Supersymmetric inflation and baryogenesis via Extra-Flat directions of the MSSM

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Abstract

One interpretation of proton stability is that it implies the existence of extra-flat directions of the minimal supersymmetric standard model, in particular $u^c u^c d^c e^c$ and $QQQL$, where the operators lifting the potential are suppressed by a mass scale Λ which is much larger than the Planck mass, $\Lambda \gtrsim 10^{26}$ GeV. Using D-term hybrid inflation as an example, we show that such flat directions can serve as the inflaton in supersymmetric inflation models. The resulting model is a minimal version of D-term inflation which requires the smallest number of additional fields. In the case where Q-balls form from the extra-flat direction condensate after inflation, successful Affleck-Dine baryogenesis is possible if the suppression mass scale is $\gtrsim 10^{31} - 10^{35}$ GeV. In this case the reheating temperature from Q-ball decay is in the range $3 - 100$ GeV, while observable baryon isocurvature perturbations and non-thermal dark matter are possible. In the case of extra-flat directions with a large t squark component, there is no Q-ball formation and reheating is via conventional condensate decay. In this case the reheating temperature is in the range $1 - 100$ TeV, naturally evading thermal gravitino overproduction while allowing sphaleron erasure of any large $B - L$ asymmetry.

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I. INTRODUCTION

Successful models of supersymmetric (SUSY) inflation should ideally satisfy a number of requirements: natural compatibility with supergravity (SUGRA), lack of fine-tuned couplings, successful post-inflation era including reheating and baryogenesis, and compatibility with unified models of particle physics. With respect to these conditions, SUSY hybrid inflation models have a particular attraction [1, 2]. They can achieve sufficient inflation without requiring very small or fine-tuned couplings, and in the case of D-term hybrid inflation they are naturally compatible with SUGRA [3]. Focusing on the D-term inflation case, a natural question is the origin of the fields in the D-term inflation sector. The $U(1)$ gauge field and charged vector pair $\Phi_\pm$ of D-term inflation might be understood as components of an extended gauge theory. However, the inflaton is usually a gauge singlet which is added to the model for no other reason[1]. If we do not add such a singlet, can D-term hybrid inflation still occur? Here we argue that it can. The vector pair will naturally couple to any gauge-invariant combination of fields in the MSSM. Such gauge-invariant products (monomials) also characterise flat directions of the MSSM. Thus a natural possibility is that a flat direction can play the role of the inflaton in D-term inflation models[2]. In this model the number of additional fields required for inflation is reduced to just a $U(1)$ gauge field and the $\Phi_\pm$ vector pair, so providing a minimal version of D-term inflation. As we will show, conventional MSSM flat directions lifted by Planck scale-suppressed gauge-invariant superpotential terms are unsuitable. This is because such terms generally lift the flat direction scalar at field strengths well below the value required for inflation. However, it is known that certain gauge-invariant superpotential terms must be suppressed by more than the Planck scale or forbidden entirely. The $d = 4$ operators $u^c u^c d^c e^c$ and $QQQL$ will lead to rapid proton decay if they are only Planck scale-suppressed [9]. One way this problem can be solved is by assuming that the underlying complete theory introduces a dynamical suppression factor into the non-renormalisable superpotential interactions, such that the effective mass scale suppressing the dangerous operators is $\Lambda \gtrsim 10^{26}$ GeV [9]. It is also possible that

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[1] Models exist which attempt to identify the inflaton with a known field, such as a right-handed sneutrino [4, 5, 6].

[2] An interesting model using MSSM flat directions as inflatons, which has a quite different philosophy with respect to fine-tunings, is given in [7]. See also [8].

this dynamical suppression will also apply to all higher-order MSSM superpotential terms lifting the flat direction, such as $(u^c u^c d^c e^c)^2$ and $(QQQL)^2$. We will refer to a flat direction for which this is true as an 'extra-flat direction'. An alternative interpretation of the absence of proton decay is in terms of a discrete symmetry which eliminates the dangerous $d = 4$ operators [10]. In this case it is possible that the higher-order operators will be unsuppressed. However, as we will show, such unsuppressed flat directions, even if higher-order, cannot serve as an inflaton. If the existence of extra-flat directions is the correct interpretation of the absence of proton decay in the MSSM, then an extra-flat direction scalar could serve as the inflaton in a D-term inflation model. The extra-flat direction potential at large field values is naturally lifted to an inflationary plateau by its gauge-invariant superpotential coupling to $\Phi_+ \Phi_-$. Reheating and possibly baryogenesis would then come from the decay of the flat direction inflaton, via either Q-ball decay or conventional homogeneous condensate decay, depending on the t squark component of the flat direction. In this paper we will study D-term inflation along an extra-flat direction of the MSSM. The paper is organised as follows. In Section 2 we discuss extra-flat directions and the resulting D-term inflation model. In Section 3 we discuss reheating and baryogenesis. In Section 4 we present our conclusions.

II. D-TERM INFLATON ALONG EXTRA-FLAT DIRECTIONS

A. Potential

We consider a flat direction Φ in the MSSM and introduce two additional fields $\Phi_\pm$ charged under a U(1) gauge group with the Fayet-Illiopoulos term ξ. The superpotential is

$$W = \frac{\lambda_1 \Phi^m}{m M^{m-3}} + \frac{\lambda_2 \Phi^n}{n M^{n-1}} \Phi_+ \Phi_- \ , \tag{1}$$

where $\lambda_{1,2}$ are Yukawa couplings and M is the reduced Planck mass, $M = M_{Pl}/\sqrt{8\pi}$ [3]. We will present results for general m and n, specialising to the case of most interest $m = n = 4$, corresponding to $\Phi^4 \sim u^c u^c d^c e^c$ or $QQQL$. Proton stability in the case of $u^c u^c d^c e^c$ or $QQQL$ requires that $\lambda_1 \lesssim 10^{-8}$, corresponding to an effective suppression mass scale $\Lambda = M/\lambda_1 \gtrsim 10^{26}$ GeV. However, λ_2 is unconstrained by phenomenology and will be determined by the inflation model. The scalar potential in the global SUSY limit is then

$$
V = \left| \frac{\lambda_2 \phi^n}{nM^{n-1}} \right|^2 (|\phi_+|^2 + |\phi_-|^2) + \left| \frac{\lambda_1 \phi^{m-1}}{M^{m-3}} + \frac{\lambda_2 \phi^{n-1}}{M^{n-1}} \phi_+ \phi_- \right|^2
$$
$$
+ \frac{g^2}{2} (\xi + |\phi_+|^2 - |\phi_-|^2)^2. \tag{2}
$$

The supersymmetric global minimum is located at[4]

$$
(\phi, \phi_+, |\phi_-|) = (0, 0, \sqrt{\xi}). \tag{3}
$$

If

$$
\left| \frac{\lambda_1 \lambda_2 \phi^{m+n-2}}{M^{m+n-4}} \right|^2 \ll (g^2 \xi)^2 \tag{4}
$$

is satisfied, the mixing between ϕ_+ and ϕ_- is negligible. The potential is then simplified to

$$
V \simeq \left| \frac{\lambda_2 \phi^n}{nM^{n-1}} \right|^2 (|\phi_+|^2 + |\phi_-|^2) + \left| \frac{\lambda_1 \phi^{m-1}}{M^{m-3}} \right|^2 + \frac{g^2}{2} (\xi + |\phi_+|^2 - |\phi_-|^2)^2. \tag{5}
$$

The critical value of ϕ is given by

$$
|\phi_c| \equiv \left(\frac{nM^{n-1}\sqrt{g^2\xi}}{|\lambda_2|} \right)^{1/n}, \tag{6}
$$

which determines the stability of the ϕ_- field at the origin. The origin is a false vacuum for $|\phi| > |\phi_c|$, while it is unstable for $|\phi| < |\phi_c|$.

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[3] A SUSY mass term $W \supset \mu \Phi_+ \Phi_-$ has not been included. This term would induce n minima with nonvanishing VEV for Φ, which consist of squark and/or slepton VEV and lead to large baryon or lepton number violation in the MSSM. Although in most cases there is no symmetry which can exclude such a term, we note that for the case $m = n$ this term can be excluded by an R-symmetry which allows the terms Φ^n and $\Phi^n \Phi_+ \Phi_-$. In addition, if μ is less than the scale of soft SUSY breaking terms, the minimum of the potential can be at $\Phi = 0$, while for larger μ there can be directions in the complex Φ plane along which the field evolution can avoid the minima with $\Phi \neq 0$.

[4] Note that there is a SUSY flat direction when $\mu = 0$ and $\phi = 0$, such that $|\phi_+|^2 - |\phi_-|^2 = \xi$. However, the minimum with $\phi_+ = 0$ is selected since ϕ_+ gains a large mass when $\phi \neq 0$ during inflation.

B. Inflationary expansion

For $|\phi| > |\phi_c|$, $\phi_- = 0$ is a local minimum and there is the false vacuum energy from the D-term, which drives inflation. The potential during inflation is given as

$$V \simeq \frac{1}{2}g^2\xi^2 \left(1 + \frac{g^2}{8\pi^2}\ln\frac{\sigma^{2n}}{\Lambda_*^{2n}}\right) , \qquad (7)$$

where $\sigma = \sqrt{2}Re(\phi)$ is the canonically normalised inflaton and Λ_* is the renormalisation scale. Inflation ends when the inflaton reaches the larger of $\sigma_c \equiv \sqrt{2}|\phi_{rmc}|$ and

$$\sigma_f \equiv \frac{\sqrt{n}gM}{2\pi} , \qquad (8)$$

where σ_f corresponds to the end of slow-roll. However, a non-vanishing F-term potential is also present in this model. Hence, we need to ensure that the condition $V_F \ll V_D$ is satisfied, which requires that

$$\left|\frac{\lambda_1\phi^{m-1}}{M^{m-3}}\right|^2 \ll \frac{1}{2}g^2\xi^2 \qquad (9)$$

is satisfied. Note that when this is satisfied, equation (4) is also satisfied. The dynamics of the inflaton field is similar to that in the minimal D-term hybrid inflation model [1]. The solution of the slow-roll field equations is

$$\sigma^2(N) = \sigma_0^2 + \frac{ng^2NM^2}{2\pi^2} . \qquad (10)$$

Here, $\sigma_0 = \max[\sigma_f, \sigma_c]$ is the expectation value of inflaton when inflation terminates. The spectral index is

$$n_s = 1 - \frac{1}{N}\left(1 + \frac{2\pi^2\sigma_0^2}{ng^2M^2N}\right)^{-1} , \qquad (11)$$

while the value of $\xi^{1/2}$ normalised to the curvature perturbation P_ζ is

$$\frac{\xi^{1/2}}{M} = \left(\frac{3nP_\zeta}{N}\right)^{1/4}\left(1 + \frac{2\pi^2\sigma_0^2}{ng^2M^2N}\right)^{-1/4} . \qquad (12)$$

In the case of $\sigma_0 = \sigma_c$, which we will show is true in examples of interest, we find

$$\frac{2\pi^2\sigma_0^2}{ng^2M^2N} = \left(\frac{\lambda_{2_c}}{\lambda_2}\right)^{2/n} \qquad (13)$$

with

$$\lambda_{2_c} = \left(\frac{4\pi^2}{ng^2NM^2}\right)^{n/2}\left(g^2\xi n^2 M^{2(n-1)}\right)^{1/2} . \qquad (14)$$

5

$\lambda_2 = \lambda_{2_c}$ corresponds to $\sigma_N = 2\sigma_c$, with $\sigma \approx \sigma_c$ throughout inflation when $\lambda_2 < \lambda_{2_c}$. When $\lambda_2 \gg \lambda_{2_c}$ as well as the case of $\sigma_0 = \sigma_c$, the spectral index is $n_s = 1 - 1/N \approx 0.98$, as in conventional D-term inflation, while the value of $\xi^{1/2}$ required to account for the observed curvature perturbation ($P_\zeta^{1/2} = 4.8 \times 10^{-5}$) is $\xi^{1/2} = 7.9 \times 10^{15} n^{1/4}$ GeV. On the other hand, in the case where $\lambda_2 \ll \lambda_{2_c}$, the spectral index approaches $n_s = 1$ while the value of $\xi^{1/2}$ is reduced by a factor $(\lambda_2/\lambda_{2_c})^{1/2n}$.

C. Comparison with observations

The spectral index observed by WMAP, $n_s = 0.958 \pm 0.016$ (1-σ) [11], is substantially smaller the D-term inflation value. In addition, WMAP data permits at most an O(10)% contribution to the CMB power spectrum from cosmic strings [12, 13, 14], which implies that $\xi^{1/2} \lesssim 4 \times 10^{15}$ GeV. (Here we have used $G\mu = 2 \times 10^{-6}$ for the $l = 10$ WMAP normalised string tension [15].) One way to interpret the WMAP observations is that they correspond to an adiabatic curvature perturbation with $n_s \approx 1$ combined with a 10% cosmic string contribution [16], which can be achieved by making λ_2 sufficiently small compared with λ_{2_c}. In this case the apparent spectral index of the combined perturbation is effectively lowered and can be in agreement with the 3-year WMAP data analysis [16]. It is a striking feature of D-term inflation models in general that they have a solution which increases n_s while decreasing the cosmic string contribution, just as required for this interpretation of the WMAP observations. With respect to this possibility, the extra-flat direction model has a possible advantage over conventional D-term inflation. The contribution of cosmic strings to the CMB power spectrum is proportional to $\mu^2 = (2\pi\xi)^2$. In the case of conventional D-term inflation with $n_s = 1$, the value of ξ^2 in the limit $\lambda_2 \ll \lambda_{2_c}$ is proportional to λ_2^2. Therefore λ_2 must lie within a rather narrow range of values for the cosmic string contribution to be O(10)%. In the case of the extra-flat direction inflaton, the dependence is $\propto \lambda_2^{2/n}$. Therefore the CMB contribution varies much more gradually with λ_2 e.g. $\xi^2 \propto \lambda_2^{1/2}$ for the case $n = 4$. Thus an O(10)% contribution is obtained for a much wider range of λ_2, making it perhaps a more natural possibility than in conventional D-term inflation.

D. Constraints from cosmic string bound, SUGRA and potential flatness

We first check that the 10% cosmic string condition $\xi^{1/2} \approx 4 \times 10^{15}$ GeV can be satisfied for reasonable values of g when $|\phi_c|^2$ is small enough compared with M^2 for SUGRA corrections to be neglected. We will require that $|\phi_c| < kM$, with $k \lesssim 0.3$, so that $|\phi_c|^2 \lesssim 0.1M^2$. From equation (6), this implies that

$$g \lesssim \frac{\lambda_2 k^n M}{n\xi^{1/2}} \ . \tag{15}$$

For the case $n = 4$ and $\xi^{1/2} \approx 4 \times 10^{15}$ GeV, equation (15) implies that

$$g \lesssim 1.2\lambda_2 \left(\frac{k}{0.3}\right)^4 \ . \tag{16}$$

Thus λ_2 should not be small compared with 1 if g is not very small compared with 1. From equation (12), to suppress $\xi^{1/2}$ from 7.9×10^{15} GeV to 4×10^{15} GeV in the case $n = 4$ we require that $\lambda_{2_c}/\lambda_2 \approx 250$, which implies that

$$\lambda_2 \approx 8 \times 10^{-7} g^{-3} \ . \tag{17}$$

Equations (16) and (17) imply that

$$g \lesssim 0.03 \left(\frac{k}{0.3}\right) \tag{18}$$

and

$$\lambda_2 \gtrsim 0.03 \left(\frac{0.3}{k}\right)^3 \ . \tag{19}$$

Thus $k \gtrsim 0.1$ is necessary when $\lambda_2 \lesssim 1$ in order to satisfy equation (19). $0.1 \lesssim k \lesssim 0.3$ then implies that $g \approx 0.01 - 0.03$ and $0.03 \lesssim \lambda_2 \lesssim 1$. Therefore, as in conventional D-term inflation in the small coupling limit, g must be somewhat smaller than the Standard Model gauge couplings [13]. In addition, λ_2 must be much larger than λ_1 [5]. We next evaluate Eq. (9) to find the condition on λ_1 for F-term corrections not to spoil the flatness of the inflaton potential. In general we find

$$\lambda_1 \ll \frac{1}{\sqrt{2}} \frac{g\xi}{k^{m-1}M^2} \ . \tag{20}$$

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[5] We have assumed that $\sigma_c > \sigma_f$. For the case $n = 4$ this requires that $\xi^{1/2}/M > |\lambda_2|g^3/\pi^4$. With $\xi^{1/2} \approx 4 \times 10^{15}$ GeV and $g \approx 0.02$ this is easily satisfied.

For the case $m = 4$ this gives

$$\lambda_1 \ll 7 \times 10^{-5} g \left(\frac{0.3}{k}\right)^3 \left(\frac{\xi^{1/2}}{4 \times 10^{15} \text{ GeV}}\right)^2 . \tag{21}$$

Thus for values of λ_1 which satisfy the proton decay constraint, $\lambda_1 \lesssim 10^{-8}$, the flat direction potential is easily sufficiently flat to serve as an inflaton. However, for the case of unsuppressed $n = m = 4$ flat directions with $\lambda_1 \sim 1$, the F-term would violate the flatness of the flat-direction inflaton potential. An alternative solution of the proton decay problem is to consider elimination of the $m = n = 4$ operators entirely by a symmetry. In this case we expect to have unsuppressed operators with $m = n = 8$, such that $\lambda_1 \sim \lambda_2 \sim 1$. However, in this case the F-flatness condition will still be violated. For $n = 8$, equation (15) implies that

$$g \lesssim 5 \times 10^{-3} \lambda_2 \left(\frac{k}{0.3}\right)^8 . \tag{22}$$

To suppress $\xi^{1/2}$ to 4×10^{15} GeV, with $n = 8$ we need $\lambda_{2_c}/\lambda_2 \approx 6.5 \times 10^4$. This implies that

$$\lambda_2 = 9 \times 10^{-12} g^{-7} . \tag{23}$$

Therefore if $\lambda_2 \lesssim 1$ we have $g \gtrsim 0.03$. Equation (23) combined with equation (22) gives

$$g \lesssim 0.02 \left(\frac{k}{0.3}\right) . \tag{24}$$

Thus $k \gtrsim 0.5$ is necessary if $g \gtrsim 0.03$. The F-term flatness condition equation (20) for $m = 8$ is

$$\lambda_1 \lesssim 8 \times 10^{-6} \left(\frac{g}{0.03}\right) \left(\frac{0.5}{k}\right)^7 \left(\frac{\xi^{1/2}}{4 \times 10^{15} \text{ GeV}}\right)^2 . \tag{25}$$

Thus the $m = n = 8$ flat direction will also need to be extra-suppressed to have a flat inflaton potential, even if the $m = n = 4$ term is completely eliminated by a discrete symmetry. Therefore extra-flat directions are essential for an MSSM flat direction to play the role of the inflaton in D-term inflation.

E. Post-inflationary evolution

Including soft SUSY breaking terms, the potential is

$$V = m_\phi^2 |\phi|^2 + A m_{3/2} \frac{\lambda_1 \phi^m}{m M^{m-3}} + \text{H.c.}$$
$$+ g^2 \xi |\phi_-|^2 \left|\frac{\phi}{\phi_c}\right|^{2n} + \left|\frac{\lambda_1 \phi^{m-1}}{M^{m-3}}\right|^2 + \frac{g^2}{2} (\xi - |\phi_-|^2)^2, \tag{26}$$

where we used equation (6). Here m_ϕ^2 is the soft SUSY breaking scalar mass. The potential has two SUSY non-renormalisable terms: the third term $g^2\xi^2(|\phi_-|^2/\xi)|\phi^n/\phi_c^n|^2$ and the fourth term $|\lambda_1\phi^m/M^{m-3}|^2$. Assuming that $m-1 < n$ (since the case of most interest will be that where $m = n$), the third term is dominant if $\phi \gtrsim \phi_*$, where

$$\phi_*^{n-m+1} \equiv \frac{\lambda_1\phi_c^n}{M^{m-3}\sqrt{g^2\xi^2}}\left(\frac{\xi}{|\phi_-|^2}\right)^{1/2}, \tag{27}$$

while the fourth term is dominant if $\phi < \phi_*$. An important point in what follows is that for $\phi \gtrsim \phi_*$, the A-term will be effectively suppressed compared with the usual case of an MSSM flat direction with potential stabilised by a non-renormalisable term. This is because the A-term is coming from the first term in the superpotential, equation (1), whereas the non-renormalisable term in the scalar potential is from the second term. As a result, the baryon asymmetry generated by the Affleck-Dine mechanism [17, 18] will be suppressed relative to the MSSM flat direction case. Once inflation ends, the ϕ_- field oscillates around the minimum $\langle\phi_-\rangle = \sqrt{\xi}$ [6]. The ϕ field will oscillate around the origin dominated by either the $|\lambda_1\phi^{m-1}/M^{m-3}|^2$ or the $|\langle\phi_-\rangle|^2 g^2\xi|\phi^n/\phi_c^n|^2$ term, depending on the amplitude. While the amplitude of the ϕ oscillation is large, the energy density of ϕ will decrease more rapidly than that of ϕ_- ($V \propto \phi^d$ implies that $\rho \propto a^{-6d/(d+2)}$, with $d \geq 6$ for ϕ oscillations and $d = 2$ for ϕ_- oscillations), so the Universe initially becomes ϕ_- dominated. In the following we will assume that the ϕ_- oscillations efficiently decay into radiation. (We will comment on how our results are altered if this is not satisfied.) Due to the ϕ_- decay, the radiation produces two distinct thermal corrections to the potential equation (26). The ϕ field is expected to acquire a thermal mass term

$$h^2T^2|\phi|^2, \tag{28}$$

with h being a coupling between ϕ and a particle in the thermal bath [19] in the case where the expectation value of the field is relatively small and the radiation temperature is high enough, and also a logarithmic term

$$\alpha T^4 \ln\frac{|\phi|^2}{T^2}, \tag{29}$$

which appears at the two-loop level through the running of couplings with non-vanishing ϕ [20]. Here, α is a constant of order of 10^{-2} and its sign can be positive or negative. For

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[6] In general, the minimum of the potential is at $|\phi_-| = \xi^{1/2}\left(1 - |\phi/\phi_c|^{2n}\right)^{1/2}$. This rapidly tends to $|\phi_-| = \xi^{1/2}$ as the ϕ oscillations are damped from ϕ_c to small amplitudes.

our purpose, hereafter we consider the case that α is negative. (For a positive α, the field oscillates around the origin by either the mass m_ϕ, the thermal mass or this two-loop effect and simply decays into radiation.) The potential with the two-loop induced logarithmic potential is

$$
\begin{aligned}
V(\phi) \;=\; & m_\phi^2 \left(1 + K \ln \frac{|\phi|^2}{\Lambda^2}\right) |\phi|^2 + A m_{3/2} \frac{\lambda_1 \phi^m}{m M^{m-3}} + \text{H.c.} + \alpha T^4 \ln \frac{|\phi|^2}{T^2} \\
& + g^2 \xi |\phi_-|^2 \left|\frac{\phi^n}{\phi_c^n}\right|^2 + \left|\frac{\lambda_1 \phi^{m-1}}{M^{m-3}}\right|^2 ,
\end{aligned}
\tag{30}
$$

where we include the radiative correction to m_ϕ with the direction dependent coefficient K. For MSSM flat directions which do not include a large top quark component, $K \simeq -10^{-2}$ [21, 22]. However, a large top squark component can drive K to positive values.

A negative K is the source of the spatial instability which leads to Q-ball formation in the gravity mediated SUSY breaking model [21]. For a negative α, as shown in Ref. [23], the thermal mass term cannot appear because of a relatively large expectation value of the field. Here, the ϕ field is trapped with nonvanishing value by the thermal logarithmic term, equation (29), and the non-renormalizable term, until the temperature decreases to a certain value. As the temperature falls, the expectation value of ϕ becomes small. When the ϕ becomes as small as

$$
|\phi_{\text{os}}|^2 \simeq \frac{(-\alpha)T^4}{m_\phi^2} ,
\tag{31}
$$

ϕ starts to oscillate around the origin with the angular momentum in the ϕ space induced by A-term, which is equivalent to the charge density (baryonic and/or leptonic) carried by ϕ [17]. Provided that the reheating by the ϕ_- decay is completed before ϕ starts to oscillate, we find from equation (31) that

$$
\left.\frac{\rho_\phi}{\rho_{\text{R}}}\right|_{t_{\text{os}}} \simeq \frac{30(-\alpha)}{\pi^2 g_*}.
\tag{32}
$$

Here, g_* is the effective total degrees of freedom of the relativistic species in the radiation. Since the ratio in equation (32) is of order of 10^{-4}, the ϕ field oscillations (or the Q-ball density formed from the ϕ condensate if $K < 0$) soon dominates the Universe.

III. REHEATING AND BARYOGENESIS

Reheating in this model is from the decay of the extra-flat direction inflaton field. The reheating temperature will therefore depend on whether or not the flat direction condensate

fragments into Q-balls, which in turn depends on the t squark content of the flat direction [22].

A. Q-ball formation and decay

The ϕ field oscillates around the origin coherently to begin with, but there is a spatial instability of its fluctuations due to the negative K. After inhomogeneities in the field grow, the coherent ϕ fragments and, as a result, Q-balls are formed [21, 24]. Here we briefly summarize properties of Q-balls in gravity mediated SUSY breaking models. The radius of a Q-ball, R, is estimated as $R^2 \simeq 2/(|K|m_\phi^2)$ [21]. By numerical calculations, it was shown that almost all the produced charge is stored inside Q-balls, and that a good fit to the Q-ball charge is

$$Q \simeq \bar{\beta} \left(\frac{|\phi_{\rm os}|}{m_\phi} \right)^2 \epsilon_Q \tag{33}$$

with

$$\epsilon_Q = \begin{cases} \epsilon & \text{for} \quad \epsilon \gtrsim \epsilon_{\rm c} \\ \epsilon_{\rm c} & \text{for} \quad \epsilon < \epsilon_{\rm c} \end{cases} , \tag{34}$$

and

$$\epsilon \equiv \left. \frac{n_q}{n_\phi} \right|_{t_{\rm os}} \simeq 2q|A| \left(\frac{m_{3/2}}{m_\phi} \right) \sin\delta \times {\rm Min} \left[\left(\frac{\lambda_1}{\lambda_{1\,*}} \right), 1 \right] \left(\frac{m_\phi}{H_{\rm os}} \right) , \tag{35}$$

where δ is the CP violating phase, $\epsilon_{\rm c} \simeq 10^{-2}$ and $\bar{\beta} = 6 \times 10^{-3}$ [25]. (For $\epsilon < \epsilon_c$ the condensate will fragment to pairs of oppositely charged Q-balls.) The last two factors in equation (35) are, respectively, the suppression of the baryon asymmetry due to the effective suppression of the A-term relative to the non-renormalisable term once $\phi \gtrsim \phi_*$ (where $\lambda_{1\,*}$ is defined below), and the enhancement due to $H_{\rm os} \ll m_\phi$ at the onset of ϕ oscillations, which allows the B violating A-term to act over many ϕ oscillations before expansion diminishes the A-term[7].

The decay temperature of Q-ball is given by [26]

$$T_{\rm d} \simeq 1\sqrt{f_{\rm s}} \left(\frac{m_\phi}{1\,{\rm TeV}} \right)^{1/2} \left(\frac{10^{20}}{Q} \right)^{1/2} {\rm GeV}, \tag{36}$$

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[7] In the case where the ϕ_- field does not decay efficiently to radiation, the Universe after inflation will be dominated by ϕ_- oscillations and onset of oscillations will be typically determined by an order H^2 correction to the ϕ mass squared due to non-minimal Kähler interactions of the form $|\phi_-|^2|\phi|^2$. In this case $m_\phi \approx H_{\rm os}$ in Eq. (35).

where $10^3 \gtrsim f_{\rm s} \geq 1$ is the enhancement factor in the decay if Q-balls can decay into final states consisting purely of scalar particles. Since Q-balls come to dominate the Universe in our scenario, the decay temperature gives the reheating temperature at the onset of radiation dominated Universe. The resultant emitted charge to entropy ratio is given by

$$\frac{n_q}{s} = \frac{3}{4}\frac{T_{\rm d}}{m_\phi}\epsilon. \tag{37}$$

B.   Affleck-Dine baryogenesis

The baryon asymmetry is generated by the B and CP violating A-term when ϕ starts to oscillate around the origin [17]. As usual in D-term inflation, there is no order H correction to the A-term before ϕ starts oscillating, since $\phi_+ = 0$ throughout [27]. In addition, in the case of a non-singlet inflaton there can be no linear coupling of the inflaton I to superpotential monomials W in the Kähler potential of the form $I^\dagger W$, which would generate an order H A-term correction [18]. Therefore the phase of the inflaton relative to the A-term at the onset of ϕ oscillations, θ, is determined by its initial random value during inflation, in which case $\sin\delta \approx (\sin 2\theta)/2$. This phase gives the CP violating phase required for Affleck-Dine baryogenesis, with $n_B \propto \theta$ for θ small compared with 1. In the estimation of the resultant baryon asymmetry produced by the Affleck-Dine mechanism, the important quantity is the amplitude of the AD field when it starts to oscillate, $\phi_{\rm os}$. For $\phi_{\rm os} \gtrsim \phi_*$, the amplitude is given by

$$m_\phi^2 \simeq ng^2\xi^2 \left|\frac{\phi_{\rm os}^{n-1}}{\phi_{\rm c}^n}\right|^2 , \tag{38}$$

where we assume $|\phi_-|^2 = \xi$. On the other hand, for $\phi_{\rm os} \lesssim \phi_*$, the amplitude is given by

$$m_\phi^2 \simeq (m-1)\left|\frac{\lambda_1\phi_{\rm os}^{m-2}}{M^{m-3}}\right|^2 . \tag{39}$$

The former applies in the case of a small $\lambda_1 \lesssim \lambda_{1\,*}$, with

$$\lambda_{1\,*}^{(n-1)} \equiv \left(\frac{\sqrt{g^2\xi^2}}{|\phi_{\rm c}|^n}\right)^{m-2} \left(\frac{m_\phi^2}{n}\right)^{\frac{n-m+1}{2}} M^{(m-3)(n-1)}, \tag{40}$$

while the latter corresponds to a large $\lambda_1 \gtrsim \lambda_{1\,*}$. For the case $\lambda_1 \lesssim \lambda_{1\,*}$ we obtain

$$\frac{|\phi_{\rm os}|^2}{m_\phi^2} = \left(\frac{1}{ng^2\xi^2}\frac{|\phi_{\rm c}|^{2n}}{m_\phi^{2(n-2)}}\right)^{\frac{1}{n-1}}. \tag{41}$$

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Equation (33) then gives

$$Q \simeq \bar{\beta} \left(\frac{1}{ng^2\xi^2} \frac{|\phi_c|^{2n}}{m_\phi^{2(n-2)}} \right)^{\frac{1}{n-1}} \epsilon_Q. \tag{42}$$

The expansion rate at the onset of ϕ oscillations during radiation domination can be obtained from equations (31) and (41),

$$\frac{H_{os}}{m_\phi} = \left(\frac{\pi^2 g_*}{90\alpha} \right)^{1/2} \frac{|\phi_{os}|}{M}, \tag{43}$$

with

$$\frac{|\phi_{os}|}{M} = \left(\frac{\sqrt{n}m_\phi}{\lambda_2 \xi^{1/2}} \right)^{\frac{1}{n-1}}. \tag{44}$$

For $n = 4$ this gives,

$$\frac{H_{os}}{m_\phi} \approx 4 \times 10^{-4} \alpha^{-1/2} \lambda_2^{-1/3} \left(\frac{m_\phi}{1 \text{ TeV}} \right)^{1/3} \left(\frac{4 \times 10^{15} \text{ GeV}}{\xi^{1/2}} \right)^{1/3}. \tag{45}$$

Therefore $H_{os}/m_\phi \approx 10^{-3}$. This justifies neglect of H corrections to the soft SUSY breaking terms at the onset of ϕ oscillations.

For $n = 4$, equation (42) becomes

$$Q \simeq 1.1 \times 10^{21} \left(\frac{0.1}{g} \right)^{2/3} \left(\frac{4 \times 10^{15} \text{ GeV}}{\xi^{1/2}} \right)^{4/3} \left(\frac{|\phi_c|}{0.3M} \right)^{8/3} \left(\frac{1 \text{TeV}}{m_\phi} \right)^{4/3} \epsilon_Q. \tag{46}$$

Then from equations (36), (37) and (46), and using $m_\phi/H_{os} \approx 10^3$, the decay temperature and baryon asymmetry are given by

$$T_d \simeq 3\sqrt{f_s} \left(\frac{0.1}{\sqrt{\epsilon_Q}} \right) \left(\frac{m_\phi}{1 \text{TeV}} \right)^{7/6} \left(\frac{0.1}{g} \right)^{-1/3} \left(\frac{4 \times 10^{15} \text{ GeV}}{\xi^{1/2}} \right)^{-2/3} \left(\frac{|\phi_c|}{0.3M} \right)^{-4/3} \text{ GeV}, \tag{47}$$

and

$$\frac{n_q}{s} \simeq 2 \times 10^{-10} \left(\frac{0.1}{\sqrt{\epsilon_Q}} \right) \left(\frac{\epsilon}{10^{-7}} \right) \sqrt{f_s} \left(\frac{m_\phi}{1 \text{TeV}} \right)^{1/6} \left(\frac{0.1}{g} \right)^{-1/3} \left(\frac{4 \times 10^{15} \text{ GeV}}{\xi^{1/2}} \right)^{-2/3} \left(\frac{|\phi_c|}{0.3M} \right)^{-4/3}. \tag{48}$$

The observed baryon asymmetry is $n_q/s = (1.8 \pm 0.1) \times 10^{-10}$. Hence $\epsilon \lesssim 10^{-7}$ is necessary to account for the observed B asymmetry. The Q-ball decay temperature, which gives the reheating temperature, is in the range 3-100 GeV for $1 \leq f_s \lesssim 10^3$. For $\lambda_1 \lesssim \lambda_{1*}$, from equation (35) we have $\epsilon \approx (0.1 - 1)(\lambda_1/\lambda_{1*})(m_\phi/H_{os})\theta$. The random phase of ϕ during inflation would be expected to be of order 1; therefore in order to generate the observed baryon asymmetry we require that $\lambda_1 \approx (10^{-10} - 10^{-9})\lambda_{1*}$. With $m = n = 4$, λ_{1*} is,

$$\lambda_{1*} \simeq 3.3 \times 10^{-8} \left(\frac{m_\phi}{1 \text{TeV}} \right)^{1/3} \left(\frac{4 \times 10^{15} \text{ GeV}}{\xi^{1/2}} \right)^{-2/3} \lambda_2^{2/3}. \tag{49}$$

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Since λ_2 should not be very small if g is not very small, $\lambda_{1*} \approx 10^{-8}$ is likely. Therefore to account for the observed baryon asymmetry with $\theta \approx 1$ we must have $\lambda_1 \lesssim 10^{-17}$. This corresponds to suppression of the $QQQL$ or $u^c u^c d^c e^c$ superpotential terms by a mass scale $\Lambda \gtrsim 10^{35}$ GeV. Thus a much larger suppression is necessary for successful baryogenesis than is required by proton stability. Note that it may be possible for Q-balls to decay at a temperature greater than that of the electroweak transition if $f_s \approx 10^3$, corresponding to Q-ball decay to purely scalar final states. In this case any dangerous baryon asymmetry will be erased by $B + L$ violating sphaleron fluctuations.

C. Baryon isocurvature perturbations

The CP violating phase δ is given by the phase of ϕ during inflation relative to the A-term, which defines the real direction. Therefore in the case where reheating is via Q-ball decay, $\epsilon \approx 10^2(\lambda_1/\lambda_{1*})\theta$ implies that $\theta \approx 10^{-9}(\lambda_{1*}/\lambda_1)$ in order to have $\epsilon \approx 10^{-7}$, as required for successful baryogenesis with $f_s = 1$. Quantum fluctuations of ϕ in the phase direction will lead to baryon isocurvature perturbations, which can be large when $\theta \ll 1$. For uncorrelated baryon isocurvature perturbations, the fractional contribution to the CMB power spectrum is given by α_{BI}, where [28, 29]

$$\alpha_{\mathrm{BI}} = \left(\frac{\Omega_B}{\Omega_{\mathrm{DM}}}\right)^2 \frac{f_\theta^2 H^2}{4\pi^2 P_{\mathrm{R}} \phi^2} \ , \tag{50}$$

with $\delta n_{\mathrm{B}}/n_{\mathrm{B}} \approx f_\theta \delta\theta$. In our case $f_\theta \approx 1/\theta$. The present observational limit is $\alpha_{\mathrm{BI}} < 0.26$ [30]. With $\theta \approx 10^{-9}(\lambda_{1*}/\lambda_1)$, $\Omega_{\mathrm{DM}} = 0.23$ and $\Omega_B = 0.04$, this gives a upper bound on H/ϕ,

$$\frac{H}{2\pi\phi} \lesssim 10^{-13}\left(\frac{\lambda_{1*}}{\lambda_1}\right) \ . \tag{51}$$

$\xi^{1/2} \approx 4 \times 10^{15}$ GeV, corresponding to O(10)% cosmic strings, implies that $H = 2.7 \times 10^{12}g$ GeV. Since we are considering $\phi \approx \phi_c \approx (0.1 - 0.3)M$, we therefore have

$$\frac{H}{2\pi\phi} \approx (0.6 - 1.8) \times 10^{-6}g \ , \tag{52}$$

Thus with $g \approx 0.01 - 0.03$, the baryon isocurvature perturbation is sufficiently small if $\lambda_1/\lambda_{1*} \lesssim 10^{-5}$. The correct baryon asymmetry then requires that $\theta \gtrsim 10^{-4}$. Thus even if the initial random phase of the flat direction field could satisfy $\theta \ll 1$, the flat direction would still have to be suppressed by $\Lambda \gtrsim 10^{31}$ GeV in order to avoid large baryon isocurvature

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perturbations. For $\Lambda \approx 10^{31}$ GeV and $\theta \approx 10^{-4}$, the correct baryon asymmetry will be generated together with a potentially observable baryon isocurvature perturbation.

D.   Non-thermal dark matter

The reheating temperature is ≈ 1 GeV for the case where Q-ball decay to purely scalar final states is kinematically suppressed, such that $f_\mathrm{s} = 1$. This low reheating temperature implies that Q-balls may decay below the freeze-out temperature of neutralino LSPs, in which case Q-ball decay will also produce non-thermal LSP dark matter particles. In fact dark matter particles are often overproduced, in particular for the standard bino-like neutralino LSP. Although several ways to avoid this problem have been proposed by taking an alternative choice of the LSP [23, 31, 32, 33], perhaps the simplest ones are to assume a Higgsino-like neutralino LSP [32], or a gravitino LSP [23] with a sneutrino NLSP to escape BBN constraints [34].

E.   Reheating from flat direction condensate decay without Q-ball formation

In the case where the inflaton corresponds to a flat direction with a large t squark component, the ϕ condensate will not fragment to Q-balls since $K > 0$ [22]. In this case reheating will occur via conventional flat direction condensate decay and a higher reheating temperature is expected. For the $B - L$ conserving $u^c u^c d^c e^c$ and $QQQL$ directions, the baryon asymmetry from Affleck-Dine baryogenesis will be erased by sphaleron $B + L$ violation so long as the ϕ condensate decays at $T > T_{ew}$. In this case it is possible for the initial phase of ϕ to take its natural value, $\theta \approx 1$, without requiring a suppression of the flat direction beyond that required to evade proton decay. Assuming that ϕ oscillations dominate the energy density when the ϕ field decays to radiation, the energy density is given by $\rho \approx m_\phi^2 \phi_\mathrm{d}^2$, where ϕ_d is the amplitude of the oscillations when they decay. $|\phi_\mathrm{d}|$ is then related to the decay temperature T_d by

$$|\phi_\mathrm{d}|^2 = \frac{k_\mathrm{d} T_\mathrm{d}^4}{m_\phi^2} \quad ; \quad k_\mathrm{d} = \frac{\pi^2 g(T_\mathrm{d})}{30} \ . \tag{53}$$

For $h|\phi_\mathrm{d}| > m_\phi$, where h is the gauge or Yukawa coupling of MSSM particles to the flat direction, particles coupling to ϕ gain masses greater than m_ϕ and so the ϕ decay is kinematically suppressed. Therefore the condensate decays once $\phi \approx m_\phi/h$, assuming that $\Gamma_\mathrm{d} > H$

when this occurs. The energy density in the field at this time is $\rho \approx m_\phi^4/h^2$. Therefore the decay temperature, which is equivalent to the reheating temperature T_R, is

$$T_{\rm d} \approx \frac{m_\phi}{\left(h^2 k_{\rm d}\right)^{1/4}} \; . \tag{54}$$

With $g(T_D) \approx 200$ we find $k_{\rm d} \approx 65$. Therefore

$$T_{\rm d} \approx 1.1 \left(\frac{m_\phi}{1 \text{ TeV}}\right) \left(\frac{0.1}{h}\right)^{1/2} \text{ TeV} \; , \tag{55}$$

where the particles with the *smallest* coupling h to ϕ will dominate the decay process, so long as $\Gamma_{\rm d} > H$. Therefore $T_{\rm d} \approx 1 - 100$ TeV in this model, assuming that the smallest coupling satisfies $0.1 \gtrsim h \gtrsim 10^{-5}$. Once $h\phi < m_\phi$ the ϕ decay rate may be estimated to be $\Gamma_{\rm d} \approx h^2 m_\phi/4\pi$, so the condition $\Gamma_{\rm d} > H \approx 5T_{\rm d}^2/M$ is easily satisfied for $T_{\rm d}$ in this range.

We have assumed that the kinematic suppression of the decay rate prevents ϕ decaying until $m_\phi \gtrsim h\phi$, in which case $T_{\rm d} \lesssim 100$ TeV. We should check that ϕ decay through heavy intermediate particles cannot cause it to decay significantly earlier. The decay rate via heavy intermediate particles of mass $h\phi$ will have the generic form

$$\Gamma_{\rm d} \approx \frac{\alpha_{\rm d} m_\phi^{1+r}}{(h\phi)^r} \; , \tag{56}$$

where $\alpha_{\rm d} < 1$ is a product of couplings and phase space factors. Since there are two heavy intermediate states, $r \geq 4$ is expected. For $r = 4$, and using equation (53), (56) and $\Gamma_{\rm d} \approx H(T_{\rm d}) \approx 5T_{\rm d}^2/M$, this gives for the decay temperature

$$T_{\rm d} \approx \left(\frac{\alpha_{\rm d}}{g^4 k_{\rm d}^2 k_T}\right)^{1/10} \left(m_\phi^9 M\right)^{1/10} \approx 35 \left(\frac{\alpha_{\rm d}}{g^4 k_{\rm d}^2 k_T}\right)^{1/10} \left(\frac{m_\phi}{1 \text{ TeV}}\right)^{9/10} \text{ TeV} \; . \tag{57}$$

Thus for typical couplings, the decay through intermediate states will also result in a reheating temperature in the range $T_{\rm R} \approx 1 - 100$ TeV. It is significant that the reheating temperature, $T_{\rm R} \lesssim 100$ TeV, is naturally compatible with the thermal gravitino upper bound, $T_R \lesssim 10^6$ GeV, without any tuning of couplings. Even though the inflaton is part of the MSSM sector, it still leads to the required low reheating temperature. Since sphaleron $B + L$ violation will erase the baryon asymmetry produced by the flat direction inflaton decay, baryogenesis must occur via some other mechanism, such as Affleck-Dine baryogenesis along an orthogonal flat direction.

IV. CONCLUSIONS

We have shown that it is possible for an MSSM extra-flat direction (one suppressed by an effective mass scale much larger than the Planck mass) of the form $QQQL$ or $u^c u^c d^c e^c$ to play the role of the inflaton in a D-term inflation model. This eliminates the otherwise unmotivated singlet inflaton, reducing the number of required additional fields and so providing a minimal version of D-term inflation. The model has all the advantages of conventional D-term inflation with respect to compatibility with SUGRA and absence of fine-tuned couplings. The nature of reheating depends on whether the extra-flat direction is unstable with respect to Q-ball formation. In the case where Q-balls form, it is possible to generate the baryon asymmetry via Q-ball decay so long as the mass scale suppressing the flat direction is sufficiently large, $\Lambda \gtrsim 10^{31} - 10^{35}$ GeV, depending on the random phase θ of the flat direction scalar during inflation. With $\Lambda \approx 10^{31}$ GeV and $\theta \approx 10^{-4}$ it is possible to generate an observably large baryon isocurvature perturbation. The reheating temperature from Q-ball decay is typically in the range $3 - 100$ GeV. As this can be less than the neutralino LSP freeze-out temperature, it is also possible to produce non-thermal dark matter from Q-ball decay. In the case where the flat direction has a large t squark component, there is no Q-ball formation. In this case the reheating temperature from decay of the homogeneous flat direction condensate is in the range $1 - 100$ TeV, ensuring sphaleron erasure of the baryon asymmetry from the $B - L$ conserving directions while remaining naturally compatible with the thermal gravitino upper bound on T_R. The fact that we are able to calculate the reheating temperature in this case is a direct consequence of the inflaton being part of the MSSM sector. Since the baryon asymmetry from the flat direction is erased, the mass scale suppressing the flat direction in this case is constrained only by proton decay, $\Lambda \gtrsim 10^{26}$ GeV. We have interpreted the WMAP observation of the spectral index as being due to an order 10% CMB contribution from cosmic strings combined with a nearly scale-invariant adiabatic curvature perturbation, $n_s \approx 1$. As in conventional D-term inflation, we can simultaneously suppress the contribution of the cosmic strings to the required level while increasing n_s by considering a small enough coupling of the inflaton to the Fayet-Iliopoulos charged fields. The extra-flat direction D-term inflation model has an advantage over conventional D-term inflation in that the range of coupling which leads to an order 10% contribution from cosmic strings is much wider, making it perhaps more natural. For this solution to work, it is also

necessary to have a $U(1)$ gauge coupling that is somewhat smaller than the known gauge couplings, $g \approx 0.01 - 0.03$. A significant feature of the model is that the superpotential coupling of the monomial $QQQL$ or $u^c u^c d^c e^c$ to $\Phi_+ \Phi_-$ must be much larger than the pure monomial superpotential coupling. This feature may serve to test the compatibility of the model with an ultra-violet complete theory, as we would naively expect all the superpotential couplings of the monomial to be strongly suppressed. Finally, we note that other solutions to the cosmic string and spectral index problems are possible, for example SUGRA corrections from a non-minimal Kähler potential [35] and/or modification of the inflaton potential by other fields, such as a RH sneutrino [36].

The existence of extra-flat directions of the MSSM is one way to interpret the empirical suppression of non-renormalisable MSSM superpotential terms demanded by proton stability. It will be important to establish whether extra-flat directions can be understood in the context of an ultra-violet complete theory and to explore more generally their role and possible signatures in cosmology.

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