On the use of the Gompertz model to predict microbial inactivation behaviour

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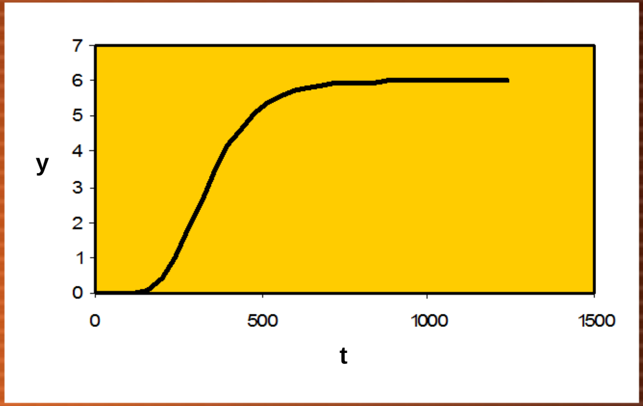
Objectives

$$y_{orig}(t) = a\ \exp(-\exp(b - ct))$$

y	population size
t	time
a,b,c	model parameters



Study of the original Gompertz equation

modifications in order to describe microbial inactivation

re-parameterization of the modified function

biological meaning

Introduction

Mathematical modelling

- Food microbiologists are giving increased attention to microbial kinetics modelling

- The use of mathematical models that properly describe microbial behaviour under specific environmental conditions is important for predictive purposes and process design

- Mathematical models require the definition of parameters (e.g. the rate constant), which should be precisely estimated

Introduction

Gompertz equation

$$y_{orig}(t) = a\exp(-\exp(b - ct))$$

- **Is an empirical sigmoidal relationship (*describe linear and nonlinear curves*)**
- **Introduced in food microbiology to describe <u>asymmetrical sigmoidal shape</u> of microbial growth**
- **Can be used to describe microbial <u>inactivation</u> behaviour (*if properly modified*)**

Introduction

However…

- information related to function mathematical study and to methodology of re-parameterization is lacking

  neglecting parameters biological meaning !

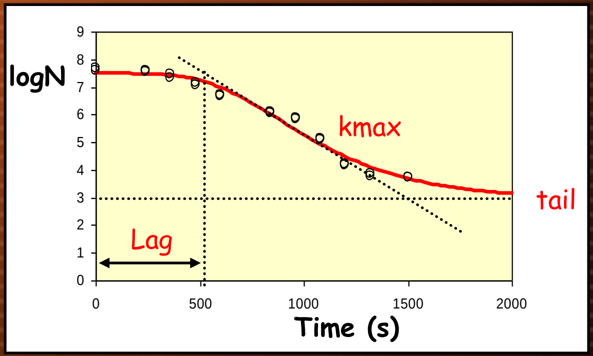
- only few works link model parameters to microbiological meaning (growth/survival rate and lag time/shoulder)

  drawbacks in microbial kinetic models

Introduction

Insightful information can be obtained with <u>re-parameterized forms</u>

Microbial survival often presents a lag, a linear phase (corresponding to a maximum inactivation rate, k_{max}) and a residual population (tail)



Those phenomena should be included in model parameters

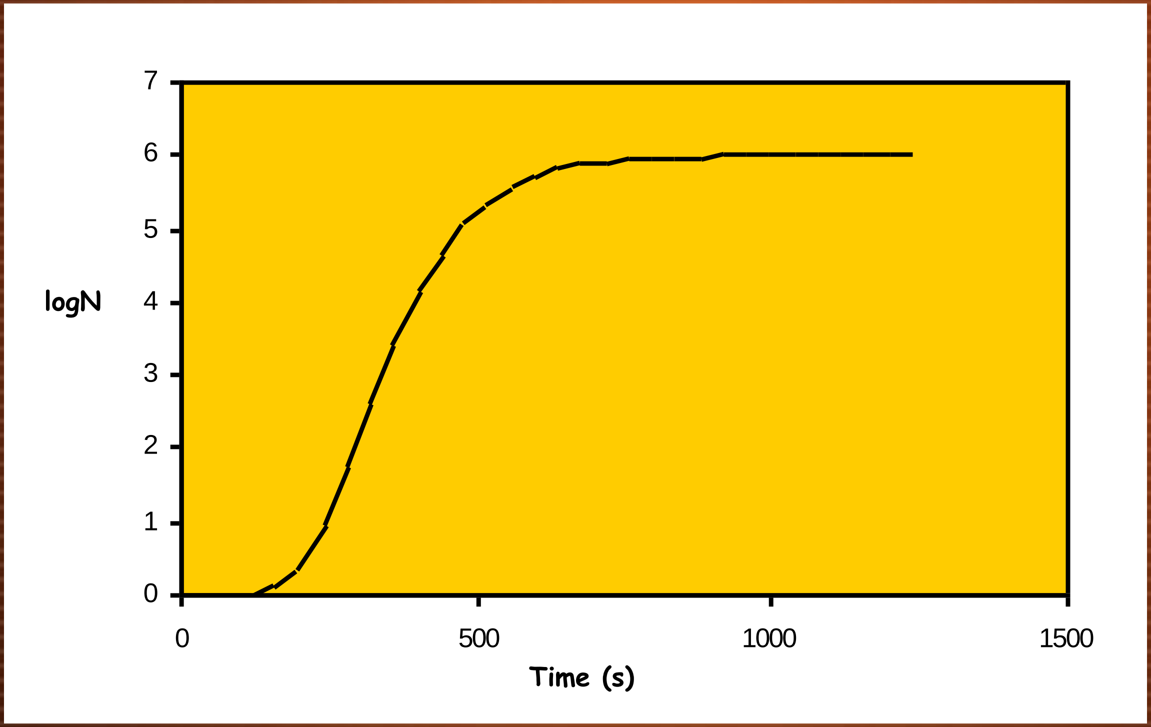
Introduction

Study aim

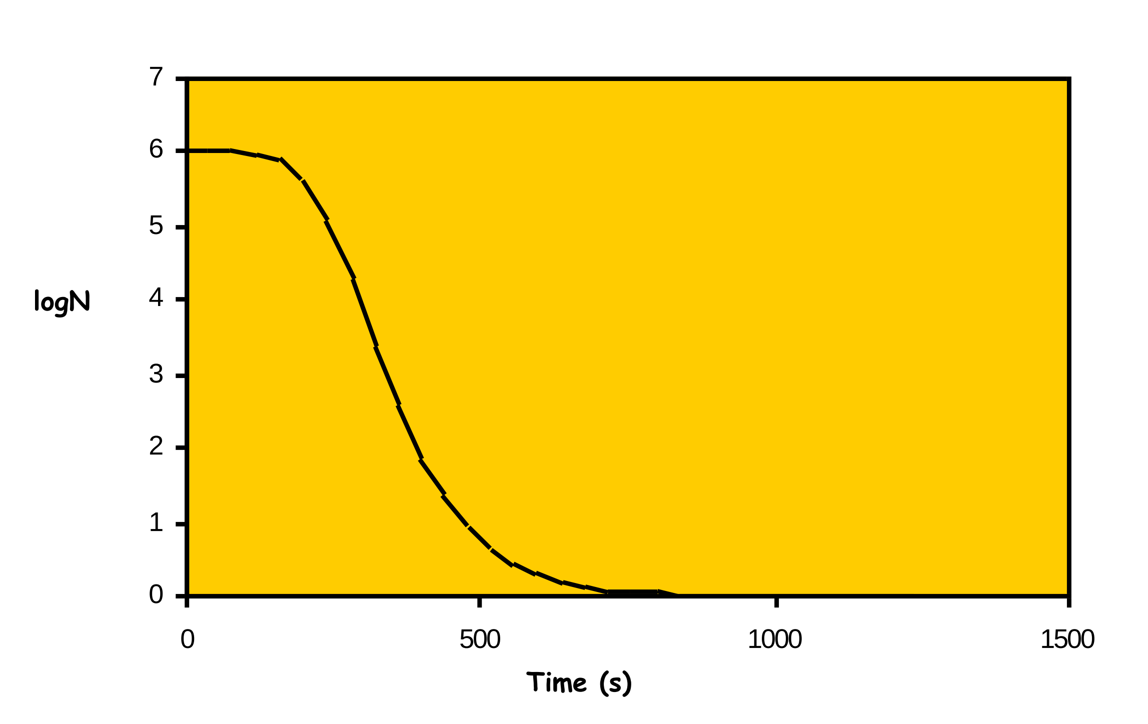
modify in order to describe inactivation

Original Gompertz model

Microbial behaviour



Microbial behaviour



Introduction

Study aim

modify in order to describe inactivation

re-parameterization based on the analytical study of the function

Original

Gompertz model

Introduction

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Original

Gompertz model

test drawbacks/advantages of the re-parameterized forms

Approaches

Two different approaches were studied in order to obtain Gompertz modified model for inactivation kinetics:

- the logarithm of the microbial load (log N) → approach 1

$$y_{inact}(t) = \log(N) = const - y_{orig}(t) \quad (1)$$

- normalized to the initial load (log N/N_0) → approach 2

$$y_{inact}(t) = \log\left(\frac{N}{N_0}\right) = -y_{orig}(t) \quad (2)$$

Re-parameterization

Analytical study of the functions

- **Function limits**

  time → ∞ tailing effect

  time → 0 initial value

- **Derivatives**

↓

Equations re-written

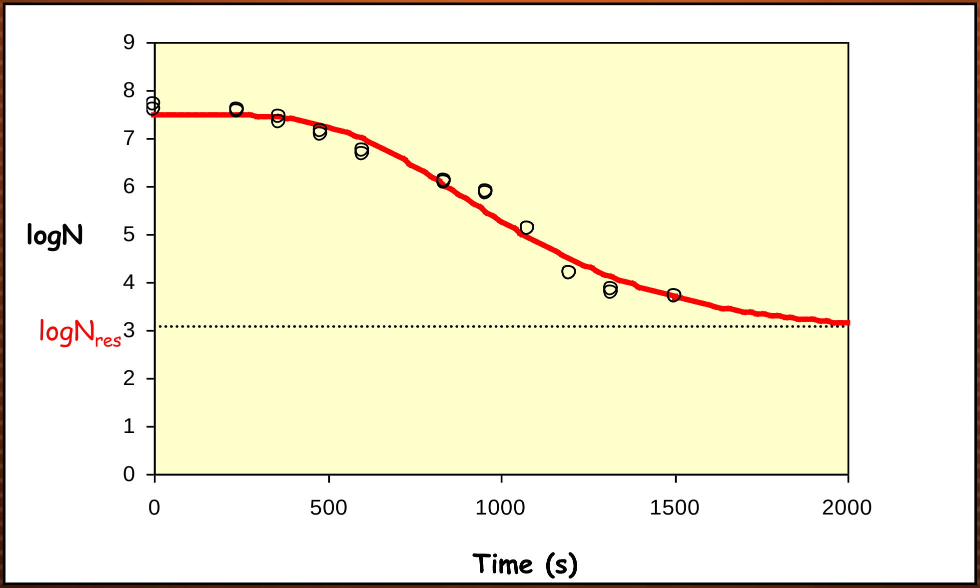
Parameters with microbiological meaning

Re-parameterization

Model parameters

Tail — asymptote of the function

$(\log N_{res})$

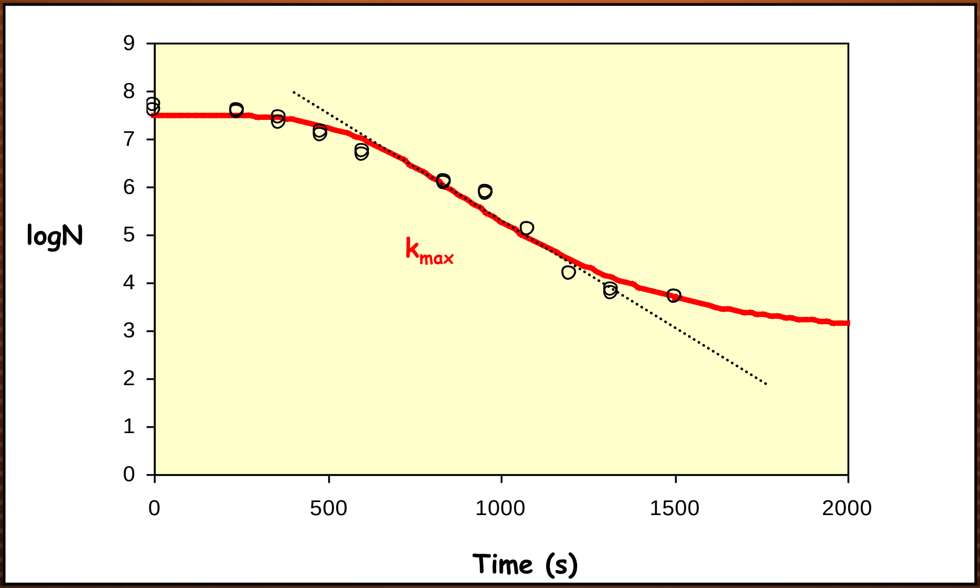


Re-parameterization

Model parameters

Maximum inactivation rate (k_{max})

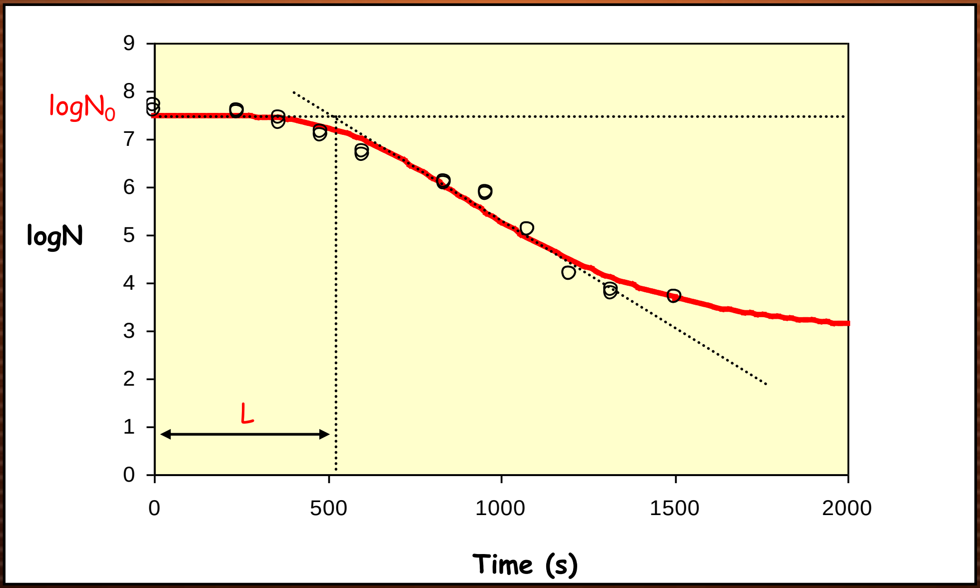
derived by calculating the 1st derivative at the curve inflexion point



Re-parameterization

Model parameters

Shoulder (L) — determined by the interception of the extrapolated tangent line with initial value (or time-axis - approach 2)



Re-parameterized forms

Approach 1

$$y_{inact}(t) = \log(N) = const - a\exp\left(-\exp\left(b - ct\right)\right)$$

$$a = \log\left(\frac{N_0}{Nres}\right)$$

$$c = -\frac{k_{\max}\exp(1)}{a}$$

$$b = L\ c + 1$$

4 parameters

$$y_{inact}(t) = \log\left(N_0\right) + \log\left(\frac{N_{res}}{N_0}\right)\exp\left(-\exp\left(\frac{k_{\max}e}{\log\left(\frac{N_{res}}{N_0}\right)}\left(L - t\right) + 1\right)\right)$$

Re-parameterized forms

Approach 2

$$y_{inact}(t) = \log\left(\frac{N}{N_0}\right) = -a\exp\left(-\exp\left(b - ct\right)\right)$$

$$a = -\log\left(\frac{N_0}{Nres}\right)$$

$$c = -\frac{k_{\max}\exp(1)}{a}$$

$$b = L\;c + 1$$

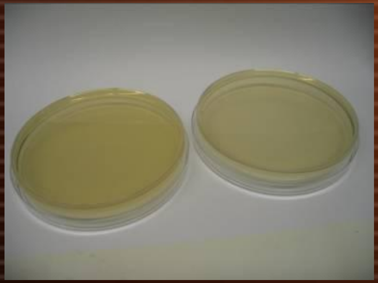
3 parameters

$$y_{inact}(t) = \log\left(\frac{N_{res}}{N_0}\right)\exp\left(-\exp\left(\frac{k_{\max}\,e}{\log\left(\frac{N_{res}}{N_0}\right)}\left(L - t\right) + 1\right)\right)$$

Case study

Studied approaches were validated on the basis of experimental thermal inactivation data of *Listeria innocua*

TSBYE



TSAYE

°C

52.5
55.0
57.5
60.0
62.5
65.0

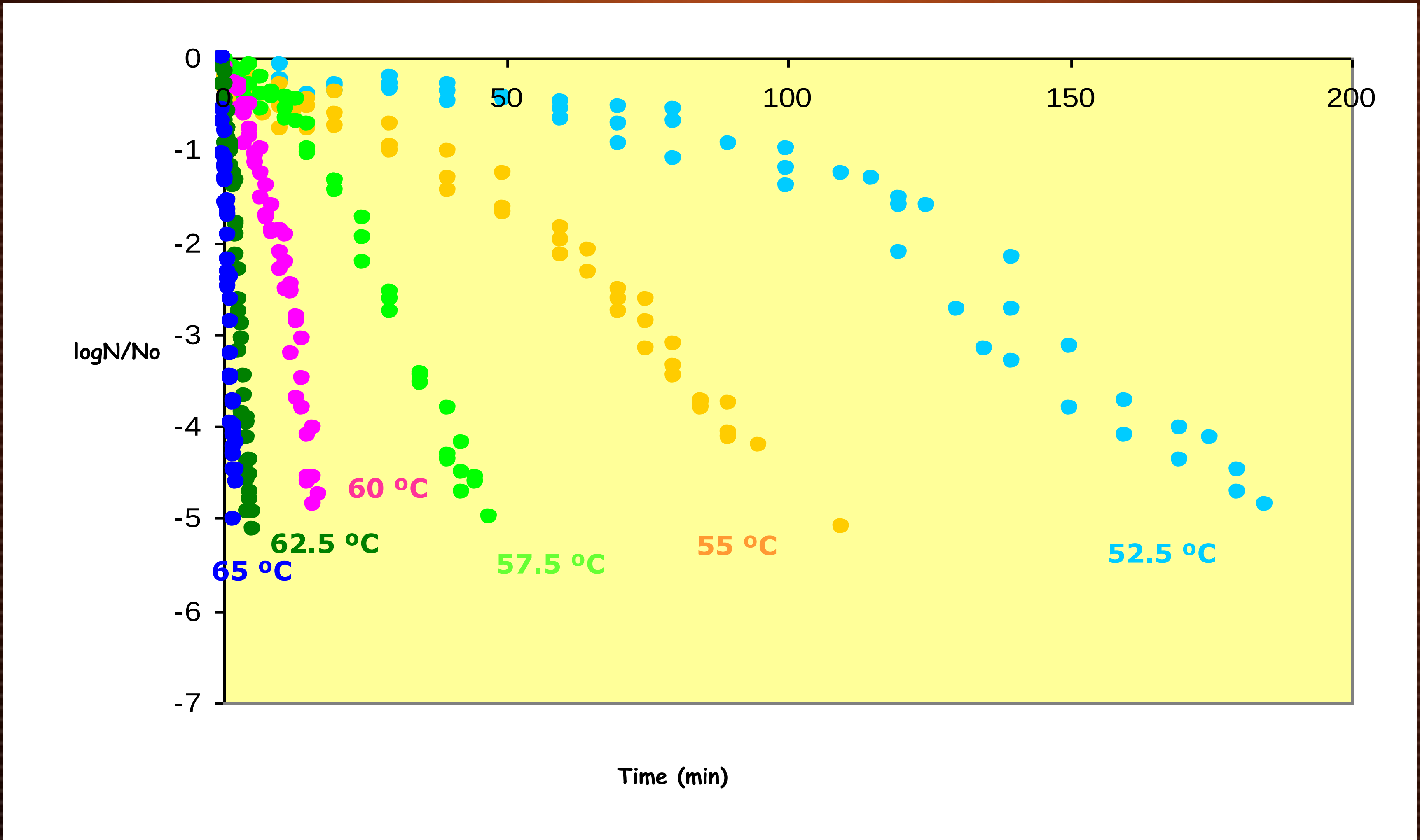
Methodology

- **Kinetic parameters were estimated by non-linear regression analysis of isothermal data**

least squares estimation

using STATISTICA TM v 6.0 software

Results

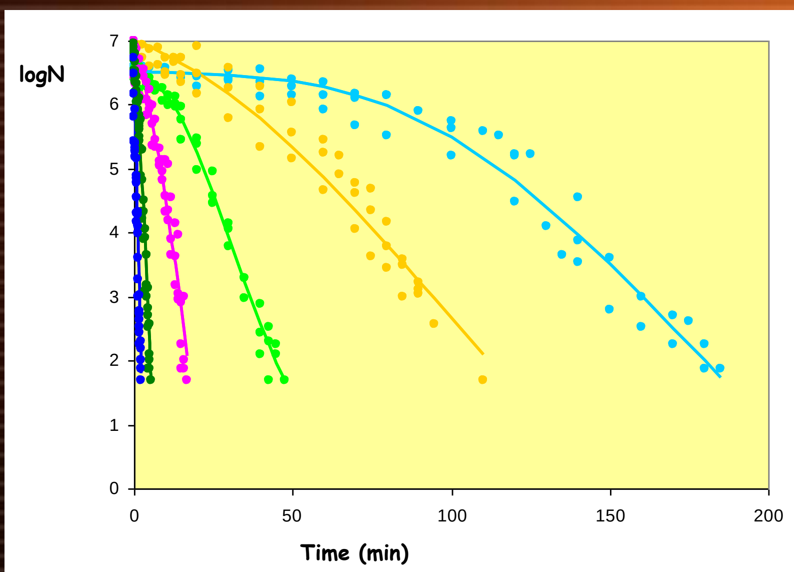


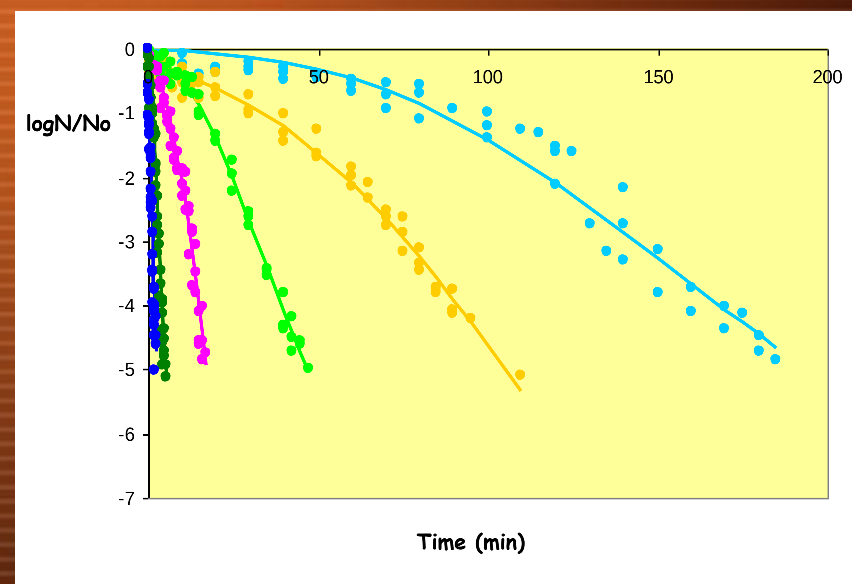
0
-1
-2
-3
-4
-5
-6
-7
0
50
100
150
200
logN/No
Time (min)
65 °C
62.5 °C
60 °C
57.5 °C
55 °C
52.5 °C

Results

approach 1

approach 2





Results

- **Gompertz modified model allowed accurate predictions of *Listeria innocua* inactivation in the temperature range considered**

- **Both approaches are quite similar**

Results

	Approach 1		Approach 2	
	$y_{inact}(t) = \log(N) = const - a\exp(-\exp(b - ct))$		$y_{inact}(t) = \log\left(\frac{N}{N_0}\right) = -a\exp(-\exp(b - ct))$	
Regression Analysis	R_{adj}^2	0.876	R_{adj}^2	0.970
	Normality	√	Normality	√
	Randomness	√	Randomness	√
	Parameter	Error (%)	Parameter	Error (%)
Parameter´s Precision	$\log N_{res}/N_0$	26 - 37	$\log N_{res}/N_0$	14 - 28
	k_{max}	11 - 17	k_{max}	7 – 13
	L	43 - 563	L	16 - 63

Results

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$$error = \frac{95\%\ \text{confidence interval}}{2} \times \frac{1}{\text{parameter estimate}}$$

Conclusions

- Gompertz allowed accurate predictions of *Listeria innocua* inactivation in the range considered

- Both approaches are quite similar

- **Parameters were estimated with precision**

- **The quality of regression and parameters estimation were improved if normalized data is used**

- **The use of accurate models is a considerable tool to predict target pathogen´s survival**

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Thank You!