A model of human performance on the travelling salesperson problem

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Abstract

A computational model is proposed of how humans solve the Travelling Salesperson Problem (TSP). Tests of the model are reported using human performance measures from a variety of 10, 20, 40 and 60 node problems, a single 48 node and a single 100 node problem. The model provided a range of solutions that approximated the range of human solutions and which conformed closely to quantitative and qualitative characteristics of human performance. The minimum path lengths of subjects and model deviated by average absolute values of 0.0%, 0.9%, 2.4%, 1.4%, 3.5% and 0.02% for the 10, 20, 40, 48, 60 and 100 node problems respectively. Because the model produces a range of solutions rather than a single solution, it may find better solutions than some conventional heuristic algorithms for solving TSPs, and comparative results are reported that support this suggestion.

Introduction

The Euclidean form of the Travelling Salesperson Problem (TSP) consists of finding the shortest path ("tour") which passes through a set of points and returns to the origin. Any particular instance of a TSP obviously has a finite set of solutions, so that there is a guaranteed method of solving any problem by exhaustive search of the solution space. However, exhaustive search becomes increasingly impractical as the number of points (n) increases. This is because the number of possible solutions increases as $(n - 1)!/2$ (ignoring direction of travel). Thus, while a computer generating 1000 solutions per second would require only three minutes to find all solutions to a 10-node problem, a 20-node problem would occupy it for almost 4 million years. The increased capacity of computers over the past ten years has made it possible to find guaranteed optimal solutions to fairly large TSPs within reasonable time periods. In 1986 a new record was set (since broken) when a 2392-city problem was solved, breaking the previous record of 532 cities, and requiring 27 hours (Sangalli, 1992). There has been a long-term interest in the field of operations research in devising economical procedures for finding solutions that approximate the optimal, and many such heuristic techniques have been developed and compared. To achieve "reasonable" approximations to an optimal solution, to within a few percentage points above the shortest path, such procedures generally need to perform in the order of n^3 calculations (Golden, Bodin, Doyle, & Stewart, 1980).

One approach to improving heuristic procedures has been to enlist the assistance of human operators (Krolak, Felts, & Marble, 1971; Michie, Fleming, & Oldfield, 1968). Krolak *et al* compared computer-generated solutions with solutions produced by a human-computer interactive approach. A number of comparisons were reported and indicated that human-computer solutions were at least as short, and often shorter, than those generated by computer alone. The results imply that human operators respond to TSPs in a manner that is not only *different* from, but more *effective* than, the heuristics used by Krolak *et al*. Otherwise, the human-computer solutions should have been the same as, or poorer than, those generated by computer only. This inference is supported by results reported by MacGregor & Ormerod (1996), indicating that the best human solutions to a range of 10-city and 20-city problems were often superior to those generated by several heuristic procedures. The results of these researches suggest that it may be worthwhile to investigate how humans solve TSPs and, if feasible, to incorporate elements of the human approach into heuristic algorithms. The present article represents a step in this direction. The article draws on characteristics of human solutions suggested by MacGregor & Ormerod (1996) to identify a model of human TSP solving behaviour. The model is tested using previously published and unpublished human data.

MacGregor & Ormerod (1996) reported the results of two experiments with small TSPs (10 and 20 nodes) which indicated the following characteristics of human performance: (1) people produce "good" solutions, with the best human performances being on average within 0.75% of the optimal solutions; (2) differences, if any, in skill levels across individuals are slight, in that the average correlation of the rank order of individuals' solutions across problems is virtually zero; (3) the complexity of problems varied directly with the number of "internal" nodes falling within the perimeter of a problem, not the total number of nodes; (4) people produced paths that connected boundary points in sequential order of adjacency (449 of 455 solutions adhered to this principle); (5) few solutions had lines which crossed (11 of 455 solutions); (6) solutions had significantly fewer "indentations" than the average allowed by the structure of the problems. Ormerod & Chronicle's (1999) results further suggest that some form of global processing may underlie human solutions and, taken together, the results provide evidence as to the kinds of processes that generate human solutions to TSPs. Below we discuss the nature of those processes, and propose a model of human TSP performance. The model is tested using data from previous experiments.

Heuristic procedures

The model is a synthesis of features taken from a number of approximate heuristic procedures that have been developed in the field of operations research. Many such procedures have now been designed and compared (Golden, Bodin, Doyle & Stewart, 1980) and there are several that might conceivably serve as models of human performance. One of the simplest is the Nearest Neighbour (NN) approach, which is the apocryphal model of human performance. For the Nearest Neighbour method an initial point is selected as the start of the path (the leading point). The unconnected point closest to the leading point is then added to the path, and becomes the leading point. This repeats until all nodes have been added, then the last and first points are joined to complete the tour. Another type of heuristic follows a "convex hull" approach, in which an initial subtour is obtained by connecting all points lying on the perimeter of the array of nodes (the convex hull) and thereafter interior points are included in the subtour sequentially in an order that adds minimally to the incremental distance. A simple analog of the process is to think of the nodes as pegs in a board with the initial subtour formed by stretching a rubber band around their perimeter. Interior pegs are then incorporated by stretching the segment of the band closest to an interior peg over that peg, and then repeating this until all of the pegs have been included. Convex hull heuristics differ in their choice of "insertion criterion", that is, in how they define "closest". For example, the "cheapest insertion" criterion finds the points *i, j* in the subtour and *k* not in the tour for which the distance i*, k* + *k,j* - *i, j* is minimal, and *k* is inserted between *i* and *j* (Golden *et al*, 1980). Norback & Love (1977) used a largest interior angle criterion, finding the unconnected point *k* for which the angle *i, j, k* is maximal, and inserting *k* between *i* and *j*. In general, the convex hull approach , as a heuristic for finding good solutions, has the advantage of capitalizing on one of the well-known characteristics of an optimal solution. That is the optimal path connects adjacent points on the boundary of the convex hull in sequence, though it may pass through interior points between adjacent boundary points (Flood, 1956). Convex hull heuristics conform to this principle. A corollary is that solutions which do not adhere to this principle will result in crossed arcs, which is clearly non-optimal.

Analysis

The experimental findings summarized previously indicated that human solutions show a high degree of consistency in a number of respects. Almost invariably, subjects connected boundary points in order of adjacency and, equally invariably, produced no crossed arcs. The quality of their solutions was influenced by the number of interior points to be incorporated. They generated solutions with few indentations. These consistent elements, together with the fact that proficiency appeared to be equally-distributed across individuals, argue that a relatively uniform process underlies performance. Furthermore, the results suggest the general form of that process to be similar to a convex hull approach. Like convex hull heuristics, human solvers join points on the boundary in order of adjacency, avoid crossing lines, and seem to be capable of producing optimal or near optimal solutions with a high degree of regularity. The facts that they produce solutions with few indentations and that the quality of their solutions erodes with increasing number of interior points further suggests that, in generating solutions, they may like to stay close to the convex hull. In addition, the idea that human solutions may be governed by the outline of the array is consistent with a global processing approach to the problem (Ormerod & Chronicle, 1999). Nevertheless, there are elements of performance that are inconsistent with convex hull heuristics. Participants appear to complete problems in a sequential way, with the terminal node of one connection becoming the origin node of the next. Of the three heuristics described above, only the Nearest Neighbour approach operates in this way. In contrast, the convex hull heuristics may jump between non-adjacent arcs seeking the next best move in a manner seemingly quite uncharacteristic of human performance. The NN approach is similar to human performance in another respect, in that it can produce a range of solutions to a single problem, while the other two heuristics typically produce a single solution. It appears from these observations that human solution processes may incorporate characteristics of each of these heuristics while being identical to none. That is, a previously unidentified heuristic may guide human solutions.

MacGregor & Ormerod (1996) compared the performances of these three heuristics with human solutions using six new 10-node problems, a standard 10-node problem, and seven 20-node problems. The results were consistent with the foregoing analysis. For the six 10-node problems the two convex hull heuristics generated only one solution each per problem. The NN procedure was repeated for each possible starting point, and therefore could produce up to 10 different solutions per problem. In terms of the best solutions found, the average for the NN, cheapest insertion and largest interior angle procedures were 1.9%, 0.9% and 4.3% above the optimal, respectively. This compared with 0% for the best human solutions. Only the human and NN approaches produced a range of solution path lengths. With respect to these distributions, the mean of human performances on each problem, averaged across problems, was 3.8%, compared with 6.0% for the NN approach. The experiment also employed the 10-node problem described by Dantzig, Fulkerson, & Johnson (1959). For this problem the best human solution was optimal, compared with solutions of 0.6, 2.7 and 3.0 percent above optimal for the NN, cheapest insertion and largest interior angle approaches respectively. The average of the human solutions was 3.8% above optimal, compared to 10.4% for the NN approach. Similar results were obtained using the seven 20-node problems. Averaged across the problems, the best solutions for the NN, cheapest insertion and largest interior angle procedures were 3.6%, 4.1% and 11.0% over optimal, respectively, compared with 1.5% for the best human solutions. The mean solutions for humans and the NN procedure were 6.3% and 11.5%, respectively. The results seem to indicate that, for these small problems at least, the best human performances were consistently superior to any of the heuristics, while the average of the human solutions was substantially superior to the average for the Nearest Neighbour heuristic (the only heuristic to provide a range of solutions).

Towards a model of human performance

As they stand, none of the three heuristics appears able to provide an adequate model of the human solution process. As a first step to creating a more adequate model, an effort was made here to incorporate the psychologically attractive elements of each of the heuristics. It appeared that there were three such elements. First, the model should retain a convex hull basis, since this ensures that boundary points are connected in order. Second, the procedure should be sequential, since this conforms to what subjects do. Third, it should have the potential to produce different paths when starting on different points, like the Nearest Neighbour procedure.

The model[1]┐ described below may be used with different insertion criteria (e.g. cheapest insertion, largest angle) and is described initially in a general way. For convenience, the term "close" is used to refer to the decision produced by the insertion rule. Informally, the model operates as follows. Initially, the arcs between adjacent boundary points are "sketched" -- not connected, but used as a guide and reference for subsequent judgements. Next a starting point and direction (clockwise or counter-clockwise) are selected randomly. If the start is an interior point, it is connected immediately to the "closest" point on the boundary in the direction of travel. From here, using the arc sketched between this point and the adjacent boundary point as a reference, the closest interior point is identified. If this point is closer to any other sketched or connected arc, then it is "passed", and the path moves to the next adjacent node on the boundary in the direction of travel. This continues until an interior point is found that is not closer to any other arc. The current node is then connected to this point, but remains

[1]Tests of a similar model are reported by Lee (1985), but as a method of generating solutions to TSPs, not as a model of human performance. Also involved in the development of the model reported by Lee were N.Lam, E.Lee and J.MacGregor

the current node. Using this newly-created arc as reference, the process repeats, until a complete tour is obtained.

To illustrate, consider how the procedure performs on the problem shown in Figure 1. First the arcs joining points on the boundary of the hull are sketched, then a starting-point and direction are selected at random, say starting with point 1 and moving in a counter-clockwise direction. The current arc is therefore 1-2, and Point 7 would be considered for inclusion. However, the test would reveal that point 7 is closer to another arc 5-1, and would be "passed". Point 6 would then be considered but, being closer to arc 2-3, would also be passed. This exhausts candidates for inclusion between points 1 and 2 and the algorithm would move on to point 2, and the arc 2-3 would become the current arc. Point 7 would be considered for inclusion but, being closer to arc 5-1, would be passed over. Next point 6 would be considered, would pass the test, and would be connected between points 2 and 3. That is, the arc 2-3 would be erased and two new arcs, 2-6 and 6-3, would be created. The arc 2-6 now becomes the current arc, and point 7 would be considered for inclusion between points 2 and 6. It would fail the test and, having exhausted all free points from this arc, the procedure will move on to point 6, and the arc 6-3 becomes the current arc. This would proceed until arc 5-1 becomes the current arc and point 7 is inserted between 5 and 1. In this case, this completes the tour, with the path 1, 2, 6, 3, 4, 5, 7, 1. A more detailed description is given in the steps below.

Step 1: Sketch connections between adjacent boundary points of the convex hull.

Step 2: Select a starting point and a direction randomly.

Step 3: If the starting point is on the boundary, then the starting node is the "current node". The arc connecting the current node to the adjacent boundary node in the direction of travel is referred to as the "current arc". Proceed immediately to Step 4. If the starting point is not on the boundary then apply the insertion rule to find the closest arc on the boundary. Connect the starting point to the end node of the closest arc which is in the direction of travel. This node becomes the "current node".

Step 4: Apply the insertion criterion to identify which unconnected interior point is closest to the current arc. Apply the insertion criterion to check whether the closest node is closer to any other arc. If not, proceed to Step 5. If it is, then move to the end node of the current arc. This becomes the current node. Repeat Step 4.

Step 5: Insert the closest node. The connection between the current node and the newly-inserted node becomes the current arc. Retaining the current node, return to Step 4, and repeat Steps 4 and 5 until a complete tour is obtained.

Tests of the model

Comparisons with data from 10 and 20 node TSPs

Two versions of the model were tested initially. One used the cheapest insertion criterion, the other the largest interior angle criterion. The first tests used the stimuli and results from MacGregor & Ormerod, (1996), who reported results for 45 participants in their first experiment and 20 in the second. Both experiments used a within-subjects design and in both cases participants were instructed to select a starting point then draw the shortest path that passed through each point and returned to the start. Subjects were tested in a group setting and were instructed to take no more than 5 minutes per problem. In testing the model, 45 replications were conducted with each basic problem from Experiment 1 to give an equal number of model and human solutions. Similarly, 20 replications were conducted for the problems from Experiment 2. The main results are presented in Table 1, which shows the minimum, mean and maximum path lengths obtained with both versions of the model as a percentage deviation from the comparable experimental results (i.e. model results minus experimental results divided by experimental results, expressed as a percentage). Summary statistics are provided in two forms, the mean absolute values of the deviations, and the mean algebraic values. The former provides a measure of goodness-of-fit, the latter an indication of whether the model tends to overestimate (positive values) or underestimate (negative values) the corresponding human results.

For the 10-node problems, it can be seen that both versions of the model produced minimum path lengths similar to the experimental results, though the cheapest insertion criterion (CI) appeared to give a more consistent performance, with a maximum deviation of 0.0%, compared with 5.2% for the largest angle criterion. Looking next at the mean path lengths, a similar picture emerges. Overall, both criteria produced solutions close to the experimental results, but more consistently with the cheapest insertion than with the largest angle criterion. Finally, the maximum path lengths resulted in the poorest fit for both criteria, with deviations in both cases as extreme as -19%. In general, the models produced maximum path lengths that were shorter than the worst human solutions.

For the 20-node problems a similar pattern emerged. For minimum and mean path lengths the cheapest insertion criterion produced solutions closer to the human results than the largest angle, though with slightly less accuracy than with the 10-node problems. Again, the poorest fit occurred for maximum path lengths, with both insertion criteria producing worst cases that were better than the corresponding worst human solutions.

Overall, the results suggested that the model may provide a promising first step to modelling human performance. From the results so far, the cheapest insertion criterion appears to be somewhat better than the largest angle for approximating human solutions, though it does not appear to provide as wide a range of path lengths at the upper end.

Comparisons with data from 10 , 20, 40 and 60 node TSPs

MacGregor & Ormerod (1996) reported results using data for 13 TSPs from Lee (1985). The data were collected from 50 participants (graduate and undergraduate students) who produced solutions to a variety of randomly-generated TSPs, ranging from

10 to 60 nodes. Several different random problems were used at each level of number of nodes. This was employed as a within-subjects factor, while number of nodes was a between-subjects factor. Subjects were tested in group settings and were instructed to draw the shortest path passing through all of the points and returning to their starting point. In his paper Lee (1985) reported only the best human solutions, and so the following tests are necessarily restricted to comparing the best model solutions with these. In conducting the tests, 50 replications of both versions of the model were employed. The results are summarized in Table 2. For each problem the table shows the number of nodes and the best paths produced by both versions of the model as (a) a distance and (b) a deviation from the corresponding best human solution.

For the 10 and 20-node problems the results were very similar to those reported above. In both cases, both insertion criteria produced best solutions close to the human results, with mean percentage deviations of 0.00 and 0.07% for the 10-node problems and -0.03% and 2.05% for the 20-node problems, for the CI and LA criteria respectively.

For the larger problems the cheapest insertion criterion continued to provide the better fit. On average, the deviations were 2.39% and 3.53% for the 40 and 60 node problems, respectively, while the corresponding results for the LA procedure were 11.91% and 10.92%, an order of magnitude poorer.

Two additional trends are discernible in the results. The goodness of fit of the models seems to decrease slightly as the size of problem increases, and the direction of error (for minimum path lengths) becomes consistently positive, indicating that the model's best solutions may become poorer than the best human performances. Nevertheless, the fit for minimum path lengths remains relatively good for problems up to 60 nodes. Combined with the trends observed above, the indications are that the model will tend to produce a relatively good fit at the lower path lengths but a narrower range of solutions than human performances. It appears that this tendency may become more pronounced as the size of problem increases. This was examined below using data for a

100 node problem. Because the cheapest insertion criterion seems to result in a better approximation to human performance subsequent tests are limited to this version.

Comparisons with data from a 100 node TSP

The following test used previously unpublished data from 8 undergraduate students who produced solutions to a single randomly-generated 100 node TSP described in Krolak *et al* (1971). The participants were tested in a group setting and instructed to choose a starting point and to draw the shortest path from that point, passing through each point and returning to the start. For comparison, 40 model solutions were generated.

The paths generated by participants ranged in length from 23788 to 53932, with a mean of 31700. The corresponding model values ranged from 23784 to 31211, with a mean of 26940. The model's best path was virtually identical to the best human solution (0.02% shorter), while the mean and worst solutions were 15.0% and 42.1% shorter, respectively. The results were fairly consistent with the trends identified above, although the model's fit to the human solutions was better than expected from the previously observed trend for the minimum path length, while being somewhat poorer than expected for the mean and maximum. The fact that the model's mean and maximum path lengths were considerably shorter than the corresponding human figures may have arisen because of the extremely poor performance of one subject, whose solution of 53932 was 5.9 standard deviations above the overall subject mean of 31700. If this person's solution is excluded, the deviations of the model from minimum, mean and maximum human path lengths were -0.02%, -5.6% and -16.8%, respectively. The pattern was similar to the trend identified above, with the model providing a very good fit to the minimum score, and a good fit to the average, but overall providing a narrower range than the human solutions.

Comparisons with data from a 48 node TSP

Data from MacGregor, Ormerod & Chronicle (1999) from a single TSP of 48 nodes provided a more detailed comparison of human and model performances. The data consisted of 103 human solutions to a problem created by randomly selecting 48 points from a 100-node problem described by Krolak *et al* (1971). Participants were tested in a group setting using the same instructions as MacGregor & Ormerod (1996). A set of 100 model solutions was generated by randomly selecting starting points and direction of travel, clockwise or counter-clockwise. Model solutions were generated using only the cheapest insertion version, which appears to provide the better approximation.

The results were as follows. The best human solution had a path length of 553. The best model solution was 561, 1.4% above this. The mean of the human solutions was 606, of the model, 602, 0.7% below the human mean. The worst human and model solutions were 697 and 644, respectively, the model worst case being 7.6% better than the human. The results were very similar to those reported earlier in the article, with the model providing a good fit to average and best human solutions, but underestimating the worst human solution (in this case, by 7.6%).

Participants in this experiment were asked to indicate the point where they started, and 99 of the 103 did so. This permits a more sensitive test of the model, since model solutions can vary depending on the starting point selected. Participants indicated a total of 22 different starting points. Seven were unique, while as many as 19 of the participants (20%) chose the most popular. (Incidentally, seventy-two percent of human solutions started on the hull, compared to an expected 29% based on the availability of hull points.) Model solutions were generated from each of the 22 starting points indicated by the participants, for each of the two possible directions of travel. A model score for each starting point was obtained by averaging these two scores. The mean participant path lengths across the 22 starting-points ranged from 567 to 628. The differences between the participant and model path lengths for each starting point were expressed as a percentage of the participant path length. The absolute values of these percentage deviations between human and model results ranged from 0% to 5.6%, with a weighted average of 1.1%.

The goodness of fit of the model seems to vary depending on whether solutions started on hull or interior points, as may be seen by the pattern of outcomes in Table 3. In the case of hull starts, the model provides a very close fit to the human path lengths, ranging from 0% to 3% with an average absolute deviation of 0.6%. In the case of interior starts, the fit is consistently poorer, ranging from -6% to +5%, with an absolute average of 2.4%. These differences in model fit perhaps arise because the human solutions cover a greater range for interior than for hull starts, and it may be that the model simply provides a better fit over the mid-range. Alternatively, the model's way of dealing with interior starts - going directly to the hull on the next move - is likely to be too simple and may fail to reflect what people do following an interior point start.

Discussion

The present article proposed and tested a model of human performance on TSPs. Because of empirical evidence that people are influenced by the convex hull in generating solutions to TSPs (MacGregor & Ormerod, 1996; Ormerod & Chronicle, 1999) the model was designed to conform in a general way to a convex hull approach. However, it differs from conventional convex hull heuristics by generating solutions from a given starting point and progressing in a specified direction (clockwise or counter-clockwise). This allows the model to generate a range of solutions for a given problem, since the solution generated can vary depending on starting point or direction. In this respect the model is unlike other convex hull approaches in particular, and heuristic algorithms in general, which typically generate a single solution only. An exception is the Nearest Neighbour procedure, which may generate different solutions depending on starting point.

However, previous results with human subjects indicated that their solutions differed qualitatively and quantitatively from the Nearest Neighbour approach (MacGregor & Ormerod, 1996).

The proposed model was tested using previously reported and unreported data on human performance with a variety of problems ranging from 10 to 60 nodes plus one 48 node problem and one 100 node problem (MacGregor & Ormerod, 1996; MacGregor, Ormerod & Chronicle, 1999). To a large extent, the results of the model conformed closely to those of the human subjects, both quantitatively and qualitatively. Two versions of the model were tested, one using a cheapest insertion criterion, the other a largest interior angle, and while both versions performed reasonably well, the results favoured the former. Nevertheless, there were consistent indications that the model produced a narrower range of solutions than the human subjects, particularly in the upper range of path lengths. That is, the worst solutions generated by the model tended to be better than the worst human solutions.

There are several reasons why this may have occurred. At one extreme, it may indicate that subjects use a different approach than the model. However, the overall goodness of fit between model and human solutions argues against this. Perhaps people use the same general approach as the model but with a different insertion criterion ñ the present model results indicate that the insertion criterion can make a considerable difference to the outcome.

Another potential difference between the model and human performance is that the latter will be subject to a degree of perceptual error in the judgement of distances, angles, or whatever other factors provide the basis for decisions. This suggests that introducing a degree of random (or systematic) error into the model's judgements may produce a wider range of solutions, that may correspond even more closely to the range of human solutions than the present results.

Another qualification to the model's generally good performance arises because of human solvers' sensitivity to pattern and regularity. For example, figural factors such as the proximity of interior points and the regularity of their arrangement appear to influence human solutions (MacGregor & Ormerod, 1996; MacGregor, Ormerod & Chronicle, 1999). There are no mechanisms within the present model to respond to factors such as these, so that the model is unlikely to produce a particularly good fit to human solutions to highly patterned TSPs. However, for random or relatively irregular TSPs, the present results suggest that the model provides a very reasonable approximation to the solutions that people produce.

Finally, the present approach of modelling human performance could be of value in identifying means of improving the solutions generated by conventional heuristic algorithms. For example, MacGregor & Ormerod (1996) tested a convex hull heuristic and the largest interior angle method (Norback & Love, 1977) on their 13 random problems, and report that on average the best path lengths produced by these procedures were, respectively, 2.5% and 7.6% longer than the optimal solutions. By comparison, the best solutions of the model described here were 1% above optimality across the same 13 test problems. The experiment also employed the highly-structured 10-node problem described by Dantzig *et al* (1959), and for this problem the best solutions were 2.7 and 3.0 percent above optimal for the cheapest insertion and largest interior angle approaches. By comparison, the present procedure found the optimal solution. These improvements over conventional procedures arise because the present approach produces a range of solutions rather than a single solution, and while many of these may be worse than those produced by conventional heuristics, some may be better.

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Table 1: Minimum, mean and maximum path lengths as a percentage deviation from the comparable experimental results for the cheapest insertion (CI) and largest angle (LA) versions of the model.

	Problem Number	Path Lengths: Minimum CI	Minimum LA	Mean CI	Mean LA	Maximum CI	Maximum LA
Exp. 1 10-nodes	1	0.0	0.0	-0.8	-0.8	-8.8	-8.8
	2	0.0	0.0	-3.4	-3.4	-18.8	-18.8
	3	0.0	5.2	-1.0	3.1	-11.2	-7.7
	4	0.0	0.0	1.0	6.4	-0.5	2.9
	5	0.0	1.0	-0.5	0.5	-7.3	-5.8
	6	0.0	1.0	-3.0	-1.4	-12.1	-8.2
Mean Absolute Difference.		0.0	1.2	1.6	2.6	9.8	8.7
Mean Difference.		0.0	1.2	-1.3	0.7	-9.8	-7.7
Exp. 2 20-nodes	1	-0.5	0.0	-2.6	-2.2	-13.3	-13.0
	2	0.4	1.9	-1.9	3.1	-10.4	2.6
	3	0.0	1.1	-0.2	3.2	-5.9	0.9
	4	-1.4	-1.4	4.8	6.6	7.2	19.3
	5	3.3	8.9	2.3	11.2	-6.0	1.1
	6	3.2	6.1	-0.5	2.7	-10.2	-1.9
	7	-0.7	1.0	-6.2	-0.3	-16.1	-5.2
Mean Absolute Difference.		1.4	2.9	2.6	4.2	9.9	6.3
Mean Difference.		0.6	2.5	-0.6	3.5	-7.8	3.8

Table 2: Minimum path lengths for the cheapest insertion (CI) and largest interior angle (LA) versions of the model and their percentage deviations from human performance for 10, 20, 40 and 60 node problems.

		CI Model		LA Model	
Problem	# Nodes	Best Path	Deviation (%)	Best Path	Deviation (%)
1	10	541	0.0	541	0.0
2	10	495	0.0	495	0.0
3	10	541	0.0	543	0.2
			Mean =0.00		Mean =0.07
4	20	650	0.0	650	0.0
5	20	708	-0.1	745	5.10
6	20	701	0.0	711	1.4
7	20	596	0.0	606	1.7
			Mean =-0.03		Mean =2.05
8	40	1000	0.5	1106	11.2
9	40	919	1.7	1014	12.2
10	40	1006	3.4	1076	10.5
11	40	906	4.0	990	13.7
			Mean =2.39		Mean =11.91
12	60	1114	1.2	1248	13.4
13	60	1143	5.8	1171	8.5
			Mean =3.53		Mean =10.92

Table 3: A comparison of human and model path lengths for a 48-node problem, for starting point on the hull versus interior starting points.

HULL START			INTERIOR START		
Human path length	Model Path length	% difference	Human path length	Model Path length	% difference
595	597	0%	567	599	-6%
598	598	0%	589	597	-1%
603	605	0%	590	606	-3%
607	607	0%	590	589	0%
607	607	0%	591	610	-3%
607	607	0%	595	602	-1%
607	603	1%	608	595	2%
609	607	0%	609	599	2%
610	610	0%	616	602	2%
616	597	3%	619	608	2%
620	603	3%	628	598	5%

Figure Caption

Figure 1: Illustration of the model's solution to a travelling sales problem when starting from Point 1 and progressing in a counter-clockwise direction.