A Multi-objective Optimization for the Design and Periodic Scheduling of Multipurpose Facilities under uncertainty

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Abstract

Like most real-world problems, the design of multipurpose batch facilities involves multiple objectives. However the existing literature on the subject has been mainly centred on mono-criterion objectives (Barbosa-Povoa, 2007). Therefore, multi-objective optimisation is a modelling approach that requires further study when applied to such facilities. The best way to deal with various goals simultaneously is to define the efficient frontier which offers the optimal solutions found by multi-objective optimization. In this work, the inspection of the efficient frontier allows the decision maker to select the most satisfactory plant topology with the respective equipment design and storage policies that minimizes the total cost of the system, while maximizing production, subject to operational restrictions. The approach to the detailed design of multipurpose batch facilities with periodic mode of operation, as proposed by Pinto et al. (2005), is now extended to address the problem of uncertainty associated with demand and the incorporation of economic aspects. The uncertainty is treated through a two-stage stochastic model, leading to a MILP formulation. A scenario is set up where the demand is represented by a discrete probability function and a cyclic operation is considered. The ε-constraint method is employed to handle the multi-objective optimization. An example, where different situations are evaluated is solved and a topology analysis is made.

Keywords: Design, scheduling, batch, multi-objective, RTN, uncertainty.

1 Introduction

In multipurpose batch facilities, a wide variety of products can be produced via different processing recipes by sharing all available resources, such as equipment, raw material, intermediates and utilities. Sometimes this type of facilities operates under conditions of relatively stable production demands, over extended periods of time. In such situations it is useful to establish a regular periodic operating schedule, in which the same sequence of operations is carried out repeatedly, thus simplifying the operation and control of such facilities.
The periodic mode of operation applied to the design/schedule of multipurpose-batch plants has been so far centred on mono-criterion. However, within the design problem is often the case that multiple objectives are present and should therefore be accounted for. Adding up to this aspect, the presence of uncertainties should also be explored at the design level, since uncertain environments do characterise design problems (Barbosa-Povoa 2007). One of the reasons that explain the lack of studies dealing with uncertainty within multipurpose batch plants is the increased complexity introduced into the problem. For this reason most of the work published deals essentially with a deterministic approach, where all the parameters are known. Uncertainty may be related with raw material availability, raw material prices, machine reliability and market requirements, which vary with respect to time and are often subject to unexpected deviations.
For these reasons, in the present paper the work of Pinto *et al.* (2005) is generalized to account for the presence of demand uncertainty within a multi-objective approach. The multi-objective is based on a ε-constraint method and the uncertainty is modelled through a two-stage model. This allows the identification of a range of plant topologies, design facilities and storage policies that minimize the total cost of the system, while maximizing production, subject to total uncertainty demand and operational constraints. A topology analysis is made from the solution of one example, which is solved for two different cases. In case a) is presented the efficient frontier for the design and periodic scheduling of multipurpose facilities under a deterministic environment, while case b) characterizes the efficient frontier for the design and periodic scheduling of multipurpose facilities under demand uncertainty.

2 Design Problem

The optimal plant design can be obtained by solving the following problem:

Given:
- Process description, through a Resource-Task Network (RTN) representation;
- The maximal amount of each type of resource available, its characteristics and costs;
- Time horizon of planning and cycle time;
- Demand over the time horizon (production range);
- Task and resources operating cost data;
- Equipment and connection suitability;
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Determine:
- The amount of each resource used;
- The process scheduling;
- The optimal plant topology as well as the associated design for all equipment and connectivity required.

Mixed storage policies, shared intermediated states, material recycles and multipurpose batch plant equipment units with continuous sizes, are allowed. In terms of operation mode, a periodic operation is considered where the concept of cycle time T is used. This is taken as the shortest interval of time at which a cycle is repeated, where the cycle represents a sequence of operations involving the production of all desired products and the utilization of all available resources.

Since all cycles are equal, the problem is formulated over a single cycle where it is guaranteed that the operation of the plant is the same at the beginning and at the end of the cycle. The execution of a task is allowed to overlap successive cycles and, since a cycle is repeated over the time of planning, its execution is modelled by wrapping around to the beginning of the same cycle. To do so, the wrap-around operator as defined by Shah *et al.* (1993) is used:

$$\Omega(t) = \begin{cases} t & \text{if } t \geq 1 \\ \Omega(t+T) & \text{if } t \leq 0 \end{cases} \tag{1}$$

When this is applied, for instance to the variable $N_{k,\Omega(t-\theta)}$ for t–θ ≤ 0, it leads to an identical equipment resource allocation which will start at time t–θ +T.

In terms of time modelling, a discretization of time is used, where the cycle time is divided into T intervals of equal duration. The start of the cycle is defined as time t=1, and the end as t=T+1. The latter coincides with the starting point of the next cycle. A planning horizon (H) is assumed which is divided into n equal cycles of duration (T). A cycle is divided into a number of elementary time steps of fixed duration (δ), as shown in Figure 1.

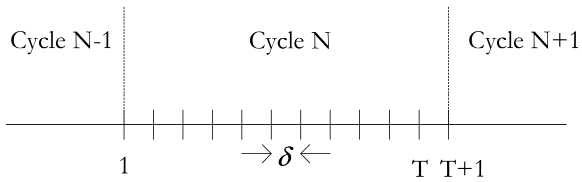

Figure 1: Time discretization for a single cycle

3 Multi-objective Optimization

In general those models requiring a single objective have a clear, quantitative way to compare alternative feasible solutions. In many applications a single objective is sufficient to model realistically the actual decision process. However, decisions become much more problematical when dealing with complex engineering designs, where more than one objective is to be evaluated. For such cases, as referred above, a multi-objective optimization model is required to capture all the possible perspectives. This is the case of batch plants design, addressed in this work, where two objectives are under consideration – maximization of revenue (that is, production) and minimization of cost. Contrary to common practice, where each of this term has an assigned equal weight and is merged into a single index such as profit, revenue and cost are now handled separately since the desired trade-off between them is meant to be open to the decision maker. The multi-objective optimization can be generically represented as:

$$\begin{aligned} & \textit{Maximize } f_m(x) \qquad m = 1,2,...,M; \\ & s.t. \\ & \qquad g_j(x) \leq 0 \qquad j = 1,2,...J; \\ & \qquad h_k(x) = 0 \qquad k = 1,2,...,K; \\ & \qquad x_i^{(L)} \leq x_i \leq x_i^{(U)} \quad i = 1,2,...,n. \end{aligned} \tag{2}$$

where m identifies the objective function $f(x) = (f_1(x), f_2(x),\ldots,f_m(x))^T$ and j and k are, respectively, the number of inequality and equality constraints. A solution will be given by a vector X of n decision variables: $X=(x_1, x_2,\ldots, x_{n-1}, x_n)^T$.

However, no solution vector X exists that maximizes all objective functions simultaneously. A feasible vector X is called an optimal solution if there is no other feasible vector that increases one objective function without causing a reduction in at least one of the others. It is up to the decision maker to select the best compromising solution among a number of optimal solutions in the efficient frontier. There are several methods to define this frontier, but one of the most popular methods is the ε-constraint, which is very useful since it overcomes duality gaps in convex sets.

3.1 The ε- Constraint

In the current work the multi-objective design of batch plants is formulated using the ε-constraint method where, in each step, the revenues are maximized subject to a limit defined by the cost objective function (maximum budget available). The procedure starts with the maximization of the revenues without restrictions on the cost function. This generates the pair maximum revenues (RV_{max}) and the cost necessary for RV_{max} (C_{max}). This is followed by a similar procedure applied to the cost indicator with no restrictions on the revenues, determining the minimum cost for the facility design, C_{min}, and the associated value of revenue. In our case, this is defined as (0, 0): if the minimum equipment capacity available is 0, there is no equipment installed and the respective revenue is null. The constraint on the objective function that defines the cost is then activated. This restriction initially takes the value of C_{max} decreased by a small value, $\Delta\varepsilon$, in successive optimization steps ($k=1,\ldots, n$), until C_{min} is reached and the entire range of interest covered. The algorithm can be summarized as follows:

1. Solve revenue objective function, RV_{max} = max RV to determine C_{max}
2. Solve cost objective function, C_{min} = min cost to determine C_{min}
3. $\Delta\varepsilon = (C_{max} - C_{min})/n$
4. Solve

$$\begin{aligned} &\max RV \\ &s.t.\ \ Cost \le C_{\max} - k\Delta\varepsilon \qquad k = 0,\ldots,n. \end{aligned}$$

This procedure is applied to the formulations presented in Pinto *et al.*(2005). Applying the ε-constraint, formulation (2) becomes:

$$\begin{aligned} &Maximize\ f_u(x) \\ &s.t. \quad f_m(x) \le \varepsilon_m \quad m = 1,2,\ldots,M \ and\ m \ne u \\ &\qquad\ \ g_j(x) \le 0 \quad j = 1,2,\ldots; \\ &\qquad\ \ h_k(x) = 0 \quad k = 1,2,\ldots,K; \\ &\qquad\ \ x_i^{(L)} \le x_i \le x_i^{(U)} \end{aligned} \qquad (3)$$

where ε_m represents an upper bound of the value of f_m. This technique suggests handling one of the objectives and restricting the others within user-specified values. First the upper and lower bounds are determined by the maximization of the total revenue and minimization of the cost, respectively. Next, varying ε, the optimization problem (maximization) is implemented with the objective function being the total revenue and the cost being a constraint varying between its lower and upper bounds. The efficient frontier is obtained, which allows the decision maker to select any solution depending on the relative worthiness of each objective.

3.2 The Two-Stage Stochastic Model

To model the uncertainty a two-stage stochastic model was developed. Consider M as the set of all possible scenarios and m ∈ M a particular scenario. Let all first-stage variables be included in vector *y* and all second-stage variables in vector *x*. Let *f* be the vector of the fixed costs related to the choice of a certain resource *c* and *p* the vector containing the remaining coefficients in the objective function. The deterministic model for a particular scenario *m* is defined as:

$$\begin{aligned} \max \quad & p_m x - c_m x - fy \\ s.t. \quad & A_m x \le a_m \\ & B_m x \le Cy \\ & y \in \{0,1\}, x \ge 0, x \in R \end{aligned}$$

where A_m and B_m are matrixes and p_m, a_m vectors.
The solution of this model gives the optimal design and scheduling for any individual scenario.

Two-stage stochastic model:
The above formulation can be modified to represent a two-stage stochastic linear model with recourse:

$$\begin{aligned} \max \quad & E\left[\Theta(y,m)\right] - f\,y \\ s.t. \quad & y \in \{0,1\} \end{aligned}$$

with

$$\begin{aligned} \max \quad & \Theta(y,m) = p_m x_m - c_m x_m \\ s.t. \quad & A_m x_m \le a_m \\ & B_m x_m \le Cy \\ & x_m \ge 0, x_m \in R \end{aligned}$$

Where $E[\cdot]$ is the expected value of $[\cdot]$ over *m* and p_m, c_m are associated with the distribution function J_m .

4 Examples

The General Algebraic Modelling System (GAMS) was used coupled with the CPLEX 11.0. Both examples used a Pentium (R) 4, 3.6 GHz, 2 GB RAM.

The algorithm proposed above is applied to the example presented by Barbosa-Povoa and Macchietto (1994), where the maximization of the revenue with the production of three final products, S4, S5 and S6 is made. The final products are S4, S5 and S6 using two raw materials, S1 and S2. In terms of equipment suitability, reactors R1, R2 and R3 are multipurpose equipment. Task T1 may process S1 during two hours in R1 or R2 producing the unstable material S3; Task T2 may process S2 during two hours in R2 or one hour in R1 producing S4, which is both an intermediate material and a final product; Task T3 processes 0.5 of S3 and 0.5 of S4 in R3 during three hours producing S5, which, like S4, is both an intermediate and a final product; Task T4 may process 0.5 of S3 and 0.5 of S5 in R3 during two hours producing the final product S6. Each vessel is suitable to store only one material State. The capacity is defined for all equipments, in a continuous range between upper and lower bounds.

This example is solved for two cases a) and b). In case a) the process operates in a periodic mode over a time horizon of 8 hours/day in a campaign of 100 days and is solved deterministically. The demand for each product is presented in Table 1. In case b) and like in case a) the process still operates in a periodic mode, using a time horizon of 8 hours/day in a campaign of 100 days, but now uncertainty associated to the product demand is also considered. In here three scenarios are contemplated namely, expected, optimistic and pessimistic, with an associated probability value, respectively of 0.5, 0.4 and 0.1. The product demand for each scenario is presented in Table 2.

Table 1: Range of product demands, for case a).

Demands	min: max [tons]
S4	0: 11000
S5	0: 6000
S6	0: 6000

Table 2: Range of product demands, for case b).

Scenarios	Pessimistic min:max [tons]	Expected min:max [tons]	Optimistic min:max [tons]
S4	0: 5500	0: 11000	0: 16500
S5	0: 3000	0: 6000	0: 9000
S6	0: 3000	0: 6000	0: 9000

4.1 Case a): Design and Periodic Scheduling of Multipurpose Facilities

The obtained efficient frontier is depicted in Figure 2 where it is possible to verify that the plant topology used between A and D requires one single reactor, R1, and produces only 25 tons of S4. At point D, the topology requires one additional reactor, R3, and is able to produce 110 tons of S4 and 58.8 of S5. This topology characterizes the range between the point D and F. As the revenue increases, the costs also increase, mainly because of the equipment capacity expansion (to allow a higher production) and the operational costs associated. This topology differs from the next one in one tank, V6 (Table 3), which allows the storage of 7.1 tons of the final product S6. The final topology presents three reactors R1, R2 and R3, whose equipment design characteristics are presented in Table 3.

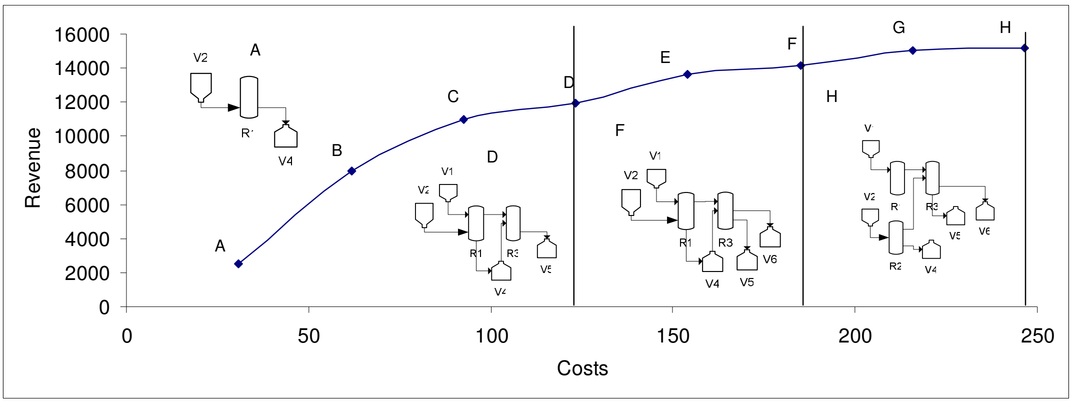

Figure 2: The efficient frontier for case a).

Table 3- Case a): Optimal design for the main equipment.

Equipment	A	B	C	D	E	F	G	H
R1	25.01	79.43	82.21	58.82	65.67	67.57	71.2	81.81
R2	-	-	-	-	-	-	-	50.4
R3	-	-	-	19.08	53.35	62.85	80.99	84
V4	25.01	79.43	110	110	110	110	110	110
V5	-	-	-	19.08	53.35	60	60	60
V6	-	-	-	-	-	7.13	52.48	60

This model requires 166 binary of a total of 649 variables, and 1069 constraints.

4.2 Case b): Design and Periodic Scheduling of Multipurpose Facilities under demand uncertainty

Using the same example, but with some adaptations to the design of a multipurpose batch facility operating under demand uncertainty, the multi-objective problem is again solved.

The efficient frontier obtained from the model operating under demand uncertainty is visible in Figure 3. The first topology, at point A, only requires reactor, R1, producing 33.9 tons of S4. In the next topology, point C, the process equipment was switched to R2 and the production of S4 (99.6 tons) increases. At point D, the obtained topology is able to produce 165 tons of S4 and 23 tons of S5, with one more vessel (V5) and reactors with higher capacity. The next topology using the same equipment with higher capacities is able to produce one more final product (S6). The final topology, at point H, requires three reactors R1, R2 and R3 producing 165 tons of S4 and 90 tons of S5 and S6.
All the equipment design, for the range of optimal topologies presented in Figure 3, are visible in
Table 4.

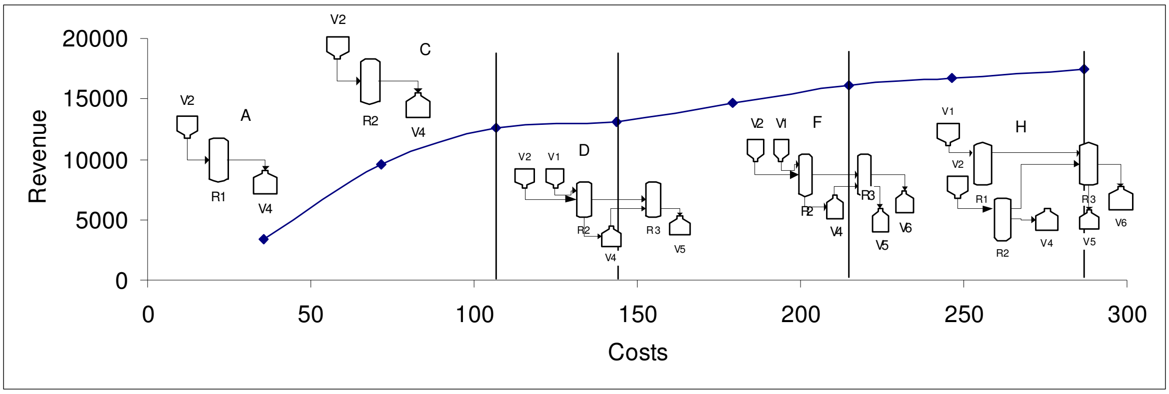

Figure 3: The efficient frontier for case b).

Comparing the optimal topology resulting from cases a) and b), it is possible to verify that, between points A and C, the topology is the same in both cases. The optimal topology only requires one reactor, R1, as process equipment. In case b) the topology at point C, changes the process equipment to reactor R2 (which presents a higher capacity) when operating under uncertainty. Analyzing the other topologies, it is visible that in both cases the number of equipment units increases with the revenue for points D, F and H. It can also be noted that for both cases, the selection of reactors is the same for these points, with the exception of reactor R1, which is always replaced in case b) by reactor R2, see Figure 3.

Table 4- Case b): Optimal design for the main equipment.

Equipment	A	B	C	D	E	F	G	H
R1	33.9	99.6	-	-	-	-	-	53
R2	-	-	165	165	165	165	165	165
R3	-	-	-	23.8	43.6	90	90	90
V4	33.9	99.6	165	165	165	165	165	165
V5	-	-	-	23	45.6	90	90	90
V6	-	-	-	-	-	23.5	60	90

As expected the number of binary, variables and constraints are higher in case b) than case a). This model requires 454 binary in a total of 18411 variables and 3137 constraints.

5 Conclusions

The design of multipurpose batch plants under a periodic mode of operation is studied, two contexts are explored: (1) multi-objective deterministic problem; (2) multi-objective problem under uncertainty in the final product demands. The models were developed as MILPs and the multi-objective method used was the ε-constraint. The obtained efficient frontier defined the optimal solutions, allowing the identification of a range of plant topologies that permits the evaluation of a topology change caused by the demand uncertainty.

The scenario approach is based on a two-stage stochastic programming. Due to the increase number of variables and constraints associated to scenarios approach, we adopted a simplistic approach using three possible scenarios defined as: optimistic, expected and pessimistic. The probabilistic values employed for each scenario justifies a close evaluation, in order to evaluate the performance of the model subject to these parameters variability.

The proposed methodology allows the decision makers to evaluate the relationship between revenues and costs when designing batch facilities, operating in a periodic mode under a products demand uncertainty environment.

6 Referencies

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