Consistency and Efficiency of Ordinary Least Squares, Maximum Likelihood, and Three Type II Linear Regression Models

A Monte-Carlo Simulation Study

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Abstract. Type I linear regression models, which allow for measurement errors only in the criterion variable, are frequently used in modeling research in psychology and the social sciences. Although there are frequently measurement errors and large natural variation both in the criterion and predictor variables, type II regression methods that account for these errors are seldom used in these fields of study. The consistency and efficiency of three type II regression methods (reduced major axis, Kendall's robust line-fit and Bartlett's three-group) were evaluated in comparison to ordinary least squares (OLS) and the maximum likelihood with known variance ratio used frequently in biometrics and econometrics. When predictors are measured with error, OLS slope estimates are biased toward zero, and the same bias was observed with both Kendall's and Bartlett's methods. Reduced major axis produced consistent estimates even for small sample sizes, whenever the measurement errors in X are similar in magnitude to measurement errors in Y, but there was a consistent bias when the measurement error in X was smaller/greater than in Y. Maximum likelihood estimates behaved erroneously for small sample sizes, but for larger sample sizes they converged to the expected values.

Keywords: measurement errors in predictors, errors-in-variables, type II regression, regression analysis

Introduction

The simple linear regression model is one of the most interesting data analysis tools that a researcher in the social sciences may use. This type of model allows for the establishment of a functional linear relationship between one dependent or criterion variable (Y_i) and one independent or predictor variable (X_i) ($i = 1,\ldots, n$) on a relatively straightforward and simple mathematical model:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \tag{1}$$

In Model 1, the variation in Y_i is explained by a constant value (β_0 – the intercept) plus an additive term that transports the influence of X_i over Y_i translated by the regression coefficient β_1 (slope) and a random term (ε_i) that is associated with both measurement error and natural variation of Y. Inference about β_1 allows for conclusions relative to the statistical significance of the influence of X_i over Y_i, as well as for using the model to estimate mean expected values for Y_i from fixed values of X_i:

$$\hat{Y}_i = \beta_0 + \beta_1 X_i \tag{2}$$

However, inference about Model 1 and estimation with Model 2 requires that a set of assumptions must be valid. First of all, the X_i ($i = 1, \ldots, n$) observations must be measured without significant error or if the error or natural variation is present it must be controlled by the researcher to a range much smaller than the error range in Y. Second, errors associated with measurement and natural variation of Y must be independent and identically distributed with normal distribution with zero mean and constant variance, i.e., $\varepsilon_i \sim IIN(0,\sigma)$. Finally, the relation between Y and X must be linear in form. When these assumptions of Model 1 are valid, the model is said to be a *type I linear regression* model. For this type of model, the regression coefficients are best estimated by ordinary least squares (OLS).

For several research scenarios in the social sciences (quasi-experimental studies, correlational studies, etc.), the independent variables are measured and not set by the researcher and so the assumption of perfectly reliable predictors is often difficult to justify (Schuster, 2004). When predictor variables are plagued with measurement errors and natural variation that is not controlled by the researcher the slope coefficients obtained by OLS are biased toward zero (Cheng & Van Ness, 1999; Fuller, 1987; Isaac, 1970; Ray-

DOI 10.1027/1614-2241.3.2.81

ner, 1985; Riggs, Guarnieri, & Addelman, 1978), introduce imprecision in the statistical inference process, and reduce the power of the tests. Linear regression models, where both the dependent and independent variables are subject to error and conjointly normally distributed, are designated as *type II linear regression* (Sokal & Rohlf, 1995) or as *errors-in-variables* models (Cheng & Van Ness, 1999; Fuller, 1987). Although the problems associated with measurement error in the independent variables are not new in several research fields, for example in biology (Kendall & Stuart, 1961; Ricker, 1973; Riggs et al., 1978), agronomy (Fuller, 1987), economy (Li, 2002), and biomedicine (Carrol & Ruppert, 1996), they are seldom acknowledged, both in these fields of study (Quinn & Keough, 2002) and in the social sciences and psychology. A search conducted in the PsychARTICLES and PsychINFO databases (as of March 25, 2006) with keywords *model II regression* or *errors-in-variables* returned only a few significant matches for the last 15 years (Jaccard & Wan, 1995; Klauer, Draine, & Greenwald, 1998; Miller, 2000; Schuster, 2004). This is not to say that the problems associated with errors-in-variables in the psychology and social sciences fields have not been previously acknowledged. Isaac (1970) pointed to a particular misuse of the OLS regression analysis in situations involving linear structural relations when both predictor and criterion variables are measured with errors. He discussed the fact that failure to consider the X error-variability can distort the estimates of the linear parameter β_1, typically by underestimating it. He then expands and discusses the application of Kendall and Stuart's (1961) unbiased slope estimators to psychology and social sciences research scenarios when error variances in Y, X, Y and X, and only the error-variance ratios are known.

When errors-in-variables are present, it is common practice in psychology and the social sciences to model these variables as latent constructs that define the observed variables (indicators) measured with error (Jöreskog & Sörbom, 1982; Rock, Werts, Linn, & Jöreskog, 1977; Weston & Gore, 2006). Regression paths between the constructs are then analyzed by structural equation modeling (SEM) methods. However, although it is possible to have single indicator constructs, its use is quite controversial, and multiple (three or more) indicator constructs are normally required (Bollen, 1989; Jaccard & Wan, 1995; Marsh, Hau, Balla, & Grayson, 1998). Additionally, constrains on error-variances must be imposed to identify the model, which requires previous knowledge of the phenomenon under study. This preknowledge constitutes a severe limitation to the application of structural equation modeling to the errors-in-variables problem; since it is not always possible to have several indicators defining a latent construct; nor to know in advance construct reliability and/or the variance of measurement errors and latent constructs. Simultaneously, the use of type II regression models has been the subject of some research. For example, Klauer et al. (1998) proposed an error-in-variables variant of the regression method that accommodated measurement-error in the predictor of subliminal perception. However, Miller (2000), presented simulation analyses of the instrumental variable method and the modified error-in-variables method of Klauer et al. (1998) for detecting unconscious cognition and argued that both methods exhibited statistical biases and neither provided valid statistical tests for the non-zero intercept that is required as a proof for unconscious cognition. The estimation of regression coefficients when predictors are measured with error is still controversial and several methods for the estimation of the regression coefficients are available (Freedman, Fainberg, Kipnis, Midthune, & Carroll, 2004; Klauer et al., 1998; Kulathinal, Kuulasmaa, & Gasbara, 2002; Miller, 2000; Quinn & Keough, 2002; Rayner, 1985; Ricker, 1984; Ryu, 2004; Sokal & Rohlf, 1995). However, these methods are specific for different types of data, errors-in-variables, or specific data distribution assumptions, and definitive recommendations are difficult to make. This is, in part, because of the lack of information on the efficiency, consistency, and distributional characteristics of the different estimators as well as of the robustness of these estimators (Carrol & Ruppert, 1996; Cheng & Van Ness, 1999; Miller, 2000; Sokal & Rohlf, 1995). While the distributional properties of OLS estimators are well known, making it possible to demonstrate theoretical biases in their estimates, for most type II models the distributional properties of sample estimators are not known and, thus, sampling distributions of estimates are best, and easily, obtainable by simulation methods.

In this paper, building on Fuller (1987), Sokal and Rohlf (1995), and Cheng and Van Ness's (1999) seminal works, a set of Monte Carlo simulation studies are presented to evaluate the efficiency and consistency of the slope and intercept estimators for three type II linear regression models estimators (reduced major axis [RMA], Kendall's robust method, Bartlett's three groups) as compared with OLS and the maximum likelihood with known error-variances ratio estimators.

Type II Linear Regression or "Error-in-Variables" Model

Suppose one is interested in modeling the linear relationship between the criterion (η_i) and predictor (ξ_i) variables, but we are not able to observe these variables without measurement error. That is, one measures (Y_i, X_i), which are the true (latent) unobserved variables (η_i,ξ_i) ($i = 1, \ldots, n$) plus additive measurement errors (ϑ_i, δ_i):

$$Y_i = \eta_i + \vartheta_i \tag{3}$$

and

$$X_i = \xi_i + \delta_i \tag{4}$$

Assume that $\vartheta_i \sim N(0, \varepsilon_\vartheta)$, $\delta_i \sim N(0, \varepsilon_\delta)$ and that the error terms are independent from each other and from the unobserved variables. Thus, the true linear relationship between η_i and ξ_i may be written as

$$\eta_i = \beta_0 + \beta_1\xi_i + \theta_i \quad (5)$$

The term θ_i stands for the equation error, and reflects the fact that η_i and ξ_i may not be perfectly related if there are variables other than ξ_i that may also be responsible for the variation in η_i. Although failure to account for θ_i may result in overestimation or underestimation of the true regression slope (Carrol & Ruppert, 1996), I assume, for simplicity and without loss of generality, that there is no equation error in the linear relationship between η_i and ξ_i (that is θ_i = 0). In this case the model is known as the no-equation error model (Cheng & Van Ness, 1999) and can be written as

$$\eta_i = \beta_0 + \beta_1\xi_i \quad (6)$$

The analysis objective is to estimate the regression coefficients β_0 and β_1 from the measured (Y_i, X_i), so, by substituting (3) and (4) into (6), the model can be written as

$$Y_i - \vartheta_i = \beta_0 + \beta_1(X_i - \delta_i) \quad (7)$$

Since ϑ and δ are independent, we may condense them and write Model 7 as

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad (8)$$

where $\varepsilon_i = \vartheta_i - \beta_1\delta_i$ and $\varepsilon_i \sim N(0, \sigma_\vartheta + \beta_1^2\sigma_\delta)$. Model 8 is known as the error-in-variables model (EVM), and it has been widely discussed in the literature (Carrol, Ruppert, & Stefanski, 1995; Cheng & Van Ness, 1999; Fuller, 1987; Isaac, 1970). If X is a set of unknown constants, then the model is called the *functional* error-in-variables model; if X is a random variable, the model is called the *structural* error-in-variables model (Cheng & Van Ness, 1999; Fuller, 1987). At first sight, the EVM may look like the simple linear, or type I, regression model, but this only holds for the trivial case ($\beta_1 = 0$) or when $\delta = 0$. For the other scenarios, X is correlated with ε, as $Cov(X, \varepsilon) = -\beta_1\sigma_\delta^2$, and if one attempts to use OLS to estimate the regression coefficients, then one obtains inconsistent estimates biased toward 0 (Cheng & Van Ness, 1999; Fuller, 1987; Isaac, 1970; Kendall & Stuart, 1961). That is, one obtains biased estimates of the true slope that do not converge, in probability, to the true population slope as the sample size (n) tends to infinity.

Monte-Carlo Simulations

Error-free data for ξ (representing the true unobserved predictor population) was generated with the RndNormal(σ) function (with $\mu = 0$ and $\sigma = 1$) from STATISTICA 7 (StatSoft, Tulsa, OK) and scaled to a range of –6 to 6 for ~95% of the values. η data (representing the true unobserved criterion population) was generated as

$$\eta = 1 + 1\xi \quad (9)$$

Random normally distributed error terms with zero mean and fixed standard deviation were then added to both the predictor and criterion to mimic measurement and natural variation errors. The simulated "observed" values of X_i and Y_i were given by:

$$X_i = \xi_i + \delta_i \text{ with } \delta_i \sim N(0, \sigma_\delta) \quad (10)$$

$$Y_i = \eta_i + \vartheta_i \text{ with } \vartheta_i \sim N(0, \sigma_\vartheta) \quad (11)$$

where δ_i and ϑ_i were generated with the RndNormal(σ_δ) and RndNormal(σ_ϑ) functions, respectively, are independently normally distributed and independent of X_i and Y_i. The degree of error in the predictors used in the simulations is estimated by the *reliability ratio* $\varkappa_\xi = Var(\xi)/Var(X)$ (Cheng & Van Ness, 1999) for comparison with traditional psychological constructs reliability.

A total of 10,000 samples were interactively generated from the above models (9–11), with sample size (n) 4, 6, 8, . . ., 50 and then 100, 200, 300, and 400 following the observation of Jaccard and Wan (1995) that the median sample size across studies in the psychology sciences is around 175, with large sample sizes around 400. Slope and intercept estimates for each of the 10,000 samples were calculated with the different models' estimators described below and data is given as averages and the 2.5 and 97.5 percentiles of the calculated estimates. Choice of models tested was based on their wide application in other fields, namely in econometry and biometry, as well as its easy mathematical calculations, which may be advantageous for researchers in psychology and the social sciences. Validity of the software code[1] implemented in STATISTICA VBASIC (STATsoft, Tulsa, OK) to perform the simulation and data calculations was assessed with the reference data sets given in Cheng and Van Ness (1999, p. 25–26) and Sokal and Rohlf (1995, p. 546–549).

Slope and Intercept Estimators

The OLS, estimators for the slope and intercept in the simple linear regression model are respectively:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})(X_i - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} = \frac{S_{XY}}{S_{XX}} \quad (12)$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1\overline{X} \quad (13)$$

These estimators have been extensively studied and are well known to produce unbiased estimates when predictors are error-free. If predictors are measured with error; slope

1 Available upon request from the author

estimates are biased toward zero. However, if a predictor's reliability is known (which happens frequently with psychological predictors), unbiased slope estimates are easily obtained as $\hat{\beta}_1^* = \beta_1/\varkappa_\xi$ (Cheng & Van Ness, 1999; Fuller, 1987).

For the RMA or geometric mean type II model, the estimators for the slope and intercept are respectively (Ricker, 1984):

$$\hat{\beta}_{1_{RMA}} = Sign[S_{XY}] \times \sqrt{b_{Y.X} \times \frac{1}{b_{X.Y}}} =$$

$$Sign\,[S_{XY}]\sqrt{\frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}} =$$

$$Sign[S_{XY}] \times \sqrt{\frac{S_{YY}}{S_{XX}}} = Sign[Cov(X,Y)\ \frac{S_Y}{S_X}] \tag{14}$$

$$\hat{\beta}_{0_{RMA}} = \bar{Y} - \hat{\beta}_{1_{RMA}}\bar{X} \tag{15}$$

These estimators are a particularization of the major axis method and give the same estimates if both variables are in the same units of measurement. If variables have different units of measurement, as is frequently the case in the social sciences, the major axis slope estimates are meaningless and the RMA technique should be used (Legendre & Legendre, 1988; Rayner, 1985). The RMA slope estimator is simply the ratio of standard deviations, affected by the sign of the covariance of both variables. Thus, its independence from whatever conjoint distribution *X* and *Y* may show, which has drawn some early criticism of this method (Jolicouer, 1975). However, as we shall see, it performs quite well when errors in both predictor and criterion variables are of the same magnitude.

The Theil (1950) or Kendall's robust method requires the ranking of (X_i, Y_i) data by the magnitude of X_i and the calculation of the $n(n-1)/2$ slopes for adjacent data points (Kendall & Gibbons, 1990):

$$S_{j+1,j} = \frac{Y_{j+1} - Y_j}{X_{j+1} - X_j} \tag{16}$$

The slope estimator is the median of $S_{j+1,j}$ ($j = 1, \ldots, n-1$):

$$\hat{\beta}_{1_K} = Med(S_{j+1,j}) \tag{17}$$

while the intercept estimator is

$$\hat{\beta}_{0_K} = Med(a_i = Y_i - \hat{\beta}_{1_K} X_i) \tag{18}$$

For the Bartlett's three groups (Sokal & Rohlf, 1995) or Wald method (Pakes, 1982), the data, after ranking by order of *X*, is divided into three equal-sized groups (if that is not possible, than the 1st and 3rd group must have the same number of observations). The slope and intercept are estimated from the (*X, Y*) data in Group 3 and Group 1 as:

$$\hat{\beta}_{1_B} = \frac{\bar{Y}_3 - \bar{Y}_1}{\bar{X}_3 - \bar{X}_1} \tag{19}$$

$$\hat{\beta}_{0_B} = \bar{Y} - \hat{\beta}_{1B}\bar{X} \tag{20}$$

Ranking of data using the *X* predictor measured with error has raised concerns about bias in slope estimates (Cheng & Van Ness, 1999; Kuhry & Marcus, 1977).

In the maximum likelihood method (ML) for the errors-in-variables model with known error-variances ratio $\lambda = \sigma_\vartheta^2/\sigma_\delta^2$, the slope and intercept estimators are (Fuller, 1987; Kendall & Stuart, 1961):

$$\beta_{1_{ML}} = \frac{S_{YY} - \lambda S_{XX} + \sqrt{(S_{XX} - \lambda S_{YY})^2 + 4S_{XY}^2}}{2S_{XY}} \tag{21}$$

$$\hat{\beta}_{0_{ML}} = \bar{Y} - \hat{\beta}_{1_{ML}}\bar{X} \tag{22}$$

The (21) estimator is also the Cheng and Van Ness (1999, p. 85–86) modified least squares estimator (which produce the same estimates as maximum likelihood, but without requiring the normality assumption), as well as the Ryu (2004) rectangular regression estimator.

Rayner (1985) showed that OLS, RMA, and ML methods all are special cases of the general structural relationship and discuss its application whenever measurement error variances are either known, the ratio known or both unknown.

Results

Slope and intercept estimates were obtained for 10,000 replicates of different sample sizes, using the estimators of the three type II regression models, as well as, for comparative proposes, OLS and ML. Figure 1 shows the average, percentile 2.5 (lower-bound) and percentile 97.5 (upper bound) for the 10,000 estimates of the intercept and slope obtained with *Y* affected by random normal errors with $\mu = 0$ and $\mu_\vartheta = 2$ to mimic measurement errors in the criterion variable only. When only *Y* is affected by measurement errors normally distributed with 0 mean and *X* is perfectly reliable ($\varkappa_\xi = 1$), OLS are the best linear unbiased estimators. Both Bartlett and Kendall methods produced nonbiased estimates; however, these estimators are less efficient than OLS as they produce estimates with larger variance than the OLS estimators. The RMA slope estimator consistently overestimated the slope, while the intercept was underestimated. Finally, the ML estimators are efficient only for large samples, especially as compared with the OLS estimators.

If both *Y* and *X* are affected by random normal errors with 0 mean and equal constant variance ($\sigma_\vartheta = \sigma_\delta = 2$; $\varkappa_\xi = 0.6$) (Figure 2), the OLS, Bartlett, and Kendall slope estimators underestimated the true slope, while the intercept

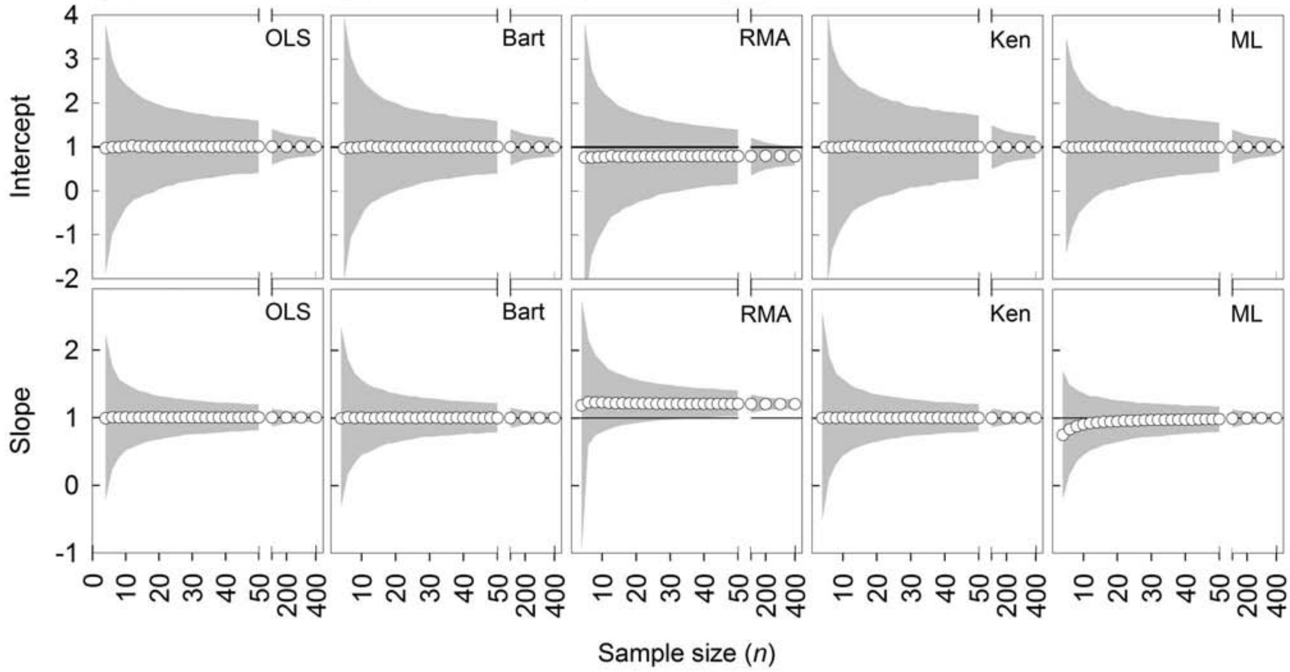

Figure 1. Intercept and slope estimates for 10,000 samples with different sample size (n) ranging from 4 to 400 obtained with OLS, Bartlett three groups method (Bart), reduced major axis (RMA), Kendall robust method (Ken), and maximum likelihood (ML) fitted to (Y,X) data generated as $Y = \eta + \vartheta$ with $\vartheta \sim N(0, 2)$ and $X = \xi + \delta$ with $\delta \sim N(0, 0)$, where $\eta = 1 + 1\xi$ as described in the "Monte Carlo Simulations" section. Reliability ratio for X is $\varkappa_\xi = 1$. Data are shown as the mean estimates (circles) with the 95% percentile interval (gray area). For the ML method $\delta \sim N(0, 0.001)$. For clarity's sake, data from sample sizes 50 to 100 have been omitted.

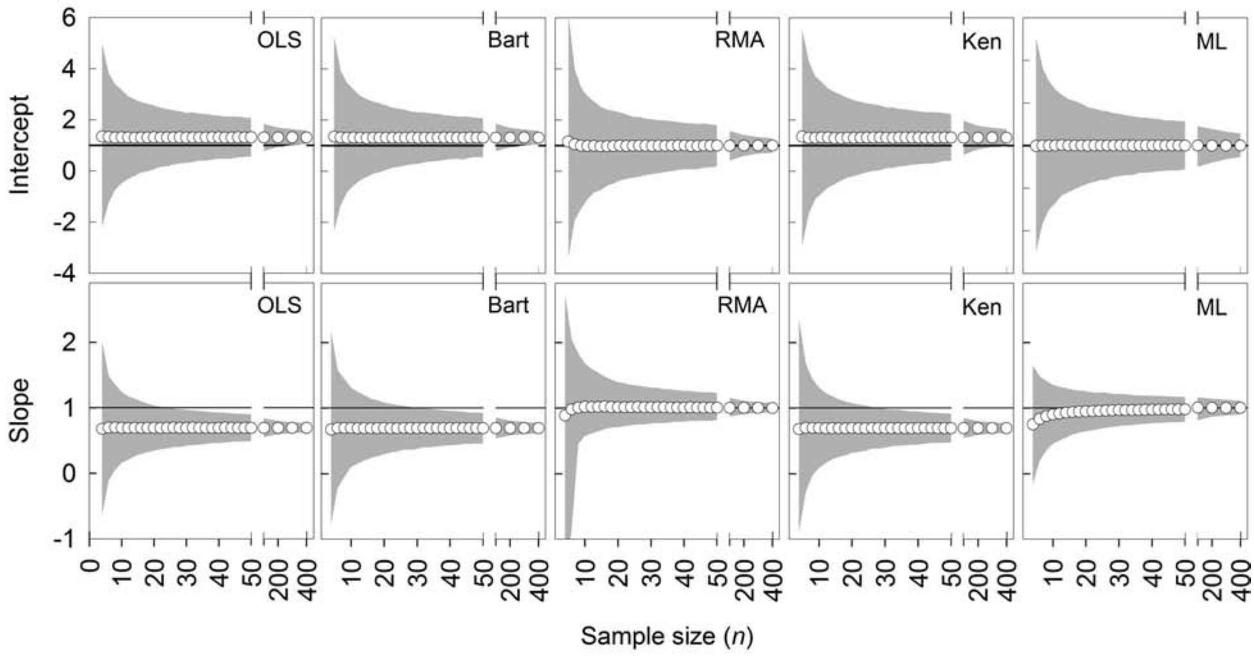

Figure 2. Intercept and slope estimates for 10,000 samples with different sample size (n) ranging from 4 to 400 obtained with OLS, Bartlett's three groups method (Bart), reduced major axis (RMA), Kendall's robust method (Ken), and maximum likelihood (ML) fitted to (Y,X) data generated as $Y = \eta + \vartheta$ with $\vartheta \sim N(0, 2)$ and $X = \xi + \delta$ with $\delta \sim N(0, 2)$, where $\eta = 1 + 1\xi$ as described in the "Monte Carlo Simulations" section. Reliability ratio for X is $\varkappa_\xi = 0.6$. Data are shown as the mean estimates (circles) with the 95% percentile interval (gray area). For clarity's sake, data from sample sizes 50 to 100 have been omitted.

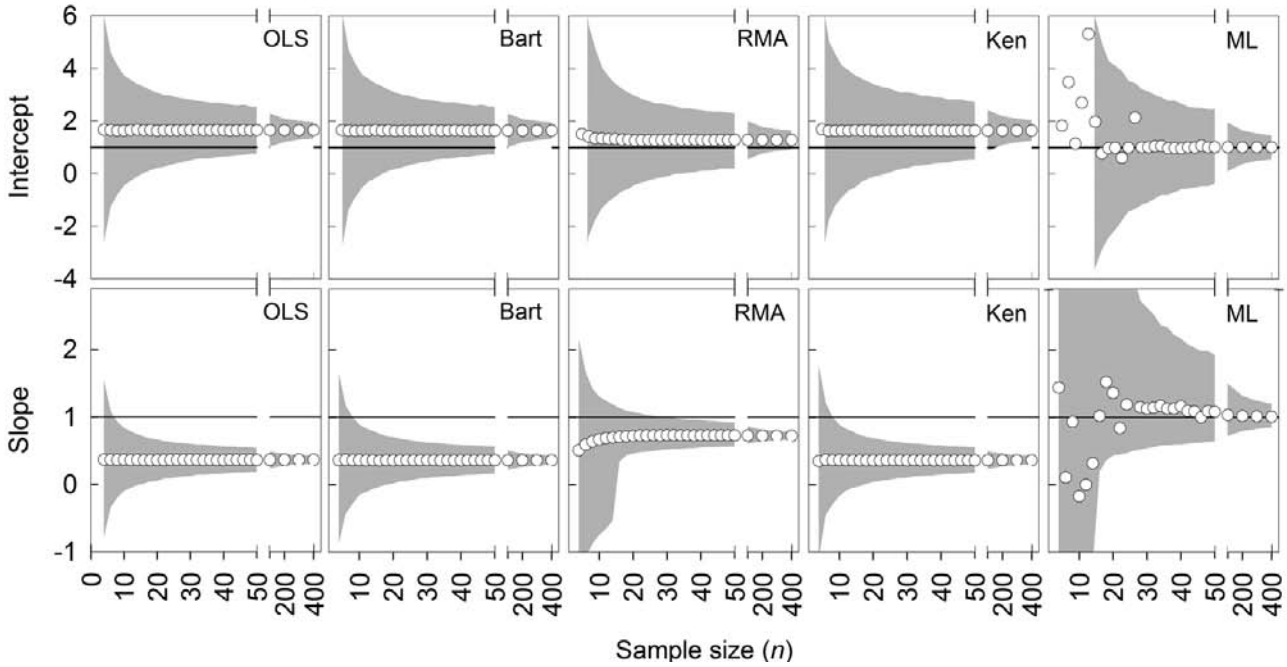

Figure 3. Intercept and slope estimates for 10,000 samples with different sample size (n) ranging from 4 to 400 obtained with OLS, Bartlett's three groups method (Bart), reduced major axis (RMA), Kendall's robust method (Ken), and maximum likelihood (ML) fitted to (Y,X) data generated as $Y = \eta + \vartheta$ with $\vartheta \sim N(0, 2)$ and $X = \xi + \delta$ with $\delta \sim N(0, 4)$, where $\eta = 1 + 1\xi$ as described in the "Monte Carlo Simulations" section. Reliability ratio for X is $\varkappa_\xi = 0.4$. Data are shown as the mean estimates (circles) with the 95% percentile interval (gray area). For clarity's sake, data from sample sizes 50 to 100 have been omitted.

estimator overestimated the true intercept. Both the RMA and ML estimates are unbiased, with the RMA estimators being more efficient than the ML estimators, as well as more robust to small sample sizes.

If both Y and X are affected by random normal errors with 0 mean and constant variance but the variation in X ($\sigma_\delta = 4$; $\varkappa_\xi = 0.4$) is greater than the variation in Y ($\sigma_\vartheta = 2$) (in this simulation, twice as great), OLS, Bartlett, and Kendall slope estimates are biased toward 0 (Figure 3). RMA tends to converge to the true population slope, but when errors in magnitude in X are greater than in Y, this estimator consistently underestimates the true population slope, and this underestimation increases as the magnitude of the error ratio of X to Y increases (data not shown). The best estimator for the slope is the ML estimator; however, it produces unbiased estimates only for moderate large samples, behaving erratically for small sample sizes.

Discussion

Slope estimates from OLS are biased toward 0, underestimating the slope if the true slope is positive; or overestimating the slope if the true slope is negative, when the X is measured with error and this has been well documented (Cheng & Van Ness, 1999; Fuller, 1987; Isaac, 1970; Morton-Jones & Henderson, 2000; Ricker, 1984; Schuster, 2004; Sokal & Rohlf, 1995). In addition, the bias increases as measurement error in X increases. Failure to acknowledge bias in OLS when predictors are measured with error is quite frequent in psychology and the social sciences. This may result in increased type II error rates regarding inference about the regression slope, deeming as nonsignificant a linear relation that, indeed, is present in the population under study. Type II regression models have been proposed to overcome bias in the linear slope estimation process when predictors are measured with error. However, results presented in this paper confirm early observations that Kendall's and Bartlett's slope estimates also show bias toward 0, with lower efficiency than OLS (Ricker, 1973, 1984). Bartlett's three-group method was reported to produce unbiased estimates, but only when ranking of data is done on ξ rather than on X (Cheng & Van Ness, 1999). Unfortunately, practical situations where ξ is known are rare and, thus, ranking has to be done on X for most research scenarios in the psychological and social sciences. If ranking is done on X, Bartlett's slope estimator shows a bias similar to the OLS slope estimator (Pakes, 1982). Both Bartlett's and Kendall's methods perform poorly for small $\varkappa_\xi$, while the RMA method is efficient and consistent if X and Y are measured with equal reliabilities. ML with known error-variances ratio is not affected by reliability but estimates are consistent only for large sample sizes. For $\varkappa_\xi \approx 1$, OLS estimators are the most efficient and, if $\varkappa_\xi$ is known, $\beta_1^* = \beta_1/\varkappa_\xi$ is an unbiased estimator for the OLS slope (see Cheng & Van Ness, 1999; Fuller, 1987). If errors in X are smaller than errors in Y, the RMA slope estimator overestimates the true slope, while if the errors in X are greater than the errors in Y, the slope is underestimated if the true slope being estimated is positive. Otherwise, one will obtain underestimates or overestimates of the true negative slope whenever errors in X are larger or smaller, respectively, than errors in Y. For the particular case for which errors in X and Y are of the same magnitude, as may be the case when standardized measures are used (a common practice in psychology and the social sciences), the RMA slope estimator gives consistent and efficient estimates even for moderate sample sizes (> 15) (see also Ricker, 1984; Sokal & Rohlf, 1995). Finally, ML estimates with known $\lambda = \sigma_\vartheta^2/\sigma_\delta^2$ give erroneous estimates for small sample sizes, which, however, converge to the true population value for larger sample sizes (> 30). Consistency and efficiency of ML estimators are inversely related to the error magnitude in X.

Consistency of Bartlett's and Kendall's methods is as poor as OLS when the variables are affected by significant measurement errors. When X is not affected by measurement errors, Bartlett's and Kendall's methods are consistent but less efficient than OLS, while the RMA estimators produce under- or overestimates. When measurement errors in X are of the same magnitude as errors in Y, the RMA estimators are consistent and efficient especially for small to moderate samples sizes, as compared with other methods. Thus, if one can assume that measurement errors in X and Y are of the same magnitude, the RMA method is quite easy to implement and gives efficient and consistent estimates even for relative small sample sizes. ML is the method that produces consistent estimates in every scenario considered for the measurement errors in X and Y, although its efficiency is penalized by the magnitude of the errors and, especially, by small sample sizes for which estimates are unreliable and erratic. However, its calculations requires knowledge of $\lambda = \sigma_\vartheta^2/\sigma_\delta^2$ which, in turn, requires a set of replicated measurements of every Y_i and X_i to estimate σ_ϑ^2 and σ_δ^2, respectively (Fuller, 1987; Morton-Jones & Hederson, 2000) or prior knowledge, or calibration, of the phenomenon under study (Carrol & Ruppert, 1996; Morton-Jones & Hederson, 2000; Schuster, 2004). While replicated measurements are common in experimental and/or quasi-experimental studies, they are improbable for correlational studies and, thus, prior knowledge of λ is a limitation to the applicability of this method for these types of studies. An alternative to type II regression methods is SEM, which allows for the modeling of structural relations between latent variables or constructs accessed by multiple observed indicators. SEM methods are used extensively in psychology and the social sciences. However, for most research scenarios requiring regression analysis, there are no multiple observed indicators per latent construct, nor prior knowledge of the measurement-error variances required to make the SEM model identifiable. For ML, it suffices to know the error variances ratio; thus, ML is applicable to a larger set of research scenarios than SEM. Researchers dealing with measurement errors in both predictor and criterion variables should consider the use of the RMA regression and/or ML (with replicated measurements, if possible) to remove bias from the linear slope estimates.

Acknowledgments

I acknowledge the helpful comments of the two anonymous reviewers. This research was funded by CII-ISPA research grant PI-04/004.

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