BUILDING ON *TEACHABLE MOMENTS*: ISSUES FOR TEACHER EDUCATION[1]

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Keywords: Didactical knowledge, communication, proof, teacher education.

DIDACTICAL KNOWLEDGE IN TEACHER EDUCATION CONTEXTS

Several authors have given attention to the notion of didactical knowledge, which is not consensual (Ponte, in press). We consider didactical knowledge as being related to aspects of teachers' practices, "essentially oriented towards action" (Ponte, 1999, p. 61), and involving four dimensions: knowledge of the curriculum, knowledge of mathematics, knowledge of students and their learning processes, and knowledge instructional processes in the classroom (Ponte & Oliveira, 2002). The didactical knowledge has a dynamic character because the experiences teachers encounter in their practice constantly shape it (Ponte & Santos, 1998).

Portugal is currently undergoing the generalization of a new mathematics program for basic education (ME, 2007), that is, for 1st through 9th grade (children aged 6 through 14 years-old). As with any curricular change, teachers face several challenges, such as providing students with rich and diversified mathematical experiences and orchestrating productive mathematical discussions in the classroom (Stein, Engle, Smith & Hughes, 2008). Coping with such challenges demands teachers to have a strong didactical knowledge.

The importance of addressing the teacher's role in provoking and sustaining a mathematically sound communication in the classroom, in both initial and ongoing teacher education contexts, is widely recognized (Bishop & Goffree, 1986; Brendefur & Frykholm, 2000; Ruthven, Hofmann & Mercer, 2011). With this perspective in mind, it is often seen as a promising strategy to engage teachers in thinking and discussing about the challenges involved in that role by analyzing classroom episodes (Bishop & Goffree, 1986; Ruthven et al., 2011).

A Classroom Episode on Prime Numbers

At the end of a mathematics methods course (in which issues of classroom communication were addressed, amongst others), a cohort of prospective teachers at a large university in Portugal, in a written individual assignment, was asked to analyze a classroom episode (Boavida, 2001, adapted from Prince, 1998) and to give

[1] This work is supported by national funds through FCT – Fundação para a Ciência e Tecnologia, under the projects *Professional Practices of Teachers of Mathematics* (Grant PTDC/CPE-CED/098931/2008) and PEst-C/MAT/UI0144/2011, and by FEDER funds through COMPETE.

suggestions about how it could be continued, taking into account the current curricular orientations for mathematics teaching (ME, 2007). This episode pictured some instructional decisions of a 7th grade teacher. The students had been asked to write down all prime numbers up to 50 and Rita told her teacher she had noticed that all the prime numbers she had found so far, bigger than 5, ended in 1, 3, 7 or 9.

The teacher then made several instructional decisions amongst which we may find the following: (1) she refrained from validating Rita's conjecture, assigning that role to the whole class; (2) she discussed the meaning of *conjecture* with the students and constantly questioned them about how they could ensure the truthfulness of Rita's conjecture for *all* prime numbers (the students were seriously convinced of that truthfulness because all the numbers they had tried fit the conjecture); (3) she wrote on the board the numbers from 0 to 9, circled the numbers indicated by Rita and, together with her students, excluded the remaining numbers from being possible unit digits of prime numbers (thus proving Rita's conjecture); and (4) she wrote on the board the reverse implication of Rita's finding, pushing her students to understand the difference between the two conjectures and letting them convince each other that the reverse conjecture could not be true.

In sum, the teacher's didactical knowledge proved to be fundamental when, though starting with a low cognitive level task, she was able to recognize the potential of Rita's finding to explore complex notions and processes. In addition, she took on that *teachable moment*, building on the students' contributions to the classroom discourse and orchestrating a productive discussion around issues of proof: the meaning of a conjecture, what it takes to prove a conjecture, the difference between reverse implications, and the role of counter-examples. The teacher's didactical knowledge allowed her to raise significantly the cognitive level of the initial task (Stein & Smith, 1998), providing her students with an opportunity to discuss complex issues of elementary logical thinking.

When challenged to analyze the episode, some prospective teachers showed serious trouble in making sense of it. Many of the difficulties that emerged are anchored in a poor knowledge of the mathematics involved. In fact, some prospective teachers did not recognize the presence of reverse implications in the episode. They also did not realize that Rita's conjecture had, in fact, been proved during the classroom discussion. Not surprisingly, the same prospective teachers did not understand the instructional decisions taken in the episode and were unable to provide adequate suggestions for continuing the episode from an instructional point of view.

We also asked the same questions of some practicing teachers, with diversified backgrounds and years of experience. Yet, the responses were similar to the prospective teachers', suggesting an inadequate understanding of proof or a conception of proof associated with a rigid and formal character.

Rethinking the Role of Teacher Education

With the recent readjustment of teacher education programs, as teacher educators, we question ourselves about how those programs are promoting prospective teachers' didactical knowledge, in particular knowledge of mathematics and knowledge of instructional processes. The data we have analyzed have suggested that we must reflect upon the adequacy of both the 1st and the 2nd cycles of studies, in which prospective teachers gain their mathematics major and their teacher certification, respectively. We intend to present an in depth analysis of some prospective and practicing teachers' reactions to the episode mentioned above, and emphasize the complex nature of didactical knowledge and its relation to teacher education.

REFERENCES

Bishop, A., & Goffree, F. (1986). Classroom organization and dynamics. In B. Christiansen, A. G. Howson & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 309-365). Dordrecht: Reidel.

Boavida, A. M. (2001). Um olhar sobre o ensino da demonstração em Matemática. *Educação e Matemática, 63*, 11-15.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two prospective teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*, 125-153.

ME (2007). *Programa de Matemática do Ensino Básico*. Lisboa: ME-DGIDC.

Ponte, J. P. (1999). Didácticas específicas e construção do conhecimento profissional. In J. Tavares, A. Pereira, A. Pedro & H. Sá (Orgs.), *Investigar e formar em educação – Actas do XI EIEM* (pp. 59-72). Porto, Portugal: SPCE.

Ponte, J. P. (in press). Estudando o conhecimento e o desenvolvimento profissional do professor de matemática. In N. Planas (Ed.), *Educación matemática: Teoria, critica y prática*. Barcelona, Spain: Graó.

Ponte, J. P., & Santos, L. (1998). Práticas lectivas num contexto de reforma curricular. *Quadrante 7*(1), 3-33.

Ponte, J. P., & Oliveira, H. (2002). Remar contra a maré: A construção do conhecimento e da identidade profissional na formação inicial. *Revista de Educação, 11*(2), 145-163.

Prince, A. (1998). Prove it! *Mathematics Teacher*, *91*(8), 726-728.

Ruthven, K., Hofmann, R., & Mercer, N. (2011). A dialogic approach to plenary problem synthesis. In B. Ubuz (Ed.), *Proceedings of the 35th Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 81-88). Ankara, Turkey: PME.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313-340.