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AcF 706 – Assessing Default Risk of a Public Company

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Abstract

The dissertation presents the determinants of credit spread, evolution of credit risk modeling and empirically evidence over the period, as well as models based on accounting information. The study explores performance of the firm with accounting and share price information. It also evaluates the predictive of two credit risk models: Merton (1974) and Leland (1994), using accounting and market variables. The finding is that both models tend to underestimate credit risk spreads, though most of the previous literature points out that Leland model usually overestimates credit spread. Further research may focus on market and industrial component of models.

Keywords: credit spread, corporate bankruptcy, credit risk models

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1. Introduction

Credit risk arises when the firm fails to meet its financial liabilities. For public companies, default probabilities and credit spread changes are of concerns to investors, creditors as well as company itself for financing strategy and risk management. Suo and Wang (2005) points out that the significance of credit risk factor is reinforced in the Basel II, which requires financial institutions to apply credit risk models that are empirically validated.

Merton (1974) proposed the first structural model to estimate the default probabilities and claimed that default is triggered at the time the assets fall below the outstanding debt at maturity. In later years, there are a couple of extensions of Merton models to correct its oversimplified assumptions with the introduction of stochastic interest, tax, bankruptcy cost, agency cost and debt renegotiation, etc. Many studies aim to empirically test the performance of structural models. However, the results show that almost all existing models share poor performance and accuracy problems especially low predictability of credit spread change.

The purpose of this paper is to present the evolution of the value-based models of credit risk as well as calibration of two models for the UK public company National Express Plc. The Merton and Leland models are adopted because they are basic and widely applied models. Variables are collected from the database and calculation and then applied to the formula. The results are analyzed considering the inherent defects of the approaches and the extensions of the models. The structure of the paper is as follows:

The first section literature review gives an overview of both theoretical and empirical research on corporate credit risk. It illustrates sources of corporate risk and determinants of credit spreads, important measurements of credit risk. Afterwards, it presents the evolution of the firm value-based models of credit risk as well as the empirical performance. Merton (1974) and Leland (1994) are two main ones. Other bankruptcy prediction models based on accounting information such as Z score (Altman, 1968) and O score (Ohlson, 1980) are also introduced. The second section focuses on a single company analysis. It starts with the descriptive analysis of the firm, followed by accounting ratio and share price analysis. Later, the calibration of Merton and Leland models are explained with a comparative analysis. Finally, the summary and the findings of the paper are given.

2. Literature Review

2.1 Sources of corporate credit risk

From Lando (2004), Credit risk, also known as default risk, is the risk that the counterparty fails to repay a loan or otherwise fails to fulfill the obligation stated in the financial contract. For a public company that issues a loan or a bond, the credit risk is usually linked to the downgrade of credit ratings, difficulty to repay the principals and interest, and variations of credit spread. There are two main sources of corporate credit risk: systematic risk and firm specific risk.

There is empirical evidence that the macroeconomic factors are closely related to the development of credit risk. Elements such as GDP growth, interest rate changes and industrial production are all influential factors on the credit ratings. In general, a holder of corporate bond bears the consequence that market value of the bond decreases as a result of the increase in the interest rates. Bonfim (2009) investigates the relationship between credit risk and macroeconomic developments at an aggregate level. The correlation metrics between a cyclical component of credit overdue and a number of macroeconomic variables prove the significance of systematic factors. The researchers come to the conclusion that in the periods of economic growth, there may be strong credit growth as well as the trend for excessive risk taking.

The corporate credit risk also arises from the firm specific factors such as financial leverage, accounting data and organizational structure. For instance, accounts receivables, notes receivables representing current and long-term liabilities while financial derivatives may be subject to foreign exchange risk, interest rate risk and commodities risk. When companies issue bonds, they are obligated to pay interest and principal and may also be restricted by bond covenants. Bonfim (2009) also performs estimates of credit risk based on the internal determinants of credit risk for more than 30,000 firms. He finds that explanatory variables, like sales growth, profitability, solvency ratio, and investment rates exhibit negative coefficients against default probabilities while firms with high leverage are liable to high default probabilities. They also find that the past experience of default should be taken into consideration since firms with default history are more likely to default in the future.

Corporates may use credit risk derivatives to transfer or single out the credit risk of any asset. Cossin and Pirotte (2001) gives an example of a default swap, where a company pays the dealer a fixed payment in exchange for contingent payment based on another country's equity index or bonds.

2.2 Determinants of credit spreads

The credit spread measures the return of a corporate bond in excess of risk free rate, which is usually assumed to come from the yield of Treasury bond. A large credit spread is recognized as a reliable measure of corporate financial distress (Bielecki and Rutkowski, 2004). The common determinants of credit spreads are as follows

- Leverage

From Merton (1974), the increase in leverage leads to a higher probability of default since the default threshold has been raised. Sarig and Warga (1989) provide the first empirical description of risk structures of interest rates with pure discount bonds. To calculate the yield spreads, the yield to a zero coupon bond is subtracted from the yield of a corporate bond of the same maturity every month and these are averaged. The result shows that the for low leverage firms, credit spreads are low but the term structure is upward sloping; for high leverage firms, term structure of credit spreads are high but downward sloping; for intermediate leverage firm, spreads are humped. The same conclusion is also arrived by Leland and Toft (1996). It should be noted that an upward term structure implies higher future interest rate thus lower credit spreads and vice versa.

- Changes in spot rates

According to Collin-Dufresne and Goldstein (2001), credit risk can be written as $CS_{(T)} = CS\ (V_{t,} r_t, \{X_t\})$ where V is value of the firm, r_t is risk-free rate and X_t represents all the other “state variables”. A higher spot rate improves the risk neutral drift of firm value process, resulting in lower probability of default and lower spreads. Longstaff and Schwartz (1995) hold the same opinion that an increase in the spot rate reduces the component of credit risk, tightens the credit spreads and increases the firm value. They also empirically prove the negative relationship.

- Implied volatility

Structural approach asserts that the debt claim resembles the short position in a put option. Considering the positive relationship between volatility and option values, an increase in implied volatility raises credit spread. Furthermore, Collin-Dufresne and Goldstein (2001) conducts a regression of credit spread on changes in VIX index and finds the asymmetric reaction of credit spreads to changes in implied volatility, i.e. implied volatility increase has significant influence on credit spreads but not for a decrease.

- Macroeconomic factors

Similar to the source of credit risk, the change in credit spread is also closely related to the systematic factors. Collin-Dufresne and Goldstein (2001) assert that a large systematic component lies outside of the structural model framework and serve as an explanation. Particularly, Amato and Luisi (2005) find that the variations of financial conditions have a profound influence on the price of default risk of BBB-rated bonds, which behave in a rather volatile manner.

- Stock return momentum

The momentum effect of equity returns researched by Jegadeesh and Titman (1993) conclude that past "winner" stocks continue to outperform the "loser" stocks in the short and medium term. Lando (2004) implies that companies that exhibit sound performance in the past are likely to have low default probabilities and lower credit spreads.

2.3 Evolution of firm value-based credit risk mode

2.3.1 Merton model (1974)

Hanke (2003) point out that Merton (1974) is the first to provide a formal application of contingent claim valuation of pricing corporate debt based on Black and Scholes (1973) model. The original Merton model is based on simple capital structure with no transaction cost, taxes. The value of the firm is independent of the capital structure and can be described by stochastic process:

$$dV = (\alpha V - C)dt + \sigma V dz \qquad (1)$$

V = value of the firm

α = expected return on the assets

C= total dollar payout per unit time

σ = asset volatility

dz = Wiener process

When pricing discount bonds, the Merton model assumes that there are only two sources of capital: equity (E) and debt (D) while the debt is a zero-coupon bond with face value B and maturity T. If the payment is not met, i.e. the value of the firm is less than B; bondholders take over the company while shareholders have no residual claim.

Furthermore, when pricing discount bonds, the firm is not able to issue new senior claims, repurchase shares nor pay cash dividend prior to debt maturity. Therefore, the firm's equity is treated as a European call option written on the asset value of the firm. The risk-free rate, asset volatility and risk premium are all assumed to be constant. With V corresponding to the stock price and B to the exercise price, under option pricing model equations, it can be deduced that:

$$E(V,t) = VN(d_1) - Be^{-r(T-t)}N(d_2) \tag{2}$$

$$d_1 = \frac{In\left(\frac{V}{B}\right) + (T-t)(r + 0.5\sigma^2)}{\sigma(T-t)} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \tag{4}$$

$$D = V - E(V,t) \tag{5}$$

N(d) = cumulative density function of a standard normal distribution

r = risk-free rate

T= time to maturity

t = initial time

Lando (2004) infers that the bond price is positively related to firm value, bond value while negatively related to interest-free rate, time to maturity and volatility. In a risk-neutral world where all assets have the same expected return with risk-free rate i.e. $\alpha = r$, default probability can be written as:

$$P(V_t < B) = N(-d_2) = 1 - N(d_2) \tag{6}$$

Then, Merton discovers that the Modigliani and Miller (1985) theory also applies in the presence of bankruptcy in the Merton model. Where there are two firms sharing the same investment decisions when one of them issues bond and the other does not, the investor can obtain a security with the same payoff of risky bond by mixing equity and debt in a portfolio. The prerequisite is that there are no differential tax benefits or transaction costs.

In the last section, Merton presents how to price bonds with risky coupons. The formula for risky coupon non-callable bonds is rather complex since once the firm defaults on a coupon payment, the rest of the coupons are also presumed to defaulted on. Based on the assumption that coupon payments are made continuously, calculations and empirical tests are feasible for the solution of a differential equation. In the final illustration, bonds with callable features are taken into account. The boundary condition changes and for each τ (first passage time of a process), there will be value $\bar{V}_{(\tau)}$ that the firm will redeem the bond when $V_{(\tau)} = \bar{V}_{(\tau)}$. While no explicit closed-form solution is provided, it is implied that numerical methods are feasible to solve value functions

Cossin and Pirotte (2001) point out that the simple structural approach allows an easy pricing framework regarding the exposition of option pricing. Whereas the shortcomings are obvious: it fails to consider coupon-paying bond with finite maturity and complex capital structures. In addition, according to Bielecki et al. (2007), predictions of default and credit spreads appear too low compared with those obtained from the market.

2.3.2 Leland model (1994)

As an important extension of Merton model, Leland model takes into account of tax and bankruptcy effect. The firm's capital structure is endogenous and the optimal capital structure depends on the trade-off between tax deductibility of interest expense and bankruptcy costs. As Merton (1974), Leland (1994) assumes independent and static capital structure. The value of the firm's assets follows a diffusion process:

$$dV = \mu V dt + \sigma V dW \qquad (7)$$

μ = expected return on firm per unit time

σ = asset volatility

dW = standard Brownian motion

For debt valuation, it is assumed that constant and perpetual coupon, C , is paid, V_B denotes asset value where bankruptcy is declared and αV_B equals to bankruptcy cost. Applying boundary conditions, value of debt (D (V)) is obtained as:

$$D(V) = \frac{C}{r} + \left[(1-\alpha)V_B - \frac{C}{r}\right][\frac{V}{V_B}]^{-x} \tag{8}$$

D (V) = value of debt

r = risk-free rate & x = $\frac{2r}{\sigma^2}$

As the present value of bankruptcy cost equals to αV_B, and $\lim_{V\to\infty} BC(V) = 0$, the bankruptcy cost is calculated as:

$$BC(V) = \alpha V_B[\frac{V}{V_B}]^{-x} \tag{9}$$

A similar approach is applied to tax benefit (TB) calculation where τ denotes tax rate and tax benefit equals to τC. The calculation is written as:

$$TB(V) = \frac{\tau C}{r} - \frac{\tau C}{r}(\frac{V}{V_B})^{-X} \tag{10}$$

Since the tax benefit of interest increases firm value while bankruptcy cost reduces value, the total value of the firm (*V*) is :

$$v(V) = V + \left(\frac{\tau C}{r}\right)[1 - \left(V - \left(\frac{V}{V_B}\right)^{-X}\right] - \alpha V_B(V/V_B)^{-X} \tag{11}$$

The value of equity (E (V)):

$$E(V) = V - \frac{(1-\tau)C}{r} + [\frac{(1-r)C}{r} - V_B][\frac{V}{V_B}]^{-x} \tag{12}$$

Then Leland studies the value of the firm, which has no protective covenant and bankruptcy, occurs when the equity value falls to zero. The firm fails to make coupon payment with issuance of extra equity. Then the company can choose to maximize firm value with V_B set as low as possible. After differentiation, V_B is found to be:

$$V_B = \left[\frac{(1-r)C}{r}\right]\left[\frac{X}{1+X}\right] = (1-\tau)C/(r+0.5\sigma^2) \qquad (13)$$

Substituting equation (13) into (8), (11), (12), it can be obtained that:

$$D(V) = \left(\frac{C}{r}\right)[1-(C/V)^X k] \qquad (14)$$

$$v(V) = V + \left(\frac{\tau C}{r}\right)[1-\left(\frac{C}{v}\right)^X h] \qquad (15)$$

$$E(V) = V - \ (1-\tau)(\frac{C}{r})[1-\left(\frac{C}{V}\right)^X m] \qquad (16)$$

The parameters of m, h, k are as follows:

$$m = [\frac{(1-\tau)X}{r}(\frac{1+X)]^X}{1+X})]$$

$$h = \left[1 + X + \frac{\alpha(1-\tau)X}{\tau}\right]m$$

$$k = [1 + X - (1-\alpha)(1-\tau)X]m$$

As a result, the credit spread, R-r, is defined as

$$R - r = r\left(\frac{C}{V}\right)^X k/[(1-\left(\frac{C}{V}\right)^X k] \qquad (17)$$

According to the result of Lando (2004), the default boundary of Leland model decreases with lower coupon or higher tax because shareholders maintain the option at the expense of coupon while tax reduces the effect.

In later sections, the author distinguishes two situations: unprotected bonds, where optimal bankruptcy threshold is determined endogenously and protected bonds, where they are protected by positive net-worth covenants. The distinctions are obvious: optimal leverage, interest rate paid at the optimum leverage and maximum values of the firm are all lower under protected bonds. Afterwards, he concentrates on the role of protective bond in mitigating agency problems. He comes to the conclusion that without positive covenants, equity is strictly convex in V and equity holders will endeavor to increase benefit at risk. Whereas when the debt is protected, both debt and equity value are lowered. Moreover, issuing debt without protective covenants leads to greater tax benefits than ones with protective covenants.

While Leland (1994) introduces bankruptcy cost and tax benefits to make assumptions more realistic compared with Merton (1974), Hanke (2003) asserts that one of the model's limitation is a restriction on the barrier level, which lacks explicit explanation or further discussion. One more distinction between two models inferred from Lando (2004) is that while under the Merton model, debt value can be raised as close as to the assert value for optimally levered firm, there is a maximum debt capacity for Leland model.

2.3.3 Mella-Barral & Perraudin (1997)

Considering that bankruptcy cost may motivate equity holders to act strategically and debt holders to accept deviations from contractual payment, Mella-Barral and Perraudin (1997) incorporates the renegotiation of debt into pricing model. The model is based on assumptions of frictionless capital markets and risk neutrality and follows a geometric Brownian motion:

$$dp_t = \mu p_t + \sigma p_t dW_t \qquad (18)$$

p_t = price sold of each unit of output

μ, σ = constant parameters

W_t = standard Brownian motion

The authors begin by analyzing all equity firms with value of W_t and income flow of $p_t - w$ with w denotes cost per period. The value of equity follows that:

$$rW = 0.5\sigma^2 p^2 W''(p) + p - \text{w} + \mu p W'(p) \qquad (\,)$$

Under the circumstance of no arbitrage, the value of the boundary conditions must satisfy conditions that $W_{(p_c)} = \gamma$ (scrapping value at closure) while p_c refers to trigger point for closure and maximize total firm value, i.e. $W'_{(p_c)} = 0$. Hence, the optimal liquidation threshold is:

$$p_c = (\frac{\lambda}{\lambda-1})(\frac{w}{r}+\gamma)(r-\mu) \qquad (19)$$

The value equals to:

$$W(p) = \frac{p}{r-\mu} - \frac{w}{r} + [\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}](\frac{p}{p_c})^\lambda \qquad (20)$$

Where λ = negative root of the equation $\lambda(\lambda-1)\sigma^2 + \lambda\mu = r$

Then, they consider situations where the firm is financed with both equity and debt where debt is captured by perpetual coupon payment b. Bankruptcy occurs when the output price p_t, falls below the level p_b. The boundary condition is:

$$p_b = \frac{\lambda}{\lambda-1}\frac{w+b}{r}(r-\mu) \qquad (21)$$

The value of debt D, and value of equity E:

$$D_{(p)} = \frac{b}{r}+[X_{(p_b)} - \frac{b}{r}](\frac{p}{p_b})^\lambda \qquad (22)$$

$$E_{(p)} = \frac{p}{r-\mu} - \frac{w+b}{r} - [\frac{p_b}{r-\mu} - \frac{w+b}{r}](\frac{p}{p_b})^\lambda \qquad (23)$$

Also, it should be noted that when $\frac{b}{r}<\gamma$, there are adequate assets to meet firm liabilities and bankruptcy will not happen. When $\frac{b}{r} > \gamma$, bondholders may bear the risk of bond and the value of the firm is:

$$W_{(p)} = \frac{p}{r-\mu} - \frac{w}{r} + [X_{(p_b)} - \frac{p_b}{r-\mu} + \frac{w}{r}](\frac{p}{p_b})^{\lambda} \quad (24)$$

Then, they discuss pricing negotiations and reach the conclusion that as long as debt is risky, there is scope for renegotiation and bankruptcy costs would not be incurred. At last, the authors generate a model so as to consider the tradeoffs between renegotiation costs and tax benefits to achieve maximum firm value. In discussing renegotiation and risk premier, they discover that under high volatility levels, the impact on renegotiation becomes nearly negligible.

2.3.4 Other extensions and generalization of models

Following Merton (1974), Black and Cox (1976) is regarded as a first passage time model that allows default before debt maturity. The firm defaults when the asset value falls below a default barrier. Bondholders have a right to exercise a “safety covenant” as the firm to liquidate or reorganize whenever the value reaches the specified threshold. Bielecki et al. (2008) postulate that equity holders receive dividend payments continuously, which are in proportion to the asset value. Longstaf and Schwartz (1995) extend the Black-Cox model in that the risk-free rate follows Vasicek process stochastically.

Another well-known extension of Merton model is KMV (Kealhofer McQuown and Vasicek) model, developed by Moody’s. It is based on the structural approach to calculate EDF (expected default frequency) and is appropriate for public traded companies and efficient liquid market according to Kowk (2012). Moreover, it takes the company stock price as one parameter. Unlike Merton (1974) where default occurs exclusively on principal payment, Charitou elt al. (2013) states that KMV recognizes that default may be triggered by nonpayment of interest as well. After loads of research and studies, one generally accepted approach to default point is firm's short-term debt plus half of its long-term debt.

Leland and Toft (1996) relaxes assumptions of Leland model by allowing debt with finite maturity. They assume that default occurs when the asset value falls to an endogenous default boundary, which heavily relies on the maturity of debt. For long-term debt, equity holders may resort to bankruptcy when the asset level surpasses debt principal. Their study exemplifies the tradeoff among tax benefits, bankruptcy costs and agency costs. Considering agency costs, risky firms prefer to issue shorter-term debt as well as issuing less debt. Firms with higher bankruptcy costs opt for long-term debt.

As an extension of Mella-Barral model, Fan and Sundaresan (2000) attributes more bargaining power to creditors by defining two bargaining scenarios: a debt-to-swap version and strategic debt services. In the presence of strategic debt service, both debt and equity holders can enjoy potential tax benefits. Therefore, the bargaining power is higher under strategic debt services when equity holders share more tax benefits. On top of that, the authors introduce dividend reinvestment to the value generation process. Equity holders tend to maximize dividend payments if they can optimally default. However, when the cash flow-based bond covenant is introduced, they will cut dividend voluntarily to avoid violating covenants. For both scenarios, the authors derive optimal trigger points for firm value. The values of debt and equity are dependent on the bargaining power of holders.

One innovation of Huang and Huang (2003) is the incorporation of jumps in the firm value process. The authors come up with a new structural model with a jump-diffusion asset value process. Therefore, the firm value is assumed to follow the process:

$$\frac{dV_t}{V_{t^-}} = (\pi^V + r - \delta)dt + \sigma_v dW_t^v + d[\sum_{i=1}^{N_t}(Z_i - 1)] - \lambda\varepsilon dt$$

δ = firm payout ratio

σ_v = diffusion volatility

W_t^v = standard Brownian motion

N_t = Poisson process

λ = constant intensity

Z_i' = random variable for jump diffusion

With the new model, the authors are able to accurately model asset risk premia, which are closely related to the predicted bond default probability. Also, they study the extent to which the credit risk is reflected in yield spreads. The credit risk accounts for only a small fraction for investment grade bonds but a much higher fraction for junk bonds.

Vassalou and Xing (2004) is the first study that uses Merton (1974) to examine default risk on equity returns. What makes them different from Merton is that they use default boundary adopted by KMV, rather than using the face value of debt at maturity. However, Charitou et al. (2013) point out that their estimation of expected growth rates are usually negative, in conflict with the asset pricing theory.

Overall, the basic Merton model is too parsimonious with assumptions that are simplified and unrealistic. Researchers endeavor to take more factors into consideration to make it more applicable and accurate in prediction. Apart from the ones mentioned, there are other extensions and generalizations such as inclusion of stochastic interest rates from Longstaff and Schwart (1995), target leverage ratio from Collin-Dufresne and Goldstein (2001) and costs of new equity issuance from Acharya, et al. (2006).

2.4 Empirical evidence on performance of credit risk models

One of early day empirical evidence come from Jones et al. (1994) who aims to test the predictive power of CCA (contingent claim analysis) model of different capital structures. The fact that companies’ capital structures consist of different classes of equity and debts makes the test difficult. As a result, samples of companies with relatively simple capital structures are chosen. They find that CCA models seem to have advantages over naïve models in their power of explanation for non-investment bonds, but not for investment bonds. There is also evidence that incorporating stochastic interest rates and taxes would enhance model performance.

Another significant research result is that the Merton model tends to underestimate credit spreads. The same conclusion is strengthened by Huang and Huang (2003). They notice that all structural models tend to under predict spreads sharply if calibrated to historical default experience.

Sarig and Warga (1989) presents the first paper to investigates the risk structure of interest rates and obtain the results that resemble the time profile provided from Merton (1974). Yield of zero-coupon bonds are used to compute yield spreads over zero-coupon U.S. Treasury bonds and the empirical hump shape corresponds to the one predicted from Merton model.

Compared with ones that focus on only one model, Eom et al. (2004) illustrates a more comprehensive test, which involves five structural bond pricing models. The results show that all five models exhibit credit spread estimation errors but the errors are different in sign and magnitude. Among five models, Merton (1974) and Geske (1977) tend to under estimate credit spreads, while Geske (1977) suffers less. One explanation is the introduction of an endogenous default boundary. On the contrary, Leland and Toft (1996) tends to overestimate credit spreads due to the assumption of a continuous coupon payment. It should be noted that the authors point out the overemphasis of previous papers about maturity as a cause of mispricing.

In a more recent research, Schaefer and Strebulaev (2008) also support the underestimation of credit spreads. They find that despite the poor prediction of prices and underestimation of credit spreads and returns, structural models show good performance in predicting sensitivity, hedge ratio and debt to equity levels. Furthermore, the sensitivities do not seem to be related to credit exposure measures, like credit rating and asset volatility. Then they test the Merton model with stochastic interest rates incorporated. The regression results reveal that the model explains the equity sensitivity well, with or without stochastic interest rates. One more finding is that market-wide factors, such as SMB and HML affect bond prices and returns, but in a way not predicted by structural models.

2.5 Other bankruptcy models

Apart from market-based models, there are also other bankruptcy models such as ones based on accounting information.

The Z-score proposed by Altman (1968) attempts to predict corporate bankruptcy based on financial ratios and discriminant analysis. The MDA (multiple discriminate analysis) was chosen as the statistical technique with multiple regression analysis. The final discriminant function is:

$$Z = 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5 \quad (25)$$

Where X_1 = working capital/total assets

X_2 = retained earnings/total assets

X_3 = earnings before interest and taxes/total assets

X_4 = market value equity/book value of total debt

X_5 = sales/total assets

Bankruptcy occurs when the firm's asset value falls below the total liabilities. Through an F test, it is found that firms with higher bankruptcy potential have lower discriminant scores. The implication is that the Z-score is an accurate predictor up to two years before bankruptcy. After that, the accuracy decreases substantially. After application, the cut-off points are decided as follows: firms with Z scores greater than 2.99 are classified as non-bankrupt; firms with Z scores between 1.81 and 2.99 are susceptible to bankruptcy and the area is known as "grey area"; firms with Z scores lower than 1.81 are liable to bankruptcy. One limitation of the Z-score research is that the examples mainly come from manufacturing corporations whose financial ratios are harder to price and the possibility of insolvency is relatively low. Therefore, it is suggested that the analysis should extend to smaller companies or unincorporated entities.

Another measurement of probability of financial distress based on accounting ratios is O-score, put forward by Ohlson (1980). The formula is as follows:

$O - Score =$
$-1.32 - 0.407 log X_1 + 6.03X_2 - 1.43X_3 + 0.076X_4 -$
$1.72(1\ if\ total\ liabilities > total\ assets, 0\ if\ otherwise) - 2.37X_5$ -
$1.83X_6$+0.285(1 if a net loss for the last two years, 0 otherwise)-0.521X_7. (26)

Where X_1= total assets

X_2= total liabilities/total assets

X_3= working capital/total assets

X_4= current liabilities/current assets

X_5= net income/total assets

X_6= funds from operations/total liabilities

X_7= $(net\ income_t - net\ income_{t-1})/(|net\ income_t| + |net\ income_{t-1}|)$

Griffin and Lemmon (2001) study the relationship between book-to-market equity, distress risk and stock returns. The findings show that different to previous findings, firms with high O-scores, low BE/ME ratio do not suffer from high financial distress risk. For firms with the highest O-scores, the large difference between high and low BE/ME stocks cannot be explained by the model. One possible explanation is that stocks with high book-to-market ratios are underpriced while stocks with low book-to-market ratios are overpriced. It should be noted that firms with high financial distress have features that make them susceptible to mispricing.

Agarwal and Taflfer (2006) compare the performance of the Z-score and market based models in UK over the period 1985 to 2001 using ROC (receiver operating characteristics). While the two approaches capture different aspects of bankruptcy risk, they do not exhibit much difference in terms of predictability. However, in the presence of competitive pricing regime and loan market, the Z-score approach leads to high risk-adjusted revenue and profit.

Compared with market-based models which are based on market information and factors, accounting-ratio based models focus more on the past performance and historical cost. On one hand, the conservatism rooted in accounting numbers may lead to true values remarkably different from books values. Also, accounting numbers are subject to manipulations and lack accuracy against market values. On the other hand, the fact that accounting-based models empirically perform better than market-based models shows their importance. Agarwal and Taffler (2006) claim that the corporate bankruptcy is a process of accumulation of year-by-year bad performance, which can be captured by financial statements. Furthermore, bond covenants are usually based on accounting information. Therefore, they are better reflected in accounting-based models.

3. Assessing default risk of a UK company

3.1 Descriptive analysis

National Express Plc. is a leading provider of transport services including bus, coach and rail across UK, Spain, North America and Morocco. In UK, it is recognized as the largest operator of scheduled coach services and the leader of the UK urban bus market outside London. In Spain, it is the largest provider of public transportation and operates long distance, regional and urban bus and coach service across Spain and Morocco. In North America, it is the second largest private operator and focuses on student transportation.

National Express is mainly operating in the outsourced public transportation market, which is driven by social stability and GDP growth. As shown from the 2012 National Express Annual Report, the UK market size is £ 4.8 billion out which 97% is privatized. Private transportation helps to provide more standard and high-quality service.

In terms of business risk, economic conditions and political and regulatory changes are the most influential ones. A gloomy economy in Europe and North America may lead to reduce economic activities and products. In Spain particularly, the sustainable low income, high unemployment rate and low government budget arising from Euro Crisis may affect the transportation division (National Express Annual Report, 2012). Also, political and regulatory environment changes can exert a profound influence on the market activities. For instance, the withdrawal of the Government's 16 million senior citizen concession scheme in 2012 leads to a loss of subsidy and 2% decline of UK coach revenue. Apart from that, the fuel cost risk cannot be ignored since the fuel costs account for around 9% of total costs and the fluctuations in price give rise to changes in profitability. As a solution, the company enters into swap and forward contracts to mitigate risk as a hedging strategy. One more notable and inherent risk is contractual risk, since a high proportion of the business comes from the biddings and contracts. The wrong assumptions in bidding process have adverse consequence on the financial performance.

In May 2012, the company acquired the American company Petermann Partners Inc which provides student transportation service. There are also other several business combinations during the year. This leads to a significant increase in acquisition cost in cash flow but also plays a key role in portfolio diversification and risk mitigation.

3.2 Accounting ratio analysis

	Ratios	2010	2011	2012
Profitability	Gross margin	6.84%	7.69%	6.42%
	Net profit margin	2.89%	4.52%	3.28%
	Return on assets	2.57%	4.19%	5.39%
Liquidity	Current ratio	0.6	0.7	0.5
	Cash flow/Current liabilities	0.3	0.3	0.2
Solvency	Interest coverage	6.9	7.2	6.7
	Net debt/EBITDA	2.1	1.9	2.5
Efficiency	Total asset turnover	0.9	0.9	0.8
	Receivable turnover	9.37	12.2	9.4
Z score		3.02	3.06	2.59

Table 1-Company Accounting Ratios (Source: Thompson One Banker)

Overall, the group has exhibits stable and strong performance over the past three years. In 2011, the company performs better than 2010 with yield improvement and organic growth in all divisions. The results for 2012 become slightly worse but stay consistent especially considering the challenging social and economic environment. The ratio analysis is focused on the following aspects:

- Profitability

The total revenue is £2238m for 2011, a 5% increase from 2010 and £1831.2m for 2012. In 2012, operating profit margin in North America increases to 10.2%, outperforming competitors and resulting from the successful integration of Petermann acquisition, which is indicated from National Express 2012 results. While the operating profit drops notably due to loss of £16m government concession subsidy as well as the fierce competition from the rail industry. In addition, there are £42.6m exceptional charge, which includes acquisition expense, bid cost and restructuring cost. There is no big change in the total assets. Therefore, the gross margin and net profit margin drop in 2012 while the return on assets improves.

- Liquidity

The payment of part of the bank loans in 2011 decreases the current liabilities and increases the current ratio compared with 2010. In 2012, the firm accelerates payment of trade payables and receipts the trade receivables, so that both current assets and current liabilities decline in 2012. However, the company increases the bank loan by approximately £100m to finance activities, which leads to a decrease in current ratios. As to the cash flow, the large increase in the purchase of property, plant and equipment is because of a series of business combinations lowers the total cash flows.

- Solvency

The covenant of the firm requires that net debt/EBITDA should be below 3.5 and the interest cover be greater than 3.5, which is illustrated in 2012 National Express Annual Report. Over the years, the company keeps the ratios high above the covenant, which implies relatively low solvency risk. In 2012, the firm increases total debt with more bank loans and bonds and the interest expense increases correspondingly. However, the group is still regarded strong in financial strategy and may consider future gearing and a less conservative dividend policy.

- Efficiency

With constant total asset figures, a decline in the total revenue leads to a decrease in total asset turnover. Moreover, a decrease of trade receivables is larger than that of total sales, so that the receivable turnover decreases as well. That may suggest company’s lower efficiency at using assets to generate revenue.

- Z score

The Altman Z scores seeks to measure the company’s bankruptcy risk two years before the event as explained in the previous section. For the National Express, Z scores in 2010 and 2011 are above 2.99, which belong to safe zone. For 2012, the score drops to

2.58 and falls to "grey zone". It suggests that the group suffers higher bankruptcy risk than previous years. One possible explanation is the heavier finance with bank loans and weaker financial performance during the year.

3.3 Share price analysis

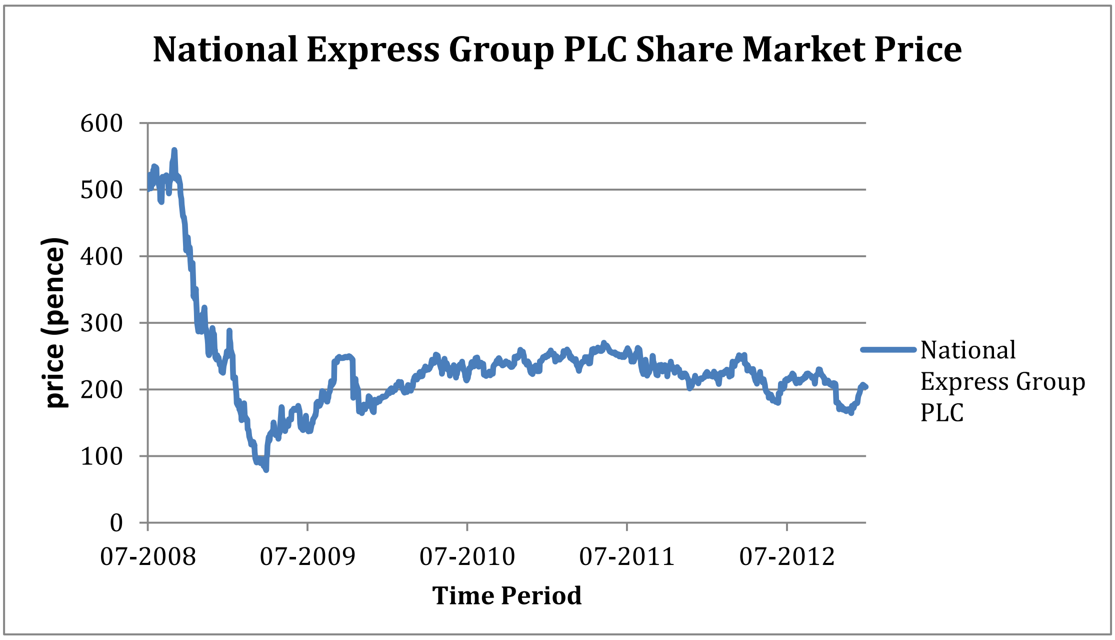

Figure 1-Company Share Market Value Chart

	2010	2011	2012
Average yearly stock return	0.12%	-0.03%	-0.02%
Closing price return	27.27%	-10.91%	-9.85%
Dividend yield	2.39%	4.26%	4.34%
P/E ratio	20.92	11.21	9.12
Earnings per share	23.5p	26.9p	25.4p

Table 2-Company Share Price Ratios

As can be seen from the graphs, the National Express share price experienced a striking plunge during 2008-2009 and then stabled since early 2010 with few fluctuations. From

National Express Annual Report 2012, the total shareholder return changes resemble that of FTSE all-shareholder travel & leisure index. Therefore, it can be inferred that part of the decline comes from the effect of the market such as high fuel cost and pension expense. In terms of the firm specific analysis, the pre-tax profit for 2009 is negative and falls below expectations. In 2009 specifically, the group was hit by several bid failures. The First Group, UK's largest transport company, pulled out of the all-share merger in the early year. Then the company waived the potential merger with its competitor Stagecoach, which led to a 12% decrease of share price.

After a couple of eventful years, the company shows healthy growth and is on the right track with year 2011 considered as the turning point. Under the government budget cut, soaring fuel cost and rising train fares, more and more passengers choose the relatively cheap coach and bus, which helps to boost revenue and confidence of the company. In addition, a series of successful business combinations and acquisitions diversify the investment portfolios, enlarge business areas and reduce risk exposure. Therefore, the closing price return for 2010 is rather high and the average yearly stock return stables in the year after.

Regarding the dividend policy, the company has increased shareholder dividend continuously, from £15.2m in 2010 to £49.3m in 2012. With the high quality of its business strategy, the company is able to generate improved return on capital. Also, the Board has a sustainable dividend policy and aims to pay dividend that is covered twice by the bus and coach earnings, according to National Express 2011 Annual Report. Higher dividend per share gives rise to higher dividend yields and benefits the shareholders. However, since earnings per share remain stable for the past three years, jumps in the share price results in lower P/E ratio. One possible explanation is that investors may be more conservative on the performance of the National Express.

3.4 Bond price and credit risk analysis

Data was collected from DataStream for daily share price, number of shares, market value of traded bonds, risk free rate and FTSE 100 index during the period 2010 to 2012. It should be noted that the group started to issue bonds in 2010. One was issued on 13th January 2010, with coupon rate of 6.25%, maturity date on 13th January 2017 and face value of £350m. The other was issued on 16th June 2010, with coupon rate of 6.625%, maturity date on 16th June 2020 and face value of £225m. The total market value of the liabilities equals to the sum of market value of two bonds and the equity market value equals to the daily shares price time number of shares traded. The yield of 10-year Government long-term bond serves as the risk-free rate for the period. The annual risk free rate implemented in each year is the average of the daily yield.

3.4.1 Merton model (iterative approach)

The two approaches of calibration of Merton model follow the way used by Loeffler and Posch (2010): iterative approach and equation approach.

For year 2010 for example, the calculation starts with the initial values of the asset value (iterative k), which is the sum of the market value of equities and market value of debt. Based on the equation (3), which derives from Black-Scholes formula, the VBA-function BSd (S, x, h, r, sigma) is applied to get d_1:

$$BSd = \frac{\log\left(\frac{S}{x}\right) + (r + 0.5 * sigma^2) * h}{sigma * h^{0.5}}$$

$\mathrm{S} = \text{value underlying}$

x = strike price

h = T-t time to maturity

r = risk-free rate

sigma = volatility underlying

Then the log return of the asset values (iterative k) are calculated as $Ln\ (\frac{price_t}{price_{t-1}})$ and the annual asset volatility is obtained from the standard deviation of log returns times the square of number of trading days 260 to get 19.81%. Afterwards, the asset value (iterative k+1) is computed with the equation (2) and (5). The column of asset value (iterative k+1) replaces that of asset value (k) as long as the squared difference is above 10^{-10}. Afterwards, the CAPM (capital asset pricing model) is applied to estimate expected asset returns:

$$E[R_i] - R = \beta_i(E[R_M] - R)$$
$$R = \exp(r) - 1$$

Return of FTSE 100 is taken as the proxy for R_M (market return) and beta 0.62 is obtained from the regressing the asset return and market return. Therefore, the expected asset return is calculated as 7.58% and the drift rate is 7.31%, computed from $Ln\ (1 + expected\ asset\ return)$. Asset value liabilities are both taken from the value at the date of 31/12/2010. Time to maturity 6.43 years is obtained as the weighted average of the maturity years of two bonds based on their market values. The distance to default (d_2) 2.61 is got using equation (4) and the default probabilities is 0.45% with equation (6).

The same process is adopted for year 2011 and 2012 and the summary of the results is as bellows:

		2010	2011	2012
Estimates	Asset value (m)	1654.20	1772.01	1705.21
	Asset volatility	19.81%	11.03%	18.56%
	Asset drift rate	7.31%	2.82%	0.73%
Balance sheet data	Liabilities (m)	627.65	647.28	676.36
Default probabilities	Distance to default	2.61	3.24	1.44
	Default probabilities	0.45%	0.06%	7.45%

Table 3 - Merton Model Iterative Approach Result

When the asset values and proportion increase, default probabilities decrease. However, it should be noted that the simple assumption of zero-coupon bond, risk neutrality in the above approach have inherent inaccuracy and make the default probabilities small.

3.4.2 Merton model (equity approach)

The equity approach uses equity values and equity volatilities and share the same assumptions with the previous one. The asset volatility σ calculation is based on the equation from Cooper and Davydenko (2007) who propose to estimate based on equity volatility:

$$\sigma_E = \sigma N(d_1) A_t / E_t \qquad (27)$$

For year 2010 for instance, the equity volatility 26.18% is the standard deviation of daily log return of share price times the square of trading days. With the assumption N $(d_1) = 1$, the $\sigma = \sigma_E * E_t / A_t$=17.77%. Then the equity value is computed with equation (2) and the solver is adopted with the aim of minimizing the sum of squared difference between model values and observed values with asset value and asset volatility set as changing variables. The asset returns, drift rates and default probabilities are obtained in the same way as the iterative approach:

		2010	2011	2012
Estimates	Asset value (m)	1886.27	1765.59	1705.18
	Asset volatility	17.77%	18.26%	18.53%
	Asset drift rate	7.31%	2.82%	0.73%
Balance sheet data	Liabilities (m)	627.65	647.28	676.36
Default probabilities	Distance to default	3.40	2.54	2.13
	Default probabilities	0.03%	0.55%	1.65%

Table 4-Merton Model Equity Approach Result

Under equity approach, the asset volatility and leverage are assumed to be constant, which is contrary to the iterative approach. Changes in asset volatility give rise to changes in default probabilities.

3.4.3 Merton model (multi-period)

The multi-period approach considers the accrued interest and dividend payment and they have higher priority than bond principal. Dividend is assumed to be paid annually and grow at annual rate of g. Take the year 2012 for example, D_0 (dividend just paid) is £49.3m, D_{0-1} (dividend paid last year) and c (average coupon rate) is 6.5%, therefore g (growth rate) equals 7.64%. For years with zero dividend payment because of the investment strategy, growth rate is replaced by increase in EPS. The present value of D (value of dividend stream) and I (value of interest payment) are as follow:

$$\mathrm{D} = \sum_{\tau=t+1}^{T} D_0\,(1+g)^{\tau-t} \exp\,(r(T-\tau) \qquad (29)$$

$$\mathrm{I} = \sum_{\tau=t+1}^{T} c * L * exp(r(T-\tau)) \qquad (30)$$

Suppose that the accrued interest and dividend have the same seniority and both are paid prior to the principal, the payoff of equity is a replicate of call options and investment in the assets: share of D/(D+I) of assets, share of D/(D+I) in a short call on assets and a call on assets with strike price L+D+I. Implementing the Black-Scholes pricing formula to get equity value:

$$E_t = A_t * N(d_1) - (L + D + I)e^{(-r(T-t)}N(d_2) + D/(D + I)(A_t - A_t N(k_1) + (D + I)e^{-r(T-t)}N(k_2) \quad (31)$$

$$d_1 = \frac{In\left(\frac{A_t}{L + D + I}\right) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

$$k_1 = \frac{In\left(\frac{A_t}{D + I}\right) + (r + \sigma^2)(T - t)}{\sigma\sqrt{T - t}}$$

$$k_2 = k_1 - \sigma\sqrt{T - t}$$

It extends the original model formula with the equity payoff option. Correspondingly, the equity volatility equals:

$$\sigma_E = \sigma \frac{A_t}{E_t}(N(d_1) + \frac{D}{D+I}(1\text{-N }(k_1)) \quad (32)$$

The later steps resemble that from equity approach , which involves use of Solver and asset drift rates from equity approach to get default probabilities for the years until maturity is 31.21%. For the annual default probability, the 1-(1-0.3121)^(1/6.43)=5.65%.

For the credit spread analysis, $B_{(t)}$ (current value of bond) is the discount value of the future payment (L+I). Consequently, the yield spread is:

$$(\frac{L+I}{A_t - E_t})^{1/(t-T)} - 1 - (\exp(r) - 1) \quad (33)$$

$$(\frac{676.359+247.939}{726.46})^{1/6.43} - 1 - (\exp(0.0182) - 1) = 1.985\%$$

Below is the summary result:

		2010	2011	2012
Estimates	Asset value (m)	1952.92	1874.94	1767.49
	Asset volatility	17.52%	18.70%	20.98%
	Accrued interest (m)	217.29	228.03	247.91
	Accrued dividend (m)	247.73	488.23	405.53
	Debt maturity	6.19	6.43	6.43
Default probabilities		1.42%	4.32%	5.65%
Credit spread		0.17%	0.85%	1.99%

Table 5 - Merton Model Multi-period Result

3.4.4 Leland model calibration

The prerequisite is that only the situation when debt has no protective covenants is considered for calibration. First, the asset volatility σ is obtained from Merton model multi-period approach to get 2.294. Take year 2010 for example, the effective tax rate obtained from Annual report is used for tax benefits calculation. Bankruptcy cost parameter α is assumed to be constant at 30% for all years. Coupon payment is the total interest payment of two traded bonds: $350\text{m} * 6.625\% + 225\text{m} * 6.625\% = 37.25\text{m}$.

Since the Leland model assumes perpetual coupon payment, the value of perpetual coupon is $37.25/3.52\% = 1058\text{m}$. Then the tax shield is deducted to get the value after tax cost 800m. The boundary value V_B, which triggers bankruptcy, is obtained with equation (13):$(1 - 24.4\%) * 1058.075 * 2.294/(1 + 2.294) = 557\text{m}$.

The original asset value of the firm (V) £1654m equals the sum of equity and total liabilities , then the coupon payment after tax cost is subtracted to get value after tax cost V-ATCPV. Letting p_B denotes the $(\frac{V}{V_B})^{-X}$, which represents the present value of £1 based on the future bankruptcy, then the equation to calculate value of debt under boundary conditions can be written as:

$$D(V) = (\frac{C}{r})(1 - P_B) + [(1\text{-}\alpha)V_B]P_B \quad (34)$$

Consequently, the value of debt is £1003m. The value of equity is contingent on the value of V compared with V_B , when V> V_B , the residual equity claim is V-ATCPV+(perpetual coupon after tax cost-V_B)*P_B; otherwise the value is equity value is 0.

In terms of the bankruptcy cost and tax benefit contained in the Leland model, equation (9) and (10) are implemented. It should be noted that the value the firm v (V)=D+E=V-BC(V)+TB(V)= £1877.354m. To get the credit spread, equation (17) is applied. A simpler way is assuming yield of debt equal to C/D, thus the yield spread becomes C/D-r=0.2%

The default probability calculation is based on the equation below, which is from the extension of Leland model:

$$DP_{(0,T)} = N\left(-\frac{IN\frac{V_0}{V_B}+\gamma T}{\sigma\sqrt{T}}\right) + \left(\frac{V_0}{V_B}\right)^{-\frac{2\gamma}{\sigma^2}} N(\frac{-IN\left(\frac{V_0}{V_B}\right)+\gamma T}{\sigma\sqrt{T}}) \quad (35)$$

Where $\gamma = \pi + r - \delta - 0.5\sigma^2$

π= asset risk premium

r= risk-free rate

δ= asset payout ratio

σ= asset volatility

The asset risk premium can be obtained from the CAPM calculation in the Merton approach: beta * market risk premium. The asset payout ratio is equal to the historical weighted average of the dividend yield and the average historical coupon rate. (Huang and Huang, 2002) Consequently, with an average dividend yield of 4.61% and coupon rate of 6%, the asset payout ratio is 5.55% and is then applied for all years. The time to maturity T is observed from the one used by Merton multi-period approach. With all parameters, the outcome of probability is 0.51% for year 2010.

The same steps are implemented for the year 2011 and 2012 and the summary is:

		2010	2011	2012
Estimates	Value of debt	1003.06	1113.44	1281.48
	Value of equity	874.30	874.26	522.72
	Value of firm	1877.35	1987.70	1804.20
	P_B	0.08	0.16	0.50
Default probability		0.51%	4.19%	22.08%
Credit spread		0.2%	0.3%	1.1%

Table 6 - Leland Model Result

3.4.5 A comparative of the results of two models

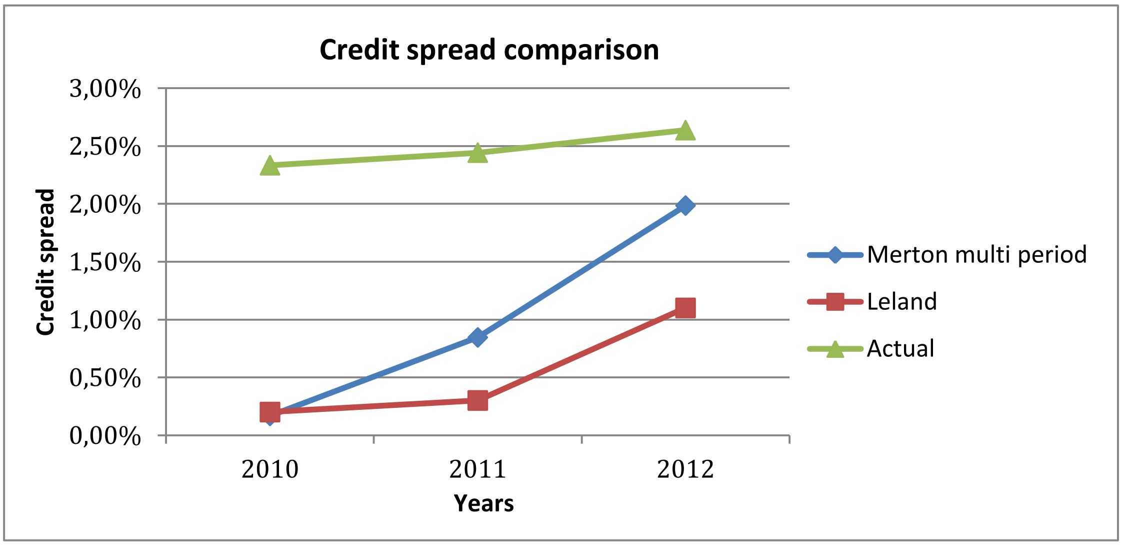

Figure 2 - Credit Spread Comparison

The empirical credit spread is calculated for analyzing the predictability of both models. The risk free rate calibrated in both models serve as the benchmark interest rate and is subtracted from the yield collected from DataStream to obtain the actual yield of the bond. As can be seen from the graph, all expect an increasing trend of credit spread during the period 2010 to 2012 while both Merton and Leland models underestimate the credit spreads. The underestimation of Merton model is consistent with the finding from Eom et al. (2004). Furthermore, the low leverage of the company makes the underestimation worse which matches the finding from Sarig and Warga (1989) that the low leverage leads to a lower default probability and lower credit spread. Nevertheless, the underestimation of the Leland model is in sharp contrast with the conclusion from the paper ,which claims that Leland model usually overestimates bond spread due to the simple assumption of perpetual coupon payment. What is consistent with most research is that as time to maturity is becoming closer, the credit spread widens.

Both models have inherent limitations and are problematic for the prediction accuracy and the respective merit and shortcomings are apparent. One shared problem is that both models only consider firm-specific factors while in fact the credit risk is not the only determinant of credit spread such as systematic factors, corporate regulation and strategy changes.

For the Merton model, which assumes default occurs only at maturity and the value of equity is treated as a European call option, the implementation is straightforward and simple. Also, the sensitivity of the credit spread change contingent on leverage change is strong and observable, which can be seen from the Figure 3. On the other hand, the naïve assumptions make the model far from realistic. The default probabilities for each year are quite low while the credit rating for the company is BBB- for 3 year. The model excludes analysis of capital structure without introduction of debt, tax, and bankruptcy cost, factors that are common in real life.

Regarding the Leland model, the firm's capital structure is endogenous. The inclusion of tax benefits and bankruptcy cost makes it more reasonable. It assumes that default can occur any time before debt maturity if the equity value becomes negative. Therefore, the value of equity is a represented by an American option. However, the assumptions of perpetual coupon, static capital structure and unchanged firm activities are unrealistic and usually cause an overestimation of credit spread. Furthermore, the model fails to take into account that the bankruptcy leaves scope for renegotiation of debt. Therefore, one of the extensions of Leland model is the strategic debt service model, such as one proposed by Fan and Sundaresan (2000).

4. Conclusion

The paper studies the empirical performance of two structural credit risk models: Merton (1974) and Leland (1994). The results show that both models predict rather low default probabilities of National Express during the period 2010 to 2012 while the credit spread experiences an upward trend. The same conclusion is reached from the accounting ratio analysis, where the Z scores of the firm fall from safe zone in 2010 to grey zone in 2012. However, compared with the real credit spread, both models underestimate the credit spread with Merton model prediction exhibits greater volatility. Since the company has prudent gearing, the underestimation becomes more serious. As pointed by most research, credit risk models are problematic inherently and have low predictability in estimating credit spread.

In fact, the credit spread not only reflects credit risk component, but also is contingent on market conditions and other unknown factors. For National Express specifically, government policy, regulatory changes and macroeconomic factors have profound influence on the financial performance and investment strategy. Apart from issuing bonds, the group also has finance lease properties, interest rate and foreign exchange derivatives. Further empirical research may need to extend market and industrial analysis of the models, comparing performance under different market conditions and countries.

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