Can Redistribution be a Reason to Tax Capital?

by

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Abstract

What is the optimal tax on capital when agents differ in wealth and income profiles? In this thesis, I develop a model of agent heterogeneity to consider optimal Ramsey taxation of labor, capital and consumption. When the only source of heterogeneity is initial wealth, and abstracting from the initial confiscation, the optimal tax on capital is zero, provided some relevant elasticities are constant. When, instead, differences are also in terms of labor characteristics, it may, in general, be desirable to use capital taxes. This follows from the imperfection of the tax system, resulting from the restriction that the same income tax must be levied on the different types of labor. This is then related to the findings of the representative agent literature with incomplete set of instruments. The results suggest that labor differences may provide a strong rational in favor of using capital taxes.

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Resumo

Qual é o nível óptimo de tributação de capital quando os agentes diferem em termos de riqueza e perfis de rendimento? Nesta tese, é desenvolvido um modelo de agentes heterogéneos para considerar tributação óptima de trabalho, capital e consumo, numa estrutura de Ramsey. Quando a única fonte de heterogeneidade é o nível de riqueza, e abstraindo do confisco inicial, o imposto óptimo sobre rendimentos de capital é zero, se as elasticidades relevantes forem constantes. Quando as diferenças se reflectem também em termos de características de trabalho pode, em geral, ser desejável usar impostos sobre o capital. Isto resulta das imperfeições do sistema fiscal, que são uma consequência da restrição de tributar ambos os trabalhos à mesma taxa de imposto. Estes resultados são comparados a resultados da literatura de agente representativo com um conjunto incompleto de instrumentos fiscais. As conclusões sugerem que diferenças em termos das características do trabalho dos agentes poderão ser um argumento a favor da utilização de impostos sobre o capital.

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Contents

1 Introduction

The results of Judd (1985) and Chamley (1986) on optimal taxation of capital have been puzzling economists for the last 30 years. Working in different settings, they come to the same conclusion that it is optimal not to tax capital in the steady state.

Chamley (1986) uses an infinitely-lived representative agent model and assesses the efficiency consequences of imposing a tax on capital. In every period, the representative household can decide how much to work, consume and save in public bonds or capital. There are only capital and labor income taxes and, to prevent expropriatory effects, an upper bound on capital taxation is imposed. He concludes that the optimal tax on capital income is asymptotically zero, when the economy converges to an interior steady state.

In a model with capitalists and workers, Judd (1985) analyzes the issue of agent heterogeneity and redistribution. The paper concludes that in a steady state, the inefficiency associated with capital income taxation would always outweigh the gains from redistributing, and therefore capital should not be taxed.

The result of Judd (1985) is especially striking. It strongly suggests that the thorough elimination of capital taxation in the steady state increases welfare for all agents, even for those that do not invest in capital. This result obviously clashes with the common perception of the redistributive role of taxes.

For this reason, a branch of the literature has focused on understanding the effects of eliminating the tax on the distribution of income. Correia (2010), in particular, shows that the elimination of capital income taxation coupled with consumption taxes not only improves efficiency but can also improve the distribution in the economy.

More recently, Straub and Werning (2015) have shown that both Judd's and Chamley's models may present, under some conditions, permanent capital taxation. Their analysis in Chamley's model focuses on the relevance of the upper-bound constraints on capital income taxes and provides conditions under which those bind forever. In their analysis of Judd (1985), Straub and Werning show that, for certain specifications of preferences, the optimal path for taxation of capital may be increasing forever.

Chari, Nicolini and Teles (2016) argue that the assumptions on what the planner can do in time zero are especially important. They show that if the planner is restricted in the initial confiscation, the Chamley-Judd result resurges. This is done by setting the value of initial holding to an exogenous value. The same constraint can be found in Armenter (2008). I follow the same approach in limiting the initial confiscation.

As a benchmark case, I review the case of Chari, Nicolini and Teles (2016) in which the two agents only differ in the initial endowment. Should capital taxes be used to tax the wealthy? The conclusion is that, if not for confiscation, capital taxes should be set to zero.

I then proceed to consider that the households also differ in the type of labor they supply, assessing the relevance of the two sources of heterogeneity in the optimal taxation of capital. When there is a restriction of non-discrimination and the two labor types must be taxed at the same rate, it will, in general, be optimal to tax capital to try to overcome the constraint. Nevertheless, I provide conditions under which this tax should not be used, always abstracting from the initial confiscation. When lump-sum redistribution is available, and preferences are separable and isoelastic in consumption and labor, the tax on capital should be set to zero, meaning that the planner will choose not to distort the intertemporal margins. Constant intratemporal wedges are imposed. If those lump-sum transfers do not exist, the capital tax may be used for redistribution.

Furthermore, under certain conditions on the production function, the wedge on the intertemporal margin for consumption should be set to zero. These are conditions under which taxing capital will not affect the allocations in labor.

The fact that there may be restrictions on how the labor income tax may be used for different types of labor is especially relevant. It is linked to the ideas of incomplete factor taxation in Correia (1996), who shows that if there is a third factor of production which may not be directly taxed, capital taxation may be used to overcome the restriction. It is also related to Jones, Manuelli and Rossi (1997), in which two different labor types, in a representative agent setting, must be taxed at the same rate. Reis (2011) also considers a model in which a representative agent supplies two different types of labor. Nevertheless, the second labor, entrepreneurial labor, has to be taxed at the same rate as capital. She shows that this restriction will justify positive capital taxation. I review and compare these results.

The analysis in this thesis, as in the literature discussed above, is restricted to optimal policies with linear tax schemes, under a Ramsey framework.[1] The Social Planner will be thought of as maximizing a Paretian Welfare function. The tax system is assumed to have taxes on consumption, capital and labor.

The thesis proceeds as follows: Section 2 introduces the model of agent heterogeneity. Section 3 uses the model to compute optimal policy. Initially, I abstract from heterogeneity in labor and consider the redistributive purpose of taxation. Then, it is shown that, in general, capital taxation should be used when different types of labor are restricted to being taxed at the same rate. Section 4 relates the results to the analysis of Correia (1996), Jones, Manuelli and Rossi (1997) and Reis (2011). Finally, Section 5 concludes.

2 The Model

To study the case of optimal taxation under heterogeneity, I consider two different households indexed by i. Households differ in terms of the initial endowment of wealth and the type of labor that they supply.

[1]For a treatment of the theory of Ramsey taxation in Dynamic Representative Agent Economies, see Chari and Kehoe (1999) or Lucas and Stokey (1983).

2.1 Households

Each household i has infinite horizon and decides, each period, how much to consume, work and save in public bonds or capital so as to maximize the utility function $\sum_{t=0}^{\infty} \beta^t u(c_t^i, n_t^i)$, where c_t^i is the consumption of the household and n_t^i is the amount of labor supplied. The utility function is assumed to be separable in consumption and labor. In particular, I assume the following function with constant elasticities in consumption and labor.[2]

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i, n_t^i) = \sum_{t=0}^{\infty} \beta^t \Big[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \chi \frac{(n_t^i)^{1+\zeta}}{1+\zeta}\Big] \tag{1}$$

The household starts period t with capital, k_t^i, and bonds, b_t^i. It rents capital to firms, earning the rental rate U_t,[3] capital income is taxed at the rate τ_t^k, with an allowance for depreciation, δ. The interest rate on a one-period bond bought at time $t-1$ is given by r_t^b. The household also supplies labor, earning net income $(1-\tau_t^n)w_t^i$, where τ_t^n is the tax on wage income, and decides how much to consume, being taxed at the rate τ_t^c. In every period t it must verify the flow of funds condition

$$(1+\tau_t^c)c_t^i + b_{t+1}^i + k_{t+1}^i \leq (1-\tau_t^n)w_t^i n_t^i + [1+(1-\tau_t^k)(U_t-\delta)]k_t^i + (1+r_t^b)b_t^i, \forall t \geq 1. \tag{2}$$

Period zero's constraint is different. The household is endowed with a level of bonds, b_0^i, and capital, k_0^i and an initial levy, l_0, may be raised on the initial wealth. Furthermore, a transfer, which may be positive or negative, is given to household i, gross of consumption taxes.

$$(1+\tau_0^c)c_0^i + b_1^i + k_1^i \leq (1-\tau_0^n)w_0^i n_t^i + (1-l_0)\big[[1+(1-\tau_0^k)(U_0-\delta)]k_0^i + (1+r_0^b)b_0^i\big] + \mathbb{T}_0^i(1+\tau_0^c). \tag{3}$$

[2]Each household is also assumed to have the same utility function.

[3]The capital of each household is a perfect substitute in production to that of the other family, implying the rental rate of capital is the same.

Furthermore, the household must verify a no-Ponzi games condition. Let $Q_t \equiv \frac{1}{(1+r_1^b)\ldots(1+r_t^b)}$ and $Q_0 \equiv 1$ and using a non-arbitrage condition between returns to capital and bonds, the intertemporal budget constraint is given by:

$$\sum_{t=0}^{\infty} Q_t \big[(1+\tau_t^c)c_t^i - (1-\tau_t^n)w_t^i n_t^i\big] \leq (1-l_0)\mathbb{W}_0^i + \mathbb{T}_0^i(1+\tau_0^c), \tag{4}$$

where $\mathbb{W}_0^i \equiv \big[[1+(1-\tau_0^k)(U_0-\delta)]k_0^i + (1+r_0^b)b_0^i\big]$.

The household's problem is to maximize utility subject to (4). The first-order conditions include

$$\frac{u_{ci}(t)}{\beta u_{ci}(t+1)} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}[1+(1-\tau_{t+1}^k)(U_{t+1}-\delta)],\ \forall t \geq 0, \tag{5}$$

$$\frac{u_{ci}(t)}{u_{ni}(t)} = -\frac{1+\tau_t^c}{(1-\tau_t^n)w_t^i},\ \forall t \geq 0, \tag{6}$$

$$[1+(1-\tau_t^k)(U_t-\delta)] = 1+r_t^b,\ \forall t \geq 1. \tag{7}$$

Furthermore, the intertemporal budget constraint is satisfied with equality.

2.2 Firms

There is a representative firm that transforms capital, K_t, and the different labor types, n_t^1 and n_t^2, into units of goods, according to the production function

$$Y_t = F(K_t, n_t^1, n_t^2), \tag{8}$$

in which F is the Constant Returns to Scale (CRS) production function. A particular specification of this production function will have both labor types being perfect substitutes.

The profit maximizing conditions imply that the remuneration of factors is equal to their marginal productivity.[4]

$$U_t = F_K(t), \tag{9}$$

$$w_t^i = F_{ni}(t),\ i = 1, 2. \tag{10}$$

In this economy, with a CRS production function, profits are zero.

2.3 The Government

The government taxes the households to finance a stream of public expenditures $\{G_t\}_{t=0}^{\infty}$. The tax instruments available to the government in each period t are proportional capital income, consumption and labor income taxes. Furthermore, the government is able to raise a tax on initial wealth, l_0. The government starts every period t with an initial level of debt gross of interest payments, $B_t(1+r_t^b)$, and it can issue debt maturing next period, B_{t+1}. The intertemporal budget constraint of the government is given by:

$$\sum_{t=0}^{\infty} Q_t G_t + (1+r_0^b)B_0 = \sum_{t=0}^{\infty} Q_t \Big[\sum_{i=1}^{2} [\tau_t^c c_t^i + \tau_t^n w_t^i n_t^i + \tau_t^k (U_t - \delta) k_t^i] \Big] + \sum_{i=1}^{2} l_0 \mathbb{W}_0^i. \tag{11}$$

An important characteristic of the heterogeneity problem is limiting the availability of discriminatory taxation. This imposes that when choosing optimal policies, the planner cannot set taxes individually and must always consider the impact over all agents of imposing any specific tax. During this thesis the analysis is restricted to non-discriminatory proportional taxes.

The reason for non-discrimination comes from the idea that tax systems should not distinguish on the basis of who the agents are, as this may present

[4]The firm is thought of as being able to differentiate between agents, therefore it is able to set a different wage to each worker.

an impossible task either politically or operationally. I, therefore, impose that all agents should have an equal treatment in terms of taxation.[5]

The only discriminatory tax instrument is the initial redistributive transfer. However, to limit lump-sum taxes the transfers must sum out to zero. When considering discriminatory lump-sum transfers the purpose will be to separate efficiency from redistributive concerns.

2.4 Market Clearing

In equilibrium all markets must clear, implying the following conditions, respectively, for the markets of goods, capital, bonds[6]

$$\sum_{i=1}^{2} c_t^i + G_t + K_{t+1} = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t, \tag{12}$$

$$K_t = \sum_{i=1}^{2} k_t^i, \tag{13}$$

$$B_{t+1} = \sum_{i=1}^{2} b_{t+1}^i. \tag{14}$$

2.5 Equilibrium

In a competitive equilibrium, households maximize utility, taking prices as given and subject to their budget constraint. Firms maximize profits, taking prices as given and subject to the production function. The government verifies its budget constraint. Finally, markets clear.

A competitive equilibrium is a sequence for allocations, prices and taxes, such that, given k_0^i, b_0^i, for $i = 1, 2$, r_0^b, the following conditions are satisfied:

[5]A branch of literature has researched the discrimination of agents based on endogenous decisions and the problem of private information on optimal taxation. An analysis of this type of issues can be found in Stiglitz (1987).

[6]The equilibrium in labor markets is implicitly being assumed.

$$\frac{u_{c1}(t)}{\beta u_{c1}(t+1)} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}(1+r_{t+1}^b), \tag{15}$$

$$\frac{u_{c1}(t)}{\beta u_{c1}(t+1)} = \frac{u_{c2}(t)}{\beta u_{c2}(t+1)} \tag{16}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)}F_{n1}(t) = -\frac{1+\tau_t^c}{(1-\tau_t^n)}, \tag{17}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)}F_{n1}(t) = \frac{u_{c2}(t)}{u_{n2}(t)}F_{n2}(t), \tag{18}$$

$$\sum_{t=0}^{\infty} Q_t\big[c_t^i - (1-\tau_t^n)w_t^i n_t^i\big] = (1-l_0)\mathbb{W}_0^i + \mathbb{T}_0^i(1+\tau_0^c), \tag{19}$$

$$[1+(1-\tau_{t+1}^k)(F_K(t+1)-\delta)] = 1 + r_{t+1}^b, \tag{20}$$

$$\mathbb{T}_0^1 + \mathbb{T}_0^2 = 0 \tag{21}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t, \tag{22}$$

where $\mathbb{W}_0^i \equiv [1+(1-\tau_0^k)(F_K(0)-\delta)]k_0^i + (1+r_0^b)b_0^i$, and $Q_t \equiv \frac{1}{(1+r_1)\ldots(1+r_t)}$ and $Q_0 \equiv 1$.

The Government's budget constraint is left out and is satisfied if all these are satisfied, according to Walras' Law.

3 Optimal Fiscal Policy

3.1 The First Best

In this section I describe the first best for the economy. The planner chooses the optimal allocations to maximize a Paretian welfare function, i.e. a weighted average of utilities. Define ω_1 and ω_2 as the weights given to household 1 and 2, respectively, such that $\omega_1 + \omega_2 = 1$ and $\omega_i \geq 0$.

The first best is the solution to the problem of maximizing social welfare subject to the resource constraints, (22). Let $\beta^t \mu_t$ be the multipliers of those constraints. The necessary conditions for optimality are

$$\omega_i u_{ci}(t) = \mu_t, \tag{23}$$

$$\omega_i u_{ni}(t) = -\mu_t F_{ni}(t), \tag{24}$$

$$\mu_t = \mu_{t+1}(F_K(t+1) + 1 - \delta). \tag{25}$$

It can be seen that, for $i = 1, 2$, $\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}}$ and $\frac{u_{ci}(t)}{u_{ni}(t)} = -\frac{1}{F_{ni}(t)}$. The conditions that the agents face the same marginal prices and taxes are always verified in this solution. The planner's first best sets the marginal rates equal across agents. Also, the marginal rate of intertemporal substitution is equal to the return on capital investment, $\frac{u_{ci}(t)}{\beta u_{ci}(t+1)} = F_K(t+1) + 1 - \delta$.

Furthermore, the ratio of marginal utilities of consumption, in each period, is equal to the inverted ratio of weights, i.e. $\frac{u_{c1}(t)}{u_{c2}(t)} = \frac{\omega_2}{\omega_1}$. The higher is the weight given to one of the agents, the higher will the consumption of that agent be relatively to the other.

How can the first best be implemented? In the literature considering a single agent, if there are lump-sum taxes the first best is always achievable, as these allow to finance expenditures without any other costs. When considering an economy populated by more than one agent, a simple lump-sum tax treating all agents equally does not achieve the same result. This is because the planner needs to be able to discriminate to perform redistribution.

The possibility of redistributive tools allows for a separation of efficiency and equity concerns. The problem of achieving higher efficiency becomes independent of the question of which agents are benefiting and which are losing from those policies.

Furthermore, if the conditions for Gorman Aggregation (GA) are met, the aggregates are also independent of the redistribution performed. The combination of the assumptions of GA and redistributive tools are the conditions under which a single agent may be considered. Since the aggregates are not affected by redistribution of wealth in the economy, we can consider the optimization for aggregate allocations on a single agent economy, i.e. a representative agent. The solution may always lead to a Pareto improvement. This can be analyzed more in depth in Appendix 6.4.

The economy considered here will not, however, always verify the assumptions of Gorman Aggregation. Therefore, I will continue to explicitly consider the population of two heterogeneous agents.

3.2 Implementability Conditions

The unavailability of lump-sum taxes imposes constraints on policy. The planner will then have to set up distortionary taxation to finance government expenditures, as well as the initial level of debt. I seek to understand whether differences in initial wealth levels or labor types justify capital taxation to improve the wealth distribution in the economy.

Following the primal approach to dynamic Ramsey taxation, the budget constraints of the two agents can be written as follows:

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t) c_t^1 + u_{n1}(t) n_t^1] = u_{c1}(0) \frac{(1-l_0)\mathbb{W}_0^1}{1+\tau_0^c} + u_{c1}(0)\mathbb{T}_0^1, \tag{26}$$

$$\sum_{t=0}^{\infty} \beta^t [u_{c2}(t) c_t^2 + u_{ni}(t) n_t^2] = u_{c2}(0) \frac{(1-l_0)\mathbb{W}_0^2}{1+\tau_0^c} - u_{c2}(0)\mathbb{T}_0^1. \tag{27}$$

Any competitive equilibrium must satisfy these constraints, as well as the following conditions:

$$\frac{u_{c1}(t)}{u_{c1}(t+1)} = \frac{u_{c2}(t)}{u_{c2}(t+1)}, \tag{28}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)} F_{n1}(t) = \frac{u_{c2}(t)}{u_{n2}(t)} F_{n2}(t), \tag{29}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t. \tag{30}$$

The first two conditions, (28) and (29), result from the fact that the two agents face the same taxes, therefore the relevant marginal rates of substitution must be equated for the two agents. (30) are the resource constraints.

Proposition 1. *The implementable set for consumption and labor for both agents and capital, as well as τ_0^c, τ_0^k, l_0 and $\mathbb{T}_0^1$ is characterized by the constraints* (26)*,* (27)*,* (28)*,* (29) *and* (30)*.*

Proof. To show that the conditions are necessary is simple. The conditions were derived using the equilibrium conditions and must, therefore, be verified. Following the primal approach, as in Lucas and Stokey (1983) or Chari and Kehoe (1999), (26) and (27) represent the budget constraints with the prices and taxes replaced by the marginal conditions for optimality. (28) and (29) result from the fact that agents face the same prices and marginal taxes and (30) is the market clearing condition in the goods markets. To argue that they are sufficient conditions, we must show that all other conditions can be satisfied by other variables. To see this, notice that condition (15) can be satisfied by some consumption tax τ_{t+1}^c, given τ_t^c. (16) is imposed. (17) is satisfied by some τ_t^n and (18) is imposed. The budget constraint for each individual, (19), is imposed. (20) can find some r_{t+1}^b and (22) is imposed. Finally, the constraint that transfers add up to zero, (21), can be satisfied by some $\mathbb{T}_0^2$. □

Corollary 1.1. *The implementation of the optimal solution needs not use the tax on capital, implying it can always be set to zero, $\tau_t^k = 0$.*

The reason for this corollary is that the consumption tax can replace the use of the income tax in the intertemporal margin. The zero taxation of capital is to be interpreted as a zero wedge in that margin.

I now proceed to solve the Ramsey problem in steps. First, I discuss the initial confiscation, in line with the approach of Chari, Nicolini and Teles

(2016). I then proceed to compute optimal taxation with labor homogeneity; this part is, to a large extent, a review of their results. Finally, I proceed to solve the problem for the case in which labor is heterogeneous.

3.3 Time Zero

Ramsey problems are characterized by time inconsistency. In period zero there is no previous commitment, but the government perfectly commits to a path for policies from then on.

The treatment the planner would like to do to the outstanding wealth in period zero is different from the one in all other periods, because taxation for that exogenous level of wealth is non-distortionary. The planner wishes to minimize distortions, and taxing that initial wealth is very tempting.

To prevent full initial expropriation, Chamley (1986) and Judd (1985) imposed constraints on the capital income taxes. But the inclusion of the upper bound raises a new problem, that of using future capital income taxes to tax the initial stock of wealth. While Chamley (1986) and Judd (1985) assumed that these effects would be only transitory, and the steady state analysis would be free from confiscatory intents, Straub and Werning (2015) provide examples on how the upper bound may bind forever in the model of Chamley and in an hybrid model in which they consider Judd's framework with unconstrained government debt.

As shown by Chari, Nicolini and Teles (2016) there are better alternatives to the initial confiscation, instead of full capital taxation. They show that with consumption taxes, the planner will choose a relatively high consumption tax for $t = 1$, compared to $t = 0$, and this will be enough to deal with the initial confiscation. If capital income taxes were unconstrained, the same could be achieved with the capital income tax in $t = 1$. However, once constrained it is possible that the needed tax is too high and the constraint is binding forever.

To prevent these initial effects, I impose a restriction on how much the household can keep initially, measured in terms of the initial utility. This implies that

$$u_{ci}(0)\frac{(1-l_0)}{1+\tau_0^c}\mathbb{W}_0^i = \mathbb{U}_0^i. \tag{31}$$

This is the same restriction that is imposed in Chari, Nicolini and Teles (2016). It is also present in the work of Armenter (2008), who argues that this makes the optimal allocations being dependent only on current conditions, which is interpreted as being "timeless".

Nevertheless, the results would be kept the same if instead the constraints were imposed in the initial taxes, τ_0^c, τ_0^k and l_0. In that case the consumption tax in $t=1$ would be higher than that of $t=0$, to be able to affect the value of initial holding. Since the consumption tax is unconstrained, it would always solve the problem.

3.4 Optimal Taxation with Labor Homogeneity

The model provides a framework to analyze optimal policy when agents differ in initial wealth. For now, assume that labors are perfect substitutes. The production function is given by $F(K_t, N_t)$, in which $N_t \equiv n_t^1 + n_t^2$. Since the goal is analyzing whether capital should be taxed for the purpose of redistribution, the initial transfers are eliminated, as those transfers would always be a better instrument.

The planner maximizes a Paretian welfare function. Let λ_i be the multiplier of the budget constraints, respectively for $i = 1, 2$. Define $\beta^t\eta_t^c$ and $\beta^t\eta_t^n$ to be the multipliers of (28) and (29), respectively. Finally, let $\beta^t\mu_t$ be the multiplier on the resource constraints.

Theorem 1 (Chamley (1986), Judd (1985))**.** *In the two agent economy, when the labor of both agents are perfect substitutes, if the Ramsey solution converges to an interior steady state, capital should not be taxed in that steady state.*

Proof. The first order condition of the Ramsey problem with respect to capital accumulation is given by

$$\mu_t = \beta\mu_{t+1}(1 - \delta + F_K(t+1)).$$

If $\mu_t \to \mu$ and $K_t \to K$, then the condition is in the limit

$$1 - \delta + F_K = 1/\beta.$$

Since in a competitive equilibrium the steady state must have, from the combinations of (15), (17) and (20) , that

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c}[1 + (1-\tau_{t+1}^k)(F_K - \delta)] = 1/\beta$$

and

$$\frac{1-\tau_t^n}{1-\tau_{t+1}^n}[1 + (1-\tau_{t+1}^k)(F_K - \delta)] = 1/\beta.$$

This means that there should be no wedge in the intertemporal margins in the steady state. Therefore, a possible implementation keeps the taxes on labor and consumption constant and sets a zero tax on capital in the steady state. □

Theorem 1 echoes the results of both Judd (1985) and Chamley (1986). If the economy converges to an interior steady state capital should not be taxed.

Proposition 2 shows that this result of zero taxation of capital can actually be obtained away from the steady state provided elasticities are constant. The proof can be found in the appendix.

Proposition 2. *In the two agent economy, when the labor of both agents are perfect substitutes, if the relevant preference elasticities are constant, then the optimal plan does not distort any of the intertemporal margins, implying that a possible implementation sets taxes on labor and consumption constant over time, with a zero tax on capital income.*

When the elasticities for consumption are constant, the marginal rate of intertemporal substitution for consumption should not be distorted. This is due to the fact that, with constant elasticities, uniform taxation of consumption in each period is optimal. Therefore, the planner does not want to distort intertemporally. The implication is that if the consumption tax is constant, the tax on capital income should be set to zero. Furthermore, if preferences are such that the elasticity of labor is also constant, none of the intertemporal margins should be distorted, i.e. capital should not be taxed.

The agents in this economy differ only in the amount of initial wealth. The conclusion is that taxation should not be used for wealth redistribution. In general, the ratio of consumptions is different from that of the first best, which had $\frac{c_t^2}{c_t^1} = \left(\frac{\omega_2}{\omega_1}\right)^{1/\sigma}$. That implies that the planner would want more redistribution, but in the second best it is not optimal to do so.

In the appendices, I study four other alternatives for redistributive instruments in this economy and show that, just as in the one I have chosen to present here, capital should never be taxed. Appendix 6.3.1 assumes that an unconstrained transfer to one of the agents exists. This is the framework most comparable to the first model of Judd (1985), as he assumed that a lump-sum transfer to workers existed. Then, Appendix 6.3.2 considers the case in which this transfer is constrained to being non-negative, while still maintaining its discriminatory characteristic. Then, Appendix 6.3.3 considers the case in which the transfer has to be given to both agents on a non-discriminatory basis, to show that, once more, capital should not be taxed. Finally, Appendix 6.3.4 considers the case in which the transfer is unconstrained but non-discriminatory, to show that even though distortionary taxation will be used, capital should not be taxed. Unconstrained discriminatory transfers to both agents are not considered, as those would just lead to the first best. Constrained discriminatory transfers are also not explicitly considered. This is because the planner is only interested in sending transfers to the relatively poorer agent,[7] and, as a consequence, the transfer to the richer agent would

[7] Also considering the weights of each agent.

be zero.

This suggests that the reason for the results in Straub and Werning (2015) is the initial confiscation. If confiscation is limited, the optimal plan will always have zero taxation of capital. Why? Being unable to confiscate more, the best policy for the planner is not to distort the intertemporal choice, because doing so leads to lower accumulation of capital, reverting towards labor productivity.

It is optimal to set a constant wedge in the marginal rate of substitution between consumption and labor. However, constant elasticities imply that the optimal solution does not distort intertemporally either consumption or labor.

3.5 Optimal Taxation with Heterogeneous Labor

How should policy be conducted when agents differ both in initial wealth and labor types? A key issue is the imperfection in the tax system. Taxes are restricted to be non-discriminatory. In this linear tax system, that means that both labor types must be taxed at the same rate. In general, these imperfections may justify the use of capital taxation as a correction measure.

The planner maximizes a Paretian welfare function. Let λ_i be the multiplier of the budget constraints, respectively for $i = 1, 2$. Define $\beta^t \eta_t^c$ and $\beta^t \eta_t^n$ to be the multipliers of (28) and (29), respectively. Finally, let $\beta^t \mu_t$ be the multiplier on the resource constraints.

3.5.1 With Initial Redistribution

Let us first analyze how policy should be when there are discriminatory lump-sum transfers. The goal is to keep the analysis based on efficiency concerns rather than redistribution. In this setup, if labor types are perfect substitutes, capital should not be taxed. This is the case even if agents differ in terms of efficiency.[8] However, when the different types are not substitutes, in general, the capital income tax may be used to partially overcome the restriction that they have the same tax rate.

In order for capital income taxes to be used to correct the tax imperfection, two conditions are needed. The first is that capital taxes are able to affect the allocations in labor. As shown later, this depends on the characteristics of the production function and on how each labor interacts with capital.

The second condition is whether the optimal plan would set different wedges on the margins for each labor. If the optimal plan is to have constant distortions for the different types, constraining the economy to an homogeneous tax does not involve a welfare loss.

Proposition 3. *In the two agent economy with heterogeneous labor, in which the planner is constrained to set the same tax on both labor types, if the Ramsey problem converges, capital taxation is, in general, used.*

Proof. The first order condition with respect to capital accumulation, is given by

$$\mu_t = \beta\Big[\mu_{t+1}(1 + F_K(t+1) - \delta) + \eta^n_{t+1}\frac{u_{c1}(t+1)}{u_{n1}(t+1)}F_{n1}(t+1)\Big(\frac{F_{n1K}(t+1)}{F_{n1}(t+1)} - \frac{F_{n2K}(t+1)}{F_{n2}(t+1)}\Big)\Big], \quad (32)$$

Which in a steady state implies a wedge between pre-tax return on capital and the intertemporal margin:

$$1 - \delta + F_K = \frac{1}{\beta} - \frac{\eta^n}{\mu}\frac{u_{c1}}{u_{n1}}F_{n1}\Big(\frac{F_{n1K}}{F_{n1}} - \frac{F_{n2K}}{F_{n2}}\Big). \quad (33)$$

□

[8] In the appendix I discuss how optimal policy, under the assumption of Gorman Aggregation, can be understood from a representative agent framework when agents differ in efficiencies.

The case with labor heterogeneity is significantly different from that in which agents supply the same type of labor. The restriction of setting the same proportional tax on both types may justify the use of capital taxes even in the steady state. This can be easily seen from the extra term in equation (33). Being, in general, different from zero it implies an optimal third best distortion on the intertemporal margin for consumption, which is a sufficient condition to argue that capital should be taxed in this setting.

The extra term can be studied to understand for which cases the intertemporal margin should not be distorted. One condition for this holds whenever the planner would optimally set the same intratemporal wedge for both agents. This would imply that $\eta = 0$ and thus the intertemporal margin would not be distorted.

Another condition to have no distortion in the intertemporal margin, is when the difference inside parenthesis is zero. If an absolute increase in the capital stock leads to the same proportionate growth in productivity for each type of labor, it holds true. Under such conditions, a distortion in the intertemporal margin for consumption would not have any effect on the allocations for each labor, and thus it should be optimally set to zero.

Proposition 4. *In the two agent economy, if types of labor are heterogeneous but the production function is weakly separable in labor types,* $F(K_t, N(n_t^1, n_t^2))$, *consumption should not be distorted intertemporally.*

Proof. With this production function, we have that

$$F_{ni}(t) = F_N(t) N_{ni}(t),$$

and

$$F_{niK}(t) = F_{NK}(t) N_{ni}(t).$$

Which implies that

$$\frac{F_{n1K}(t+1)}{F_{n1}(t+1)} - \frac{F_{n2K}(t+1)}{F_{n2}(t+1)} = \frac{F_{NK}(t+1)}{F_N(t+1)} - \frac{F_{NK}(t+1)}{F_N(t+1)} = 0.$$

And thus,

$$\mu_t = \beta\mu_{t+1}(1 + F_K(t+1) - \delta).$$

□

A Cobb-Douglas production function on the three arguments, for example, verifies the assumption of Proposition 4. However, for a general CES production function the assumption is not valid.

The result of non-zero capital taxation when the set of tax instruments is restricted is similar to the result in the representative agent economies of Correia (1996) or Reis (2011). However, these two settings only consider labor and capital income taxes. There, the introduction of consumption taxes recovers the complete set of instruments and the asymptotic zero tax on capital.

This is not true when the tax imperfection is setting the same tax to different labor types. Indeed, when the tax system has a restriction of this sort, the consumption tax cannot help overcoming the constraint, because it cannot be independently used to target the extra margin. This also applies to the representative agent framework of Jones, Manuelli and Rossi (1997), as will be discussed in Section 4.

Nevertheless, for the particular case of preference specifications we have been considering, in which utility is isoelastic both in consumption and labor and equal across agents, the planner will not tax capital income. This result is independent of the production function, as is stated in Proposition 5.

Proposition 5. *In the two agent economy with heterogeneous types of labor, and preferences which are separable in consumption and labor and isoelastic in each, if initial redistribution is allowed, capital should not be taxed, and the multipliers on constraints* (28) *and* (29) *are zero.*

The proof can be found in the appendix.

The intuition for the result is straightforward. Since both agents have the same constant elasticity of consumption and labor the planner wants to set the marginal rates of each agent equal, which then implies that the constraints (28) and (29) are verified in the optimal solution. It echoes a similar argument for uniform taxation of sources of income in a representative agent setting, discussed in the next section. Therefore, when lump-sum transfers are available, the planner chooses not to distort any of the intertemporal margins, implying capital should not be taxed. For other preference specifications, with variable elasticities, capital may be taxed, as long as elasticities remain different across agents, even if constant.

3.5.2 Without Initial Redistribution

Should capital taxation be used even if elasticities are constant but when there are no transfers to perform the needed redistribution? Clearly, whenever the production function verifies the assumptions of Proposition 4, capital taxation needs not be used to distort the intertemporal consumption margin.

Nevertheless, for general production functions the capital tax may be used even if agents have the same constant elasticities. This can be easily seen from the dependence of the proof of Proposition 5 on the availability of initial transfers.

Proposition 6. *In the two agent economy with heterogeneous labor types, and preferences which are separable in consumption and labor and isoelastic in each, it is, in general, optimal to use capital taxation.*

Whenever the planner does not have lump-sum redistributive tools, the conditions to tax capital may be met, even when elasticities are constant. This is because the planner would like to tax more one of the agents, suggesting that strong differences in labor characteristics may justify the use of capital taxes to perform redistribution.

3.6 Judd (1985) and Straub and Werning (2015)

How do these findings relate to Judd (1985) and Straub and Werning (2015)?

Judd (1985) first suggested that capital should not be taxed in the steady state for reasons of redistribution. The first model of Judd (1985) considers the case in which there are two classes of agents: workers and capitalists. Capitalists make decisions on how much to consume and save, but do not supply labor. On the other hand, workers supply labor inelastically. They are also excluded from participating in the assets market and, therefore, live hand-to-mouth.

Capital is taxed and a lump-sum transfer/tax is set on workers. In this setting all taxes are discriminatory, taxing capital will only tax capitalists and the lump-sum transfer only targets workers. Furthermore, the government cannot hold assets/debt, i.e. the government must run a balanced budget. Judd (1985) shows that if the economy converges to an interior steady state, the capital income tax should converge to zero.

Nevertheless, Straub and Werning (2015) show that, when the intertemporal elasticity of substitution is below one, the optimal path for these taxes is increasing forever, and the economy does not converge to an interior steady state. The intuition provided is that, with these preference characteristics, the income effect dominates the substitution effect. Therefore, promising higher future capital taxes decreases the consumption of capitalists, which in their case is desirable since the planner is only concerned with the consumption of the worker.

In order to understand these results, the hypothesis on the initial confiscation are essential, as shown by Chari, Nicolini and Teles (2016). When considering heterogeneous agents the problem becomes even more extreme.

Since the problem with restricted government debt is less clear, consider first the hybrid model presented by Straub and Werning (2015), with unconstrained government debt. Define agent 1 as being the capitalist and agent 2 the worker. The planner is only concerned with the worker, which implies that the weight of capitalists is zero.

Suppose that the initial levy is restricted to allow the capitalist to have a given amount of wealth, i.e. $\frac{(1-l_0)}{1+\tau_0^c}([1+(1-\tau_0^k)(U_0-\delta)]K_0+(1+r_0^b)B_0) \geq W_0$.

This restriction will always bind, for $\omega_1 = 0$. The implementability condition is given by

$$\sum_{t=0}^{\infty} \beta^t u_{c1}(t) c_t = u_{c1}(0) W_0. \tag{34}$$

To the planner, capitalist's consumption is a cost, because, everything else equal, it decreases the amount of worker's consumption. As argued by Straub and Werning (2015), reducing the consumption of capitalists is desirable.

As shown in their work, for $\sigma > 1$, promising higher future taxes will decrease consumption of the capitalist in all periods. Therefore, the planner can eliminate the capitalist's consumption by promising infinite capital taxes in period 1, while respecting (34), and maximizing the welfare of workers. This is taxing the initial amount of wealth, in the sense of reducing the relative valuation of that wealth today.

Suppose that the initial levy was unrestricted. Setting $l_0 = 1$ would achieve exactly the same effect, and would be optimal for any level of σ. The government would confiscate the whole amount of wealth, act as a saver and smooth out transfers to workers.

How will the optimal policy differ if, instead, the government cannot issue debt? Note that in the setting discussed, the government can borrow and lend in order to smooth transfers to workers. However, when the government must run a balanced budget, it is unable to do so. For instance, if the government were to fully tax the initial endowment of capitalists, the only possibility would be to increase the consumption of workers in period 0. In that setup, capitalists are the only savings technology.

It is the interconnectedness of the desire to expropriate initial holdings with the impossibility of government savings that makes the problem significantly different from the previous one.

We can then conclude that the model with capitalists and workers and unconstrained government debt yields similar results to the model in which both agents work and save. The participation constraints, in the assets or labor markets, do not change the conclusions. The reason to tax capital is the initial expropriation. In the model with a balanced budget, the dynamics become significantly different, however the ideas remain.

4 On the Incompleteness of the Set of Instruments

In the analysis before I assumed that the two types of labor had to be taxed at the same rate. This is a restriction on the tax system which in a representative agent framework, in general, corresponds to an incomplete set of instruments, as discussed in Chari and Kehoe (1999). In this section I discuss this and other such restrictions and how consumption taxes may recover the complete set of instruments. I am going to assume that there is a representative household, which is allowed to have a third factor of production (interpreted as a different labor). I consider unrestricted taxes on labor, capital and consumption and abstract from the initial confiscation effects.

Correia (1996) develops a model in which a third factor of production is used but cannot be taxed by the government. The consequence of having too few instruments in the tax system justifies a long-run capital tax (or subsidy).

A different situation occurs if the third factor has to be taxed at the same rate as some other source of income. The planner has to choose a common tax on both factors. Reis (2011) considers such a case. In her model there is a second labor, entrepreneurial labor, which has to be taxed at the same rate as capital. This justifies a positive long-run tax on capital.

I show that these results will change if taxes on consumption are considered. The intuition is as follows: while taxes on capital and labor are taxes on sources of income, the consumption tax targets the use of all wealth. As such, it can be used to target all income sources, even the extra one. This has the potential to correct the imperfection of the tax system.

However, for this to be true, it must be that the consumption tax is an independent instrument. This means that the planner has to be able to use the consumption tax to independently affect the extra margin.

If the third factor of production has to be taxed at the same rate as labor, consumption taxes provide no further improvement because they are not independent of the tax on labor. This last example is, in fact, the representative agent analog to the model developed with two different types of labor in the previous section.

4.1 A Third Factor that Cannot be Taxed

Correia (1996) restricts the tax system instruments to be able to target only two of the three sources of income. In this model the introduction of consumption taxes can be an improvement.

Following Correia (1996), but allowing for consumption taxes the household's problem is the following

$$\max U = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, \Im_t),\ s.t. \tag{35}$$

$$(1+\tau_t^c)C_t + K_{t+1} + B_{t+1} \leq [1+(1-\tau_t^k)(U_t-\delta)]K_t + (1-\tau_t^n)w_t N_t + s_t \Im_t + (1+r_t)B_t,$$

together with a terminal condition. The production function is CRS using all three inputs, and thus $w_t = F_N(t)$, $U_t = F_K(t)$ and $s_t = F_\Im(t)$. This therefore implies that the equilibrium in this economy is described by

$$\frac{U_C(t)}{\beta U_C(t+1)} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}[1+(1-\tau_{t+1}^k)(F_K(t+1)-\delta)], \tag{36}$$

$$\frac{U_C(t)}{U_N(t)} = -\frac{1+\tau_t^c}{(1-\tau_t^n)F_N(t)}, \tag{37}$$

$$\frac{U_C(t)}{U_\Im(t)} = -\frac{1+\tau_t^c}{F_\Im(t)}, \tag{38}$$

$$\sum_{t=0}^{\infty} Q_t[(1+\tau_t^c)C_t - (1-\tau_t^n)F_N(t)N_t - F_\Im(t)\Im_t] = \mathbb{W}_0, \tag{39}$$

$$1 + r_{t+1} = 1 + (1 - \tau_{t+1}^k)(F_K(t+1) - \delta), \tag{40}$$

$$C_t + G_t + K_{t+1} = F(K_t, N_t, \Im_t) + (1 - \delta)K_t. \tag{41}$$

where $\mathbb{W}_0 \equiv [1 + (1 - \tau_0^k)(F_K(0) - \delta)]K_0 + (1 + r_0)B_0$, and $Q_t \equiv \frac{1}{(1+r_1)...(1+r_t)}$ and $Q_0 \equiv 1$.

The implementability condition can be written as

$$\sum_{t=0}^{\infty} \beta^t [U_C(t)C_t + U_N(t)N_t + U_{\Im}(t)\Im_t] = -\frac{U_{\Im}(0)\mathbb{W}_0}{F_{\Im}(0)}. \tag{42}$$

Proposition 7. *The set of implementable allocations can be described by* (41) *and* (42)*. These are the necessary and sufficient conditions for an equilibrium in the quantities and initial tax on capital.*

Proof. To show that they are necessary notice that they are obtained from the equilibrium conditions. In fact (42) is the budget constraint with the prices replaced by the marginal conditions of households and firms. To show that they are sufficient, notice that all other conditions can be satisfied by other variables: (36) by a tax on capital τ_{t+1}^k, (37) by τ_t^n, and (38) by τ_t^c. (39) and (41) are imposed. (40) can be satisfied by some r_{t+1}. □

Following standard results in Ramsey literature, to avoid uninteresting solutions, think of the initial tax on capital as being set at some exogenous level.

The set of implementable allocations is defined by the implementability condition plus the resources constraint. This is what characterizes a Ramsey problem with a complete set of instruments. While without consumption taxes the benevolent planner will choose to use the tax on capital income to also target the extra production factor, once consumption taxes are introduced, the planner can use these to tax all income sources and correct for the needed margins with the other taxes.

Proposition 8. *In the model of Correia (1996), in which a third factor of production cannot be taxed, and considering the existence of consumption taxes, the tax on capital income converges asymptotically to zero.*

Proof. The first order conditions of the planner's problem of maximizing the agents utility subject to (41) and (42), with respect to C_t, N_t, $\Im_t$ and K_{t+1} are, respectively,

$$U_C(t)[1 + \lambda(1 - \sigma^{CC}(t) - \sigma^{NC}(t) - \sigma^{\Im C}(t))] = \mu_t,$$

$$U_N(t)[1 + \lambda(1 - \sigma^{NN}(t) - \sigma^{CN}(t) - \sigma^{\Im N}(t))] = -\mu_t F_N(t),$$

$$U_\Im(t)[1 + \lambda(1 - \sigma^{\Im\Im}(t) - \sigma^{C\Im}(t) - \sigma^{N\Im}(t))] = -\mu_t F_\Im(t),$$

$$\mu_t = \beta\mu_{t+1}(1 - \delta + F_K(t+1)),$$

where λ denotes the multiplier on (42), and $\beta^t\mu_t$ the multiplier on constraint (41), in each period. Furthermore $\sigma^{xy}(t) \equiv -\frac{U_{xy}(t)}{U_y(t)}x_t$.

In an interior steady state we must have that $\sigma^{xy}(t) \to \sigma^{xy}$. Therefore,

$$\frac{U_C(t)}{U_C(t+1)} = \frac{\mu_t}{\mu_{t+1}} = \beta(1 - \delta + F_K), \tag{43}$$

and

$$\frac{U_C(t)}{U_\Im(t)}F_\Im(t) = -\frac{1 + \lambda(1 - \sigma^{\Im\Im} - \sigma^{C\Im} - \sigma^{N\Im})}{1 + \lambda(1 - \sigma^{CC} - \sigma^{NC} - \sigma^{\Im C})}. \tag{44}$$

(44) implies that $\tau_t^c \to \tau^c$, and then (43) implies that $\tau_t^k \to 0$. □

We can conclude that if consumption taxes are available to the planner it is not optimal to distort intertemporally. In this setup, consumption taxes are constant and the capital tax should be set to zero when elasticities are constant.

4.2 A Third Factor Taxed at the Same Rate as Capital

Suppose that the third source of income can be taxed. However this must be done at the same rate as one of the remaining factors of production. This is the problem of Reis (2011) or Jones, Manuelli and Rossi (1997). Consider first that the income of factor $\Im_t$ is taxed at the same rate as capital and consumption taxes are available.

The representative household solves the following problem

$$\max U = \sum_t \beta^t U(C_t, N_t, \Im_t),\ s.t. \tag{45}$$

$$(1+\tau_t^c)C_t + K_{t+1} + B_{t+1} = [1+(1-\tau_t^k)(U_t-\delta)]K_t + (1-\tau_t^n)w_t N_t + (1-\tau_t^k)s_t \Im_t$$
$$+(1+r_t)B_t,$$

together with a terminal condition. The production function is CRS using all three inputs, and thus $w_t = F_N(t)$, $U_t = F_K(t)$ and $s_t = F_\Im(t)$. The equilibrium in this economy is described by

$$\frac{U_C(t)}{\beta U_C(t+1)} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\Big[1 + (1-\tau_{t+1}^k)(F_K(t+1)-\delta)\Big], \tag{46}$$

$$\frac{U_C(t)}{U_N(t)} = -\frac{1+\tau_t^c}{(1-\tau_t^n)F_N(t)}, \tag{47}$$

$$\frac{U_C(t)}{U_\Im(t)} = -\frac{1+\tau_t^c}{(1-\tau_t^k)F_\Im(t)}, \tag{48}$$

$$\sum_{t=0}^{\infty} Q_t[(1+\tau_t^c)C_t - (1-\tau_t^n)F_N(t)N_t - (1-\tau_t^k)F_\Im(t)\Im_t] = \mathbb{W}_0, \tag{49}$$

$$1 + r_{t+1} = 1 + (1-\tau_{t+1}^k)(F_K(t+1)-\delta), \tag{50}$$

$$C_t + G_t + K_{t+1} = F(K_t, N_t, \Im_t) + (1-\delta)K_t, \tag{51}$$

where $\mathbb{W}_0 = (1 + (1 - \tau_0^k)(F_K(0) - \delta)K_0 + (1 + r_0)B_0)$, and $Q_t \equiv \frac{1}{(1+r_1)...(1+r_t)}$ and $Q_0 \equiv 1$.

The implementability condition can be written as

$$\sum_{t=0}^{\infty} \beta^t [U_C(t)C_t + U_N(t)N_t + U_{\Im}(t)\Im_t] = -\frac{U_{\Im}(0)\mathbb{W}_0}{(1 - \tau_0^k)F_{\Im}(0)}. \tag{52}$$

Proposition 9. *The set of implementable allocations can be described by* (51) *and* (52)*. These are the necessary and sufficient conditions for for an equilibrium in the quantities and initial tax on capital.*

Proof. To show that they are necessary notice that they are obtained from the equilibrium conditions. In fact (52) is the budget constraint with the prices replaced by the marginal conditions of households and firms. To show that they are sufficient, notice that all other conditions can be satisfied by other variables: (46) by a capital tax τ_{t+1}^k, (47) by τ_t^n, and (48) by τ_t^c. (49) and (51) are imposed. (50) can be satisfied by some r_{t+1}. □

The same result of Proposition 8 is applicable in this case. Therefore, capital should not be taxed in the steady state.

4.3 A Third Factor Taxed at the Same Rate as Labor

Consider, instead, that the income of factor $\Im_t$ has to be taxed at the same rate as labor and consumption taxes are available.

The representative household solves

$$\max U = \sum_t \beta^t U(C_t, N_t, \Im_t),\ s.t. \tag{53}$$

$$(1+\tau_t^c)C_t + K_{t+1} + B_{t+1} \leq [1+(1-\tau_t^k)(U_t - \delta)]K_t + (1-\tau_t^n)w_t N_t + (1-\tau_t^n)s_t \Im_t$$
$$+(1+r_t)B_t,$$

together with a terminal condition. The production function is CRS using all three inputs, and therefore $w_t = F_N(t)$, $U_t = F_K(t)$ and $s_t = F_{\Im}(t)$. The equilibrium conditions can be described by

$$\frac{U_C(t)}{\beta U_C(t+1)} = \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\Big[1+(1-\tau_{t+1}^k)(F_K(t+1)-\delta)\Big], \tag{54}$$

$$\frac{U_C(t)}{U_N(t)} = -\frac{1+\tau_t^c}{(1-\tau_t^n)F_N(t)}, \tag{55}$$

$$\frac{U_N(t)}{U_{\Im}(t)} = \frac{F_N(t)}{F_{\Im}(t)}, \tag{56}$$

$$\sum_{t=0}^{\infty} Q_t[(1+\tau_t^c)C_t - (1-\tau_t^n)F_N(t)N_t - (1-\tau_t^n)F_{\Im}(t)\Im_t] = \mathbb{W}_0, \tag{57}$$

$$1+r_{t+1} = 1+(1-\tau_{t+1}^k)(F_K(t+1)-\delta), \tag{58}$$

$$C_t + G_t + K_{t+1} = F(K_t, N_t, \Im_t) + (1-\delta)K_t, \tag{59}$$

where $\mathbb{W}_0 = (1+(1-\tau_0^k)(F_K(0)-\delta)K_0 + (1+r_o)B_0)$, and $Q_t \equiv \frac{1}{(1+r_1)...(1+r_t)}$ and $Q_0 \equiv 1$.

The implementability condition can be written as

$$\sum_{t=0}^{\infty} \beta^t[U_C(t)C_t + U_N(t)N_t + U_{\Im}(t)\Im_t] = U_C(0)\frac{\mathbb{W}_0}{1+\tau_0^c}. \tag{60}$$

Furthermore, since no tax instrument can be independently used to impose a wedge in the marginal rate of substitution between labor types, another restriction must be included to the Ramsey problem. This condition is given by (56). The extra restriction imposed on the planner's problem means that the set of instruments is incomplete.

Proposition 10. *If there is a third factor of production and its has to be taxed at the same rate as labor income, then in general the set of implementable allocations are restricted by* (56), (59) *and* (60). *These are the necessary and* *sufficient conditions for an equilibrium in the quantities and initial taxes on consumption and capital.*

Proof. To show that they are necessary notice that they are obtained from the equilibrium conditions. In fact (60) is the budget constraint with the prices replaced by the marginal conditions of households and firms. To show that they are sufficient, notice that all other conditions can be satisfied by other variables: (54) by a consumption tax τ_{t+1}^c, given τ_t^c, (55) by τ_t^n. (56), (57) and (59) are imposed. (58) can be satisfied by some r_{t+1}. □

Following standard practice in the Ramsey literature, to avoid uninteresting solutions, think of the initial taxes on capital and consumption as being set at some exogenous level.

In general, condition (56) presents a restriction on the set of implementable allocations, as no tax instrument can be independently used to affect this margin in the optimal way. Note that, unlike the previous cases, the consumption tax cannot help overcoming the restriction. This is what is meant by not having an independent instrument.

There is a special case for which (56) is not binding. This is when preferences are weakly separable and homogeneous in labor types: $U(C_t, G(N_t, \Im_t))$, where G is an homogeneous function. These preferences imply uniform taxation of each labor and therefore the wedge between labor types is optimally zero. The proof can be found in Appendix 6.5. In this case, restricting the planning problem to set the same tax on both sources of income has no effect and therefore the tax on capital should be asymptotically zero, as before. A special case is when the utility function depends on the sum N_t and $\Im_t$. However, since this does not generally hold, the condition must be included to the planning problem.

Proposition 11. *In the case in which the income of the third factor is restricted to being taxed at the same rate as labor income, in general, capital income may be subsidized or taxed in the steady state.*

Proof. Let $\beta^t\eta_t$ be the multiplier of (56), λ the multiplier of restriction (57) and $\beta^t\mu_t$ the multiplier of (59). The marginal conditions of the Ramsey problem for consumption are given by

$$U_C(t)[1+\lambda(1-\sigma^{CC}(t)-\sigma^{NC}(t)-\sigma^{\Im C}(t))]=\mu_t.$$

In an interior steady state, because elasticities are constant, this implies that

$$\frac{U_C(t)}{U_C(t+1)}=\frac{\mu_t}{\mu_{t+1}}.$$

The first order conditions to capital accumulation are given by

$$\mu_t=\beta\Big[\mu_{t+1}(F_K(t+1)+1-\delta)+\eta_{t+1}U_N(t+1)F_{\Im}(t+1)\Big(\frac{F_{\Im K}(t+1)}{F_{\Im}(t+1)} - \frac{F_{NK}(t+1)}{F_N(t+1)}\Big)\Big]. \quad (61)$$

This shows that the intertemporal margin will, in general, be distorted, as long as η and the term $\left(\frac{F_{\Im K}(t+1)}{F_{\Im}(t+1)}-\frac{F_{NK}(t+1)}{F_N(t+1)}\right)$ are different from zero. □

This representative agent framework is analogous to the problem with agent heterogeneity. Similar considerations on the production function apply. The reason to distort the intertemporal margin for consumption is to try to overcome the restriction that the two types of labor must be taxed at the same rate. In order for that to be possible it must be that the tax on capital can affect the two differently.

This result can also be found in Jones, Manuelli and Rossi (1997). They assume that a representative agent supplies two different types of labor but a single tax rate on labor must be used. Their result also depends on the assumption that labor types are not perfect substitutes in preferences, such that uniform taxation is not optimal.

5 Conclusions

Should capital be taxed when agents differ in terms of the amount of initial wealth and labor characteristics? To answer that question, this thesis develops a model assuming two households differing in these characteristics. The labor of each household is, in general, a different productive input. Households have the same preferences over consumption and labor, being isoelastic in both arguments.

I derive the equilibrium for this economy and use it to compute optimal fiscal policies. A key issue is that the government does not distinguish between agents when setting up taxation. As it turns out, for the implementation of the optimal plans the capital tax is never needed, as time-varying consumption taxes can be used to distort the intertemporal margin. Capital not being taxed is to be interpreted as those margins not being distorted.

As a benchmark case, it is shown that, abstracting from the initial levy, as in Chari, Nicolini and Teles (2016), redistributive capital taxation is not to be used when the only source of heterogeneity is the size of initial wealth.

Considering that households also differ in their labor characteristics leads to a different conclusion. This is because the non-discriminatory characteristics of the tax system imposes that the tax rate on the two different labor inputs be the same. In general, this inefficiency in the tax system will justify the use of capital taxation.

Two conditions must be met in order for capital taxation to be optimal. The first is that the planner would actually want to tax labor types differently. It is shown that, when there are discriminatory lump-sum transfers, the planner chooses not to tax capital. This is because the restriction is not binding, as the optimal solution sets the same wedge in the intratemporal margin for both agents. However, without these transfers, the planner will choose to tax capital to redistribute.

In any case, a second condition implies that the distortion on the intertemporal margin for consumption must be able to affect differently each labor type. This depends on particular conditions on the technology used in the economy. The planner will, therefore, only raise a wedge on the intertemporal margin for consumption if and only if this helps targeting the labor types differently, thus helping in overcoming the restriction.

These results are compared to results of past literature, that derive implications for capital taxation in the presence of restrictions on the taxes. One such case is Correia (1996), who shows that when a third factor of production cannot be taxed it may be optimal to tax capital. As shown here, the introduction of consumption taxes will allow the planner to tax the third source of income, overcoming the restriction.

I also consider the case in which the third factor has to be taxed at the same rate as some other factor, capital or labor. When the tax on capital and the third factor must be the same, as in Reis (2011), the result that the steady state taxation of capital is zero is also recovered. Again, this is because consumption taxes are an independent instrument in that setting.

Instead, when the third factor has to be taxed at the same rate as labor income (as in Jones, Manuelli and Rossi (1997)), I show that the consumption tax provides no improvement. This is the representative agent analog to the model of heterogeneity.

The conclusions suggest that the imperfections in the tax system on the treatment of different types of labor may be an argument in favor of the use of capital taxes, both for efficiency and redistribution concerns.

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6 Appendices

6.1 Proof of Proposition 2

The first-order conditions for the consumption of each agent are given by

$$u_{c1}(t)[\omega_1+\lambda_1(1-\sigma)]+\frac{u_{c1c1}(t)}{u_{c1}(t)}\left[-\beta^{-1}\eta^c_{t-1}\frac{u_{c1}(t-1)}{u_{c1}(t)}+\eta^c_t\frac{u_{c1}(t)}{u_{c1}(t+1)}+\eta^n_t\frac{u_{c1}(t)}{u_{n1}(t)}\right]=\mu_t,$$

$$u_{c2}(t)[\omega_2+\lambda_2(1-\sigma)]-\frac{u_{c2c2}(t)}{u_{c2}(t)}\left[-\beta^{-1}\eta^c_{t-1}\frac{u_{c2}(t-1)}{u_{c2}(t)}+\eta^c_t\frac{u_{c2}(t)}{u_{c2}(t+1)}+\eta^n_t\frac{u_{c2}(t)}{u_{n2}(t)}\right]=\mu_t.$$

We can multiply the first equation by c^1_t and the second by c^2_t, and add both conditions to obtain that

$$u_{c1}(t)c^1_t[\omega_1+\lambda_1(1-\sigma)]+u_{c2}(t)c^2_t[\omega_2+\lambda_2(1-\sigma)]=\mu_t(c^1_t+c^2_t).$$

An important characteristic of these preferences is that the equality between marginal rates of intertemporal substitution implies that the ratio of consumptions $\frac{c^2_t}{c^1_t}$ is constant over time. Define this ratio as γ. We can then conclude that

$$u_{c1}(t)[\omega_1+\lambda_1(1-\sigma)]+u_{c2}(t)\gamma[\omega_2+\lambda_2(1-\sigma)]=\mu_t(1+\gamma).$$

Finally, since $u_{c2}(t)\gamma=u_{c1}(t)\gamma^{1-\sigma}$, this implies that

$$u_{c1}(t)\left[\omega_1+\lambda_1(1-\sigma)+\gamma^{1-\sigma}[\omega_2+\lambda_2(1-\sigma)]\right]=\mu_t(1+\gamma).$$

Which, in turn, implies that $\frac{u_{c1}(t)}{u_{c1}(t+1)}=\frac{\mu_t}{\mu_{t+1}}$.

The first order condition to capital accumulation has that the ratio of multipliers is equal to the pre-tax return on capital

$$\frac{\mu_t}{\mu_{t+1}}=\beta(F_K(t+1)+1-\delta).$$

The first order conditions to each labor, already multiplied by n_t^i, are given by

$$u_{n1}(t)n_t^1[\omega_1 + \lambda_1(1+\zeta)] - \zeta\eta_t^n \frac{u_{c1}(t)}{u_{n1}(t)} = -\mu_t F_N(t) n_t^1,$$

$$u_{n2}(t)n_t^2[\omega_2 + \lambda_2(1+\zeta)] + \zeta\eta_t^n \frac{u_{c2}(t)}{u_{n2}(t)} = -\mu_t F_N(t) n_t^2.$$

Since

$$\frac{u_{c1}(t)}{u_{n1}(t)} = \frac{u_{c2}(t)}{u_{n2}(t)} \Leftrightarrow u_{n2}(t) = \gamma^{-\sigma} u_{n1}(t) \Leftrightarrow \frac{n_t^2}{n_t^1} = \gamma^{-\sigma/\zeta},$$

adding both constraints implies the following condition:

$$u_{n1}(t)\big[\omega_1 + \lambda_1(1+\zeta) + \gamma^{-\sigma}[\omega_2 + \lambda_2(1+\zeta)]\big] = -\mu_t F_N(t)(1+\gamma^{-\sigma/\zeta}).$$

This shows that that

$$\frac{u_{n1}(t)}{u_{n2}(t+1)} = \frac{\mu_t F_N(t)}{\mu_{t+1} F_N(t+1)}$$

.

Using these relations it can now be understood that the decentralization of the optimal plan can be done with constant consumption and labor taxes and a zero tax on capital. This can be seen from the following conditions

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c}[1 + (1-\tau_{t+1}^k)(F_K(t+1) - \delta)] = [1 + (F_K(t+1) - \delta)],$$

$$\frac{1-\tau_t^n}{1-\tau_{t+1}^n}[1 + (1-\tau_{t+1}^k)(F_K(t+1) - \delta)] = [1 + (F_K(t+1) - \delta)],$$

Then a possible implementation has $\tau_t^c = \tau^c$ and $\tau_t^n = \tau^n$, with $\tau_{t+1}^k = 0$.

6.2 Proof of Proposition 5

Let us guess that the constraints on the marginal rates of substitution are verified and need not be imposed. Then $\eta_t^c = \eta_t^n = 0$. The first-order condition with respect to consumption of each individual, from period 1 onwards, are thus given by

$$u_{ci}(t)[\omega_i + \lambda_i(1-\sigma)] = \mu_t.$$

This then determines that $\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}}$ for all $t \geq 1$, and then the equality between marginal rates of intertemporal substitution is satisfied, i.e. $\eta_t^c = 0$. From this we define once more that $\frac{c_t^2}{c_t^1} \equiv \gamma$.

The first-order condition with respect to labor for every period is given by

$$u_{ni}(t)[\omega_i + \lambda_i(1+\zeta)] = -\mu_t F_{ni}(t).$$

Note that we must have that

$$\frac{u_{c1}(t)}{u_{n1}(t)} F_{n1}(t) \frac{[\omega_1 + \lambda_1(1-\sigma)]}{[\omega_1 + \lambda_1(1+\zeta)]} = \frac{u_{c2}(t)}{u_{n2}(t)} F_{n2}(t) \frac{[\omega_2 + \lambda_2(1-\sigma)]}{[\omega_2 + \lambda_2(1+\zeta)]}.$$

Condition (29) is verified if and only if

$$\frac{\omega_1 + \lambda_1(1-\sigma)}{\omega_1 + \lambda_1(1+\zeta)} = \frac{\omega_2 + \lambda_2(1-\sigma)}{\omega_2 + \lambda_2(1+\zeta)}$$

Which holds true if

$$(\lambda_2\omega_1 - \lambda_1\omega_2)(\zeta + \sigma) = 0 \implies \frac{\lambda_2}{\lambda_1} = \frac{\omega_2}{\omega_1}.$$

We can now show that this always holds with initial redistribution. The first order condition with respect to $\mathbb{T}_0^1$ shows that

$$u_{c1}(0)\lambda_1 = u_{c2}(0)\lambda_2 \implies \frac{\lambda_2}{\lambda_1} = \gamma^\sigma.$$

The first-order conditions to consumption imply that

$$u_{c1}(t)[\omega_1 + \lambda_1(1-\sigma)] = u_{c2}(t)[\omega_2 + \gamma^\sigma\lambda_1(1-\sigma)].$$

Which using the fact that $u_{c2}(t)\gamma^\sigma = u_{c1}(t)$, shows that indeed, $\eta_t^n = 0$.

$$u_{c1}(t)\omega_1 = u_{c2}(t)\omega_2 \implies \frac{\omega_2}{\omega_1} = \gamma^\sigma.$$

We have not analyzed the problem of period 0. The first-order conditions with respect to consumption in period 0 are given by

$$u_{c1}(0)[\omega_1 + \lambda_1(1-\sigma)] - \frac{u_{cc1}(0)}{u_{c1}(0)} u_{c1}(0)\mathbb{T}_0^1 \lambda_1 = \mu_0,$$

$$u_{c2}(0)[\omega_2 + \lambda_2(1-\sigma)] + \frac{u_{cc2}(0)}{u_{c2}(0)} u_{c2}(0)\mathbb{T}_0^1 \lambda_2 = \mu_0.$$

Then by multiplying by c_t^i each condition and adding both we have that

$$u_{c1}(0)c_0^1[\omega_1 + \lambda_1(1-\sigma)] + u_{c2}(0)c_0^2[\omega_2 + \lambda_2(1-\sigma)] = \mu_0(c_0^1 + c_0^2).$$

Dividing by c_0^1 and using the fact that $u_{c2}(0) = \gamma^{1-\sigma}u_{c1}(0)$, yields that

$$u_{c1}(0)[\omega_1 + \lambda_1(1-\sigma) + \gamma^{1-\sigma}\omega_2 + \gamma^{1-\sigma}\lambda_2(1-\sigma)] = \mu_0(1+\gamma).$$

And using the fact that $\omega_1 = \gamma^{-\sigma}\omega_2$ and $\lambda_1 = \gamma^{-\sigma}\lambda_2$ we can write that

$$u_{c1}(0)[\omega_1 + \lambda_1(1-\sigma)] = \mu_0.$$

Furtheremore,

$$u_{c2}(0)[\omega_2 + \lambda_2(1-\sigma)] = \mu_0.$$

Which concludes the proof to show that $\frac{u_{ci}(0)}{u_{ci}(1)} = \frac{\mu_0}{\mu_1}$ and thus $\eta_0^c = 0$.

6.3 Redistributive Taxation

6.3.1 Unrestricted Discriminatory Transfer

Suppose an unrestricted lump-sum transfer to one of the agents existed (assume without loss of generality it is to agent 2).[9] If such is the case the planning problem is restricted by one less condition - the budget constraint of agent 2 - because that can be met by this transfer.

The planning problem is restricted by the following constraints

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t) c_t^1 + u_{n1}(t) n_t^1] = \mathbb{U}_0^1, \tag{62}$$

$$\frac{u_{c1}(t)}{u_{c1}(t+1)} = \frac{u_{c2}(t)}{u_{c2}(t+1)}, \tag{63}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)} = \frac{u_{c2}(t)}{u_{n2}(t)}, \tag{64}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t. \tag{65}$$

The necessary conditions for optimality, with respect to each consumption, are given by

$$u_{c1}(t)[\omega_1 + \lambda_1(1-\sigma)] + \frac{u_{c1c1}(t)}{u_{c1}(t)} \Big[-\beta^{-1} \eta_{t-1}^c \frac{u_{c1}(t-1)}{u_{c1}(t)} + \eta_t^c \frac{u_{c1}(t)}{u_{c1}(t+1)} + \eta_t^n \frac{u_{c1}(t)}{u_{n1}(t)} \Big] = \mu_t,$$

$$u_{c2}(t)\omega_2 - \frac{u_{c2c2}(t)}{u_{c2}(t)} \Big[-\beta^{-1} \eta_{t-1}^c \frac{u_{c2}(t-1)}{u_{c2}(t)} + \eta_t^c \frac{u_{c2}(t)}{u_{c2}(t+1)} + \eta_t^n \frac{u_{c2}(t)}{u_{n2}(t)} \Big] = \mu_t.$$

Following the same argument we can see that

$$u_{c1}(t) \big[\omega_1 + \lambda_1(1-\sigma) + \gamma^{1-\sigma}\omega_2 \big] = \mu_t(1+\gamma).$$

[9] This follows the ideas of Judd (1985).

Which yields that

$$\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}}$$

.

Therefore, if consumption taxes are set constant over time, the capital tax should be set to zero.

Furthermore, the tax on labor should also be set constant over time. The first order conditions with respect to each labor, already multiplied by each labor level, are given by

$$u_{n1}(t)n_t^1[\omega_1 + \lambda_1(1+\zeta)] - \zeta\eta_t^n \frac{u_{c1}(t)}{u_{n1}(t)} = -\mu_t F_N(t) n_t^1,$$

$$u_{n2}(t)n_t^2\omega_2 + \zeta\eta_t^n \frac{u_{c2}(t)}{u_{n2}(t)} = -\mu_t F_N(t) n_t^2.$$

Adding both constraints and using the previous relation between marginal utilities of labor, implies that

$$u_{n1}(t)\big[\omega_1 + \lambda_1(1+\zeta) + \gamma^{-\sigma}\omega_2\big] = -\mu_t F_N(t)(1+\gamma^{-\sigma/\zeta}).$$

Showing that

$$\frac{u_{ni}(t)}{u_{ni}(t+1)} = \frac{\mu_t F_N(t)}{\mu_{t+1} F_N(t+1)}.$$

6.3.2 Restricted Discriminatory Transfer

Suppose, instead, that the lump-sum transfer is restricted to being positive, but still idiosyncratic to agent 2. This is different from the previous result because the planning problem will be restricted by the budget constraint of household 2.

The planning problem is restricted by the following constraints

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t)c_t^1 + u_{n1}(t)n_t^1] = \mathbb{U}_0^1, \tag{66}$$

$$\sum_{t=0}^{\infty} \beta^t [u_{c2}(t)c_t^2 + u_{n2}(t)n_t^2] = \mathbb{U}_0^2 + u_{c2}(0)\mathbb{T}_0^2, \tag{67}$$

$$\frac{u_{c1}(t)}{u_{c1}(t+1)} = \frac{u_{c2}(t)}{u_{c2}(t+1)}, \tag{68}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)} = \frac{u_{c2}(t)}{u_{n2}(t)}, \tag{69}$$

$$\mathbb{T}_0^2 \geq 0 \tag{70}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t. \tag{71}$$

The necessary conditions for optimality, with respect to each consumption from period 1 onwards are unchanged, implying that

$$\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}},\ \forall t \geq 1.$$

This states that from period one onwards there should be no distortion on the marginal rate of intertemporal substitution for consumption.

In period zero the condition for consumption of household 1 is exactly the same (with $\eta_{-1}^c = 0$). However, the first order condition to consumption of household 2 is somehow different. It is given by

$$u_{c2}(0)c_0^2[\omega_2 + \lambda_2(1-\sigma)] + \sigma\left[\eta_0^c \frac{u_{c2}(0)}{u_{c2}(1)} + \eta_0^n \frac{u_{c2}(0)}{u_{n2}(t)}\right] + \sigma\lambda_2 u_{c2}(0)\mathbb{T}_0^2 = \mu_0.$$

The Kuhn-Tucker condition with respect to the choice of $\mathbb{T}_0^2$ is given by:

$$\lambda_2 \mathbb{T}_0^2 u_{c2}(0) = 0.$$

.

One of two scenarios are possible: (1) $\mathbb{T}_0^2 = 0$ or (2) $\mathbb{T}_0^2 > 0$. (1) immediately implies that the first order condition with respect to c_0^2 can be written as

$$u_{c2}(0)c_0^2[\omega_2 + \lambda_2(1-\sigma)] + \sigma\left[\eta_0^c \frac{u_{c2}(0)}{u_{c2}(1)} + \eta_0^n \frac{u_{c2}(0)}{u_{n2}(0)}\right] = \mu_0,$$

from which the previous arguments hold exactly. If the transfer is strictly positive, scenario (2), then $\lambda_2 = 0$, since $u_{c2}(0) > 0$. Under this condition, the optimal plan has the same characteristics of the optimal plan of 6.3.1, and thus capital should not be taxed.

6.3.3 Restricted Non-Discriminatory Transfer

Suppose that the lump-sum transfer is given to both agents, on a non-discriminatory way, i.e. both receive the same transfer. Despite its practical appeal, as one would think that non-discriminatory instruments are more likely to exist in real economies, this case is more restrictive than the previous one. It should yield roughly the same results.

The planning problem is restricted by the following constraints

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t) c_t^1 + u_{n1}(t) n_t^1] = \mathbb{U}_0^1 + u_{c1}(0)\mathbb{T}_0, \tag{72}$$

$$\sum_{t=0}^{\infty} \beta^t [u_{c2}(t) c_t^2 + u_{n2}(t) n_t^2] = \mathbb{U}_0^2 + u_{c2}(0)\mathbb{T}_0, \tag{73}$$

$$\frac{u_{c1}(t)}{u_{c1}(t+1)} = \frac{u_{c2}(t)}{u_{c2}(t+1)}, \tag{74}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)} = \frac{u_{c2}(t)}{u_{n2}(t)}, \tag{75}$$

$$\mathbb{T}_0 \geq 0 \tag{76}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t. \tag{77}$$

The necessary conditions for optimality, with respect to each consumption from period 1 onwards are unchanged, implying that

$$\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}}, \ \forall t \geq 1.$$

This states that from period one onwards there should be no distortion on the marginal rate of intertemporal substitution for consumption.

In period zero, the necessary conditions for optimality with respect to the consumption of each household are given by

$$u_{c1}(0)c_0^1[\omega_1 + \lambda_1(1-\sigma)] - \sigma\left[\eta_0^c \frac{u_{c1}(0)}{u_{c1}(1)} + \eta_0^n \frac{u_{c1}(0)}{u_{n1}(0)}\right] + \sigma\lambda_1 u_{c1}(0)\mathbb{T}_0 = \mu_0 c_0^1,$$

$$u_{c2}(0)c_0^2[\omega_2 + \lambda_2(1-\sigma)] + \sigma\left[\eta_0^c \frac{u_{c2}(0)}{u_{c2}(1)} + \eta_0^n \frac{u_{c2}(0)}{u_{n2}(0)}\right] + \sigma\lambda_2 u_{c2}(0)\mathbb{T}_0 = \mu_0 c_0^2.$$

Adding both constraints yields the following condition

$$u_{c1}(0)[\omega_1+\lambda_1(1-\sigma)+\gamma^{1-\sigma}[\omega_2+\lambda_2(1-\sigma)]]+\sigma\mathbb{T}_0(\lambda_1 u_{c1}(0)+\lambda_2 u_{c2}(0)) = \mu_0(1+\gamma)$$

Since the Kuhn-Tucker condition with respect to the transfer shows that

$$\mathbb{T}_0(\lambda_1 u_{c1}(0) + \lambda_2 u_{c2}(0)) = 0,$$

we can rewrite the condition as

$$u_{c1}(0)[\omega_1 + \lambda_1(1-\sigma) + \gamma^{1-\sigma}[\omega_2 + \lambda_2(1-\sigma)]] = \mu_0(1+\gamma).$$

Therefore,

$$\frac{u_{ci}(0)}{u_{ci}(1)} = \frac{\mu_0}{\mu_1},$$

implying that capital should not be taxed.

6.3.4 Unrestricted Non-Discriminatory Transfer

Suppose that this transfer is now unconstrained, meaning it can be a lump-sum tax. This is the case under which the government has a non-distortionary tool available. However, since it is not able to discriminate between individuals, the economy cannot, in general, achieve the first-best and distortionary taxation will be raised to finance the government.

The planning problem is restricted by the following constraints

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t) c_t^1 + u_{n1}(t) n_t^1] = \mathbb{U}_0^1 + u_{c1}(0)\mathbb{T}_0, \tag{78}$$

$$\sum_{t=0}^{\infty} \beta^t [u_{c2}(t) c_t^2 + u_{n2}(t) n_t^2] = \mathbb{U}_0^2 + u_{c2}(0)\mathbb{T}_0, \tag{79}$$

$$\frac{u_{c1}(t)}{u_{c1}(t+1)} = \frac{u_{c2}(t)}{u_{c2}(t+1)}, \tag{80}$$

$$\frac{u_{c1}(t)}{u_{n1}(t)} = \frac{u_{c2}(t)}{u_{n2}(t)}, \tag{81}$$

$$c_t^1 + c_t^2 + K_{t+1} + G_t = F(K_t, n_t^1, n_t^2) + (1-\delta)K_t. \tag{82}$$

The necessary conditions for optimality, with respect to each consumption from period 1 onwards are unchanged, implying that

$$\frac{u_{ci}(t)}{u_{ci}(t+1)} = \frac{\mu_t}{\mu_{t+1}}, \ \forall t \geq 1.$$

This states that from period one onwards there should be no distortion on the marginal rate of intertemporal substitution for consumption.

In period zero, the necessary conditions for optimality with respect to the consumption of each household are given by

$$u_{c1}(0) c_0^1 [\omega_1 + \lambda_1(1-\sigma)] - \sigma \Big[\eta_0^c \frac{u_{c1}(0)}{u_{c1}(1)} + \eta_0^n \frac{u_{c1}(0)}{u_{n1}(0)} \Big] + \sigma \lambda_1 u_{c1}(0)\mathbb{T}_0 = \mu_0 c_0^1,$$

$$u_{c2}(0)c_0^2[\omega_2 + \lambda_2(1-\sigma)] + \sigma\left[\eta_0^c \frac{u_{c2}(0)}{u_{c2}(1)} + \eta_0^n \frac{u_{c2}(0)}{u_{n2}(0)}\right] + \sigma\lambda_2 u_{c2}(0)\mathbb{T}_0 = \mu_0 c_0^2.$$

Adding both constraints yields the following condition

$$u_{c1}(0)[\omega_1+\lambda_1(1-\sigma)+\gamma^{1-\sigma}[\omega_2+\lambda_2(1-\sigma)]]+\sigma\mathbb{T}_0(\lambda_1 u_{c1}(0)+\lambda_2 u_{c2}(0)) = \mu_0(1+\gamma)$$

Since the first-order condition with respect to the transfer shows that

$$\lambda_1 u_{c1}(0) + \lambda_2 u_{c2}(0) = 0,$$

we can rewrite the condition as

$$u_{c1}(0)[\omega_1 + \lambda_1(1-\sigma) + \gamma^{1-\sigma}[\omega_2 + \lambda_2(1-\sigma)]] = \mu_0(1+\gamma).$$

Therefore,

$$\frac{u_{ci}(0)}{u_{ci}(1)} = \frac{\mu_0}{\mu_1},$$

implying that capital should not be taxed.

6.4 On Gorman Aggregation

Correia (2010) develops a model of heterogeneous agents that is still amenable to Gorman aggregation. In that model agents differ both on initial wealth and labor efficiencies. However, to keep aggregation labor types are still perfect substitutes.[10]

In that paper, Correia does not compute optimal policies, but rather shows that setting the tax on capital income to zero while increasing the taxes on consumption and labor, increases both efficiency and equity. I seek to show that in an economy amenable to Gorman aggregation, optimality can be computed from a representative agent framework if discriminatory lump-sum transfers exist.

In a Gorman aggregable economy with different labor efficiencies, whenever the indirect utility function can be written as

$$v_i(p) = \alpha(p)g(E_i) + \beta(p)A_i, \tag{83}$$

we can define a representative agent where the indirect utility function is given by

$$v_r(p) \equiv \sum_{i=1}^{N} \frac{\alpha(p)g(E_i) + \beta(p)A_i}{N} = \alpha(p)E_r + \beta(p)A_r, \tag{84}$$

defining $\sum_i \frac{g(E_i)}{N} \equiv E_r$ and $A_r \equiv \sum_i \frac{A_i}{N}$.

Proposition 12. *If the economy is amenable to Gorman Aggregation, and the indirect utility can be written as $v_i(p) = \alpha(p)g(E_i) + \beta(p)A_i$, when discriminatory lump-sum transfers are available the solution to the representative agent problem can always lead to a Pareto improvement.*

Proof. The proof is done in 2 steps. First, I argue that the solution to the Representative Agent problem can be represented by an increase in the

[10] In Correia (2010) the market wage is w_t, however the agent's wage is given by $E_i w_t$, where E_i is the measure of labor efficiency for agent i.

initial wealth of the agent, and then show that with discriminatory lump-sum transfers we can find a Pareto improvement.

(a) Equivalent Wealth Increase

Optimality in the representative agent problem, implies that

$$v(p^*) \geq v(p'), \forall p' \in F, \tag{85}$$

where F represents the set of feasible price vectors, p.

We define the equivalent wealth, A_i^*, as the wealth that would yield the same welfare of p^* at the price vector level of p', this implies that

$$v(p^*) = \alpha(p^*)E_r + \beta(p^*)A_r = \alpha(p')E_r + \beta(p')A_r^* \geq \alpha(p')E_r + \beta(p')A_r.$$

(85) implies that $A_r^* \geq A_r$.

We can find an expression for the equivalent wealth

$$A_r^* = \frac{\alpha(p^*)E_r - \alpha(p')E_r + \beta(p^*)A_r}{\beta(p')}. \tag{86}$$

(b) Pareto Improvement

To show that this can lead to a Pareto improvement, we fix the wealth of all $N-1$ first agents to be such that the utility they attain with the price vector p^* is the same level of utility they had with the alternative p'. Given this we can show that the utility for the $N-th$ agent increases. Therefore, defining by v_i^* the utility for each agent at the price level p^* and after redistribution, where final wealth is A_i^*, we have that

$$v_i^* = \alpha(p^*)g(E_i) + \beta(p^*)A_i^* = \alpha(p')g(E_i) + \beta(p')A_i, \; \forall\, i = 1, ..., N-1, \tag{87}$$

$$v_N^* = \alpha(p^*)g(E_N) + \beta(p^*)A_N^* = \alpha(p')g(E_N) + \beta(p')A_N', \tag{88}$$

where $A_N' = NA_r^* - \sum_{i=1}^{N-1} A_i$.

To show that welfare is increased, we need to show the $N-th$ agent is better off, (1) $A'_N \geq A_N$. Furthermore, it must be feasible, hence initial aggregate wealth should be unchanged, (2) $\sum_{i=1}^{N} A_i^* = \sum_{i=1}^{N} A_i$.

(1) immediately follows from the fact that $A_r^* \geq A_r$, which implies that the equivalent aggregate wealth increases $NA_r^* \geq NA_r$.

To prove (2) we must show that at the final prices, initial wealth does not increase

$$A_i^* = \frac{(\alpha(p') - \alpha(p^*))g(E_i) + \beta(p')A_i}{\beta(p^*)},\ \forall\, i = 1, ..., N-1, \tag{89}$$

$$A_N^* = \frac{(\alpha(p') - \alpha(p^*))g(E_N) + \beta(p')A'_N}{\beta(p^*)}. \tag{90}$$

$$\sum_{i=1}^{N} A_i^* = \sum_{i=1}^{N-1} \frac{(\alpha(p') - \alpha(p^*))g(E_i) + \beta(p')A_i}{\beta(p^*)} + \frac{(\alpha(p') - \alpha(p^*))g(E_N) + \beta(p')A'_N}{\beta(p^*)}$$

$$= N\frac{(\alpha(p') - \alpha(p^*))E_r}{\beta(p^*)} + \frac{\beta(p')}{\beta(p^*)}\Big[\sum_{i=1}^{N} A_i + A'_N\Big].$$

By definition of A'_N, the above equation can be written as

$$N\frac{(\alpha(p') - \alpha(p^*))E_r + \beta(p')A_r^*}{\beta(p^*)} = N\frac{(\alpha(p') - \alpha(p^*))E_r + \beta(p')\left[\frac{(\alpha(p^*) - \alpha(p'))E_r + \beta(p^*)A_r}{\beta(p')}\right]}{\beta(p^*)}$$

$$= NA_r = \sum_{i}^{N} A_i.$$

□

Gorman aggregation does, however, underly a key assumption on how each agent's labor efficiency relates to capital. In specific, is implies that an increase in the capital stock of this economy will lead to the same proportional increase in all labor productivities.

6.5 Uniform Taxation of Labor Types

Suppose the utility function of the representative agent is given by

$$U = U(C_t, G(N_t, \Im_t))$$

and G is homogenous of degree k. We can then write that, for $i = N, \Im$:

$$-\sigma^{Ci}(t) = \frac{U_{Ci(t)}}{U_i(t)} C_t = \frac{U_{CG}(t)}{U_G(t)} C_t,$$

$$-\sigma^{Ni}(t) = \frac{U_{GG}(t)}{U_G(t)} G_N(t) N_t + \frac{G_{Ni}(t)}{G_i(t)} N_t,$$

$$-\sigma^{\Im i}(t) = \frac{U_{GG}(t)}{U_G(t)} G_\Im(t) \Im_t + \frac{G_{\Im i}(t)}{G_i(t)} \Im_t.$$

Therefore,

$$-\sigma^{Ci}(t) - \sigma^{Ni}(t) - \sigma^{\Im i}(t) = \frac{U_{CG}(t)}{U_G(t)} C_t + \frac{U_{GG}(t)}{U_G(t)} G(t) + (k-1).$$

Which implies that

$$\frac{[1 + \lambda(1 - \sigma^{C\Im}(t) - \sigma^{N\Im}(t) - \sigma^{\Im\Im}(t))]}{[1 + \lambda(1 - \sigma^{CN}(t) - \sigma^{NN}(t) - \sigma^{\Im N}(t))]} = 1.$$

This shows that the optimal plan has

$$\frac{U_N(t)}{U_\Im(t)} = \frac{F_N(t)}{F_\Im(t)},$$

implying condition (56) is not binding.