive demand is the amount of intellectual activity required to perform a task. Several studies have looked at the level of cognitive demand of tasks as they are written and implemented in the classroom. Looking at the cognitive demand is important because it helps identify the level of thinking processes that occur in a student's mind. It is important for students to think and reason about mathematics instead of simply memorizing facts and practicing procedures so that they can use mathematics meaningfully. Then students can develop critical thinking skills to succeed in mathematics and in life. These higher-level processes are connected to high-level cognitive tasks. In this way, cognitive demand is a way to find out whether or not meaningful learning is happening.

There are several ways to categorize tasks based on cognitive demand. One hierarchy of cognitive levels is Bloom's Taxonomy of the Cognitive Learning Domain. From lowest to highest, the categories are knowledge, comprehension, application, analysis, synthesis, and evaluation. Knowledge requires recall memory. Comprehension requires an understanding of concepts. Application requires the use of concepts in different contexts. Analysis requires the breaking down of concepts. Synthesis requires building or creating a structure from previous knowledge. Evaluation requires making judgements about the concept or problem (Clark, 2010). These categories have been used to describe not only mathematical tasks but cognitive tasks in general.

In coding how mathematical tasks were set up in reform classrooms, Stein, Grover, and Henningsen (1996) classified tasks as one of three categories: "memorization, the use of formulas, algorithms, or procedures without connection to concepts, understanding, or meaning; the use of formulas, algorithms, or procedures with connections...; and cognitive activity that can be characterized as 'doing mathematics,' including complex mathematical thinking and reasoning activities." Because the levels are not mutually exclusive, they coded a task with two or more categories whenever necessary.

Similar to Stein, Grover, and Henningsen's classifications are the three categories used to analyze the results of the Third International Mathematics and Science Study (TIMSS) 1999 Video Study. The three categories used for the analysis were using procedures, stating concepts, and making connections (Hiebert et al., 2005).

Another research group used the idea of cognitive demand in analyzing elementary school mathematics textbooks published in the twentieth century. The levels consisted of six levels in a continuum going from more concrete to more abstract. These were: general strategy description, nonconceptual strategy, physical representation and counting, representation with fairly concrete items, moderately abstract representation, conceptually based shortcuts, and conceptual use of properties of operation and number (Baker et al., 2010). This was yet another way of setting up a scale to analyze the cognitive demand level of a task. However, this type of categorizing does not work in all mathematical tasks. It has been specified to elementary curricula.

Finally, Schoenfeld highlighted the difference between problem solving heuristics versus algorithms and described what is considered a higher level of problem solving. Many textbooks provide sections with strategies for solving word problems. The students are then given practice on these strategies, and subsequently assessed on them. He claims that "when strategies are taught this way, they are no longer heuristics... they are mere algorithms." (Grouws, 1992) When strategies are explicitly taught to students, they use those in an algorithmic way, which requires little cognitive demand. Heuristics require students to solve problems based on experience and previous knowledge, essentially making connections among different concepts, ultimately requiring a high level of cognitive demand.

Chapter 3

Methods and Results

For my thesis, I investigated an absolute value equation lesson from each of these textbooks: Holt McDougal *Algebra 1*, Glencoe McGraw-Hill *California Algebra 1: Concepts, Skills, and Problem Solving*, *Center for Mathematics Education (CME) Project Algebra 1*, and College Preparatory Mathematics (CPM) *Algebra Connections California Edition*. These textbooks were chosen based on availability and the lesson comes from the California Algebra 1 Content Standard 3.0: Students solve equations and inequalities involving absolute values (California State Board of Education, 2007). I chose a standard that I personally enjoyed and a mathematics topic that all the textbooks taught. I decided to look at only the first part of the standard because the size of this task.

Algebra 1 by Holt McDougal is a traditional textbook with typical chapters and sections. However, in the 2011 edition, labs, multistep problems, study guides, chapter tests, quizzes, and writing are placed in several parts of the text (Burger et al., 2011). This edition is based on the Common Core State Standards (California State Academic Content Standards Commission, 2010).

California Algebra 1: Concepts, Skills, and Problem Solving by Glencoe McGraw-Hill is another traditional textbook that is traditionally organized as well. This textbook offers study guides, chapter reviews and tests, internet resources, and many other useful supplements (Holliday et al., 2008). It also provides the correlations to the California Content Standards (California State Board of Education, 2007).

The CME Project curriculum can be seen as a hybrid of reform and traditional curricula. However, I call it a reform curriculum because it differs from the traditional in its fundamental ideas. Although the organization of the textbook is similar to that of a traditional textbook, the lessons promote more discovery of concepts and less direct instruction (Cuoco, 2009). Their claim is that understanding develops "as a result of independent (or guided) investigations and as the result of reading, discussing, and internalizing mathematical exposition (Cuoco, 2010)." They also emphasize "reasoning by continuity, abstracting regularity from repeated calculations, developing theories based on numerical evidence, and using thought-experiments (Cuoco, 2010)." These are examples of habits of mind, essential thinking skills that mathematics can help develop. Whether or not students continue in mathematics, these habits help develop critical that can be helpful in other subject areas as well.

CPM's *Algebra Connections California Edition* was created by middle and high school teachers working in collaboration with college professors. Although the structure of the book is not tradtional, it is structured so that learning is consistent. The book is organized in a way that reviews and builds on previous material by integrating several topics throughout the book (Dietiker and Baldinger, 2008). The curriculum supports collaboration and discussion of mathematics, communication of mathematics, and a deeper understanding of the material (College Preparatory Mathematics, 2011). There are several tasks in which students work in pairs or teams, share their ideas, and communicate the mathematics through writing. In addition, the end of every lesson has closure, in which the students reflect on the lesson (Dietiker and Baldinger, 2008). All of these activities encourage a deeper understanding of mathematical ideas.

3.1 Identification of Tasks

In order to code tasks, I first had to identify them. I consider any statement or question that required cognitive function related to the mathematics at hand by students to be a task.

Instead of taking one numbered or lettered problem, I took parts of problems because each part may require a different level of cognitive demand. For example, if a problem is stated as, "How can you graph a line with slope 0? Explain," I considered "how can you graph a line with slope 0?" and "explain" as different tasks and therefore, coded them separately. Tasks included questions from the teacher's guide. Furthermore, any task that asked students about other mathematical topics not related to the lesson at hand was not considered. In addition, a factor that influenced the coding was context. Whenever I coded a task, the previous lessons, the sequence of instruction, and any examples provided beforehand influenced the cognitive demand that a student will most likely experience. For this reason, I made the following assumptions: teachers follow the teacher's guide thoroughly, teachers assign all problems in the textbook, and students have some understanding of the content previously covered. Because of the size of the task, I ignored any extraneous instructional material that did not appear in the textbook.

3.2 Levels of Cognitive Demand

As mentioned in Section 2.2, there have been task analyses using different scales. I could not apply the scale used by Baker et al. (2010) because it did not apply beyond elementary mathematics. I also did not use Bloom's Taxonomy of the Cognitive Learning Domain because whenever I tried to categorize a task, it seemed that the task fit into more than one category.

My next course of action was to simply take one task at a time and describe its nature. After finding tasks related to the standard, I wrote those tasks on separate papers, and later grouped them according to similar task types.

I gave each group a brief description as follows, representing increasing levels of cognitive demand: tasks require recall memory and are in direct correspondence with definitions, formulas, and examples; tasks require understanding of concepts and are able to give an example of the concept using some representation (e.g., sketch); tasks require comparison of equations, graphs, and so on. They also require the ability to look for patterns and describe those patterns; tasks require that students apply concepts to solve problems; tasks require the interpretation of equations and meanings of results in context; tasks require creation of a context around the equation or results; tasks require the ability to find errors, articulate them, and fix them; tasks require justification of results, thoughts, or conjectures. However, this method led to problems. I realized the categories were often too narrow.

For the scale I used, I extended the idea Schoenfeld described about problem solving (Grouws, 1992) as described in Section 2.2. I also integrated the procedures with and without connections that Stein et al. (1996) used in their research. The foundation of the scale can be seen in terms of the procedures. Level 1 tasks require no procedures. Level 2 tasks require the use of previously known or taught procedures. Level 3 tasks require


EXAMPLE 1

• What does *absolute value* mean?

Figure 3.1 An example of a Level 1 task from the Holt McDougal textbook (Burger et al., 2011). This task is considered Level 1 because it is a recall of facts.

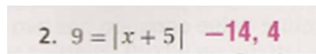
2. $9 = |x + 5|$ $-14, 4$

Figure 3.2 An example of a Level 2 task from the Holt McDougal textbook (Burger et al., 2011). This task is considered Level 2 because students must use the procedures explicitly taught to them at the beginning of the lesson in order to solve the equation.

the development of own procedures. Level 4 tasks require the application of learned or developed procedures in different contexts. Level 5 tasks require the mathematical justification of the procedures. A more detailed description and examples of each level follows:

1. Tasks require no procedural knowledge; only recall memory. Most are in direct correspondence with definitions, formulas, and examples. See Figure 3.1.

2. Tasks require use of procedures previously known or explicitly taught. They also require applications of definitions, formulas, and so on. See Figure 3.2.

3. Tasks require own development of new procedures whether or not by combination of two or more old procedures. They also require that students be able to articulate concepts in their own words. Thus they are able to compare and contrast ideas (making connections among concepts). See Figures 3.3 and 3.4.

4. Tasks require application of procedures from Level 2 or 3 in different contexts. They also require the creation of a problem or situation given a solution. See Figure 3.5.

5. Tasks require mathematical justification of results or steps in a procedures. See Figure 3.6.

Whenever tasks did not fit the exact descriptions of my categories, I used my best judgement to label the task a number from one to five depending on what I thought was indicative of the level of cognitive demand.

10-60. Consider the equation $|2x-5|=9$.

a. How many solutions do you think this equation has? Why? **[While students may think there is only one solution, there are actually two solutions.]**

b. Which of the three solution approaches do you think will work best for this equation? **[Looking inside offers a quick solution.]**

c. With your team, solve $|2x-5|=9$. Record your work carefully as you go. Check your solution(s). **[Since $2x-5=9$ or -9, then $x=7$ or $x=-2$.]**

Figure 3.3 The boxed problem in this figure shows an example of a Level 3 task from the CPM textbook (Dietiker and Baldinger, 2008). Students have not been taught a procedure to solve absolute value equations. They have to develop it on their own.

- How is the absolute-value sign similar to parentheses? How is it different?

Figure 3.4 Two examples of a Level 3 task from the Holt McDougal textbook (Burger et al., 2011). Both tasks require students to understand the concept of absolute value in order to compare and contrast absolute value to parentheses.

33. **Construction** A brick company guarantees to fill a contractor's order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.

Figure 3.5 The boxed task in this figure shows an example of a Level 4 task from the CPM textbook (Dietiker and Baldinger, 2008). Students must use the concept of absolute value in a different context. Furthermore, they are asked to write an equation.

b. Likewise, create an equation with an absolute value that will have only one solution. **Justify** why it will have only one solution. **[Answers vary, but it should contain an absolute-value expression equal to zero.]**

Figure 3.6 The boxed task in this figure shows an example of a Level 5 task form the CPM textbook (Dietiker and Baldinger, 2008). Students must justify their example of an absolute value equation with only one solution.

Textbook	n	Level 1	Level 2	Level 3	Level 4	Level 5
Holt McDougal	82	16%	52%	9%	12%	11%
Glencoe McGraw-Hill	28	18%	50%	0%	14%	18%
CME Project	40	5%	55%	10%	17%	13%
CPM	18	0%	55%	17%	11%	17%

Table 3.1 Percentage of tasks that were Level 1, 2, 3, 4, or 5 for each of the analyzed textbooks. n represents the number of tasks.

Textbook	n	Low-Level	High-Level
Holt McDougal	82	68%	32%
Glencoe McGraw-Hill	28	68%	32%
CME Project	40	60%	40%
CPM	18	55%	45%

Table 3.2 Percentage of tasks that were either low-level or high-level for each of the analyzed textbooks. n represents the number of tasks.

3.3 Results

After tabulating the codes, I calculated the percentage of tasks that were Level 1, 2, 3, 4, or 5 (see Table 3.1). A complete list of the results are tabulated in Appendix A. As expected, the two traditional textbooks had the highest proportion of Level 1 tasks. Unexpectedly, Glencoe McGraw-Hill's textbook had the highest proportion of Level 5 tasks with CPM's textbook as a close second.

To look at the balance between low and high-level cognitive tasks, I categorized Levels 1 and 2 as low-level tasks and Levels 3, 4, and 5 as high-level tasks. I calculated the percentage of tasks that were either low-level or high-level for each of the textbooks (see Table 3.2). From this broader look at cognitive demand, the results show that the two traditional textbooks have a slightly higher proportion of low-level tasks.

Chapter 4

Conclusion

Due to the scale of this analysis, it is unclear to what extent these findings generalize. If the findings do generalize then it would appear that all four textbooks have similar proportions of level one through five tasks. However, there is a difference in the number of tasks. Holt McDougal's textbook had a high number of procedural skill-based problems. From an overall view of the textbooks, I have found from the Holt McDougal to Glencoe McGraw-Hill to the CME Project to CPM, there is an increasing level of cognitive demand in the lesson portion of absolute value equations. The two traditional textbooks stated the definitions and provided examples of how to solve absolute value equations before the student was asked to practice problems. In contrast, the CPM textbook guided the students toward developing the procedures for solving absolute value equations. In this way, I believe that further comprehensive examination of all the lessons within these textbooks and other textbooks would result in conclusive evidence that reform-based textbooks have tasks with high levels of cognitive demand.

4.1 Implications

This research is relevant for mathematics education policy, especially for textbook adoption. The more evidence is gathered in this area, the easier the push for more reform textbooks in the classrooms. With this, also comes better preparation for teachers in implementing tasks that require higher levels of cognitive demand. Teachers can modify their daily instruction according to their students' needs by raising or lowering the cognitive demand of the tasks in the textbooks they are required to use.

A more comprehensive investigation of the cognitive demand of tasks in textbooks can be connected to research done on task implementation in the classroom and its correlation to student achievement. Once that connection is made, it will be easier to place better textbooks in the classroom that will provide instruction based on reform values. Ultimately, this can lead to higher student achievement.

Appendix A

Coding Results

The tables represent the coding results from all four textbooks. Each table contains the page number, task number, task type, code, and the reason for the code (for when I thought there would be confusion).

A.1 Holt McDougal

Table A.1 The following table represents the codes for the tasks in the Holt McDougal textbook (Burger et al., 2011).

Page	Task	Type	Code	Reason
113	1a	CW	2	Use of explicitly taught procedures.
	1b	CW	2	Use of explicitly taught procedures.
	example 1 bullet 1	TGQ	1	Recall of definition.
	example 1 bullet 2 task 1	TGQ	3	Comparing.
	example 1 bullet 2 task 2	TGQ	3	Contrasting.

Continued on Next Page. . .

Table A.1 – Continued

Page	Task	Type	Code	Reason
	example 1 bullet 3	TGQ	1	Possibly a 3 if the student says it in their own words that show understanding. This is said in the guided instruction, so it gets a 1. Possible that students make a connection to answer the question.
	2a	CW	2	Use of explicitly taught procedures.
	2b	CW	2	Use of explicitly taught procedures.
	example 2 bullet 1	TGQ	1	Recall of short blurb in middle of page 113.
	example 2 bullet 2	TGQ	1	Recall of short blurb in middle of page 113.
114	3 task 1	CW	2	Very similar to Example 3
	3 task 2	CW	2	Use of explicitly taught procedures.
	example 3 bullet 2	TGQ	3	Similar to short blurb in example 3. However, the question is more general, so it gets a 3.
	Think and Discuss 1	CW	3	Student must articulate the steps that they were explicitly taught. They are not required to justify anything, just explain how to do the steps.
	Think and Discuss 2	CW	4	Although students know what kind of absolute value equations have none, one, or two solutions, they must come up with their own examples.

Continued on Next Page…

Table A.1 – Continued

Page	Task	Type	Code	Reason
	Close task 1	CW	1	Students most likely will copy the steps highlighted on page 113.
	Close task 2	CW	1	Recall of definition.
115	1	CW	1	Requires hardly any thinking.
	2	CW	2	
	3	CW	2	
	4	CW	2	
	5	CW	2	
	6	CW	2	
	7	CW	2	
	8	CW	1	Requires hardly any thinking.
	9	CW	2	Application of fact that if absolute value expression equals a negative number, then there is no solution.
	10	CW	2	
	11	CW	2	
	12	CW	2	
	13 task 1	CW	3	Because of Example 3, this gets a 3 instead of a 4.
	13 task 2	CW	2	
	14	HW	1	Requires hardly any thinking.
	15	HW	2	
	16	HW	2	
	17	HW	2	
	18	HW	2	
	19	HW	2	
	20	HW	2	
	21	HW	2	
	22	HW	2	
	23	HW	2	
	24	HW	2	

Continued on Next Page…

Table A.1 – Continued

Page	Task	Type	Code	Reason
	25	HW	2	
	26	HW	2	
	27	HW	2	
	28	HW	2	
	29 task 1	HW	4	
	29 task 2	HW	2	
	30 task 1	HW	2	They only have to solve.
	31 task 1	HW	3	Given problem 30, it gets a 3.
	31 task 2	HW	2	
	32 task 1	HW	4	
	32 task 2	HW	2	
	33 task 1	HW	4	
	33 task 2	HW	2	
	34 task 1	HW	2	
	34 task 2	HW	5	Must be able to interpret the meaning of the answer.
116	39 task 2	HW	5	Must give a justification for their answer.
	40 task 2	HW	5	Must give a justification for their answer.
	41 task 2	HW	5	Must give a justification for their answer.
	42 task 1	HW	4	
	42 task 2	HW	2	
	43a	HW	4	
	43b	HW	2	
	43c task 2 (Explain)	HW	5	
	43d	HW	5	
	44a	HW	4	
	44b	HW	4	
	44c	HW	2	
117	45 task 2	HW	5	
	46 task 2	HW	5	
	47	HW	4	

Continued on Next Page. . .

Table A.1 – Continued

Page	Task	Type	Code	Reason
	48	HW	2	
	49	HW	4	
	50 task 1	HW	2	
	50 task 2	HW	5	
	51 - 2	HW	1	Recall of properties.
	51 - 4	HW	1	Recall of properties.
	51 - 5	HW	1	Recall of properties.
	52	HW	3	Students must come up with some way to solve the equation with absolute values on both sides.
	Journal task 1	CW	1	Recalling the steps.
	Journal task 2	CW	2	

A.2 Glencoe McGraw–Hill

Table A.2 The following table represents the codes for the tasks in the Glencoe McGraw–Hill textbook (Holliday et al., 2008).

Page	Task	Type	Code	Reason
322	TEACH bullet 3	TGQ	4	Coming up with how to represent situation with an absolute value equation.
323	1a	CW	2	Solving by procedures explicitly taught.
	1b	CW	1	Like example 1.
	add'l example 1a	TGQ	2	
325	1 task 1	CW	2	
	2 task 1	CW	2	

Continued on Next Page...

Table A.2 – Continued

Page	Task	Type	Code	Reason
	3 task 1	CW	1	
	7	HW	2	
	8	HW	2	
	9	HW	1	
	10	HW	2	
	11	HW	2	
	12	HW	1	
	13	HW	1	
	14	HW	2	
	15	HW	2	
	16	HW	2	
	17	HW	2	
	18	HW	2	
326	38	HW	4	Coming up with an equation—implicit 2 when student must solve it.
	41	HW	4	Coming up with a situation for an equation.
	42 task 2	HW	2	Application of knowledge of absolute value.
	43 task 2	HW	5	
	45 task 2	HW	5	
	46b	HW	4	
	47 task 2	HW	5	

A.3 CME Project

Table A.3 The following table represents the codes for the tasks in the CME Project textbook (Cuoco, 2009).

Page	Task	Type	Code	Reason
204	8	CW	2	
	9	CW	2	Application of knowledge of absolute value and how they can't equal something negative.
207	3a task 1	CW	4	
	3a task 2	CW	2	
	3b	CW	2	
	4a	CW	2	
	4b	CW	2	
	4c	CW	3	Discovery of what happens when the equation equals 0.
	4d	CW	1	
	4e	CW	2	
	4f	CW	2	
	6a	CW	4	
	6b	CW	4	
	6c	CW	4	
	6d	CW	4	
	6e	CW	4	
	6f	CW	4	
208	9	HW	5	
	10a task 2	HW	5	They must either explain reasoning or provide a counterexample.
	10b task 2	HW	5	
	10c task 2	HW	5	
	10d task 2	HW	5	
209	11a	HW	2	
	11b	HW	2	
	11c	HW	2	

Continued on Next Page…

Table A.3 – Continued

Page	Task	Type	Code	Reason
	11d	HW	2	
	11e	HW	2	
	12	HW	2	
	13	HW	3	They must combine what they learned about abs value equations and extend it.
	14	HW	1	
	15a	HW	2	
	15b	HW	2	
	15c	HW	2	
	15d	HW	2	
	15e	HW	3	Articulating a pattern.
	16a	HW	2	
	16b	HW	2	
	16c	HW	2	
	16d	HW	2	
	16e	HW	3	Articulating a pattern.

A.4 CPM

Table A.4 The following table represents the codes for the tasks in the CPM textbook (Dietiker and Baldinger, 2008).

Page	Task	Type	Code	Reason
781	10-51b	HW	2	They are applying their knowledge of absolute value.
782	10-52a	HW	2	
	10-52b	HW	2	
	10-52c	HW	2	
	10-52d	HW	3	
786	10-60a task 2	CW	5	

Continued on Next Page...

Table A.4 – Continued

Page	Task	Type	Code	Reason
	10-60c task 1	CW	3	Combination of absolute value and using what they know about solving equations.
	10-61a task 1	CW	2	
	10-61a task 2	CW	4	
	10-61a task 3	CW	5	
	10-61b task 1	CW	4	
	10-61b task 2	CW	5	
	10-62 Learning Log	CW	3	Making connections between the number of solutions of equations in perfect square form and those with absolute value.
790	10-69a	CW	2	
	10-69d	CW	2	
	10-69f	CW	2	
792	10-73b	HW	2	
	10-73d	HW	2	

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