

An Analysis On The Interaction Between Migration And Redistributive Policies

Written by: Vasco Maria Ferraz da Costa Maldonado Passanha

Academic Advisors: Professora Teresa Lloyd Braga, Professor Manuel Leite Monteiro

Dissertation submitted in partial fulfilment of the requirements for the degree of MSc in Economics at Católica-Lisbon School of Business & Economics, September 2015

ABSTRACT

This study tries to explain a cause and effect chain, how does a redistributive policy impacts on migration, and how does migration impacts on the wages of the different agents in the two economies under analysis.

The model in use has two countries identical in all aspects except that one has a tax/subsidy scheme and the other does not. There are two types of workers, skilled and unskilled, and only the skilled workers have the possibility to migrate. The tax is charged proportionally on the skilled workers' wage and then distributed, lump sum, among the unskilled workers.

Findings show that there is a range of values for the tax to maximize the unskilled workers wage of the country where the tax is launched that goes in line with the ones reported by OECD (2012). Another finding is that there is a reduction on the wage gap between the two types of workers in the country where there is no tax.

RESUMO

Este estudo pretende explicar uma relação de causa efeito entre diferentes variáveis, qual o impacto de uma política redistributiva na migração, e qual o impacto da migração nos salários dos diferentes agentes das suas economias em análise.

O modelo em uso tem dois países em tudo idênticos com excepção de que num deles existe um mecanismo de imposto/subsídio e no outro não. Existem dois tipos de trabalhadores, qualificados e não qualificados, sendo que só os trabalhadores qualificados têm oportunidade de emigrar. O imposto é aplicado porporcionalmente no salário dos trabalhadores qualificados e distribuído de forma igualitária, por todos, os trabalhadores não qualificados.

Os resultados mostram que existe um intervalo de valores para o imposto de tal forma que este maximize o salário dos trabalhadores não qualificados onde o imposto é aplicado, estes valores estão de acordo com os valores reportados pela OCDE (2012). Outro resultado é que existe uma redução na diferença entre o salário pago aos dois tipos de trabalhadores no país onde não existe imposto.

i

ACKNOWLEDGEMENTS

The first acknowledge I sincere want to make is to the two Professors that make all this possible by being my academic advisors Professor Teresa Lloyd Braga and Professor Manuel Leite Monteiro. I show my gratitude due to all the guidance, the motivational support and specially all the insights throughout this entire period of the Master program.

I would also particularly like to thank Professor Fernando Branco for the opportunity I had to enrol on another of his courses and Professor Leonor Modesto for all her support during these five years at Católica.

Apart from the University, I want to thank my family that played the most important role through all this academic process.

At last I am really thankful to my friends that always believed in me and had a clear understanding of what it meant to me writing a master thesis.

In particular to Joana for helping my time management in the most useful way, my coffee group, my Italian friend, for all the laughs and conversations during this period and to my "not so study buddies" Sofia and João for all those study sessions at home.

TABLE OF CONTENTS

ABSTRACT	i
Acknowledgements	ii
Table Index	iv
1. Introduction	1
2. Literature Review	3
2.1. Migration and its evolution through time	
2.2. Brain Drain	5
2.3. Redistributive Policies	7
2.4. Relevant Empirical Studies	9
3. THE MODEL	10
3.1. Closed Economy	11
3.1.1. Taxation	16
3.2. Open Economy	17
3.2.1. Effects of Labour Mobility	
3.2.2. Effects of Taxation	
3.2.2.1. Effects of taxation on Country B	
3.2.2.2. Effects of Taxation on Country A	
3.3. Optimal Taxation	
4. Conclusions	39
4.1. Limitations and suggestions for further research	40
5. References	41

TABLE INDEX

Table 1: Optimal Taxation, elasticity of substitution: 1	37
Table 2: Optimal Taxation, elasticity of substitution: 1,5	38
Table 3: Optimal Taxation, elasticity of substitution: 2	38

1. INTRODUCTION

The following research work is developed with the purpose of studying the current phenomenon common to many European countries that is migration of skilled workers and how governments are still able to raise enough revenues for redistributive aims. With this goal in mind we built a model that can be applied to any country where migration and redistributive policies are present. We do not focus particularly on any country but Portugal can be used as a suitable example for the model presented next. Portuguese history has always had some migration flows especially at the current times. Due to the financial crisis the government is showing some concerns with the topic.

Portugal has been attracting many migrants from several countries around the world; the current situation, the financial crisis, has added pressure on the emigrants' flow of workers, Portuguese people moving out of Portugal. Anyway there has always been emigration among the Portuguese people, especially to countries like France and Switzerland. This was usually emigration of unskilled workers; it was a mass movement in the decade of 1950. Currently emigrants are much more qualified, we have skilled workers emigration.

On top of this what is crucial to be noticed is that in 2002, around 27.358 people emigrated and in 2014 the number increased abruptly to 134.624 in the country. (Pordata, 2015)

With the world crisis of 2008 the country found itself on a different situation.

Since then, most of the economic indicators had worse values, particularly the unemployment rate. It increased from around 2% in 2002 to around 14% in 2014 (Pordata, 2015). But the most problematic unemployment, as stated by the government, is the unemployment rate of young people leaving university that was around 35% in 2014 (Eurostat, 2015) and according to OECD (2015) data, in 2015, Portugal is the fifth country with the highest unemployment rate among youth in the Euro Area.

Students who complete a master degree are considered skilled workers. Graduates expect, then, good job positions where their effort on university is compensated not only

by the wage received but also by the working environment and recognition. This reasoning is explained in section 3 and it goes by the name of skilled premium.

When adding the country situation to a university student looking out for a job position the immediate question that arises is whether or not the student will be able to find a job that matches his expectations, and if the answer to the question is no, then the first best solution is often to look for a job position on another country.

This is part of what motivates my interest on the topic.

The Portuguese government is concerned with the so called "brain drain", a situation that occurs when a significant fraction of skilled workers, formed in Portugal, leave the country to search for better life/job opportunities beyond borders. But if citizens with capacity to generate welfare for the country as a whole, the ones with the highest capabilities are leaving, how is the system going to be sustainable? Who is going to pay enough taxes to keep our government expenditures on education, health system and social pensions? The aim of this research is precisely to show that a problem is being created on the way countries will be able to redistribute money from the skilled to the unskilled workers if the first category is being reduced at this path.

Even if this is a recent phenomenon governments and institutions start now to worry about this mass migration, and even though this is not the aim of this research, further research can be done on how we can revert the trend, how we can promote better job opportunities to avoid all this migration.

This master thesis main objective is to focus on the relationship between the tax and migration and because the interaction is not unilateral, one has effects on the other and vice-versa. This is developed through a general equilibrium model with two similar countries. They both have the same types of inputs, skilled and unskilled workers, they produce in the exact same way the same output. The only difference between countries is that on one of them we have a proportional income tax that is then distributed lump sum. The tax is charged on the skilled workers wage leading to a higher income to the poorer unskilled ones. Migration is allowed for the skilled workers.

In the next section a review of the literature is presented divided in accordance to the model under use on section 3, being this section divided into two subsections: closed economy and open economy. In these subsections the effects of the tax/subsidy policy

on net incomes are presented and they lead to some conclusions summarised on section 4. This same section also presents some limitations and suggestions for further research.

2. LITERATURE REVIEW

Since the beginning countries worldwide have tried to improve their inhabitants' quality of life and this passes not only but also, through the increase of the income per capita. From time to time people have tried to achieve better living standards that include simple aspects as peace, freedom, security but also monetarily ones such as income level, public services and well functioning of institutions. To help on this process of achieving and providing better conditions for inhabitants governments have promoted economic growth for an overall improvement of the country's socio economic conditions. When everyone does not reach the welfare improvement, governments introduce programs and systems of funds to help the ones in need; this topic is also covered on the model through a very simple tax scheme that is launched on the income of the richer to give through lump sum subsidy to the poorer. So there are many different possibilities that lead to the final goal of better living standards. There are no pre-defined formulas although much literature has been written on the effects and causalities of relevant factors on welfare improvements. In this research project the focus is made on migration and its redistributive policy effects, specifically on the welfare of the unskilled workers that stay on the country where emigration occurs.

2.1. Migration and its evolution through time

There have been hard times for globalization mainly linked with migration restrictions. As stated by Hamilton and Whalley (1982) the possibilities linked to migration were vast, the restrictions implemented at the time were reducing the potential welfare increase to much lower values than the ones that could have been reached by simply reducing some of these restrictions. This reasoning is made based on what happened next, after eliminating part of the restrictions, social welfare started to increase. To better sustain the argument, a model based on different examples of migration policies is developed and ends up finding that, "small changes in global migration restrictions are especially important since their marginal effects are large".

And so there has been some evolution regarding this policy gap we had earlier in time.

As stated in Bordo, Taylor and Williamson (2003) the main question is linked to the effects of migration on the wages paid on the two countries involved in the process and through their book they reached five main conclusions presented by history: not also there is an effect on wages caused by migration but also an effect on migration caused by wages, mainly net wages. Net wages are one of the most important drivers of migration. But the focus of this study was made on inequality and the conclusion derived on the mentioned book was that inequality has been reduced. "World incomes would still be unequal under complete global integration, as they are in any large integrated national economy. But they would be less unequal in a fully integrated world economy than in one fully segmented." Bordo, Taylor and Williamson (2003).

The number of people migrating has increased and it is still increasing. The point made by OECD (2013) is that not only the amount of people migrating is increasing but also the people migrating are different. Years ago people migrating were unskilled, and they were moving mainly from non-developed and developing countries to the developed ones. This is changing, in 2013 "About 30% of all migrants in the OECD area were highly educated and one-fifth of them were came from India, China or the Philippines." OECD (2013)

There are several explanations for the increase of this number, costs of migration are lower and economies become better integrated from time to time. So what we have been experiencing at current times is the possibility to study abroad by not only the elites but by almost everyone that can get financial assistance for an overseas training as mentioned by Nguyen (2006), which is seen as one of the causes for the increase in migration flows.

It must be noted that, as reported by OECD (2013), migration keeps increasing but when we compare data before and after 2008, there is a difference at the amount of migrants, now higher, mainly caused by the crisis that characterized 2008.

2.2. Brain Drain

As mentioned before what is increasing sharply is the proportion of highly educated immigrants in OECD countries. This goes along with the idea of a brain drain. The term brain drain was first used by the British Royal Society to describe the emigration of "scientists and technologists" from Europe to North America after the World Wars. The term has been used nowadays to describe what is currently happening, the migration of skilled individuals instead of the unskilled ones as we were used to see.

The literature developed so far has been focused on the links between the brain drain and economic inequality. The question being answered many times is whether it is the brain drain causing inequality or inequality causing the brain drain.

So the first point is to understand what are the drivers of the brain drain, what motivates people to move from one country to another? Ever since human race was described as human race we have seen people moving from place to place in search of better living conditions. At this time we are no different. According to some previous literature there are some standard checkpoints migrants think about before taking the decision. Below there are some of them but these are general to both skilled and unskilled workers. As mentioned above, and citing OECD (2013), there is a factor explaining the choice to migrate to a specific country/region, the correlation between "skilled recent immigrants (having lived in the region for less than five years) and skilled established migrants (with more than four years of residence in the region)". So there is evidence of a network effect, the migrants already on a country do have an important role to attract new skilled migrants. It is a challenge for countries where skilled migration is not common to be able to attract these migrants. There are already certain places known to be easy for skilled migrants to find jobs and this is linked to the mentioned network effect. As stated in Docquier et al. (2007) there are many factors to be taken into account while deciding to migrate. The major factors described in here are linked to the country itself, more than to the person who wants to migrate. The factors listed are the economic ones such as the lack of economic growth and the existence of bad institutions and the more social ones like discriminations or political repression. These are the most common factors that migrants take into account while deciding to leave their own country and also the factors they look for when deciding which country to move in. It also mentioned a particularity linked to the people that want to migrate, and not the country dimension, the skilled workers, they are much more sensible to the economic push-pull factors when compared to the unskilled ones.

Wildasin (2014) focuses the analysis on the migrant, and discusses the impact of the labour market conditions on the migration decision. It is mentioned the fiscal systems, the aging populations and the years of schooling of the people.

In Ehrenberg and Smith, 1988, cited in Wildasin (2014), the age is taken as an important factor contributing to migration, it is a phenomenon much more common in younger generations than in elder ones (OECD, 2013), and this leads to better understand which generation is moving. When it comes to understand, in the same generation, who is more likely to be moving it is more common to have the more educated to migrate first, Docquier et al. (2009). This goes in line with the last argument presented on an early paper by the same authors, Docquier et al. (2007).

In many countries of the world, education is provided by the government and represents a large fraction of its national budget. The question that immediately arises is whether it is an investment with high returns or not for the country if all the brains do leave after schooling. There are many studies developed on this area trying to understand if the outcome of a migrant is positive or negative for the origin country. Docquier and Rapoport (2012) tried to understand what is the relationship between migrants, their level of education and remittances. The first though is that the higher the level of education, the higher the salary the migrant will be able to collect at the country he moved hence the higher the remittances. On the other hand highly educated individuals are able to move the entire family to a new country leading to no remittances to the origin country. In case the remittances do exist, the next question is on whether they are used to invest back on education or not. Docquier and Rapoport (2012) state that:

"Remittances sent by skill migrants may help overcome liquidity constraints, stimulate education investments, and reduce poverty at origin. The size of the effect depends on the amounts transferred and on their distributional impact."

This has been discussed a lot, especially when developing countries are concerned due to level of corruption and the complications associated to these countries on this state of development. The impact of the fiscal policy mentioned above is presented in the next sub section, putting a clear focus on this aspect since it is crucial for the model.

2.3. Redistributive Policies

Agents in the economy, from households to firms, do adjust to the different fiscal policies a government might implement. As stated in Wildasin (2012) the impact of these fiscal programs depends on many parameters but mostly on the mobility of the resource being taxed. If the economy is an open economy, taxation on capital is very easy to avoid by simply moving capital from the country where the tax is launched to another one where that tax does not exist. This degree of factor mobility is a concept that has no specific value; it depends very much on the period of the study, since nowadays it is easier to move from one place to another and also on the integration of the economy. Wildasin (2012) developed a dynamic model where it accounts for both the cost of adjustment and degree of factor mobility. The main conclusion of this paper is that the higher the degree of factor mobility the lower the impact on the tax revenue, so the lower the transfers the government is able to make from one input category to the other.

Another dimension that is also discussed is the fiscal impact of immigration. To answer to this question, Jean et al. (2007) focus on the impact migrants have throughout their lives and moreover their descendants. For immigrants to be a beneficial part of the fiscal system of the country it is necessary that their contributions overcome social benefits they receive. What is discussed on the paper is that this evaluation depends on the present value of the contributions and the assistance this people receive. And the final result depends on some known parameters such as age, if the migrants are still at school than the expense on education will rise, if they are skilled migrants then the tax revenue that directly comes out of their wages will be higher. So the fiscal policy at the country level is highly linked to migration, Jean et al. (2007).

All facts on migration stated above were general and could be applied to any kind of migrants. The present paper presents a much more specific approach, closer to the topic of the study, the direct impact of skilled workers migration. Cremer et al. (1996) refer another consequence of the skilled workers migration. It is known that redistributive

policies rely mostly on collectable wages of this type of workers. As migration starts to become more frequent, the taxable amounts will be reduced which will be a problem when redistributive policies are concern. Skilled workers will leave the country while the government, keeping everything else constant, will have a lower amount of tax to redistribute among the unskilled leading to a less effective redistributive policy. The same is also possible, if, on the contrary, migration was only possible for the unskilled workers. The government would experience an increase of unskilled workers into their country looking for a more favourable redistributive policy.

One thus sees that redistribution becomes unsustainable. Note that subsidizing skilled labour would be even worse, as it would attract skilled immigrants but at a high cost for the poorer immobile individuals. Also if the unskilled workers are mobile instead of the skilled, the same unsustainability result arises: regions offering relatively generous transfers will experience an influx of unskilled workers and this, once again, undermines the financial feasibility of the initial redistributive program.

As mentioned in the past two subsections of the literature review, brain drain and redistributive policies are the main drivers of this thesis, which is based on a general paper on the topic, Wildasin, D. '*Human Capital Mobility: Implications for Efficiency, Income Distribution, and Policy*', 2014.

On Wildasin (2014), the focus is made on the different labour market conditions, taxes and some other fiscal policies in relationship with migration. Another important point mentioned on this paper is the aging population effect and the link that this problem has to the government when concerning budget for age-sensitive programs like the health care system or the public pensions. But what drives the problem in here is the way the government collects most of its revenue, through income taxes, so it is immediate that the less people a country has working, the riskier it becomes for the system to collapse. Emigration contributes positively to this risk, the more people emigrating, the less people working on that country, the lower the tax revenue, the shorter the budget. Note that this affirmation relies on the assumption that the cost for the government of these emigrants is lower than the benefits they bring to it. On the other hand, if countries are able to stimulate the entrance of young skilled immigrants, immigrants whose benefits for the country are higher then the expenses, this might be a good way to overcome the above mentioned problem, since the reasoning goes the other way around. The main results of the paper describe the impact of a fiscal transfer from the skilled workers to the unskilled ones based on a dynamic model that allows for analysis along time where a difference between the short run, the transitional period and the long run is made. They found some of the determinants of human capital mobility that can be used by the policy maker to influence the decision. The first driver of the human capital mobility is an upper bound that can be established to control the number of immigrants a country is able to integrate, this can be made through different policies such as regulation on employment access, housing or border controls. There are also some other determinants that influence the cost of relocation which are not directly controlled by the policy maker such as the language and the culture of the country where to migrate.

Another main conclusion of the paper under analysis is that the level of education and migration are highly interconnected in a positive way, meaning that the more educated people are the more likely people migrate. The analysis made after is on the temporal aspect mentioned before. They found out that the more mobile the resources, the faster the effects of taxation in the economy.

2.4. Relevant Empirical Studies

In the model used in this thesis, values for some parameters at use will be needed. We will use European Commission (2007) and Behar (2009) to obtain estimates for those values.

The first parameter to be needed on the model is the labour income share, depending on the type of worker being skilled or unskilled. The European Commission in 2007 has developed a study about the Employment in Europe where they present some values for these parameters. The calculation is made based on the wage bill and the GDP of the country under analysis. There was a positive trend on the labour income share until the 1970's, after this it started to decrease in almost all EU-15 countries until the 80's and now is quite stable. The point is that it varies a lot from country to country and it is difficult to come up with a value that suits the different realities of the countries in the EU. When we specify the labour according to the type of worker than the paths are divergent, while unskilled labour share of income decreased through time, the skilled labour share of income increased. On the study there is a higher specification than the one we make here on the research project, they divided the type of workers into low, medium and high skilled ones. On the present research we only divide workers into two groups, the unskilled and the skilled ones. We therefore assume that medium skilled workers could be split evenly into the two groups. For the 10 Member state countries used in European Commission (2007) the values are around 0,45 for the unskilled labour share of income and 0,55 for the skilled labour share of income. On section 3, these values will be used as the reference ones, but a wider interval for them will also be considered.

Another parameter that will be needed is the elasticity of substitution between factors, being the factors skilled and unskilled workers. "The elasticity of substitution parameter influences what impact schooling will have on the wage premium." Behar (2009). The wage premium will be discussed later on page 14 of this thesis. The first analysis made on Behar (2009) is a revision of the values used by other authors and it seemed to have no consensus since it varies a lot. What was proven on the paper is that the value is around 2 and for this a regression estimating the wage premium using four sets of different data was made. So this is the value that is now accepted as the most right and it is, then, the one we will use later while estimating the model. Also at the European Commission (2007) the value is stated to be around 2, and there are further developments on the topic, concerning union power and institution level at the countries' level that can actually set the elasticity of substitution parameter into values different from 2 but still around 2.

3. THE MODEL

This study uses a model of general equilibrium to evaluate the impacts of redistributive taxation on country A on the two types of workers present on both countries A and B. The first assumption is that those countries are identical in everything; in each country there is an equal fixed number of skilled (\overline{H}) and unskilled (\overline{L}) workers. There is a unique output. The production technology, homogeneous of degree one, is also equal on the two countries. The first part presents the model and the tax implemented on one of the countries. This analysis is developed under the assumption of a closed economy. On the second part, and since the purpose of this model is to also evaluate migration, we had to focus the analysis on an open economy to better understand the impacts of this

tax on migration but also the impacts of the tax as a redistributive policy.

The increase in the tax as a redistributive policy can be analyzed under a closed economy environment. It is something that happens at the country level, is a relationship between the two types of workers on the same economy. To assess the influence of migration, we need to consider two countries in the model. In this model we restrict the analysis to migration of only one category of workers; however, through general equilibrium effects, it will have an impact on both categories of workers on both countries.

3.1. Closed Economy

In this first section the model is solved under a closed economy setting. Since both countries are equal, we end up having the exact same equilibrium for both of them. In this model we use a general production function with two inputs, skilled (H) and unskilled (L) workers.

$$Y = F(L; H)$$

where *Y* is the number of the single units of output produced.

And since it is assumed to be a homogeneous function of degree one with marginal productivities of the factors being both positive but decreasing with the factor, we can write it as:

$$Y = L f\left(\frac{H}{L}\right)$$

$$f' > 0 \quad and \quad f'' < 0$$

$$F_L = f - \frac{H}{L^2} L f' = f - \frac{H}{L} f'; \quad F_H = f'$$

$$\frac{F_L}{F_H} = \frac{f - \frac{H}{L}f'}{f'} = \frac{f}{f'} - \frac{H}{L}$$

Assuming we have perfectly competitive markets both at the input and output levels, and that firms' objective is to maximize profits, we can state that the price we pay per unit of input used, the wage, will be equal to the marginal productivity of the production factor, in this model the two types of workers. The price of the product is assumed to be unitary.

$$\max \Pi = Y - (w_H H + w_L L)$$

$$\begin{cases} \frac{\partial \Pi}{\partial L} = 0 \iff F_L = w_L \\ \frac{\partial \Pi}{\partial H} = 0 \iff F_H = w_H \end{cases}$$

When a firm's maximization problem is stated like this, we want to maximize the production at the lower cost, this means we will end up with different combinations of skilled and unskilled workers that do lead to the exact same amount of production. The one with the lowest cost is the chosen one by the firms. We can compute the share of skilled labour income, which is the total amount paid to skilled workers divided by the level of total production. This expression, also with the elasticity of substitution, will now be computed to be used later on the development of the model.

$$s \equiv \frac{w_H H}{F} = F_H \frac{H}{F} = f' \frac{1}{Lf} H = \frac{f' \frac{H}{L}}{f}$$

. .

Now we can compute the elasticity of substitution between factors, being the effect of a percentage change in the skilled and unskilled ratio of workers to a percentage change

in the marginal rate of technical substitution or the ratio of unskilled workers versus skilled workers under perfect competition.

$$\sigma \equiv \frac{d\left(\frac{H}{L}\right)}{d\left(\frac{F_L}{F_H}\right)} \frac{F_L}{H} \Leftrightarrow \frac{1}{\sigma} = \frac{d\left(\frac{F_L}{F_H}\right)}{d\left(\frac{H}{L}\right)} \frac{H}{F_H}$$

$$\frac{1}{\sigma} = \frac{\left(f' - f' - \frac{H}{L}f''\right)f' - f''\left(f - \frac{H}{L}f'\right)}{f'^2} \frac{H}{L} \frac{f'}{f - f'\frac{H}{L}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\sigma} = \frac{\left(-\frac{H}{L}f''\right)f' - f''\left(f - \frac{H}{L}f'\right)}{f'} \frac{H}{L} \frac{1}{f - f'\frac{H}{L}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\sigma} = \frac{-f''f}{f'} \frac{H}{L} \frac{1}{f - f'\frac{H}{L}} = \frac{-f''f\frac{H}{L}}{f'\left(f - f'\frac{H}{L}\right)} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\sigma} = -\frac{1}{\frac{f'(f - f'\frac{H}{L})}{f''f\frac{H}{L}}} = -\frac{1}{\frac{f'f\left(1 - \frac{f'}{H}\frac{H}{L}\right)}{f''f\frac{H}{L}}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\sigma} = -\frac{1}{\frac{f'f(1 - s)}{f''f\frac{H}{L}}} = -\frac{f''\frac{H}{L}}{f'(1 - s)} = -\frac{\frac{f''}{\frac{f'}{L}}}{\frac{f''}{1 - s}}$$

If technology is of a constant elasticity of substitution type, such that σ is a constant parameter we can go further on this discussion taking into consideration this parameter:

$$\left\{ \begin{array}{l} \sigma \to 0 \text{ , Leontief Production Function, only producing in} \\ \sigma \to \infty \text{ , The inputs are perfectly substitutes, and there will} \\ \text{be no skilled labour premium} \\ \sigma \to 1 \text{ , Cobb Douglas case} \end{array} \right.$$

The production function used in this model is a general function so we can consider any values for σ . As stated on the literature review, the value for the elasticity of substitution between skilled and unskilled labour is around 2. We shall assume that relative wage of skilled versus unskilled workers is higher than one so that redistributive policies are meaningful. The relative wage is computed as follows,

$$\omega = \frac{w_L}{w_H} = \frac{F_L}{F_H} = \frac{f - \frac{H}{L}f'}{f'} = \frac{f}{f'} - \frac{H}{L}$$
$$\omega = \frac{w_L}{w_H} = \frac{f - \frac{H}{L}f'}{f'} < 1$$

Since this ratio is the inverse of the skill premium, it means that the wage of unskilled workers is lower than the wage of skilled workers. This is what may actually motivate workers to become skilled. Since being a skilled worker requires some investment, money, schooling and so on, if people are willing to make this investment they will be compensated and therefore the skilled workers' wage must be higher than the unskilled workers' one. For simplicity, we assume that the income of the skilled is always higher than the income of the unskilled.

We can also write the wage premium as a function of the skilled labour share of income,

$$\omega = \frac{f}{f'} - \frac{H}{L} = \left(\frac{f}{f'\frac{H}{L}} - 1\right)\frac{H}{L} = \frac{H}{L}\left(\frac{1}{s} - 1\right)$$

And we can state that, as the share of skilled labour income increases, the inverse of the premium decreases.

Another variable that impacts on the premium is the ratio of skilled and unskilled workers.

...

...

$$\frac{\partial \omega}{\partial \left(\frac{H}{L}\right)} = \frac{\left[f' - \left(f' + \frac{H}{L}f''\right)\right]f' - \left(f - \frac{H}{L}f'\right)f''}{f'^2} = \frac{\partial \omega}{\partial \left(\frac{H}{L}\right)} = \frac{f'f' - f'f' - \frac{H}{L}f''f' - ff'' + \frac{H}{L}f'f''}{f'^2} = \frac{\partial \omega}{\partial \left(\frac{H}{L}\right)} = -\frac{ff''}{f'^2} = \frac{(1-s)}{s\sigma} > 0$$

As the ratio of skilled workers increases, the wage paid to them decreases since they become relatively more abundant. So the ratio of unskilled workers' wage over skilled workers increases meaning that the derivative is positive.

And this means that when the ratio of skilled/unskilled workers increases, the premium decreases. This is intuitive. The scarcer is a resource, the higher the value paid to use it. Providing an example, consider we have a small country where the individuals either have one year of schooling or none. The individuals with one year of schooling are said to be the skilled workers while the ones with zero years of schooling are the unskilled ones. The wage of the skilled workers must be higher than the wage of the unskilled ones, if suddenly all of the workers at that country have one year of schooling, which means are all skilled, then the premium tends to zero. The resource will no longer be as scarce as it was and therefore firms wont need to pay more to those skilled workers because all of them actually are skilled now. So as the number of skilled workers

increases the premium they receive decreases, being that the reason why the derivative is positive.

We can also think about this in terms of the elasticity of substitution between factors. If sigma is close to zero, it means that the inputs are almost non-substitutes hence if we have a slight reduction on one of the inputs; its remuneration will increase a lot.

3.1.1. Taxation

In this section, we no longer have the two countries identical in all aspects; we introduce a specification that allows for differentiation between the two. In one of the countries the government decided to introduce a proportional tax on skilled workers' wages so that it could be distributed later, through lump sum subsidies, to the unskilled ones. So this tax/subsidy scheme is a redistributive policy, the government takes from the skilled to deliver to the poorer unskilled workers. We assume both workers and firms know about this policy *a priori* and therefore they take this into account when we open the labour market between the two countries. The tax (τ) is a percentage of the wage paid to the skilled workers and therefore the sum of all the taxes is the government revenue (*G*) which is redistributed to the unskilled workers. The lump sum subsidy per unskilled worker is given by:

$$T = \frac{G}{L}$$
$$T = \tau w_H \frac{H}{L}, \qquad 0 < \tau < 1$$

The profit maximization problem solved by the firms will be the same; since they take into account the gross wages they have to pay to the workers.

$$\max \Pi = Y - (w_H H + w_L L)$$

$$\begin{cases} \frac{\partial \Pi}{\partial L} = 0 \iff F_L = w_L \\\\ \frac{\partial \Pi}{\partial H} = 0 \iff F_H = w_H \end{cases}$$

However, net income received by workers will now be different. In the end we know that there will be a redistributive effect where the skilled workers will be taxed and the unskilled workers will be subsidized. So the total amount collected from the skilled workers will be distributed, evenly, to all the unskilled workers as a subsidy.

The skilled workers' income, tax revenues collected and unskilled workers' income are given by the following expressions:

$$y_{H} = w_{H}(1 - \tau) \iff y_{H} = f'(1 - \tau)$$

$$T = \tau w_{H} \frac{H}{L}$$

$$y_{L} = w_{L} + T \iff y_{L} = w_{L} + \tau w_{H} \frac{H}{L} \iff$$

$$\iff y_{L} = f - \frac{H}{L} f' + \tau \frac{H}{L} f' =$$

$$= f - \frac{H}{L} f'(1 - \tau)$$

3.2. Open Economy

In this section we will analyse the effects on each country caused by the opening of the labour market for the skilled workers. We assume that unskilled workers, by some exogenous reason, are not able to migrate. The initial assumption is that skilled workers can freely move from one country to another, meaning that there are no costs associated with migration. Under perfect labour mobility a non-arbitrage condition arises, stating that at the equilibrium level, skilled workers' wages of the two countries must be equal, otherwise we are not at the equilibrium. This happens because workers will move from

the country with the lowest wage to the one with higher one; this will lead to an adjustment at the wage level. Once wages are equal, workers will no longer have incentives to migrate. Since frontiers are closed for the unskilled ones, their wages at equilibrium do not have to be the same. Of course if taxation were the same in both countries, there would be no incentive for migration of any type of worker.

For this model the tax was introduced in country A, while country B remains with no labour taxation of any kind. If there was no tax on skilled workers' wages, the closed and open economy equilibriums would be the same, since everything else would be equal across countries, namely $\overline{L}^A = \overline{L}^B = \overline{L}$ and $\overline{H}^A + \overline{H}^B = 2\overline{H}$.

Now we introduce a distortion meaning that the skilled workers are now receiving less than before in the country where the tax is being charged, country A. This creates an incentive for skilled workers in country A to migrate to country B.

Let x^i be defined as,

$$x^i = \frac{H^i}{L^i}$$
, $i = A, B$

where H^i and L^i are the amount of skilled and unskilled services used in production, respectively. In an open economy we have that $H^A \neq \overline{H}^A$ and $H^B \neq \overline{H}^B$ while $L^A = \overline{L}^A$ and $L^B = \overline{L}^B$ since no migration is allowed for unskilled workers, with $H^A + H^B = 2\overline{H}$ and $L^A + L^B = 2\overline{L}$. Recalling the following expressions that will be used from now on,

$$s \equiv \frac{f'x}{f} \wedge \frac{1}{\sigma} \equiv -\frac{\frac{f''}{f'}x}{1-s}$$

Before the tax was introduced we had:

$$w_L^A = f(x^A) - f'(x^A) x^A$$
 and $w_H^A = f'(x^A)$

$$w_L^B = f(x^B) - f'(x^B) x^B$$
 and $w_H^B = f'(x^B)$

And since both countries are symmetric we could also say that:

$$w_L^B = w_L^A$$
 and $w_H^B = w_H^A$

With the introduction of a tax we no longer have an equality condition because skilled workers' wages in country A will be different from the ones in country B. In this case since the tax is charged on skilled workers wage, skilled workers will gain in net terms less in country A while unskilled workers will gain more at that same country. If $H^A = \overline{H}^A$ and $H^B = \overline{H}^B$,

$$y_L^B < y_L^A$$
 and $y^B > y_H^A$

The non-arbitrage condition implies that H^A and H^B at equilibrium must be the such that, wages of the (mobile) skilled workers, net of taxes, must be equal across countries:

$$y_H^B = y_H^A$$

So in country A, by applying the tax to skilled workers, the income is lower than the productivity, which means workers receive less than they would be under undistorted labour markets. On the other hand unskilled workers' income is higher than would be under the same undistorted labour markets.

$$F_L^A < y_L^A$$
 and $F_H^A > y_H^A$

So this is what drives migration of skilled workers from country A to B and therefore we can expect that:

$$\frac{dH^B}{d\tau} > 0$$

Meaning that as the tax in country A increases, skilled workers from country A will move to B, increasing the total amount of skilled workers in country B, H^B . This will be shown below.

3.2.1. Effects of Labour Mobility

From the previous section we recognize that the first impact of the redistributive policy on net wages of the skilled workers is to reduce them in country A, hence we expect these workers to move to country B since there is labour mobility between the two countries.

So we can now evaluate the impact of this skilled workers' movement on net wages paid to both types of workers in both countries. *A priori* we can state that in country B, the tax will impact only through the number of skilled workers that migrate. On country A, they will impact through their number, as in country B, and moreover through the collected tax revenue.

Since there is no mobility for the unskilled workers, the values of L^A and L^B are kept constant \overline{L} . On the contrary, the values H^A and H^B will vary.

For the analysis of the impact of the tax on workers, we need the following expressions, being them the incomes and the tax,

$$y_{H}^{A} = (1 - \tau)f'(x^{A})$$
$$y_{L}^{A} = f(x^{A}) - f'(x^{A})x^{A} + T^{A}$$

$$T^{A} = \tau f'(x^{A})x^{A}$$
$$y^{B}_{H} = f'(x^{B})$$
$$y^{B}_{L} = f(x^{B}) - f'(x^{B})x^{B}$$

Due to labour mobility,

$$y_H^A = y_H^B$$

And as stated before, we end up having H^A and H^B such that the skilled workers' wages equal on both countries.

Since,

$$H^A = 2\overline{H} - H^B$$

we have,

$$(1-\tau)f'\left(\frac{2\overline{H}-H^B}{\overline{L}}\right) = f'\left(\frac{H^B}{\overline{L}}\right)$$

This expression implicitly determines H^B .

We will now consider the effects of an increase of the tax rate on skilled workers wage on country A taking into account induced migration. So taking into account the nonarbitrage condition we can state that,

$$f''(x^A)\left(-\frac{1}{\overline{L}}\right)dH^B\left(1-\tau\right) - f'(x^A)d\tau = f''(x^B)\left(-\frac{1}{\overline{L}}\right)dH^B \Leftrightarrow$$

$$\Leftrightarrow \frac{dH^B}{\overline{L}} [f''(x^B) + f''(x^A)(1-\tau)] = -f'(x^A)d\tau \Leftrightarrow$$
$$\Leftrightarrow dH^B \left[\frac{f''(x^B)}{f'(x^A)} \frac{1}{\overline{L}} + \frac{f''(x^A)}{f'(x^A)} \frac{1}{\overline{L}} (1-\tau) \right] = -d\tau \Leftrightarrow$$

From the non arbitrage condition,

$$f'(x^{A}) = \frac{f'(x^{B})}{(1-\tau)}$$
$$\Leftrightarrow \frac{dH^{B}}{H^{B}} \left[\frac{f''(x^{B})}{f'^{(x^{B})}} \frac{H^{B}}{\overline{L}} + \frac{f''(x^{A})}{f'^{(x^{A})}} \frac{H^{A}}{\overline{L}} \frac{H^{B}}{H^{A}} \right] = -\frac{d\tau}{\tau} \frac{\tau}{(1-\tau)}$$

As shown in section 3.1.,

$$\frac{f''(x^i)}{f'(x^i)} = -\frac{1-s^i}{\sigma^i}, i = A, B$$

$$\begin{split} \frac{dH^B}{H^B} & \left[\frac{1-s^B}{\sigma^B} + \frac{1-s^A}{\sigma^A} \frac{H^B}{H^A} \right] \frac{(1-\tau)}{\tau} = \frac{d\tau}{\tau} \Leftrightarrow \\ \frac{dH^B}{H^B} & \left[\frac{\sigma^A (1-s^B) + \sigma^B (1-s^A) \frac{H^B}{H^A}}{\sigma^B \sigma^A} \right] \frac{(1-\tau)}{\tau} = \frac{d\tau}{\tau} \Leftrightarrow \\ \mathcal{E}_{H^B;\tau} & = \frac{\tau}{(1-\tau)} \frac{\sigma^B \sigma^A}{\sigma^A (1-s^B) + \sigma^B (1-s^A) \frac{H^B}{H^A}} > 0 \end{split}$$

This elasticity is positive, meaning that when the tax level is increased the number of skilled workers on country B increases. The intuition for this result is that, the higher the tax, the higher the difference between wages of skilled workers in country A and B

and therefore the higher the number of skilled workers migrants from country A, where the tax is launched, to country B. Since the initial endowments were the same, if a single skilled worker moves from country A to country B, the ratio of workers in country B versus A will be higher than 1.

Before checking what is the effect on the income levels, it is also important to check what happens to government revenue. This because governments want to maximize their revenues and this is done with an optimal taxation scheme, in this model an optimal tax.

The question to be answered here is how does the tax revenue vary with the tax; under which conditions do we have increasing or decreasing revenue.

$$T^{A} = \tau f' \left(\frac{2\overline{H} - H^{B}}{\overline{L}}\right) \frac{2\overline{H} - H^{B}}{\overline{L}}$$

$$\begin{aligned} \frac{dT^{A}}{d\tau} &= f'(x^{A})x^{A} + \tau x^{A}f''(x^{A})\left(-\frac{1}{\overline{L}}\right)\frac{dH^{B}}{d\tau} + \tau f'(x^{A})\left(-\frac{1}{\overline{L}}\right)\frac{dH^{B}}{d\tau} \Leftrightarrow \\ \Leftrightarrow \frac{dT^{A}}{d\tau}\frac{\tau}{T^{A}} &= \frac{\tau f'(x^{A})x^{A}}{T^{A}} - \frac{\tau x^{A}f''(x^{A})}{\tau x^{A}f'(x^{A})}\frac{1}{\overline{L}}\frac{dH^{B}}{d\tau}\tau - \frac{\tau f'(x^{A})\frac{1}{\overline{L}}\frac{dH^{B}}{d\tau}}{\tau x^{A}f'(x^{A})}\tau \Leftrightarrow \\ \Leftrightarrow \frac{dT^{A}}{d\tau}\frac{\tau}{T^{A}} &= 1 - \tau \frac{f''(x^{A})}{f'(x^{A})}\frac{H^{A}}{\overline{L}}\frac{dH^{B}}{d\tau}\frac{1}{H^{A}} - \frac{\tau}{H^{A}}\frac{dH^{B}}{d\tau} \Leftrightarrow \\ \varepsilon_{T^{A};\tau} &= 1 + \frac{1 - s^{A}}{\sigma^{A}}\frac{dH^{B}}{d\tau}\frac{\tau}{H^{B}}\frac{H^{B}}{H^{A}} - \frac{\tau}{H^{B}}\frac{dH^{B}}{d\tau}\frac{H^{B}}{H^{A}} = 1 + \varepsilon_{H^{B};\tau}\frac{H^{B}}{H^{A}}\left[\frac{1 - s^{A}}{\sigma^{A}} - 1\right] = \\ &= 1 + \frac{\tau}{(1 - \tau)}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1 - s^{B}) + \sigma^{B}(1 - s^{A})}\frac{H^{B}}{H^{A}}\frac{H^{B}}{\sigma^{A}}}{H^{A}} - \frac{1}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1 - s^{B}) + \sigma^{B}(1 - s^{A})}\frac{H^{B}}{H^{A}}H^{B}} = 1 + \frac{\tau}{(1 - \tau)}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1 - s^{B}) + \sigma^{B}(1 - s^{A})}\frac{H^{B}}{H^{A}}H^{A}} = 1 + \frac{\tau}{\sigma^{A}}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}} + \frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{H^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1 - s^{B}) + \sigma^{B}(1 - s^{A})}\frac{H^{B}}{H^{A}}H^{A}} = 1 + \frac{\tau}{\sigma^{A}}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}} + \frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1 - s^{B}) + \sigma^{B}(1 - s^{A})}\frac{H^{B}}{H^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{B}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} = 1 + \frac{\tau}{\tau}\frac{\sigma^{A}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} + \frac{\tau}{\tau}\frac{\sigma^{A}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} + \frac{\tau}{\tau}\frac{\sigma^{A}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}H^{A}} + \frac{\tau}{\tau}\frac{\sigma^{A}\sigma^{A}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{\sigma^{A}}\frac{H^{B}}{$$

$$= 1 + \frac{\tau}{(1-\tau)} \frac{\sigma^{B} \frac{H^{B}}{H^{A}} (1-s^{A})}{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}}} - \frac{\tau}{(1-\tau)} \frac{\sigma^{B} \sigma^{A} \frac{H^{B}}{H^{A}}}{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}}}$$
$$= 1 + \frac{\tau}{(1-\tau)} \frac{\sigma^{B} \frac{H^{B}}{H^{A}} (1-s^{A})}{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}}} - \frac{\tau}{(1-\tau)} \frac{\sigma^{B} \sigma^{A} \frac{H^{B}}{H^{A}}}{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}}}$$
$$= \frac{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}} + \frac{\tau}{(1-\tau)} \frac{H^{B}}{H^{A}} \sigma^{B} [(1-s^{A}) - \sigma^{A}]}{\sigma^{A} (1-s^{B}) + \sigma^{B} (1-s^{A}) \frac{H^{B}}{H^{A}}}$$

For the elasticity to be positive we have to ensure that both the numerator and the denominator are either positive or negative. Since the denominator is always positive we only need to ensure that the numerator is also positive.

$$\begin{split} & \mathcal{E}_{T^{A};\tau} > 0 \Leftrightarrow \sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}} + \frac{\tau}{(1-\tau)}\frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A}) - \sigma^{A}] > 0 \\ & \Leftrightarrow \frac{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}(1-\tau) + \tau\frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A}) - \sigma^{A}]}{(1-\tau)} > 0 \end{split}$$

And, since $(1 - \tau) > 0$, for the ratio to be positive the numerator must be also positive.

$$\begin{split} \sigma^{A}(1-s^{B}) &+ \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}(1-\tau) + \tau\frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A})-\sigma^{A}] > 0 \Leftrightarrow \\ \Leftrightarrow \sigma^{A}(1-s^{B}) &+ \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}} \\ &- \tau\left(\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}} - \frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A})-\sigma^{A}]\right) > 0 \Leftrightarrow \\ \Leftrightarrow \tau < \frac{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}} - \frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A})-\sigma^{A}]}{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}} - \frac{H^{B}}{H^{A}}\sigma^{B}[(1-s^{A})-\sigma^{A}]} \equiv \bar{\tau} \end{split}$$

This is in line with the Laffer curve. This curve expresses the variation of the revenue with the tax level charged on the country. It is positively sloped initially, according to the model until $\bar{\tau}$, it has a maximum at $\bar{\tau}$ and then it starts to decrease after the $\bar{\tau}$. This is what happens in many situations, the taxes start to increase and it means the revenue is also increasing but there is a certain limit where the incentives to produce anything that can be taxable do not exist. So there must be a tax that maximizes this revenue:

$$\begin{cases} \mathcal{E}_{T^{A};\tau} < 0 \Longrightarrow \bar{\tau} < \tau \\ \mathcal{E}_{T^{A};\tau} = 0 \Longrightarrow \bar{\tau} = \tau \\ \mathcal{E}_{T^{A};\tau} > 0 \Longrightarrow \bar{\tau} > \tau \end{cases}$$

Accordingly, the sign of $\mathcal{E}_{T^A;\tau}$ might be either positive or negative. This is due to the double effect the tax has on the tax revenue. The direct effect has a positive impact, the higher the tax, the most we collect out of the skilled workers' wages. The indirect impact is that linked to the migration effect. The higher is the tax the lower the number of skilled workers in the country where we have the tax because they will tend to migrate from country A to country B. The sign of the elasticity depends on which effect dominates. If the direct effect dominates, the elasticity is positive; if the indirect dominates the elasticity is negative; if by any chance the two effects have the exact same impact, the elasticity will be zero, meaning that what the tax is able to increase on the tax revenue, migration compensates by the same amount.

Of course it does not make any sense the government setting a tax higher than $\overline{\tau}$ since this would mean that government revenues would be lower than with a lower tax rate.

A sufficient condition for the elasticity to be positive is that,

$$1 - s^{A} - \tau \sigma^{A} > 0 \Leftrightarrow$$
$$\Leftrightarrow \tau < \frac{1 - s^{A}}{\sigma^{A}} \equiv \tilde{\tau}$$

From this definition of $\tilde{\tau}$, we can state that the higher the elasticity of substitution the lower the value for the tax that maximizes government revenue. The same happens with the share of skilled labour income. This is intuitive, keeping everything else constant, the higher the taxable income the lower the tax that maximizes government spending. If there is a larger fraction of skilled workers, we will be able to collect the same amount with a lower tax. Regarding the elasticity of substitution, and keeping everything else constant, if it is high, it means that for a firm it is easy to substitute one input by the other, and this has a negative impact on the tax.

To better understand the impacts of the elasticity of substitution we can focus on the analysis of the two extreme cases.

If we had a Leontief production function, the sigma would take a value of zero meaning that the tax that maximizes government revenues would tend to 100%. In this case, the inputs will be used in almost fixed proportions meaning that an increase on the tax level would not have significant impact on skilled workers reduction. Since the tax revenue is the product of the tax and the number of workers, if, when we increase the tax the number of workers remains constant, then the tax revenue is always increasing. Therefore the condition will be $\tau < \tilde{\tau} = 1$.

If the production function were such that inputs were perfect substitutes, meaning that sigma would be infinite, the tax that maximizes government revenues would be zero. This because there would be no distinction between the two inputs and therefore it would be very easy to change one input for the other.

Using the data stated in the literature review we can reach some further conclusion regarding the sign of the elasticity that previously was not well defined. If we consider the elasticity of substitution to be around a value equal to 2 and the shares of skilled labour income and unskilled labour income to be values around 0,55 and 0,45, respectively than we can immediately state that:

$$\frac{1-s^A}{\sigma^A} \equiv \tilde{\tau} = \frac{1-0.55}{2} = 0.225 = 22.5\%$$

$$\begin{cases} \mathcal{E}_{T^{A};\tau} < 0 \Longrightarrow \tau > 0,225 \\ \mathcal{E}_{T^{A};\tau} = 0 \Longrightarrow \tau = 0,225 \\ \mathcal{E}_{T^{A};\tau} > 0 \Longrightarrow \tau < 0,225 \end{cases}$$

This tax depends on two variables, the share of skilled labour income and the elasticity of substitution. It depends negatively on both of them. The higher the skilled labour income the lower the optimal tax, because we have a higher proportion of skilled income and therefore the tax can be lower to achieve the optimal collectable revenue to be then distributed to the unskilled. So as long as the tax is no higher than 22,5%, revenues increase with the tax.

3.2.2. Effects of Taxation

On this section we want to describe the effects on workers' wages, both skilled and unskilled, from both countries, A and B, caused by the tax.

In country B we have an immediate result while on country A, as in the tax revenue, we have a combination of two contrary effects.

3.2.2.1. Effects of taxation on Country B

The effect of taxation on the unskilled workers' wage is derived on the following expression:

$$\frac{\partial y_L^B}{\partial \tau} = \frac{\partial y_L^B}{dH^B} \frac{dH^B}{d\tau} = \left[f'(x^B) \frac{1}{\overline{L}^B} - \left[f''(x^B) \frac{1}{\overline{L}^B} x^B + f'(x^B) \frac{1}{\overline{L}^B} \right] \right] \frac{dH^B}{d\tau} = \left[-f''(x^B) \frac{1}{\overline{L}^B} x^B \right] \frac{dH^B}{d\tau} > 0$$

For an easier understanding we can write the previous expressions with elasticities.

$$\begin{aligned} \frac{\partial y_L^B}{\partial \tau} \frac{\tau}{y_L^B} &= \left[-f^{\prime\prime}(x^B) \frac{1}{L^B} x^B \right] \frac{dH^B}{d\tau} \frac{\tau}{f(x^B) - f^\prime(x^B) x^B} \frac{H^B}{H^B} \Leftrightarrow \\ &\Leftrightarrow \mathcal{E}_{y_L^B,\tau} = -\frac{f^{\prime\prime}(x^B) x^B}{f(x^B) - f^\prime(x^B) x^B} x^B \mathcal{E}_{H^B,\tau} = \\ &= -\frac{f^{\prime\prime}(x^B) x^B}{\frac{f^\prime(x^B)}{x^B} - f^\prime(x^B)} \mathcal{E}_{H^B,\tau} = \\ &= -\frac{\frac{f^{\prime\prime}(x^B) x^B}{f^\prime(x^B) x^B} - 1}{\mathcal{E}_{H^B,\tau}} \mathcal{E}_{H^B,\tau} = \\ &= -\frac{-\frac{1 - s^B}{\sigma^B}}{\frac{f(x^B)}{f^\prime(x^B) x^B} - 1} \mathcal{E}_{H^B,\tau} = \\ &= -\frac{-\frac{1 - s^B}{\sigma^B}}{\frac{1 - s^B}{s^B} - 1} \mathcal{E}_{H^B,\tau} = \\ &= -\frac{\frac{1 - s^B}{\sigma^B}}{\frac{1 - s^B}{s^B}} \mathcal{E}_{H^B,\tau} = \end{aligned}$$

$$\mathcal{E}_{y_L^B,\tau} = \frac{s^B}{\sigma^B} \mathcal{E}_{H^B,\tau} > 0$$

For unskilled workers that live in country B the tax on country A impacts positively on their wages. As the tax increases, skilled workers on country A move to country B and therefore the relative number of unskilled workers decreases in B, leading to a relative scarcity of unskilled workers and therefore to a wage increase. We can state that taxation on country's A skilled workers benefits the unskilled workers on country B.

The effect of taxation on the skilled workers' wage is derived on the following expression:

$$\frac{\partial y_{H}^{B}}{\partial \tau} = \frac{\partial y_{H}^{B}}{\partial H^{B}} \frac{dH^{B}}{d\tau} = f^{\prime\prime}(x^{B}) \frac{1}{\overline{L}^{B}} \frac{dH^{B}}{d\tau} < 0$$

Again, for an easier understanding we can write the previous expressions with elasticities.

$$\frac{\partial y_{H}^{B}}{\partial \tau} \frac{\tau}{y_{H}^{B}} = f''(x^{B}) \frac{1}{\overline{L}^{B}} \frac{dH^{B}}{d\tau} \frac{\tau}{f'(x^{B})} \frac{H^{B}}{H^{B}} \Leftrightarrow$$

$$\Leftrightarrow \mathcal{E}_{y_{H,\tau}^{B}} = -\frac{f''(x^{B})x^{B}}{f(x^{B}) - f'(x^{B})x^{B}} x^{B} \mathcal{E}_{H^{B},\tau} =$$

$$= \frac{f''(x^{B})x^{B}}{f'(x^{B})} \mathcal{E}_{H^{B},\tau} =$$

$$= -\frac{1 - s^{B}}{\sigma^{B}} \mathcal{E}_{H^{B},\tau}$$

$$\mathcal{E}_{y_{H,\tau}^{B}} = -\frac{1 - s^{B}}{\sigma^{B}} \mathcal{E}_{H^{B},\tau} < 0$$

On the other hand, for skilled workers in country B, the wages are expected to decrease. The amount of skilled workers in country B will increase, due to migration movements from country A, a tax consequence, and therefore skilled workers will be less scarce and their wages will be reduced.

It is also interesting to understand that the tax policy in country A contributes to the reduction of the difference between skilled workers' wage and unskilled workers' wage in country B. Recalling the concept of skill premium we can state that it decreases in gross terms in country B. This is true because wages for the skilled are reduced while wages for the unskilled workers are increase, since the gap between wages is lower then the skill premium is also lower.

3.2.2.2. Effects of Taxation on Country A

The effect of taxation on the skilled workers' wage is derived on the following expression:

$$\begin{split} \frac{\partial y_{H}^{A}}{\partial \tau} &= \frac{\partial y_{H}^{A}}{\partial \tau} + \frac{\partial y_{H}^{A}}{\partial H^{B}} \frac{dH^{B}}{d\tau} = \\ \frac{\partial y_{H}^{A}}{\partial \tau} \frac{\tau}{y_{H}^{A}} &= \frac{\tau}{y_{H}^{A}} \left[\frac{\partial y_{H}^{A}}{\partial \tau} + \frac{\partial y_{H}^{A}}{\partial H^{B}} \frac{dH^{B}}{d\tau} \right] = \\ &= \left[-f'(x^{A}) + (1-\tau) \left[f''(x^{A}) \left(-\frac{1}{L^{A}} \right) \right] \frac{dH^{B}}{d\tau} \right] \frac{\tau}{(1-\tau)f'(x^{A})} \\ &= -\frac{\tau}{(1-\tau)} + \frac{f''(x^{A})}{f'(x^{A})} \left(-\frac{H^{B}}{L^{A}} \right) \mathcal{E}_{H^{B},\tau} \\ &= -\frac{\tau}{(1-\tau)} - \frac{\tau}{(1-\tau)} \frac{f''(x^{A})x^{A}}{f'(x^{A})} \frac{H^{B}}{H^{A}} \frac{\sigma^{B}\sigma^{A}}{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})} \frac{H^{B}}{H^{A}} \\ &= -\frac{\tau}{(1-\tau)} \left[1 - \frac{\sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}}{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}} \right] \\ &= -\frac{\tau}{(1-\tau)} \left[\frac{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}}{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}} \right] \end{split}$$

$$\mathcal{E}_{y_{H,\tau}^{A}\tau} = -\frac{\tau}{(1-\tau)} \left[\frac{\sigma^{A}(1-s^{B})}{\sigma^{A}(1-s^{B}) + \sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}} \right] < 0$$

For this group of workers in country A we have two effects. A direct one, that results from the proportional tax on wages, and an indirect one, which results from the increase

in gross income due to emigration of skilled workers, making them scarcer and therefore increasing wages for the ones that remain on country A. Since we have an open economy and there is mobility for the skilled workers, we know that skilled workers' wages across countries will be equal. Since skilled workers' wages in country B decrease, and there is a non-arbitrage condition, wages of skilled workers in country A will also have to decrease.

The effect of taxation on the unskilled workers' income is derived on the following expression:

$$\frac{\partial y_L^A}{\partial \tau} = \frac{\partial y_L^A}{\partial \tau} + \frac{\partial y_L^A}{\partial H^B} \frac{dH^B}{d\tau}$$

For this derivative, as for the elasticity, three cases might happen. In here, the initial first term is always positive while the second one is always negative. The first fraction is the increase from the subsidy on the wage generated by the tax; it is the redistribution from the skilled to the unskilled workers. The second term is the indirect impact through migration: when skilled workers migrate, unskilled labour becomes relatively more abundant and therefore their wage is revised down. If we believe that the latter is stronger than the first, we would have:

$$\frac{\partial y_L^A}{\partial \tau} < \frac{\partial y_L^A}{\partial H^A} \frac{d H^A}{d \tau} \Rightarrow \frac{\partial y_L^A}{\partial \tau} < 0$$

If we believe that the contrary is true, we would have the direct impact stronger than the indirect one and hence:

$$\frac{\partial y_L^A}{\partial \tau} > \frac{\partial y_L^A}{\partial H^A} \frac{d H^A}{d \tau} \Rightarrow \frac{\partial y_L^A}{\partial \tau} > 0$$

We have a third possible scenario, when both effects have the exact same impact, in this case there is no impact on wages of unskilled workers in country A caused by the tax.

$$\frac{\partial y_L^A}{\partial \tau} = \frac{\partial y_L^A}{\partial H^A} \frac{dH^A}{d\tau} \Rightarrow \frac{\partial y_L^A}{\partial \tau} = 0$$

Since there are three possible scenarios, the following expressions allow understanding under which conditions we might have one or another.

$$\begin{aligned} \frac{\partial y_L^A}{\partial \tau} &= \frac{\partial y_L^A}{\partial \tau} + \frac{\partial y_L^A}{\partial H^B} \frac{dH^B}{d\tau} = \\ &= f'(x^A)x^A + \left[f'(x^A) \left(-\frac{1}{\overline{L}^A} \right) - (1-\tau) \left[f''(x^A) \left(-\frac{1}{\overline{L}^A} \right) x^A + f'(x^A) \left(-\frac{1}{\overline{L}^A} \right) \right] \right] \frac{dH^B}{d\tau} \\ &= f'(x^A)x^A + \left[f'(x^A) \left(-\frac{1}{\overline{L}^A} \right) - (1-\tau)f''(x^A) \left(-\frac{1}{\overline{L}^A} \right) x^A \right. \\ &- (1-\tau)f'(x^A) \left(-\frac{1}{\overline{L}^A} \right) \right] \frac{dH^B}{d\tau} \\ &= f'(x^A)x^A + \left[(1-\tau)f''(x^A) \left(\frac{1}{\overline{L}^A} \right) x^A - \tau f'(x^A) \left(\frac{1}{\overline{L}^A} \right) \right] \frac{dH^B}{d\tau} \\ &\text{Note that: } y_L^A = f(x^A) \left(1 - \frac{f'(x^A)}{f(x^A)} x^A (1-\tau) \right) = f(x^A) (1-s^A(1-\tau)) \end{aligned}$$

So the previous derivative can be written in elasticity,

$$\mathcal{E}_{y_{L}^{A},\tau} = \frac{\partial y_{L}^{A}}{\partial \tau} \frac{\tau}{y_{L}^{A}} =$$
$$= \frac{\tau}{y_{L}^{A}} \left[f'(x^{A})x^{A} + f'(x^{A})x^{A} \left[(1-\tau)\frac{f''(x^{A})}{f'(x^{A})} \left(\frac{1}{\overline{L}^{A}}\right) - \tau \left(\frac{1}{H^{A}}\right) \right] \frac{dH^{B}}{d\tau} \right] =$$

$$\begin{split} &= \frac{\tau}{y_L^A} \bigg[f'(x^A) x^A + f'(x^A) x^A \bigg[\frac{(1-\tau)}{\tau} \frac{f''(x^A)}{f'(x^A)} \bigg(\frac{H^B}{\overline{L}^A} \bigg) - \bigg(\frac{H^B}{H^A} \bigg) \bigg] \mathcal{E}_{H^B,\tau} \bigg] \\ &= \frac{\tau}{(1-s^A(1-\tau))} \frac{f'(x^A) x^A}{f(x^A)} \bigg[1 + \bigg[\frac{(1-\tau)}{\tau} \frac{f''(x^A) x^A}{f'(x^A)} \bigg(\frac{H^B}{H^A} \bigg) - \bigg(\frac{H^B}{H^A} \bigg) \bigg] \mathcal{E}_{H^B,\tau} \bigg] \\ &= \frac{\tau s^A}{(1-s^A(1-\tau))} \bigg[1 + \bigg[-\frac{(1-\tau)}{\tau} \frac{1-s^A}{\sigma^A} \bigg(\frac{H^B}{H^A} \bigg) - \bigg(\frac{H^B}{H^A} \bigg) \bigg] \mathcal{E}_{H^B,\tau} \bigg] \\ &= \frac{\tau s^A}{(1-s^A(1-\tau))} \bigg[1 - \frac{1-s^A}{\sigma^A} \bigg(\frac{H^B}{H^A} \bigg) \frac{\sigma^B \sigma^A}{\sigma^A(1-s^B) + \sigma^B(1-s^A)} \frac{H^B}{H^A} \bigg] \\ &- \bigg(\frac{H^B}{H^A} \bigg) \frac{\tau}{(1-\tau)} \frac{\sigma^B \sigma^A}{\sigma^A(1-s^B) + \sigma^B(1-s^A)} \frac{H^B}{H^A} \bigg] = \end{split}$$

$$=\frac{\tau s^{A}}{\left(1-s^{A}(1-\tau)\right)}\left[1-\frac{\left(\frac{H^{B}}{H^{A}}\right)\frac{\tau}{(1-\tau)}\sigma^{B}\sigma^{A}+\sigma^{B}(1-s^{A})\left(\frac{H^{B}}{H^{A}}\right)}{\sigma^{A}(1-s^{B})+\sigma^{B}(1-s^{A})\frac{H^{B}}{H^{A}}}\right]$$

The first term of the previous expression is always positive. The sign of the elasticity then depends on the sign of the second term. To decide whether what is in brackets is positive or negative, we have to understand under which conditions the second fraction is bigger or lower than 1. If it is bigger, the elasticity is negative, otherwise it is positive. If it is equal to 1 then the elasticity is zero meaning that a marginal change on the tax rate has no impact on the incomes of unskilled workers on country A.

$$\begin{cases} \mathcal{E}_{y_L^A,\tau} < 0 \Longrightarrow \left(\frac{H^B}{H^A}\right) \frac{\tau}{(1-\tau)} \sigma^B > (1-s^B) \\ \mathcal{E}_{y_L^A,\tau} = 0 \Longrightarrow \left(\frac{H^B}{H^A}\right) \frac{\tau}{(1-\tau)} \sigma^B = (1-s^B) \\ \mathcal{E}_{y_L^A,\tau} > 0 \Longrightarrow \left(\frac{H^B}{H^A}\right) \frac{\tau}{(1-\tau)} \sigma^B < (1-s^B) \end{cases}$$

From the previous expression we can understand that this elasticity depends on more than one variable. Keeping everything else constant, if migration occurs in a large fraction of the skilled workers, this elasticity is more likely to be negative – if all skilled workers migrate, limit case, there would be no tax revenue and the unskilled would have no subsidy to receive. The same reasoning applies for the tax, if the tax is high, it is more likely for the elasticity to be negative – the reasoning in here is linked to the indirect effect of taxation, the higher the tax, the higher the migration flow from A to B and therefore the less collectable revenue the lower the subsidy to be then distributed among unskilled workers.

On country A, the tax on skilled workers is used to finance a lump-sum subsidy to the unskilled workers. A question arises. When the tax is introduced and we have an open economy this creates an incentive for skilled workers to migrate. When they migrate we have less people to tax and therefore we will collect less to deliver to the unskilled. On the other hand the wages of these skilled workers will be revised upwards meaning we have higher wages to tax.

A possible extension that could introduce some further discussion on the sign of this derivative could be made through the introduction of another variable to help on measure the two effects stated above. If we consider, for example, labour market rigidity we could say that in countries where the labour market is rigid and therefore wages are sticky, migration takes long to occur. Being this, the tax revenue will be higher and therefore the derivative is positive. When we introduce a tax which objective is to redistribute income, if the country is an open economy with a rigid labour market then this might be an effective policy. What can also be done is a dynamic model where things evolve over time. We might say that initially migration does not occur and therefore the impact of a rising tax on unskilled workers wage is positive, but after a certain amount of time, skilled workers start moving and here we have another effect. The equilibrium point will always depend on which of two effects described above is the dominant one.

3.3. Optimal Taxation

On the previous section we defined the tax that maximizes the government revenue, which could be a criteria for optimality and the value was, $\tilde{\tau} = 22,5\%$.

Regarding this other elasticity, $\mathcal{E}_{y_L^A,\tau}$ we can only simplify it a bit more. It is not possible to come up with an optimal value for the tax because there is not enough specification for all the parameters needed in both economies. We can state that, the ratio skilled in country B / skilled in country A is always higher than 1. This is true because the initial assumption states that countries are equal in all parameters at the beginning meaning that initial endowments of workers are equal. Once the tax is introduced and frontiers are open we know that the skilled workers in A will move to B leading to a ratio higher than 1.

Recalling the expression from the previous section,

$$\begin{cases} \mathcal{E}_{y_{L,\tau}^{A}} < 0 \Longrightarrow \left(\frac{H^{B}}{H^{A}}\right) \frac{\tau}{(1-\tau)} \sigma^{B} > (1-s^{B}) \\ \mathcal{E}_{y_{L,\tau}^{A}} = 0 \Longrightarrow \left(\frac{H^{B}}{H^{A}}\right) \frac{\tau}{(1-\tau)} \sigma^{B} = (1-s^{B}) \\ \mathcal{E}_{y_{L,\tau}^{A}} > 0 \Longrightarrow \left(\frac{H^{B}}{H^{A}}\right) \frac{\tau}{(1-\tau)} \sigma^{B} < (1-s^{B}) \end{cases}$$

We can simplify the previous expressions to better understand the taxation.

$$\begin{split} \mathcal{E}_{y_{L,\tau}^{A}} &< 0 \Longrightarrow \left(\frac{H^{B}}{H^{A}}\right) \frac{\tau}{(1-\tau)} \sigma^{B} > (1-s^{B}) \Leftrightarrow \frac{1-\tau}{\tau} < \frac{\left(\frac{H^{B}}{H^{A}}\right) \sigma^{B}}{(1-s^{B})} \Leftrightarrow \\ & \Leftrightarrow \frac{1}{\tau} < \frac{\left(\frac{H^{B}}{H^{A}}\right) \sigma^{B} + 1 - s^{B}}{(1-s^{B})} \Leftrightarrow \tau > \frac{1-s^{B}}{\left(\frac{H^{B}}{H^{A}}\right) \sigma^{B} + 1 - s^{B}} \end{split}$$

$$\begin{cases} \mathcal{E}_{y_L^A,\tau} < 0 \Longrightarrow \tau > \frac{1-s^B}{\left(\frac{H^B}{H^A}\right)\sigma^B + 1 - s^B} \\ \mathcal{E}_{y_L^A,\tau} = 0 \Longrightarrow \tau = \frac{1-s^B}{\left(\frac{H^B}{H^A}\right)\sigma^B + 1 - s^B} \\ \mathcal{E}_{y_L^A,\tau} > 0 \Longrightarrow \tau < \frac{1-s^B}{\left(\frac{H^B}{H^A}\right)\sigma^B + 1 - s^B} \end{cases}$$

And this then depends on both the tax level and the amount of skilled workers at the end on each country.

After computing these expressions mentioned above we can define what is an optimal tax scheme. But the definition of optimality is not trivial since there is not a single objective; we can have several different definitions of optimality depending on the concepts we choose to be optimal. In here, we will assume that the problem under the optimality condition is the welfare of the unskilled workers, which is measured by their income level. This follows the line that governments in some developed and mostly developing countries do have problems on the public sector due to lack of resources partially explained by the reduction of the skilled workers. So the best way, not to prevent the problem but to solve this, is to identify the optimal tax that does maximize the unskilled workers income. This optimality criteria is close to the Rawlsian Social Welfare Function, maximizes the utility of the poorest. In this study we focus on the utility of a worker. There are other alternatives that could be implemented in this study such as the Utilitarian Criteria that measures welfare according to the maximization of the sum of the utilities of each and every agent of the economy. In this study we opted for the Rawlsian one because it is a criteria that goes in line with the developments of the model. This model can be used with this purpose, to understand what happens to the ones with fewer possibilities, in this case the no migration option.

Another interesting reasoning is to think what would happen under a closed economy. If we had a closed economy the optimal taxation to be implemented by the government would be to set both income levels equal. Meaning that the amount taken from the skilled workers would be the necessary to set the unskilled worker's income equal to the skilled ones. Recalling that,

$$\frac{\partial y_L^A}{\partial \tau} = 0 \iff \tau = \frac{1 - s^B}{\left(\frac{H^B}{H^A}\right)\sigma^B + 1 - s^B}$$

Now to reach the optimal tax we have to assume values for the parameters. The elasticity of substitution is around 2. The share of skilled labour income is around 0,55 and the ratio of skilled workers between the two countries knowing that it is between 1, when no worker moves, and 2, when all workers move. Since it is not possible, given the analysis, to reach a single value for the tax we, we will present possible intervals for it.

The following tables assume an elasticity of substitution equal to 1, 1,5 and 2. 1 is the lowest value considered in this research. The optimal tax rates vary between 0,21 and 0,41. The lower values assume a very high share skilled labour income and associated with also a very high skilled workers ratio this would imply low taxation. Because the taxable wages would be a large value leading to low taxes charged by governments. This reasoning applies independently of the value of the elasticity of substitution meaning that this interpretation can be made to all the three tables below.

Skilled workers ratio	1	1,1	1,2	1,3	1,4	1,5
0,3	0,41	0,39	0,37	0,35	0,33	0,32
0,35	0,39	0,37	0,35	0,33	0,32	0,3
0,4	0,38	0,35	0,33	0,32	0,3	0,29
0,45	0,35	0,33	0,31	0,3	0,28	0,27
0,5	0,33	0,31	0,29	0,28	0,26	0,25
0,55	0,31	0,29	0,27	0,26	0,24	0,23
0,6	0,29	0,27	0,25	0,24	0,22	0,21

Table 1: Optimal Taxation, elasticity of substitution: 1

Skilled workers ratio	1	1,1	1,2	1,3	1,4	1,5
0,3	0,32	0,3	0,28	0,26	0,25	0,24
0,35	0,3	0,28	0,27	0,25	0,24	0,22
0,4	0,29	0,27	0,25	0,24	0,22	0,21
0,45	0,27	0,25	0,23	0,22	0,21	0,2
0,5	0,25	0,23	0,22	0,2	0,19	0,18
0,55	0,23	0,21	0,2	0,19	0,18	0,17
0,6	0,21	0,2	0,18	0,17	0,16	0,15

Table 2: Optimal Taxation, elasticity of substitution: 1,5

Skilled workers ratio	1	1,1	1,2	1,3	1,4	1,5
0,3	0,26	0,24	0,23	0,21	0,2	0,19
0,35	0,25	0,23	0,21	0,2	0,19	0,18
0,4	0,23	0,21	0,2	0,19	0,18	0,17
0,45	0,22	0,2	0,19	0,17	0,16	0,15
0,5	0,2	0,19	0,17	0,16	0,15	0,14
0,55	0,18	0,17	0,16	0,15	0,14	0,13
0,6	0,17	0,15	0,14	0,13	0,13	0,12

Table 3: Optimal Taxation, elasticity of substitution: 2

The first table presented starts with a limit value for the elasticity of substitution between factors. The second and the third tables have values for the elasticity of substitution closer to the values used on previous literature. If we recap the value for $\tilde{\tau}$, it was (given the statistics from the literature review) a value around the 22,5%. According to the tables this value is not far from the hypothesis stated.

4. CONCLUSIONS

With the presented model we can derive some relevant conclusions in accordance to the main goal of this research project.

To facilitate the reasoning, below are the conditions reached in the previous section, the sum up of the effect of the tax on the several wages we have:

$$\begin{cases} \frac{\partial y_L^B}{\partial \tau} > 0\\\\ \frac{\partial y_H^B}{\partial \tau} < 0\\\\ \frac{\partial y_H^A}{\partial \tau} < 0\\\\ \frac{\partial y_L^A}{\partial \tau} < = > 0 \end{cases}$$

Since the markets are open for the skilled workers, we immediately know that if in country B the wage of the skilled workers is decreasing, then for the skilled workers in country A the wage is also going to decrease. And this happens in country B because the relative amount of skilled workers increases and therefore their wages decrease. The contrary is true for the unskilled workers in country B, they become relatively scarcer and therefore their wages increase.

When we move to the unskilled workers' wage, the same reasoning is not applicable because frontiers are closed for this type of workers.

So at the country level it is difficult to state whether the country as a whole is better or worst off. It does not only depend on the sign of the derivatives but also on the criteria we use. We can state that the welfare is a sum of all workers welfare or use another valid criteria. In this case it will depend not only on the parameters used on the tables, elasticity of substitution, share of skilled labour income and skilled workers ratio, but also on the amount of workers the country ends up with. So the focus of this research is not on the country's utilitarian welfare but on the optimal tax that maximizes the wage of an unskilled worker.

To reach some relevant value for the optimal tax, we used a range of different values for the parameters on which the tax depended.

A possible range of values for the tax level that maximizes the wage of unskilled workers in country A is 0,15 and 0,30. These values of taxes are in accordance to the values reported by OECD (2012) on the OECD countries. So we could state that with a tax on skilled workers' wage around 22% the country, with an open frontier exclusively on this labour market, would be maximizing the net income of its unskilled workers.

Another interesting conclusion is that the political tax scheme implemented on country A ends up being favourable for country B, it increases the income of the unskilled and reduces the gross wage *premium*. Meaning that income inequality is reduced on country B.

4.1. Limitations and suggestions for further research

The model used in this research is a very simple model. The main limitation is to achieve an absolute conclusion on whether the income of unskilled workers at the country where the tax is implemented decreases or increases with the tax. It is nevertheless a very good model to understand the economic mechanisms behind the final result. The model could incorporate more variables such as unemployment, imperfect labour markets or imperfect mobility.

Another suggestion would be to introduce some country specific data to compare the different fiscal systems implemented around Europe, for example.

Another relevant aspect, directly linked to the topic studied in here is public finance. One of the main concerns, especially in Portugal, is the available budget of the government to invest on education. It would be interesting to understand how much does a government expend per university student and how many years, on average, a student must work to repay the investment made by the country. This would be a direct measure of the cost and benefit of raising skilled workers.

5. REFERENCES

BEHAR, A. (2009) Directed technical change, the elasticity of substitution and wage inequality in developing countries, *Department of Economics, University of Oxford*.

BORDO, M., TAYLOR, A., WILLIAMSON, J., (eds.) (2003) Globalization in Historical Perspective. *University of Chicago Press*.

CREMER, H., FOURGEAUD, V., LEITE-MONTEIRO, M., MARCHAND, M. and PESTIEAU, P. (1996) Mobility and Redistribution: a Survey, *Public Finance*, *51: 325–352*

EUROSTAT (2015) Unemployment statistics [2015] Available from:

http://ec.europa.eu/eurostat/statisticsexplained/index.php/Unemployment_statistics#You th_unemployment_trends [Accessed: 8 September 2015]

EUROPEAN COMMISSION (2007) Employment in Europe 2007, Luxembourg

HAMILTON, B. and WHALLEY, J. (1982) Efficiency and distributional implications of global restrictions on labour mobility. *Journal of Development Economics 14 (1984)* 61-75 North-Holland.

JEAN et al. (2007) Migration in OECD Countries: Labour Market impact and integration issues. Economics Department Working Papers No. 562, Organization for Economic Co-operation and Development

NGUYEN, C. (2006) Brain Drain or Brain Gain? The Revitalization of a Slow Death, Can Tho University

OECD (2013) World Migration in Figures

OECD (2012) Statutory personal income tax rate at average wage earnings

PORDATA (2015) *Emigrantes total e tipo - Portugal*. [2013] Available from: http://www.pordata.pt/Portugal/Emigrantes+total+e+por+tipo-21 [Accessed: 16 August 2015]

WILDASIN, D. E. (2012) Fiscal Competition, Redistributive Transfers, and Factor Mobility in a Dynamic Context, *University of Kentucky Lexington, KY 40506-0027* USA

41

WILDASIN, D. E. (2014) Human Capital Mobility: Implications for Efficiency, Income Distribution, and Policy, *Discussion Paper No. 8199 May 2014 IZA*, Germany in Gérard, M. and Ubelmesser, S. (eds), The Mobility of Students and the Highly Skilled: Implications for Education Financing and Economic Policy. MIT Press: Cambridge, MA, December 2014.