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The modulation of disjunctive assertions

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Abstract

The theory of mental models postulates that disjunctions of the sort, *A or B*, where *A* and *B* are sensible everyday clauses, have a core meaning that allows an inclusive interpretation, referring to three possibilities: *A and not-B*, *not-A and B*, and *A and B*. The meaning of the clauses and knowledge can modulate this meaning by blocking the construction of at least one model of a possibility, e.g., "Rui is playing tennis or he is surfing" blocks the model of Rui doing both activities. This theory is implemented in a computer program. Three experiments investigated the core interpretation and

interpretations in which the contents of the clauses should block the model of *A and B* (as in the preceding example), the model of *A and not-B*, or the model of *not-A and B*. In Experiment 1, the participants listed the possibilities for each of the four sorts of disjunction. The results corroborated the predicted modulations. In Experiment 2, these predicted interpretations governed the conclusions that participants accepted from disjunctions and categorical premises. In Experiment 3, the predicted interpretations yielded reliable effects on the conclusions that the participants drew for themselves. We relate these results to theories of reasoning.

Keywords: *deductive reasoning*, *disjunctions*, *logic*, *mental models*, *modulation*

Word count: Abstract – 199 words; Text – 7984 words

In logic, sentential connectives corresponding to *if*, *or*, and *and*, have constant meanings, which map the truth values of their clauses onto a truth value for the compound as a whole, e.g., an *inclusive* disjunction *A or B or both* is true if at least one of its clauses is true (Jeffrey, 1981). In daily life, the meanings of connectives vary from one assertion to another (Evans, Newstead, & Byrne, 1993), and understanding does not concern truth values (Johnson-Laird & Byrne, 1991). Consider, for instance, the difference between these two disjunctions:

Filomena is eating clams or Paulo is eating sardines.

and:

Luís is eating gazpacho or he is eating vegetable broth.

Typically, individuals are biased towards an inclusive interpretation of the first example, which includes the possibility that both of its clauses are true, but a sizeable minority prefers an exclusive interpretation (Newstead & Griggs, 1983). In contrast, individuals tend to concur that the second example is an *exclusive* disjunction because it

excludes the possibility that Luis is eating both sorts of soup at the same time. The theory of mental models – the "model theory" for short – postulates that understanding depends on constructing mental representations of the possibilities to which assertions refer. These representations are models that insofar as possible have a structure isomorphic to the structure of the situations to which the assertions refer. And the theory distinguishes between *mental* models, which represent only clauses that hold in each possibility, and *fully explicit* models, which also represent clauses that do not hold (see, e.g., Johnson-Laird & Byrne, 2002; Johnson-Laird, 2006; Khemlani & Johnson-Laird, 2013). Because mental models do not represent clauses that do not hold in a possibility, they predict the occurrence of "illusory" inferences in certain cases, i.e., inferences from disjunctions that are systematically fallacious. Such errors are robust (Khemlani & Johnson-Laird, 2009). Perhaps the simplest example is that most reasoners judged that pairs of assertions, such as the following two:

Either the pie is on the table or else the cake is on the table.

Either the pie isn't on the table or else the cake is on the table.

could both be true at the same time (Johnson-Laird, Lotstein, & Byrne, 2012). In fact, they cannot be: their fully explicit models have no possibility in common.

In cognitive science, a standard view of disjunctions is that the inclusive meaning is basic and is taken for granted unless a sentence makes explicit that a disjunction is exclusive (Barrett & Stenner, 1971; Grice, 1989; Kamp & Reyle, 1993, p. 191-2), and evidence shows that an exclusive interpretation can take more time – at least in the evaluation of truth or falsity (Chevallier, Noveck, Nazir, Bott, Lanzetti, & Sperber, 2008). Theories of reasoning based on formal logic likewise take the inclusive sense as basic (e.g., Rips, 1994; Braine & O'Brien, 1998). One problem for this standard view, however, is that inferences from exclusive disjunctions are easier

than those from inclusive disjunctions – a point that we illustrate below. In contrast to the standard view, the model theory allows an initial representation of disjunctions that is compatible with both an inclusive and an exclusive interpretation (Johnson-Laird & Byrne, 1991, p. 45; Johnson-Laird, Byrne, & Schaeken, 1992). An inclusive interpretation of a disjunction, such as: *Filomena is eating clams or Paulo is eating sardines*, elicits three mental models of possibilities, as depicted in the following diagram:

Filomena eating clams

Paulo eating sardines

Filomena eating clams Paulo eating sardine

We use descriptions for convenience in such diagrams; real mental models represent the world. An exclusive interpretation of the disjunction omits the third model in which both events occur. The fully explicit models of the inclusive interpretation are:

Filomena eating clams ¬ Paulo eating sardines

¬ Filomena eating clams Paulo eating sardines

Filomena eating clams Paulo eating sardines

where "¬" denotes the negation of a clause. Intuitions rely on single mental models, whereas deliberative reasoning calls for fully explicit models (see, e.g., Johnson-Laird, 1983, Ch. 6; Khemlani & Johnson-Laird, 2013). To think about several possibilities takes time, and so when individuals are short of time, they tend to revert to intuitions from mental models, and to be less likely to make inferences that depend on fully explicit models (Quelhas, Johnson-Laird, & Juhos, 2010). In simple tasks, such as listing the possibilities to which an assertion refers, individuals can list those that correspond to fully explicit models.

The core meaning of a disjunction, *A or B*, where *A* and *B* are sensible clauses, allows for an inclusive interpretation, which elicits a set of mental models representing the following possibilities (Johnson-Laird, Lotstein, & Byrne, 2012):

A

B

A B

But, in tasks such as the listing of possibilities, individuals should be able to flesh out these models into fully explicit models, which represent clauses that hold and clauses that do not hold in each possibility:

A ¬ B

¬ A B

A B

Inclusive disjunctions can be expressed explicitly, using "or both":

A or B, or both.

Likewise, an exclusive disjunction can be expressed explicitly, using "but not both":

Either A or else B, but not both.

It elicits just two mental models:

A

B

and accordingly has two fully explicit models:

A ¬ B

¬ A B

The basic principle of reasoning is that individuals draw conclusions that their models of the premises support. Hence, their interpretation of compound assertions, such as disjunctions, is critical for the conclusions that they infer. Because disjunctions

can call for three models, reasoning from them is notoriously difficult (see, e.g., Evans et al., 1993). One corroboration of the use of models is that an inference of this sort:

A or B.

Not A.

Therefore, B.

is easier to infer from an exclusive disjunction, which calls for only two models, than from an inclusive disjunction, which calls for three models (Johnson-Laird, et al., 1992; Bauer & Johnson-Laird, 1993). Another corroboration of the use of models is that disjunctive inferences based on negative categorical premises, such as the preceding example, are more difficult than those that depend on affirmative categorical premises (e.g., Roberge, 1976; Johnson-Laird et al., 1992). One cause of the difficulty is that the negative is inconsistent with a model and calls for its elimination (Johnson-Laird, 1991, p. 55). This inconsistency causes even greater difficulty when an affirmative categorical premise contradicts a negative clause in the disjunction (Johnson-Laird & Tridgell, 1972). Likewise, the elimination of a mental model of a disjunction leaves no conclusion to be drawn, and so reasoners need to flesh out their models into fully explicit ones in order to make an inference.

The model theory postulates that each sentential connective has a core meaning, but that the contents of clauses and knowledge can modulate this meaning (Johnson-Laird & Byrne, 2002). The core meaning consists of a set of possibilities, and modulation can have two effects. First, it can block the construction of models of possibilities. Second, it can add relations between models of the two clauses, such as a temporal or spatial relation (Juhos, Quelhas, & Johnson-Laird, 2012), which can affect reasoning in similar ways to explicit assertions of relations (Goodwin & Johnson-Laird, 2005).

Previous studies have investigated the effects of content on disjunctive reasoning, but have concerned differences between abstract and concrete meanings, which can affect the difficulty of inferences (e.g., Van Duyne, 1974; Roberge, 1977). Previous studies have also corroborated both sorts of effects of modulation on conditional assertions of the sort, *If A then B*, which have the mental models:

A B

. . .

where the ellipsis represents the possibilities in which *A* does not hold. As Quelhas et al. (2010) showed, individuals tend to list the fully explicit models for unmodulated conditionals, such as: "If the dish is kidney beans then its basis is beans":

Kidney beans	basis beans	(A B)
Not kidney beans	basis not beans	(¬ A ¬ B)
Not kidney beans	basis beans.	(¬ A B)

But, for a conditional such as, "If a plumber repairs the pipes then he must be paid," individuals tended to list only two possibilities (*A and B*, *not-A and not-B*) equivalent to a biconditional interpretation of the conditional. The inferences that reasoners made from conditionals likewise reflected the effects of modulation.

The second sort of modulation adds information to models. A conditional, such as, *If Carla printed the report, then the toner ran out*, elicits the interpretation that the printing occurred before the toner ran out, whereas the conditional, *If Jessica visited Lisbon, then Leonel invited her*, yields the interpretation that Jessica's visit occurred after Leonel's invitation (Quelhas et al., 2010). Modulation can also introduce spatial relations, and for some conditionals, such as: *If the maid cleans the desk, then she puts the folders on the floor*, it yields temporal and spatial relations. In this case, it implies that the maid first moved the folders from the desk to the floor, and then cleaned the

desk (Juhos et al., 2012). Modulation according to the model theory is semantic, depending on the meanings of clauses, or pragmatic, depending on knowledge in long term memory, or both.

We have written a computer program to demonstrate how modulation occurs, and we outline its main procedures here. As an example, consider the assertion:

It's raining or it's pouring.

Its interpretation as an inclusive disjunction would, in principle, yield the following fully explicit models of possibilities:

raining ¬ pouring

¬ raining pouring

raining pouring

Reasoners know, however, that it can't pour without raining, i.e., if it's pouring then it's raining. The program can build models of this proposition, but, because the theory postulates that knowledge is often represented in models, it contains fully explicit models of the proposition in its knowledge base:

pouring raining

¬ pouring ¬ raining

¬ pouring raining

The program forms a conjunction of the set of models for the assertion and the set of models in knowledge. In essence, the procedure constructs all pairwise combinations of the two sets except for those that are inconsistent with one another, e.g., one model represents *raining* and another model in a conjunction represents its negation: ¬ *raining*. The conjunction in the present case blocks the model of an impossibility to which the assertion would otherwise refer:

¬ raining pouring

The program therefore yields an interpretation in which it's raining and may, or may not, be pouring:

raining ¬ pouring

raining pouring

Given the further premise:

It's not pouring

it follows validly:

It's raining.

Logicians often suggest instead that this sort of inference is an enthymeme, that is, it merely calls for a missing premise in order to yield a proof of the conclusion. We consider this alternative to modulation after we have reported our experimental results.

The aim of the present studies was to investigate whether modulation can block the construction of models of disjunctions. The studies focus on four sorts of disjunction. For a disjunction, such as:

1. Ana is in Portugal or Rui is in Spain

modulation should not block the construction of any models, and so the disjunction can receive an interpretation corresponding to an *inclusive* interpretation. It refers to three possibilities, which mental models represent as follows:

Ana in Portugal

Rui in Spain

Ana in Portugal Rui in Spain

For a disjunction, such as:

2. Ana is in Portugal or she is in Spain

modulation should rule out the possibility in which one person – Ana – is in two different countries at the same time. Hence, it should receive an *exclusive* interpretation:

Ana in Portugal

Ana in Spain

For the disjunction:

3. Ana is in Lisbon or she is in Portugal

the first clause implies the second clause, because Lisbon is in Portugal, and so modulation should yield an interpretation with a *forwards* interpretation:

Ana in Lisbon Ana in Portugal

Ana in Portugal

For the disjunction:

4. Ana is in Portugal or she is in Lisbon

the second clause implies the first, and so modulation should yield a *backwards* interpretation:

Ana in Portugal

Ana in Portugal Ana in Lisbon

Hurford (1974) argued that disjunctions such as the forwards and backwards ones are unacceptable, because one clause entails the other. Yet, despite his intuitions, such disjunctions do occur in daily life. For example, the following backwards disjunction occurs on YouTube: "We have a commitment to see you the same day or within 24 hours." Such disjunctions may be odd because they violate the convention that discourse should be informative (cf. the maxim of quantity in Grice, 1989). They can be ameliorated by context or by the addition of a phrase, such as "at least," acknowledging their redundancy, e.g., "We have a commitment to see you the same day

or at least within 24 hours". Nevertheless, our experiments used disjunctions without such qualifying phrases. Table 1 below summarizes the predicted possibilities for each of the four sorts of disjunction.

In what follows, we report three experiments designed to test whether naive individuals – those who know nothing about logic – list the possibilities appropriate to these interpretations, evaluate inferences that follow from them, and draw their own appropriate conclusions from them. Finally, we discuss the implications of the results for current theories of reasoning.

Experiment 1

Our first experiment examined the four sorts of disjunction summarized in Table 1 below in order to test whether modulation blocked the predicted models of possibilities. The participants' task was to list what is possible given disjunctions of the four sorts (inclusive, exclusive, forwards, and backwards).

Method

Participants

The participants were 44 psychology undergraduates from ISPA, in Lisbon, who volunteered to take part in the experiment. They were 38 women and six men, average aged 20 years (*SD* = 5.304).

Design

The participants acted as their own controls, and listed possibilities for six different contents for each of the four sorts of disjunction (inclusive, exclusive, forwards, and backwards) in a total of 24 trials.

Material and Procedure

We devised contents concerning six topics from everyday life, which could be used for each of the four sorts of disjunction. These topics were locations, sporting activities, work, cultural activities, leisure, and cuisine. Supplemental Material A shows the contents translated into English, and Supplemental Material B shows them in Portuguese, which was the native language of the participants and the language in which the experiment was carried out. Each disjunction referred to one or two people using proper nouns, with an equal number of male and female names in the contents as a whole.

The participants were tested in a group, and the experiment was presented in a booklet. Its first page asked for some simple demographic information – the age and gender of the participant. It then instructed the participants to imagine that they were finalists to enter the Portuguese secret service agency, and that their task was part of their admissions exam. They would be presented with a true statement, which was part of a conversation, and they had to decide which situations were possible, and which situations were not possible, given the truth of the statement. The second page showed an example of a problem. The rest of the pages in the booklet were problems from the experiment proper. Figure 1 shows a translation into English of a typical problem.

(Figure 1 about here)

There were two versions of the printed booklet, which presented the problems in two different random orders. Within each booklet, the order of the four conjunctions to be evaluated were in one random order for half the problems, and in another random order for the other half of the problems. Each participant received a block with 24

problems in a different randomised order. The participants were allowed to take as long as they liked to complete the experiment.

Results and discussion

Nine of 44 participants responded that the situations in which both clauses of the disjunction were negative were possible for more than half the problems. Such egregious errors suggest that they were not paying full attention, and so we dropped their data from the analyses. The purely chance probability that a participant's list of possibilities on a trial matches the theory's prediction is 1/16, and in fact 15 of these possible patterns occurred in the experiment. The only pattern that did not occur was one in which the only possibility for *A or B* was *not-A and not-B*. It is therefore reasonable to assume a chance probability of 1/15 for a predicted pattern, any participant who matched the predictions on 5 or more trials is doing so in a statistically significant way (Binomial $p < .02$), and all 35 participants had at least 5 such matches, with a mean of 12.5 matches on 24 trials (Sign test, $p = .5^{35}$, i.e., $p < 1$ in a billion). As in this test, we used nonparametric ("distribution free") statistical tests throughout the present paper because they obviate problems of distribution, and because they allow us to test the reliability of predicted rank-order trends. Unlike analysis of variance, which can test for linear trends, quadratic trends, and so forth, non-parametric tests can assess a monotonic increase from one condition to another. Nonparametric tests are less powerful than parametric tests such as analysis of variance (see, e.g., Siegel & Castellan, 1988, Sec. 3.4.1), and so they are less likely to lead to an incorrect rejection of the null hypothesis (a Type I error).

Table 1 presents the predominant patterns of judgment for each of the four sorts of disjunction, where the predominant pattern for a participant is the one that

occurs most frequently over the six trials for a given sort of disjunction. The numbers do not always sum to 35 because some participants did not have a predominant pattern for a particular sort of disjunction.

The table reveals two principal phenomena. First, modulation occurred reliably, as shown by the frequencies in bold. Given the a priori probability for the occurrence of a predicted pattern, each disjunction yielded a reliable percentage of the predicted patterns of evaluation. Second, the degree to which individuals fit the predictions of modulation had a reliable concordance: as Table 1 shows, their fit with

(Table 1 about here)

the predictions had the following trend (with mean ranks in parentheses): exclusive disjunctions (3.7), forwards and backwards disjunctions (each 2.3) and inclusive disjunctions (1.6; Kendall's W = .49, $\chi^2_{df=3,} = 51.8$, $p < 1$ in a million). This trend reflects, on the one hand, the tendency for the participants to make an exclusive interpretation even in the case of the unmodulated disjunctions. But, seven participants did make the inclusive interpretation, whereas no participant made this interpretation for exclusive disjunctions (Fisher-Yates exact test, $p < 0.01$, one tail). As an anonymous reviewer pointed out, one factor that may have encouraged the exclusive interpretation of the unmodulated disjunctions was that, unlike the other disjunctions, it referred to two different individuals, e.g.: "Paula is running or Daniel is swimming". On the other hand, the trend also reflects the less clear-cut results for the forwards and backwards disjunctions. One reason may be that, as we noted earlier, disjunctions, such as:

Andre is in Lisbon or he is in Portugal

seem pragmatically odd in the absence of context. We suspect that participants were slightly confused by them, and so as a result they tended to revert to more typical interpretations or to idiosyncratic ones.

Experiment 2

The modulated interpretations of the four sorts of disjunctions corroborated in Experiment 1 should elicit different patterns of inference. For example, an exclusive interpretation of *A or B* yields the mental models:

A

B

and so if the disjunction is combined with the categorical premise, *A*, reasoners should infer: *not B*. But, a forwards interpretation of *A or B* yields the mental models:

A B

B

and so if the disjunction is combined with the categorical premise, *A*, reasoners should infer: *B*. The experiment combined each of the four sorts of disjunction with each of the four sorts of categorical premise: *A*, *not-A*, *B*, and *not-B*. Table 2 below summarizes the model theory's predictions for each of the 16 sorts of inference. As the table shows, there are two cases in which a categorical premise is inconsistent with the predicted models of the disjunction. In logic, any conclusion whatsoever follows from such a contradiction (Jeffrey, 1981). But, in the context of multiple disjunctions, reasoners are likely to be biased towards a conclusion based on the core interpretation, *not-A*, which follows from both an inclusive and an exclusive disjunction.

Method

Participants

The participants were 88 students from ISPA who took part voluntarily. They were 70 women and 18 men, average age 19 years (*SD* = 2.8).

Design

The participants acted as their own controls and chose one of three possible conclusions to inferences based on 3 different contents for each of the 16 sorts of inference (4 sorts of disjunction depending on modulation and four sorts of categorical premise: *A*, *not-A*, *B*, and *not-B*). A typical trial, translated from the Portuguese was as follows, and both premises were presented in bold:

Someone, who tells the truth, asserts that:

José ate seafood or he ate shrimp.

In the mean time you know that:

José ate seafood.

What conclusion can you draw?

a) José ate shrimp.

b) José did not eat shrimp.

c) José may or may not have eaten shrimp.

In order to use all six contents from the previous experiment, but to avoid a task that was too long, we used the following assignment of contents to the sorts of inference. Half the participants had the following assignment:

1. Contents 1-3 for inferences with categorical premises *A* and *not-A* and inclusive and forwards disjunctions, and contents 4-6 for inferences with categorical premises *B* and *not-B* and inclusive and forwards disjunctions.
2. The converse assignments for inferences from exclusive and backwards disjunctions.

Half the participants had the opposite assignment of contents.

Materials and Procedure

We prepared two sorts of booklet containing one page of instructions, and each of the 48 problems on a separate page in a different random order for each booklet. One sort of booklet had one assignment of contents to problems, and the other sort of booklet had the other assignment of contents to problems. The key instruction was that the participants should choose one of the three possible responses (as shown above), depending on whether one or other conclusion followed of necessity or neither of them did. We gave each participant one of the two sorts of booklet at random, with the constraint that the two sorts occurred equally often in the experiment as a whole. The participants were allowed to take as much time as they needed to complete the booklets.

Results and discussion

The difference between the two assignments of contents had no reliable effect on the percentages of predicted responses (68% vs. 71% correct, Mann-Whitney test, $z = 1.055$, $p > .25$, two-tailed), and so we amalgamated the results from the two groups for further analyses. Table 2 shows the percentages of predicted inferences from the four categorical premises combined with the four sorts of disjunction. It is immediately apparent from the table that the contents of the inferences affected the inferences that the participants drew. In other words, modulation influenced inferences. Its predicted evaluation for any inference has a prior chance probability of 1/3. Overall, the participants mean percentage of evaluations fitting the theory's predictions was 64%, which was significantly better than chance (Wilcoxon test, $z = 8.153$, $p < .0001$; in fact, p is less than one in a million for $z < 8.0$). Likewise, the participants fit the predictions better than chance for each of the four sorts of disjunction:

(Table 2 about here)

Inclusive: 51% (Wilcoxon test, $z = 6.616$, $p < .0001$).

Exclusive: 79% (Wilcoxon test, $z = 8.139$, $p < .0001$).

Forwards: 61% (Wilcoxon test, $z = 7.846$, $p < .0001$).

Backwards: 75% (Wilcoxon test, $z = 7.987$, $p < .0001$).

Inferences from disjunctions are difficult, and a robust finding is that they are harder from negative categoricals than from affirmative categoricals (see, e.g., Johnson-Laird & Byrne, 1991). The stringent comparison is from exclusive disjunctions, because only in this case are both sorts of inference valid, and the difference, which is shown in Table 2, was highly reliable (Wilcoxon test, $z = 5.837$, $p < .0001$). However, it was also reliable for forwards inferences (Wilcoxon test, $z = 6.251$, $p < .0001$) and for backwards inferences (Wilcoxon test, $z = 4.962$, $p < .0001$).

For each of the 16 different sorts of inference, a participant evaluated three inferences for different contents. If the response was the same for at least two of these inferences, then it was a predominant one. We examined the predominant inferences for the different sorts of disjunction. With inclusive disjunctions, the most frequent pattern of evaluations treated the disjunction as exclusive or was one evaluation away from it (24% of participants), but many participants understood that nothing follows from an affirmative categorical (19%), and many made this evaluation of all four categoricals (22%). With exclusive disjunctions, most participants drew the predicted inferences or were only one inference away from them (72%). With forwards disjunction, most participants made the predicted inferences or were only one inference away from them (42%), many inferred *B* from the categorical *A* but made no other

definite inferences (17%), and most of the remainder had no predominant response or made idiosyncratic evaluations (31%). With backwards disjunctions, most participants made the predicted inferences or were only one inference away from them (67%), and there were no other frequent patterns. In the case of forwards and backwards inferences, participants often relied on knowledge in inferences from negative categoricals, e.g.: from the premises:

Sofia is in France or she is in Paris.

Sofia is not in Paris.

The participants tend to ignore the disjunction and to rely on the knowledge that a person who is not in Paris could be anywhere:

Sofia may or may not be in France.

Disjunctive inferences are difficult, and the present participants had difficulty with them. Nevertheless, modulation exerted reliable effects on their performance.

Experiment 3

When participants are asked to choose among a set of options, as they were in the previous experiment, they may be inclined to guess their response. Hence, our final experiment examined what conclusions participants drew for themselves from disjunctive premises combined with categoricals. Disjunctions with no modulation tended to be interpreted as exclusive in the previous studies, and only a minority of participants interpreting them as inclusive. We therefore dropped them from the present experiment, which accordingly examined only three sorts of disjunction: those for which modulation should yield exclusive, forwards, and backwards interpretations. These disjunctions were combined on separate trials with the four sorts of categorical premise: *A*, *not-A*, *B*, and *not-B*.

In order to examine the model theory's predictions, we used the pattern of predicted conclusions shown in Table 2 above, and as a base line those predicted for the exclusive disjunction. For each categorical premise, the model theory predicts a trend in the likelihood of the exclusive conclusion over the three sorts of disjunction. For all four sorts of categorical, the exclusive disjunction should yield the most exclusive conclusions. The categorical premise, *Sofia is in Portugal*, together with a backwards disjunction, such as: *Sofia is in Portugal or she is in Porto*, implies nothing about whether or not she is in Porto, and so participants should tend to respond that nothing follows or perhaps to make the exclusive response. But, for a forwards disjunction, such as *Andre is in Lisbon or he is in Portugal*, the categorical premise, *Andre is in Lisbon*, should elicit the conclusion that Andre is in Portugal, which is the opposite to the exclusive conclusion. Hence, there should be the following trend for the categorical, *A*: exclusive disjunctions, *A or B*, should yield more exclusive responses than backwards disjunctions, which should yield more of them than forwards disjunctions. With the categorical *not-A*, exclusive disjunctions are most likely to receive exclusive conclusions, and the order of backwards and forwards should switch round, so the trend prediction should be: exclusive disjunctions yield more exclusive conclusions than forwards disjunctions, which should yield more of them than backwards disjunctions. With the categoricals, *B* and *not-B*, the orders of backwards and forwards disjunctions in the trends switch round from the previous predictions, but exclusive disjunctions always remain the most likely to elicit exclusive patterns of inference. The summary of the trend predictions for the frequencies of exclusive interpretations is accordingly:

Categorical premise, *A*: Exclusive > Backwards > Forwards

Categorical premise, *not-A*: Exclusive > Forwards > Backwards

Categorical premise, *B*: Exclusive > Forwards > Backwards

Categorical premise, *not-B*: Exclusive > Backwards > Forwards

The experiment tested these trend predictions in the participants' spontaneous inferences from the disjunctions and the categorical premises.

Method

Participants

Eighty psychology students from ISPA's laboratory pool, 71 male and 9 female, aged 17 to 47 years (M = 22 years; SD = 6.7 years), participated in the experiment in exchange for partial fulfillment of a course requirement.

Design

The participants acted as their own controls and carried out the three instances of each of the 12 sorts of inference based on three sorts of disjunction (exclusive, forwards, and backwards) and four sorts of categorical premise (*A*, *not-A*, *B*, and *not-B*), i.e., a total of 36 trials. We tested two separate groups of participants in to order to counterbalance the contents of the premises. The order of the trials was random for each participant.

Materials and Procedure

The experiment used three sorts of contents from the sets in the previous experiments: locations, culture, and food. We created two sets of these materials (see Supplemental Material C and D), and assigned each participant at random to one of them. The participants carried out the experiment interacting with a computer in an individual cubicle, running an E-prime program that controlled the experiment.

The instructions framed the task in terms of the test for spies used in the previous experiments. It explained that the task was to write down what conclusion followed of necessity from the premises, that is, if the premises were true it must be true too. The instructions also explained that the conclusion could be affirmative, or negative (giving hypothetical examples of each), and that the participants could also

respond "nothing follows" in case they thought that there was no conclusion that followed from the premises.

There were two practice problems, which were simple inferences based on conditional premises – we designed them as a filter in order to exclude any participants who failed to draw their correct "modus ponens" conclusions. The experiment proper followed them. A typical trial appeared on the computer's screen as follows:

Someone, who tells the truth, asserts that:

Ana is in Portugal or she is in France.

In fact, you know that:

Ana is not in France.

Please write down what conclusion follows of necessity from the premises.

_____

Results

Table 3 presents the percentages of the principal conclusions that the participants drew for themselves from the 12 sorts of premise. It is based on the inferences drawn by 80 of the participants. We excluded the data from 2 participants, because they failed to draw the two simple modus ponens conclusions in the practice problems. Two independent judges evaluated the participants' responses as affirming or denying the disjunct other than the categorical premise, allowing that it may or may not occur, drawing no conclusion, or falling into some other miscellaneous category. The judges were in close agreement (Cohen's $k = .96$, $p < .001$). The difference between the two sets of materials had no reliable effect on the percentages of predicted responses (62% vs. 56% correct, Mann-Whitney test, $z = 1.750$, $p > .08$, two-tailed), and so we amalgamated the results from the two groups for further analyses.

As Table 3 shows, there is a highly salient difference between the uniformity of the conclusions to the exclusive disjunctions as opposed to the variety of conclusions to the forwards and backwards disjunctions. To test the model theory, we examined its trend predictions in terms of the closeness to the pattern of inferences for exclusive disjunctions, which we outlined earlier. Overall, the inferences that the participants drew corroborated the predicted rank orders, and the mean observed ranks were 1.41, 1.84, 2.76 (Page's L = 1068.0, z = 8.5, p < .0000003). The percentages of exclusive responses and their mean rank orders over the participants for each of the categorical premises were as follows for the three sorts of disjunction:

(Table 3 about here)

Premise, *A*: Exclusive (80% 1.59) > Backwards (44% 1.83) > Forwards (32% 2.59)

Premise, *not-A*: Exclusive (80%, 1.56) > Forwards (54% 1.92) > Backwards (40% 2.53)

Premise, *B*: Exclusive (79% 1.62) > Forwards (39% 1.78) > Backwards (32% 2.60)

Premise, *not-B*: Exclusive (76% 1.58) > Backwards (51% 1.89) > Forwards (35% 2.53)

Page's L for these trends ranged from 1036 to 1040, z ranged from 6.0 to 6.3, each with p < .0000003). Overall, modulation had a highly reliable effect, and distinguished the three sorts of disjunction. But, as in the previous study, forwards and backwards disjunctions tended to yield a greater variety of conclusions than exclusive disjunctions.

General Discussion

The logician Bar-Hillel described the lack of application of logic to the analysis of everyday inferences as " one of the greatest scandals of human existence" (Bar-Hillel, 1969, p. 256). Nearly fifty years later, there is still no algorithm for such

analyses. One reason is that sentential connectives, such as disjunctions, have a constant meaning in formal logic, whereas in natural language they vary in their interpretation (e.g., Evans et al., 1993). Several potential explanations for this variation exist, e.g., connectives could be ambiguous just as many words are, or their interpretation could be enthymemic, depending on other unstated premises that individuals call to mind. However, according to the model theory, connectives are neither ambiguous nor enthymemic. They have a core meaning, but knowledge can modulate this meaning (Johnson-Laird & Byrne, 2002). Previous studies, as we discussed in the introduction, established the modulation of conditionals (Quelhas, et al., 2010; Juhos et al., 2012). Our present studies corroborated its occurrence for disjunctions. A disjunction such as:

Miguel is at the beach or Leonor is in the swimming pool.

should not elicit any modulation, and so it should be interpretable in the core sense of disjunctions, which includes the possibility that both its clauses are true. But, a disjunction such as:

Cristina is at the beach or she is at home.

is modulated by the knowledge that one person cannot be in two different places at the same time. It cannot receive an inclusive interpretation, but demands an exclusive interpretation in which one clause holds when the other does not. It accordingly refers to two possibilities. Modulation can in principle block the construction of any of the three possibilities to which the core interpretation refers. A disjunction such as:

Sara is eating bass or she is eating fish.

has a first clause that implies the second clause. It should elicit a forwards interpretation in which there are two possibilities: Sara is eating bass and (therefore)

fish, and Sara is not eating bass but eating fish. Hence, Sara is eating fish, which may or may not be bass. When the order of the two clauses is reversed, as in the disjunction:

Sara is eating fish or she is eating bass

the analogous backwards interpretation should also yield two possibilities in which Sara is eating fish, either bass or not.

When participants in Experiment 1 were asked to list what was possible given disjunctions of these four sorts (inclusive, exclusive, forwards, and backwards), they tended to list the possibilities that modulation predicts (see Table 1 above). The interpretations were clear-cut for exclusive disjunctions: 97% of trials with this disjunction elicited the predicted interpretation. Only a small proportion of trials (20%) yielded the inclusive interpretation for the unmodulated disjunction, but this interpretation never occurred for the exclusive disjunctions, and the difference was reliable. Nothing prevents an exclusive interpretation for such disjunctions – a fact that has led to controversy about the basic meaning of disjunctions (cf. Fillenbaum, 1974; Newstead, Griggs, & Chrostowski, 1984; Chierchia et al., 2001; Noveck et al., 2002; Chevallier et al., 2008; and for a review, Johnson-Laird et al., 2012). The forwards and backwards disjunctions were evidently a little confusing – they are perhaps pragmatically odd without an appropriate context, because they seem redundant (cf. Hurford, 1974). Yet, they led to 51% of forwards interpretations, and 49% of backwards interpretation (where chance is about 1/16), and their other interpretations were inclusive, exclusive, or idiosyncratic.

Modulation also predicted the inferences that individuals drew from disjunctions. Experiment 2 combined disjunctions with a categorical premise in inferences of the sort:

A or B.

A.

What conclusion can you draw?

a) B

b) Not B.

c) B may be or may not be.

and *A* and *B* had the same contents as before. The results corroborated the effect of modulation. For inclusive interpretations, the most frequent response with the categorical, *A*, was: *B may or may not follow*. For exclusive interpretations, it was: *Not B*. For forwards interpretations, it was: *B*. And for backwards interpretations, it was: *B may or may not follow*. In general, the results bore out the prediction that participants should draw inferences appropriate to modulated interpretations of disjunctions. They also replicated the previous findings that inferences from disjunctive premises are difficult (García-Madruga et al., 2001), and that inferences from disjunctions and negative categoricals are more difficult than those from disjunctions and affirmative categoricals (see, e.g., Johnson-Laird & Byrne, 1991; Bauer & Johnson-Laird, 1993).

When participants drew their own conclusions for inferences from disjunctions paired with categorical premises, the task was just as difficult. The model theory predicts trends over the different disjunctions in terms of the frequencies with which a conclusion should be the same as one from an exclusive disjunction. For example, given a backwards disjunction, such as: *José is eating seafood or he is eating shrimp*, the categorical premise that *José is not eating seafood* implies that he is not eating shrimp, which is the opposite to the conclusion that follows from an exclusive disjunction, whereas for the forwards disjunction: *José is eating shrimp or he is eating seafood,* the categorical premise that *he is not eating shrimp* allows that he may or may not be eating seafood. It follows that the inference corresponding to an exclusive

interpretation should show the following declining trend: exclusive disjunctions > backwards disjunctions > forwards disjunctions. The model theory yields trends for each of the four sorts of categorical premise (*A*, *not-A*, *B*, *not-B*), and Experiment 3 corroborated these trends. Overall, the experimental results bore out the hypothesis that modulation has robust effects on the interpretation of disjunctions, which in turn yield predictable patterns of inference from them.

The effects of modulation in our studies concern the role of knowledge in blocking the construction of a model of a single possibility from the three possibilities to which a core disjunction, *A or B*, can refer to:

A ¬ B

¬ A B

A B

where "¬" denotes negation. In principle, however, modulation can also prevent the construction of models of two possibilities, so that "or" refers to only a single possibility, as illustrated in these three cases, which show the single remaining possibility and examples of corresponding disjunctions:

A ¬ B She's married or I'm a Dutchman.

¬ A B I'm a Dutchman or she's married.

A B In my leisure, I paint pictures or I make up new recipes.

The obvious falsity of the speaker being a Dutchman rules out two possibilities in the core interpretation. What is of greater interest is the last example, in which *or* has the same meaning as *and*. This meaning is common in disjunctions connecting phrases rather than clauses, such as: "I drink red or white wine," which means that the speaker drinks both sorts of wine. Why would disjunction have a conjunctive interpretation? It signals that the two disjuncts do not hold at the same time. In the last of the three

examples, the speaker both paints pictures and makes up recipes, but not at the same time. And, in the phrasal example, the speaker likes both sorts of wine, but doesn't drink them at the same time. These three interpretations in which disjunctions refer to only a single possibility seem convincing enough not to merit empirical confirmation.

Are the phenomena of modulation open to an alternative explanation? One recent development has been the rise of psychological theories based on probabilistic considerations (see, e.g., Evans, Handley, & Over, 2003; Pfeifer & Kleiter, 2005; Oaksford & Chater, 2007, and for a review, Johnson-Laird, Khemlani, & Goodwin, 2015). But, these theories have yet to address the problem of the different interpretations of sentential connectives.

Another theoretical approach is that reasoning is based on formal rules of inference akin to those of a logical calculus, and that inferences are often enthymemes, i.e., they depend on knowledge in the form of additional premises (e.g., Rips, 1994; Braine & O'Brien, 1998; Stenning & Van Lambalgen, 2008). Often, a major difficulty is to determine an appropriate missing premise. The problem arises when modulation introduces a temporal relation in the interpretation of conditionals, which can affect the tense of the verbs in participants' conclusions (Juhos et al., 2012). But, it become acute in the case of disjunctive inferences, such as:

The fault is in the software or it is in the printer, or both.

Therefore, possibly the fault is in the software.

Most people accept that the conclusion must be true given the premise (Hinterecker, Knauff, & Johnson-Laird, 2016). It has the grammatical form:

A or B or both.

Therefore, possibly A.

The inference is not valid in any modal logic (Hughes & Cresswell, 1996), because *A* could be self-contradictory, and self-contradictions cannot be possible. In the present example, the premise would be true if *B* is true, but the conclusion would be false, because self-contradictions are impossible. The proof of the inference in logic therefore depends on an additional premise to rule out the case in which *A* is impossible. So, an obvious candidate is the premise:

It is not the case that not possibly A.

But, this double negative is equivalent to the conclusion to be drawn:

Possibly A.

The argument is circular, and the original premise – the disjunction about the fault – has no role to play in the inference.

Another approach, compatible with a probabilistic account and with formal rules of inference, is due to Grice (1989). He emphasized that speakers communicate more

than they say. For instance, a remark such as:

Ana is in Portugal or she is in Spain

conveys that the speaker does not know which of the two countries Ana is in. Otherwise, the speaker would have named a single country. The inference that the speaker does not know Ana's exact whereabouts is a "conversational implicature" that follows from the cooperative nature of conversation. One sign of a conversational implicature is that it is deniable without creating a contradiction:

Ana is in Portugal or she is in Spain, but I am not allowed to tell you which.

This remark implies that the speaker does know where she is. Conversational implicatures have been implemented in many complex systems in formal semantics, game theory, and Bayesian probabilities. They have also been used to explain the

conjunctive interpretation of disjunctions, such as: "He likes red or white wine (Franke, 2011), and inferences from disjunctions to possibilities (Sauerland, 2004), e.g.:

Ana is in Portugal or she is in Spain.

So, possibly she is in Portugal, and possibly she is in Spain.

However, claim about a possibility, *A*, cannot be denied by asserting either that *A* holds, or does not hold, because its possibility is compatible with both these cases. Its denial calls instead for an assertion that *A* is not possible, e.g.:

Ana is in Portugal or she is in Spain, and it is impossible that she is in Portugal, and it is impossible that she is in Spain.

But, this assertion is self-contradictory. It shows that the inference about Ana's possible locations is not a conversational implicature, but a valid deduction. Grice allowed for inferences that depend on the meanings of terms, which he called "conventional implicatures". And they can be denied only on pain of contradiction. Our computer program implementing modulation could be treated as an inference engine for Gricean conventional implicatures.

Overall, modulation yields seven different interpretations of "or", including the three conjunctive senses above. A corollary is the difficulty of the recovery of the logical form of assertions in everyday discourse – the form that matches that of the formal rules of inference. We have already encountered a typical difficulty. An assertion such as:

At work, she talks to clients or she briefs programmers

has the surface form of a disjunction. But, it has the force of a conjunction: she talks to clients *and* she briefs programmers. The disjunctions under investigation in the present studies all present analogous challenges to any algorithm designed to recover logical form. Not surprisingly, no such algorithm exists. In contrast, logical form plays a part

neither in the model theory nor in its computer implementation, which depends on the surface grammar of sentences (Khemlani & Johnson-Laird, 2013). We do not claim that an enthymemic account is impossible. But, it won't be easy, and no-one has proposed such an account, let alone an algorithm implementing it, since the publication of modulation over a decade ago.

Any satisfactory account of disjunctions needs to deal with five principal phenomena. Two of them concern unmodulated disjunctions:

• Reasoning with disjunctions is harder than reasoning with conjunctions (e.g., García-Madruga et al., 2001).

• Deductions and inferences about consistency yield illusory inferences from unmodulated disjunctions (e.g., Khemlani & Johnson-Laird, 2009; Johnson-Laird et al., 2012).

The remaining three phenomena are reported in the present paper:

• Modulation has an impact on the interpretations of disjunctions (see Table 1), and therefore on the inferences that individuals draw from them (see Tables 2 and 3).

• The same valid deductions are easier to evaluate from exclusive disjunctions that modulation yields than from inclusive disjunctions (see Table 2).

• Valid deductions from exclusive, forwards, and backwards disjunctions are easier with affirmative categorical premises than with negative categorical premises (see Table 2).

The model theory predicts each of these phenomena (see our account in the Introduction). Exclusive disjunctions have two mental models, whereas inclusive disjunctions have three mental models. Affirmative categoricals allow an inference to be drawn from a single model of a disjunction whereas negative categories rule out a model and call for an examination of an alternative model. As far as we can tell no other theory presently accounts for all five of the phenomena.

In conclusion, differences in the interpretation of the disjunctive sentential connective, *or*, arise from the influence of the contents of clauses and of knowledge. The connective has a core meaning, which allows for an inclusive interpretation. But, modulation affects this interpretation, and it can do so by blocking the construction of models of various possibilities. In theory, it can block any single possibility, and any pair of possibilities, from the three models of possibilities that the core meaning allows. Our studies have shown that modulation yields three different interpretations: exclusive, forwards, and backwards disjunctions. These modulations, in turn, led to differences in the inferences that disjunctions yield. Reasoning with disjunctions is difficult – if only because it usually calls for reasoners to take into account more than one model of a possibility, and multiple models transcend intuition and place considerable demands on deliberation. The inferences that individuals judge to be valid and that they draw for themselves depend on the possibilities to which the premises refer. As modulation changes these possibilities from one sort of interpretation of a disjunction to another, so, too, do the inferences that individuals make.

Supplemental Material

Supplemental Material A - Materials for Experiment 1 and 2 in English.

Supplemental Material B – Materials for Experiments 1 and 2 in the original Portuguese.

Supplemental Material C – Materials for Experiment 3 in English.

Supplemental Material D – Materials for Experiment 3 in original Portuguese.

The Supplemental Material can be found at the address

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Figure 1. A typical problem from Experiment 1.

Henrique is in Lisbon or he is in Rome.	
Given that this assertion is true, your task is to decide whether each of the following situations is possible or impossible:	
Henrique is in Lisbon and he is in Rome.	Possible ☐ Impossible ☐
Henrique is in Lisbon and he is not in Rome.	Possible ☐ Impossible ☐
Henrique is not in Lisbon and he is in Rome.	Possible ☐ Impossible ☐
Henrique is not in Lisbon and he is not in Rome.	Possible ☐ Impossible ☐
Note: The number of "possible" and "impossible" responses need not be equal.	

Table 1: The frequencies of predominant patterns of judgments for the four sorts of disjunction in Experiment 1 (n = 35). A predominant pattern is one that occurs most frequently in a participant's six trials with a given sort of disjunction, and so any total less than 35 occurred because some participants had no single predominant pattern in their judgments. Miscellaneous evaluations are patterns of responses that no more than three participants made in the experiment as a whole. The symbol "¬" denotes a negative clause, and the frequencies in bold are those for the evaluations that the model theory predicts.

	The predominant patterns of evaluation				
Sort of disjunction	A ¬ B ¬ A B A B	A ¬B ¬ A B A B	¬A B A B	A ¬B A B	Miscellaneous	TOTAL
Inclusive	**7**	24			4	35
Exclusive		**34**			0	34
Forwards	2	3	**18**		7	30
Backwards	2	4		**17**	6	29

Table 2: The predicted responses and their percentages in Experiment 2 for each of the 16 sorts of inference. The symbol "?" denotes the response option that the categorical conclusion may, or may not, follow, and the symbol "¬" denotes negation. In two cases, as the asterisk shows, the categorical premise is inconsistent with the predicted interpretation of the disjunction.

		The sort of categorical premise			
Sort of disjunction	Possibilities to which the disjunction should refer	A	Not-A	B	Not-B
Inclusive	A ¬ B ¬ A B A B	? 54	B 50	? 50	A 49
Exclusive	A ¬ B ¬ A B	Not-B 96	B 66	Not-A 92	A: 62
Forwards	A B ¬ A B	B 74	B 34	? 73	not-A* 64
Backwards	A B A ¬ B	? 81	Not-B* 70	A 71	A 39

Table 3: The percentages of the participants' principal spontaneous conclusions for each of the twelve sorts of inference in Experiment 3, where "?" designates "nothing follows", and "*" indicates that the categorical premise is inconsistent with the predicted interpretation of the disjunction.

		The sort of categorical premise											
Sort of disjunction	Possibilities to which the disjunction should refer	A			Not-A			B			Not-B		
Exclusive	A ¬ B ¬ A B	∴ not-B 80			∴ B 80			∴ not-A 79			∴ A 76		
Forwards	A B ¬ A B	∴ B 45	∴ not-B 32	? 10	∴ B 54	∴ not-B 12	? 31	∴ A 9	∴ not-A 39	? 35	∴ A* 35	∴ not-A 29	? 25
Backwards	A B A ¬ B	∴ B 5	∴ not-B 44	? 32	∴ B* 40	∴ not-B 29	? 23	∴ A 44	∴ not-A 32	? 10	∴ A 51	∴ not-A 12	? 32