Instituto de Engenharia de Sistemas e Computadores de Coimbra

Institute of Systems Engineering and Computers

INESC - Coimbra

Paula Sarabando

Luis C. Dias

Rudolf Vetschera

Approaches to suggest potential agreements:

Perspectives of mediation with incomplete information

No. 11 2009

ISSN: 1645-2631

Instituto de Engenharia de Sistemas e Computadores de Coimbra

INESC - Coimbra

Rua Antero de Quental, 199; 3000-033 Coimbra; Portugal

www.inescc.pt

Approaches to suggest potential agreements: Perspectives of mediation with incomplete information

Paula Sarabando
INESC Coimbra and Departamento de Matemática, Instituto Superior Politécnico de Viseu
Campus Politécnico de Viseu, 3504-510 Viseu, Portugal.
e-mail: psarabando@mat.estv.ipv.pt

Luís C. Dias
INESC Coimbra and Faculdade de Economia da Universidade de Coimbra
Av Dias da Silva 165, 2004-512 Coimbra, Portugal.
e-mail: ldias@inescc.pt

Rudolf Vetschera
Faculty of Business, Economics and Statistics University of Vienna
Bruenner Strasse 72, A-1210 Vienna, Austria.
e-mail: Rudolf.Vetschera@univie.ac.at

June 22, 2009

Abstract

In bilateral Negotiation Analysis, the literature often considers the case of complete information. In this context, since the negotiators know the value functions of both parties, it is not difficult to calculate the Pareto frontier and the Pareto efficient solutions for the negotiation. Thus rational negotiators can reach agreement on this frontier. However, these approaches are not applied in practice when the parties do not have complete information. The research question of our work is "It is possible to help negotiators achieving an efficient solution if they do not have complete information regarding the different parameters of the model?". We propose to obtain information regarding the preferences of negotiators during the negotiation process, in order to be able to propose alternatives close to the Pareto frontier. During this work we will present three approaches to help a mediator proposing a better solution than the compromise the negotiators have reached or are close to reach.

Key words: Incomplete Information, Negotiation, Mediation, Integrative Negotiation, "Dance of the Packages"

1 Introduction

A large majority of the most complex decisions is taken and implemented by groups of people. Jelassi et al. [11] distinguished between four types of procedures for deciding when several decision makers are involved: (i) individual decision-making in a group setting, (ii) hierarchial or bureaucratic decision making, (iii) group decision-making or one-party decision-making, and (iv) multi-party decision-making or negotiation. In the first situation, one decision maker has the responsibility for the decision, but utilizes knowledge of experts, advisers or stakeholders during the process. In hierarchial decision making, there are two cases to consider: centralized and decentralized. In the centralized setting, there is one set of objectives representing the interests of a top-level decision maker, who has full control over the lower-level members. In the decentralized case, each member independently controls subsets of the decision variables and objectives and is responsible for his decision which serves as input to the higher level. In group decision-making, each group member participates in the process and is partly responsible for the final decision. There usually is an overall goal which is accepted by all the members, but they differ in the ways of how this goal should be achieved. In negotiation, each decision maker represents one party and is responsible for the decision before this party and not before the other one(s). There is a conflict of interests because parties have separate and conflicting objectives and they have different needs which they want to satisfy. Negotiation is the chosen way to resolve a conflict out of necessity and not out of effectiveness or efficiency. Note that, although the distinction between group decision-making and negotiation is not always clear, it is possible to point out some differences (see for example [6]). In this work we will be concerned with negotiation, more specifically bilateral (two-party) negotiation.

In negotiations usually there exists the possibility to bring in the help of a third party, as a mediator or an arbitrator. A mediator is a person who should be acceptable, impartial, and neutral and, despite not having power of authoritarian decision, should assist the negotiation of the other parties, establishing a positive climate. An arbitrator is a neutral and impartial person who takes a decision in the negotiation process by comparing previous results, using justice criteria or by other methods. An arbitrator's decision may be binding or not binding.

The general goal of our work is to contribute new methodologies to support a mediator in advising negotiators (Raiffa's externally prescriptive perspective [23]). However, the methodology developed in this work can also be adapted to support one of the parties based on a description of the other party's behavior (Raiffa's asymmetrically descriptive-prescriptive perspective).

We consider integrative negotiations over multiple issues, which are the ones most likely to benefit from the efforts of a mediator. Integrative (or win-win) negotiation (see for example [32]) assumes the integration of resources and capabilities of parties to generate more value. This contrasts with distributive (or win-lose) negotiation where the aim is typically the division of a single good and the main concern of negotiators is to get the largest possible share of the "pie". In integrative negotiation, successful strategies include cooperation, information sharing and joint resolution of problems. A typical form of negotiation between two parties is the "dance of packages" [23], in which offers and counter-offers are successively presented by both parties. Imagine that party 1 prepares a proposal that he finds appealing and hopes the party 2 would accept. Then, party 2 will answer with a complete proposal of his own. As one would expect, party 1's initial proposal might be wonderful for party 1 and unacceptable for party 2. The counter-offers from party 2 might have the opposite characteristics. Now there are two proposals on the table, and each side describes the merits of its own offer and possibly criticizes the other. The dance of the packages proceeds by making concessions seeking a compromise. In a slight variation of this procedure, the parties might not offer proposals in sequence, and instead both of them might simultaneously put offers on the table.

According to Raiffa [23], in integrative negotiation it is necessary to construct and evaluate proposals covering various issues. The construction of these proposals consists in the identification of issues to solve, in the specification of the possible levels of resolution for each subject, and in the specification of the scores of each possible combination of levels (scores which can be obtained through an aggregation method, e.g. the additive value model). The existence of a value-based evaluation model allows that each party evaluates their potential proposals, evaluates the other party's proposals, and evaluates their BATNA (best alternative to a negotiated agreement), it also allows that someone with complete information can say whether an agreement is Pareto efficient or not. Recall that a solution is Pareto efficient if it is not possible to move improve the position of one party without worsening the value to one of the other parties.

In bilateral Negotiation Analysis, the literature often considers the case with complete information. If the mediator knows the value functions of both parties, then he can calculate the set of Pareto efficient solutions. Thus, the mediator can suggest an agreement from this set, where the choice among the Pareto efficient solutions can be based on additional criteria like the fairness of the proposed compromise.

However, these approaches are not applied in practice where neither the parties nor an outside mediator have complete information about the preferences of all parties (see for example [17]). In many cases, parties might not even have complete information about the parameters describing their own preferences, because the assumption that parameter values can be precisely elicited is often unrealistic or, at least, there may be advantages in working with less precise information (see for example [16] and [33]). For a mediator in a negotiation, obtaining information about the value functions of the parties is even more difficult, since parties might have incentives to strategically distort the preference information they provide [29].

Some approaches in the literature deal with incomplete information in the context of negotiation problems. An important objective in negotiation processes is to achieve a win-win solution (or integrative solution), a solution that improves the position of both parties with respect to the present situation. According to the Dual Concern model [22, 27], these solutions can be achieved if negotiators have a high concern about the both their own preferences and the preferences of their opponents, which requires also information about the opponent's preferences. Typically, this information is not complete [30].

Clímaco and Dias [4] proposed an extension of the methodology of the software VIP-G [6] for bilateral negotiation processes, based on the concept of convergence paths in the weights space. While assuming that negotiators make decisions based on their value functions, constructing these functions is not trivial when there are multiple attributes. Lai et al. [17] presented a model that considers Pareto efficiency and computational efficiency, for situations where information is incomplete, the value functions are not linear and are not explicitly known. The authors refer that one of the main problems associated with multi-attribute negotiation is the difficulty of making decisions in an n-dimension space. To reduce this problem, a process was proposed that enables negotiators, in each period, to negotiate based on a line, called negotiation base line. To implement this model, a mediator needs to be available. Though it is not difficult to involve such a mediator in automated negotiations between software agents, there may exist situations where a mediator is not trusted or hard to be implemented. Thus, Lai and Sycara [18] focused on developing mechanisms for Pareto-efficient multi-attribute negotiations without the presence of a mediator.

Ehtamo et al. [7] presented a class of methods called constraint proposal methods, which are interactive methods to find Pareto efficient solutions through common tangent hyperplanes. This process supports negotiations of two parties with two or more continuous issues. A mediator tries to find a hyperplane, through some reference points, so that the most preferred alternative for both parties in this hyperplane coincide. Heikanen [9] proposed an interactive process to determine Pareto efficient solutions in negotiations with multiple parties about continuous issues, with help from a mediator. In this method it is not required that negotiators know the value functions of other parties or that someone outside the negotiation knows all the value functions.

In this paper we assume that the preferences of both parties can be roughly modelled by an additive value function, as in Raiffa's Negotiation Analysis [23]. However, we do not make the assumption that each party's value function is precisely known, i.e., we will not assume that the parties will indicate explicitly the parameters values that fully define their model. The information that is available about the negotiator's preferences can come from one of two sources: incomplete information obtained implicitly through the offers or the decisions and incomplete information explicitly provided by the negotiators (e.g. intervals of parameter values). The information we will use is mainly based on comparisons of proposals that are implicitly or explicitly made by the parties. We will consider different levels of such incomplete information, in particular the case where some parameters of the evaluation model are known (value functions, weights of the value functions), and the case where no parameters of the model are exactly known.

The main contribution of this paper is to propose and compare three new approaches to support a mediator under incomplete information: the first is based on robust conclusions, the second is based on inferred approximations, and the third uses a domain-based analysis. These approaches will allow the mediator to assess how each proposal he may put forward would be received by the parties, namely if they would consider it as better than the ones they have already considered (or even accepted as a compromise), and to know which would the most promising proposals be according to some well-known arbitration criteria.

We envision two scenarios in which these methods could be applied:

1. The parties have reached a potential compromise and want to improve it.

2. The parties have not (yet) reached a compromise. There are two offers on the table, which provide different utilities to the two parties.

In the latter case, each party can at least obtain the utility which it would receive from the current offer made by the opponent. We therefore consider the utility levels offered by each parties' proposal to the other side as the status quo in such negotiations.

This paper is structured as follows. In section 2 we will present a framework for negotiations under incomplete information. In section 3 we will propose three different approaches to suggest potential agreements. In section 4 we will present an illustrative example where we use the approaches presented in section 3. We will finish in section 5 with some conclusions and thoughts on future research.

2 A Framework for negotiations under incomplete information

2.1 Information levels

In this paper, we consider several different levels of information about the negotiators' preferences over multiple issues that might be available to an outside mediator. To formally characterize this information, we assume that the true (but possibly unknown) preferences of a negotiator can be represented by an additive value function of the form

$$V^k(x) = \sum_{j=1}^{n} V_j^k(x) = w_1^k v_1^k(x_1) + w_2^k v_2^k(x_2) + ... + w_n^k v_n^k(x_n) \tag{1}$$

where $V_j^k(x) = w_j^k v_j^k(x)$, $v_j^k(x)$ represents the value of the proposal x related to the j^{th} issue and w_j^k represents the scale coefficient or "weight" of the value function $v_j^k(.)$, for party k, and n represents the number of issues. Without loss of generality, we further assume that the value function is standardized so that:

$$0 \leq w_j^k \leq 1, j = 1, ...n \text{ and } \sum_{j=1}^{n} w_j^k = 1 \tag{2}$$

and

$$0 \leq v_j^k(x) \leq 1. \tag{3}$$

An additive value function imposes certain restrictions on the preferences that can be represented, most notably preferential independence between the issues being considered [12, 31]. While the additive form allows us a certain simplification in the models we are going to formulate (like the use of linear programming rather than nonlinear programming), our approach does not rely very strongly on additivity of the value function and can with some adaptations also be extended to other forms like bilinear or multilinear functions [12].

Function (1) allows for a classification of different types of information levels. As a benchmark, we consider the case of *complete information* in which all components of the value function are assumed to be known. By relaxing this assumption, we consider three possible levels of incomplete information:

1. The weights w_j^k are unknown, while the values $v_j^k(x_j)$ are known.
2. Both weights and values are unknown, but the value function $v_j^k(x)$ is assumed to be approximately linear; hence, exact (but approximate) values can be used.
3. Both weights and values are unknown, and no further assumptions about the shape of the value function $v_j^k(x)$ are made.

In case 2, we essentially replace the true values $v_j^k(x_j)$ by their linear approximation

$$v^{lin}(x_j) = \frac{x_j - \underline{x_j}}{\overline{x_j} - \underline{x_j}} \tag{4}$$

where $\overline{x_j}$ and $\underline{x_j}$ represent the best and worst possible outcome in attribute x_j, respectively.

In case 3, we restrict possible value functions by a lower and an upper bound. If we can exclude increasing marginal values (which is reasonable and can easily be assessed by asking simple questions to each party) the lower bound will be formed by the linear function (4) and the upper bound will be formed by a concave value function

$$v^{con}(x_j) = a + b(-e^{-c*x_j}) \tag{5}$$

where parameters a and b are chosen to scale the function to values between zero and one, and parameter c determines the degree of concavity of the function. Values are thus restricted to $v^{lin}(x_j) \leq \hat{v}(x_j) \leq v^{con}(x_j)$, where we use $\hat{v}$ to indicate that this is an approximation of the unknown true value. For example, in Figure 1 we would consider that $m \leq \hat{v}_{price}(11) \leq M$.

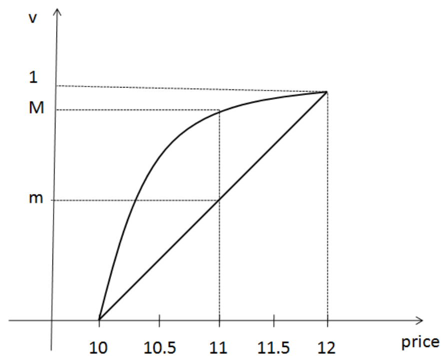

Figure 1: Shape of a value function of the issue price (for a seller).

2.2 Representation of incomplete information

In all three cases outlined above, the mediator is not necessarily completely ignorant about the weights and/or values, but might be able to get at least some information about them. Such information can be obtained in two ways: (i) it can explicitly be provided by the negotiators, or (ii) it can be inferred from observing their behavior during the process of the negotiation, in particular from the offers that each of them makes and their reactions to offers from the opponent.

In both cases, the information obtained by the mediator is most likely in the form of statements of preference or indifference between different alternatives, where each alternative is characterized by a value for each issue. If information is directly provided by the negotiators, the mediator could ask whether they would consider another alternative to be about as good as a proposed alternative, or when the mediator makes a proposal, he could also ask if this proposal is indeed better than an offer already on the table, thus inferring the direction of preference between these two alternatives.

In a "Dance of packages" negotiation process, preferences between alternatives can also be inferred from the offers made by negotiators during the process [30]. For instance, in a scenario where the negotiators have already reached a tentative compromise and wish to improve upon it, one can safely assume that a negotiator will prefer that compromise to all other offers made by the opponent during the negotiation. Otherwise, it would in most cases be possible to revert to that previous offer from the opponent (which the opponent could hardly reject, since it was him who originally proposed it). Furthermore, in a "Dance of packages" negotiation process, negotiators typically start with offers very favorable to themselves and then successively make concessions in the course of the negotiation. Thus we can assume that a negotiator prefers all offers made by himself to the compromise and also prefers his earlier offers to the offers he made later in the process. From transitivity, it also follows that a negotiator will prefer all offers made by himself to all offers made by the opponent. This last condition will hold also if no compromise has been reached (yet).

Information about preferences of negotiators will therefore be available in the form of statements of preference or indifference between alternatives. A statement that alternative $x^{(1)}$ is preferred to alternative $x^{(2)}$ can be represented by the condition (see, e.g., [10, 30]):

$$\sum_{j=1}^{n} V_j^k(x_j^{(1)}) \geq \sum_{j=1}^{n} V_j^k(x_j^{(2)}), \tag{6}$$

while a statement of indifference can be represented by the constraint

$$\left| \sum_{j=1}^{n} V_j^k(x_j^{(1)}) - \sum_{j=1}^{n} V_j^k(x_j^{(2)}) \right| \leq \epsilon \tag{7}$$

where ϵ is a suitably small tolerance value.

The specification of V_j^k depends on the information level being considered. For the case of unknown weights and known values, it is defined as

$$V_j^k(x_j) = w_j^k v_j^k(x_j) \tag{8}$$

i.e. the unknown weight is combined with the known value function of negotiator k. In the case of unknown weights and the value function considered as linear it is defined as

$$V_j^k(x_j) = w_j^k v_j^{lin}(x_j). \tag{9}$$

In these two cases, the constraints are linear and define a feasible set of weights W_k (a polytope) which can be considered as possible preference parameters of negotiator k.

In the third case, with unknown weights and unknown values, the values for $V_j^k(x_j)$ can directly be used as variables in the model, as in [8]. Let s_j denote the number of different values for x_j considering all the potential alternatives $x^{(1)}$, ..., $x^{(m)}$ (m denoting the number of alternatives). Let us define a vector of $s_1 + ... + s_n$ variables $v_{i,j}^k = V_j^k(x_j^{(i)})$. These variables can be used in constraints of type (6) and (7). Furthermore, if $\overline{x_j^k}$ represents the best possible outcome in attribute x_j for party k, then considering $v_j^k(\overline{x_j^k}) = 1$ we will have $V_j^k(\overline{x_j^k}) = w_j^k$.

If we assume that values are ordered in decreasing order of preference, i.e. that $v_{1,j}^k$ represents $V_j^k(\overline{x_j^k})$, we can express (2) as:

$$1 \geq w_j^k = v_{1,j}^k > v_{2,j}^k > \ldots > v_{s_j,j}^k \geq 0 \text{ and } \sum_{j=1}^{n} v_{1,j}^k = 1. \tag{10}$$

Thus, we are dealing with linear problems even in the case where both the weights and the values are unknown.

When each attribute's value function is considered to be known or is replaced by a linear approximation, the constraints (6), (7), and (2) (plus possibly other ones) define a polytope W_k of admissible weights. When each attribute's value function is considered to be unknown, the constraints (6), (7), and (10) (plus possibly other ones) define a feasible set of values we denote by M_k (also a polytope). In either case, the polytope can be considered as the set of possible preference parameters of negotiator k. It should be noted that we assume here that all actions of a negotiator, and all preference statements provided by a negotiator, are consistent with a true value function of the form (1). If this is not the case, and constraints derived from the negotiator's choices contradict each other, these sets might become empty.

In the next section will use a general notation $(w, v) \in (W_k, M_k)$ with the following meaning:

$$(w, v) \in (W_k, M_k) \Leftrightarrow \begin{cases} (w_1^k, ..., w_n^k) \in W_k & \text{if } v_j^k(.) \text{ is known or is replaced} \\ & \text{by a linear approximation.} \\ (v_{1,1}^k, ..., v_{s_1,1}^k, ..., v_{s_n,n}^k) \in M_k & \text{if } v_j^k(.) \text{ is unknown.} \end{cases} \tag{11}$$

2.3 Criteria for selecting alternative solutions

The mediator can use information of one of the types presented above to suggest one or several alternative solutions to the negotiators, with a good potential to be accepted by both. To guide these proposals, several criteria can be applied. We start by presenting these criteria, and then in the next section we will present three different approaches how such proposals can be obtained.

If several alternatives are to be proposed, the dominance criterion is a natural starting point. In this case, the mediator could identify all alternatives which dominate the currently proposed compromise

$x^{(c)}$ or the status quo of the negotiation (we are using this term for the case of two offers on the table). Conversely, alternatives which are dominated by the proposed compromise or the status quo can be eliminated from further consideration.

Let $x^{(r)}$ denote the reference (or reservation) point below which the negotiators will not accept any alternative. If a compromise has been reached then $x^{(r)} = x^{(c)}$. If a compromise has not been reached and the two last offers on the table (the status quo) are $x^{(o_1)}$ (offered by negotiator 1) and $x^{(o_2)}$ (offered by negotiator 2), then $x^{(r)}$ will refer to the $(V^1(x^{(o_2)}), V^2(x^{(o_1)}))$ point in the value space, i.e., a fictitious alternative yielding for each negotiator the amount that was offered by the opponent. The alternatives to be proposed by the mediator should, for both negotiators, be better than $x^{(r)}$. Since preference information is incomplete, one can distinguish here between alternatives which surely dominate $x^{(r)}$ (i.e. which are better for both parties under all preference parameters still considered possible), alternatives which possibly dominate $x^{(r)}$ (i.e. which are better for both parties for at least one vector of preference parameters for each party), and alternatives that cannot dominate $x^{(r)}$.

As a second criterion, the alternatives to be proposed should also be Pareto efficient concerning the value they yield to each party. Once again, under incomplete information we can talk about alternatives that are surely efficient, alternatives that are possibly efficient, and alternatives that cannot be efficient (because they are surely dominated by another alternative).

If the negotiator wants to present only one (or a small number) of alternatives, additional criteria can be used to guide this selection. Several such criteria can be developed, depending on whether the mediator is more interested in finding an efficient solution (which maximizes total value creation) or an equitable solution, which tries to balance the interests of the parties involved. In this paper, we consider the following *mediation criteria* [23]:

1. The *max-sum criterion*, which maximizes the sum of values of both parties and thus selects the alternative which is best according to total efficiency.

2. The *max-min PoP criterion*, which maximizes the minimum payoff, i.e. the payoff to the negotiator who receives the lowest payoff from the negotiation result. To make payoffs comparable between negotiators, they are standardized within the possible range by calculating the Proportion of Potential (PoP).

Thus, the max-sum criterion selects the alternative (x) which maximizes

$$V^1(x) + V^2(x) \tag{12}$$

and the max-min PoP criterion maximizes

$$\min_k \frac{V^k(x) - V^k_{min}}{V^k_{max} - V^k_{min}} \tag{13}$$

where V^k_{max} is the best payoff that player k could achieve considering the set of alternatives being considered (better for both parties than $x^{(r)}$), and V^k_{min} is a lower limit on the payoffs considered for player k for the same set of alternatives.

Naturally, other mediation criterion that can be used, e.g., maximizing the product of excesses relatively to $x^{(r)}$, or the Nash bargaining solution. Although generalization to those other criteria is straightforward, we will restrict our analysis in this paper to the max-sum criterion and the max-minPoP criterion, because they lead to linear programming models while other criteria would require nonlinear models.

When we consider incomplete information on preferences we can distinguish different classes of alternatives: Alternatives that are surely optimal for a criterion maximize that criterion for all possible preference parameters. Alternatives that are potentially optimal maximize the criterion for some of the possible preference parameters, while the maximum is obtained with another alternative for some other possible parameter values. Alternatives are called surely sub-optimal if no preference parameter $(w, v) \in (W_k, M_k)$ exists at which the alternative maximizes the criterion under consideration.

3 Approaches to suggest potential agreements

3.1 Extreme parameters approach

As a first approach, we formulate optimization models to detect which alternatives surely meet the mediator's requirements described in the previous section, as well as which alternatives surely do not meet these requirements. Other alternatives can exist that will meet each requirement, or not, depending on the parameter values. We call this first method the "extreme parameters" approach, because we are looking for parameter values which lay on the boundary of the feasible set, leading to extreme value differences.

To find out whether an alternative is surely better or surely worse than the reference, a Linear Program (LP) can be solved. Recall that $V^k(x)$ is the value of alternative x for negotiator k ($k = 1, 2$). Let m^k_{ij} denote the solution of the following LP:

$$\begin{array}{l} \max\{V^k(x^{(i)}) - V^k(x^{(j)})\} \\ (w, v) \in (W_k, M_k) \end{array} \tag{14}$$

Whenever $m^k_{ij} < 0$, there is no possible combination of parameters which would make alternative $x^{(i)}$ at least as good as $x^{(j)}$ for negotiator k, thus we can say that $x^{(j)}$ is *surely better* than $x^{(i)}$ (or $x^{(i)}$ is *surely worse* than $x^{(j)}$) with respect to the available information about negotiator k's preferences.

Given the sets of feasible parameter values (W_1, M_1) and (W_2, M_2), it is possible to determine, for each negotiator, which alternatives are surely better than the reference $x^{(r)}$ and which alternatives are surely worse than this reference. The mediator would like to propose an alternative $x^{(i)}$ such that $m^1_{ri} < 0$ and $m^2_{ri} < 0$. The problem is that it can happen that there are no alternatives that are surely better than the reference for both negotiators. Nevertheless, this approach is a good starting point: if there are alternatives that are surely worse than the reference point for one of the negotiators, then the mediator can discard these alternatives, i.e., we can eliminate the alternatives $x^{(i)}$ for which $m^1_{ir} < 0$ or $m^2_{ir} < 0$. These calculations are analogous to those proposed by Dias and Clímaco [5] to obtain binary robust conclusions. Hence, only the alternatives that are potentially at least as good as the reference for both negotiators are candidates to be proposed to them.

The LP (14) can also be used to compare any other pairs of alternatives, besides pairs containing the reference $x^{(r)}$. This allows to check for Pareto efficiency. For a pair $(x^{(i)}, x^{(j)})$, if $m^1_{ji} < 0$ and $m^2_{ji} < 0$, then $x^{(j)}$ is surely worse than $x^{(i)}$ for both negotiators and hence $x^{(j)}$ is surely not Pareto efficient. Thus, it can also be discarded.

Let P denote the index set of the remaining candidate alternatives, after discarding alternatives surely worse than the reference point for any of the negotiators and alternatives surely not belonging to the Pareto frontier. To discriminate between alternatives in P the mediator might also try to identify which ones can be potentially optimal according to a mediation criterion. For the max-sum criterion the following LP is solved for each alternative $x^{(i)} \in P$:

$$\begin{array}{l} \max \delta \\ V^1(x^{(i)}) + V'^2(x^{(i)}) - [V^1(x^{(j)}) + V'^2(x^{(j)})] - \delta \geq 0, \forall j \in P, j \neq i \\ (w, v) \in (W_1, M_1) \\ (w', v') \in (W_2, M_2) \\ \delta \text{ free} \end{array} \tag{15}$$

If this LP yields $\delta \geq 0$ at the optimal solution then $x^{(i)}$ is potentially optimal according to the max-sum criterion; otherwise, it cannot be the best one according to that criterion. Let us note that if we tried to maximize the sum of the values, this would not lead to acceptable results (for more details see Appendix A). To perform a similar analysis considering another mediation criteria requires introducing binary variables (for the criterion of maximizing the minimum PoP) or nonlinear programming (for criteria involving products).

3.2 Central parameters approach

A second approach the mediator might follow, pursuing the objective of finding good potential alternatives, consists in inferring a representative combination of parameter values from (W_1, M_1) and (W_2, M_2), and then use these surrogate values to find alternatives that are better than the reference point for both negotiators, are efficient, and optimal according to a mediation criterion. Of course, the conclusions that hold for such a surrogate parameter vector do not necessarily hold for the true parameter values that would be set in the course of a thorough and explicit elicitation process. Nevertheless, studies in the context of additive value functions (e.g., [1, 24]) show that using a combination of parameter values that is central with respect to the feasible set boundaries yields good approximations. The more information the mediator has, in terms of constraints to the parameter values, the more accurate this approximation will be.

One possible approach to find a central combination of parameter values is to solve a LP of the max-min type to find a point such that the smallest slack in a constraint of the form (6) is as large as possible. This is an approach used for inferring parameters of multicriteria aggregation approaches (e.g., [10, 21]). Let A_k denote a coefficient matrix and let b denote a right-hand side vector such that $(w, v) \in (W_k, M_k)$ if and only if $A_k.(w, v) \leq b$. Let s be a vector containing one constant per constraint, equal to 1 if the constraint is of type (6) and equal to 0 otherwise. The following LP can then be used to infer a central parameter vector with respect to the inequality preference statements, for $k \in \{1, 2\}$:

$$\begin{aligned} &\max \Delta^k \\ &A_k.(w, v) + s\Delta^k \leq b \end{aligned} \tag{16}$$

The variables of this problem are the Δ^k scalar (representing the smallest slack to be maximized), the weights, and possibly the values. The optimal solution will be a kind of "safest" vector, which is as far as possible from any boundary. Because of that, our objective is to maximize the slack. Note that all constraints are formulated in terms of multi-attribute value, which is scaled between zero and one, and thus have a comparable magnitude. This makes it possible to compare deviations across constraints without further rescaling.

A different approach for obtaining a central combination of parameter values is to compute the centroid of (W_k, M_k) in an exact manner or using an approximation. Exact methods exist for some types of polytopes [25]. An approximation to the centroid of any polytope can however be easily obtained using Monte-Carlo simulation, as in the computation of central weights used in the SMAA method [15].

Let $(w^1, v^1)^*$ denote the central parameter vector obtained for negotiator 1, and let $(w^2, v^2)^*$ denote the analogous result obtained for negotiator 2. Using $(w^1, v^1)^*$ and $(w^2, v^2)^*$ as surrogate parameter values it is possible to compute which alternatives are better than the reference point for both negotiators, and which one among those maximizes the mediation criterion. In contrast to the extreme parameters approach, maximizing the minimum PoP or criteria involving products is straightforward in the central parameters approach, because it is only necessary to compute the respective objective function for each alternative using one parameter vector. In addition to the optimal alternative for the mediation criterion, the set of all other efficient solutions for the central parameter vector can also be determined easily.

Since the parameter vector used in this approach is only an approximation, it might not reflect the true preferences of negotiators. Thus, a situation can arise in which an negotiator finds an alternative x proposed by the mediator unacceptable. From such a statement, the mediator can conclude that alternative x has inferior value than the reference point $x^{(r)}$ for negotiator k. In this case it is possible to change (W_k, V_k) by introducing this new information in the form of a constraint $V^k(x) < V^k(x^{(r)})$ and compute new central parameter values for the new smaller polytope.

3.3 Domains approach

The domain criterion, introduced by Starr [26], uses the volume of parameters space in which an alternative remains optimal to indicate the sensitivity of a solution. The use of this criterion for multi-attribute decision problems was proposed by Charnetski and Soland [3], who used Monte Carlo simulation to obtain approximations for the volume of the domain. SMAA methods [16] are also based on exploring the weight space in order to describe the preferences that would make each alternative the most preferred one, or that would give a certain rank for a specified alternative. The method proposed by Vetschera [30] to measure the extent to which information about the preferences is available during the negotiation is also based on the domain criterion.

Our third approach also consists in exploring the parameter space in order to measure the relative volume of the feasible set of parameter values where some conditions are verified. Let S denote the set of feasible parameter values for the two parties given the information currently available:

$$S = \{(w^1, v^1, w^2, v^2) \in (W_1, M_1) \times (W_2, M_2)\}.$$

Let $S(\tilde{C})$ denote the subset of S where condition $\tilde{C}$ holds:

$$S(\tilde{C}) = \{(w^1, v^1, w^2, v^2) \in (W_1, M_1) \times (W_2, M_2) : \tilde{C} \text{ is true}\}.$$

Let $Vol(S(\tilde{C}))$ denote the volume of set $S(\tilde{C})$ and let $Vol(S)$ denote the volume of set S. The expression

$$Vol(S(\tilde{C}))/Vol(S)$$

then denotes the relative volume of the subset in which condition $\tilde{C}$ holds as compared to the volume of the entire feasible region. If we further assume that parameter vectors are uniformly distributed, this ratio can be interpreted as the probability that condition $\tilde{C}$ is fulfilled for any randomly drawn feasible parameter vector.

The relative volume of the parameter set in which each alternative $x^{(i)}$ is at least as good as the reference for both negotiators can be computed as

$$Vol\left(S\left(V^1(x^{(i)}) \geq V^1(x^{(r)}) \wedge V^2(x^{(i)}) \geq V^2(x^{(r)})\right)\right)/Vol(S).$$

Note that this relative volume is equal to zero for alternatives that are surely worse than $x^{(r)}$, and is equal to one for alternatives that are surely better than this reference. This approach therefore complements the extreme values approach by providing additional information about alternatives which are between the two extreme cases of being definitely better or definitely worse than the reference alternative and indicates the probability that, given the preference information collected so far from the negotiators, both negotiators will prefer alternative $x^{(i)}$ over the reference alternative.

It is also interesting to determine the relative volume of the parameter set in which each alternative is Pareto efficient: $Vol\left(S(x^{(i)} \text{ is efficient})\right)/Vol(S)$.

The same approach can also be used to determine the relative volume of the subset of parameter space where each alternative $x^{(i)}$ is optimal according to the different mediation criteria (maximizing the sum of the values, maximizing the minimal PoP, etc.) as $Vol\left(S(x^{(i)} \text{ is optimal})\right)/Vol(S)$. For the sum of values criterion this relative volume is:

$$Vol\left(S(V^1(x^{(i)}) + V^2(x^{(i)}) \geq V^1(x^{(j)}) + V^2(x^{(j)}), \forall j \neq i)\right)/Vol(S).$$

The domains approach can also be used interactively in a similar way as the central parameters approach. If a negotiator does not accept one alternative it is possible to redefine S by introducing a new constraint to eliminate this alternative and calculate again the domain volumes.

As the mediator should be informed of the relative volumes of many different results, we suggest to use Monte-Carlo simulation to approximate volumes. Exact methods for computing volumes also exist (see, e.g., [19, 20, 28]), but are more computationally demanding and can be used only for one question at a time.

The simulation generates a large number n_{iter} of random instances of the two negotiators's parameter values satisfying all the constraints. For each vector, all properties $\tilde{C}$ of interest can be evaluated simultaneously, i.e. which alternatives are better than the reference $x^{(r)}$ for both of the negotiators, which alternatives are efficient, and which alternative is the best one according to each mediation criterion (as it is also possible to analyze several mediation criteria simultaneously). Considering the results for all these instances, it is possible to indicate, for each alternative x, the proportion of instances where each of the above mentioned conditions was verified for that particular alternative. In order to allow for (relative) volumes to be interpreted as probabilities, a uniform distribution of parameter vectors must be used for the simulations. In the experiments described in the next section, scaling weights were generated according to an uniform distribution using the process described in [2].

When we interpret the volumes as probabilities, it might also be interesting to compute conditional probabilities, e.g., the probability that an alternative is optimal for a mediation criterion under the condition that it is better than the reference point and efficient. Such conditional probabilities can also be obtained from the simulation by recording the number of instances in which both conditions are fulfilled and calculating

$$p(\tilde{C}|\tilde{D}) = p(\tilde{C}\&\tilde{D})/p(\tilde{D}) \tag{17}$$

where $\tilde{C}$ and $\tilde{D}$ represent the two conditions to be analyzed.

3.4 Comparison of the three approaches

In the last subsections, we proposed three approaches to help a mediator who observes a dance of the packages with incomplete information. Table 1 summarizes the different intervention possibilities for a mediator, which altogether can constitute a process with three steps.

Concept / Approach	**Extreme**	**Central**	**Domains**
Step 1: Comparison to reference point in value space	**1a.** identify alternatives which are surely better than the reference point for both negotiators **1b.** eliminate alternatives which are surely worse than the reference point for one negotiator (or both)	**1'.** identify alternatives which are better than the reference point for both negotiators, assuming the central parameter values (LP solution or centroid)	**1".** find the probability that each alternative is better than the reference point for both negotiators
Step 2: Pareto Efficiency	**2a.** identify alternatives which are surely Pareto efficient **2b.** eliminate alternatives which are surely not Pareto efficient	**2'.** identify alternatives which are Pareto efficient, assuming the central parameter values (LP solution or centroid)	**2".** find the probability that each alternative is Pareto efficient
Step 3: Optimal alternative using mediation criterion	**3a.** identify alternatives which are surely optimal for the mediation criterion (for all parameter vectors) **3b.** identify alternatives which might be optimal for the mediation criterion (at least for one parameter vector)	**3'.** identify alternatives which are optimal for the mediation criterion, assuming the central parameter values (LP solution or centroid)	**3".** find the probability that each alternative is optimal for the mediation criterion

Table 1: Summary of the different analyses that can be performed

The rows of Table 1 express the complementary concerns of a mediator. A mediator would like to propose an alternative likely to be accepted, hence better than the reference point for both negotiators. Three approaches can then be used:

- The extreme parameters approach will compute exactly which alternatives are surely better (i.e. better for all parameter vectors) than the reference point for both negotiators simultaneously (analysis 1a). However, it might turn out that no such alternatives exist. The same approach can be used to determine exactly which alternatives are possibly better (i.e. better for at least one parameter vector) than the reference point for both negotiators simultaneously, allowing to eliminate all those alternatives that cannot achieve this condition (analysis 1b). The advantage of this approach is that the conditions of being surely better or surely worse are exactly determined. Its disadvantage is that it requires solving n_{alt} LPs for each analysis (where n_{alt} represents the number of possible alternatives, i.e. the number of different combinations of issue levels).

- The central parameters approach will find which alternatives are better than the reference point for both negotiators simultaneously, assuming a central parameter vector (analysis 1'). This vector can be computed solving a LP maximizing the minimum slack or computing a centroid. An advantage of this approach is that only two LPs need to be solved (one for each negotiator), or only two centroids have to be computed. Another advantage is that it provides a clear-cut partition of the alternatives set: those better than the reference, and those worse than the reference. Its disadvantage is that the central vector is just an approximation, which can be a rather coarse one if information is scarce. Hence, there is no guarantee that the supposedly better alternatives will really have higher value than the reference point, for both negotiators.

- The domains approach will compute the probabilities that each alternative is better than the reference point for both negotiators simultaneously (analysis 1"). Some alternatives will have a very low probability and might be discarded from further analysis. The advantage of this approach is that it is straightforward to compute the probabilities using Monte-Carlo simulation, with a confidence level as high as needed (it is a matter of how many iterations are used for the simulation). Its disadvantage is that the result will not be as clear cut as in the previous case.

Concerning the second row of Table 1, a mediator would like to propose an alternative on the Pareto efficient frontier. Again, the same three approaches can then be used:

- The extreme parameters approach will compute exactly which alternatives are surely efficient (analysis 2a), or surely not efficient (analysis 2b), allowing to eliminate the latter. Its disadvantage is that it requires solving $2 * n_{alt} * (n_{alt} - 1)$ LPs for each analysis (this is a worst case bound, because as soon as an alternative is deemed surely inefficient it is no longer necessary to include it in the subsequent comparisons). The advantage of this approach is that the conditions of being surely efficient or surely inefficient are exactly determined.

- The central parameters approach will find which alternatives are efficient, assuming a central parameter vector computed solving a LP or computing a centroid (analysis 2'). An advantage of this approach is that only two LPs or centroid computations are needed. It also provides a clear-cut partition of the alternatives set between efficient and inefficient ones. Its disadvantage is that the central vector is just an approximation, and therefore there is no guarantee that the partition is perfectly accurate.

- The domains approach will compute the probabilities that each alternative is efficient (analysis 2"). Alternatives with a very low probability of being efficient might be discarded from further analysis. The advantage and the disadvantage are the same as for analysis 1".

Finally, concerning the third row of Table 1, a mediator could have the requirement of proposing, among efficient alternatives, an alternative that would be optimal according to a mediation criterion such as the sum of the values (pursuing maximal enlargement of the pie) or the minimum PoP (pursuing equity):

- The extreme parameters approach will compute exactly if there exists an optimal alternative for all parameter values (analysis 3a), which is not very likely, and will determine exactly which alternatives might be optimal (analysis 3b). As an advantage, the conditions of being surely optimal or potentially optimal are exactly determined. However, the most likely result will be finding a set of potentially optimal alternatives, with no way of knowing which one is better. Furthermore, the use of linear programming is limited to the case where the mediation criterion is the sum of values.

- The central parameters approach will find the optimal alternative, assuming a central parameter vector computed solving a LP or computing a centroid (analysis 3'). An advantage of this approach is that a single alternative will be identified (except for rare cases where multiple alternative optima might exist). A second advantage is that it is not computationally difficult to use the minimum PoP as a mediation criterion, or a criterion involving products. However, as the central vector is just an approximation, there is no guarantee that the supposedly optimal alternative is indeed optimal.

- The domains approach will compute the probabilities of each alternative being optimal (analysis 3"). Alternatives with a very low probability of being efficient might be discarded from further analysis. The advantage of this approach is that the probabilities can be straightforward to compute using Monte-Carlo simulation, and the probabilities will allow to identify the most promising alternatives among the set of potentially optimal ones, even if the the mediation criterion is the minimum PoP or a criterion involving products. A clear cut result is not very likely, but we still have more information than in analysis 3b, because we know the probabilities.

All the presented approaches provide interesting and diversified results. The choice of the approaches to be used depends on the mediator's goals, but we suggest to use different approaches complementarily in sequence.

The mediator can start by addressing the concern of finding alternatives that are considered by both negotiators to be an improvement relatively to the reference point. The mediator can reduce the set of potentially interesting alternatives, eliminating those surely worse than the reference for one of the negotiators (analysis 1b), or those with a very low probability of being better than the reference for both negotiators (analysis 1"). In a second step, the mediator can eliminate alternatives that are surely inefficient (analysis 2b) or very unlikely to be efficient (analysis 2"), the latter approach being preferable if there remain many alternatives. To detect efficiency, each of the alternatives that was not eliminated in the previous step would be compared with the original set of alternatives. Finally, to choose a single alternative to propose to both negotiators, the mediator can use analysis 3' to propose the optimal solution using central parameters, or use analysis 3" to pick the alternative that is optimal with highest probability. For this purpose, more than one mediation criterion can be considered.

As referred previously, this type of overall approach can be used in an interactive way. If the alternative proposed by the mediator is accepted, the negotiation ends successfully. However, it can happen that the alternative proposed by the mediator is not accepted by one negotiator (or both). If a negotiator k states that a proposed solution $x^{(p)}$ is not better than the reference $x^{(r)}$, then the constraint $V^k(x^{(p)}) < V^k(x^{(r)})$ can be added to the definition of (W_k, M_k). Then, the analysis can be repeated to find a new solution. It might happen that negotiator was insincere (acting strategically) when saying $x^{(p)}$ is not better than the reference, hoping a better alternative is proposed by the mediator. However, since the mediator will incorporate the constraint $V^k(x^{(p)}) < V^k(x^{(r)})$ that is contrary to this negotiator's preferences, possibly excluding the negotiator's true vector of preference parameters from the feasible region, it might happen that the following alternatives proposed will not be as good as the previous one. For this reason, manipulation attempts might eventually lead to miss an opportunity to improve the first proposal by the mediator.

Considering the three types of approaches corresponding to the three columns of Table 1, it is to be noted that running a Monte-Carlo simulation can be sufficient to implement them all. Indeed, Monte-Carlo simulation can be used to compute probabilities (relative domains) for different conditions with high accuracy. For an accuracy that can be as high as needed (the large the number of iterations, the higher is the accuracy), the conditions with probability equal to 0 or to 1 will correspond to conditions that "surely" do not hold or that to conditions that "surely" hold, respectively. This corresponds to the extreme parameters approach. On the other hand, averaging the instances of random input values for each parameter generated generated in the simulation over the number of iterations will define an accurate approximation of the centroid of (W_1, M_1) and (W_2, M_2). Using these centroids corresponds to the central parameters approach.

4 Illustrative Example

4.1 Nelson vs Amstore case

4.1.1 Introduction

To illustrate the approaches presented in last section let us consider an example introduced by Raiffa [23]. In this example, there are two parties in a negotiation: Nelson and Amstore. Nelson has a construction firm and he negotiates with a retail chain (Amstore) to build a new store for them. There are three issues: price (10, 10.5, 11, 11.5 or 12 thousand dollars), design (basic or improved) and time (20, 21, 22, 23, 24, 25 or 26 days). Combining these issue levels yields a total of 70 possible alternatives (see Table

2). For Nelson, price and time are maximizing issues and design is a minimizing one, while for Amstore it is the opposite. Therefore, the preferred alternative for Amstore is alternative 1 ($x^{(1)}$), whereas the preferred alternative for Nelson is alternative 70 ($x^{(70)}$).

Alternative	Price	Design	Time	Alternative	Price	Design	Time	Alternative	Price	Design	Time
1	10	Improved	20	25	10.5	Basic	23	49	11.5	Improved	26
2	10	Improved	21	26	10.5	Basic	24	50	11.5	Basic	20
3	10	Improved	22	27	10.5	Basic	25	51	11.5	Basic	21
4	10	Improved	23	28	10.5	Basic	26	52	11.5	Basic	22
5	10	Improved	24	29	11	Improved	20	53	11.5	Basic	23
6	10	Improved	25	30	11	Improved	21	54	11.5	Basic	24
7	10	Improved	26	31	11	Improved	22	55	11.5	Basic	25
8	10	Basic	20	32	11	Improved	23	56	11.5	Basic	26
9	10	Basic	21	33	11	Improved	24	57	12	Improved	20
10	10	Basic	22	34	11	Improved	25	58	12	Improved	21
11	10	Basic	23	35	11	Improved	26	59	12	Improved	22
12	10	Basic	24	36	11	Basic	20	60	12	Improved	23
13	10	Basic	25	37	11	Basic	21	61	12	Improved	24
14	10	Basic	26	38	11	Basic	22	62	12	Improved	25
15	10.5	Improved	20	39	11	Basic	23	63	12	Improved	26
16	10.5	Improved	21	40	11	Basic	24	64	12	Basic	20
17	10.5	Improved	22	41	11	Basic	25	65	12	Basic	21
18	10.5	Improved	23	42	11	Basic	26	66	12	Basic	22
19	10.5	Improved	24	43	11.5	Improved	20	67	12	Basic	23
20	10.5	Improved	25	44	11.5	Improved	21	68	12	Basic	24
21	10.5	Improved	26	45	11.5	Improved	22	69	12	Basic	25
22	10.5	Basic	20	46	11.5	Improved	23	70	12	Basic	26
23	10.5	Basic	21	47	11.5	Improved	24				
24	10.5	Basic	22	48	11.5	Improved	25				

Table 2: Alternatives.

4.1.2 Complete information

Suppose that we have complete information, i.e., the mediator knows the value of each level and knows the weights of the three issues, for both negotiators (see Table 3). Thus, the value of each alternative is known. To simplify we multiply the value of each issue level, for each party, by 100. So, the global value of each alternative, for each party, is a value between 0 and 100. Alternatives 1-5, 9, 15-19, 22-27, 37-42, 51-56 and 67-70 are efficient.

		Parcial value		Value*100	
Issue	**Level**	**Nelson**	**Amstore**	**Nelson**	**Amstore**
Price	10	0	1	0	70
	10,5	0,41667	0,85714	25	60
	11	0,66667	0,64286	40	45
	11,5	0,91667	0,35714	55	25
	12	1	0	60	0
	w_1	***0,6***	***0,7***		
Design	Basic	1	0	20	0
	Improved	0	1	0	10
	w_2	***0,2***	***0,1***		
Time	20	0	1	0	20
	21	0,4	0,95	8	19
	22	0,6	0,9	12	18
	23	0,75	0,8	15	16
	24	0,85	0,6	17	12
	25	0,95	0,35	19	7
	26	1	0	20	0
	w_3	***0,2***	***0,2***		

Table 3: Complete Information.

Suppose that Amstore begins proposing alternative 1, Nelson answers proposing alternative 70, Amstore proposes alternative 8, and so on. The process ends when Nelson proposes alternative 44, which Amstore accepts (see Table 4). Alternatives 27-28, 37-42, 45-56 and 58-70 are better than the compromise solution for Nelson and alternatives 1-40 and 43 are better than the compromise solution for Amstore. Considering the complete information in Table 3 was available, we would know that alternative 44 is inefficient, and that alternatives better for both parties than the compromise alternative exist: alternatives 27-28 and 37-40 (see Figure 2). Among these alternatives, only alternative 28 is not efficient.

In this illustration we consider the mediation criteria maximizing the sum of the values and maximizing the minimal PoP (relatively to the compromise alternative). Between the efficient alternatives that

Amstore	Nelson
$x^{(1)}$	$x^{(70)}$
$x^{(8)}$	$x^{(67)}$
$x^{(17)}$	$x^{(53)}$
$x^{(20)}$	$x^{(42)}$
$x^{(31)}$	$x^{(46)}$
$x^{(32)}$	$x^{(44)}$

Table 4: Sequence of proposals.

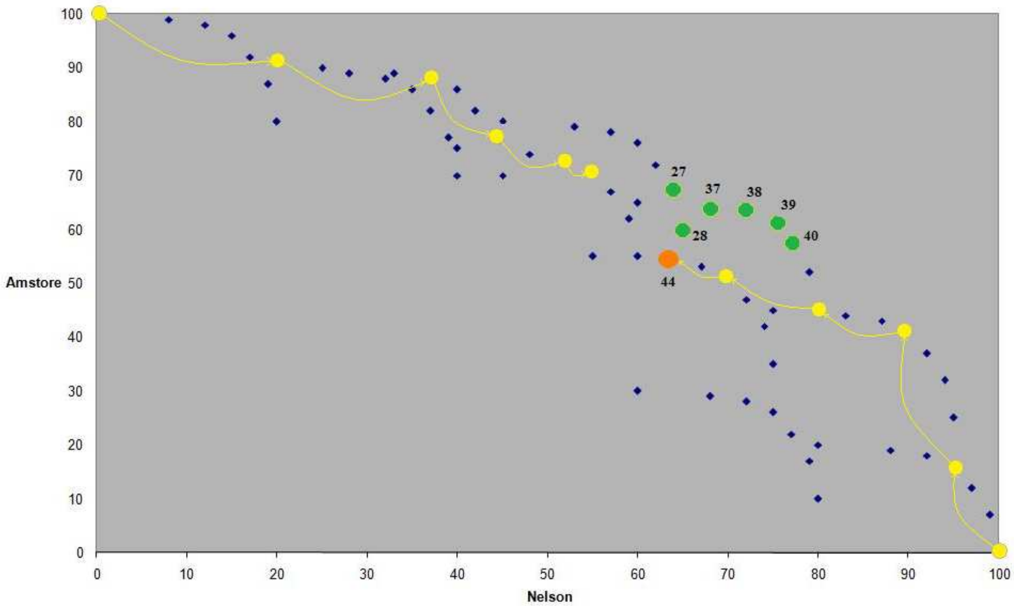

Figure 2: Dance of the Packages.

are better for both parties than the compromise solution, depending on the objective, it is possible to recommend different alternatives. If the objective is to maximize the sum of the values, alternative 39 is the best one, with a sum of the values equal to 136 (alternative 25 has also sum of the values equal to 136, but it is not better for both parties than the compromise solution). If the objective is to maximize the minimal PoP (relatively to the compromise alternative), alternative 38 is the best one, with a minimal PoP of 0.64. To determine the minimal PoP we used the maximum value attained by each party considering that the chosen alternative needs to be better for both parties than the compromise solution. In Table 5 it is possible to see the value (value for Nelson, value for Amstore, sum of the values and minimal PoP) of the alternatives that are better for both parties than the compromise solution. Given these results, and depending on his judgement, a mediator would propose alternative 38 or alternative 39 to the negotiating parties.

	Nelson	Amstore	Sum of the values	PoP
$x^{(27)}$	60	67	131	0.07
$x^{(28)}$	65	60	125	0.14
$x^{(37)}$	68	64	132	0.36
$x^{(38)}$	72	63	135	**0.64**
$x^{(39)}$	75	61	**136**	0.54
$x^{(40)}$	77	57	134	0.23

Table 5: Values of the alternatives that are better for both parties than the compromise solution.

4.1.3 Incomplete Information

Let us now consider the analysis of a mediator who knows the available alternatives (Table 2) and witnesses the sequence of proposals (Table 4), but does not know the exact parameter values of each negotiator displayed in Table 3. We will consider the three levels of uncertainty presented in subsection 2.1: only weights uncertain with known values (level 1), weights uncertain with value function assumed to be linear (level 2), and uncertain weights and value functions (level 3).

In the second case, we approximate the issue values considering that the value functions of each negotiator are linear. For Nelson we have, for price: $V^N_{price}(10) = 0$ and $V^N_{price}(12) = 1$, thus we can conclude that $V^N_{price}(x) = 0.5x - 5$ (so $V^N_{price}(10.5) = 0.25$, $V^N_{price}(11) = 0.5$ and $V^N_{price}(11.5) = 0.75$). For time: $V^N_{time}(20) = 0$ and $V^N_{time}(26) = 1$, thus we can conclude that $V^N_{time}(x) = \frac{1}{6}x - \frac{10}{3}$ (so $V^N_{time}(21) = \frac{1}{6}$, $V^N_{time}(22) = \frac{1}{3}$, $V^N_{time}(23) = 0.5$, $V^N_{time}(24) = \frac{2}{3}$ and $V^N_{time}(25) = \frac{5}{6}$). Similarly for Amstore we have $V^A_{price}(x) = -0.5x + 6$ and $V^A_{time}(x) = -\frac{1}{6}x + \frac{13}{3}$.

In the third case, we consider that value functions lie within two limits, taking the linear function as a lower limit and the concave function $v(x) = a + b(-e^{-c*x})$ as the upper limit. For example, for Nelson and relatively to price:

$$\begin{cases} V^N_{price}(10) = 0 \\ V^N_{price}(12) = 1 \end{cases} \Leftrightarrow \begin{cases} a + b(-e^{-c*10}) = 0 \\ a + b(-e^{-c*12}) = 1 \end{cases} \Leftrightarrow \begin{cases} a = \frac{1}{e^{-10c} - e^{-12c}} * e^{-10c} \\ b = \frac{1}{e^{-10c} - e^{-12c}} \end{cases}$$

We decided to use $c = 5$ for the upper limit so that the real values belong to the interval with the linear function as lower limit and the concave function as upper limit, but without considering that value functions could be extremely concave. Figure 3 shows the true value functions of Nelson and Amstore and also the inferior and superior limits.

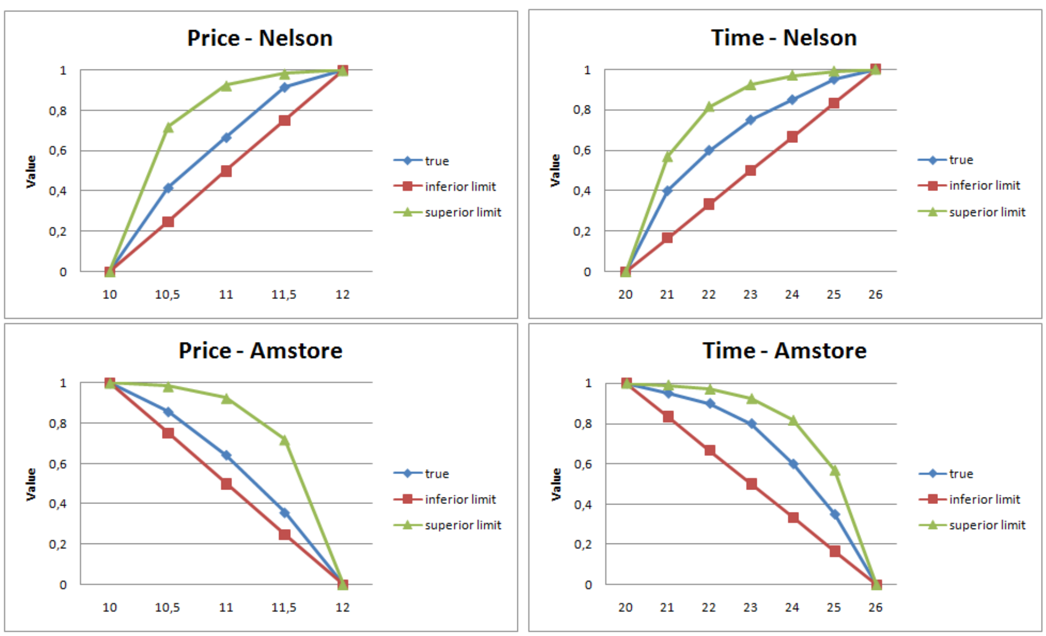

Figure 3: Value functions.

Assuming that a mediator would know the possible alternatives (Table 2), as well as the offers that had been made leading to a compromise (Table 4), he could set additional constraints. To start with, we know which are the best and the worst alternatives for Nelson and Amstore:

$V^N_{price}(10) + V^N_{design}(\text{improved}) + V^N_{time}(20) = 0$,
$V^N_{price}(12) + V^N_{design}(\text{basic}) + V^N_{time}(26) = 100$,

$V^A_{price}(12) + V^A_{design}(\text{basic}) + V^A_{time}(26) = 0$,

$V^A_{price}(10) + V^A_{design}(\text{improved}) + V^A_{time}(20) = 100.$

For the case with unknown values, we assume preferences are monotonous, such that [1]:

$V^N_{price}(10) < V^N_{price}(10.5) < V^N_{price}(11) < V^N_{price}(11.5) < V^N_{price}(12),$
$V^N_{design}(\text{basic}) > V^N_{design}(\text{improved}),$
$V^N_{time}(20) < V^N_{time}(21) < V^N_{time}(22) < V^N_{time}(23) < V^N_{time}(24) < V^N_{time}(25) < V^N_{time}(26),$

$V^A_{price}(10) > V^A_{price}(10.5) > V^A_{price}(11) > V^A_{price}(11.5) > V^A_{price}(12),$
$V^A_{design}(\text{basic}) < V^A_{design}(\text{improved}),$
$V^A_{time}(20) > V^A_{time}(21) > V^A_{time}(22) > V^A_{time}(23) > V^A_{time}(24) > V^A_{time}(25) > V^A_{time}(26).$

We assume that each proposal presented by a negotiator has for him lower value than the last proposal presented earlier by himself. We obtain the following restrictions of type (6):

$$V^N(x^{(70)}) > V^N(x^{(67)}) > V^N(x^{(53)}) > V^N(x^{(42)}) > V^N(x^{(46)}) > V^N(x^{(44)}), \tag{18}$$

$$V^A(x^{(1)}) > V^A(x^{(8)}) > V^A(x^{(17)}) > V^A(x^{(20)}) > V^A(x^{(31)}) > V^A(x^{(32)}). \tag{19}$$

We also assume that a negotiator prefers his proposals to the proposals of the opponent. For Nelson and Amstore we have [2]:

$$\begin{array}{l} V^N(x^{(44)}) > V^N(x^{(32)}), \\ V^N(x^{(44)}) > V^N(x^{(31)}), \\ V^N(x^{(44)}) > V^N(x^{(20)}), \\ V^N(x^{(44)}) > V^N(x^{(17)}), \\ V^N(x^{(44)}) > V^N(x^{(8)}), \\ V^N(x^{(44)}) > V^N(x^{(1)}), \end{array} \tag{20}$$

$$\begin{array}{l} V^A(x^{(32)}) > V^A(x^{(44)}), \\ V^A(x^{(32)}) > V^A(x^{(46)}), \\ V^A(x^{(32)}) > V^A(x^{(42)}), \\ V^A(x^{(32)}) > V^A(x^{(53)}), \\ V^A(x^{(32)}) > V^A(x^{(67)}), \\ V^A(x^{(32)}) > V^A(x^{(70)}). \end{array} \tag{21}$$

To illustrate the use of explicit preference information, suppose that Nelson says that alternative 25 is as good as alternative 36, and alternative 39 is as good as alternative 50. Let us also assume that Amstore says that alternatives 50 and 42 are equivalent, and the same occurs for alternatives 36 and 37. We obtain the following restrictions of type (7):

$$|V^N(x^{(25)}) - V^N(x^{(36)})| \leq \epsilon, \tag{22}$$

$$|V^N(x^{(39)}) - V^N(x^{(50)})| \leq \epsilon, \tag{23}$$

$$|V^A(x^{(50)}) - V^A(x^{(42)})| \leq \epsilon, \tag{24}$$

[1] To ensure that the inequality is strictly we consider that, for example, $V^N_{price}(10) < V^N_{price}(10.5) \Rightarrow V^N_{price}(10) + 1 \leq V^N_{price}(10.5)$.

[2] Also for constraints (18)-(20) we consider that the inequality is strict.

$$|V^A(x^{(36)}) - V^A(x^{(37)})| < \epsilon. \quad (25)$$

For these constraints we used $\epsilon = 10$.

Our objective is, with the incomplete information indicated above, to try to suggest to the negotiators one efficient alternative, better for both parties than the compromise alternative. The results obtained by following the three approaches we proposed are presented in the next sections.

4.2 Extreme Parameters Approach

Remember that in this approach it is necessary to solve $2 \times (70 - 1)$ LPs of the type (14) to check if there are alternatives that are surely better than the compromise solution and to eliminate alternatives that are surely worse. The same type of LP can be used to compare any other pair of alternatives to check Pareto efficiency. Considering the set of the remaining alternatives - set P (after discarding alternatives surely worse than the compromise solution and alternatives surely not belonging to the Pareto frontier) it is possible to solve LPs of the type (15) to check if each alternative in P is potentially optimal according to the criterion of maximizing the sum of the values. We considered the three types of incomplete information, the constraints regarding the sequence of proposals (constraints of type (6)) and the constraints (22)-(25) regarding the equivalence of alternatives (constraints of type (7)). In Tables 6 and 7 it is possible to see the alternatives that are surely worse and alternatives that are surely better than the compromise solution, respectively, considering constraints of type (6) and constraints of type (6)+(7), and the three types of incomplete information.

Considering that only the weights are uncertain (values known), and considering the constraints related to the sequence of proposals, the set of the non eliminated alternatives consists of of 24 alternatives (alternatives 9-14, 22-28, 33-42 and 50), about 34% of the initial number of alternatives. For example, $\max\{V^N(x^{(1)}) - V^N(x^{(44)})\} = -42.1436$, so, alternative $x^{(1)}$ can be eliminated, because it is surely worse than the compromise for Nelson. Considering constraints related to the sequence of proposals and the equivalence of alternatives the results are very similar. The set of the non eliminated alternatives is made of 21 alternatives (alternatives 12-14, 22-28, 33-42 and 50). Thus, the additional information obtained from explicit indifference statements has very little effect in this case.

So far, we have identified alternatives which are potentially (i.e. for at least one parameter vector) better than the compromise. We now analyze which alternatives are surely (i.e. for all parameter vectors) better than the compromise (Table 5). We can see that alternatives 27, 37 and 38 are pointed as being surely better than the compromise solution for Amstore, alternative 40 is pointed as being surely better than the compromise solution for Nelson, alternative 28 is not pointed as being surely better than the compromise solution for none of the parties and alternative 39 is pointed as being surely better than the compromise solution for both parties. As alternative 39 is pointed as being better than the compromise for both parties, this alternative can be a good suggestion. However, it should be noted that in realty there are more alternatives which are better than the compromise for both parties. But since there are still some parameter vector considered possible for which these alternatives appear to be worse than the compromise, they are not indicated as being surely better than the compromise.

In a second step, we also want to compare all the pairs of the remaining alternatives, to check Pareto efficiency. For example, considering constraints of type (6)+(7) it is also possible to eliminate alternative 13 (e.g. $\max\{V^k(x^{(13)} - V^k(x^{(23)})\} < 0$, k=1,2), alternative 14 (e.g. $\max\{V^k(x^{(14)}) - V^k(x^{(23)})\} < 0$, k=1,2), alternative 34 (e.g. $\max\{V^k(x^{(34)}) - V^k(x^{(38)})\} < 0$, k=1,2), alternative 35 (e.g. $\max\{V^k(x^{(35)}) - V^k(x^{(39)})\} < 0$, k=1,2), alternative 36 (e.g. $\max\{V^k(x^{(36)}) - V^k(x^{(26)})\} < 0$, k=1,2) and alternative 50 (e.g. $\max\{V^k(x^{(50)}) - V^k(x^{(40)})\} < 0$, k=1,2). None of the alternatives is surely efficient. Between the 15 non-eliminated alternatives of the set P (alternatives 12, 22-28, 33 and 37-42), all can maximize the sum of the values, because for all $x^{(i)} \in P$, $\max \delta > 0$.

The results obtained considering known values are very similar to the ones obtained considering unknown values (with the main difference that in the two cases where we considered unknown values, alternative 39 is not pointed as being better than the compromise solution for both parties), we will not comment the second results in detail. We will only refer that, for example, considering weights uncertain and value functions with unknown parameters and using the two types of constraints at the same time, comparing all the pairs of the remaining alternatives, to check Pareto efficiency, did not give any additional information, because all can be Pareto efficient and all can maximize the sum of the values.

	Values known				Linear value function				Value function with unknown parameter			
	Constraints (6)		Constraints (6)+(7)		Constraints (6)		Constraints (6)+(7)		Constraints (6)		Constraints (6)+(7)	
Alternatives	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore
1	X		X		X		X		X		X	
2	X		X		X		X		X		X	
3	X		X		X		X		X		X	
4	X		X		X		X		X		X	
5	X		X		X		X		X		X	
6	X		X		X		X		X		X	
7	X		X		X		X		X		X	
8	X		X		X		X		X		X	
9			X				X					
10			X									
11			X									
12												
13												
14												
15	X		X		X		X		X		X	
16	X		X		X		X		X		X	
17	X		X		X		X		X		X	
18	X		X		X		X		X		X	
19	X		X		X		X		X		X	
20	X		X		X		X		X		X	
21	X		X		X		X					
22												
23												
24												
25												
26												
27												
28												
29	X		X		X		X		X		X	
30	X		X		X		X		X		X	
31	X		X		X		X		X		X	
32	X		X		X		X		X		X	
33					X		X					
34												
35												
36												
37												
38												
39												
40												
41												
42												
43	X		X		X		X		X		X	
44												
45		X		X		X		X		X		X
46		X		X		X		X		X		X
47		X		X		X		X		X		X
48		X		X		X		X		X		X
49		X		X		X		X		X		X
50												
51		X		X		X		X		X		X
52		X		X		X		X		X		X
53		X		X		X		X		X		X
54		X		X		X		X		X		X
55		X		X		X		X		X		X
56		X		X		X		X		X		X
57	X	X	X	X		X		X				X
58		X		X		X		X		X		X
59		X		X		X		X		X		X
60		X		X		X		X		X		X
61		X		X		X		X		X		X
62		X		X		X		X		X		X
63		X		X		X		X		X		X
64		X		X		X		X				X
65		X		X		X		X		X		X
66		X		X		X		X		X		X
67		X		X		X		X		X		X
68		X		X		X		X		X		X
69		X		X		X		X		X		X
70		X		X		X		X		X		X

Table 6: Alternatives that are surely worse than the compromise.

4.2.1 Some comments

In this example changing the type of incomplete information did not affected too much the results. Furthermore, considering the constraints of type (6) and those of type (7), the results are not also very different from the ones obtained considering only constraints of type (6). In all the cases there are many alternatives that can be eliminated because they are surely worse than the compromise one. Only in the case in which the values were considered known we were able to find one alternative that is surely better than the compromise alternative for both parties. Note that, even considering known values and using the two types of constraints, comparing all the pairs of the remaining alternatives, to check Pareto efficiency, did not help to eliminate a lot of alternatives (considering weights uncertain and value functions with unknown parameters it was not possible to eliminate any alternative). Comparing the alternatives which can be better than the compromise and can be Pareto efficient, and trying to see which of these alternatives can maximize the sum of the values, did not give any additional information. In

	Values known				Linear value function				Value function with unknown parameter			
	Constraints (6)		Constraints (6)+(7)		Constraints (6)		Constraints (6)+(7)		Constraints (6)		Constraints (6)+(7)	
Alternatives	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore	Nelson	Amstore
1		X		X		X		X		X		X
2		X		X		X		X		X		X
3		X		X		X		X		X		X
4		X		X		X		X		X		X
5		X		X		X		X		X		X
6		X		X		X		X		X		X
7		X		X		X		X				
8		X		X		X		X		X		X
9		X		X		X		X		X		X
10		X		X		X		X				X
11		X		X		X		X				X
12		X		X		X		X				
13		X		X		X		X				
14		X		X				X				
15		X		X		X		X		X		X
16		X		X		X		X		X		X
17		X		X		X		X		X		X
18		X		X		X		X		X		X
19		X		X		X		X		X		X
20		X		X		X		X		X		X
21		X		X		X		X				
22		X		X		X		X				
23		X		X		X		X				
24		X		X		X		X				
25		X		X								
26		X		X								
27		X		X								
28												
29		X		X		X		X		X		X
30		X		X		X		X		X		X
31		X		X		X		X		X		X
32		X		X		X		X		X		X
33		X		X		X		X				
34		X		X								
35												
36		X		X								
37		X		X								
38		X		X								
39	X	X	X	X			X					
40	X		X		X		X					
41	X		X		X		X					
42	X		X		X		X		X		X	
43		X		X		X		X		X		X
44												
45	X		X		X		X		X		X	
46	X		X		X		X		X		X	
47	X		X		X		X		X		X	
48	X		X		X		X		X		X	
49	X		X		X		X		X		X	
50							X					
51	X		X		X		X		X		X	
52	X		X		X		X		X		X	
53	X		X		X		X		X		X	
54	X		X		X		X		X		X	
55	X		X		X		X		X		X	
56	X		X		X		X		X		X	
57					X		X					
58	X		X		X		X		X		X	
59	X		X		X		X		X		X	
60	X		X		X		X		X		X	
61	X		X		X		X		X		X	
62	X		X		X		X		X		X	
63	X		X		X		X		X		X	
64			X		X		X					
65	X		X		X		X		X		X	
66	X		X		X		X		X		X	
67	X		X		X		X		X		X	
68	X		X		X		X		X		X	
69	X		X		X		X		X		X	
70	X		X		X		X		X		X	

Table 7: Alternatives that are surely better than the compromise.

this case, in the end of the analysis, the mediator would have a set with 15 alternatives to recommend to the negotiators. Note that this is the case in which more information is required from negotiators. Even in this case, comparing the non-eliminated alternatives to check Pareto efficiency and seeing what alternatives can be optimal according the criterion maximizing the sum of the values did not give a lot of additional information, so we did not repeat this analysis for the other cases. As presented in Section 3.4 this approach can be a good starting point, eliminating alternatives surely worse than the compromise for both parties. To recommend one alternative, between the remaining alternatives, it is better to use one of the other two approaches.

4.3 Central Parameters Approach

The central parameters approach consists in inferring a representative combination of the parameters values, from the sets of admissible values, and use these values to find alternatives that are better for both parties than the compromise solution, are efficient and optimal according to a mediation criterion. In this subsection we consider two cases of central parameter vectors: vectors obtained solving a linear problem of the type (16) maximizing the minimal slack (therefore called "LP" or "inferred" vectors) and vectors obtained approximating the centroids (therefore called "centroid" vectors). To approximate the centroids we averaged 5000 parameter vectors generated in a Monte-Carlo simulation (the same vectors will be used in Section 4.4).

In Table 8 it is possible to compare the LP weights and the centroid weights, for Nelson and for Amstore, considering the three types of incomplete information and considering the constraints of type (6) and (6)+(7). To simplify the comparison of parameter vectors, we also calculated the Euclidean distance between the true and the LP weights and between the true and the centroid weights.

Table 9 shows the results considering the Central Parameters Approach, using the LP parameter values and the centroid parameter values. In this table it is possible to see: which alternatives are better than the compromise solution for Nelson, for Amstore, and for both; which of the alternatives that are better for both parties than the compromise solution are not efficient; which alternative maximizes the sum of the values and which alternative maximizes the minimal PoP. The presented results were obtained without using the process interactively.

			Values Known							
			Constraints (6)				Constraints (6) + (7)			
	TRUE weights		Nelson		Amstore		Nelson		Amstore	
	Nelson	Amstore	inferred	centroid	inferred	centroid	inferred	centroid	inferred	centroid
w_1	0.6	0.7	0.6316	0.5261	0.7339	0.7176	0.6316	0.5482	0.7339	0.7191
w_2	0.2	0.1	0.1579	0.2093	0.0242	0.0532	0.1579	0.1992	0.0242	0.0553
w_3	0.2	0.2	0.2105	0.2645	0.2419	0.2292	0.2105	0.2526	0.2419	0.2256
	Euclidean Distance		0.0537	0.0985	0.0930	0.0579	0.0537	0.0738	0.0930	0.0549

			Linear value function							
			Constraints (6)				Constraints (6) + (7)			
	TRUE weights		Nelson		Amstore		Nelson		Amstore	
	Nelson	Amstore	inferred	centroid	inferred	centroid	inferred	centroid	inferred	centroid
w_1	0.6	0.7	0.6667	0.6150	0.6957	0.7305	0.6471	0.5824	0.6957	0.7119
w_2	0.2	0.1	0.1334	0.2194	0.0435	0.0971	0.1588	0.2347	0.0435	0.1073
w_3	0.2	0.2	0.2000	0.1656	0.2609	0.1724	0.1941	0.1829	0.2609	0.1808
	Euclidean Distance		0.0943	0.0422	0.0832	0.0412	0.0628	0.0425	0.0832	0.0237

			Value function with unknown parameter							
			Constraints (6)				Constraints (6) + (7)			
	TRUE weights		Nelson		Amstore		Nelson		Amstore	
	Nelson	Amstore	inferred	centroid	inferred	centroid	inferred	centroid	inferred	centroid
w_1	0.6	0.7	0.5769	0.5486	0.7018	0.6891	0.5769	0.5671	0.5769	0.7289
w_2	0.2	0.1	0.1923	0.2059	0.0861	0.0613	0.1923	0.1932	0.1923	0.0630
w_3	0.2	0.2	0.2308	0.2455	0.2121	0.2495	0.2308	0.2389	0.2308	0.2082
	Euclidean Distance		0.0392	0.0689	0.0185	0.0638	0.0392	0.0514	0.0392	0.0477

Table 8: Comparison of the LP weights and the centroid weights.

4.3.1 Only weights uncertain, values known

In the first case of central parameters vectors, we have inferred the weights solving the LP (16) for each party. For example, for Nelson we have $V^N(x^{(67)}) < V^N(x^{(70)}) \Leftrightarrow w_1^N * 100 + w_2^N * 100 + 0.75 * w_3^N * 100 < 0.6667 * w_1^N * 100 + w_2^N * 100 + w_3^N * 100$. So, one of the constraints we include in the linear problem is: $(1 - 0.6667) * w_1^N + 0 * w_2^N + (0.75 - 1) * w_3^N + \Delta^N \leq 0$ (remember that $x^{(67)}$=(12, basic, 23) and $x^{(70)}$=(11, basic, 26)).

	Values Known			
	Constraints (6)		Constraints (6) + (7)	
Alternatives:	inferred	centroid	inferred	centroid
Better for Nelson than the compromise	38-42, 45-56, 58-70	25-28, 34-35, 37-42, 45-56, 58-70	38-42, 45-56, 58-70	25-28, 34-35, 37-42, 45-56, 58-70
Better for Amstore than the compromise	1-34, 36-41, 43	1-34, 36-41, 43	1-34, 36-41, 43	1-34, 36-41, 43
Better for both than the compromise	38-41	25-28, 34 37-41	38-41	25-28, 34 37-41
Not Efficient	none	34 and 37	none	34 and 37
Best considering the sum	39	25	39	25
Best considering the PoP	39	38	39	38

	Linear value function			
	Constraints (6)		Constraints (6) + (7)	
Alternatives:	inferred	centroid	inferred	centroid
Better for Nelson than the compromise	39-42, 45-70	27-28, 37-42, 45-56, 58-70	35, 38-42, 45-70	25-28, 36-42, 45-56, 58-70
Better for Amstore than the compromise	1-33, 36-39, 43, 50	1-41, 43	1-33, 36-39, 43, 50	1-41, 43
Better for both than the compromise	39 and 50	27-28, 37-41	38-39, 50	25-28, 36-40
Not Efficient	none	28	none	25
Best considering the sum	50	39	50	27
Best considering the PoP	39	38	39	37

	Value function with unknown parameter			
	Constraints (6)		Constraints (6) + (7)	
Alternatives:	inferred	centroid	inferred	centroid
Better for Nelson than the compromise	24-28, 37-42, 45-70	25-28, 35, 37-42, 45-56, 58-70	24-28, 37-42, 45-70	25-28, 35, 37-42, 45-56, 58-70
Better for Amstore than the compromise	1-34, 36-38, 43	1-34, 36-40, 43	1-34, 36-38, 43	1-41, 43
Better for both than the compromise	24-28, 37-38	27-28, 37-40	24-28, 37-38	25-28, 35, 37-41
Not Efficient	37 and 38	37	37 and 38	35, 37 and 38
Best considering the sum	24	25	24	25 and 26
Best considering the PoP	26	27	26	28

Table 9: Results of the Central Parameters Approach.

Considering the constraints regarding the sequence of proposals, for Nelson a better approximation is obtained considering the LP weights, while for Amstore it is the opposite and the centroid weights provide better results. Using the LP weights, between the alternatives that are efficient and better for both parties than the compromise solution, alternative 39 is the one which maximizes the sum of the values and also the minimal PoP. Remember that alternative 39 is in reality better for both parties than the compromise solution and it is efficient. Alternative 39 is the one which in reality maximizes the sum of the values and alternative 38 is the one which in reality maximizes the minimal PoP. Alternatives 27, 28 and 37 that in reality are better for both parties than the compromise solution did not appear here as being better for both parties; the opposite happened with alternative 41. Using the centroid weights, alternative 25 is the one which maximizes the sum of the values, and alternative 38 is the alternative which maximizes the minimal PoP. Note that alternative 25 is, in reality, one of the two alternatives that maximizes the sum of the values, but it is not better for Nelson than the compromise solution, so probably, Nelson would not accept this alternative. If this happens it is necessary to include the constraint $V^N(x^{(25)}) < V^N(x^{(44)})$ and compute a new centroid. With this new centroid, alternatives 28, 34 and 38-41 are pointed as being better for both parties than the compromise solution. Between these, the alternative 39 is pointed as maximizing the sum of the values and the minimal PoP.

Considering the constraints regarding the sequence of proposals and the equivalence of alternatives, and considering the LP weights, the results are exactly the same. It is not surprising that we get the same results, since we still use the true values here and the constraints regarding the equivalence of alternatives are based on alternatives which have the same value (based on the true weights, which are close to the weights we get from the model, and the true values), and we did not include the slack, Δ^K, in the equivalence constraints. Considering the centroid weights the results are exactly the same although the weights are somewhat different. The Euclidean distances are smaller considering both types of constraints.

4.3.2 Weights uncertain, value function assumed to be linear

Except for Nelson, and using the LP weights, the Euclidean distances are smaller considering linear value functions than considering the true values as knwon.

Considering the constraints regarding the sequence of proposals, and using the LP weights, alternative 50 is the one which maximizes the sum of the values, and alternative 39 is the alternative which maximizes the minimal PoP. Note that alternative 50 is worse for Amstore than the compromise solution and it is not efficient, so, probably, Amstore would not accept this alternative. If this happens, it is necessary to include the constraint $V^A(x^{(50)}) < V^A(x^{(44)})$, and infer again the weights for Amstore. With these new weights alternative 39 is the only one pointed as being better for both parties than the compromise solution. Using the centroid weights, alternative 39 is the one which maximizes the sum of the values, and alternative 38 is the alternative which maximizes the minimal PoP.

Considering the constraints regarding the sequence of proposals and the equivalence of alternatives, and using the LP weights, alternative 50 is the one which maximizes the sum of the values, and alternative 39 is the alternative which maximizes the minimal PoP. Using the centroid weights,

alternative 27 is the one which maximizes the sum of the values, and alternative 37 is the alternative which maximizes the minimal PoP.

4.3.3 Weights uncertain, value function with unknown parameter

In Figure 4 it is possible to compare the true values, the LP and the centroid values, for Nelson and Amstore, considering the constraints regarding the sequence of proposals. As it is possible to see in the figures, the centroid value functions are closer to the true value function than the LP value functions (this is just opposite as for the weights). Note that the shape of the centroid values function did not really depend on the value of c. Considering another value for c (e.g., $c = 20$) the centroid value functions are very similar to those we obtained considering $c = 5$.

Considering the constraints regarding the sequence of alternatives, and using the LP weights and values, alternative 24 is the one which maximizes the sum of the values, and alternative 26 is the alternative which maximizes the minimal PoP. Note that Nelson should not accept these alternatives because they have inferior value for him than the compromise solution. It is necessary to solve again the linear problem and infer new weights and new values, including the two additional constraints. Inferring the weights with the two new constraints, alternative 38 is the only one pointed as being better for both parties than the compromise solution. Using the centroid weights and values, alternative 25 maximizes the sum of the values, and alternative 27 maximizes the minimal PoP.

Considering the constraints of type (6)+(7) the values are very similar to the ones obtained considering only constraints of type (6). Once again, the centroid values are a better approximation than the LP values. Considering the LP weights and values, the results are equal to the ones obtained considering only the constraints regarding the sequence of alternatives. Using the centroid weights and values, alternatives 25 and 26 are the ones which maximizes the sum of the values, and alternative 28 is the alternative which maximizes the minimal PoP. If the objective is to maximize the sum of the values, probably Nelson would not accept neither alternative 25 nor alternative 26, because these alternatives are worse for him than the compromise solution. If this happens it is possible to include two new constraints and generate again the weights and values. The alternatives pointed as being better than the compromise solution are alternatives 37-41. Between these alternatives, alternatives 38, 39 and 40 maximize the sum of the values and alternative 39 maximizes the minimal PoP.

4.3.4 Some comments

Considering the values are known, it would always possible to advice negotiators with an efficient alternative, better for both parties than the compromise solution. However, in some cases these alternatives would not be found directly, but it would be necessary to use the process interactively and include new constraints in the problem after first proposals are rejected by the negotiators. In such cases the information which is initially available is not sufficient and additional information must be acquired by interaction with the negotiators. The results considering the centroid weights and the LP weights were not very different, and using constraints of type (6) or of type (6)+(7) gave exactly the same results. Using the three types of incomplete information, and inferring the vectors solving the LPs, the set of

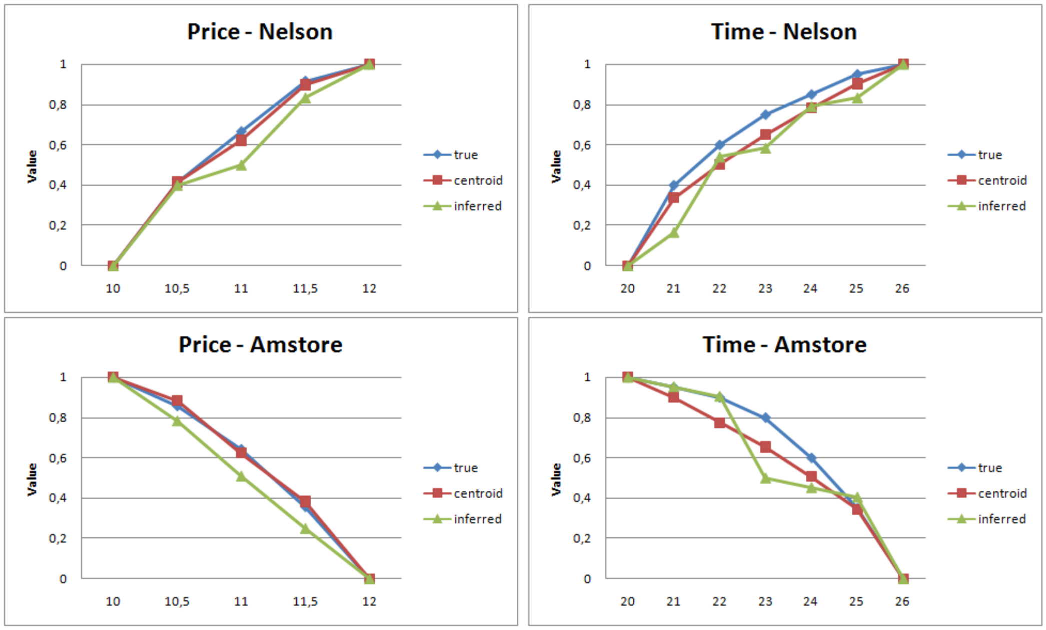

Figure 4: Comparison of the true values, the LP values and the centroid values - value function with unknown parameter (constraints of type (6)).

alternatives recommended as being better for both parties than the compromise solution did not include all the alternatives that are in reality better for both parties. This never happened considering the centroid vectors. The results are very similar considering only constraints of type (6) and constraints of type (6)+(7). Both solving the LPs and approximating the centroids, there are cases where alternatives that in reality are efficient were pointed as not being efficient. The opposite also happened. Centroid values seem to be closer to the true parameters than the LP values. This is not really surprising, since the max-min LP optimizes only the smallest slack, ignoring the slacks for the remaining constraints.

4.4 Domains Approach

Our implementation of the domains approach is based on a simulation, in which one generates a large number of random instances of the two negotiators's parameter values, satisfying all the constraints. In our experiments, we generated 5000 such parameter vectors. For each vector, we determined which alternatives were better than the compromise solution for both negotiators, which alternatives were efficient and which alternative was optimal according to each mediation criterion. We considered the three types of incomplete information and the constraints of type (6) and (7). In all the cases, we started eliminating not only alternatives for which the probability of being better than the compromise solution for both parties was equal to 0, but also other alternatives for which this probability was lower than 0.05.

In Table 10 it is possible to see the probability of each alternative being better than the compromise one for Nelson, for Amstore and for both parties. Since parameter vectors generated for the two parties are independent random variables, the probability that an alternative is better than the compromise for both parties is equal to the product of the probabilities for the two parties. Table 11 shows the probability of each alternative being efficient. For this analysis, we consider an alternative as not efficient if it is dominated by any other alternative. One could also calculate efficiency considering only dominance by non-eliminated alternatives, but we consider the fact that an alternative is dominated by any other alternative as important, even when the dominating alternative is eliminated because e.g. it is worth less than the compromise to one party.

Tables 12 and 13 refer to the probability of each alternative being the best according the criterion sum of the values and to the probability of each alternative being the best according the criterion minimal PoP, respectively. In all these tables we present the results obtained considering constraints of type (6) and results obtained considering constraints of type (6) + (7), and the three types of incomplete information.

	Values known						Linear value function						Value function with unknown parameter					
	Constraints (6)			Constraints (6)+(7)			Constraints (6)			Constraints (6)+(7)			Constraints (6)			Constraints (6)+(7)		
Alternatives	Both	Nelson	Amstore	Both	Nelson	Amstore	Both	Nelson	Amstore	Both	Nelson	Amstore	Both	Nelson	Amstore	Both	Nelson	Amstore
9													0.0502	0.0502	1			
10	0.0840	0.0840	1				0.0612	0.0612	1				0.0866	0.0866	1			
11	0.1460	0.1460	1				0.1024	0.1024	1				0.1194	0.1194	1			
12	0.1808	0.1808	1				0.1444	0.1444	1	0.0998	0.0998	1	0.1536	0.1536	0.9994			
13	0.2168	0.2168	1				0.1886	0.1886	1	0.1900	0.1900	1	0.2002	0.2006	0.9984	0.0702	0.0702	1
14	0.2360	0.2360	1				0.2408	0.2412	0.9988	0.2824	0.2824	1	0.2044	0.2396	0.8630	0.1108	0.1112	0.9970
21													0.0502	0.0570	0.8698			
22	0.1820	0.1820	1				0.2258	0.2258	1	0.1030	0.1030	1	0.1906	0.1906	1			
23	0.3198	0.3198	1	0.1310	0.1310	1	0.2820	0.2820	1	0.2146	0.2146	1	0.3320	0.3320	1	0.1786	0.1788	0.9998
24	0.4428	0.4428	1	0.3514	0.3514	1	0.3338	0.3338	1	0.3490	0.3490	1	0.4316	0.4316	0.9982	0.3564	0.3566	0.9996
25	0.5548	0.5548	1	0.5856	0.5856	1	0.3862	0.3898	0.9889	0.6428	0.6752	0.9972	0.5462	0.5486	0.9944	0.5512	0.5518	0.9988
26	0.6580	0.6580	1	0.7038	0.7038	1	0.4348	0.4604	0.9458	0.6428	0.6752	0.9504	0.6376	0.6500	0.9834	0.7154	0.7206	0.9920
27	0.7678	0.7678	1	0.8268	0.8268	1	0.4746	0.5272	0.8946	0.7176	0.8106	0.8856	0.6954	0.7376	0.9462	0.8192	0.8344	0.9812
28	0.8272	0.8292	0.9980	0.8552	0.8552	1	0.5126	0.6142	0.8300	0.7410	0.9132	0.8134	0.5272	0.7928	0.6664	0.8058	0.9004	0.8934
33	0.3358	0.3358	1	0.2208	0.2208	1							0.1246	0.1518	0.8140	0.1204	0.1224	0.9842
34	0.7172	0.7172	1	0.4534	0.4534	1	0.1900	0.2360	0.8150	0.2458	0.2816	0.8832	0.2618	0.4556	0.5752	0.3624	0.4082	0.8800
35	0.2396	0.8758	0.2746	0.2254	0.6408	0.3480	0.2748	0.4246	0.6614	0.3532	0.5620	0.6352	0.1774	0.6722	0.2648	0.2762	0.6290	0.4428
36	0.3512	0.3512	1	0.2422	0.2422	1	0.5492	0.6064	0.9024	0.7476	0.8350	0.8974	0.3774	0.3962	0.9612	0.3272	0.3370	0.9710
37	0.7076	0.7076	1	0.8506	0.8506	1	0.5848	0.6954	0.8342	0.7984	0.9816	0.8136	0.5946	0.6510	0.9192	0.6448	0.6790	0.9478
38	0.9870	0.9870	1	1	1	1	0.5792	0.7716	0.7488	0.7026	1	0.7026	0.6552	0.8130	0.8092	0.7872	0.8742	0.9018
39	1	1	1	1	1	1	0.5848	0.8938	0.6532	0.5630	1	0.5630	0.6158	0.9286	0.6640	0.8032	0.9730	0.8256
40	1	1	1	1	1	1	0.5190	1	0.5190	0.4206	1	0.4206	0.4696	0.9876	0.4750	0.7026	0.9968	0.7044
41	0.6992	1	0.6992	0.6490	1	0.6490	0.3978	1	0.3978	0.2828	1	0.2828	0.3142	1	0.3142	0.5064	1	0.5056
42							0.3090	1	0.3090	0.1854	1	0.1854	0.1470	1	0.1470	0.1806	1	0.1806
50							0.0910	0.9928	0.0920	0.0902	1	0.0902	0.1046	0.8544	0.1196	0.0670	0.9490	0.0690

Table 10: Probability of each alternative being better than the compromise one (not displaying the alternatives for which the probability of being better, for both parties, than the compromise solution is lower than 0.05).

	Values known		Linear value function		Value function with unknown parameter	
Alternatives	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)
9					0.6202	
10	0.9294		0.8446		0.4838	
11	0.8788		0.8510		0.3624	
12	0.6178		0.7530	0.8974	0.2596	
13			0.7040	0.8012	0.1708	0.2484
14			0.6644	0.7694	0.0570	0.0928
21					0.0308	
22	0.3216		0.5596	0.4736	0.6372	
23	0.9128	0.9972	0.6210	0.5696	0.8662	0.8636
24	0.9954	1	0.7164	0.7392	0.9028	0.9280
25	0.9998	1	0.8510	0.9714	0.9156	0.9638
26	0.9964	1	0.7530	0.8974	0.9006	0.9550
27	0.9580	0.9724	0.7040	0.8012	0.8432	0.9140
28	0.0044		0.6644	0.7694	0.5116	0.6636
33	0.0682	0.0100			0.0718	0.0294
34	0.0036		0.1584	0.0334	0.0640	0.0296
35			0.1708	0.0320	0.0178	0.0062
36			0.5596	0.4736	0.1610	0.1440
37	0.3108	0.5592	0.6210	0.5696	0.3372	0.3168
38	0.9954	1	0.7164	0.7392	0.4574	0.4484
39	0.9998	1	0.8510	0.9714	0.5874	0.6256
40	1	1	0.7530	0.8974	0.6310	0.7890
41	0.9998	1	0.7040	0.8012	0.6072	0.8728
42			0.6646	0.7696	0.3408	0.5834
50			0.5596	0.4736	0.2002	0.1122

Table 11: Probability of each alternative being efficient.

4.4.1 Only weights uncertain, values known

Considering the constraints regarding the sequence of proposals, and eliminating the alternatives with probability of being better than the compromise solution for both parties lower than 0.05, we reduce the set of the alternatives to 21 alternatives (30% of the initial number of alternatives). The set of the non eliminated alternatives is very similar to the one obtained using the extreme parameters approach. Remember that the alternatives that in reality are better for both parties than the compromise solution

	Values known		Linear value function		Value function with unknown parameter	
Alternatives	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)
9			0.0125		0.0011	
10			0.0225		0.0004	
11			0.0209	0.0113	0.0032	
12			0.0109	0.0226	0.0045	
13			0.0109	0.0241	0.0055	0.0077
14			0.1276	0.2028	0.0008	0.0028
21					0.0019	
22			0.0956	0.0134	0.0078	
23			0.0216	0.0283	0.0759	0.0238
24			0.0153	0.0442	0.0880	0.0598
25	0.5548	0.5856	0.0158	0.0371	0.1474	0.1200
26	0.0528	0.0330	0.0131	0.0409	0.1695	0.2240
27	0.0038		0.0125	0.0329	0.1703	0.2583
28			0.1752	0.3001	0.0250	0.0413
33	0.0024				0.0081	0.0014
34			0.0140	0.0027	0.0146	0.0020
35			0.0614	0.0031	0.0028	
36			0.1637	0.1674	0.0066	0.0039
37			0.0254	0.0363	0.0562	0.0563
38			0.0238	0.0096	0.0566	0.0572
39	0.3862	0.3814	0.0251	0.0031	0.0626	0.0620
40			0.0151	0.0029	0.0454	0.0448
41			0.0122	0.0027	0.0342	0.0336
42			0.0872	0.0122	0.0023	0.0004
50			0.0178	0.0021	0.0091	0.0006

Table 12: Probability of each alternative being the best according the criterion sum of the values.

	Values known		Linear value function		Value function with unknown parameter	
Alternatives	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)
13					0.0002	
21					0.0011	
25			0.0192	0.0248	0.0206	0.0041
26	0.0094		0.0159	0.0420	0.0882	0.0163
27	0.1064	0.0084	0.0371	0.0648	0.2482	0.1392
28			0.1523	0.2927	0.1700	0.3067
33					0.0070	0.0002
34	0.0004		0.0179	0.0040	0.0136	0.0028
35			0.0567	0.0021	0.0042	0.0008
36			0.0833	0.0652	0.0144	0.0110
37	0.0028		0.1311	0.1170	0.0748	0.0556
38	0.2258	0.1890	0.1441	0.2013	0.1067	0.1260
39	0.4672	0.6902	0.1406	0.1539	0.1028	0.1647
40	0.1880	0.1124	0.1007	0.0312	0.799	0.1160
41			0.0632	0.0011	0.0542	0.0560
42			0.0326		0.0115	0.0006
50			0.0058		0.0025	

Table 13: Probability of each alternative being the best according the criterion minimal PoP (compromise as reference).

are alternatives 27, 28, 37, 38, 39 and 40. All these alternatives exhibit high probabilities (higher than 0.70) of being better than the compromise solution for both parties. Considering the criterion maximizing the sum of the values, the two alternatives with highest probabilities are alternatives 25 (with probability equal to 0.5548) and 39 (with probability equal to 0.3862). Remember that these two alternatives are the ones that in reality maximize the sum of the values. For alternative 39 the probability of being better than the compromise solution is equal to 1 and the probability of being efficient is equal to 0.9998. The probability of alternative 25 being efficient is also equal to 0.9998, but only in 55.48% of the cases this alternative is better for both parties than the compromise solution. In this case we consider that alternative 39 is the best option for the mediator to propose. Considering the criterion maximizing the minimal PoP, alternative 39 has the highest probability (0.4672). Also considering the criterion maximizing the minimal PoP, the mediator should suggest alternative 39. Note, however, that the alternative that in reality maximizes the minimal PoP is alternative 38 which presents here a probability, of maximizing the minimal PoP, equal to 0.2258. The real minimal PoP of alternative 39 is equal to 0.54 (vs. 0.64 for alternative 38).

Considering constraints of type (6) + (7), we retain about 22% of the alternatives (15 alternatives). All the alternatives that, in reality, are better for both parties than the compromise solution present high probabilities (above 0.82). The alternatives that in reality are better for both parties than the compromise solution are the ones with highest probabilities, which is an improvement to the results obtained when considering only the sequence of proposals. Considering the criterion maximizing the sum of the values, the two alternatives with highest probabilities are also alternatives 25 (with probability equal to 0.5856) and 39 (with probability equal to 0.3814).

Considering the criterion maximizing the minimal PoP, alternative 39 is the one with highest probability (0.6902). Alternative 38 (the really best alternative considering this criterion) presents a probability equal to 0.1890. Once again we consider that given these results the mediator should suggest alternative 39.

4.4.2 Weights uncertain, value function assumed to be linear

Considering only the sequence of proposals it is possible to eliminate 58 alternatives (i.e, about 32% of the initial number of alternatives are retained). The alternatives that in reality are better for both parties than the compromise solution do not present high probabilities (between 0.4746 and 0.5848), however these alternatives are the ones which present highest probabilities. Considering the criterion maximizing the sum of the values, there are 23 alternatives that can be optimal considering this criterion (but only 6 of them with probability higher than 0.05). The three alternatives with highest probabilities are alternative 14 (with probability equal to 0.1276), alternative 28 (with probability equal to 0.1752) and alternative 36 (with probability equal to 0.1637). Only in 24.08% of the cases alternative 14 is better than the compromise solution for both parties. The corresponding percentage is equal to 51.26% for alternative 28 and equal to 54.92% for alternative 36. The probabilities of alternatives 14, 28 and 36 being efficient are equal to 0.6644, 0.6644 and 0.5596, respectively. Note that, in reality, alternatives 14 and 36 are not better for both parties than the compromise solution, neither efficient. Alternative 28 is better for both parties than the compromise solution but it is not efficient. If the mediator suggest these alternatives, the negotiators will probably chose alternative 28 because it is the only one that is better for both parties than the compromise solution.

Alternatives that appeared as being the best according to the sum of the values, and considering known values, have probabilities lower than 0.05. Alternatives pointed here as the ones with highest probabilities of being the best ones have probability equal to zero considering known values. This happens because when we approximate the values using linear value functions we are using an inferior value in all the cases except for the highest and lowest value in each issue. In Table 14 it is possible to see the difference between the true value of each issue level and the linear value. Alternatives that have extreme levels are the ones for which the loss of value caused by the linear approximation is smallest. The opposite happens with alternatives with levels that are in the middle of the scale. Alternatives 25 (10.5; basic; 23) and 39 (11; basic; 23) lose a lot with this approximation, which does not happen for alternatives 14 (10; basic; 26), 28 (10.5; basic; 26) and 36 (11; basic; 20). Alternative 44 (the compromise) is one of the alternatives that loses with the linear approximation. Hence, using the linear approximation, alternative 44 looks worse than what it really is.

Thus, the linear approximation of value functions creates a systematic bias in favor of alternatives having extreme values in at least some attributes. However, it should be noted that this bias is a result of our assumption that value functions are concave. For convex value functions, the bias would work in the opposite direction and favor alternatives having values in the middle of the possible range.

Considering the criterion maximizing the minimal PoP, there are 14 alternatives that can maximize the minimal PoP (8 of them with probability superior than 0.05). The four alternatives with highest probabilities are alternative 28 (with probability equal to 0.1523), alternative 37 (with probability equal to 0.1311), alternative 38 (with probability equal to 0.1441) and alternative 39 (with probability equal to 0.1406). The percentage of cases where these alternatives are better for both parties than the compromise solution are 51.26%, 58.48%, 57.92% and 58.48%, respectively. The probability of alternatives 28, 37, 38 and 39 being efficient are equal to 0.7040, 0.6644, 0.7164 and 0.8510, respectively. All these alternatives are, in reality, better for both parties than the compromise solution and only alternative 28 is not efficient. Between these four alternatives it is not easy to known what alternative the mediator should suggest.

The results are not very different considering only constraints of type (6) and considering constraints of type (6) + (7). So we will not comment on the second results.

		Difference of values	
Issue	Level	Nelson	Amstore
Price	10	0	0
	10,5	0,1667	0,1071
	11	0,1667	0,1429
	11,5	0,1667	0,1071
	12	0	0
Design	Basic	0	0
	Improved	0	0
Time	20	0	0
	21	0,2333	0,1167
	22	0,2667	0,2333
	23	0,25	0,3
	24	0,1833	0,2667
	25	0,1167	0,1833
	26	0	0

Table 14: Difference between the true values and the linear values.

4.4.3 Weights uncertain, value function with unknown parameter

Considering only the constraints regarding the sequence of proposals, the set of the alternatives is reduced to 25 alternatives (compared to 21 alternatives considering the values to be known). The probabilities of the alternatives that in reality are better for both parties than the compromise solution, vary between 0.4696 and 0.6954. There are alternatives that are pointed as being better than the compromise solution for both parties that did not appear considering known values, but these alternatives have low probabilities. Between the 25 alternatives that have positive probability of being the best according the sum of the values (8 of them with probability superior than 0.05), alternatives 25, 26 and 27 have the highest probabilities (alternative 25 with probability equal to 0.1474, alternative 26 with probability equal to 0.1695 and alternative 27 with probability equal to 0.1703). The probability of these alternatives being better than the compromise solution for both parties are equal to 0.5462, 0.6376 and 0.6954, respectively. The probabilities of being efficient are 0.9156, 0.9006 and 0.8432, respectively. Remember that all these alternatives have positive probability of maximizing the sum of the values considering known values, but the probabilities are now quite lower. Alternatives 25, 26 and 27 are efficient but only alternative 27 is better for both parties than the compromise solution. If the mediator suggest these three alternatives to the negotiators they probably will agree and chose alternative 27 because it is the only one that is better for both parties than the compromise solution. Note that alternative 39, pointed as being the best considering known values, has in this case a very low probability equal to 0.0626. There are 17 alternatives that can maximize the minimal PoP (8 of them with probability superior than 0.05). Between these alternatives, alternative 27 is the one with highest probability (probability equal to 0.2482). This alternative also has a positive probability considering known values.

Considering the constraints regarding the sequence of proposals and the equivalence of alternatives, there are 19 alternatives with positive probability of being better for both parties than the compromise solution (comparing with the 15 considering known values). The probabilities of alternatives 27-28, 37-40 being better for both parties than the compromise solution, vary between 0.6448 and 0.8192. If the objective is to maximize the sum of the values, there are 18 alternatives with positive probability (7 of them with probability superior than 0.05). The alternatives with highest probabilities are alternatives 26 and 27 (with probabilities equal to 0.2240 and 0.2583, respectively). Note that alternative 27 has probability equal to zero of maximizing the sum of the values, considering known values. Alternative 26 has probability equal to 0.7154 of being better for both parties than the compromise solution, and probability equal to 0.9550 of being efficient. The corresponding probabilities for alternative 27 are 0.8192 and 0.9140, respectively. Alternative 28 is the one which has highest probability of maximizing the minimal PoP (with probability equal to 0.3067), between the 14 alternatives with positive probability (6 of them with probability superior than 0.05). Alternative 28 has probability equal to zero considering known values. The probability of alternative 28 being better for both parties than the compromise solution is 0.8058 and the probability of being efficient is equal to 0.6636. In reality this alternative is better for both parties than the compromise solution but it is not efficient. Alternatives 27, 38, 39 and 40 also appear with high probabilities of maximizing the minimal PoP.

4.4.4 Some comments

Considering the domains approach the results are very different according to the type of the (incomplete) information used. Considering known values, it was always possible to advise negotiators with an efficient alternative better for both parties than the compromise solution. Considering an approximation of the values using linear value functions, the suggestions obtained are different from the true optimal alternatives, mainly considering the criterion maximizing the sum of the values. Considering value functions with unknown parameters, the results are better than the ones obtained using linear value functions but not so good (obviously) as the ones obtained considering known values. In this last case, it may happen that the suggested alternative is not efficient, or that the set of the suggested alternatives contain alternatives that are not better for both parties than the compromise solution. Considering the two types of constraints at the same time the results are improved (e.g., minimizing the number of suggested alternatives), but not by much. In all the cases, the sets of the retained alternatives are very similar considering the extreme parameters approach and considering the domains approach. However, in the domains approach it is always possible to recommend one alternative, which does not happen considering the extreme parameters approach. If one, or both, negotiators do not agree with the suggested alternative it is possible to include new constraints and to calculate again the probabilities.

4.5 Comparison of the recommendations provided by the different approaches

In Tables 15 and 16 it is possible to compare the results of the three approaches considering known values and considering value functions with unknown parameters, respectively, and using the constraints of type (6)+(7). We do not present the results using linear value functions because, as we have already explained, the results were not very promising. We choose to present the results using the two type of constraints at the same time because the results are better than the ones obtained considering only constraints regarding the sequence of alternatives. The results presented in the tables are the ones which we obtain without using the approaches interactively.

Considering known values, in all the approaches, the mediator should recommend alternative 39. With the central parameters approach and the centroid weights it is possible to come to this conclusion after using the approach interactively. Alternative 39 is in reality better for both parties than the compromise solution and it is efficient.

Considering value functions with unknown parameters, and using the extreme parameters approach it is only possible to recommend a set of 25 alternatives that can be better for both parties than the compromise solution, can be efficient and can maximize the sum of the values. Considering the central approach and solving the LPs it is possible to recommend alternative 38 (after using the approach interactively), both in the criterion maximizing the sum of the values and the criterion maximizing the minimal PoP. This alternative is in reality better for both parties than the compromise solution and efficient. Using the centroid values and weights if the objective is to maximize the sum of the values the mediator should suggest alternatives 38, 39 and 40, if the objective is to maximize the minimal PoP the mediator should suggest alternative 39 (after using the approach interactively). All these alternatives are efficient and better for both parties than the compromise solution. Considering the domains approach, if the objective is to maximize the sum of the values the mediator should suggest alternatives 26 and 27 (both alternatives are efficient, but alternative 26 is not better for Nelson than the reservation level), if the objective is to maximize the minimal PoP the mediator should suggest alternative 28 (but in reality this alternative is not efficient).

4.6 No compromise is reached (yet)

If a compromise has not been reached, the minimum defined by the last offers from each negotiator will be considered as the reference level. In this subsection we illustrate how the approaches can be used before reaching a compromise, using the domains approach and considering known values.

We consider that the last two offers on the table are alternative 32, for Amstore and alternative 46, for Nelson. The reference point used instead of the compromise is a fictitious alternative yielding $V^N(x^{(32)})$, for Nelson and $V^A(x^{(46)})$, for Amstore. Alternatives that in reality are better for both parties than the reference point are alternatives: 24-28, 33-41 and 44-45. In Table 17 it is possible to see the sum of the values and the minimal PoP of the alternatives better than the reference point for both parties.

Concept / Approach	Extreme	Central	Domains
Comparison to reference point in value space	1. identify alternatives which are surely better than the compromise for both negotiators: **alternative 39** 2. eliminate alternatives which are surely worse than the compromise for one negotiator: **alternatives 1-11, 15-21, 29-32, 43-49, 51-70**	3. identify alternatives which are better than the compromise for both negotiators: **LP - alternatives 38-41** **centroid - alternatives 25-28, 34, 37-41**	4. Identify alternatives with probability superior than 50% of being better than the compromise solution for both parties: **alternatives 25-28, 37-41**
Pareto Efficiency	5. identify alternatives that are surely Pareto efficient: **none** 6. identify alternatives that are surely not Pareto efficient: **alternatives 13, 14, 34, 35, 36, 50**	7. identify alternatives which are Pareto efficient: **LP - alternatives 38-41** **centroid - alternatives 25-28, 38-41**	5. identify the probability that each alternative is Pareto efficient: **alternatives 25-26, 38 - 41 (1)** **alternative 27 (0.9724), alternative 37 (0.5592)**
Optimal alternative using mediation criterion	9. identify alternatives wich are surely optimal for the sum of the values: **none** 10. identify alternatives wich might be optimal for the sum of the values: **alternatives 12, 22-28, 33, 37-42**	8. identify alternatives which are optimal for the mediation criterion: **LP sum, LP PoP - alternative 39** **centroid sum - alternative 25** **centroid PoP - alternative 38**	6. find the probability that each alternative is optimal for the mediation criterion: **sum - alternatives 25 (0.5856) and 39 (0.3814)** **PoP - alternative 39 (0.6902)**
Mediator Recommendation	**alternative 39**	**LP sum, LP PoP - alternative 39** **centroid sum - alternative 25** **centroid PoP - alternative 38**	**sum and PoP - alternative 39**

Table 15: Comparison of the three approaches considering known values and using constraints of type (6)+(7).

Concept / Approach	Extreme	Central	Domains
Comparison to reference point in value space	1. identify alternatives which are surely better than the compromise for both negotiators: **none** 2. eliminate alternatives which are surely worse than the compromise for one negotiator: **alternatives 1-8, 15-20, 29-32, 43, 45-49, 51-70**	3. identify alternatives which are better than the compromise for both negotiators: **LP - alternatives 24-28, 37-38** **centroid - alternatives 25-28, 35, 37-41**	4. Identify alternatives with probability superior than 50% of being better than the compromise solution for both parties: **alternatives 25-28, 37-41**
Pareto Efficiency	5. identify alternatives that are surely Pareto efficient: **none** 6. identify alternatives that are surely not Pareto efficient: **none**	7. identify alternatives which are Pareto efficient: **LP - alternatives 24-28** **centroid - alternatives 25-28, 39-41**	5. identify the probability that each alternative is Pareto efficient: **alternatives 25 (0.9638), 26 (0.9550), 27 (0.9140), 28 (0.6636), 39 (0.6256), 40 (0.7890), 41 (0.8728)**
Optimal alternative using mediation criterion	9. identify alternatives wich are surely optimal for the sum of the values: **none** 10. identify alternatives wich might be optimal for the sum of the values: **alternatives 9-14, 21-28, 33-42, 50**	8. identify alternatives which are optimal for the mediation criterion: **LP sum - alternative 24, LP PoP - alternative 26** **centroid sum - alternative 25 and alternative 26** **centroid PoP - alternative 28**	6. find the probability that each alternative is optimal for the mediation criterion: **sum - alternatives 26 (0.2240) and 27 (0.2583)** **PoP - alternative 28 (0.3067)**
Mediator Recommendation	**Not conclusive**	**LP sum - alternative 24, LP PoP - alternative 26** **centroid sum - alternative 25 or alternative 26** **centroid PoP - alternative 28**	**sum - alternative 26 or alternative 27** **PoP - alternative 28**

Table 16: Comparison of the three approaches considering the value functions with unknown parameters and using constraints of type (6)+(7).

Considering that the sequence of proposals is the same considered previously (see Table 18). We obtain the following restrictions of type (6):

$$V^N(x^{(70)}) > V^N(x^{(67)}) > V^N(x^{(53)}) > V^N(x^{(42)}) > V^N(x^{(32)}), \tag{26}$$

$$V^A(x^{(1)}) > V^A(x^{(8)}) > V^A(x^{(17)}) > V^A(x^{(20)}) > V^A(x^{(31)}) > V^A(x^{(46)}). \tag{27}$$

We also have:

$$V^N(x^{(32)}) > V^N(x^{(31)}),$$

	Sum of the values	PoP
$x^{(24)}$	135	0.08
$x^{(25)}$	**136**	0.21
$x^{(26)}$	134	0.29
$x^{(27)}$	131	0.38
$x^{(28)}$	125	0.33
$x^{(33)}$	124	0.08
$x^{(34)}$	121	0.17
$x^{(35)}$	115	0.15
$x^{(36)}$	125	0.21
$x^{(37)}$	132	**0.48**
$x^{(38)}$	135	0.44
$x^{(39)}$	**136**	0.37
$x^{(40)}$	134	0.22
$x^{(41)}$	131	0.04
$x^{(44)}$	117	0.11
$x^{(45)}$	120	0.07

Table 17: Values of the alternatives that are better for both parties than the reference point, considering the different criteria.

Amstore	Nelson
$x^{(1)}$	$x^{(70)}$
$x^{(8)}$	$x^{(67)}$
$x^{(17)}$	$x^{(53)}$
$x^{(20)}$	$x^{(42)}$
$x^{(31)}$	$x^{(46)}$
$x^{(32)}$	

Table 18: Sequence of proposals - compromise not yet reached.

$$\begin{aligned} V^N(x^{(32)}) &> V^N(x^{(20)}), \\ V^N(x^{(32)}) &> V^N(x^{(17)}), \\ V^N(x^{(32)}) &> V^N(x^{(8)}), \\ V^N(x^{(32)}) &> V^N(x^{(1)}), \end{aligned} \tag{28}$$

$$\begin{aligned} V^A(x^{(46)}) &> V^A(x^{(42)}), \\ V^A(x^{(46)}) &> V^A(x^{(53)}), \\ V^A(x^{(46)}) &> V^A(x^{(67)}), \\ V^A(x^{(46)}) &> V^A(x^{(70)}). \end{aligned} \tag{29}$$

Table 19 refers to the probability of each alternative being better than the reference point for Nelson, for Amstore and for both parties, the probability of each alternative being the best according the criterion sum of the values, the probability of each alternative being the best according the criterion minimal PoP and the probability of each alternative being efficient. As it is possible to see, the results before the compromise are very interesting, as they are similar to the corresponding results after the compromise. This indicates that our methods are not very sensitive to the choice of a reference level.

5 Conclusions

In this paper, we have looked at three ways to deal with incomplete information in the context of negotiations:

1. the extreme parameters approach,
2. the central parameters approach, and

Alternatives	Better than the reference point						Efficient		Maximize the sum of the values		Maximize the minimal PoP	
	Constraints (6)			Constraints (6)+(7)								
	Both	Nelson	Amstore	Both	Nelson	Amstore	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)	Constraints (6)	Constraints (6)+(7)
9	0.0752	0.0752	1				0.6142					
10	0.1268	0.1268	1				0.7358					
11	0.1694	0.1694	1				0.7186					
12	0.2086	0.2086	1				0.4446					
13	0.2428	0.2428	1									
14	0.2688	0.2688	1	0.0654	0.0654	1						
22	0.1512	0.1512	1				0.1202					
23	0.3106	0.3106	1	0.3590	0.3590	1	0.6306	0.9974				
24	0.4240	0.4240	1	0.5954	0.5954	1	0.8016	1				
25	0.5196	0.5196	1	0.8070	0.8070	1	0.9332	1	0.5086	0.7564		
26	0.6184	0.6184		0.9684	0.9684	1	0.9022	0.9986	0.0602	0.0032	0.0094	
27	0.7154	0.7154	1	1	1	1	0.7376	0.9532	0.0140		0.1466	
28	0.7748	0.7748	1	1	1	1						
33	1	1	1	1	1	1	0.2802	0.0302	0.1384			
34	1	1	1	1	1	1	0.1554				0.0840	
35	0.9320	1	0.9320	0.9094	1	0.9094	0.0396				0.0004	
36	0.3362	0.3362	1	0.5574	0.5574	1						
37	0.6254	0.6254	1	1	1	1	0.1816	0.5182			0.0015	
38	0.8466	0.8466	1	1	1	1	0.5930	1			0.1297	0.2120
39	1	1	1	1	1	1	0.9332	1	0.2688	0.2404	0.3969	0.7848
40	1	1	1	1	1	1	0.9582	1			0.2090	0.0032
41	0.9178	1	0.9178	0.8932	1	0.8932	0.9366	1			0.0224	
43	0.0890	0.0890	1	0.0528	0.0528	1	0.0642					
44	0.7244	0.7244	1	1	1	1	0.0642					
45	1	1	1	1	1	1	0.1650		0.0100			
50	0.1378	0.7154	0.1964	0.1862	1	0.1862						
51	0.1960	0.9958	0.1964	0.1862	1	0.1862	0.1600	0.4656				

Table 19: Probability of each alternative being better than the reference point (eliminating the alternatives for which the probability of being better, for both parties, than the reference point is inferior than 0.05), probability of being efficient, probability of being the best according the criterion sum of the values, probability of being the best according the criterion minimal PoP (reference point as reference) - values known.

3. the domains approach,

and analyzed how they can be applied to different levels of information that might be available about the preferences of negotiators.

The three methods we have discussed reflect two important trade-offs in dealing with incomplete information. The first trade-off, which can best be illustrated by comparing the extreme parameters approach to the domains approach, can be labeled as *ambiguity* vs. *lack of universality*. The domains approach generates only probability statements, which sometimes can be rather vague and might be hard to interpret. This contrasts with the very clear statements generated by the extreme parameters approach. If an alternative is definitely better than another alternative according to the extreme parameters approach, there is no doubt how the two alternatives are to be seen, while the domains approach might create statements like there is a 55 percent probability that one alternative i is better than another alternative j. However, the advantage of the extreme parameters approach in terms of lower ambiguity comes at a price: The domains approach is able to generate a (probabilistic) statement about any two alternatives, the extreme parameters approach might be unable to state whether one alternative is definitely better than the other or vice versa.

The central parameters approach overcomes this dilemma. It will always deliver a unique result, but does so by ignoring much of the information that is available and focusing on only one out of possibly many possible parameter vectors. Thus, it illustrates another important trade-off between *information richness* and *uniqueness of results*. Figure 5 illustrates this relationship.

The two dimensions represented in Figure 5 represent trade-offs, both ends of these axes have their advantages and disadvantages. Consequently, there is no method which is clearly better than the others, all methods have their particular strengths which make them suitable for some tasks. We therefore argue for a mix of methods, which should preferably be implemented in the form of an interactive process. The first step of such a process consists in a pre-selection of alternatives based on the extreme parameters approach. Depending on the purpose of the analysis, further choice between these alternatives can be based on the central parameters approach to obtain specific results, or on the domains approach to better exploit the rich, but potentially ambiguous information available. This integration can probably best be

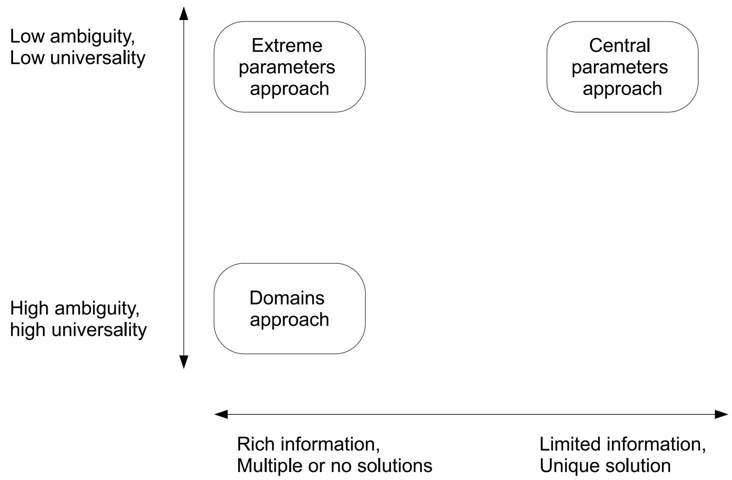

Figure 5: Trade-offs between approaches to deal with incomplete information in negotiations.

achieved using simulation methods, which make it possible to follow a central parameters approach and a domains approach simultaneously.

Incomplete information makes the ranking of alternatives uncertain. Our methods also represent different ways of handling uncertainty. The domains approach in a way relates to decision criteria under risk like the expected value, which explicitly take into account probabilities. The extreme parameters approach can be compared to a pessimistic min-max criterion, which only looks at the baseline which can be obtained under any circumstances.

When outcomes are uncertain, there are two kinds of errors which can be made: on one hand, an alternative can be indicated as optimal or as better than another alternative while in reality it is not, and on the other hand, the method might fail to identify an alternative which is good in reality. All methods might lead to the second kind of error. The first kind of error, declaring an alternative erroneously as optimal, is a particular problem for methods which focus on particular elements of the available information. This is the case for the central parameters approach. The same risk also exists when marginal utility functions are replaced by linear functions. The example has shown that this approximation can introduce distortions which could lead to a positive evaluation of alternatives which in reality are inferior.

When information is incomplete, there is also the possibility to obtain additional information to improve the quality of results. In particular from the examples, we can draw two conclusions with respect to this topic. On one hand, the results indicate that the information which can be inferred from choices made during the negotiation is not enough for reliable results, and the results can significantly be improved by adding at least a few preference or indifference statements directly obtained from the negotiators. On the other hand, just a few equivalence statements are sufficient to obtain results which are very close to the true preferences of negotiators. Thus it seems that one need not obtain much additional information from the decision makers.

While our study has led to some interesting results concerning the advantages and disadvantages of the methods we studied, it also has several limitations which indicate the need for future research. First, and perhaps most importantly, we have only applied our methods to one single case for illustrative purposes. An important next step in our research will therefore consist in creating a larger empirical basis, both by applying the methods to real data from (experimental) negotiations and perhaps by using more comprehensive simulations to study our methods in a wider range of settings. Such studies could be particularly useful to clarify the relationship between observed preference information and information which is explicitly provided by the negotiators and the impact of additional preference information.

Apart from broadening the empirical basis, there are also some interesting topics for theoretical improvements of our methods. So far, we have assumed that all information obtained from negotiators either implicitly or explicitly is consistent and reflects the same true utility function of a negotiator. But in reality, negotiators might make mistakes during the negotiation by proposing incorrect offers or incorrectly accepting or rejecting offers from their opponents, or they might provide inconsistent information when explicitly asked about their preferences. It is therefore necessary to extend our methods to deal with such inconsistencies.

Inconsistencies in the responses of negotiators might be the result of an error, but they might also be the result of deliberate manipulation. In particular when our methods are used by a mediator to suggest potential agreements to negotiators, or even by an arbitrator to calculate a binding solution, there are incentives for parties for strategic misrepresentation of their preferences. While the complexity of the calculations involved would make it difficult for negotiators to manipulate their answers in an optimal way, parties could nevertheless successfully try to improve their situation even by simplistic methods [29]. These possibilities and their impact on the quality of results could also be analyzed in computational studies.

Apart from these theoretical and empirical developments, further work is needed to enable the practical application of our methods. This includes the development of actual scenarios for their use. While we have discussed the use of the proposed methods mainly as tools for a mediator or arbitrator in the present paper, this is not the only setting in which our proposed methods could be useful: they could also be applied as tools in an asymmetric setting for the support of one party in a negotiation. Of course, in such a setting the quality of information available about the preferences of the two parties will be different, since a negotiator could provide quite exact information about his or her own preferences, but would be restricted to information implicitly obtained from observed behavior concerning the preferences of the opponent. Application of our method in such a setting would also require different objective functions to pursue the interests of one party rather than to provide fair solutions in terms of the concepts discussed here. However, the general methodology could also be applied in such a setting.

Another important topic which needs to be clarified before application is acceptability of the proposed methods by users. There is some empirical evidence that negotiators are reluctant to accept solutions proposed by an automated system, even if it would improve their situation [13, 14]. Thus it is not clear how negotiators would react to the proposals generated by our methods. This could also be a topic of future empirical research aimed at transforming the theoretical concepts introduced here into practical tools for actual negotiations.

Acknowledgements:
This work benefited from support of FCT/FEDER grant POCI/EGE/58371/2004.

References

[1] F. Barron and B. Barrett. Decision quality using ranked attribute weights. *Management Science*, 42(11):1515–1523, 1996.

[2] J. Butler, J. Jianmin, and J. Dyer. Simulation techniques for the sensitivity analysis of multi-criteria decision models. *European Journal of Operational Research*, 103:531–546, 1997.

[3] J. Charnetski and R. Soland. Multiple-attribute decision making with partial information: The comparative hypervolume criterion. *Naval Research Logistics Quarterly*, 25:279–288, 1978.

[4] J.N. Clímaco and L.C. Dias. An approach to support negotiation processes with imprecise information multicriteria additive models. *Group Decision and Negotiation*, 15(2):171–184, 2006.

[5] L.C. Dias and J.N. Clímaco. Additive aggregation with variable interdependent parameters: the VIP analysis software. *Journal of the Operational Research Society*, 51(9), 2000.

[6] L.C. Dias and J.N. Clímaco. Dealing with imprecise information in group multicriteria decisions: A methodology and a gdss architecture. *European Journal of Operational Research*, 160:291–307, 2005.

[7] H. Ehtamo, R. Hämäläinen, P. Heiskanen, J. Teich, M. Verkama, and S. Zionts. Generating pareto solutions in a two-party setting: Constraint proposal methods. *Management Science*, 45(12):1697–1709, 1999.

[8] S. Greco, V. Mousseau, and R. Slowinski. Ordinal regression revisited: multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):415–435, 2008.

[9] P. Heikanen. Decentralized method for computing pareto solutions in multiparty negotiation. *European Journal of Operational Research*, 117:578–590, 1999.

[10] E. Jacquet-Lagreze and J. Siskos. Assessing a set of additive utility functions for multicriteria decision-making, the uta method. *European Journal of Operational Research*, 10:151–164., 1982.

[11] T. Jelassi, G. Kersten, and S. Zionts. An introduction to group decision and negotiation support. In C. Bana e Costa, editor, *Readings in Multiple Criteria Decision Aid*, pages 537–568. Springer Verlag, Berlin, 1998.

[12] R. Keeney and H. Raiffa. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. J. Wiley and Sons, New York, 1976.

[13] G.E. Kersten and S.J. Noronha. Rational agents, contract curves, and inefficient compromises. *IEEE Transactions on Systems, Man and Cybernetics*, 28(3):326–338, 1998.

[14] P. Korhonen, J. Phillips, J. Teich, and J. Wallenius. Are pareto improvements always preferred by negotiators? *Journal of multi-Criteria Decision Analysis*, 7(1):1–2, 1998.

[15] R. Lahdelma, J. Hokkanen, and P. Salminen. SMAA - stochastic multiobjective acceptability analysis. *European Journal of Operational Research*, 106(1):137–143, 1998.

[16] R. Lahdelma, K. Miettinen, and P. Salminen. Ordinal criteria in stochastic multicriteria acceptability analysis (SMAA). *European Journal of Operational Research*, 147(1):117–127, 2003.

[17] G. Lai, C. Li, and K. Sycara. Efficient multi-attribute negotiation with incomplete information. *Group Decicion and Negotiation*, 15:511–528, 2006.

[18] G. Lai and K. Sycara. A generic framework for automated multi-attribute negotiation. *Group Decicion and Negotiation*, 18:169–187, 2009.

[19] J.B. Lasserre. An analytical expression and an algorithm for the volume of a convex polyhedron in rn. *Journal of Optimization Theory and Application*, 39(3):363–377, 1983.

[20] J. Lawrence. Polytope volume computation. *Mathematics of Computation*, 57:259–271, 1991.

[21] V. Mousseau and L. Dias. Valued outranking relations in electre providing manageable disaggregation procedures. *European Journal of Operational Research*, 156(2):467–482, 2004.

[22] D.G. Pruitt. Strategic choice in negotiation. *The American Behavioral Scientist*, 27(2):167–194, 1983.

[23] H. Raiffa, J. Richardson, and D. Metcalfe. *Negotiation analyis: the science and art of collaborative decision making*. Belknap Press of Harvard, University Press, Cambridge (Ma), 2002.

[24] P. Sarabando and L. Dias. Comparison of different rules to deal with incomplete information: perspectives of mediation. Technical report, 2009. Research Reports of INESC Coimbra, No. 2.

[25] T. Solymosi and J. Dombi. A method for determining the weights of criteria: the centralized weights. *European Journal of Operational Research*, 26:35–41, 1986.

[26] M.K. Starr. *Product Design and Decision Theory*. Prentice Hall, Englewood Cliffs, 1962.

[27] K.W. Thomas. Conflict and conflict management: Reflections and update. *Journal of Organizational Behavior*, 13:265–274, 1992.

[28] R. Vetschera. A recursive algorithm for volume-based sensitivity analysis of linear decision models. *Computers & OR*, 24(5):477–491, 1997.

[29] R. Vetschera. Strategic manipulation of preference information in multi-criteria group decision methods. *Group Decision and Negotiation*, 14:393–414, 2005.

[30] R. Vetschera. Learning about preferences in electronic negotiations - a volume based measurement method. *European Journal of Operational Research*, 194:452–463, 2009.

[31] D. von Winterfeldt and W. Edwards. *Decision Analysis and Behavioral Research*. Cambridge University Press, Cambridge, 1986.

[32] R.E. Walton and R.B. McKersie. *A Behavioral Theory of Labor Negotiations*. McGraw-Hill, New York, 1965.

[33] M. Weber. Decision making with incomplete information. *European Journal of Operational Research*, 28(1):44–57, 1987.

A Some notes

This appendix illustrates why it is not a good idea that a mediator suggests the alternative that maximizes the sum of the values considering the constraints regarding the parameters values. The alternative which maximizes the sum of the values can be obtained solving the following linear problem:

$$\begin{array}{l} \max V^1(x) + V^2(x) \\ (w_1, v_1, w_2, v_2) \in (W_1, M_1, W_2, M_2) \end{array} \tag{30}$$

We will show what happens using the example presented in section 4, and approximating the values using linear value functions. We chose this case to better explain some results obtained in subsection 4.4.2 (results of the domains approach). Remember that alternatives that are better for both parties than the compromise solution (alternative 44) are alternatives 27, 28, 37, 38, 39 and 40. Between these alternatives, only the alternative 28 is not efficient.

Considering that negotiators have linear value functions and using the constrains of type (6), and also the constraints of type (6)+(7), solving the LP (30) the alternative that the mediator should recommend is the alternative 28. To better explain why the recommended alternative is the 28 one, which is not efficient neither maximizes the sum of the values using the true values, we will make a graphical analysis.

To study the problem we transformed the inicial 3 dimensions problem in a 2 dimensions one. For Nelson let us fix $w_2^N = 0.2$ (so $w_3^N = 1 - 0.2 - w_1^N$), and for Amstore let us fix $w_2^A = 0.1$ (so $w_3^A = 1 - 0.1 - w_1^A$). We can now draw some pictures varying w_1^N and w_1^A (for simplicity we used weights between 0 and 100). Figure 6 refers to Nelson, varying w_1^N (which is a value between 0 and 80) and to Amstore varying w_1^A (which is a value between 0 and 90). In the left side of the figure we indicate real values and the right side is constructed supposing that value functions are linear. The objective of this figure is only to see the differences between the values for Nelson and for Amstore considering real values and considering approximated values using linear value functions.

In Figure 7 we depict the sum of the values of both parties. In each picture we varied the value of w_1^N. For w_1^A equal to 0, 10 and 20, the alternative which maximizes the sum of the values is the alternative 37. For $w_1^A \geq 40$ the alternative which maximizes the sum of the values is the alternative 28. Remember that the real weights are $w_1^N = 60$ and $w_1^A = 70$. Neither alternative 28 nor alternative 37 maximize the sum of the values considering true values. Figure 7 also enables us to see why alternative 28 has high probability of maximizing the sum of the values when we consider unknown weights and approximate the values using linear value functions (subsection 4.4.2).

In the presented example, solving the linear problem (30), and approximating the values using linear values functions, does not guarantee the achievement of a good alternative. Thus, as it is possible to see, suggesting the alternative that maximizes the sum of the values may not be a good idea. Remember that using approximated values, the extreme approach (subsection 4.2) enables the mediator to suggest a set of alternatives that can be better for both parties than the compromise solution, can be efficient and can maximize the sum of the values. The central parameters approach (subsection 4.3.2) enables the mediator to suggest one efficient alternative better for both parties than the compromise solution based on inferred weights. Using the domains approach (subsection 4.4.2) the mediator can suggest the alternative with highest probability of maximizing the sum of the values (alternative 28) but can conjugate this result with the probability of this alternative being efficient and the probability of being better for both parties than the compromise solution.

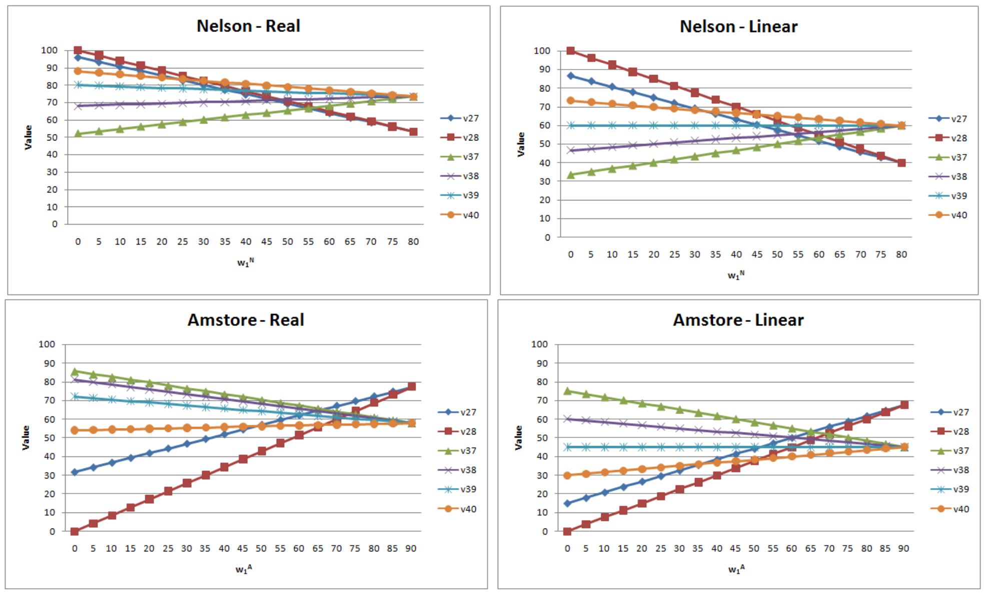

Figure 6: Real values and approximation of values using linear functions, for Nelson and Amstore.

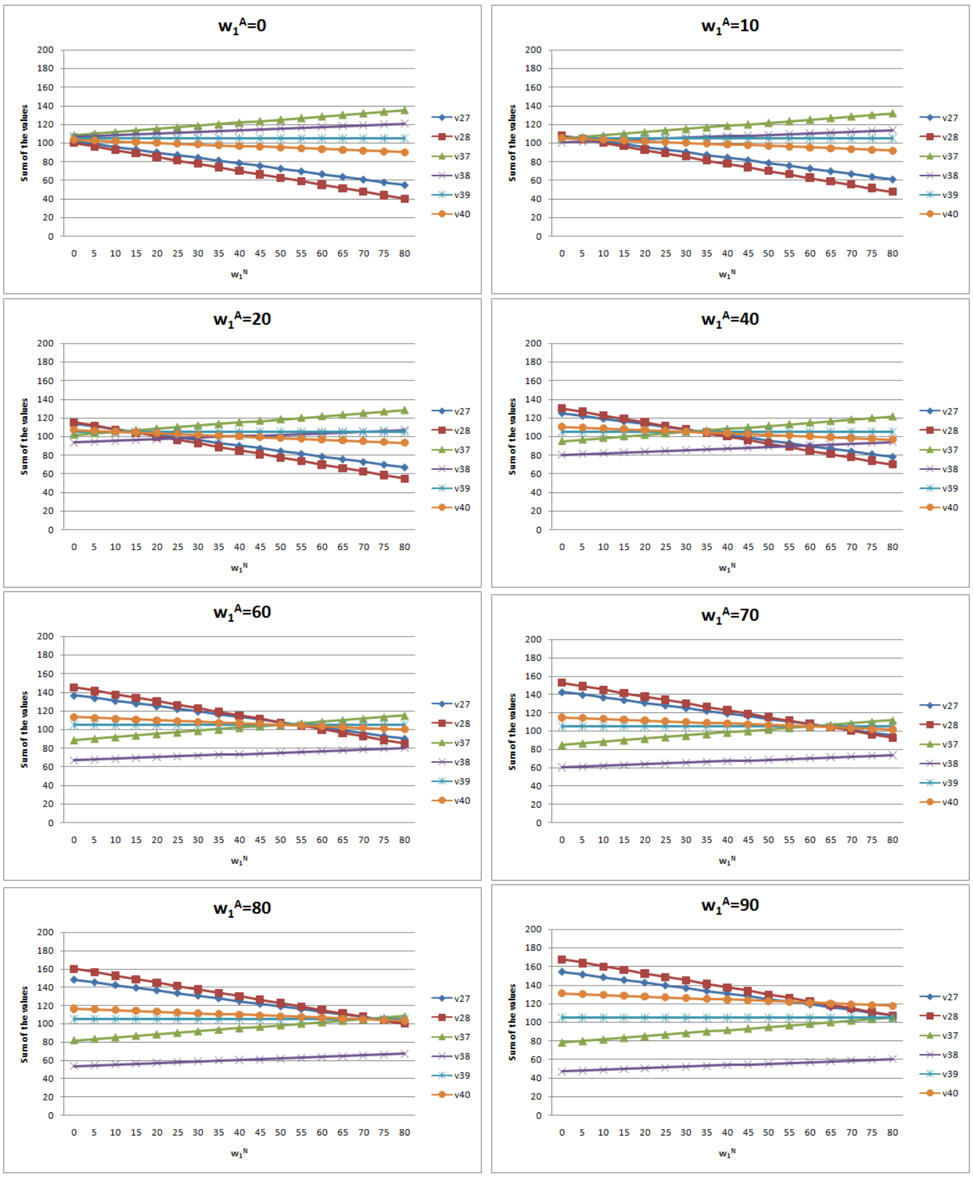

Figure 7: Sum of the values.