Variance Improved Performance

Nuno Clara

January 2013

Abstract

We propose a simple and efficient way of forecasting the term structure of swap rates and we demonstrate how an investor might benefit from (i) the variance swap as an asset; and (ii) from the implied information present on the swap rate. We show that the Nelson-Siegel model is enough to capture the dynamics of the swap rate term-structure and that the three factors may be interpreted as the level, slope and curvature of the curve. Further, we show that the expected change in the swap rate predicts the one-month forward market return with an OOS R^2 of 2.9%. An investment strategy in both the variance swap and the underlying yields out-of-sample annualized Sharpe ratios around 1.89 which are robust across several different portfolios.

Professor José Faias

Supervisor

Dissertation submitted in partial fulfillment of requirements for the degree of Master of Science in Finance, at Universidade Católica Portuguesa, January 2013.

Acknowledgements:

I am very grateful for the opportunity I had of studying Empirical Finance which gave me a new perspective on Finance and inspired me on this dissertation.

First, I would like to thank my supervisor Professor José Faias for his continuous support and guidance even before the start of this thesis. Thank you for the many fruitful (and long) discussions which allowed me to overcome all the mishaps, deadlocks and dead ends that arose during my path. I honestly hope to keep working with Professor José Faias during my career.

Second, I would like to thank all my friends from the teaching's assistant room and the 3rd floor for all the great moments we shared during the past couple of years. Further, thanks to Miguel Salgado for being my *3rd floor buddy* during the writing of this dissertation; Duarte Ribeiro for useful comments; and Fundação para a Ciência e Tecnologia (FCT) and Católica-Lisbon Research Unit for financial support.

Third, a special thank you to Martinha for her support during the entire dissertation and for being there during the countless frustrating moments of this work.

Finally, I must thank my parents for constant guidance and support during the amazing five years that I spent at Católica-Lisbon.

Contents

1 Introduction

Variance as an asset has gained importance over the past decade with trading volume on VIX futures increasing 113% on average per year since 2008 (according to the Chicago Board of Options Exchange). This volume is likely to increase exponentially during the next couple of years, as variance swaps start being traded on regulated exchanges (such as the CBOE) in addition to the over-the-counter markets.[1] Volatility is indeed interesting as an asset class for an investor because it tends to increase when uncertainty and risk increase, it is mean reverting (Schwert (1989)), and it is negatively correlated with the stock or index level (e.g. Ang et al. (2006); Carr and Wu (2009)), providing an effective hedge against market crashes. Carr and Madan (1998) propose three ways for an investor to trade realized volatility: either through a static position in a straddle, hedging through options the price risk, or investing directly on an over-the-counter (OTC) variance swap which pays the difference between the realized variance and the swap rate. Clearly, the first alternative suffers from significant price exposure if the underlying moves away from its value when the position was opened (one way to avoid this would be to engage in a costly delta-hedge) and the second option suffers from having a price dependent path profit/loss. The variance swap, in its turn, only has pure volatility exposure and might be valued through an option replicating portfolio without relying on the restrictive assumptions of the Black-Scholes model (Britten-Jones and Neuberger (2000); Jiang and Tian (2005)).

The average profit or loss for one dollar investment in a variance swap is given by the difference between the realized variance and the swap rate. This difference, also called the variance risk premium, has been thoroughly documented to be negative for aggregate stock indexes (e.g. Carr and Wu (2009); Han and Zhou (2012)) and there is a mixed evidence on individual stocks as some researchers document a negative variance risk premium (e.g. Carr and Wu (2009)) and others a slightly positive (e.g Driessen et al. (2009); Han and Zhou (2012)). We find that for the S&P 100 the variance-risk premium is negative for all maturities (ranging from one month to twenty-four months), and slightly positive for the

[1] http://www.bloomberg.com/article/2012-10-01/amkZBt2qVqYM.html .

average individual stocks in the index. This asymmetry, led Driessen et al. (2009) to argue that correlation risk is priced in the market. Carr and Wu (2009) showed that the index and firm specific variance risk-premium cannot be explained by the standard risk-factors such as the CAPM and the Fama and French (1993) factors implying that either there is some inefficiency in the market for variance or that the variance risk is another risk factor heavily priced by the market.

We find that a simple curve-fitting model (the Nelson-Siegel exponential components) is enough to model the term-structure of the variance swap rates with good in-sample fit and good out-of-sample (OOS) forecasts of the next period's term-structure. This result is robust for both the index and the individual stocks. Our approach clearly contrasts with the popular approaches to variance swap rates term-structure modeling (e.g. Aït-Sahalia et al. (2012), Egloff et al. (2010), Buehler (2006)) which belong to the affine class of term-structure modeling. We show that our three-parameter model evolving dynamically (which imposes a structure on factor loadings) is able to capture with high computational efficiency the term-structure of swap rates and that each parameter may be interpreted as the level, slope and curvature of the term-structure.

Our research is also related to the return predictability strand of literature. We propose a new predictor and find that expected changes in the swap rate (which may be interpreted as expected changes in the market volatility or as a proxy for the expected variance swap return) predict the monthly S&P 100 returns with an OOS R^2 of 2.9% (as defined in Goyal and Welch (2008)). Other authors use market variance related variables to predict the stock market return, the most prominent example being Pollet and Wilson (2010) who find that correlation predicts the stock market monthly return with an OOS R^2 of 1.26% but that the average variance has no forecasting power whatsoever.

The final strand of literature to which our dissertation relates is the asset allocation. We show that investing in both the market index and the one-month variance swap yields large Sharpe ratios and certain equivalents even during a period in which the market Sharpe ratio was negative. Our thesis does not fit on the pure stock asset allocation strategies (such as

DeMiguel et al. (2009)) nor on the pure variance swap allocation strategies (Madan (2009)), as we allow the investor to allocate on both the stocks and the variance derivative. Egloff et al. (2010) uses a term-structure affine model to find the optimal weights on the S&P 500 index and the two and twenty-four month variance swaps, whereas Hafner and Wallmeier (2008) use a mean-variance analysis to allocate between both the DAX index and the ESX index and the corresponding 45 days variance swap. We use a mean-variance framework to allocate between the stocks and variance swaps because it has the advantage over Egloff et al. (2010) model of allowing the optimal weights to evolve dynamically over time.

We find that it is optimal for the investor to short the shorter maturity S&P 100 variance swap due to the high negative variance risk premium. Investing on both the index, its swap and the risk-free allows the investor to achieve a Sharpe ratio of 1.89 and certain equivalent of 37.38%. Yet, we find that the investment performance can be enhanced by sorting stocks of the S&P 100 on portfolios based on their previous month variance risk premium. In fact, our deciles approach clearly show that for individual stocks it is optimal to be long (short) on the variance swap if the variance risk premium has been positive (negative) on the previous month. The extreme portfolios Sharpe ratios (i.e. the ones built based on stocks with the highest or lowest variance risk premium) achieve annualized Sharpe ratios around 2.48 (0.21) and certain equivalents around 84.86% (3.37%) for the bottom (top) variance risk premium sorted portfolios.

The remainder of the study is organized as follows. Section 2 describes the methodology used to estimate the variance swap rates and the variance risk premiums. Section 3 describes the data used. Section 4 investigates the variance swap rates term structure. Section 5 investigates market returns prediction using swap rates. Section 6 presents two asset allocation strategies that allow the investor to profit from the variance swap as an asset. Section 7 concludes.

2 The term Structure of Variance Swap Contracts

A variance swap contract is an over-the-counter (OTC) instrument which allows investors to trade future variance of an asset. At maturity T, the payoff of an investor who is long on a variance swap is given by:

$$(RV_{t,T} - SW_{t,T}) \times n \tag{1}$$

where $RV_{t,T}$ is the annualized realized variance over the life of the contract and $SW_{t,T}$ is the swap rate defined at t, and n is the amount invested. In absence of arbitrage, the variance swap rate must equal the risk neutral expected value of the realized variance under some risk neutral measure $\mathbb{Q}$:

$$SW_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}] \tag{2}$$

Our methodology to approximate the variance swap rate follows closely the model-free estimate proposed by Demeterfi et al. (1999) and Carr and Madan (1998) who show that if one owns a portfolio of options across all strikes inversely weighted by the squared strike then one gets a variance exposure that does not depend on the price, which is exactly what is needed to trade variance. We assume that the stock price path evolves continuously, though the approximation error induced by jumps is negligible (Carr and Wu (2009)). The variance swap rate is approximated by:

$$SW_{t,T} = \frac{2}{B(t,T)(T-t)} \left(\int_0^{S_t} \frac{P(t,T,K)}{K^2} dK + \int_{S_t}^{\infty} \frac{C(t,T,K)}{K^2} dK \right) \tag{3}$$

where $B(t,T)$ is a zero-coupon bond expiring in T, and $P(t,T,K)$ and $C(t,T,K)$ are respectively the prices of a put and call options with maturity T and strike K. In practice, a continuum of option strikes does not exist, so one needs to interpolate and extrapolate strikes and implied volatilities for the remaining moneyness levels. Using the same approximation as Trolle and Schwartz (2010), we truncate the first integral at $K_{min} = F(t,T)e^{-d\times\sigma\sqrt{T-t}}$ and the second at $K_{max} = F(t,T)e^{d\times\sigma\sqrt{T-t}}$ where σ is the implied volatility of the option closest to be at-the-money (ATM) and d is approximately the number of standard deviations that the log strike is away from the log future price. In a lognormal setting:

$$d = \frac{log(X/F(t,T))}{\sigma\sqrt{T-t}} \tag{4}$$

We fix $d = 10$ (Trolle and Schwartz (2010)) and create a fine grid of 1.000 strikes (integration points). We then interpolate and extrapolate implied volatilities for each strike: for moneyness levels above (below) the highest (lowest) available strike we use the implied volatility of the highest (lowest) strike. However, unlike Trolle and Schwartz (2010) and Carr and Wu (2009) who linearly interpolate implied volatilities for the remaining strikes, we instead fit smooth cubic splines to the volatility smile (the results are not much sensitive to this assumption). The differences may be seen on Figure 1.

Throughout the analysis we define the variance risk premium as the difference between the realized variance, $RV_{t,T}$, over the life of the contract and the swap rate defined at the inception where:

$$RV_{t,T} = \frac{252}{T-t}\sum_{i=1}^{T}\left(\frac{F_{t+i,t+T} - F_{t+i-1,t+T}}{F_{t+i-1,t+T}}\right)^2 \tag{5}$$

The daily future prices are synthetically computed through no-arbitrage conditions. There is no standard way of computing realized volatility for a variance swap as term sheets from different brokers vary on whether to use log or simple returns and on whether to use the 365/day or the 252/day annualisation convention. Notwithstanding, we find no evidence on the finance industry on the use of intraday data to compute realized volatility.

3 Data

We use data from both equity options and stock markets on all stocks included on the S&P 100 and the index itself.[2] The options data is from OptionMetrics and the sample period starts on January of 1996 and ends in December of 2011. We start by using the

[2] Except for the following stocks Accenture, Metlife, Monsanto Co and Visa in which we found inconsistencies on the OptionMetrics data (options data started earlier than the IPO).

raw data on options from OptionMetrics, but after filtering the data, and excluding all observations with bid prices equal to zero or higher than ask prices, excluding observations with no implied volatilities we are left with several days with less than 3 strikes for one of the maturities thus creating several gaps on the series. To overcome this problem we use instead the OptionMetrics implied volatility surface file, which contains a smoothed volatility surface for a range of maturities and strikes. Using this surface also has the advantage of making this study more easily replicable by other researchers.

All options on individual stocks are American so OptionMetrics employs a binomial tree approach that adjusts the implied volatilities for the early exercise premium. Everyday we only keep out of the money calls and puts which are more liquid instruments, and option dates that match the underlying trading days. So we were left with about 13 observations per day per stock at 6 different maturities (1, 2, 3, 6, 12 and 24 months).

The stock data is from Bloomberg and we retrieve two price sets for each stock: the raw prices to determine which options are out of the money and prices adjusted for dividends and stock splits to compute returns. Finally, the risk-free rate is the one-month T-bill rate from Ibbotson available on the Kenneth French's data library.[3]

Our dataset consists, on average, of 4,027 estimated daily variance swap rates (191 monthly rates) for each of the 6 maturities under analysis (for each stock).[4] The term-structure of variance swap rates can have several shapes ranging from upward sloping to downward sloping, humped, and even some intermediate shapes (Panel A of Figure 2). For most of the sample the term-structure is upward sloping for both the S&P 100 and the individual stocks. Usually the short-term variance swap rates spike during crisis periods (e.g. 2009) which implies that the term-structure gets downward sloping. Taking a glance at Panel B of Figure 2 it may be seen that swap rates share some of the variance stylized facts (Schwert (1989)), such as clustering and mean-reversion.

Taking a look at Panel A from Table 1, we can see that the swap rates term structure for S&P 100 was on average almost flat for maturities higher than 60 days between 1996

[3]The data library is available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[4]The stocks that started trading after 1996 have fewer quotes.

and 2011. Further, glancing at Panel C from Table 1 we can conclude that, in line with most literature on variance swaps, the one-month variance risk premium on the S&P 100 is statistically negative and on average equal to -1.3% between 1996 and 2011 (Carr and Wu (2009), Driessen et al. (2009) and Han and Zhou (2012)). As a robustness check we compare our S&P 100 one-month variance risk premium estimate between 1996 and 2003 with the one estimated by Driessen et al. (2009) (who used the OptionMetrics raw options data) and we obtain the same estimate. Not only the one-month variance risk premium for the index is significantly negative but for the remaining maturities as well.

On the other hand, for the individual stocks the average swap rate was much higher and clearly decreasing in maturity (Panel B from Table 1) and the variance risk premium statistically higher than zero for all maturities. This positiveness for the individual stocks variance risk premium is in accordance with the findings from Han and Zhou (2012) and Driessen et al. (2009) but not with those from Carr and Wu (2009) who find a statistically significant negative variance risk premium for individual stocks. Further, as one should expect individual stocks swap rates show much higher standard deviation and autocorrelation than the index swap rates. Finally, both swap rates and variance risk premiums show large persistence even after twelve months as shown by the large Ljung-Box statistic.

4 Forecasting the term-structure of variance swaps

Few models have been proposed to forecast the term-structure of variance swaps. One exception is the two affine factor model from Egloff et al. (2010) whose out of sample (OOS) forecasts for mid-term maturities are fairly accurate. However, Egloff et al. (2010) only try to forecast the market variance swap curve providing no evidence on their model performance on individual swap rates. On the contrary, we try to model not only the market term structure but the individual stock variance swap term structure as well. Our model is much simpler than the one proposed by Egloff et al. (2010) as it relies on only 3 parameters. Following Diebold and Li (2006) we use the Nelson-Siegel (NS) exponential components to forecast the variance swaps term-structure as it imposes a structure on factor loadings thus reducing the estimation error. Each month we fit the following curve to the observed swap rates:

$$SW_{t,T} = \beta_{0,t} + \beta_{1,t} e^{-T/\theta} + \beta_{2,t} \frac{T}{\theta} e^{-T/\theta} \tag{6}$$

Here the parameters are easy to interpret, for long-term maturities swap rates approach asymptotically β_0; then β_1 represents the deviation from the asymptote; β_2 determines the hump that happens at time T. The parameter θ governs the decay, so a high (low) value of θ allows for a better fit for short (long) maturities (following Diebold and Li (2006) we fix $\theta = 0.25$ to maximize the loading of the medium term factor at three months which is when the hump occurs on average). Panel A from Figure 3 depicts the factor loadings which illustrates the wide variety of shapes that the fitted curve may have, thus being able to capture the swap rate term structure. We find that for most months one hump is enough to completely model the swap rate curve. However, for robustness we also fit a Svensson model to the S&P 100 swap rates, which allows the curve to have one more hump. We find that the Svensson model (not reported) in spite of having a better in-sample fit the OOS forecast of the yield curve is worse.

Let us define our performance measures (root mean squared error and mean absolute error) as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\widehat{SW}_{t,T} - SW_{t,T})^2} \quad and \quad MAE = \frac{1}{n}\sum_{t=1}^{n}|\widehat{SW}_{t,T} - SW_{t,T}| \tag{7}$$

where $\widehat{SW}_{t,T}$ is the fitted swap rate at time t with maturity T and $SW_{t,T}$ is the actual swap rate. Panel A of Table 2 reports the residual statistics from in-sample estimation of Equation (6) for the S&P 100. Notice that the average error is constant and fairly low for all maturities, implying that indeed our model succeeds to fit the entire swap rate curve for all maturities. Further, the error seems to be persistent from one month to the next, but it vanishes through time, making it not worthwhile to include that information on the next periods fit. This error persistence might be due to the lack of liquidity of this sort of instruments, or to some estimation bias in our risk-neutral approach to approximate swap rates. Panel B from Table 2 reports the same but as an average for all individual stocks in our sample. The model still performs quite well for individual stocks, with an average of root mean squared error slightly higher than the ones from the S&P 100. As the shape of the term structure of individual stocks changes more often than the one from the index, the short-term correlation of the errors is lower and not statistically significant (for shorter maturities).

Further, we may interpret the β coefficients as the level, slope and curvature. Define the level, β_0, as the long-term swap rate ($SW_{t,t+24}$); the slope, β_1, as the difference between the twenty-four-month swap rate and the one-month swap rate ($SW_{t,t+24} - SW_{t,t+1}$); and the curvature, β_2, as the difference between twice the three-month swap rate and the sum of the one-month swap rate with the twenty-four-month swap rate ($2 \times SW_{t,t+3} - SW_{t,t+1} - SW_{t,t+24}$); we show in Figure 4 that the empirical levels of level, slope and curvature closely track our estimated coefficients.

Finally, we try to use the Nelson-Siegel model to forecast the term-structure of variance swap rates one-month ahead. As on a Nelson-Siegel framework the variance swap curve only depends on $\{\beta_0, \beta_1, \beta_2\}$, forecasting the swap rates is equivalent to forecasting the coefficients. Therefore, we estimate the model coefficients β_i, $i = 0, 1, 2$ for the next month using a simple AR(1) regression framework:

$$\hat{\beta}_{i,t} = \alpha + \psi \hat{\beta}_{i,t-1} + e_t \tag{8}$$

We choose an AR(1) to forecast the coefficients for two reasons: first, because an AR(1) is one of the most simple predictive frameworks available and second, because the coefficients show some persistence. Our one-month ahead forecast of the NS coefficients is given by: $\widehat{\hat{\beta}}_{i,t+1} = \hat{\alpha} + \hat{\psi}\hat{\beta}_{i,t}$ where the double-hat beta denotes the forecasted beta from the AR(1) process using past betas estimated using the Nelson-Siegel framework. We use both rolling and expanding window estimates but decide to keep the expanding window as it minimizes the forecasting errors. We use as our initial estimation period the period starting in January of 1996 and ending in December of 1998 and start the term-structure forecast in January of 1999.

The estimated swap-rate one month ahead is:

$$\widehat{SW}_{t+1,T+1} = \widehat{\hat{\beta}}_{0,t+1} + \widehat{\hat{\beta}}_{1,t+1}e^{-(T+1)/\theta} + \widehat{\hat{\beta}}_{2,t+1}\frac{T+1}{\theta}e^{-(T+1)/\theta} \tag{9}$$

Table 3 reports the OOS performance of the Nelson-Siegel model both for the S&P 100 index and the individual stocks compared with a standard benchmark (naive) model under which the swap rate at period $t+1$ is equal to the swap rate at period t: $\widehat{SW}_{t+1,T+1} = SW_{t,T}$. The use of a naive benchmark to race a model against is common practice in the literature and several authors on different applications have done so (e.g. Goyal and Welch (2008) on predicting market returns, Diebold and Li (2006) on predicting interest rates, Hansen and Lunde (2005) on predicting volatility and DeMiguel et al. (2009) on benchmarking asset allocation models). We define the forecast error as $(\widehat{SW}_{t+1,T+1} - SW_{t+1,T+1})$, and measure the forecasting performance of the model using the root mean squared error (RMSE) and the mean squared error (MAE). We find that the Nelson-Siegel model clearly outperforms our naive benchmark in predicting next period swap rates both for individual stocks and the S&P 100 index. In spite of outperforming the naive model across all maturities, the NS out-performance is more pronounced for shorter maturities. This might be due to the higher short-term swap rate volatility as it may be seen on Table 1. Finally, the forecasting error is persistent, but trying to include this persistence into our forecast would not decrease our out of sample forecasting error. Table 4 reports the forecasting performance for two different sub-samples: the first from January of 1996 to December of 2003 and the second from January of 2004 to December of 2011. We find that the model is robust through time as our predictive model beats the naive forecast on both sub-samples.

5 Predictive Regressions

Asset predictability has been one of the main finance research concerns during the past decade (Goyal and Welch (2008); Campbell and Thompson (2008); Lettau and Nieuwerburgh (2008); Drechsler and Yaron (2011); Ferreira and Santa-Clara (2011)). We propose a new predictor of the stock market return which relies on the empirical fact than changes in the market implied one-month volatility (i.e. $r_{SW_{t,T}} = SW_{t,T}/SW_{t-1,T-1} - 1$) are contemporaneously strongly negatively correlated with the market return (e.g. Ang et al. (2006)). Given our estimate of the swap rate next period we may indeed try to exploit this correlation by making the following regression:

$$r_{t,T} = \alpha + \beta \mathbb{E}_t \left[r_{SW_{t+1,T+1}} \right] + e_t \tag{10}$$

where $r_{t,T}$ is the return between month t and T, $\mathbb{E}_t \left[r_{SW_{t+1,T+1}} \right] = \mathbb{E}_t(SW_{t+1,T+1})/SW_{t,T} - 1$ and we replace $\mathbb{E}_t(SW_{t+1,T+1})$ by our Nelson-Siegel estimate $\widehat{SW}_{t+1,T+1}$. To conduct this exercise we need to proceed in several steps: first we fit a Nelson-Siegel model to the swap rates; then we use an AR(1) model to forecast the term-structure of swap rates; finally, given the estimated swap rates, we run Equation (10) to predict market returns. To carry this analysis we need two estimation periods: the first from March of 1996 until December of 1998 which is used as the initial estimation period for the AR(1), the second from January of 1999 until December of 2000 which is used as the initial estimation period for Equation (10) (then we use an expanding window).

The results are reported in Table 5. The one-month return prediction out-of-sample R^2 defined as $R^2 = 1 - {}^{MSE_A}/_{MSE_N}$ (where MSE_A is the average squared prediction error of our forecast and MSE_N is the mean squared error of the naive forecast (historical average)) is positive and around 3% therefore passing the test proposed by Goyal and Welch (2008). The average β coefficients are also significant and have the expected negative sign, meaning that expected changes in the market one-month implied volatility can indeed predict stock market returns. We also find that common used predictors in the literature such as earnings-price ratio do not increase our out-of-sample R^2 (not reported). Figure 5 represents the cumulative sum of the differences between our model forecasting error and the naive model forecasting error (for the one-month prediction). Whenever the line increases the prediction error of our model is lower than the prediction error of the naive model. Therefore, whenever the line increases our model predicts better and whenever the line decreases the naive model predicts better. Although the graph units have no interpretation, it provides a valuable tool to evaluate our model performance through time (the grey bands represent US recessions as defined by the National Bureau of Economic Research - NBER). Notice that our model clearly beats the naive model during recessions periods, which implies that expected changes in volatility influence expected returns during turmoil periods. The opposite is true during normal periods, in which the naive model is a better predictor of the S&P 100 returns. For robustness, we also try to predict longer horizon returns (two and three month returns) using longer maturities swap rates. As expected, the OOS R^2 are higher: 4.61% for the two-month prediction and 7.87% for the three-month prediction (we use overlapping returns).

6 Asset allocation

Another way to exploit the negative correlation between the stock return and its variance return is by investing on both the stock and the variance swap (which serves as an hedge for the stock). The profit (loss) on a variance swap is its variance risk premium as defined on Equation (1).

We test a simple mean-variance strategy which allocates on both stocks and its variance swaps, and evaluate their OOS return, Sharpe ratio and certain equivalent. We make this for eleven portfolios: the index (S&P 100) and ten equally weighted portfolios. Each month we sort stocks in ten equally weighted portfolios according to their one-month variance risk premium and keep the portfolios next month variance swap return for the six maturities and the stocks return. For the variance swaps with maturity higher than one month we approximate its one-month profit (loss) using the following (Egloff et al. (2010)):

$$n \times (\omega RV_{t,T_1} + (1-\omega) SW_{T_1,T_2} - SW_{t,T_2}) \tag{11}$$

where n is the amount invested, $\omega = (T_1 - t)/(T_2 - t)$ denotes the time passed since the inception of the swap rate contract, RV_{t,T_1} is the realized variance between T_1 and t, and SW_{T_1,T_2} is the swap rate of a contract which starts at T_1 and ends at T_2.

As an example, if we compute the one-month profit (loss) of a twelve-month variance swap contract, one month after its inception, then its profit (loss) comes from two sources (Egloff et al. (2010)): the realization of the return variance over the past month and the new variance swap rate at the same expiry date. So, in our example, RV_{t,T_1} is the realized variance over that month, SW_{T_1,T_2} is a swap rate of a contract starting at $T_1 = 1$ with eleven months remaining until maturity ($T_2 = 12$) and SW_{t,T_2} is the original twelve-month swap rate. In order to compute Equation (11) the only unknown value is SW_{T_1,T_2} (the eleven-month swap rate in our example above) which we approximate using a linear, in total variance, interpolation:

$$SW_{T_1,T_2} = \frac{1}{T_2 - T_1}\left[\frac{SW_{t,T_3}(T_3 - T_1)(T_4 - T_1) + SW_{t,T_4}(T_4 - t)(T_1 - T_3)}{T_4 - T_3}\right] \tag{12}$$

we used T_3 equal to 1-month and T_4 equal to 12-month to interpolate the 11-month swap rate.

Each month we maximize the utility of a mean-variance investor and allow him to allocate between two sets of assets: (1) the stock return, its one-month variance swap and the risk-free asset; (2) the stock return, its one- and twelve-month variance swap and the risk-free. The problem of the investor is to choose the weights, ω_i $i = 1, 2, 3$ that maximizes the following equation:

$$\underset{\omega}{Max} \quad \omega_t^T \mu - \frac{\gamma}{2}\omega_t^T \Sigma\, \omega_t \tag{13}$$

where ω is a vector of weights, μ is a vector of expected returns and Σ is the variance-covariance matrix. We set the degree of risk aversion, γ, relatively high and equal to 10 to compensate for the high variance risk premium - this is just a shrinking factor that does not alter our conclusions (in fact Rosenberg and Engle (2002) estimate a coefficient of risk aversion between 2.26 and 12.55 for S&P 500 options between 1991 and 1995 (average of 7.36)). We use the period between March of 1996 until December of 2000 as our initial estimation period (then we use an expanding window), and the period starting in January of 2001 and ending December of 2011 as our performance evaluation period. Our benchmark is the S&P 100 index which during the period achieved an average monthly return of -0.24%

with an annualized Sharpe ratio of -0.29 and an annualized certain equivalent of -24.60%. On the other hand, a mean-variance strategy on both the index and a one-month variance swap yields an average monthly return of 6.09% with an annualized Sharpe ratio of 1.89 and a certain equivalent of 37% (Table 6). The average position on the S&P 100 one-month variance swap is VS_{1Month}=-21% and does not change much from one period to the next. The inclusion of a third asset on our strategy (a twelve-month variance swap) actually worsens the Sharpe ratio. Figure 6 shows the return and cumulative return of an investment on the index, the one-month variance swap and the two together. An investor who had invested on the S&P 100 and the risk-free at the beginning of 2001 would have had a return close to 5% at the end of 2011 (or 0.06% per month). On the contrary, if he had invested on the variance swap as well he would have ended with ten times his investment.

We also analyse the performance of our mean-variance strategy on portfolios sorted by variance risk premiums. The results are reported on Table 7. As expected, as the variance risk premiums show some persistence, our mean-variance investor prefers to short the variance swap on the portfolio with lowest variance risk premium and be long on the variance swap on the portfolios with higher variance risk premium. The top (bottom) portfolios have Sharpe ratios around 2.40 (0.32) and certain equivalents of 82.32% (1.68%). As it happened with the index, including the 12-month variance swap did not change much the investor's Sharpe ratio, thus it is not worthwhile to include it as an asset on the portfolio. Figure 7 shows the returns and cumulative returns of an investing in either the top or the bottom decile. An investor who had invested his money on the stocks and its variance swaps from the bottom decile would have ended up with a large sum (a return of 11% per month). The two large drops that occurred on October 2008 and August 2011 were due to investors fear. On October 2008 the variance swap market almost dried up as a result of large moves on stocks prices that made dealers exposed to much more vega than a hedging strategy would permit.[5] On August 2011 the market participants were caught of guard by a sudden peak in volatility that led to several positions being closed (according to Reuters).

On Table 8 we report the performance of the bottom and top deciles against a mean-

[5]Carr and Lee (2008).

variance strategy on the index and the risk-free during recession periods. It is indeed remarkable that on both during the recession of 2001 and 2007-2009 our variance swap strategy performs quite well always having, on average, positive returns and Sharpe ratios above 0.3.

Finally, for robustness we sort portfolios on the 24-month variance risk premium instead of the one-month and check whether that would change our allocation. We find that this change has slight impact on the portfolios weights and consequently no impact on the final return and Sharpe ratio and certain equivalent of the investor (not reported).

7 Conclusion

In this dissertation we model the term-structure of variance swap rates and propose two ways of profiting from the variance swap.

We show that the Nelson-Siegel model is enough to estimate with a good in-sample fit the term-structure of variance swap rates and that the three factors evolving dynamically may be interpreted as level, slope and curvature of the term-structure. Further, we find that the next month term structure can be estimated OOS with accuracy just by forecasting the Nelson-Siegel parameters under an AR(1) regression framework. We then use our swap rate forecast to estimate the expected change in the swap rate, and show that it predicts OOS market returns with accuracy.

We also find that investors may indeed benefit from investing in variance through a variance swap. We show that during a period in which the market Sharpe ratio was -0.29 investing in both the market and the one-month variance swap under a mean-variance optimization allows the investor to achieve a Sharpe ratio of 1.89. This result is robust across several portfolios sorted by the variance risk premium.

References

Aït-Sahalia, Y., Karaman, M., Mancini, L., 2012. The term structure of variance swaps, risk premia and the expectation hypothesis. EPFL, Working Paper.

Ang, A., Hodrick, R. J., Xing, Y., Zhang, X., 2006. The cross-section of volatility and expected returns. Journal of Finance 61 (1), 259–299.

Britten-Jones, M., Neuberger, A., 2000. Option prices, implied price processes, and stochastic volatility. The Journal of Finance 55 (2), 839–866.

Buehler, H., 2006. Consistent variance curve models. Finance and Stochastics 10 (2), 178–203.

Campbell, J. Y., Thompson, S. B., 2008. Predicting excess stock returns out of sample: Can anything beat the historical average? Review of Financial Studies 21 (4), 1509–1531.

Carr, P., Lee, R., 2008. Robust replication of volatility derivatives. Courant Institute of Mathematica Schiences, Working Paper.

Carr, P., Madan, D., 1998. Towards a Theory of Volatility Trading, in R. Jarrow (ed). Risk Book on Volatility, New York, pp. 417–427.

Carr, P., Wu, L., 2009. Variance risk premia. Review of Financial Studies 22, 1311–1341.

Demeterfi, K., Derman, E., Kamal, M., Zou, J., 1999. A guide to volatility and variance swaps. The Journal of Derivatives 6, 9–32.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Review of Financial Studies 22 (5), 1915–1953.

Diebold, F. X., Li, C., 2006. Forecasting the term structure of government bond yields. Journal of Econometrics 130 (2), 337–364.

Drechsler, I., Yaron, A., 2011. What's vol got to do with it. Review of Financial Studies 24 (1), 1–45.

Driessen, J., Maenhout, P. J., Vilkov, G., 2009. The price of correlation risk: Evidence from equity options. The Journal of Finance 64 (3), 1377–1406.

Egloff, D., Leippold, M., Wu, L., 2010. The term structure of variance swap rates and optimal variance swap investments. Journal of Financial and Quantitative Analysis 45 (5), 1279–1310.

Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 3 (1), 3–56.

Ferreira, M. A., Santa-Clara, P., 2011. Forecasting stock market returns: The sum of the parts is more than the whole. Journal of Financial Economics 100 (3), 514–537.

Goyal, A., Welch, I., 2008. A comprehensive look at the empirical performance of equity premium prediction. Review of Financial Studies 21 (4), 1455–1508.

Hafner, R., Wallmeier, M., 2008. Optimal investments in volatility. Financial Markets and Portfolio Management 22 (2), 147–167.

Han, B., Zhou, Y., 2012. Variance risk premium and cross section of stock returns. Working Paper.

Hansen, P. R., Lunde, A., 2005. A forecast comparison of volatility models: Does anything beat a GARCH(1,1)? Journal of Applied Econometrics 20 (7), 873–889.

Jiang, G., Tian, Y., 2005. Model-free implied volatility and its information content. Review of Financial Studies 18 (4), 1305–1342.

Lettau, M., Nieuwerburgh, S. V., 2008. Reconciling the return predictability evidence. Review of Financial Studies 21 (4), 1607–1652.

Madan, D. B., 2009. Variance swap portfolio theory. Robert H. Smith School, Working Paper.

Pollet, J. M., Wilson, M., 2010. Average correlation and stock market returns. Journal of Financial Economics 96 (3), 364–380.

Rosenberg, J. V., Engle, R. F., 2002. Empirical pricing kernels. Review of Financial Economics 64 (3), 341–372.

Schwert, G. W., 1989. Why does stock market volatility change over time? The Journal of Finance 49 (5), 1115–1153.

Trolle, A. B., Schwartz, E. S., 2010. Variance risk premia in energy commodities. The Journal of Derivatives 17 (3), 15–32.

Table 1: Descriptive Statistics Variance Swaps and Variance Risk Premiums

This table presents summary statistics for swap rates (panel A and panel B) and variance risk premiums (panel C and panel D) as defined on the first section. Panel A and C show the statistics for the S&P 100 and panel B and C the statistics as an average for the stocks in the index. Kurtosis is the excess kurtosis. ρ_1 is the first-order correlation coefficient and Q_{12} is the Ljung-Box statistics with 12 lags. The symbols ***, ** and * denote the statistical significance of the coefficient at 1%, 5% and 10% significance level respectively. The sample period starts in January of 1996 and ends in December of 2011.

	Panel A: S&P 100 Swap Rates						Panel B: Individual Stocks Swap Rates					
T	Mean	St Dev	Skew	Kurt	ρ_1	Q_{12}	Mean	St Dev	Skew	Kurt	ρ_1	Q_{12}
1M	5.08%	4.71%	3.98	24.25	0.75***	264.1***	14.27%	12.66%	2.80	13.08	0.78***	489.0***
2M	5.14%	4.30%	3.53	20.09	0.78***	308.1***	14.14%	12.10%	2.68	11.99	0.81***	539.2***
3M	5.15%	3.93%	3.16	16.65	0.81***	364.2***	13.64%	10.77%	2.41	9.56	0.85***	627.2***
6M	5.13%	3.35%	2.39	9.69	0.85***	471.0***	13.01%	9.51%	2.21	8.12	0.88***	712.5***
12M	5.13%	2.88%	1.56	3.92	0.87***	566.2***	12.67%	8.38%	1.88	5.70	0.91***	815.6***
24M	5.16%	2.62%	1.09	1.59	0.87***	629.4***	12.43%	7.43%	1.54	3.57	0.92***	924.9***

	Panel C: S&P 100 Var. Risk Premium						Panel D: Individual Stocks Var. Risk Premium					
T	Mean	St Dev	Skew	Kurt	ρ_1	Q_{12}	Mean	St Dev	Skew	Kurt	ρ_1	Q_{12}
1M	-1.29%***	4.98%	2.98	37.50	0.31***	29.6***	0.59%**	17.19%	3.94	35.46	0.18***	36.3***
2M	-1.33%***	5.05%	3.70	33.94	0.53***	64.9***	0.68%***	15.29%	3.30	23.93	0.51***	78.3***
3M	-1.39%***	4.80%	3.47	27.14	0.64***	94.4***	1.12%***	14.45%	3.13	18.78	0.66***	125.0***
6M	-1.34%***	4.40%	2.42	11.37	0.82***	239.8***	1.71%***	13.36%	2.15	8.73	0.83***	276.9***
12M	-1.29%***	3.95%	1.05	3.57	0.90***	406.0***	2.08%***	12.63%	0.98	3.31	0.91***	508.5***
24M	-1.10%***	3.67%	0.21	0.66	0.92***	637.5***	2.63%***	11.66%	0.16	0.59	0.94***	804.5***

Table 2: Nelson-Siegel In-Sample Performance

This table presents the in-sample performance of the Nelson-Siegel model in fitting the swap rate term-structure. Each month we fit Equation (6) to the observed swap rates. Panel A reports the residual statistics for the S&P 100 fit and Panel B reports the residual statistics for the S&P 100 stocks as an average across stocks. The residual at time t for the swap rate with maturity T is defined as: $\hat{e}_{t,T} = \widehat{SW}_{t,T} - SW_{t,T}$. The first two columns present the average residuals for each maturity and their standard deviation. The MAE and RMSE are the performance measures as defined on Equation (7). ρ_1 and ρ_{12} are the order one and twelve auto-correlation coefficients respectively.

	Panel A: Nelson-Siegel IS Performance (S&P 100)							
T	Average	St Dev	Max	Min	MAE	RMSE	ρ_1	ρ_{12}
1M	0.000	0.001	0.044	-0.021	0.001	0.001	0.61	0.09
2M	-0.000	0.001	0.049	-0.020	0.001	0.001	0.55	0.12
3M	-0.000	0.002	0.048	-0.021	0.001	0.001	0.45	0.05
6M	0.000	0.003	0.049	-0.022	0.001	0.001	0.52	0.07
12M	0.000	0.003	0.049	-0.020	0.001	0.001	0.53	0.02
24M	-0.000	0.003	0.048	-0.021	0.001	0.001	0.73	0.04

	Panel B: Nelson-Siegel IS Performance (average for S&P 100 stocks)							
T	Average	St. Dev	Max	Min	MAE	RMSE	ρ_1	ρ_{12}
1M	0.000	0.005	0.025	-0.040	0.002	0.005	-0.01	0.08
2M	-0.001	0.009	0.033	-0.076	0.004	0.009	-0.08	0.11
3M	0.001	0.009	0.068	-0.030	0.004	0.009	0.00	0.11
6M	0.000	0.006	0.030	-0.035	0.003	0.006	0.18	0.08
12M	-0.001	0.006	0.023	-0.040	0.002	0.006	0.28	0.07
24M	0.000	0.005	0.031	-0.022	0.002	0.005	0.22	0.07

Table 3: Nelson-Siegel Out-of-Sample Performance

This table presents the out-of-sample performance of the Nelson-Siegel model (Panel A and Panel C) and of a naive model (Panel B and Panel D) in forecasting variance swap rates. Panel A reports the residual statistics for the S&P 100 Nelson-Siegel forecast (using the methodology described in the text) and Panel B reports the residual statistics for the S&P 100 naive forecast ($\widehat{SW}_{t+1,T+1} = SW_{t.T}$). Panel C and D report the same but for as an average for the individual stocks of the S&P 100. The residual at time $t+1$ for the swap rate with maturity $T+1$ is defined as: $\hat{e}_{t+1,T+1} = \widehat{SW}_{t+1,T+1} - SW_{t+1,T+1}$. The first two columns present the average residuals for each maturity and their standard deviation. The MAE and RMSE are the performance measures as defined on Equation (7). ρ_1 and ρ_{12} are the order one and twelve auto-correlation coefficients respectively.

	Panel A: Nelson-Siegel (S&P 100) residuals						Panel B: Naive forecast (S&P 100) residuals					
T	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	0.001	0.038	0.017	0.038	0.02	0.02	0.000	0.048	0.023	0.048	0.35	-0.01
2M	0.001	0.033	0.016	0.033	0.03	-0.02	0.000	0.042	0.020	0.042	0.38	-0.02
3M	0.001	0.028	0.014	0.028	0.04	0.02	0.000	0.036	0.018	0.036	0.40	-0.03
6M	0.001	0.020	0.011	0.020	0.16	0.05	0.000	0.028	0.015	0.027	0.43	-0.01
12M	0.001	0.014	0.009	0.014	0.23	0.08	0.000	0.021	0.012	0.021	0.46	0.00
24M	0.000	0.012	0.008	0.013	0.18	0.08	0.000	0.017	0.011	0.017	0.49	-0.02

	Panel C: Nelson-Siegel (stocks) residuals						Panel D: Naive forecast (stocks) residuals					
T	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	-0.012	0.106	0.054	0.108	0.37	0.05	0.150	0.028	0.153	0.156	0.86	0.62
2M	-0.005	0.094	0.048	0.095	0.38	0.05	0.160	0.109	0.165	0.201	0.41	0.09
3M	0.001	0.077	0.042	0.078	0.40	0.07	0.147	0.097	0.152	0.182	0.42	0.09
6M	0.003	0.061	0.035	0.063	0.41	0.08	0.133	0.080	0.137	0.161	0.43	0.11
12M	0.000	0.048	0.027	0.048	0.33	0.10	0.127	0.065	0.131	0.147	0.45	0.13
24M	0.002	0.039	0.022	0.040	0.18	0.08	0.127	0.053	0.131	0.143	0.39	0.17

Table 4: Nelson-Siegel Out-of-Sample Performance (sub-samples)

This table presents the out-of-sample performance of the Nelson-Siegel (Panels A and Panels C) and a naive model (Panels B and Panels D) in forecasting variance swap rates. Panels A and C report respectively for the S&P 100 and the individual stocks the residual statistics of the Nelson-Siegel forecast (using the methodology described in the text). Panels B and D report respectively for the S&P 100 and the individual stocks the residual statistics of naive forecast model ($\widehat{SW}_{t+1,T+1} = SW_{t.T}$). The residual at time $t+1$ for the swap rate at maturity $T+1$ is defined as: $\hat{e}_{t+1,T+1} = \widehat{SW}_{t+1,T+1} - SW_{t+1,T+1}$. The first two columns present the average residuals for each maturity and their standard deviation. The MAE and RMSE are the performance measures as defined on Equation (7). ρ_1 and ρ_{12} are the order one and twelve auto-correlation coefficients respectively. Panels A1, B1, C1 and D1 statistics correspond to the sub-sample ranging from January of 1996 until December of 2003 whereas panel A2, B2, C2 and D2 correspond to the subsample ranging from January of 2004 until December of 2011.

	Panel A1: Nelson-Siegel (S&P 100) residuals						Panel B1: Naive forecast (S&P 100) residuals					
T	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	-0.001	0.017	0.013	0.017	0.22	0.06	-0.001	0.025	0.019	0.025	0.45	0.03
2M	-0.001	0.016	0.012	0.016	0.25	0.05	-0.001	0.024	0.018	0.023	0.47	0.04
3M	-0.001	0.014	0.011	0.014	0.17	0.08	-0.001	0.021	0.016	0.021	0.46	0.05
6M	0.000	0.012	0.009	0.012	0.03	0.05	-0.001	0.017	0.013	0.017	0.44	0.00
12M	0.000	0.012	0.009	0.012	0.06	0.01	-0.001	0.017	0.013	0.017	0.44	-0.01
24M	-0.001	0.012	0.009	0.012	0.07	0.02	-0.001	0.018	0.014	0.018	0.43	-0.01

	Panel C1: Nelson-Siegel (stocks) residuals						Panel D1: Naive forecast (stocks) residuals					
T	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	-0.017	0.081	0.058	0.086	0.28	0.09	0.170	0.020	0.172	0.174	0.98	0.59
2M	-0.011	0.071	0.051	0.074	0.29	0.08	0.184	0.082	0.186	0.205	0.28	0.06
3M	-0.003	0.058	0.043	0.061	0.29	0.08	0.169	0.071	0.172	0.187	0.30	0.05
6M	0.000	0.046	0.036	0.050	0.32	0.10	0.152	0.058	0.154	0.165	0.30	0.06
12M	-0.003	0.035	0.027	0.038	0.25	0.10	0.143	0.047	0.146	0.153	0.33	0.09
24M	-0.001	0.031	0.022	0.032	0.15	0.09	0.144	0.036	0.145	0.150	0.29	0.09

T	Panel A2: Nelson-Siegel (S&P 100) residuals						Panel B2: Naive forecast (S&P 100) residuals					
	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	0.002	0.047	0.021	0.046	0.00	0.02	0.001	0.058	0.026	0.058	0.34	-0.01
2M	0.003	0.040	0.018	0.040	0.00	0.01	0.001	0.050	0.022	0.050	0.37	-0.03
3M	0.002	0.034	0.016	0.034	0.02	0.01	0.001	0.043	0.020	0.043	0.39	-0.04
6M	0.001	0.024	0.012	0.024	0.18	0.05	0.001	0.033	0.016	0.033	0.42	-0.01
12M	0.001	0.015	0.009	0.015	0.29	0.11	0.001	0.023	0.012	0.022	0.46	0.01
24M	0.001	0.011	0.007	0.011	0.25	0.13	0.001	0.017	0.009	0.017	0.52	0.00

T	Panel C2: Nelson-Siegel (stocks) residuals						Panel D2: Naive forecast (stocks) residuals					
	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}	Average	St. Dev	MAE	RMSE	ρ_1	ρ_{12}
1M	-0.009	0.107	0.053	0.111	0.41	0.00	0.141	0.023	0.144	0.146	0.69	0.42
2M	-0.002	0.097	0.047	0.099	0.41	0.00	0.149	0.109	0.155	0.194	0.41	0.02
3M	0.004	0.080	0.041	0.082	0.43	0.02	0.136	0.099	0.142	0.177	0.41	0.01
6M	0.005	0.064	0.035	0.066	0.42	0.03	0.124	0.082	0.130	0.156	0.43	0.03
12M	0.002	0.050	0.027	0.051	0.34	0.06	0.119	0.066	0.124	0.143	0.44	0.05
24M	0.003	0.040	0.023	0.041	0.19	0.06	0.120	0.053	0.125	0.138	0.38	0.09

Table 5: Return predictive regressions

This table presents the average coefficients of the one, two and three month returns predictive regressions (for the S&P 100 returns). The dependent variable is the one defined in Section 6 where $\mathbb{E}(r_{SW_{t+1,T+1}})$ is the expected return of a variance swap with maturity $(T-t)$. The second a third equations were estimated using overlapping returns. The R^2 is the OOS performance measure as defined in Goyal and Welch (2008). The first estimation period is from March of 1996 to December of 1998 where we estimate the AR(1) as defined in Equation 8 and the second estimation period is from January of 1998 to January of 2001 where we estimate Equation (10). The first estimation (Nelson-Siegel) is done on a period-by-period basis whereas both the AR(1) and the predictive regression estimations are done using an expanding window.

	Predictive Regression				
Return Horizon (months)	constant	$\mathbb{E}(r_{SW_{t+1,t+2}})$	$\mathbb{E}(r_{SW_{t+1,t+3}})$	$\mathbb{E}(r_{SW_{t+1,t+4}})$	R^2
$r_{t,1}$	0.001	-0.004			2.82%
$r_{t,2}$	-0.003		-0.012		4.61%
$r_{t,3}$	-0.005			-0.060	7.87%

Table 6: Investing on S&P 100 return and variance return

This table presents the OOS return, standard deviation and Sharpe ratios of a mean-variance investor who may invest in 4 different portfolios (P_i). The initial estimation period ranges from March of 1996 until December of 2000 (expanding window). The first portfolio (P1) is a passive strategy on the S&P 100 index. On the second portfolio (P2) the investor is allowed to invest on both the S&P 100 and the risk-free using a MV strategy. On the third portfolio (P3) the investor also has access to the one-month variance swap. The fourth portfolio (P4) also includes a the 12-month variance swap. γ is set to 10 to compensate the large VRP. CE is the certainty equivalent. The average return and standard deviation are monthly figures. The Sharpe ratio and certainty equivalent are annualized.

	S&P 100 portfolios			
	P1	P2	P3	P4
Average (%)	-0.24	0.06	6.09	1.84
St Dev (%)	4.88	0.68	10.89	3.72
SR	-0.29	-0.56	1.89	1.56
CE ($\gamma = 10$) (%)	-24.60	0.69	37.38	10.95
OEX (%)	100.00	8.68	-77.15	-15.73
VS_{1Month} (%)			-21.24	1.78
$VS_{12Month}$ (%)				-73.56
$Risk\text{-}free$ (%)		91.32	198.39	187.51

Table 7: Mean-Variance Stock and Variance Swap Portfolios (sorted by VRP)

This table presents the OOS return, standard deviation and Sharpe ratios of a mean-variance investor who may invest in 10 different portfolios (P_i). Each month we sort the S&P 100 stocks into 10 equally-weighted portfolios according to their one-month variance risk premium. We then keep the next month portfolio return and variance return. Given this each month the investor chooses how much to allocate to the stocks and to the variance swaps. The initial estimation period ranges from March of 1996 until December of 2000 (expanding window). γ is set to 10 to compensate the large VRP. CE is the certainty equivalent. The average return and standard deviation are monthly figures. The Sharpe ratio and CE are annualized. Panel A reports the results when the investor may invest on both the stocks, the one-month variance swap and a risk-free asset. Panel B reports the same but allowing the investor to invest on the twelve-month swap as well.

	Panel A: Stocks and 1-M Variance Swaps Strategy w/ risk-free									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Average (%)	9.91	0.50	0.30	0.29	0.23	0.19	0.11	0.01	0.18	0.46
St Dev (%)	14.09	2.75	2.58	2.79	2.49	2.24	2.86	1.48	3.33	3.12
SR	2.40	0.42	0.17	0.16	0.09	0.03	-0.07	-0.38	0.01	0.32
CE ($\gamma = 10$) (%)	82.32	2.26	-0.37	-1.02	-2.55	-1.00	-3.27	-1.32	-4.09	1.68
OEX (%)	6.92	42.44	53.27	54.92	55.42	36.07	49.79	19.62	36.53	2.47
VS_{1Month} (%)	-36.91	-2.43	-1.08	-1.30	1.58	2.17	1.03	1.66	5.98	3.69
$Risk\text{-}free$ (%)	129.99	59.99	47.81	46.39	43.00	61.75	49.19	78.72	57.49	93.84

	Panel B: Stocks, 1-M and 12-M Variance Swaps Strategy w/ risk-free									
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Average (%)	11.35	-0.18	-0.45	-0.69	0.10	-0.24	-0.16	0.11	0.71	1.21
St Dev (%)	20.11	3.93	5.21	7.31	2.90	4.10	4.97	5.62	5.74	7.81
SR	1.93	-0.31	-0.41	-0.41	-0.08	-0.34	-0.23	-0.04	0.33	0.46
CE ($\gamma = 10$) (%)	63.38	-11.23	-14.66	-17.26	-5.27	-11.59	-17.83	-14.75	-11.62	-12.35
OEX (%)	23.29	58.72	70.32	62.17	57.54	51.70	73.38	40.01	69.17	38.21
VS_{1Month} (%)	-71.37	-15.31	-14.16	-15.72	1.26	-9.46	-26.84	-30.48	-21.99	-31.89
$VS_{12Month}$ (%)	370.76	167.97	160.01	175.22	18.98	155.33	296.92	353.13	301.77	401.56
$Risk\text{-}free$ (%)	-222.68	-111.38	-116.16	-121.67	22.22	-97.56	-243.46	-262.66	-248.95	-307.88

Table 8: Mean-Variance Stock and Variance Swap Portfolios (sorted by VRP)

This table presents for robustness the OOS return, standard deviation and Sharpe ratios of a mean-variance investor who may invest in the top/bottom deciles sorted by VRP across two different sub-samples.

	Full Sample			Recession I			Recession II		
	Jan-01/Dec-11			Mar-01/Nov-01			Dec-07/Jun-09		
VRP Deciles	Bottom	Top	P2	Bottom	Top	P2	Bottom	Top	P2
Average (%)	9.91	0.46	0.06	20.48	1.12	-0.14	2.66	1.71	-0.10
St Dev (%)	14.09	3.12	0.68	19.38	9.23	1.49	17.71	3.54	0.45
SR	2.40	0.32	-0.56	3.61	0.36	-0.72	0.49	1.52	-2.06
CE ($\gamma = 10$) (%)	82.32	1.68	0.69	436.70	5.45	-1.70	-23.94	21.36	-1.22
OEX (%)	6.92	2.47	8.61	22.97	5.54	23.07	-14.41	2.01	2.98
VS_{1Month} (%)	-36.91	3.69		-44.89	9.73		-37.07	2.18	
$Risk\text{-}free$	129.99	93.84	90.63	121.91	84.73	76.93	151.48	95.81	97.02

Figure 1: Implied Volatility Smile

This figure illustrates the difference between interpolating the implied volatility smile using linear interpolation or by fitting smooth cubic splines to the available implied volatilities. We use flat extrapolation for moneyness levels above (below) the highest (lowest) available strike.

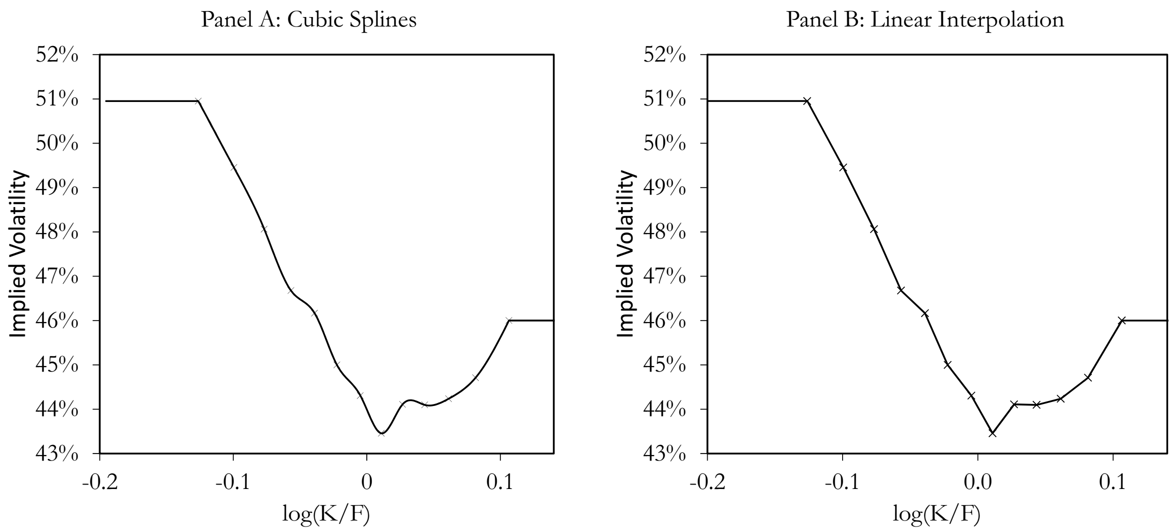

Figure 2: The term-structure of swap rates and variance risk premiums

Panel A illustrates several shapes that the term-structure of the S&P 100 swap rates had at 3 different points in time. Panel B illustrates the S&P 100 one-month realized variance and variance swap rates since 1996. The difference between the realized variance line and the swap rate line is the variance risk premium.

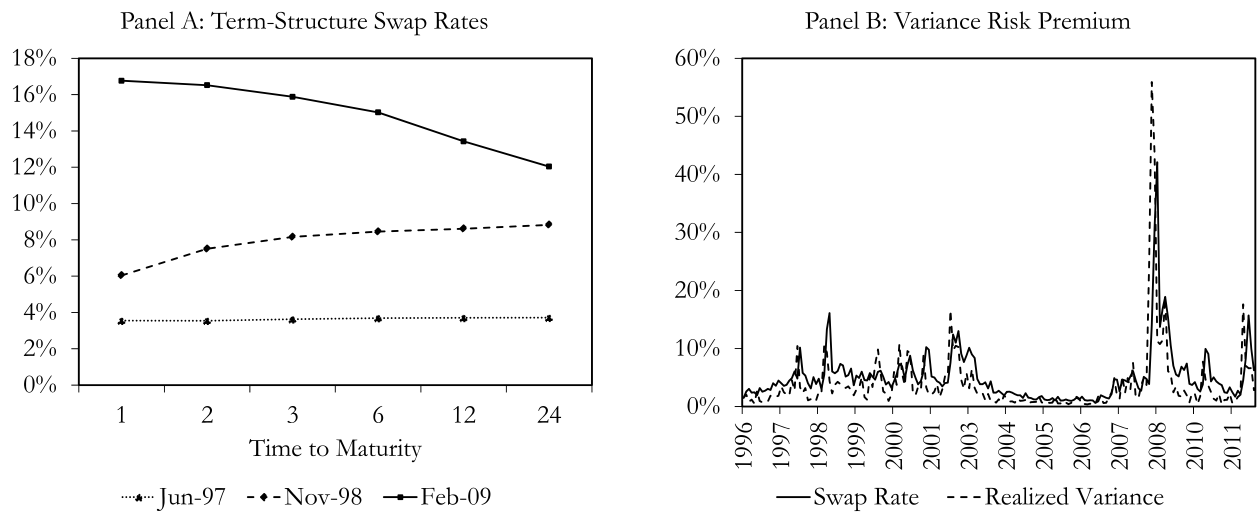

Figure 3: Factor Loadings and factor evolution for the S&P 100

Panel A depicts the factor loadings of the Nelson-Siegel model depending on maturity. Panels B to D shows how the three estimated Nelson-Siegel factors (β_0,β_1, β_2) for the S&P 100 evolved through time.

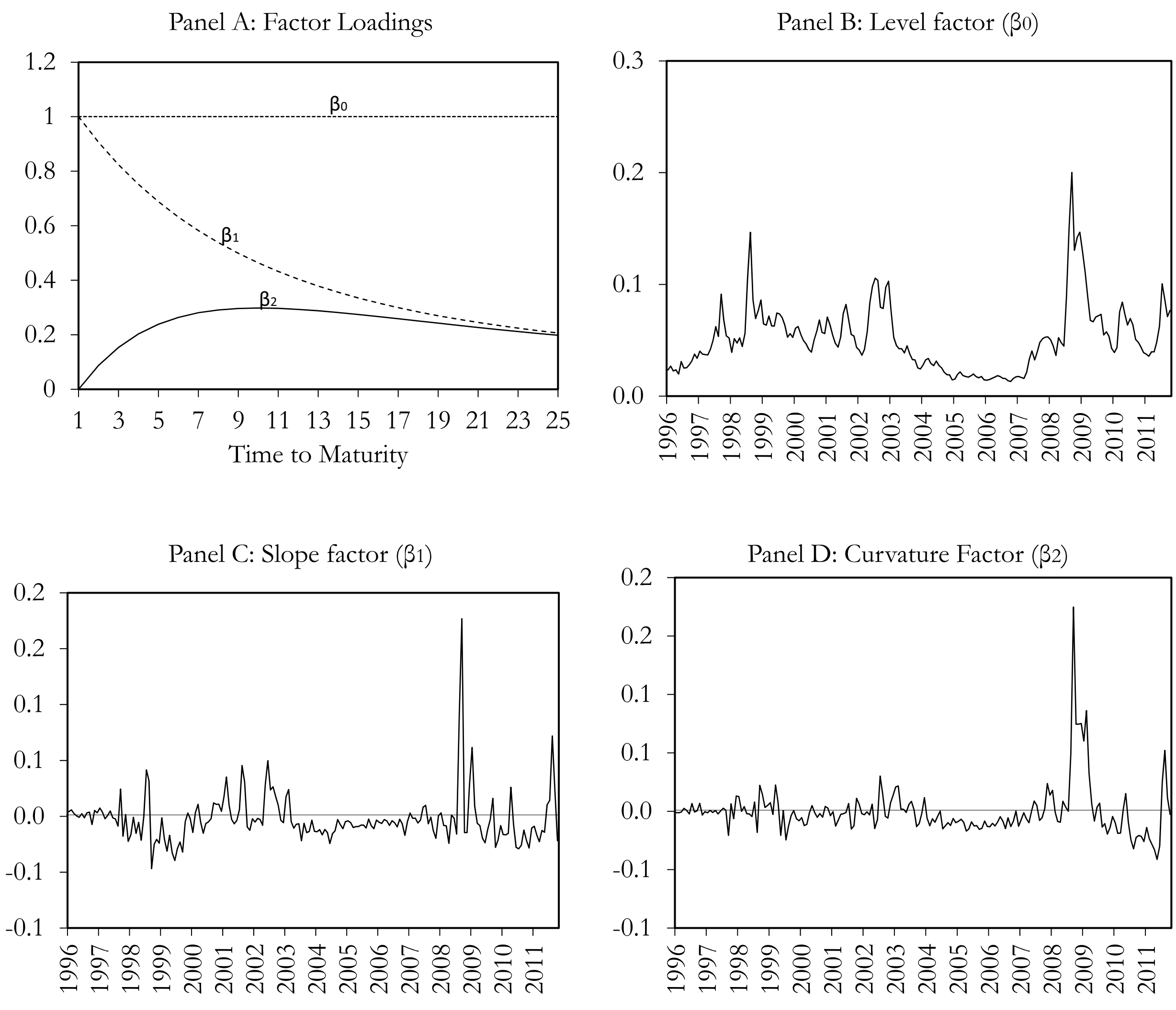

Figure 4: Level, Slope and Curvature of Term-Structure

This figure represents the model based level, slope and curvature (β_0, β_1 and β_2) versus the empirical level, slope and curvature for the S&P 100. We define the level β_0 as the long-term swap rate ($SW_{t,t+24}$), the slope β_1 as the difference between the two-year swap rate and the one-month swap rate ($SW_{t,t+24} - SW_{t,t+1}$), and the curvature β_2 as the difference between the twice the three-month swap rate and the sum of the one-month swap rate and the two-year swap rate ($2 \times SW_{t,t+3} - SW_{t,t+1} - SW_{t,t+24}$)

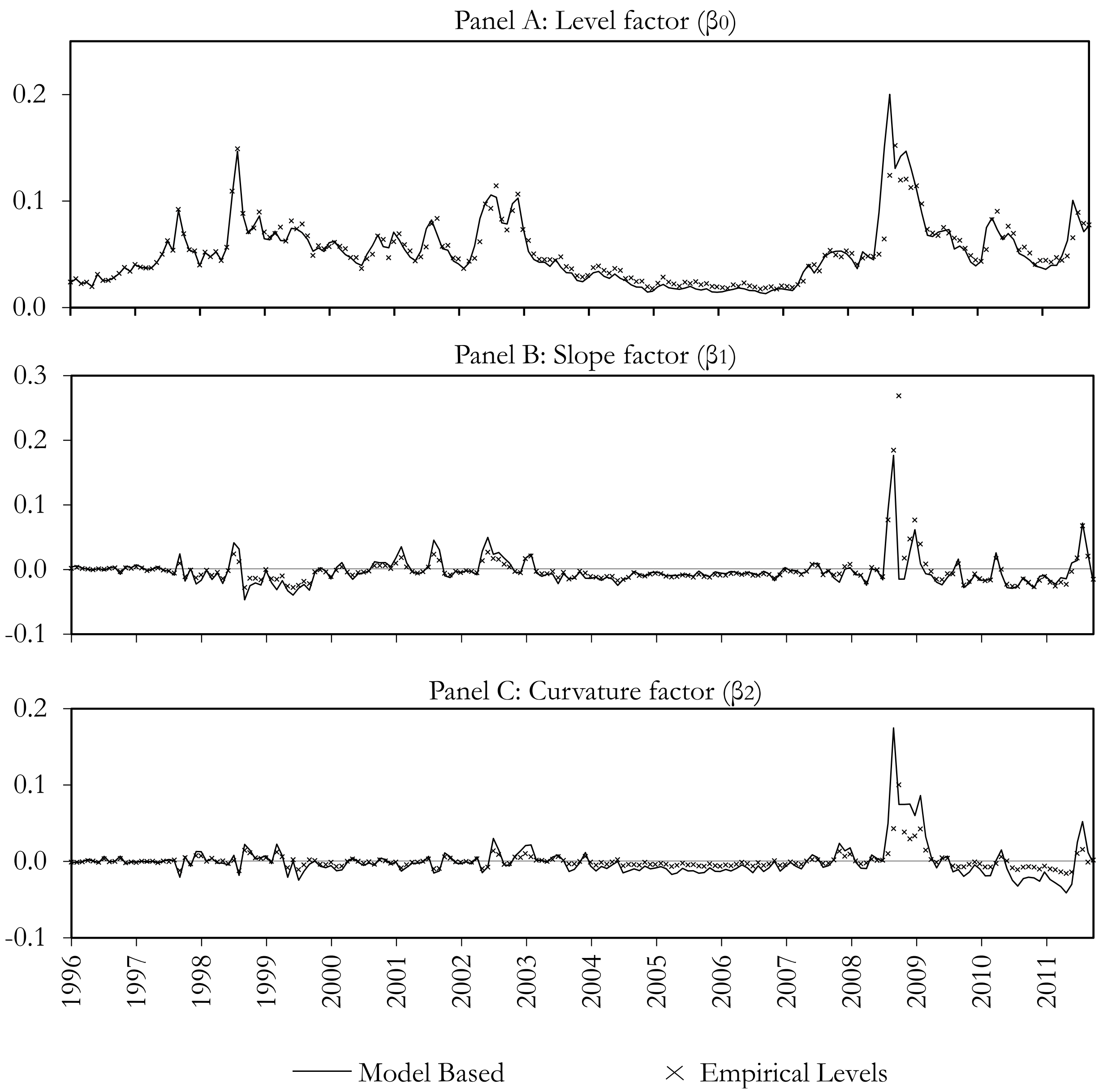

Figure 5: Predictive Regression (conditional mean versus unconditional mean)

This figure represents the OOS performance of the one-month predictive regression (for the S&P 100 returns). Specifically, these are the cumulative squared prediction errors of the model defined in Equation (10) minus the cumulative squared prediction error of the naive model. Equation (10) forecasts the one-month market return using the expected change on the one-month variance swap rate whereas the naive model assumes that the historical average is the best forecast. When the line increases (decreases) the conditional (naive) model predicted better. The grey bands are recessive periods as defined by the NBER.

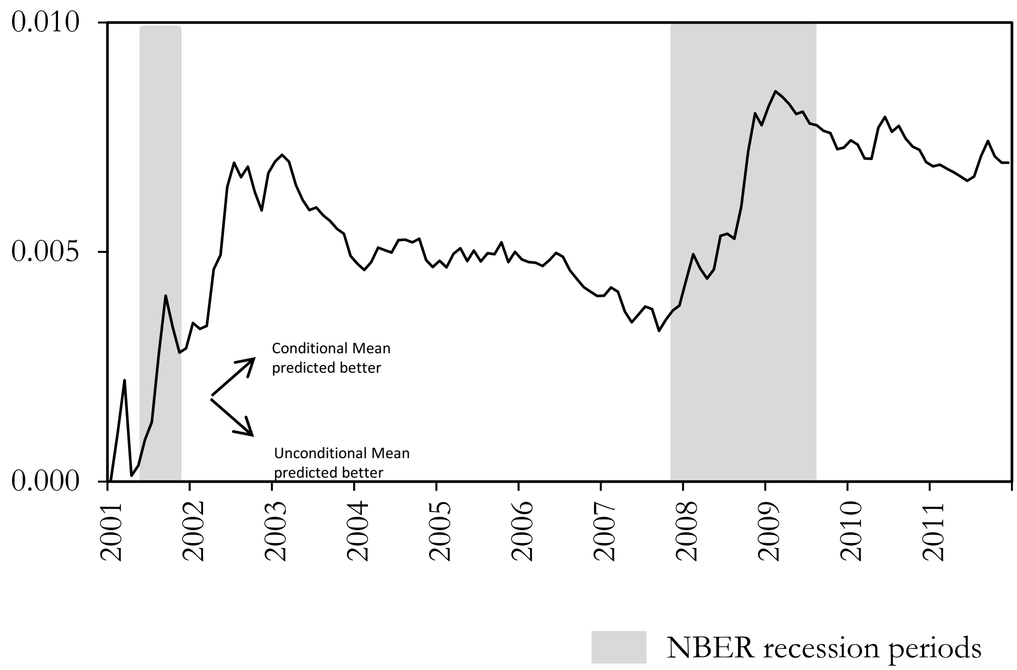

Figure 6: Returns from S&P 100 stock, S&P 100 1-month variance swap

This figure represents the returns and cumulative performance of several S&P 100 allocations during the period January of 2001 until December of 2011. Panel A reports the return of the S&P 100, Panel B reports the return of a mean-variance portfolio on the both the S&P 100 and the risk-free, Panel C shows the return of a mean-variance portfolio on the risk-free, the S&P 100 and the one-month variance swap. Panels D to E show the cumulative performance assuming an initial investment of 100. The returns are discrete returns rather than continuous.

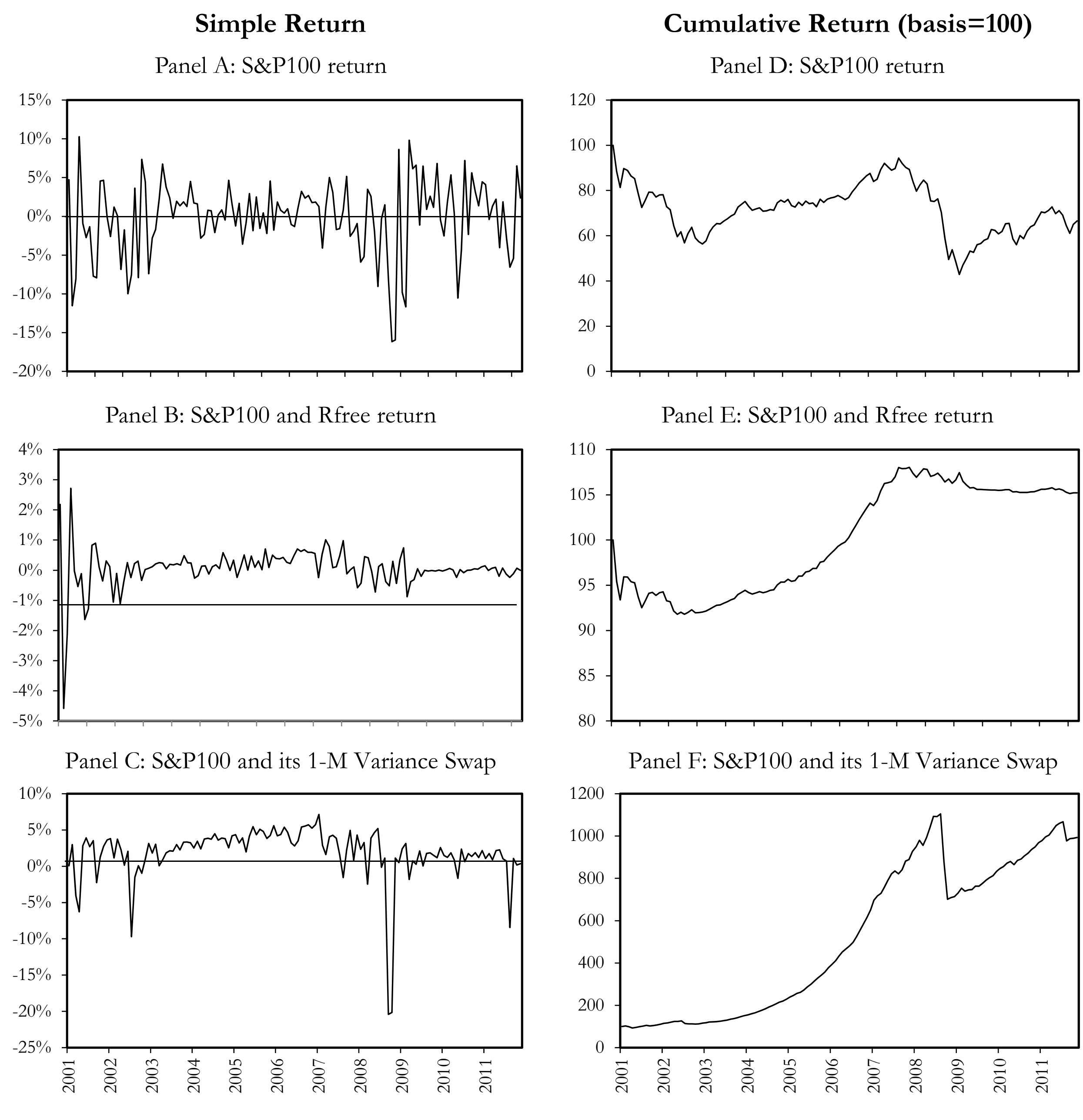

Figure 7: Returns from top and bottom deciles sorted by VRP

This figure represents the returns of the main portfolios analysed during the period January of 2001 until December of 2011.. Each month we sort the S&P 100 stocks into 10 equally-weighted portfolios according to their one-month variance risk premium. We then keep the next month portfolio return and variance return. Panel A and B show the returns of a mean-variance strategy on the two extreme portfolios which include stocks, risk-free and a one-month variance swap .Panels C and D show the cumulative performance assuming an initial investment of 100. The returns are discrete returns rather than continuous.