Financial Intermediation and Credit Spreads

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Abstract

This dissertation presents the numerical solution of the model developed in Correia, I., F. De Fiore, P. Teles, O. Tristani (2012). In this framework, financial intermediation takes place with private intermediaries facing endogenously determined balance sheet constraints. I compute the approximate solution of the problem of a Ramsey planner in response to several exogenous shocks. The response to these shocks under optimal policy isolates the financial sector from the rest of the economy so that the financing cost of firms does not increase and allocations are not distorted. Furthermore, I show that for a given price level on impact there is always a nominal interest rate path that satisfies the financial constraint and replicates the first best allocations. In this framework, indeterminacy in price level leads to multiple solutions for the optimal nominal interest rate policy.

Keywords: financial intermediation, financial frictions, credit costs, optimal monetary policy

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Part I
Introduction

Over the past few years, most of the industrialized world has experienced a major financial crisis, the worst of the post-war era. This crisis was followed by a global recession, whose effects are still evident in many economies. A distinguishing feature of the present downturn was the significant disruption of financial intermediation.

During financial crises, financing costs tend to increase. After the Lehman Brothers collapse the credit costs peaked, which was considered to be a major factor in the fall of durable goods spending in the last quarter of 2008. This in turn was followed by a contraction in output and a rise in unemployment.

In the attempt to stabilize credit markets, authorities embraced a variety of unconventional monetary and fiscal policies. The Federal Funds was cut until the zero bound was reached, and the Federal Reserve began to act directly by injecting funds into private markets, aiming to reduce credit costs. While expanding central bank intermediation, the Fed attempts to offset the disruption of private financial intermediation.

Motivated by these events, there as been a growing literature that incorporates financial factors within the quantitative macroeconomic framework. In Gertler and Kiyotaki (2010) much of this work is surveyed. In order to illustrate how a disruption in financial intermediation can induce a downturn in the economy, but also to analyze quantitatively how the unconventional credit policies of the central banks might work during a financial crisis, they develop a baseline framework that incorporates financial intermediation. As in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), an agency problem between borrowers and lenders is introduced. This agency problem endogenizes financial market frictions. In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) (GKa, henceforth) a distinct approach is used when it is assumed that the credit constraints are not faced by the non-financial borrowers, but by financial intermediaries instead. Here the credit spread depends on their balance sheets, in particular on the level of their internal funds.

The analysis in GKa is focused on the policies adopted by public authorities in the United States in response to the 2008 financial crisis. In that framework, the central bank acts as a financial intermediary during crisis periods. During a financial crisis, with a disruption of private credit channels, a central bank intervenes to compensate for that disruption.

Private intermediation on the other hand, is more efficient during normal times. These intermediaries obtain funds from depositors and channel them to non-financial firms. In addition, an agency problem is introduced so that borrowing and lending depends on intermediaries capital structure. The credit costs in the economy, that impact real activity, are also affected by intermediaries own funds. This feedback between the financial and the real sector is usually referred to as the "financial accelerator" effect. In a crisis event with a deterioration of intermediaries funds, financing costs increase. This depresses real activity, that in turn further deteriorates the balance sheet of the banks.

The setup used in Correia et al. (2012) builds on the framework developed in GKa. However here the central bank does not work as a financial intermediary. The assumptions on the structure of private intermediaries are nevertheless identical. In this economy, firms need funds in order to pay the wage bill. Financial intermediaries provide these funds, using deposits and their own internal funds. They accumulate internal funds because of a costly enforcement problem: the managers of the "banks" have the possibility, in the beginning of each period, to divert a fraction of their assets (claims on non-financial firms). Accordingly, in order for intermediaries to be willing to continue to operate, the fraction of assets they can divert in each period has to be balanced by the discounted value of the retained earnings from their activity. As we will see in more detail in the next section as long as this constraint is binding, the credit spread (defined as the difference between the return on loans to the firms and the nominal interest rate) is positive. Without this incentive compatibility constraint, intermediaries would expand their assets until the rates of return adjust so that the difference between nominal interest rate and the return on financial assets was zero.

In this MSc dissertation, I will introduce this model and present the numerical solutions of the responses to various exogenous shocks. With this it is possible to evaluate how the financial friction introduced affects the economy.

The impulse responses refer to an equilibrium in which policy is described by the first order conditions of a Ramsey planner. With this exercise it was possible to understand that in this setup there are multiple optimal policy responses to a shock: it is possible, for any given initial price level, to choose a path for the nominal interest rate that is consistent with first best allocations. Indeterminacy in price level is not a surprising result in a standard monetary model, but in this framework nominal funds of intermediaries are predetermined. These intermediaries are financially constrained and so real assets required for production every period are restricted by banks' balance sheets. Here indeterminacy in price level implies multiple optimal decisions for the nominal interest rate. I will present this result with an illustration in the concluding section of this dissertation.

The structure of this paper is as follows. In Part II, I start by outlining the environment and describing the equilibrium conditions. Also, I briefly describe a steady state of the model. In Part III, I provide the numerical results of the impluse responses to three different shocks. I also discuss the result that in this framework there are multiple optimal policy decisions and provide an illustration. In Part IV, I conclude with some final comments.

Part II
The Model

The model that will be analyzed throughout this text is presented in Correia et al. (2012). In this economy, firms need external funds to finance production. They raise these funds in the beginning of each period in order to pay for the wage bill.

There is single good produced with a linear technology that uses labor only. Aggregate productivity is stochastic, and there are no idiosyncratic shocks.

The representative household has a continuum of members of measure unity, and has preferences over consumption and labor. These members can be of two different types: "workers" or financial intermediaries, "bankers". Workers supply labor to production; bankers manage a financial intermediary.

The financial intermediaries channel funds from depositors to firms. They finance production with deposits from households and their own internal funds. Because of a costly enforcement problem, banks need to have internal funds. If the bankers would be able to operate forever, they could accumulate enough funds so that they would eventually become financially unrestricted. Accordingly in this setup it is assumed to occur a shift between occupations of the household members: every period there is a certain fraction of bankers that become workers, and of workers that become bankers. In this way expected lifetime of bankers is finite.

In the following subsections, we will look with more detail into the structure of this economy.

1. Production

Before describing the economy with financial intermediaries, let us describe the physical environment. The aggregate uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$, where S_t is the set of possible realizations of s_t. The history of its realizations up to period t (state at t), $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. We assume that s_t has a discrete distribution. $Pr(s^{t+1}|s^t)$ is the probability of state s^{t+1} conditional on state s^t. Variables indexed by t are thus a function of the state s^t.

There is a representative firm endowed with a stochastic technology that transforms N_t units of labor into $A_t N_t$ units of output. $\ln A_t$ is an AR(1) aggregate productivity shock. In the beginning of period t, the firm needs to pay the salaries, before receiving the revenues from production. They have to raise external funds S_t in order to pay the wage bill, $W_t N_t$:

$$W_t N_t \leq S_t \tag{1}$$

In the end of each period, profits are then given by

$$\Pi_t^f = P_t Y_t - R_t^l W_t N_t$$

with Y_t representing period t output, P_t the price and R_t^l the gross interest rate paid for the loans to the firms.

The profit maximizing condition implies that in each period

$$P_t A_t = W_t R_t^l \tag{2}$$

2. Households

The household sector is structured in a way that permits maintaining the tractability of the representative agent approach.

A representative household is a continuum of members of measure unity. Within each household, there is a fraction $1 - f$ of "workers" and f of "bankers". Workers supply labor and return their labor income back to the household; bankers manage a financial intermediary and similarly transfer earnings back to the household, at the time they exit the banking activity. The entire household shares consumption.

Over time, a given individual of the household can switch between the two occupations. In particular, a member who is a banker in period t continues as a banker in the subsequent period with a given i.i.d. probability θ. As will become clear in the following sub-section, this finite time horizon for the bankers insures that they don't live enough so that they can eventually overcome the financial constraints they face, i.e. that they don't retain enough earnings so that they can finance all the production with their own funds only. As a consequence, in each period $(1-\theta)f$ bankers exit and become workers. To keep the number in each occupation constant over time, a similar number of workers randomly become bankers. When exiting, bankers transfer their accumulated earnings to the household. As will be described in more detail in the next sub-section, the household gives the new bankers some start-up funds in order for them to start operating as financial intermediaries.

In the beginning of period t, households decide on one-period riskless deposits denominated in units of currency D_t that pay $R_t D_t$ in period $t+1$. The households also decides on a portfolio of nominal state-contingent bonds, B_{t+1}, each paying one unit of currency in a particular state in period $t+1$ and each costing $Q_{t,t+1}$ units of money at t.

With wealth in the beginning of period t given by Ω_t, the households budget constraint is

$$\begin{gathered} E_t Q_{t,t+1} B_{t+1} + D_t \leq \Omega_t, \\ \Omega_{t+1} = B_{t+1} + R_t D_t - P_t C_t + W_t N_t + \Pi_t^b - T_t \end{gathered} \tag{3}$$

where E_t is the mathematical expectation conditional on the household's time t information set, C_t is time t consumption, P_t the corresponding price, N_t is hours worked and W_t nominal wage. Π_t^b are funds transferred to the household from financial intermediaries (net the transfer households give to its members that enter the banking activity in period t), and T_t are lump sum taxes charged by the government.

The household has preferences over consumption and hours worked, that are ordered by the utility function

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \right\} \tag{4}$$

where u is a strictly concave one-period utility function and $\beta \in (0,1)$ is a discount factor.

The household maximizes (4) subject to (3) and a no-Ponzi games condition. It is further assumed that u is separable in consumption and labor.

Solving this problem gives the following conditions:

$$-\frac{u_C(t)}{u_N(t)} = \frac{P_t}{W_t}, \tag{5}$$

$$\frac{u_C(t)}{\beta u_C(t+1)} = Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}}, \tag{6}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \frac{\beta u_C(t+1)}{P_{t+1}} \tag{7}$$

where $u_C(t)$ and $u_N(t)$ are the marginal utilities in state s^t for consumption and labor, respectively.

2.1. Financial Intermediaries

A financial intermediary channels funds from depositors to the firms.

Let $Z_{j,t}$ be the amount of net worth that a given bank j has at the end of period t; $D_{j,t}$ the deposits from households, and $S_{j,t}$ the financial claims on firms that the intermediary holds. A bank's balance sheet is then given by

$$S_{j,t} = Z_{j,t} + D_{j,t} \tag{8}$$

Households deposits pay the non-contingent gross return R_t, while intermediary assets pay the endogenously determined return R_t^l over period t.

Thus bank's internal funds evolve over time as the difference between earnings on assets and interest rate payments on households' deposits. Accordingly,

$$Z_{j,t+1} = R_t^l S_{j,t} - R_t D_{j,t} = (R_t^l - R_t) S_{j,t} + R_t Z_{j,t} \tag{9}$$

Since financial intermediaries face a financing constraint, they are willing to retain earnings until exiting activity, at which point they transfer accumulated earnings as dividends to the household. Then the intermediaries' objective is to maximize the expected value of accumulated funds:

$$V_{j,t} = E_t \sum_{s=0}^{\infty} (1-\theta)\theta^s Q_{t,t+1+s} Z_{j,t+1+s}$$

or, using expression (9),

$$V_{j,t} = E_t \sum_{s=0}^{\infty} (1-\theta)\theta^s Q_{t,t+1+s} \left[(R^l_{t+s} - R_{t+s}) S_{j,t+s} + R_{t+s} Z_{j,t+s} \right] \tag{10}$$

Looking at the previous expression we know that, in any period, while the discount factor $Q_{t,t+1+s}(R^l_{t+s} - R_{t+s})$ is positive, the intermediary will want to borrow additional funds from depositors in order to increase its assets. A limit on their ability to do is motivated by an incentive compatibility constraint: at the beginning of each period, a banker can choose to divert a fraction λ of its assets.[2] If they do so, they are forced into bankruptcy and depositors can recover the remaining fraction. It is too costly however for depositors to recover the funds diverted. Accordingly, bank managers are willing to continue to operate as long as the amount they loose when diverting funds (the discounted value of retained earnings) is at least balanced by the fraction of funds they can appropriate. This condition can be written as:

$$V_{j,t} \geq \lambda S_{j,t} \tag{11}$$

As shown in Appendix A, the value function $V_{j,t}$ can be written in a separable way:

$$V_{j,t} = \upsilon_t S_{j,t} + \eta_t Z_{j,t} \tag{12}$$

where

$$\upsilon_t = E_t \left\{ (1-\theta) Q_{t,t+1} (R^l_t - R_t) + Q_{t,t+1} \theta \frac{S_{j,t+1}}{S_{j,t}} \upsilon_{t+1} \right\} \tag{13}$$

and

$$\eta_t = E_t \left\{ (1-\theta) + Q_{t,t+1} \theta \frac{Z_{j,t+1}}{Z_{j,t}} \eta_{t+1} \right\} \tag{14}$$

The weights υ_t and η_t on the value function have the interpretation of the expected marginal gain to the banker of expanding assets or net worth by one unit, respectively, while holding the other variable constant. Without the agency problem that was introduced, bankers would expand borrowing to a point where rates of return would adjust to ensure υ_t was zero. However, if the incentive constraint is binding, intermediary's claims on firms are constrained by their level of internal funds.

Making use of this property, the incentive constraint can be rewritten as

$$\upsilon_t S_{j,t} + \eta_t Z_{j,t} \geq \lambda S_{j,t} \tag{15}$$

[2] One may think, as pointed out in Gertler and Karadi (2009), that intermediaries receive deposits from families other than the one they are members of.

Assuming this constraint holds with equality, the nominal assets that an intermediary holds in each period and its predetermined internal funds are related according to:

$$S_{j,t} = \frac{\eta_t}{(\lambda - \upsilon_t)} Z_{j,t} \equiv \phi_t Z_{j,t} \tag{16}$$

where ϕ_t is what I call the leverage ratio, the ratio of intermediated assets to internal funds.

Expanding $S_{j,t}$ while holding $Z_{j,t}$ constant increases bankers incentive to divert funds. With internal funds $Z_{j,t}$ positive, the financial constraint is binding for $0 < \upsilon_t < \lambda_t$. As noted before, under this condition it is profitable for the banker to increase assets' level $S_{j,t}$, since $\upsilon_t > 0$. If υ_t increases above λ the incentive constraint does not bind since the value of the intermediary's discounted earnings, or franchise value $V_{j,t}$, always exceeds the the fraction of funds that can be appropriated.

The banks net worth can be expressed using the relations derived above. Using expressions (9) and (16) :

$$Z_{j,t+1} = \left[(R^l_t - R_t)\frac{S_{j,t}}{Z_{j,t}} + R_t\right] Z_{j,t} = \left[(R^l_t - R_t)\phi_t + R_t\right] Z_{j,t} \tag{17}$$

The growth rates of $Z_{j,t}$ and $S_{j,t}$, respectively $\zeta_{t,t+1}$ and $\xi_{t,t+1}$, can be defined as

$$\zeta_{t,t+1} = \frac{Z_{j,t+1}}{Z_{j,t}} = (R^l_t - R_t)\phi_t + R_t \tag{18}$$

$$\xi_{t,t+1} = \frac{S_{j,t+1}}{S_{j,t}} = \frac{\phi_{t+1} Z_{j,t+1}}{\phi_t Z_{j,t}} = \frac{\phi_{t+1}}{\phi_t}\left[\left(R^l_t - R_t\right)\phi_t + R_t\right] \tag{19}$$

Note that the evolution of internal funds for a given intermediary j does not depend on individual specific factors. We can then sum across individuals to obtain aggregate assets and internal funds of the financial intermediaries. However, when doing so for the internal funds of the banks, we must take into account that in each period only a fraction θ of banks survive (continue to operate). In addition, new banks start their activity each period. As a consequence, the internal funds of bankers in period t is the sum of the funds of existing banks Z^e_t and new banks Z^n_t. The share of retained earnings that continues in the market is given by

$$Z^e_t = \theta\left[(R^l_{t\text{-}1} - R_{t-1})\phi_{t-1} + R_{t-1}\right] Z_{t-1} \tag{20}$$

It is assumed that the household transfers a small fraction of the internal funds that exiting bank had in the end of the previous period. If households transfer a fraction $\frac{\omega}{1-\theta}$ of these funds in the end period $t-1$, then the "new" banks receive a start up amount of

$$Z^n_t = \omega\left[(R^l_{t\text{-}1} - R_{t-1})\phi_{t-1} + R_{t-1}\right] Z_{t-1} \tag{21}$$

and the aggregate evolution of banks net worth is according to

$$Z_t = Z_t^n + Z_t^e = (\theta + \omega)\left[(R^l_{t\text{-}1} - R_{t-1})\phi_{t-1} + R_{t-1}\right] Z_{t-1} \tag{22}$$

3. Government

The government spends G_t and finances spending collecting lump sum taxes T_t. In this setting, government spending is assumed to be an exogenous share g of production every period, as in Fiore et al (2010). The government also issues outside money held by the banks in the form of non-remunerated reserves.

The fact that $g > 0$ will make it optimal to distort the consumption-leisure margin, even if lump sum taxes are available. This parameter pins down the steady state gross return on loans to the firms. With a positive share of government expenditure, the gross return on assets is greater than 1. Also, with a binding financial constraint, the spread (the difference between return on assets and nominal interest rate) is always positive, as noted before. Thus in this setup, for g high enough, the steady state nominal interest rate is greater than zero, so we can abstract from the zero lower bound while studying optimal policy in response to small shocks.

4. Equilibria

The equilibrium conditions are given by equation (1) holding with equality, (2); equations (5) to (7) ;(13), (14) and (16), and the definition

$$\phi_t \equiv \frac{\eta_t}{(\lambda - \upsilon_t)}$$

condition (22), and the resource constraint

$$C_t + G_t = A_t N_t.$$

In the next subsection we will define a steady state with constant inflation and discuss some features of the steady state economy.

4.1. Steady State

In a steady state with constant gross inflation Π, we have that

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} = \frac{S_{t+1}}{S_t} = \frac{Z_{t+1}}{Z_t} = \Pi$$

u_C and u_N denote marginal utilities of consumption and labor evaluated at the steady state. Using the equilibrium conditions enumerated above, we can write the following:

$$R = \frac{\Pi}{\beta} \tag{23}$$

$$\frac{u_C}{u_N} = \frac{R_l}{A} \tag{24}$$

$$WN = S \tag{25}$$

$$S = \phi Z \tag{26}$$

$$\phi = \frac{\eta}{\lambda - \upsilon} \tag{27}$$

The first condition is the Euler equation given by (7) in the steady state. The second condition is the steady state condition for the consumption-labor choice (5) together with the firm's profit maximization condition, equation (2). The fourth condition represents the steady state financing restriction of firms, condition (1), that holds with equality. The last two conditions result from the incentive constraint (16) of intermediaries and the definition of the leverage ratio.

The steady state weights of the value function are

$$\upsilon = \frac{\frac{(1-\theta)}{R}(R^l - R)}{1 - \frac{\theta}{R}\left[(R^l - R)\,\phi + R\right]} \tag{28}$$

and

$$\eta = \frac{(1-\theta)}{1 - \frac{\theta}{R}\left[(R^l - R)\,\phi + R\right]} \tag{29}$$

In a steady state described by the conditions above, the aggregate amount of internal funds of the banks is constant in real terms, which means that

$$\Pi = (\theta + \omega)\left[(R^l - R)\phi + R\right] \tag{30}$$

The growth rates of assets and internal funds for each intermediary are also constant and equal to

$$\zeta = \xi = (R^l - R)\phi + R \tag{31}$$

As discussed before, as long as the balance sheet constraint on intermediaries is binding we have $\upsilon > 0$. In this instance, condition (28) implies that the spread (defined as the difference between the return on loans to the firms and the nominal interest rate) is positive. Thus one can see that for a higher steady state gross inflation, i.e. a higher nominal interest rate, the intratemporal wedge increases.

Additionally, using expression (30) together with (23) it is possible to write

$$\frac{\beta}{\theta + \omega} = \left[(\frac{R^l}{R} - 1)\phi + 1\right]$$

and once again considering a steady state with a binding intermediaries' constraint[3], i.e. if $\left(\frac{R^l}{R}-1\right)>0$ and $\phi>0$, we have that

$$\frac{\beta}{\theta+\omega}>1$$

The steady state leverage ratio can be expressed as a function of exogenous parameters only. Using conditions (27), (28) and (29),

$$\phi=\frac{\beta(1-\theta)}{\lambda\left[\theta(1-\beta)+\omega\right]}$$

We can immediately see that banks will decrease steady state leverage when the appropriation share λ increases, with everything else equal. In other words, if intermediaries have the possibility of appropriating a higher share of the assets they hold, then they will be required to have a higher level of internal funds. Accordingly, the steady state spread also increases (so that, for the same level of assets, retained earnings increase).

In the next section, I will describe the parametrization of the model and some experiments that illustrate how the economy reacts in response to shocks to three exogenous variables.

Part III

Model Analysis

5. Calibration

The model calibration requires 7 parameters, 4 of them are conventional. The parameter values are listed in Table 1. Utility is assumed to be logarithmic in consumption and linear in leisure. The discount factor β is standard. The values for the relative utility weight on labor χ and the inverse of the Frisch elasticity of labor supply φ, are in accordance with the estimate reported in Primiceri et al. (2006), as in GKa.

The parameters related with the financial intermediaries are also chosen in line with what is done in GKa, so that the following three targets are met: a steady state interest rate spread of one hundred basis points; a steady state leverage ratio of four; and an average horizon of bankers of a decade[4].

The government consumption share g is chosen high enough so that a reasonably small but positive steady state nominal interest rate is achieved (as in Fiore, Teles and Tristani, 2010).

[3] Under reasonable parameter values the constraint always binds within a local region of the steady state.

[4] As in GKa, the choice of these parameters is meant to be suggestive.

Households		
β	0.99	Discount rate
χ	3.409	Relative utility weight of labor
φ	0.276	Inverse Fisch elasticity of labor supply
Financial Intermediaries		
λ	0.28	Fraction of assets that can be diverted
ω	0.01	Proportional transfer to entering banks
θ	0.975	Survival rate of bankers
Government		
g	0.02	Output share of government expenditure

Table 1: Parameters.

Government expenditure being proportional to output every period is a technical device to abstract from the zero lower bound on the nominal interest rate. It makes it optimal to distort the consumption-labor decision even if lump sum taxes are available. This is a second best result, and although the solutions presented in the following subsections show allocations that are the same as the ones that result from the maximization of preferences subject to the resource constraint, they incorporate a policy restriction of having an exogenously given proportional government expenditure. If g was to be set optimally, then government expenditure would be zero and as a consequence the return on assets R^l would be one. With a positive spread, this would imply having negative nominal interest rates.

The experiments of the next subsection show how the economy behaves in response to three exogenous shocks. An innovation shock, a financial shock and a government expenditure shock are considered.

Optimal impulse responses result from policy described by the first order conditions of a Ramsey planner choosing allocations for periods $t \geq 1$ in accordance with the timeless perspective in Woodford (2003).

The results displayed below represent the log-linear dynamics of the model.

6. Impulse responses

Optimal policy in this setup requires setting the rate of return on financial claims higher than one as long as $g > 0$. As noted before, the calibrated model displays a positive steady state nominal interest rate, so the optimal response to small shocks that requires movements in the nominal interest rate can be studied ignoring the zero lower bound. For the numerical results displayed in the subsections below I used Dynare and Dynare++[5], that computes the first order conditions of an optimal policy given the constraints, that are then solved

[5] The package is available at http://www.dynare.org/

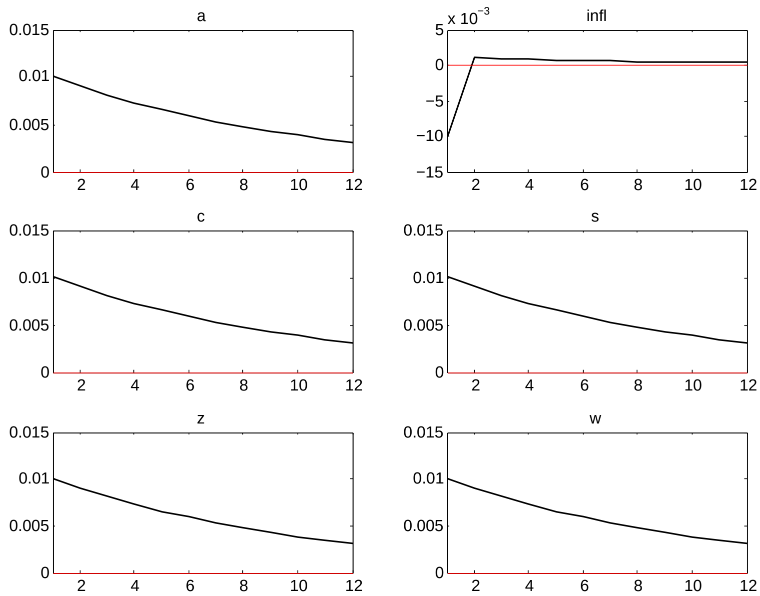

Figure 1: Impulse response to a positive technology shock under optimal policy. Correlation of the shock: 0.9.

in the usual way. I have also had access to the Dynare codes provided by the authors, that I used mostly for comparison purposes.

6.1. Technology shock

Figure 1 illustrates the impulse response of the selected variables to a positive one percent technology shock[6], with a quarterly autoregressive coefficient of 0.9. The variables plotted are the technology process $a_t \equiv \ln(A_t)$, inflation $\pi_t \equiv \ln(P_{t+1}/P_t)$, consumption $c_t \equiv \ln(C_t)$, real wage $w_t \equiv \ln(W_t/P_t)$, real assets $s_t \equiv \ln(S_t/P_t)$ and real internal funds $z_t \equiv \ln(Z_t/P_t)$.

Fig. 1 shows that allocations are as in the first best, with consumption following production exactly and hours worked (not shown) remaining constant throughout the adjustment.

In response to a positive productivity shock, price level decreases in the first period. Here a one percent decrease in price level on impact matches exactly the amplitude of the shock. In this way, and since nominal internal funds are predetermined, the financial restriction is satisfied with real assets and real funds changing by the same percentage deviation. Hence leverage stays constant during all the adjustment period.

In the subsequent periods, the adjustment is made through a smooth increase in price level. All the variables return gradually to their steady state values.

[6]A negative shock would produce symmetric results of the ones presented here.

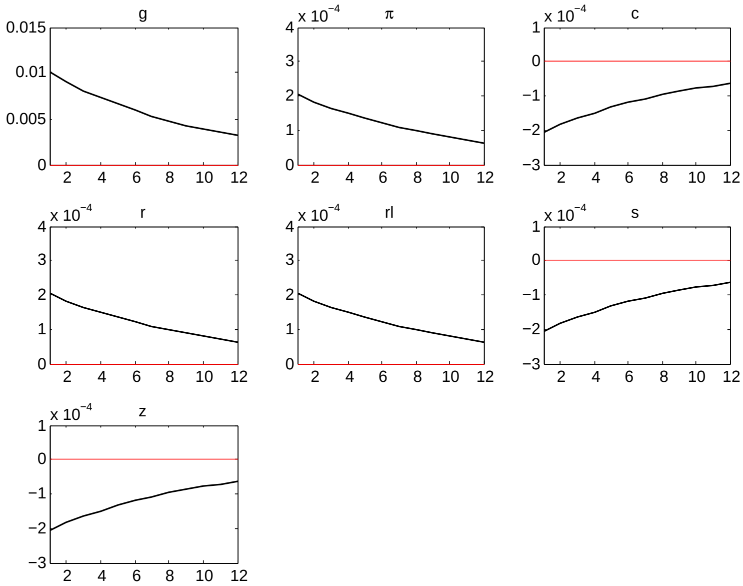

Figure 2: Impulse responses to an increase in the government expenditure share under optimal policy. Correlation of the shock: 0.9.

Inflation completely offsets the effect of the shock. With nominal wage constant during the transition, nominal assets required to finance production are constant too. The return on assets is constant, and since leverage remains constant the spread is also constant in all the periods. This in turn implies that the marginal value of assets and internal funds don't change so that the financial sector is isolated from the effect of the shock.

6.2. Government expenditure shock

Government expenditure is assumed to be exogenously given as a fraction g_t of output every period, that pins down the steady sate return on loans and consequently the steady state nominal interest rate.

The impulse response to a temporary one percent shock on g_t produces effects on inflation, nominal interest rate $r_t \equiv \ln(R_t)$, the gross rate of return on loans $r_t^l \equiv \ln(R_t^l)$, as well as real assets, real internal funds and consumption $c_t \equiv \ln(C_t)$. This can be analyzed in Figure 2.

With R_t^l moving in response to a government expenditure shock, it is optimal that both nominal interest rate and price level change on impact. Consumption choices are distorted but are the same as in a first best solution. With hours worked remaining at the steady state level, adjusting the price level to compensate for the increase in the borrowing cost of producers allows nominal wages and consequently nominal assets of intermediaries to remain constant. Since nominal internal funds are predetermined, the fact that nominal assets are able to be kept constant during the transition implies that the financial constraint

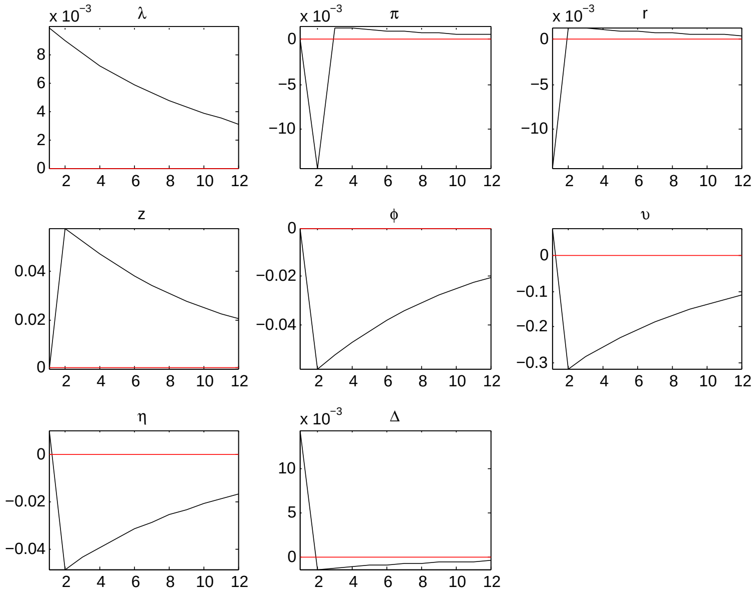

Figure 3: Impulse responses to a positive financial shock under optimal policy. Correlation of the shock: 0.9.

is satisfied with leverage remaining at the steady state. This is similar to the case of a productivity shock, but here nominal interest rates offset the change in the return on loans, so that spreads remain constant throughout all the adjustment process. In this way, the financial sector is again isolated. Production is not affected and consumption adjusts smoothly to the increase in government expenditure.

Also here the spread remains unchanged in response to the shock, and then optimal policy can be such that the financial constraint does not kick in, in the sense that leverage is always constant.

6.3. Financial shock

A financial shock is characterized as an increase (or decrease) in the appropriation parameter λ_t of the financial intermediaries. If we consider a case where there is a positive increment in this share, the result is that for the same level of assets at the moment of the shock, a higher level of internal funds is required by the depositors. But in this setup internal funds are predetermined, so an optimal response displays spreads compensating on impact for the exogenous tightening of the financial restriction on intermediaries.

In the experiment considered in Figure 3, a one percent increase in the parameter λ_t is simulated. Contrary to the previous two shocks, the spread between the lending and the deposits rate $\Delta_t \equiv \ln(R_t^l/R_t)$ and the leverage ratio $\varphi_t \equiv \ln(S_t/Z_t)$ are no longer constant during the transition. As can be seen in the figure, leverage decreases with the increase in real funds. Real assets stay constant, and nominal interest rate changes so that the spread increases on impact. This change in the spread compensates for the more demanding conditions of intermediaries financing restriction. Accordingly, the weights of the financial constraint, i.e. the relative value of assets and internal funds of intermediaries, change on impact.

It is nevertheless possible to have the adjustment occurring in the financial sector only, with allocations not distorted provided that the return on loans remains constant. In this instance there is no effect on output, consumption or hours worked.

After the second period, all variables start to smoothly converge towards the steady state.

Both in the case of a productivity shock and a government expenditure shock, since the spread is constant during the transition, the fact that internal funds are predetermined does not play a role as a restriction in the sense that since leverage stays constant, the incentive compatibility constraint is not affected. Thus the adjustment can be such that the financial sector is isolated and allocations are not distorted. In contrast, in the case of the financial shock the spread changes but the return on loans is kept constant, i.e. the intratemporal wedge is constant, and the allocations are again as in the first best.

If the spread changes in a given period, this implies nominal funds growing at a different rate for the subsequent period. Since it is optimal to have the return on loans constant or following government share of output dynamics, when spreads move nominal interest rate moves too. When the accommodation of the shock is made using price level policy only, as shown above for the productivity and government expenditure shocks, nominal interest rate either stays constant or moves accordingly so that the spread is constant during the adjustment. This is so because when inflation completely offsets the effects of shocks, nominal assets remain constant and since internal funds are predetermined, leverage stays constant. If the spread was to move on impact, the marginal value of internal funds and assets on intermediaries balance sheet would be changing, and thus leverage, so that we could no longer have nominal assets unchanged.

The first order approximations of the impulse responses under optimal policy can be performed using Dynare. We can also use Dynare++ to solve the same problem, and while doing so I obtained solutions that were not the same as the ones presented in the last subsections. The solutions using Dynare have the common feature that the accommodation of the shock is made exclusively using price level policy. With the exception of the financial shock, spreads remain constant during the convergence towards the steady state. In contrast with Dynare++ we obtain multiple optimal policy responses.

For a given price level on impact, there is a corresponding nominal interest rate path that is able to replicate the first best allocations. This was a somehow unexpected result. In this framework optimal policy does not pin down price level, a usual result in a standard monetary model, and this implies multiple optimal nominal interest rate decisions. Even though real assets required to finance production are restricted by predetermined nominal internal funds, there is always a pair of price level and nominal interest rate decisions that replicates first best allocations, so that we obtain multiple nominal interest rate paths that solve the planner's problem.

In the next subsection I present an example of this result.

6.4. An example of multiple optimal policy trajectories

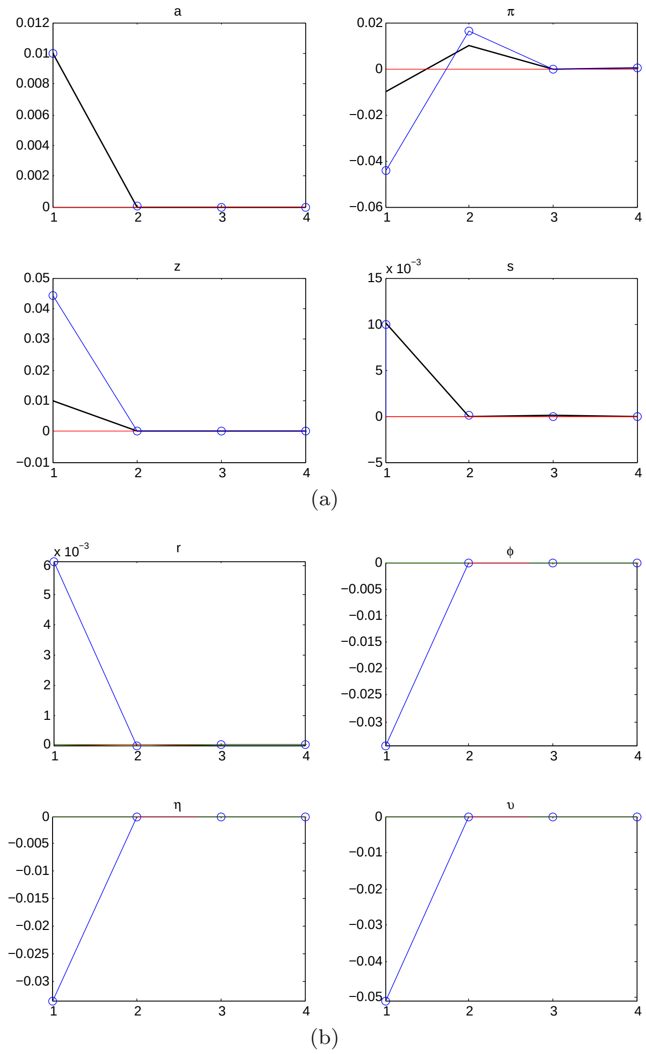

Figure 4: One percent uncorrelated productivity shock.

For simplification, let us consider an uncorrelated one percent productivity shock. Figure 4 shows two different nominal interest rate and price level paths that are optimal responses to this shock. In black, there is what I will refer to as the baseline solution. It is similar to the one seen before Figure 1, in the sense that the financial sector is isolated because spreads are constant during the transition.

The blue lines refer to a solution where that is not the case, i.e. nominal interest rate changes on impact and the accommodation of the shock is not made exclusively with inflation. Both solutions provide allocations as in the first best, i.e. hours worked remaining constant and consumption following productivity one-to-one. The gross return on loans is constant too. In the baseline case, with spreads constant during the transition, the leverage ratio and the relative weights of assets and internal funds of banks' restriction are also constant. In contrast, the alternative solution with nominal interest rate changing, leverage and the relative value of funds and assets changes. In this way, the restriction on financial intermediaries is satisfied precisely with a change in the relative value of assets and retained earnings. When inflation does not completely offset the shock, nominal assets change due to the change in nominal wage. Since internal funds of intermediaries are predetermined in each period, the only way to satisfy the condition on banks capital structure is to change the relative value of its components during the adjustment. This can be seen in Figure 4(b), noting that η and υ decrease in the first period, in contrast with the baseline solution where these weights remain constant. This variation is accompanied with a change in nominal interest rate on impact, and thus the spread, that affects the value of assets and internal funds in the intermediaries objective function.

In Appendix B, I analyze the conditions that define equilibria in this model and show that the system of equations is indeterminate, confirming the result illustrated in Figure 4.

Part IV
Concluding remarks

In this MSc dissertation, I analyzed the model presented in Correia et al. (2012).

I discussed the result that optimal policy is not uniquely determined. The results presented in subsections 6.1 and 6.2 are what I referred to as the baseline solutions. Here optimal policy requires price level to offset the effects of the shock, while the nominal interest rate path is such that spreads are always constant during the adjustment. Optimal policy in response to a financial shock has spreads changing so that the marginal value of assets and own funds compensates for the tightening of the financial restriction on banks' balance sheets. In any case, provided that the return on financial assets is constant the consumption-labor choice is not distorted and allocations are as in the first best.

I show that these solutions are not unique. In this economy firms need external funds in order to pay the wage bill. Financial intermediaries channel funds from depositors to firms, and need to accumulate internal funds during the period of their activity due to a costly enforcement problem and because they don't live forever. A positive credit spread emerges as the difference between the return on their assets and the risk free return on deposits. As noted before, the exogenously given output share of government expenditure determines the rate of return on loans to the firms, so that for positive government expenditure it is optimal to distort the intratemporal choice of the agents, even when lump sum taxation is available. Under optimal policy, the spread that affects the marginal value of the banks' balance sheet components is determined by the nominal interest rate, since the financing cost of firms is constant while government expenditure is constant. In this framework real assets are required to finance production every period. Due to an agency problem between borrowers and lenders, these assets are restricted by intermediaries' level of internal funds that are predetermined. Thus in a given period the price level determines the real value of intermediaries internal funds. I showed that for a given price there is a corresponding optimal nominal interest rate path that satisfies the financial restriction and simultaneously replicates first best allocations. This leads to multiple optimal trajectories for inflation and nominal interest rates.

Part V
Appendix

Appendix A

Bankers objective is to maximize expected terminal wealth

$$V_{j,t}(Z_{j,t}, S_{j,t}) = E_t \sum_{s=0}^{\infty} (1-\theta)\theta^s Q_{t,t+1+s} Z_{j,t+1+s}$$

Rewriting the value function:

$$\begin{aligned} V_{j,t}(Z_{j,t}, S_{j,t}) &= (1-\theta)E_t Q_{t,t+1} Z_{j,t+1} + E_t \textstyle\sum_{s=1}^{\infty} (1-\theta)\theta^s Q_{t,t+1+s} Z_{j,t+1+s} = \\ &= (1-\theta)E_t Q_{t,t+1} Z_{j,t+1} + E_t Q_{t,t+1}\theta \textstyle\sum_{s=0}^{\infty} (1-\theta)\theta^s Q_{t+1,t+2+s} Z_{j,t+2+s} = \\ &= (1-\theta)E_t Q_{t,t+1} Z_{j,t+1} + Max E_t Q_{t,t+1}\theta V_{j,t+1}(Z_{j,t+1}, S_{j,t+1}) \end{aligned}$$

The conjecture is $V_{j,t}(Z_{j,t}, S_{j,t}) = \upsilon_t S_{j,t} + \eta_t Z_{j,t}$. Assuming the incentive compatibility constraint is binding:

$$\upsilon_t S_{j,t} + \eta_t Z_{j,t} = \lambda S_{j,t}$$

Recalling the condition for the accumulation of internal funds of bank j:

$$Z_{j,t+1} = (R_t^l - R_t) S_{j,t} + R_t Z_{j,t}$$

Replacing we have

$$\upsilon_t S_{j,t} + \eta_t Z_{j,t} = (1-\theta)E_t Q_{t,t+1} \left[(R_t^l - R_t)S_{j,t} + R_t Z_{j,t}\right] + E_t Q_{t,t+1}\theta \left[\upsilon_{t+1} S_{j,t+1} + \eta_{t+1} Z_{j,t+1}\right]$$

We have defined before the growth rates of assets and internal funds as

$$\zeta_{t,t+1} = \frac{Z_{j,t+1}}{Z_{j,t}} = (R_t^l - R_t)\phi_t + R_t$$

$$\xi_{t,t+1} = \frac{S_{j,t+1}}{S_{j,t}} = \frac{\phi_{t+1} Z_{j,t+1}}{\phi_t Z_{j,t}} = \frac{\phi_{t+1}}{\phi_t} \left[\left(R_t^l - R_t\right)\phi_t + R_t\right]$$

Using these expressions we can substitute $S_{j,t+1}$ and $Z_{j,t+1}$ in order to have

$$\upsilon_t S_{j,t} + \eta_t Z_{j,t} = (1-\theta)E_t Q_{t,t+1} \left[(R_t^l - R_t)S_{j,t} + R_t Z_{j,t}\right] + E_t Q_{t,t+1}\theta \left[\upsilon_{t+1}\xi_{t,t+1} S_{j,t} + \eta_{t+1}\zeta_{t,t+1} Z_{j,t}\right]$$

Accordingly, by inspection it is possible to write the expressions for υ_t and η_t, equivalent to conditions (13) and (14):

$$\upsilon_t = E_t \left\{(1-\theta)Q_{t,t+1}(R_t^l - R_t) + Q_{t,t+1}\theta\xi_{t,t+1}\upsilon_{t+1}\right\}$$

and

$$\eta_t = E_t \left\{(1-\theta) + Q_{t,t+1}\theta\zeta_{t,t+1}\eta_{t+1}\right\}$$

Appendix B

Let us start by summarizing the conditions that define the equilibrium of the model. These conditions are the following:

$$-\frac{u_C(t)}{u_N(t)} = \frac{R_t^l}{A_t}, \tag{.1}$$

$$P_t A_t = W_t R_t^l \tag{.2}$$

$$W_t N_t = S_t \tag{.3}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \frac{\beta u_C(t+1)}{P_{t+1}} \tag{.4}$$

$$S_t = \phi_t Z_t \tag{.5}$$

$$Z_t = (\theta + \omega)\left[(R_{t\text{-}1}^l - R_{t-1})\phi_{t-1} + R_{t-1}\right] Z_{t-1} \tag{.6}$$

with

$$\phi_t \equiv \frac{\eta_t}{\lambda - \upsilon_t} \tag{.7}$$

where the weights are given by:

$$\upsilon_t = E_t \left\{ (1-\theta)\frac{\beta u_C(t+1)}{u_C(t)} \frac{1}{\Pi_{t+1}} (R_t^l - R_t) + \frac{\beta u_C(t+1)}{u_C(t)} \frac{1}{\Pi_{t+1}} \theta \left[\left(R_t^l - R_t\right)\phi_t + R_t\right] \frac{\phi_{t+1}}{\phi_t} \upsilon_{t+1} \right\} \tag{.8}$$

$$\eta_t = E_t \left\{ (1-\theta) + \frac{\beta u_C(t+1)}{u_C(t)} \frac{1}{\Pi_{t+1}} \theta \left[\left(R_t^l - R_t\right)\phi_t + R_t\right] \eta_{t+1} \right\} \tag{.9}$$

and the resource constraint

$$C_t + G_t = A_t N_t \tag{.10}$$

Considering the example in Figure .4, we can study the necessary and sufficient conditions that define this problem. Given the first best allocations, condition (.1) is satisfied provided that the gross return on loans to the firms, R_t^l, is constant and at the steady state. Condition (.2) can be satisfied with W_t and (.3) with nominal assets S_t. The Euler condition (.4) can be satisfied with prices P_t. The financial constraint, expressed in condition (.5), with the leverage ratio defined as in (.7) and the weights given by (.8) and (.9), can be satisfied with a spread that is defined by the nominal interest rate R_t. And this is for any given level of nominal internal funds Z_t, that are predetermined in period t. But while proceeding in this way it seems that we have already pinned down price level.

For simplification, let us consider now the deterministic case and describe the preferences of the household, and then proceed to investigate the existence of indeterminacy in the system of equations.

Defining the spread as

$$\Delta_t \equiv \left(\frac{R_t^l}{R_t} - 1 \right)$$

we can rewrite conditions (.8) and (.9) as

$$\upsilon_t = \left\{ (1-\theta)\Delta_t + \theta \left[\Delta_t \phi_t + 1 \right] \frac{\phi_{t+1}}{\phi_t} \upsilon_{t+1} \right\} \quad (.11)$$

and

$$\eta_t = \left\{ (1-\theta) + \theta \left[\Delta_t \phi_t + 1 \right] \eta_{t+1} \right\} \quad (.12)$$

where we have already used (.4) to simplify the two expressions.

Preferences are described by

$$u = \log C_t - \frac{\chi}{1+\varphi} N_t^{1+\varphi}$$

so that

$$u_C(t) = \frac{1}{C_t}$$

$$u_N(t) = -\chi N_t^{\varphi}$$

and government expenditure given by

$$G_t = g A_t N_t$$

For simplification $g = 0$ and $\varphi = 0$. The first best allocations are given by

$$C_t = \frac{A_t}{\chi}, \; N_t = \frac{1}{\chi}$$

Hence, from condition (.1) we have that $R_t^l = 1$ for every t.

From conditions (.2) and (.3), taking the first best allocations as above and using W_t, we have that

$$P_t C_t = S_t$$

Using this with the Euler equation

$$\frac{P_{t+1} C_{t+1}}{P_t C_t} = R_t \beta$$

$$R_t = \beta^{-1} \frac{S_{t+1}}{S_t}$$

If the financial constraint (.5) holds, we can write a condition implied by that constraint and the condition above:

$$R_t = \beta^{-1} \frac{\phi_{t+1}}{\phi_t} \frac{Z_{t+1}}{Z_t}$$

Using this with the law of motion of capital accumulation to substitute out internal funds Z_t, we have that

$$\beta = \frac{\phi_{t+1}}{\phi_t} (\theta + \omega) [\Delta_t \phi_t + 1]$$

or

$$\frac{\beta}{\theta + \omega} = \frac{\phi_{t+1}}{\phi_t} [\Delta_t \phi_t + 1] \tag{.13}$$

The left hand side of this equation can be written in terms of steady state inflation and nominal interest rate, using the conditions in subsection 2.4.1:

$$\frac{\Pi}{R} = (\theta + \omega) [\Delta \phi + 1]$$

or

$$\beta = (\theta + \omega) [\Delta \phi + 1]$$

Thus equation (.13) can be rewritten as

$$[\Delta \phi + 1] = \frac{\phi_{t+1}}{\phi_t} [\Delta_t \phi_t + 1]$$

or, rearranging

$$\frac{\phi_{t+1}}{\phi_t} = \frac{\Delta \phi + 1}{\Delta_t \phi_t + 1} \tag{.14}$$

Under these conditions, using condition (.7) and substituting the weights by (.11) and (.12), we have

$$\phi_t (\lambda - \upsilon_t) = \eta_t$$

$$\phi_t \lambda - (1 - \theta) \Delta_t \phi_t - \theta [\Delta_t \phi_t + 1] \upsilon_{t+1} \phi_{t+1} = (1 - \theta) + \theta [\Delta_t \phi_t + 1] \eta_{t+1}$$

or

$$\phi_t \lambda - (1 - \theta) [\Delta_t \phi_t + 1] = \theta [\Delta_t \phi_t + 1] (\eta_{t+1} + \upsilon_{t+1} \phi_{t+1})$$

Note that $\eta_{t+1} + \upsilon_{t+1}\phi_{t+1} = \phi_{t+1}\lambda$, by the definition (.7)
Then

$$\phi_t\lambda - (1-\theta)\left[\Delta_t\phi_t + 1\right] = \theta\left[\Delta_t\phi_t + 1\right]\phi_{t+1}\lambda$$

or

$$\phi_t\lambda - (1-\theta)\left[\Delta_t\phi_t + 1\right] = \theta\phi_t\left[\Delta_t\phi_t + 1\right]\frac{\phi_{t+1}}{\phi_t}\lambda$$

The ratio $\frac{\phi_{t+1}}{\phi_t}$, or ϕ_{t+1}, only appears once in the two conditions left. We can use the condition (.14) to substitute it out in the last one, yielding

$$\phi_t\lambda - (1-\theta)\left[\Delta_t\phi_t + 1\right] = \theta\phi_t\left[\Delta\phi + 1\right]\lambda$$

or

$$\frac{\Delta_t\phi_t + 1}{\phi_t} = \lambda\frac{(1-\theta\left[\Delta\phi + 1\right])}{1-\theta} \tag{.15}$$

Recalling that in a steady state with constant inflation, the expressions of the weights on the financial constraint give

$$\eta = \left[\frac{(1-\theta\left[\Delta\phi + 1\right])}{1-\theta}\right]^{-1}$$

and also

$$\frac{\lambda}{\eta} = \frac{\Delta\phi + 1}{\phi}$$

Condition (.15) is equivalent to

$$\frac{\Delta_t\phi_t + 1}{\phi_t} = \frac{\Delta\phi + 1}{\phi} \tag{.16}$$

This is a necessary and sufficient condition, so that for any nominal interest ratio choice at t (with R_t^l fixed) we have a corresponding leverage ratio that satisfies all the equilibrium conditions, given the allocations.

Part VI
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