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Scheduling of job shop, make-to-order industries with recirculation and assembly: discrete versus continuous time models

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Abstract This work studies the performance of two Mixed Integer Linear Programming (MILP) models to solve scheduling problems in a flexible job shop environment with recirculation and assembly using a due-date-based objective function. The models convey different approaches both in the modelling of time (discrete and continuous approaches) as well as in the assignment of jobs to machines. The comparison is carried out for a job shop system considered closer to the industrial reality than the classical job shop problem of a single machine per operation that has been extensively studied in the literature, with the mould making industry providing the motivating application.

1 Introduction

A commonly used classification divides mathematical programming models for scheduling into discrete and continuous time models. Discrete time models divide the scheduling horizon into a finite number of intervals with equal and predefined duration and allow the beginning and ending of operations to take place only at the boundaries of these time periods. In continuous time models there is no previous division of the scheduling horizon, and timing decisions are explicitly represented as a set of continuous variables defining the exact times at which the events take place. Models of both kinds have been intensively developed for scheduling problems in the process industries but their application in discrete parts manufacturing has been quite limited so far.

A mould is a specially designed tool consisting of a "base" and one or more "cavities" contoured to the exact specifications of the desired product. A moulding compound is filled into the cavities, hardens by cooling and/or by completion of a chemical reaction and is extracted once it has taken the shape of the mould. A closer look at our daily environments shows the ubiquity of moulded pieces: the automotive, household and electrical appliances industries, together with the packing, electronics, telecommunications and hardware industries are the main clients of the mould industry. Mould makers operate in a make-to-order basis, producing one-of-a-kind products (orders seldom consist of more than one copy of a mould). Machinery is organized in the shop floor as in a typical job shop. The Mixed Integer Linear Programming formulations developed model explicitly several features of the mould fabrication process:

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– machine groups composed of multiple machines operating in parallel (flexible job shop);
– generalized processing routes, with no constraints upon the machine groups that constitute them;
– recirculation: some jobs visits a machine group more than once;
– assembly of components produced separately to form an order;
– every order has a due date for delivery to the costumer.

In the ***continuous time formulation***, the decision variables in common with the model of Manne (1960) are the starting time of each job on each machine (continuous) and the sequencing variables (binary) that establish the precedence between tasks in each processing unit. The notion of global precedence is used instead of immediate precedence. The model additionally accounts for product recirculation, product combination to form an order and a due date-based objective function. Orders may correspond to a single component or to several components combined in a final assembly operation. Earliness and tardiness variables were added following the approach of Zhu and Heady (2000) for the earliness and tardiness problem in a single-stage system of non-identical parallel units. While Manne's model considers a single machine per operation, non-identical machines per operation and binary assignment variables were considered following the approach of Méndez *et al*. (2001) in solving a flowshop problem in the process industry.

Scheduling objectives are: to finish each order as close as possible to the corresponding due date and to minimize storage of unfinished components in intermediate buffers (the queues to the machines). Earliness and tardiness costs for the orders are given, as well as costs of intermediate storage of unfinished components.

The ***discrete time scheduling formulation*** developed is based on the one presented by Chang and Liao (1994) for flexible flow-shop scheduling, which comprises homogeneous machine groups, limited buffers and orders composed of several identical units that follow the same processing route. A number of units (of the same or different orders) may be processed at the same time in a machine group. Gomes *et al*. (2005) extended this model to the flexible job shop case with generalized processing routes and recirculation. The model was further generalized to consider the existence of non-homogeneous machine groups (machines in a group may have different processing times). Orders corresponding to single or assembled components are considered (the notion of "unit" is replaced by "component" in the new model) and assembly operations were added (Gomes 2007). The scheduling horizon is divided into identical time slots; all model variables are integer.

In both models, the objective function (to be minimized) is a weighted sum of order earliness, tardiness and intermediate storage time. To summarize, in the discrete time formulation, time is modelled explicitly through a time index with no binary variables being required for machine assignment. In the continuous time formulation, modelling of time is implicit and jobs are assigned to machines through binary variables.

2 Computational study

The example considered is based on data collected at a mould making plant. Figure 1 is a simplified diagram of the plant: it consists of fifteen machines (M1 to M15) and four processing routes (A,B,C,D) that the components may follow. Orders are formed either by individual components or by the assemblage of two components, with two possibilities: a component from route A combined with a component from route C (assembly 1, performed in "machine", or assembly unit M14) or a component from route B joined with a component from route D (assembly 2, performed in assembly unit M15). Routes B and C display recirculation: machines M3, M10 and M11 are visited twice.

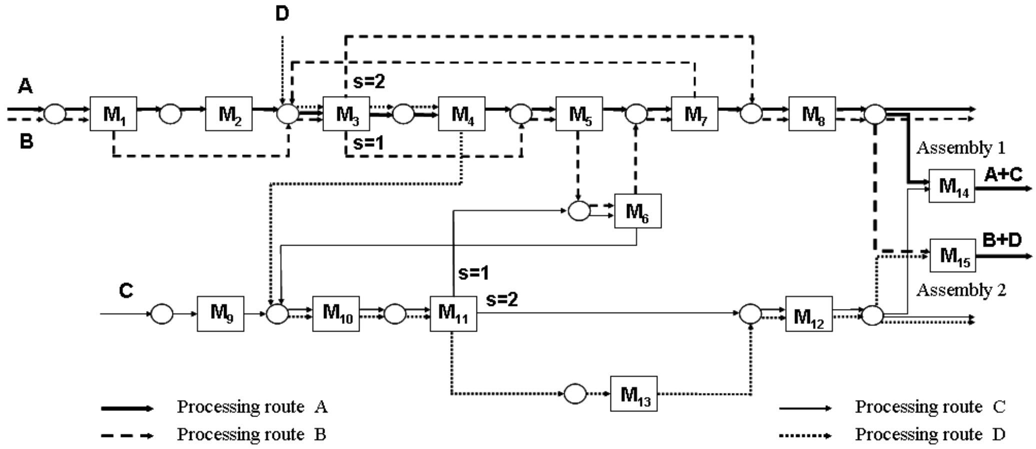

Figure 1. Processing routes in a shop producing single and assembled orders.

Five sets of orders to be scheduled were generated, involving 10, 20, 30 and 40 components (which correspond to "jobs" in the job shop terminology). Each set is composed of single and assembled orders. Two sets of 40 components were generated, which differ in the average interval between consecutive due dates: a "tight" due date set (40A) and a "loose" due date set (40B). The simplest operational scenarios considered have one or two machines per machine group. Scenario 2+(3,4) displays 34 machines in total: two machines per machine group, recirculation groups M10 and M11 with three machines each, and recirculation group M3 with four machines. Likewise for scenario 4+(5,6), with 64 machines in total: four machines per machine group, groups M10 and M11 have five machines each and group M3 has six machines. A complete description of order data and processing times can be found in Gomes (2007).

The MILP models were implemented in GAMS modelling system and solved with CPLEX on a 3 GHz Pentium IV with 512MB RAM running Windows XP Professional. CPLEX releases 9.1.2 (10 component set) and 10.0.1 (other sets) were used. Table I summarizes the results, by displaying model statistics, computational performance, the objective function value and also the total number of operations. "Gap" is the optimality gap (in percentage). For the continuous time model, a better estimate of the gap than the one returned by the solver was determined by using the optimal solution or the lower bound computed by the discrete time model, whenever it was available. For models not solved to optimality, "solution polishing" was performed (a heuristic procedure available in CPLEX 10 that improves a solution after the branch-and-bound search).

2.1 Discussion of results

When analyzing the results (see Table I) for the 10 and 20 component sets, the continuous time model performed better than its discrete counterpart when a single machine is available per operation. With the continuous time model either the optimal solution was computed in a shorter time (10 component set) or a better sub-optimal solution could be obtained with less computational effort (20 component set; 422 sec against 2701 sec).

However, when assignment to machines is considered (scenarios 2 and 2+(3,4) machines), the discrete time formulation generally outperformed the continuous one although the model size (variables plus constraints) is considerably larger. In all problem instances, except set 40A, the discrete time model was solved in CPU time less or equal to 12.7 min (759 sec). The continuous time model either took, on average, longer time to be solved or could not

Table I
Computational statistics and scheduling results

Scenario	No. of components	No. of variables	No. of constraints	Objective Function	Gap (%)	CPU (sec)	No. of operations
Continuous time model							
1 machine	10	329	477	621.8	0	4.3	71
	20	1,067	1,775	755.9	11.8	422.1	141
2 machines	10	400	801	0.8	0	192.6	71
	20	1,208	3,247	0.5	20.0	2700.2	141
2+(3,4) machines	20	1,278	4,387	0.2	0	40.7	141
	30	2,510	9,863	0.3	33.3	1200.3	210
	40A	4,198	17,845	81.8	100	1203.4	282
	40A	‘’	‘’	22.3	100	3000.3	282
	40B	‘’	‘’	1.2	100	1200.4	282
4+(5,6)	40A	4,762	30,305	0	0	356.4	282
Discrete time model							
1 machine	10	18,279	13,562	621.8	0	27.4	71
	20	52,025	33,958	859.9	22.5	2701.3	141
2 machines	10	27,233	16,356	0.8	0	9.6	71
	20	77,162	38,935	0.4	0	95.9	141
2+(3,4) machines	20	89,132	40,345	0.2	0	31.6	141
	30	132,269	55,962	0.2	0	248.8	210
	40A	178,263	73,156	—[a]	—	—	282
	40B	220,743	89,356	0	0	758.6	282

[a] An integer solution could not be found within the time limit of 30 min for branch & bound.

prove optimality within a time limit of 10 min (or longer) for branch-and-bound and 10 min (or longer) for solution polishing.

In set 40A of “tight” due dates and operational scenario 2+(3,4) no integer solution to the discrete formulation was found within 30 min of running the solver. Nevertheless, the continuous time formulation found a feasible solution in this case: a double entry “40A” is shown which corresponds to two solutions obtained with different time limits for branch-and-bound and solution polishing. A reduction of 73% in the objective function value was achieved when total run time increased from 20 to 50 min (1200 to 3000 sec).

Finally the table shows that two problem instances that could not be solved to optimality in 30 min of branch-and-bound with the continuous time model can be so if the number of machines per group is increased (and despite the increase in model size). This is the case of the 20 component set when the number of machines is increased from 2 to 2+(3,4) (41 sec to obtain the optimal solution) and the 40A component set when the operational scenario varies from 2+(3,4) to 4+(5,6) (356 sec or 5.9 min of CPU time).

As a main conclusion it can be stated that in the operational scenario of a single machine per operation the continuous time formulation performs better than its discrete counterpart while in the scenario of two or more machines in parallel per operation the discrete time formulation outperforms the continuous one. This result contrasts with the neglect of discrete time models in the fields of flow shop and job shop scheduling, where continuous time MILP formulations are considered superior based on the performance on classical models of one machine per operation (Pan (1997), Stafford *et al*. (2005), Pan and Chen (2005)).

Results also show that model performance is not directly related to the model size, but critically depends on factors like due date distribution and the number of machines available per operation. In spite of the continuous time model being generally outperformed by its discrete counterpart, there was a large size instance where no integer solution of the discrete model was found within 30 min of running the solver. In such situations the continuous time model becomes more attractive because a feasible solution can be obtained and further enhanced in relatively short time.

3 Conclusion and future developments

The development and comparison of discrete and continuous time formulations for solving the job shop scheduling problem is an original contribution of this study. Pan (1997) compares different formulations for the classical job shop problem in terms of model size complexity (number of variables and number of constraints) but no experimental study seems to have been reported like the one of Stafford *et al*. (2005) for the classical permutation flow shop problem.

The models developed apply directly to different types of job shop environments; this is the case of the mould making industry. Portugal has a relevant place in the world's mould producers. Manufacturers employ processes and techniques with a strong technological level and are internationally recognized for the technical quality and the ability to adapt to new and increasingly complex challenges. However, among others weaknesses of the sector, the following have been pointed out: poor interaction with knowledge centres, difficulty in meeting due dates and few highly competitive companies coexisting with a vast number of firms with ineffective organization and out-of-date equipments. In this context, mould makers would benefit from the use of decision support systems (DSS) to organize production. This work expects to be a contribution towards development of such decision tools.

As future developments, on the one hand the conclusions of this research should be consolidated by solving more problem instances and studying other configurations of the job shop system. On the other hand, solving larger and more realistic problems will pose the challenge of obtaining efficient and fast solutions for MILP models. Several techniques have been proposed with the aim of maintaining the number of decisions at a reasonable level in exact approaches of large-scale problems (Méndez *et al*. 2006). In the near future we intend to explore model reduction by means of heuristics. The incorporation of structural knowledge of the problem into the mathematical representation (like simple or combined dispatching rules) can lead to reduced models that capture only the critical decisions to be made and therefore to good solutions that can be generated in reasonably short time.

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