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Efficient Message Propagation In Dense Linear Ad-Hoc Networks

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EFFICIENT MESSAGE PROPAGATION IN DENSE LINEAR AD-HOC NETWORKS

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ABSTRACT

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In this thesis, the dynamics of the cooperative broadcast in high density 1-dimensional (1D) networks is studied using multiple levels of relays. The benefits of the cooperative broadcast in terms of transmission range and power efficiency are evaluated by approximating the network as a continuum of nodes. Two transmission protocols are studied, the single-shot transmission and the continuous source transmission. In the single-shot transmission, the source node transmits a single message, whereas in the continuous source transmission, it transmits independent messages periodically adding signal interference into the picture. For each case, the analytical expressions for successful broadcasting are presented as a function of source and relay powers, decoding threshold and the noise power. This study shows that the broadcast behavior highly depends on the path-loss exponent γ and the type of transmission, being bidirectional or unidirectional. It is shown that the message can propagate successfully to the entire network for all ranges of γ using the cooperative broadcast. Furthermore, the cooperative broadcast is compared with the non-cooperative multi-hop broadcast in terms of power efficiency.

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I. INTRODUCTION

A. Ad-Hoc Networks

Wireless networks can be divided into two major categories, infrastructure-based networks and *ad-hoc* networks. Infrastructure-based networks have a fixed backbone design in which devices communicate with a fixed access point. A well-known example is the cellular network which uses base stations as the interface to the wireless communications between users. In contrast, *ad-hoc* networks do not rely on an existing infrastructure. Instead, devices are equipped with networking capabilities and autonomously participate in transmission based on the network connectivity [1].

The benefits of *ad-hoc* networks were first discovered in the 1970s. In 1972, a project known as packet radio was initiated by DARPA, in which several communicating terminals had to communicate on a battlefield in the absence of base stations [2]. This application took advantage of the unique features of *ad-hoc* networks which are ease of deployment, speed of deployment and decreased dependence on the infrastructure, as compared to infrastructure-based networks.

Another motivation for using *ad-hoc* networks is mobility of communicating nodes. An existing link between two nodes may break due to mobility and the chance of sudden degradation of the channel quality. A self-organizing and adaptive network allows spontaneous formation and deformation of mobile networks. Each mobile node can act as a relay to support peer-to-peer communications, improving the performance while reducing the administrative cost [3].

Designing *ad-hoc* mobile networks is quite challenging due to constant change of network topology over time, limited power capacity, limited wireless bandwidth and the presence of varying channel quality. To catch up with frequent link changes, the event updates should be sent quite often, introducing considerable signal overhead. Furthermore, the routing tables may not always converge in response to the sudden changes of the network [3].

Packet collision is another issue in *ad-hoc* networks. Broadcasting different messages by the neighboring nodes causes interference, reducing the signal to interference plus noise (SINR) ratio at the receiving nodes. The throughput capacity of a fixed *ad-hoc* network decreases as the total number of nodes increases, making the interference a limiting factor in high density *ad-hoc* networks. The amount of interference can be reduced by setting a power constraint on the transmitting relays so that interference from nodes far away becomes negligible [4].

However, the power allocation in *ad-hoc* networks is application-specific and depends on the type of devices. Most networks include battery powered devices which have limited power capacity for wireless transmission, reception and retransmission. Although interference becomes limited by reducing the transmit power, the message propagation to the destination may fail due to pathloss attenuation over the link. This issue has motivated a great deal of research on power efficient protocols and better power management techniques.

B. Cooperative Communication

The above-mentioned challenges in *ad-hoc* networks can be overcome by means of cooperation. Rather than competing for spectral resources as in traditional point-to-point communication, neighboring nodes can share their antennas in relaying the same message to the desired destination. In fact, this technique exploits the collisions of the same copy to increase the received power for a fixed transmit power, or alternatively to reduce the transmit power for a fixed received power. Consequently, cooperative communication enhances the system performance by achieving transmit spatial diversity gains without using additional antennas [5].

The idea of cooperation in communication was first introduced in 1979 in [6] by considering a relay channel including a source node, a relay node and a destination

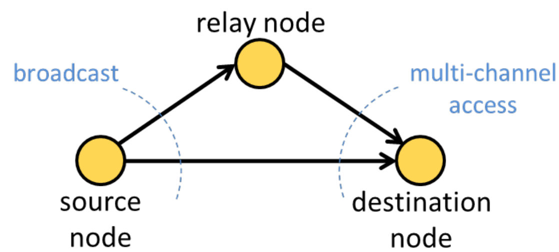


Fig. 1: Cooperative communication: relay channel model

node as shown in Figure 1. This model includes a broadcast channel where the source transmits signal to the other two nodes, and a multi-access channel where the destination receives signal from both the relay and the source. This is the basic illustration of cooperation which was proved to improve the capacity of AWGN relay channel compared to source-destination channel [6].

However, the capabilities of cooperative communication are not limited to this model. Adding relay transmission technology to the picture, cooperative communication provides a distributed virtual Multiple-Input Multiple-Output (MIMO) system. The first simple but effective user cooperation protocol was proposed by [7] in 1998, which ignited an extensive amount of research worldwide. Many have made outstanding contributions to cooperative communication research by proposing a variety of low-complexity protocols, e.g. in [8] and also coding strategies, e.g. in [9], to fully exploit spatial diversity, to improve capacity or to optimize the power efficiency.

Research on cooperation targets different applications and network setups. An example of cooperation is the periodic broadcast of routing updates and control messages which should be delivered successfully to the entire network. The network operation may be impacted by the failure to transmit these messages to the entire network. Cooperation can enhance the broadcast operation by enabling a group of nodes to retransmit the same message simultaneously upon decoding it successfully in the previous time slot. As a result, the message is more likely to reach the entire network without increasing the transmit power [10].

In high density networks, however, the advantages of cooperative communication over direct transmission or classical multi-hopping are more difficult to determine. In fact, obtaining analytical results for such networks is tedious especially when the number of nodes exceeds four. An extensive analysis of cooperation protocols has been done for 2-D (planar) high density networks in [11], [12]. In these works, authors model the network with a continuum of nodes and observe a phase transition in the propagation of the packets. Using the same technique, they analyze power efficiency of cooperative protocols over classical multi-hopping in [13], [14].

C. *Objectives*

In this thesis, the analytical study of cooperative broadcast in 1-D high density networks is performed in terms of transmission range and power efficiency. The main objective is to find the conditions for the successful propagation of the message to the entire network when a source node initiates the transmission with a decentralized structure. The dynamics of cooperative broadcast is formulated as a function of the network parameters and the channel environment. The propagation is subject to pathloss attenuation while assuming no fading. According to our findings, the broadcast behavior is highly dependent on the pathloss exponent, γ, which can be divided into three separate ranges: $\gamma < 1$, $\gamma = 1$ and $\gamma > 1$. For all the ranges of γ, the corresponding conditions for successful propagation are found and presented.

Different transmission protocols are considered in this thesis. First, single-shot transmission is studied in which the source node transmits a single message. For each range of γ, the message propagates to the entire network as long as the network parameters meet the corresponding condition found for that range of γ. Two different transmission schemes are analyzed; unidirectional transmission and bidirectional transmission protocols. In the former the message is relayed in one direction while in the latter it is relayed in two directions along the 1-D network. Under both transmission protocols, the power efficiency of cooperative broadcast is examined as compared to the non-cooperative multihop broadcast.

In the next step, continuous source transmission is studied in which the source node transmits independent messages continuously adding feed-forward interference to the picture. A new parameter is defined, named as "transmission period" denoted by *i* which is the number of time slots to transmit a single message by the source node. Choosing a smaller value of i results in a higher number of independent messages transmitted at a time and hence a greater amount of interference at each node. The worst case scenario in terms of interference is associated with $i = 1$, i.e. the source node along with the following relays transmit different messages every time slot. Analyzing this scenario provides an upper bound on the amount of interference in any cases. The presented conditions for successful propagation will also guarantee the successful propagation for any value of transmission period i. As with the single-shot transmission, the continuous source transmission exhibits different dynamics depending on the range of γ. Furthermore, the analysis includes both unidirectional transmission and bidirectional transmission protocols, for each the power efficiency of cooperative broadcast is also determined as compared with the non-cooperative multihop broadcast.

D. Motivation and Applications

Linear *ad-hoc* networks have many applications in monitoring and protecting systems, such as in above the ground as well as the underwater pipelines. The analysis of different ranges of pathloss exponent γ can be useful for such different environments. Furthermore, monitoring AC powerlines, both overhead and underground is another area of interest. In this application, the sensors acquire their power from the power line, and as a result the power efficiency is not the main concern. Nonetheless, in all applications, the propagation of the message to the entire network is of great interest especially for control and emergency messages and updates.

E. Organization

The organization of the thesis is as follows. In Chapter II, the system is analyzed under single-shot transmission and in Chapter III, the system is analyzed under continuous source transmission. Finally, the conclusion is drawn in Chapter IV.

II. Analysis of Cooperative Communication In One-dimensional Dense Ad-hoc Networks

A. Introduction

Cooperative transmission schemes have been known to be advantageous over direct transmission or classical multi-hopping in wireless adhoc networks [15, 16]. However, obtaining analytical results for such networks is very tedious especially when the number of nodes exceeds four. An approach to this issue is to look at asymptotes such as high SNR, high node density, etc. An extensive analysis of cooperation protocols has been done for 2D high-node-density networks in [11, 12] for both broadcasting and unicasting. In these works, authors model dense wireless networks using a continuum limit and observe a phase transition in the propagation of the packets, which is dependent on the node transmit powers and the decoding threshold. The authors extend their 2-D analysis further in [13] by designing and analyzing of power efficient cooperative schemes. The energy efficiency of cooperative protocols over classical multi hopping have been also analyzed in [14, 17].

In [18], the broadcast probability in cooperative 1-D and 2-D extended networks is analyzed for different ranges of pathloss exponent. The authors assume finite node density networks with infinite size. They show that the broadcast probability (the probability that the entire network gets the message correctly) is zero for pathloss exponents $\gamma > 1$ for any node density, while it is strictly greater than zero for $\gamma < 1$. In this work, the broadcast dynamics of cooperative transmission in 1-D networks such as vehicular networks are analyzed based on the continuum approach [11, 12] in 1-D networks with high-node density. We show that the broadcast probability for $\gamma > 1$ is always one for infinite density whereas the broadcast probability for $\gamma \leq 1$ can be zero under some condition.

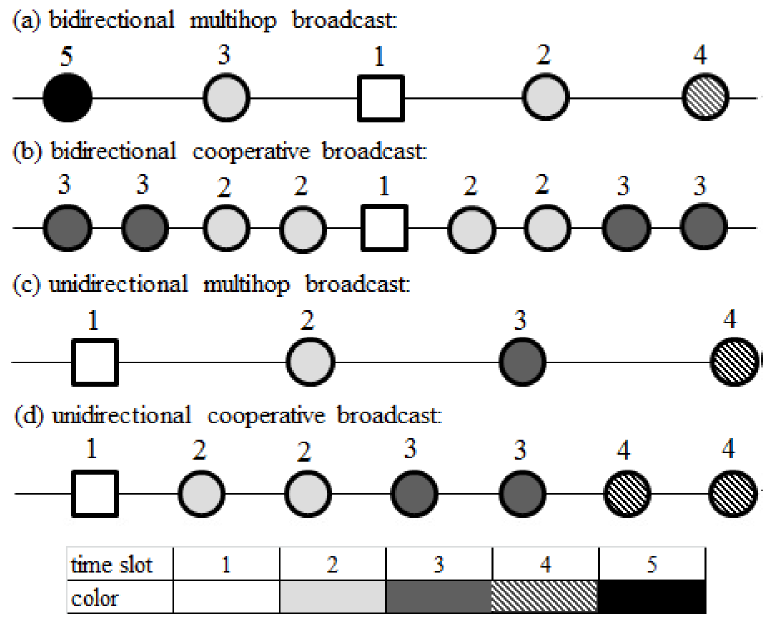


Fig. 2: Unidirectional and bidirectional multi-hop and cooperative broadcast schemes. In each figure, the order of broadcast is shown by numbers.

B. System model

1) Transmission protocol: Vehicular network can be modeled as a 1-D linear network comprised of a source node and multiple relay nodes which are distributed in a random and uniform fashion. Fig. 2 illustrates the linear network under multihop and cooperative broadcasts. In cooperative broadcast, shown in Fig. 2(b) and (d), the source node broadcasts a single packet and upon successful reception by the relays, the packet is retransmitted simultaneously by the first group of relays in the next time slot. Each group of nodes transmitting together is referred to as one level of nodes. Henceforth, the set of nodes which receive the message from level one ($\mathcal{L}_1$) form the second level ($\mathcal{L}_2$) and transmit together. The packet propagates level by level until it is heard by all of the existing nodes. On the other hand, in the multi hop broadcast shown in Fig. 2(a) and (c), only the farthest hop that receives the packet transmits. The same packet cannot be transmitted by more than one node at any given time slot to avoid interference.

Cooperative and multi hop broadcasts are analyzed under unidirectional and bidirectional schemes as shown in Fig. 2. According to this figure, under unidirectional transmission, the message is to propagate to the right-hand side of the source node. Under bidirectional transmission, the message propagates in both directions. Hence, in bidirectional cooperative broadcast, $\mathcal{L}_1$ is the union of $\mathcal{L}_1^+$ in the right-hand side and $\mathcal{L}_1^-$ in the left-hand side of the source node:

$$\mathcal{L}_k = \mathcal{L}_k^+ \bigcup \mathcal{L}_k^- \tag{1}$$

At each relay node, the received SNR (signal-to-noise ratio) is estimated using a training preamble in the message. Each node determines locally whether to broadcast the message to the following nodes. It compares the received SNR with a pre-assigned SNR threshold and only if it finds it greater or equal, it retransmits the message. The SNR threshold is denoted by parameter τ, hereafter.

We also assume that the power of simultaneously transmitted packets at the receiving node is equal to the sum of individual powers. This assumption is well suited for orthogonal channels or relays which use orthogonal space-time codes. The path-loss attenuation model adopted in this paper is the simplified pathless model [1]. The received power at distance d from a node with transmit power P_r is

$$P_r K (\frac{d_0}{d})^\gamma \tag{2}$$

where K and d_0 are assumed to be unity, just for simplicity. Thus, the power received by a single node j after the transmission of level $\mathcal{L}_{k-1}$ nodes is

$$P_j = \sum_{i \in \mathcal{L}_{k-1}} \frac{P_r}{(d_j - d_i)^\gamma} \tag{3}$$

where d_j and d_i are the relative distance from the source to node j and node i, respectively. Note that all the relay nodes transmit with equal power, denoted by P_r. The source node, however, has a distinct transmit power, referred to as P_s. We will denote the received noise power with N_0. The nodes which satisfy SNR condition ($P_j/N_0 > \tau$) will form the next level, i.e. $\mathcal{L}_k$. This is based on the assumption that the nodes with sufficient SNR will be able to decode the message correctly using an appropriate channel code.

It should be noted that the path-loss exponent γ depends on the channel environment, and its value is normally in the range of 2 - 4. Pathloss exponent $\gamma = 2$ corresponds to propagation in free space while $\gamma = 4$ corresponds to lossy environments such as wireless networks. In vehicular communications, γ has been reported to be in the range of 2.75 - 4. [10]. In this study, we consider a wide range of γ in three general categories; $\gamma < 1$, $\gamma = 1$ and $\gamma > 1$. Each range results in a unique network behavior which is studied extensively, with respect to parameters P_r, P_s and τ. Our analysis is based on the continuum model which is introduced in the following subsection.

2) Continuum model: Continuum model has been initially proposed in [1] to analyze 2-D high-node-density networks. Let $\mathbb{L} \triangleq \{x : x \leq L\}$ represent the linear network. Let $\rho = N/L$ be the node density (node/unit length). The continuum model assumes high density setup as N and ρ go to infinity while the total power $P_r N$ is fixed. Rather than working with P_r which approaches zero as $N \to \infty$, we define relay power per unit length,

$$\bar{P}_r \triangleq \frac{P_r N}{L} = \rho P_r \tag{4}$$

which is a fixed parameter. As $N \to \infty$, every infinitesimal length dx contains ρdx relays with power P_r, i.e. $P_r \rho dx = \bar{P}_r dx$ in total. As a result, Eqn. 3 is transformed to

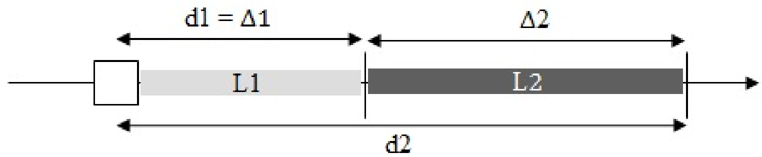


Fig. 3: Depiction of d_1, d_2 and Δ_2

an integral over level L_{k-1}

$$P_y = \int_{x \in L_{k-1}} \frac{\bar{P}_r dx}{(d_y - d_x)^\gamma} \tag{5}$$

where y is any arbitrary location for a receiving node, and L_{k-1} denotes the location of the level $\mathcal{L}_{k-1}$ nodes in the continuum limit. We refer the readers to works [11, 12] for further details of continuum approximation for 2-D networks. In following section, we analyze transmission behavior under single-shot transmission protocol with unidirectional and bidirectional schemes and different ranges of γ for 1-D networks.

C. Single-shot transmission

In this transmission mode, the source node goes to silence after transmitting a packet until the message completes its propagation to the entire network. Thus, sending a new packet will not cause any interference with previous packet anywhere along the network. Transmission behavior is studied by finding the distanced for level boundaries (d_i's) as the packet flows through the network (see Fig. 3) in the continuum limit. This is done for L_1 using Eqn. 2 simply as

$$d_1 = \left(\frac{P_s}{N_0 \tau} \right)^{1/\gamma} \tag{6}$$

where d_1 is the distance from the source node to the outer boundary of L_1 (L_1^+ and L_1^- for bidirectional transmission). Since, the protocol results in symmetric transmission behavior on both directions for bidirectional transmission, we focus our analysis only on one direction. We also define the length of level L_k as

$$\Delta_k = d_k - d_{k-1} \tag{7}$$

Thus, $d_1 = \Delta_1$ both used interchangeably throughout this paper. We now present following subsections for different ranges of γ. In the following, the proof of some of the Lemmas are avoided for brevity.

1) Cooperative transmission under pathloss exponent $\gamma < 1$: We analyze broadcast behavior under unidirectional and bidirectional transmission schemes.

Theorem 1. *Under unidirectional transmission with $\gamma < 1$, the source node broadcasts its message to the entire network, if and only if*

$$(\frac{P_s}{\tau})^{1/\gamma} \geq \left(\frac{\tau N_0(1-\gamma)}{(2^{(1-\gamma)}-1)\bar{P}_r}\right)^{1/1-\gamma} \tag{8}$$

Proof. See Appendix 1. □

Remark. *If equality in Eqn. 8 is achieved, then for all k*

$$\Delta_k = \left(\frac{\tau N_0(1-\gamma)}{(2^{(1-\gamma)}-1)\bar{P}_r}\right)^{1/1-\gamma} \tag{9}$$

which is referred to as the critical condition for continuous flow.

Lemma 1. *If Eqn. 8 is satisfied with strict greater sign, Δ_k in the unidirectional scheme grows exponentially with respect to Δ_{k-1} within the bounds*

[1]

$$\left(\frac{\bar{P}_r}{\tau N_0}\Delta_{k-1}(1-\frac{\gamma}{2})\right)^{1/\gamma} < \Delta_k < (\frac{\bar{P}_r}{\tau N_0}\Delta_{k-1})^{1/\gamma} \tag{10}$$

Compared to unidirectional scheme, the received power in bidirectional protocol

[1]Under this condition, Δ_k increases with a faster rate than exponentially with respect to Δ_1 as $\Delta_k \propto \Delta_1^{1/\gamma^k}$ since $a\log(\frac{\bar{P}_r}{\tau N_0}(1-\frac{\gamma}{2})) + b\log(\Delta_1) < \log(\Delta_k) < a\log(\frac{\bar{P}_r}{\tau N_0}) + b\log(\Delta_1)$ where $a = \sum_{n=1}^{k-1}(\frac{1}{\gamma})^n$ and $b = (\frac{1}{\gamma})^k$.

becomes greater as each relay node receives packet from levels of both directions. Unlike unidirectional, Δ_k is not always monotonic and $\Delta_2 > \Delta_1$ does not always lead to $\Delta_3 > \Delta_2$. However, we can prove that $\Delta_3 > \Delta_2$ results in $\Delta_k > \Delta_{k-1}$ for all following levels.

Theorem 2. *Under bidirectional transmission with $\gamma < 1$, if $\Delta_3 > \Delta_2 > 0$ then $\Delta_k > \Delta_{k-1}$ for $k > 3$ which implies the packet propagation to the entire network.*

Proof. A brief proof is provided Appendix 2. □

Lemma 2. *Assuming the continuous flow is achieved, in other words, Theorem 2 condition is satisfied, then d_k grows exponentially with respect to Δ_{k-1} as*

$$d_k = \left(\frac{2\bar{P}_r}{\tau N_0}\Delta_{k-1}\right)^{1/\gamma} + O\left((\frac{d_{k-1}}{d_k})^3, (\frac{d_{k-2}}{d_k})^3\right) \tag{11}$$

2) Cooperative transmission under pathloss exponent $\gamma = 1$: Transmission under $\gamma = 1$ shows similar behavior as transmission with $\gamma < 1$ in the sense that Δ_k grows to infinity under some condition. The growth rate of Δ_k, however, is not exponential anymore. Next, we define a new parameter, which will be used in the following. Let

$$\mu := e^{\tau N_0/\bar{P}_r}.$$

Theorem 3. *Under unidirectional transmission with $\gamma = 1$, for $k > 1$,*

$$\Delta_k = \left(\frac{1}{\mu - 1}\right)\Delta_{k-1}.$$

In addition, as $k \to \infty$,

$$\begin{aligned}
\Delta_k &\to \infty \quad \text{if} \quad \mu < 2, \\
\Delta_k &\to \Delta_1 \quad \text{if} \quad \mu = 2, \\
\Delta_k &\to 0 \quad \text{if} \quad \mu > 2.
\end{aligned}$$

Proof. See Appendix 3.[2] $\square$

Theorem 4. *Under the bidirectional transmission with $\gamma = 1$, the ratio $a_k := \Delta_k/\Delta_{k-1}$ converges as*

$$a_k = \frac{\Delta_k}{\Delta_{k-1}} \to a_\infty := \frac{1}{\mu - 1} + \frac{\sqrt{1 - (\mu - 1)^2}}{\mu - 1}$$

Proof. The proof is avoided for brevity. $\square$

Remark. *If we compare the bidirectional transmission and unidirectional transmission in the limit, that is as $k \to \infty$, as expected the bidirectional transmission moves faster since for bidirectional transmission $a_\infty > \frac{1}{\mu-1}$.*

3) Cooperative Transmission under pathloss exponent $\gamma > 1$:

Theorem 5. *Under both bidirectional and unidirectional transmissions with $\gamma > 1$,*

$$\lim_{k\to\infty} \Delta_k = \Delta_\infty := \left[\frac{\bar{P}_r(1 - 2^{1-\gamma})}{\tau N_0(\gamma - 1)}\right]^{1/\gamma - 1}, \tag{12}$$

and consequently the message always flows to the entire network.

Proof. The proof is avoided for brevity. $\square$

Remark. *Note that Δ_∞ is the same expression as Δ_k in the critical condition with $\gamma < 1$ in Eqn. 9. According to Eqn. 81 the length of Δ_∞ is dependent on $\bar{P}_r$, τ, N_0 and γ parameters.*

D. Power efficiency comparison of different broadcast schemes

In this section we compare unidirectional and bidirectional cooperative broadcasts with multihop noncooperative broadcast in terms of power efficiency. Since in multihop broadcast, the same packet is transmitted only by a single node, under bidirec-

[2] According to Theorem 3, Δ_k increases (decreases) exponentially with respect to Δ_1 since $\Delta_k = \left(\frac{1}{\mu-1}\right)^{k-1}\Delta_1$ if μ is smaller (greater) than 2.

tional transmission, multihop broadcast takes double the time to propagate over the same length of network than cooperative broadcast.

Assuming that each hop (covering a fixed length of r) has the same power consumption in a linear network, the power consumption over each hop

$$P_{noncoop} = \tau N_0 r^{\gamma}. \tag{13}$$

In the following subsections, we compare the power consumption of cooperative and noncooperative multihop schemes under both unidirectional and bidirectional transmissions. In order to make a fair comparison, we fix the propagation speed and the length of the network for cooperative and multihop broadcast schemes.

1) Power consumption under unidirectional transmission: For the unidirectional transmission, we set the transmission radius r of noncooperative scheme equal to the limiting step size Δ_∞ of the cooperative scheme. Hence the power consumption over the length of r for the noncooperative scheme can be expressed as

$$P_{noncoop}^{uni} = \tau N_0 r^{\gamma} = \tau N_0 \Delta_\infty^{\gamma}. \tag{14}$$

For cooperative broadcast with $\gamma < 1$ and $\gamma > 1$, the expression for total power consumption over one time-slot can be found as

$$P_{coop}^{uni} = \bar{P}_r \Delta_\infty = \frac{\tau N_0 \Delta^{\gamma} (\gamma - 1) 2^{\gamma-1}}{2^{\gamma-1} - 1} \tag{15}$$

The power gain of cooperative broadcast is

$$\begin{aligned} Gain^{uni} &= \frac{P_{noncoop}^{uni} - P_{coop}^{uni}}{P_{noncoop}^{uni}} \\ &= \begin{cases} 1 - \frac{(\gamma-1)2^{\gamma-1}}{2^{(\gamma-1)}-1} & \gamma \neq 1 \\ 1 - \frac{1}{\ln(2)} & \gamma = 1 \end{cases} \end{aligned} \tag{16}$$

which is negative for all values of γ suggesting that noncooperative broadcast is more efficient than cooperative broadcast for all pathless exponents under the unicast transmission. This result is very intuitive; Since the message flows in one direction, placing all the available power at each hop to the node whose furthest (i.e. multi hopping) will perform much better than distributing that power to the nodes which are more distant to the next group (i.e. cooperative).

2) Power consumption under bidirectional transmission: Considering the fact that cooperative broadcast flows double times faster than noncooperative broadcast under bidirectional scheme, we assign the transmission radius of the noncooperative scheme twice the step size of cooperative scheme, i.e. $r = 2\Delta_\infty$. Hence, the power consumption of noncooperative scheme can be expressed as

$$P^{bi}_{noncoop} = \tau N_0 r^\gamma = \tau N_0 (2\Delta_\infty)^\gamma. \tag{17}$$

For the bidirectional transmission, the power gain of cooperative broadcast is

$$\begin{aligned} Gain^{bi} &= \frac{P^{bi}_{noncoop} - P^{bi}_{coop}}{P^{bi}_{noncoop}} \\ &= \begin{cases} 1 - \frac{(\gamma-1)2^{\gamma-1}}{(2^{(\gamma-1)}-1)2^\gamma} & \gamma > 1 \\ 1 - \frac{1}{2\ln(2)} & \gamma = 1 \end{cases} \end{aligned} \tag{18}$$

For $\gamma < 1$, we can use the fact that $\Delta^{bi}_k \geq \Delta^{uni}_k$ for all k, which lead to a lower bound on the $Gain^{bi}$:

$$Gain^{bi} \geq 1 - \frac{1-\gamma}{(2^{1-\gamma}-1)2^\gamma}, \quad \text{for} \quad \gamma < 1. \tag{19}$$

Equations 18 and 19 suggest that cooperative broadcast is more efficient than noncooperative broadcast under bidirectional transmission.

In Fig. 4, the expressions for $Gain^{uni}$ and $Gain^{uni}$ is plotted for the three ranges of γ. It can be observed that the gain is positive in favor of cooperative broadcast in bidirectional transmission while it is negative under unidirectional transmission.

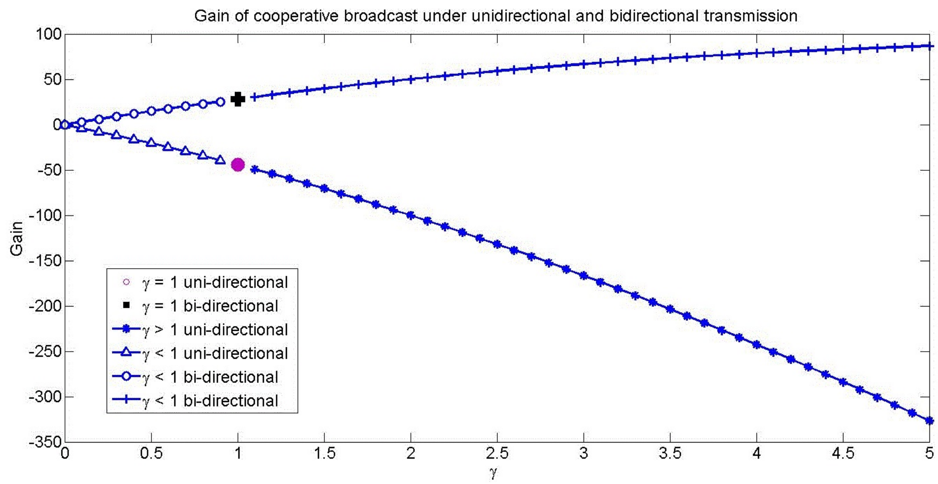


Fig. 4: The gain of cooperative broadcast compared to noncooperative multihop broadcast under unidirectional and bidirectional transmission

E. Simulation results

In this section we check the accuracy of analytical expressions for finite-density, finite-length networks. The networks consist of 20000 nodes over the length of 100, i.e. with the node density of $\rho = 200$. Fig. 5 - 8 show the normalized histogram ($:= P$) of the maximum distance from the source, x, that successfully gets the message. In Fig. 5 and 6, three cases are simulated with $\gamma = \frac{1}{2}$, while keeping P_s and N_0 fixed at unity and choosing τ from the set; 0.5, 1 and 2. Fig. 5 corresponds to unidirectional transmission while Fig. 6 corresponds to bidirectional transmission.

In Fig. 5, the middle column corresponds to $\bar{P}_r$ obtained to create critical condition for continuous flow according to Theorem 1. However, we find $P \neq 1$ for $x = 100$ in this column, mainly due to testing of asymptotic behavior for a finite-density network. The right and left columns correspond to slightly lower and higher $\bar{P}_r$ values than the middle column, respectively. We can find $P = 1$ for the right column in all cases for $x = 100$. Comparing the left and the right columns, the change in broadcast behavior can be observed over the small range around the critical $\bar{P}_r$.

Fig. 5 corresponds to bidirectional transmission. The same cases are simulated and

$\bar{P}_r$ is found according to Theorem 2 which results in $\Delta_3 > \Delta_2$. As with Fig. 5, the middle column corresponds to the critical $\bar{P}_r$ while the right and the left columns show the behavior with slightly smaller and greater $\bar{P}_r$ values. Although $P \neq 1$ for the middle column for $x = 100$, $P = 1$ for the right column in all cases for $x = 100$.

Fig. 7 and 8, show the normalized histograms for $\gamma = 1$ under unidirectional and bidirectional transmission, respectively. This time, we fix $\bar{P}_r$ at unity and change P_s from 0.1 to 1 and 5. The threshold τ is found according to Theorem 3 for Fig. 7 and Theorem 4 for Fig. 8. For both cases, it results in $\tau_{critical} = \ln(2) = 0.6913$ as shown in the middle column. It should be noted that $\tau_{critical}$ is independent from P_s. The left and the right columns correspond to $\tau = 0.59$ and $\tau = 0.79$, respectively. We can observe that for values smaller and equal to $\tau_{critical}$ (the left and the middle columns), the message propagates to the entire network with $P = 1$ for $x = 100$.

We test the third category of γ with $\gamma = 2$, 3 and 4. We simulate three cases for each γ, keeping $\bar{P}_r$ and P_s constant at unity and $\tau = 0.1$, 1 and 10. In all case, Δ_k approaches to a non-zero value as expected according to our analysis, i.e. $\Delta_k \to \Delta_\infty$ as $k \to \infty$. To obtain a reliable value of Δ_∞, each scenario is simulated for 10 different random distributions of nodes and ultimately the resulted Δ_∞ is averaged. Table I and II list the averaged Δ_∞ for unidirectional and bidirectional transmissions, together with difference percentage with respect to the corresponding value given by Theorem 5.

TABLE I: Simulation cases with $\gamma = 2$, 3 and 4, under unidirectional single-shot transmission. The difference with respect to calculated values are indicated in parenthesis

γ \ τ	0.1	1	10
2	4.9786 (-0.43%)	0.5007 (0.14%)	0.0999 (-0.1%)
3	1.9341 (-0.12%)	0.6112 (-0.2%)	0.2724 (-0.54%)
4	1.4251 (-0.26%)	0.6612 (-0.3%)	0.3861 (-0.44%)

TABLE II: Simulation cases with $\gamma = 2$, 3 and 4, under bidirectional single-shot transmission. The difference with respect to calculated values are indicated in parenthesis

γ \ τ	0.1	1	10
2	5.0307 (0.6%)	0.5018 (0.36%)	0.099 (-0.03%)
3	1.9372 (0.04%)	0.6102 (-0.36%)	0.2729 (-0.36%)
4	1.4301 (0.09%)	0.6614 (-0.27%)	0.3860 (0.46%)

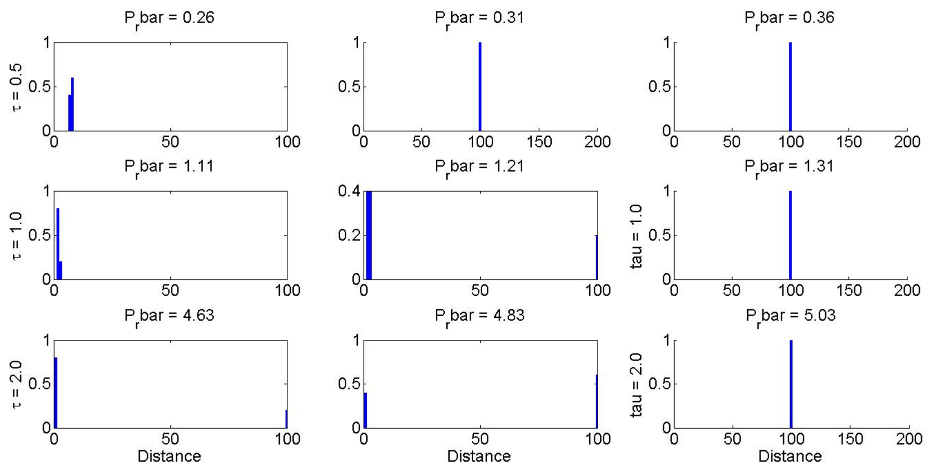


Fig. 5: Unidirectional cooperative broadcast with $\gamma = 0.5$ with $P_s = 1$, $\rho = 200$ and $\tau = 0.5$, 1 and 2

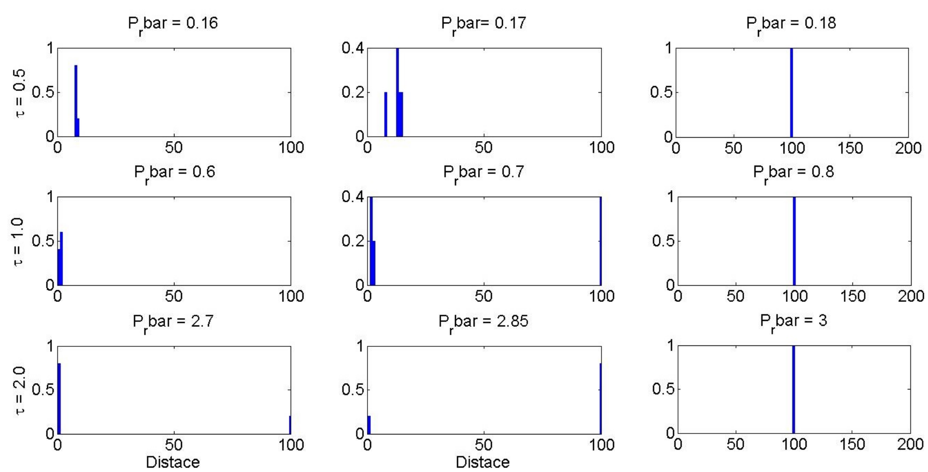


Fig. 6: Bidirectional cooperative broadcast with $\gamma = 0.5$ with $P_s = 1$, $\rho = 200$ and $\tau = 0.5$, 1 and 2

F. Conclusion

We analyzed a cooperative broadcasting scheme over multiple hops of relays in a linear network. The dynamic of single-shot transmission is analyzed for different

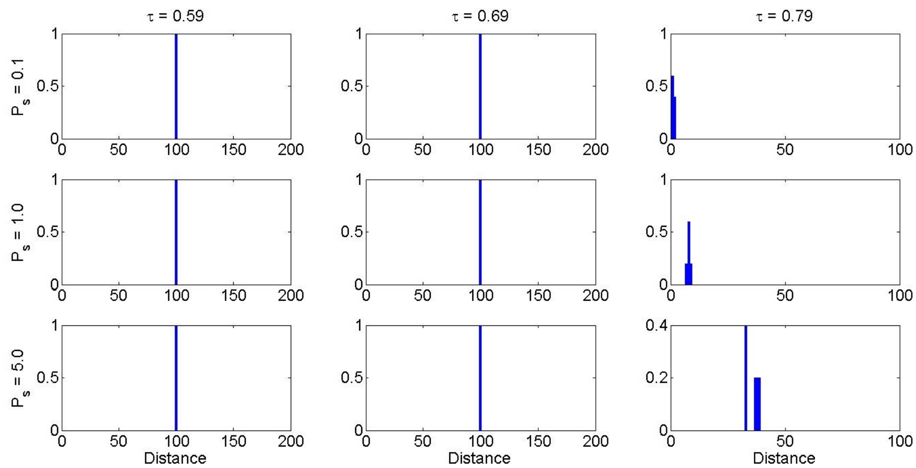


Fig. 7: Cooperative broadcast with $\gamma = 1$ with $\bar{P}_r = 1$, $P_s = 0.1$, 1 and 5

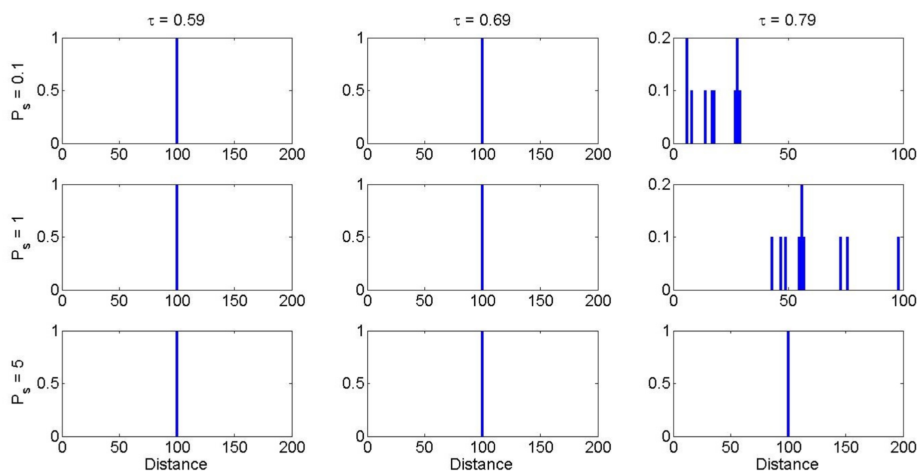


Fig. 8: Cooperative broadcast with $\gamma = 1$ with $\bar{P}_r = 1$, $P_s = 0.1$, 1 and 5

ranges of pathloss exponent γ.

Similar to 2-D network analysis in [12], we found phase transitions in the network behavior for different ranges of γ. The decoding threshold has the same critical value for $\gamma = 1$ as with $\gamma = 2$ in 2-D networks. We also found a phase transition with a decoding threshold for $\gamma < 1$ while there exists no phase transition for $\gamma > 1$. The summary of behavior of the range of cooperating nodes at each hop (Δ_k for kth hop) is listed in Table III.

We also compared the power efficiency of this cooperative scheme with the classical multihop broadcast. It was found that multihop broadcast demonstrates more efficient behavior under unidirectional transmission, whereas cooperative broadcast proves to be more efficient for all ranges of γ if bidirectional transmission is employed.

TABLE III: Broadcast behavior of cooperative linear network

	Unidirectional	Bidirectional
$\gamma < 1$	$\Delta_k \geq \Delta_{k-1}$ or $\Delta_k \to 0$ Δ_k can grow *exponentially* with respect to Δ_{k-1}	$\Delta_k > \Delta_{k-1}$ or $\Delta_k \to 0$ Δ_k can grow *exponentially* with respect to Δ_{k-1}
$\gamma = 1$	$\Delta_k \geq \Delta_{k-1}$ or $\Delta_k \to 0$ Δ_k can grow *linearly* with respect to Δ_{k-1}	$\Delta_k > \Delta_{k-1}$ or $\Delta_k \to 0$ Δ_k can grow *linearly* with respect to Δ_{k-1}
$\gamma > 1$	$\Delta_k \to c$ under no condition	$\Delta_k \to c$ under no condition

Another interesting problem is the dynamics of linear transmission when the source nodes sends new packets carrying different messages periodically. This causes interference between the source packets carrying different messages, and the analysis in this case is considerably complicated than that of the single shot transmission provided in this paper. Furthermore, we believe that the multihop diversity, that is accumulating power from $m > 1$ preceding levels, will improve the performance of cooperative broadcasting in linear networks.

APPENDICES

G. *Proof of Theorem 1*

Proof. Using Eqn. 35, the recursive equation to find d_k, i.e. the distance between the source and the outer boundary of level L_k is

$$\frac{\bar{P}_r}{1-\gamma}((d_k - d_{k-2})^{(1-\gamma)} - (d_k - d_{k-1})^{(1-\gamma)}) = \tau N_0 \tag{20}$$

Above equation can be expressed in terms of Δ_ks as

$$\frac{\bar{P}_r}{1-\gamma}((\Delta_k + \Delta_{k-1})^{(1-\gamma)} - \Delta_k^{(1-\gamma)}) = \tau N_0 \tag{21}$$

Assuming that Δ_{k-1} is known having found in the previous step, Δ_k is found as the root of the following function.

$$f(x) = \frac{\bar{P}_r}{1-\gamma}((x + \Delta_{k-1})^{1-\gamma} - x^{1-\gamma}) - \tau N_0 \tag{22}$$

The $f(x)$ is a monotonically decreasing function which results in a unique solution for Δ_k for a given value of Δ_{k-1}. Furthermore, as Δ_{k-1} decreases (increases), the root of $f(x)$, i.e. Δ_k, decreases (increases). This implies that as k increases from the beginning to the end of the network, Δ_k either increases or decreases or simply remains constant for all levels. Since $d_k = \sum_{j=1}^{k} \Delta_j$, and all Δ_j's are positive, using Cauchy-ratio test , we can claim that

$$\begin{aligned} &\text{if } \lim_{k\to\infty} |\frac{\Delta_k}{\Delta_{k-1}}| > 1 \text{ then } \Delta_k \to \infty \\ &\text{if } \lim_{k\to\infty} |\frac{\Delta_k}{\Delta_{k-1}}| < 1 \text{ then } \Delta_k \to 0 \end{aligned}$$

It can be proved that if $d_1 > \left(\frac{\tau N_0(1-\gamma)}{(2^{(1-\gamma)}-1)\bar{P}_r}\right)^{1/1-\gamma}$ then $\Delta_2 > \Delta_1$ and hence $\Delta_k > \Delta_{k-1}$. $\square$

H. Proof of Theorem 2

Under bidirectional scheme,

$$\begin{aligned} \frac{\tau N_0(1-\gamma)}{\bar{P}_r} &= (d_k + d_{k-1})^{1-\gamma} - (d_k - d_{k-1})^{1-\gamma} \\ &\quad - ((d_k + d_{k-2})^{1-\gamma} - (d_k - d_{k-2})^{1-\gamma}) \end{aligned} \tag{23}$$

According to monotonic decreasing behavior of above expression $\Delta_k > \Delta_{k-1}$ or $d_k > 2d_{k-1} - d_{k-2}$ leads to

$$\begin{aligned}&(3d_{k-1} + d_{k-2})^{1-\gamma} - (2d_{k-1})^{1-\gamma}\\&+ (2^{1-\gamma} - 1)(d_{k-1} - dk - 2) > \frac{\tau N_0(1-\gamma)}{\bar{P}_r}\end{aligned} \tag{24}$$

Assuming that above condition is satisfied, we can guarantee $\Delta_{k+1} > \Delta_k$ since

$$\begin{aligned}&(3d_k + d_{k-1})^{1-\gamma} - (2d_k)^{1-\gamma}\\&+ (2^{1-\gamma} - 1)(d_k - dk - 1) > \frac{\tau N_0(1-\gamma)}{\bar{P}_r}\end{aligned} \tag{25}$$

holds if $d_k > 2d_{k-1} - d_{k-2}$.

I. Proof of Theorem 3

Equating the received SNR from level L_{k-1} to τ leads to the general equation under unidirectional scheme

$$\bar{P}_r(\ln(d_k - d_{k-2}) - \ln(d_k - d_{k-1}) = \tau N_0 \tag{26}$$

which can also be written in terms of Δ_k

$$\ln(\frac{\Delta_k + \Delta_{k-1}}{\Delta_k}) = \frac{\tau N_0}{\bar{P}_r}. \tag{27}$$

By defining $\mu := e^{\tau N_0/\bar{P}_r}$, we get the ratio

$$a = \frac{\Delta_k}{\Delta_{k-1}} = \frac{1}{\mu - 1}. \tag{28}$$

Eqn. 27 suggests that Δ_k grows linearly as opposed to growing exponentially in case of $\gamma < 1$.

III. Interference Analysis of Cooperative Communications In One-dimensional Dense Ad-hoc Networks

A. Introduction

1) Motivation: In most *ad-hoc* networks, transmitting a message to all the existing nodes may result in significant energy consumption or a high number of transmissions. Rather than using conventional noncooperative multihop broadcast in *ad-hoc* networks, many authors have proposed cooperative broadcast with a variety of low-complexity protocols, e.g. in [20] and also coding strategies, e.g. in [9] to fully exploit spatial diversity, to improve capacity or to optimize the power efficiency. This technique is based on the transmission of the same message by several nodes to increase the received power and as a result to increase the transmission range. Rather than avoiding the collision between different transmitting nodes, the cooperative broadcast exploits it to provide transmit diversity [12]. Furthermore, cooperative broadcast results in faster message propagation with a fewer number of steps [12].

The performance of cooperative broadcast and the advantages in high density 2-D *ad-hoc* networks have been fully analyzed in [12] - [16]. Motivated by these studies, we are interested to determine if these advantages are still valid in 1-D high density wireless *ad-hoc* networks. We would like to analyze the broadcast behavior under continuous source transmission with two different transmission schemes, the unidirectional scheme and the bidirectional scheme. We would like to study the effects of network parameters as well as the wireless environment on the broadcast behavior in the presence of interference.

2) Contribution: Our primary goal is to determine if cooperative broadcast is "successful" in 1-D high density *ad-hoc* networks subject to pathloss signal power attenuation. Fig. 9 illustrates a linear network under cooperative broadcast where the source node, depicted as a white square, initiates the transmission by broadcasting a message with power P_s. The message is then forwarded simultaneously by a group of nodes which receive the message, such that it can be received successfully by another group of nodes along the network. Each group of nodes receiving or transmitting the message simultaneously is depicted with the same color. If the message is heard by the entire network, the broadcast is then regarded as "successful". We have analyzed the message propagation in [21] under the single-shot transmission, in which the source node broadcasts a single message.

In this paper, we are interested in the scenario where the source transmits independent messages periodically imposing the presence of interference in the picture. Using the continuum approximation as introduced in [11], we characterize the system behavior under two different transmission protocols, the unidirectional transmission and the bidirectional transmission protocols. We find the condition for "successful" propagation under each transmission protocol, which highly depends on the pathloss exponent γ. Unlike noncooperative multihop broadcast which dies off under the continuous source transmission with $\gamma \leq 1$, we show that the cooperative broadcast can be successful for any value of γ under the presented conditions. We also show that the broadcast is more likely to succeed under the bidirectional transmission compared to the unidirectional transmission. Furthermore, the cooperative broadcast is more power efficient than noncooperative multihop broadcast under the bidirectional transmission.

3) Related works: The analysis of cooperation protocols has been mostly focused on single- or two-hop communications, e.g. [23, 24]. The extensions of such designs are usually done using relays in parallel, i.e. multiple relays transmit simultaneously in groups as in [20], or in series where relays transmit sequentially as in [13], or a combination of these two as in [13]. In [11], the continuum analysis of cooperative communication was introduced for the first time to characterize the broadcast behavior, the total area reached by broadcast, and the critical threshold. Based on this approximation, an extensive analysis of cooperation protocols is done for 2-D high density networks in [12] and [13]. In these works, authors model dense wireless networks using a continuum limit and observe a phase transition in the propagation of the packets, which is dependent on the node transmit powers and the decoding threshold. The authors extend their 2-D analysis further in [16] by designing and analyzing power efficient cooperative schemes. They show that cooperative broadcast is significantly more power efficient than noncooperative multihop broadcast in 2-D networks. Furthermore, they determine the upper and lower bounds on the broadcast capacity in both high density and extended networks.

Many have shown that the broadcast performance depends on the power pathloss exponent of the wireless environment γ, e.g. [26, 27]. The pathloss exponent γ determines the rate of signal power attenuation with distance in an environment. In 2-D networks, there is a transition in network behavior at $\gamma = 2$ which also is observed at $\gamma = 1$ in 1-D networks. In [18], the broadcast probability in cooperative 1-D extended networks is analyzed for different ranges of γ. They showed that the broadcast probability, i.e. the probability that the entire network receives the message correctly, is zero for $\gamma > 1$ and for any node density, while it is strictly greater than zero for $\gamma \leq 1$. In [21], we analyze 1-D high density networks using the deterministic continuum model and unlike [18] show that the message propagation always continues for $\gamma > 1$ considering infinite node density while it can stop for $\gamma \leq 1$ depending on the network parameters. All these analyses have been based on the single-shot transmission in which the source transmits only a single message.

Considering the continuous source transmission, in which the source transmits independent messages continuously, introduces interference as a limiting factor in cooperative networks. Multiple access interference (MAI) created by the signals of other nodes is introduced in [22] although it is not included in the scope of their study. In [16], the continuous source transmission is analyzed and the achievable rate of multistage cooperative broadcasting is determined for 2-D high density extended networks. In [28], a hierarchical beamforming architecture is proposed for 1-D networks which is claimed to achieve optimal capacity scaling in low SNR regime for which the effect of interference is neglected. To further avoid collisions between the neighboring clusters, half of the clusters are scheduled to remain silent at a time, resulting in a reduction in the throughput by a factor of two.

Broadcast scheduling in order to avoid interference is actually a well-established approach, e.g. in [29]-[31]. However, there is a common assumption in all these studies; that the transmission range is limited and hence interference beyond that range is negligible. Usually only the interference caused by the neighboring nodes or the neighboring cluster of nodes is considered. On the other hand, we should note that the aggregate interference of a large number of far transmitting nodes, especially in high density networks, may reduce SINR significantly to less than the decoding threshold.

This fact is taken into account in [32] and accordingly a cross-layer design is proposed which includes two phases; a simple scheduling algorithm to eliminate strong levels of interference from the neighboring nodes and a distributed power control algorithm to find the optimal power vector for successful transmission. However, their approach also limits the signal power as they limit the received signal at each node from only one transmitting neighbor. We also calculate the aggregate interference, yet with no scheduling limitations. Each node can receive and transmit at the same time. However, the effect of self-interference which is caused by a node simultaneously transmitting and receiving is not considered.

The organization of the paper is as follows. In Section II, the system model with the deterministic continuum model is presented. In Section III, the network behavior is characterized under the continuous source transmission. In Section V, simulation results are presented. We conclude the paper in Section VI.

B. System model

1) Transmission Protocols: Consider a system with a single source and multiple destination nodes over a linear network. Our focus is on the high density fixed-area network where the node density increases proportionally with the number of nodes. The location of the nodes is uniformly and independently distributed. The nodes communicate over a wireless channel subject to the pathloss attenuation. It is assumed that each relay node is able to estimate the received SINR which is the only parameter locally available. The node compares the received SINR with a pre-assigned threshold and only if it finds it greater than or equal to the threshold, it retransmits the message. The SINR threshold is denoted by parameter τ, hereafter.

It is assumed that the message is channel coded, hence every node is able to decode the message correctly as long as SINR $\geq \tau$. Fig. 9 illustrates cooperative broadcast in 1-D linear network. The number associated to each transmitting node shows the order of transmission for that node. In the unidirectional transmission as in 1(a), the message is relayed in one direction whereas in the bidirectional transmission as in 1(b), it is relayed in both directions along the network. The first group of nodes which receive the message successfully, form the first *level* of nodes referred to as ($\mathcal{L}_1$). Unlike multihop broadcast, *all* nodes in ($\mathcal{L}_1$) transmit the message in the next time slot. Similarly, the nodes which receive the message successfully from ($\mathcal{L}_k$) form ($\mathcal{L}_{k+1}$). We assume that the nodes in each level only decode the message from the very preceding level. In both cooperative and noncooperative broadcasts, the propagation is regarded to be successful when the packet is heard by all of the existing nodes.

It should be noted that in the bidirectional scheme $\mathcal{L}_1$ is the union of $\mathcal{L}_1^+$ in the right-hand side and $\mathcal{L}_1^-$ in the left-hand side of the source node:

$$\mathcal{L}_k = \mathcal{L}_k^+ \bigcup \mathcal{L}_k^- \tag{29}$$

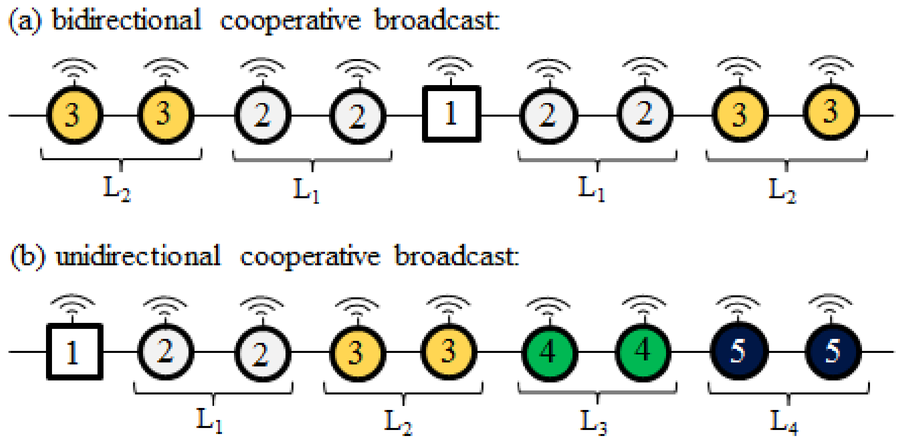


Fig. 9: The unidirectional and the bidirectional cooperative broadcast schemes

2) Transmission Criterion: We assume that the power of simultaneously transmitted packets at the receiving node is equal to the sum of individual powers. This assumption is well-suited to orthogonal channels or relays which use distributed space-time codes. Note that all the relay nodes transmit with equal power, denoted by P_r, and the source node has a distinct transmit power, referred to as P_s. The pathloss attenuation is based on the simplified pathless model [1], while neglecting the effect of small-scale fading. Thus, the power received at node j after the transmission of level $\mathcal{L}_{k-1}$ is

$$P_j = \sum_{i \in \mathcal{L}_{k-1}} P_r \ell(d_j - d_i) = \sum_{i \in \mathcal{L}_{k-1}} \frac{P_r}{(d_j - d_i)^{\gamma}} \tag{30}$$

where $\ell(.)$ is the pathloss attenuation function, d_j and d_i are distances from the source to node j and node i, respectively. Similarly, the received interference power at node j can be expressed as

$$I_j = \sum_{n=1}^{\lfloor k-2/i \rfloor} \sum_{d_i \in \mathcal{L}_{k-n \times i-1}} P_r \ell(d_j - d_i) + P_s \ell(d_j) \tag{31}$$

where i denotes the number of time slots which is required for transmitting a single message. The smaller i, the more independent messages transmitted at a given time by preceding levels and therefore the greater amount of interference at node j. Clearly, if the source transmits different messages every time slot, i.e. $i = 1$, all the preceding levels from $\mathcal{L}_1$ to $\mathcal{L}_{k-2}$ including the source contribute to the interference power while $\mathcal{L}_{k-1}$ transmits the message. For this scenario, the interference can be simplified as

$$I_j = \sum_{n=1}^{k-2} \sum_{d_i \in \mathcal{L}_n} Pr\ell(d_j - d_i) + \frac{P_s}{d_j^\gamma} \tag{32}$$

We will denote the received noise power with N_0. Consequently, all the nodes which satisfy the SINR condition

$$\frac{P_j}{(N_0 + I_j)} \geq \tau \tag{33}$$

form the next level, $\mathcal{L}_k$. It should be noted that the pathloss exponent γ depends on the channel environment. In this study, we consider γ in three general categories; $\gamma < 1$, $\gamma = 1$ and $\gamma > 1$. Each category results in a unique network behavior which is studied with respect to parameters P_r, P_s and τ. Our analysis is based on the continuum model introduced in the following subsection.

3) Continuum model: Let $\mathbb{L} \triangleq \{x : x \leq L\}$ represent the linear network. Let $\rho = N/L$ be the node density (node/unit length). The continuum model assumes high density setup as N and ρ go to infinity while the total power $P_r N$ is fixed. Rather than working with P_r which approaches zero as $N \to \infty$, we define relay power per unit length,

$$\bar{P}_r \triangleq \frac{P_r N}{L} = \rho P_r \tag{34}$$

which is a fixed parameter. As $N \to \infty$, every infinitesimal length dx contains ρdx relays with power P_r, i.e. $P_r \rho dx = \bar{P}_r dx$ in total. As a result, Eqn. 30 is transformed to an integral over level L_{k-1}

$$P_j = \int_{x \in L_{k-1}} \frac{\bar{P}_r dx}{(d_j - d_x)^\gamma} \tag{35}$$

where j is any arbitrary location for a receiving node, and L_{k-1} denotes the location of the level $\mathcal{L}_{k-1}$ nodes in the continuum limit. We refer the readers to works [11, 12] for further details of continuum approximation for 2-D networks. In the following section, we analyze the transmission behavior under the continuous source transmission protocol with the unidirectional and the bidirectional schemes and with different ranges of γ.

C. Continuous Source Transmission under $\gamma < 1$

In this transmission mode, the source node transmits an independent message after a pause of silence. We introduce *transmission period* i as the number of time slots over which the source transmits a message and then remains silent before sending the next message. This measure also reflects the severity of interference power. The most severe case of interference corresponds to $i = 1$ in which interference first appears at $\mathcal{L}_2$ originating from the source. However, in our analysis, the interference from the source is neglected since its effect becomes more negligible as the message propagates farther away from the source. Hence $\mathcal{L}_1$ and $\mathcal{L}_2$ are interference free, and the interference first appears at $\mathcal{L}_3$ from $\mathcal{L}_1$ sending a different message.

The transmission behavior is studied by finding the deterministic distance of level boundaries (d_k's) in the continuum limit (See Fig. 10). This is done for L_1 using Eqn. 30 and 33 simply as

$$d_1 = \left(\frac{P_s}{N_0\tau}\right)^{1/\gamma} \tag{36}$$

where d_1 is the distance from the source node to the outer boundary of L_1 (L_1^+ and L_1^- for the bidirectional transmission). Since, the protocol results in a symmetric transmission behavior on both directions for the bidirectional transmission, we focus our analysis only on one direction. We also define the length of level L_k as

$$\Delta_k = d_k - d_{k-1} \tag{37}$$

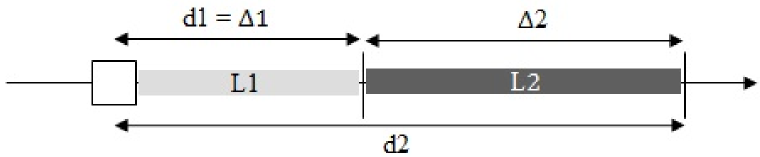


Fig. 10: Depiction of d_1, d_2 and Δ_2

Thus $d_1 = \Delta_1$ and they both are used interchangeably throughout this paper. Now, we present the following subsections for different ranges of γ.

1) Unidirectional scheme: The recursive equation to find Δ_k is expressed as

$$\frac{\bar{P}_r}{1-\gamma}\left[(\Delta_k + \Delta_{k-1})^{(1-\gamma)} - \Delta_k^{(1-\gamma)}\right] = \tau(N_0 + I_k) \tag{38}$$

where Δ_{k-1} and Δ_k are the lengths of levels L_{k-1} and L_k, respectively. I_k is the interference at the boundary of L_k and N_0 is the received noise power. Following the similar analysis as described in [4] for the single-shot transmission, we can find bounds on Δ_k

$$\frac{\bar{P}_r}{\tau(N_0 + I_k)}\Delta_{k-1}(1 - \frac{\gamma}{2}) < \Delta_k^{\gamma} < \frac{\bar{P}_r}{\tau(N_0 + I_k)}\Delta_{k-1} \tag{39}$$

given that $\Delta_k > \Delta_{k-1}$ is satisfied for every k under the continuous source transmission. According to these bounds, Δ_k can increase exponentially with respect to Δ_{k-1} provided that I_k is bounded. However, I_k initially increases with k since the number of interfering levels increases as the message flows along the network. Under $i = 1$, i.e. the most severe case of interference, I_k under the unidirectional transmission is expressed as

$$I_k^{uni} = \int_0^{d_{k-2}} \frac{\bar{P}_r dx}{(d_k - x)^{\gamma}} = \frac{\bar{P}_r}{1-\gamma}\left[d_k^{1-\gamma} - (d_k - d_{k-2})^{1-\gamma}\right] \tag{40}$$

In the following theorem, we show that for a certain condition, I_k surprisingly starts to decrease with k after an initial increase as mentioned above.

Theorem 6. *Under the unidirectional transmission with $\gamma < 1$,*

if

$$\exists k_0 \quad \textit{s.t.} \quad I_{k_0+1} \leq I_{k_0}$$

then

$$I_{k+1} < I_k \quad \textit{for} \quad \forall k > k_0$$

and

$$\lim_{k\to\infty} I_k = 0$$

Proof. We first find the condition for $I_{k_0+1} \leq I_{k_0}$. We can write

$$\int_0^{d_{k+1}} \frac{\bar{P}_r dx}{(d_{k+1}-x)^\gamma} = \int_0^{d_{k-1}} \frac{\bar{P}_r dx}{(d_{k+1}-x)^\gamma} + \int_{d_{k-1}}^{d_k} \frac{\bar{P}_r dx}{(d_{k+1}-x)^\gamma} + \int_{d_k}^{d_{k+1}} \frac{\bar{P}_r dx}{(d_{k+1}-x)^\gamma} \tag{41}$$

which according to Eqn. 38 and 40 yields to,

$$\frac{\bar{P}_r}{1-\gamma} d_{k+1}^{1-\gamma} = I_{k+1} + \tau(N_0 + I_{k+1}) + \frac{\bar{P}_r}{1-\gamma}\Delta_{k+1}^{1-\gamma} \tag{42}$$

Hence,

$$\frac{\bar{P}_r}{1-\gamma}(d_{k+1}^{1-\gamma} - \Delta_{k+1}^{1-\gamma}) = (1+\tau)I_{k+1} + \tau N_0 \tag{43}$$

using the above equation, the condition for $I_{k+1} \leq I_k$ can be found as

$$d_{k+1}^{1-\gamma} - \Delta_{k+1}^{1-\gamma} \leq d_k^{1-\gamma} - \Delta_k^{1-\gamma} \tag{44}$$

Now we show that if $I_{k_0+1} \leq I_{k_0}$, then $I_{k_0+2} < I_{k_0+1}$.

Let

$$I_k(x) := \frac{\bar{P}_r}{1-\gamma}\left[(d_{k-1}+x)^{1-\gamma} - (\Delta_{k-1}+x)^{1-\gamma}\right] \tag{45}$$

In other words, $I_k(0)$ is the interference at the closest node to the boundary of L_{k-1}

belonging to L_k, and $I_k(\Delta_k)$ is the interference at the boundary of L_k which is also denoted by I_k for brevity. We show $I_{k_0+2}(0)$ is smaller than $I_{k_0+1}(0)$ if $I_{k_0+1} \leq I_{k_0}$. The expressions for $I_{k_0}(0)$ and $I_{k_0+1}(0)$ are as follows,

$$I_{k_0+1}(0) = \frac{\bar{P}_r}{1-\gamma}\left[d_{k_0}^{1-\gamma} - \Delta_{k_0}^{1-\gamma}\right] \tag{46}$$

$$I_{k_0+2}(0) = \frac{\bar{P}_r}{1-\gamma}\left[d_{k_0+1}^{1-\gamma} - \Delta_{k_0+1}^{1-\gamma}\right] \tag{47}$$

Since $I_{k_0+1} \leq I_{k_0}$, according to Eqn. 44 we find that $I_{k_0+2}(0) \leq I_{k_0+1}(0)$. This implies that as the message starts to flow in L_{k_0+2}, there is a smaller or an equal amount of interference compared to the interference at the starting point of L_{k_0+1}. Furthermore, since $\Delta_{k_0+1} > \Delta_{k_0}$, the nodes in L_{k_0+2} receive more signal power from L_{k_0+1} compared to the power received by nodes in L_{k_0+1} from L_{k_0}. Obviously, this provides a better condition for the message to flow in L_{k_0+2} as compared to L_{k_0+1} and the message gets farther resulting in $I_{k_0+2} < I_{k_0+1}$. This behavior continues and I_k steadily decreases. Since $\frac{I_k}{I_{k-1}} < 1$,

$$\sum_{k=0}^{\infty} I_k \rightarrow c \tag{48}$$

and therefore $\lim_{k\to\infty} I_k = 0$. □

Remark. *According to Eqn. 39, the condition for* $I_{k_0+1} < I_{k_0}$ *and hence* $\lim_{k\to\infty} I_k = 0$*, has another implication, which is*

$$\Delta_{k+1} \propto \Delta_k^{1/\gamma}$$

since the term in denominator $\tau(N_0 + I_k)$ *is upper-bounded by* $\tau(N_0 + I_{max})$.

2) Bidirectional scheme: Under the bidirectional transmission with pathloss exponent $\gamma < 1$, the most severe interference is produced by all the preceding levels in both directions,

$$I_k = \frac{\bar{P}_r}{1-\gamma}\left[(d_k + d_{k-2})^{1-\gamma} - (d_k - d_{k-2})^{1-\gamma}\right] \tag{49}$$

Although this expression results in a greater amount of interference as compared to the interference under the unidirectional transmission, in the following theorem we show a successful broadcast under the unidirectional transmission also results in a successful broadcast under the bidirectional transmission.

Theorem 7. *if*

$$\exists k_0 \quad \textit{s.t.} \quad I_{k_0+1} \leq I_{k_0}$$

Under the unidirectional transmission with $\gamma < 1$*, then also under the bidirectional transmission with the same network parameters*

$$\exists k_0 \quad \textit{s.t.} \quad I_{k_0+1} \leq I_{k_0}$$

and

$$\lim_{k\to\infty} I_k = 0$$

Proof. Let's assume the same network parameters for the unidirectional and the bidirectional networks. We denote Δ_k and I_k for the unidirectional transmission and Δ'_k and I'_k for the bidirectional transmission. The similar network parameters result in $\Delta_1 = \Delta'_1$. Considering $I_2 = I'_2 = 0$, i.e. neglecting the interference from the source, according to Eq. 39,

$$\Delta_2 \propto \Delta_1^{1/\gamma} \tag{50}$$

$$\Delta'_2 \propto (2\Delta'_1)^{1/\gamma} \tag{51}$$

Hence $\Delta'_2 \propto 2^{1/\gamma}\Delta_2$. Using this fact, we proceed to the next level and prove that $I'_3 <$

I_3. We start by using the definition in Eqn. 45, and write $I_3(0)$ as

$$I_3(0) = \frac{\bar{P}_r}{1-\gamma}\left[(\Delta_1+\Delta_2)^{1-\gamma} - (\Delta_2)^{1-\gamma}\right] \tag{52}$$

which is actually equal to τN_0 based on Eqn. 38. Similarly, $I'_3(0)$ can be written as

$$I'_3(0) = \frac{\bar{P}_r}{1-\gamma}\left[(2\Delta_1+\Delta'_2)^{1-\gamma} - (\Delta'_2)^{1-\gamma}\right] \tag{53}$$

which is again equal to τN_0. Thus, $I_3(0) = I'_3(0)$ and the two levels start with the same amount of interference. On the other hand, the amount of the received power by L_3 and L'_3 is not the same. We should note that $P_3(0)$, i.e. the received power at distance zero from the previous level, cannot be defined under continuum approximation. Instead, we write the expressions for $P_3(\epsilon)$ and $P'_3(\epsilon)$, where $\epsilon << 1$,

$$P_3(\epsilon) = \frac{\bar{P}_r}{1-\gamma}\left[(\Delta_2+\epsilon)^{1-\gamma} - (\epsilon)^{1-\gamma}\right] \approx \frac{\bar{P}_r}{1-\gamma}(\Delta_2)^{1-\gamma} \tag{54}$$

In the bidirectional scheme, P'_3 is produced by two sections of Δ'_2. Neglecting the power from the farther one, we can write

$$P'_3(\epsilon) > \frac{\bar{P}_r}{1-\gamma}(\Delta'_2)^{1-\gamma} \propto 2^{\frac{1-\gamma}{\gamma}} P_3(\epsilon) \tag{55}$$

The above expressions clearly show that $P'_3(\epsilon) > P_3(\epsilon)$, which also is intuitively acceptable as $\Delta'_2 > \Delta_2$. Hence, L'_3 can be viewed as a unidirectional level like L_3, receiving the same interference as L_3, yet receiving more power. Hence, not only is $\Delta'_3 > \Delta_3$ but $I'_3 < I_3$.

By induction, we can prove that $I'_{k+1} < I_{k+1}$, if $I'_k < I_k$ and $\Delta'_k > \Delta_k$. $I'_{k+1}(0)$ and $I_{k+1}(0)$ can be expressed as

$$I'_{k+1}(0) = P'_k + I'_k = \tau(N_0 + I'_k) + I'_k = \tau N_0 + (1+\tau)I'_k \tag{56}$$

and

$$I_{k+1}(0) = P_k + I_k = \tau(N_0 + I_k) + I_k = \tau N_0 + (1+\tau)I_k \tag{57}$$

Hence $I'_k < I_k$ results in $I'_{k+1}(0) < I_{k+1}(0)$. However, we have $P'_{k+1}(\epsilon) > P_{k+1}(\epsilon)$ since $\Delta'_k > \Delta_k$. Therefore, L'_{k+1} starts with less amount of interference than L_{k+1} yet with a greater amount of power. This leads to $\Delta'_{k+1} > \Delta_{k+1}$ and consequently $I'_{k+1} < I_{k+1}$ as explained for the case of I_3 and I'_3.

□

D. *Cooperative Transmission under pathloss exponent $\gamma = 1$*

1) Unidirectional scheme: According to [4], the single-shot transmission under $\gamma = 1$ is similar to the one under $\gamma < 1$ in the sense that Δ_k grows to infinity under a particular condition. First, we define a new parameter,

$$\mu := e^{\tau N_0/\bar{P}_r}.$$

and for the unidirectional transmission we show that if $\mu > 2$ then $\Delta_k \to \infty$, if $\mu = 2$ then $\Delta_k \to \Delta_1$, otherwise ($\mu < 2$) $\Delta_k \to 0$. The same condition is valid for the bidirectional scheme, yet the bidirectional transmission moves faster as expected. Under the presence of interference, the equation for the unidirectional transmission with $\gamma = 1$ is modified as

$$\ln\left(\frac{\Delta_k + \Delta_{k-1}}{\Delta_k}\right) = \frac{\tau}{\bar{P}_r}(N_0 + I_k) \tag{58}$$

Thus,

$$\Delta_k = \frac{\Delta_{k-1}}{\mu^{N_0+I_k} - 1} \tag{59}$$

Assuming the most severe interference case, i.e. $i = 1$, and neglecting the interference from the source, the interference I_k at the boundary of L_k can be written in the form of

$$I_k = \bar{P}_r \left(\ln(d_k) - \ln(d_k - d_{k-2})\right) = \bar{P}_r \ln(\frac{d_k}{d_k - d_{k-2}}) \tag{60}$$

In the following theorem, we prove that under a particular condition I_k saturates to a non-zero value as $k \to \infty$. Before proceeding to the theorem, we define a_k as the ratio of d_k over d_{k-1}.

$$a_k := \frac{d_k}{d_{k-1}}$$

Theorem 8. *For $\gamma = 1$, under the unidirectional transmission*

$$\lim_{k\to\infty} I_k = c \tag{61}$$

where c is a nonzero value if and only if $\tau \leq \tau_c$, and τ_c is the solution to

$$\mu^{N_0}(\frac{(2\tau_c+1)^2}{(2\tau_c+1)^2-1})^{\tau_c} - 1 = \frac{1}{2\tau_c+1}$$

Proof. According to Eqn. 60, I_k is a function of d_k and d_{k-2}. Substituting d_{k-2} with its equivalent expression in terms of d_k, a_k and a_{k-1}, we have

$$I_k = \bar{P}_r \ln(\frac{d_k}{d_k - \frac{d_k}{a_k a_{k-1}}}) \tag{62}$$

Now if

$$\lim_{k\to\infty} a_k = a \tag{63}$$

then we have

$$\lim_{k\to\infty} I_k = \bar{P}_r \ln(\frac{d_k}{d_k - \frac{d_k}{a^2}}) = \bar{P}_r \ln(\frac{a^2}{a^2-1}) \tag{64}$$

We would like to find the condition under which both a_k and I_k saturate to a nonzero value. In addition to a_k, we define another ratio a'_k which is the ratio of Δ_k to Δ_{k-1}.

$$a'_k := \frac{\Delta_k}{\Delta_{k-1}}$$

It can be easily proven that If

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{d_k}{d_{k-1}} = a$$

then

$$\lim_{k\to\infty} a'_k = \lim_{k\to\infty} \frac{\Delta_k}{\Delta_{k-1}} = a$$

The expression for a'_k can be written in terms of a_k as,

$$a'_k = \frac{d_k - d_{k-1}}{d_{k-1} - d_{k-2}} = \frac{d_k - \frac{d_k}{a_k}}{\frac{d_k}{a_k} - \frac{d_k}{a_k a_{k-1}}} \tag{65}$$

if $\lim_{k\to\infty} a_k = a$, then we have

$$\lim_{k\to\infty} a'_k = \frac{d_k - \frac{d_k}{a}}{\frac{d_k}{a} - \frac{d_k}{a^2}} = a \tag{66}$$

According to Eq. 59,

$$a'_k = \frac{\Delta_k}{\Delta_{k-1}} = \frac{1}{\mu^{N_0+I_k} - 1} \tag{67}$$

Since $a'_k, a_k \to a$, we can combine Eq. 64 and 67, and write

$$a = \frac{1}{\mu^{N_0}(\frac{a^2}{a^2-1})^\tau - 1} \tag{68}$$

This nonlinear equation is solved by finding the roots of following function

$$f(a) = \mu^{N_0} \frac{a^{2\tau+1}}{(a^2-1)^\tau} - a - 1 \tag{69}$$

This function has a global minimum which depends on $\bar{P}_r$ and τ. The minimum value should not be greater than zero in order for Eq. 69 to have a real solution, i.e. $\min(f(a)) \leq$

0. We can therefore construct an equation system consisting of

$$\begin{aligned} &(1)\ f'(a_m) = 0 \\ &(2)\ f(a_m) = 0 \end{aligned} \tag{70}$$

to find the minimum a, i.e. a_m. Solving the first equation we get,

$$\begin{aligned} f'(a_m) = (2\tau+1)\mu^{N_0}\frac{a_m^{2\tau}}{(a_m^2-1)^\tau} \\ - 2a_m\tau\mu^{N_0}\frac{a_m^{2\tau+1}}{(a_m^2-1)^{\tau+1}} - 1 = 0 \\ \rightarrow \mu^{N_0}\frac{a_m^{2\tau}}{(a_m^2-1)^\tau}\left[1 - \frac{2\tau}{a_m^2-1}\right] - 1 = 0 \\ \rightarrow \mu^{N_0}\frac{a_m^{2\tau}}{(a_m^2-1)^\tau} = \frac{a_m^2-1}{a_m^2-1} \end{aligned} \tag{71}$$

Now to govern the inequality,

$$f(a) = a_m\frac{1}{\frac{a_m^2-1}{a_m^2-1}} - a_m - 1 = 0 \rightarrow a_m = 2\tau+1 \tag{72}$$

therefore,

$$a_m = \frac{1}{\mu^{N_0}(\frac{(2\tau+1)^2}{(2\tau+1)^2-1})^\tau - 1} = 2\tau+1 \tag{73}$$

and hence the condition for the successful flow is found as $\tau \leq \tau_c$, where τ_c is the solution to

$$\mu^{N_0}(\frac{(2\tau_c+1)^2}{(2\tau_c+1)^2-1})^{\tau_c} - 1 = \frac{1}{2\tau_c+1} \tag{74}$$

□

2) Bidirectional scheme: Cooperative broadcast exhibits similar behavior under the bidirectional continuous transmission. The expression for interference I_k is, however, a bit different,

$$I_k = \bar{P}_r \ln(\frac{d_k + d_{k-2}}{d_k - d_{k-2}}) \tag{75}$$

Thus, its ultimate value is equal to

$$\lim_{k\to\infty} I_k^{bi} = \bar{P}_r \ln(\frac{a^2+1}{a^2-1}) \tag{76}$$

and we have,

$$a_k^{bi} = \frac{d_k}{d_{k-1}} = \frac{1+\sqrt{1-(\mu^{I_k+N_0}-1)^2}}{\mu^{I_k+N_0}-1} \tag{77}$$

The same approach can be taken to find the condition for successful propagation. However, solving the resulting equation system would be tedious. Instead, we prove that the successful propagation under the unidirectional transmission leads to the successful propagation under the bidirectional transmission.

Theorem 9. *If under the unidirectional transmission*

$$\lim_{k\to\infty} I_k = c_1 \tag{78}$$

where c_1 *is a nonzero value then also under the bidirectional transmission with same network parameters*

$$\lim_{k\to\infty} I_k = c_2 \tag{79}$$

where c_2 *is a nonzero value.*

Proof. As with $\gamma < 1$, in the bidirectional transmission, we neglect the contribution of the farther section of level L_2 and onward in the signal power but consider its contribution in the interference power. Nonetheless, we show that the interference I'_k under the bidirectional transmission is smaller than I_k under the unidirectional scheme. The proof is very similar to the proof of Theorem 2.

Considering the fact that $2\Delta_1$ contributes to the signal power in the bidirectional scheme rather than Δ_1 in the unidirectional scheme, the relationship between Δ_2 and

Δ_2' can be expressed as

$$\ln(\frac{\Delta_1+\Delta_2}{\Delta_2}) = \ln(\frac{2\Delta_1+\Delta_2'}{\Delta_2'}) = \frac{\tau}{\bar{P}_r}N_0 \tag{80}$$

which leads to $\Delta_2' = 2\Delta_2$. Hence, $P_3(\epsilon) < P_3'(\epsilon)$ although $I_3(0) = I_3'(0)$. As a result, $\Delta_3 < \Delta_3'$ and $I_3 > I_3'$. As done similarly in the proof of Theorem 2, It can be proved by induction that $I_k > I_k'$ for the following levels, as $I_k(0) > I_k'(0)$ and $P_k(\epsilon) < P_k(\epsilon)'$. □

E. Cooperative Transmission under pathloss exponent $\gamma > 1$

1) Unidirectional and bidirectional schemes: According to [4] for the single-shot transmission,

$$\lim_{k\to\infty}\Delta_k = \Delta_\infty := \left[\frac{\bar{P}_r(1-2^{1-\gamma})}{\tau N_0(\gamma-1)}\right]^{1/\gamma-1}, \tag{81}$$

under both the bidirectional and the unidirectional transmission with $\gamma > 1$. Hence, the message always flows to the entire network as long as the the node density is sufficiently high. Similarly, under the continuous source transmission $\Delta_k \to \Delta_\infty$ as $k \to \infty$. However, Δ_∞ can be zero or negative in the presence of interference causing the transmission to break. Whether, the transmission stops or continues depends on the value of τ, transmission period i and pathloss exponent γ. Since $\gamma > 1$, we have

$$\frac{\bar{P}_r}{\gamma-1}\left[\frac{1}{(d_k-d_{k-1})^{\gamma-1}} - \frac{1}{(d_k-d_{k-2})^{\gamma-1}}\right] = \tau(N_0+I_k) \tag{82}$$

In the following theorem, we find the condition for successful propagation under the continuous source transmission considering the both unidirectional and bidirectional transmission schemes.

Theorem 10. *Under the unidirectional and the bidirectional transmission schemes with $\gamma > 1$, the continuous source transmission is successful* *iff*

$$\tau < \tau_c$$

where,

$$\tau_c = 2^{1-\gamma} - 1$$

Proof. Assuming $i = 1$, the expression for I_k as $k \to \infty$ is

$$I_k = \sum_{n=1}^{\infty} \frac{\bar{P}_r}{\gamma - 1}[\frac{1}{(d_k - d_{k-n-1})^{\gamma-1}} - \frac{1}{(d_k - d_{k-n-2})^{\gamma-1}}] \tag{83}$$

Now, assuming that $\Delta_k \to \Delta_\infty$, we can write

$$\begin{aligned} I_k &= \sum_{n=1}^{\infty} \frac{\bar{P}_r}{(\gamma - 1)\Delta_\infty^{\gamma-1}}[\frac{1}{(n+1)^{\gamma-1}} - \frac{1}{(n+2)^{\gamma-1}}] \\ &= \frac{\bar{P}_r}{(\gamma - 1)(2\Delta_\infty)^{\gamma-1}} \end{aligned} \tag{84}$$

Thus using Eqn. 82

$$\frac{\bar{P}_r}{(\gamma - 1)\Delta_\infty^{\gamma-1}}\left[1 - (1+\tau)2^{1-\gamma}\right] = \tau N_0 \tag{85}$$

which results in

$$\tau < 2^{\gamma-1} - 1 \tag{86}$$

□

the above condition guarantees the successful flow under the both unidirectional and bidirectional transmission with any value of transmission period i.

This condition specifies a maximum value for τ which is obtained based on γ and i. The value of $\bar{P}_r$ does not impose any restriction on the flow but determines the value of Δ_∞. In case of continuum model, with node density $\rho = \infty$, the cooperation should continue for any nonzero value of Δ_∞. However, in practical networks with finite node density, transmission breaks for small enough values of Δ_∞.

F. Power Efficiency Comparison of Cooperative and Non-cooperative Broadcasts

1) continuous source transmission with $\gamma < 1$: It is impossible for noncooperative broadcast to continue under any level of interference under $\gamma < 1$. If Δ_k is assumed to be constant for multihop broadcast, i.e. $\Delta_k = \Delta$, then I_k (interference at level k) can be expressed as

$$I_k = \frac{P_r}{\Delta^\gamma} \sum_{j=2}^{k} (\frac{1}{j})^\gamma \tag{87}$$

apparently for $\gamma < 1$

$$\sum_{j=2}^{\infty} (\frac{1}{j})^\gamma = \infty \tag{88}$$

which implies that interference blows up and causes the transmission to break. This also can be shown from another aspect, assuming flexible Δ_k for each level. Let's look at the equation for Δ_k at level k,

$$\frac{\bar{P}_r}{\Delta_k} = \tau(N_0 + I_k) \tag{89}$$

By Neglecting the interference from the source for cooperative broadcast, $I_2 = 0$. Since $I_3 \neq 0$, therefore $I_3 > I_2$ which according to above equation, results in $\Delta_3 < \Delta_2$. This in turn results in $I_4 > I_3$ and consequently $\Delta_4 < \Delta_3$. We can therefore conclude that $\Delta_k < \Delta_{k-1}$, meaning that Δ_k approaches zero.

2) continuous source transmission with $\gamma = 1$: The same argument can be made for multihop broadcast with $\gamma = 1$ to show that the interference diverges as k increases. Clearly

$$\sum_{j=2}^{\infty} (\frac{1}{j}) = \infty \tag{90}$$

On the other hand, we have shown the possibility of successful propagation for cooperative broadcast under both unidirectional and bidirectional schemes.

3) continuous source transmission with $\gamma > 1$: For noncooperative broadcast, I_k saturates to,

$$\lim_{k\to\infty} I_k = \frac{P_r}{\Delta^\gamma} S_{noncoop} \tag{91}$$

where

$$S_{noncoop} = \sum_{n=1}^{\infty} \left[\frac{1}{n(i+1)+1}\right]^\gamma \tag{92}$$

which is equivalent to p-series and always converges for $\gamma > 1$. Therefore,

$$P_{noncoop} = \frac{\Delta^\gamma \tau N_0}{1 - \tau S_{noncoop}} \tag{93}$$

As shown in our analysis for cooperative transmission with $\gamma > 1$, Δ_k converges to a constant value as

$$\Delta = \frac{(1 - 2^{1-\gamma} - \tau S_{coop})\bar{P}_r}{(\gamma - 1)\tau N_0} \tag{94}$$

where,

$$S_{coop} = \sum_{n=1}^{\infty} \left[\frac{1}{(n \times i + 1)^{\gamma - 1}} - \frac{1}{(n \times i + 2)^{\gamma - 1}}\right] \tag{95}$$

Thus,

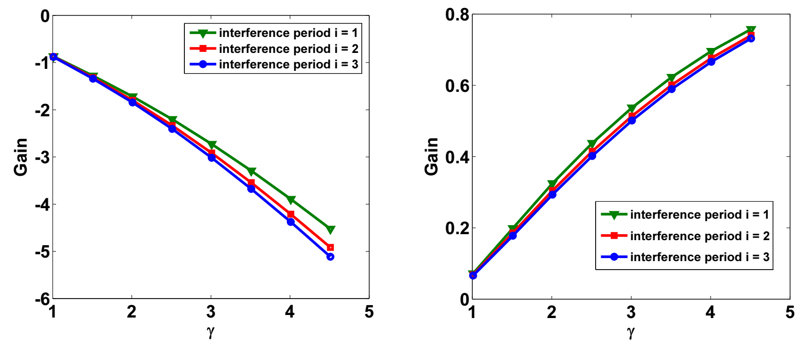
$$P_{coop} = \bar{P}_r \Delta = \frac{\Delta^\gamma (\gamma - 1)\tau N_0}{1 - 2^{1-\gamma} - \tau S} \tag{96}$$

The gain of using noncooperative broadcast over using cooperative broadcast is defined as

$$gain = \frac{P_{noncoop} - P_{coop}}{P_{noncoop}} \tag{97}$$

For the bidirectional transmission, cooperative broadcast flows double times faster than noncooperative broadcast. To have a fair comparison, we assign the transmission radius of the noncooperative scheme to be twice the step size of cooperative scheme, i.e. $r = 2\Delta_\infty$. As a result,

$$P^{bi}_{noncoop} = \frac{\tau N_0 (2\Delta)^\gamma}{1 - \tau S_{noncoop}} = 2^\gamma P^{uni}_{noncoop} \tag{98}$$



(a) unidirectional scheme (b) bidirectional scheme

Fig. 11: The gain of using cooperative broadcast instead of non-cooperative broadcast

The condition for the continuous flow in noncooperative is given as

$$\tau_c < 1/S_{noncoop}$$

Fig. 11 shows the gain for the both unidirectional and bidirectional schemes for the range of γ between (1 5]. We can observe that the gain is negative for the unidirectional scheme while it is positive for the bidirectional scheme. Under both schemes, the gain increases in favor cooperative broadcast as i decreases, i.e. the interference becomes more severe.

G. Simulation Results

1) Transmission under $\gamma < 1$: In this section we check the accuracy of the analytical expressions for the finite-density, finite-length networks. First, we perform a numerical analysis of the continuous source transmission with $\gamma = 0.25$, 0.5 and 0.75. In all the three cases, the network parameters are fixed at $P_s = 0.1$, $\bar{P}_r = 1$, $N_0 = 1$ and $i = 0$. The critical threshold τ_c for each case is found by setting the condition $\Delta_{k+1} > \Delta_k$ on k starting from $k = 1$. If the resultant τ leads to a decrease in I_k at some k then we know the propagation will be successful and we stop looking for a smaller τ, otherwise we continue on imposing similar condition for the next level

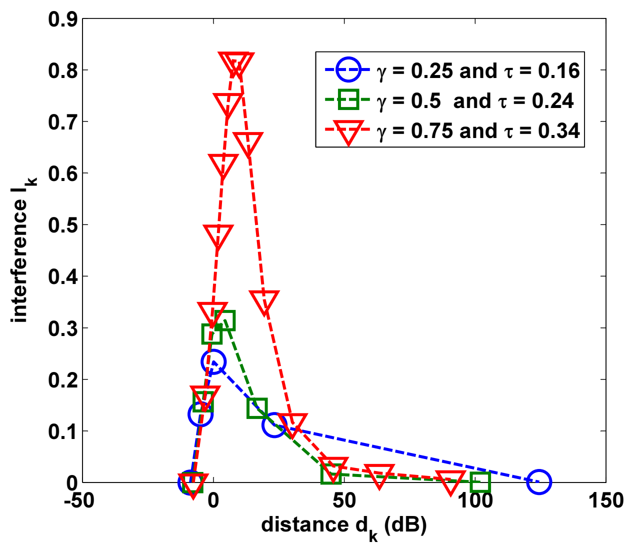


Fig. 12: The analytical propagation under continuous source transmission with $i = 0$, $\gamma < 1$, $P_s = 0.1$, $\bar{P}_r = 1$, $N_0 = 1$.

$k + 1$ and find a new τ accordingly. By means of this trial and error method, we found $\tau = 0.16$, 0.24 and 0.34 for $\gamma = 0.25$, 0.5 and 0.75, respectively, which lead to the successful broadcast.

Fig. 12 shows interference I_k versus d_k (in dB scale) for these three cases. The markers on each curve represent the levels. According to Theorem 1, I_k starts to decrease from some level converging to zero. For $\gamma = 0.25$, the interference starts to drop at level 4, i.e. $I_4 < I_3$. It can be observed that as γ increases, it takes more levels to observe the drop in I_k.

A finite network consisting 30000 nodes over the length of 100, i.e. node density of $\rho = 300$, is simulated by considering exactly the same network parameters as in the analytical study. The probability of propagation is obtained by realizing the system for 100 times. Fig. 13 shows the results for $\gamma = 0.25$, 0.5 and 0.75 each with three different values of τ; one taken equal to, one slightly below and the other slightly

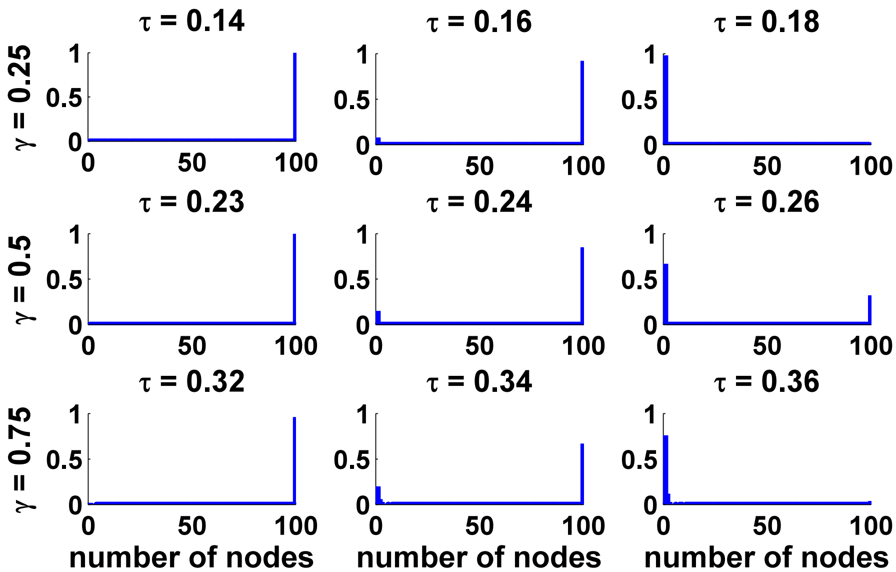


Fig. 13: Simulated propagation under continuous source transmission with $i = 0$, $\gamma = 0.25$ and 0.5, $P_s = 0.1$, $\bar{P}_r = 1$ and $N_0 = 1$.

greater than the corresponding critical value. We can see that the probability of the successful propagation changes dramatically over the narrow range of τ. This shows that for each γ and fixed network parameters, there is a critical threshold which results in a successful propagation.

As stated by Theorem 2, the bidirectional scheme is more likely to be successful than the unidirectional scheme. We show this fact by simulating the bidirectional transmission with the same network parameters as previous example only for the highest τ associated to each γ; $\tau = 0.18$, 0.26, 0.36 for $\gamma = 0.25$, 0.5 and 0.75, respectively. Unlike the probability of the successful propagation under the unidirectional scheme being almost zero, Fig. 14 shows that the bidirectional scheme results in the completely successful propagation for all the three cases.

2) Transmission under $\gamma = 1$: Following the similar process as for $\gamma < 1$, first we perform a numerical analysis of successful propagation under $\gamma = 1$. Solving the equations for $\bar{P}_r = 1$ leads to $\tau_{max} = 0.352$. This threshold is independent of P_s.

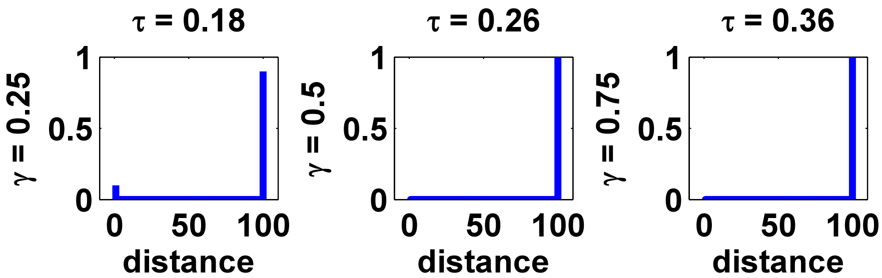


Fig. 14: Simulated propagation under the bidirectional continuous source transmission with $i = 0$, $\gamma < 1$, $P_s = 0.1$, $\bar{P}_r = 1$ and $N_0 = 1$.

Considering this threshold value and the network parameters; $\bar{P}_r = 1$, $P_s = 0.1$, N_0 and $i = 0$, we find maximum distance d_k and interference at this distance I_k for each level. Fig. 15 shows I_k versus d_k. As we expected, the interference saturates to a constant value, which can be found according to Eqn. 64 equal to 0.185.

The simulation results of a finite network is shown in Fig. 16 for the same network parameters while P_s takes two values, 0.1 and 1. The threshold τ is given three different values; being 0.325, 0.375 and 0.425. For $P_s = 0.1$, we can verify that $\tau = 0.325$ results in the successful propagation. On the other hand, the other values of τ do not result in the success probability of unity. For the second case where $P_s = 1$, a network with length $L = 200$ is simulated to show that the successful propagation is not possible for $\tau = 0.375$ and 0.425.

In Fig. 17, the bidirectional scheme is simulated with the same network parameters. As stated by Theorem 4, it is more likely to be successful compared to its unidirectional counterpart. In fact, it is completely successful for all the three values of τ which resulted in failure in the previous example for the unidirectional transmission.

3) Transmission under $\gamma > 1$: Theorem 5 states that for $\gamma > 1$, the condition for successful propagation is the same under the both bidirectional and unidirectional transmission protocols. According to this condition, the maximum threshold is $\tau = 1$ for $\gamma = 2$ and $i = 1$. As with $\gamma = 1$, P_s does not play any role in the successful

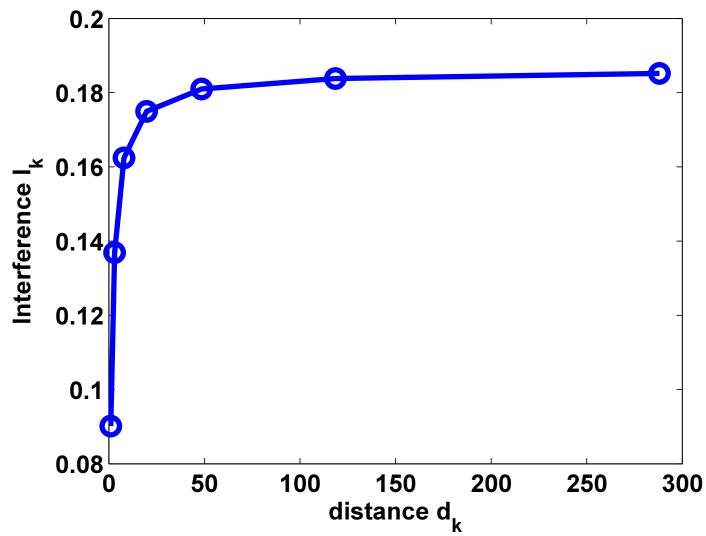


Fig. 15: The analytical propagation under continuous source transmission with $i = 1$, $\gamma = 1$, $\tau = 0.325$, $P_s = 0.1$, $\bar{P}_r = 1$, $N_0 = 1$.

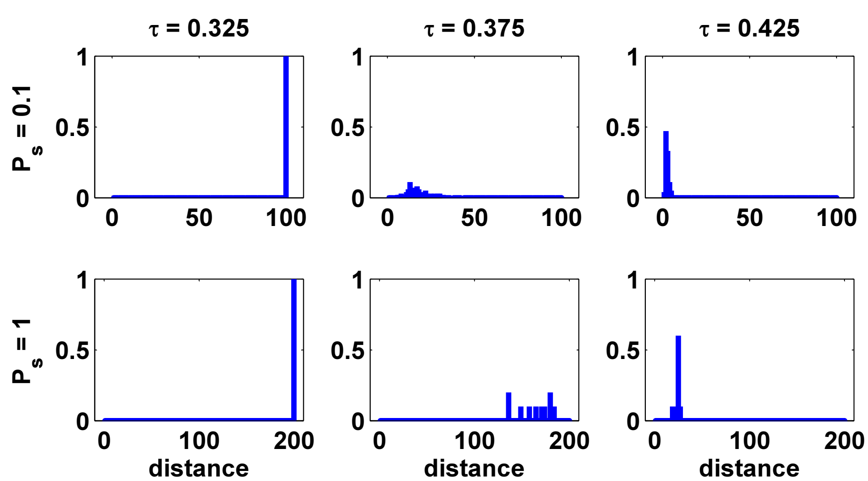


Fig. 16: Simulated propagation under the unidirectional continuous source transmission with $i = 1$, $\gamma = 1$, $\bar{P}_r = 1$ and $N_0 = 1$.

propagation. In addition, the choice of $\bar{P}_r$ does not affect the performance as long as the node density is assumed to be infinite. Its value only determines the size of Δ_∞. Thus, in the case of a finite node density, not sufficiently high $\bar{P}_r$ can result in a break

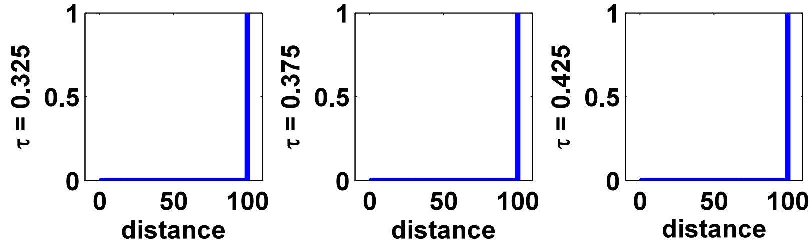


Fig. 17: Simulated propagation under the bidirectional continuous source transmission with $i = 0$, $\gamma = 1$, $\bar{P}_r = 1$ and $N_0 = 1$.

in transmission. We simulated the same setup ($\rho = 300$, $L = 100$, $\bar{P}_r = 1$, $P_s = 0.1$, and $N_0 = 1$)for $\gamma = 2$ and $\tau = 0.9$ which results in $\Delta_\infty = 0.055$. The results show that the density $\rho = 300$ is not enough to allow a successful propagation since there are 16 nodes in average in each level. Increasing $\bar{P}_r$ to 10, results in $\Delta_\infty = 0.55$ allowing 166 nodes in average in each level. With this setup, we are able to obtain successful propagation with probability of unity for $\tau = 0.9$ and smaller probabilities for $\tau = 1$ and 1.1. Fig. 18 shows the results for the bidirectional transmission. In this figure, we can also see the results for $\gamma = 3$ which requires $\tau < 3$ for the successful propagation with $i = 1$. Fig. 19 show the simulated result for the finite network with $\gamma = 2$ under the bidirectional transmission. As expected, we observe the similar behavior as with the unidirectional transmission.

H. Conclusion

In this paper, we analyzed a cooperative broadcasting scheme over multiple hops of relays in a high density linear network. The dynamics of the continuous source transmission is characterized for different ranges of pathloss exponent γ.

For $\gamma < 1$, we found that interference can either blow up or approaches zero depending on the network parameters. We provided the condition for interference to drop at some level compared to the previous level. We proved that once the interference drops, it continues decreasing until it gets zero. This behavior guarantees the success-

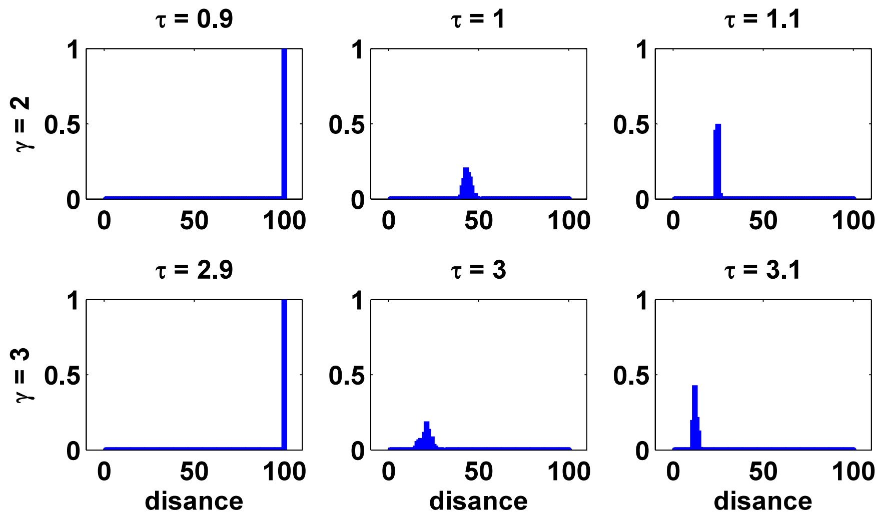


Fig. 18: Simulated propagation under the unidirectional continuous source transmission with $i = 1$, $\gamma = 2$ and 3, $\bar{P}_r = 10$ and $N_0 = 1$.

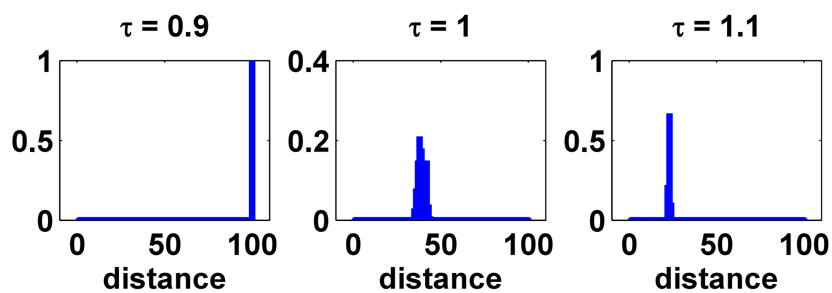


Fig. 19: Simulated propagation under the bidirectional continuous source transmission with $i = 1$, $\gamma = 2$, $\bar{P}_r = 10$ and $N_0 = 1$.

ful propagation which also was verified by the simulation results.

Similarly, for $\gamma = 1$, we showed that interference can either blow up or saturates to a constant value. Under a certain condition, the length of each level grows linearly with respect to the previous level, resulting in an increase in the received power by the next level, while the amount of interference converges to a constant value. Thus, the propagation is guaranteed to be successful when the interference converges to a constant value. We provided the condition for bounded interference with $\gamma = 1$ under the unidirectional transmission. We proved that the bidirectional transmission is successful given that it is successful under the unidirectional transmission.

For $\gamma > 1$, we showed the bidirectional and the unidirectional transmission exhibit similar dynamics. Interference is always bounded, however, it can still hinder the transmission since the length of levels converges to a constant value as well. We presented the condition for successful propagation which is the same for the unidirectional and the bidirectional transmission.

We also compared the power efficiency of this cooperative scheme with the classical multihop broadcast. First, we should note that multihop broadcast is interference limited for $\gamma \leq 1$, in the sense that transmission simply breaks due to the accumulated amount of interference. Only for $\gamma > 1$, transmission can be successful under a certain condition. We compared the power efficiency of cooperative broadcast and noncooperative multihop broadcast when both satisfy their corresponding conditions for the successful propagation. It was found that multihop broadcast exhibits a more efficient behavior under the unidirectional transmission, whereas cooperative broadcast proves to be more efficient for all ranges of γ if the bidirectional transmission is employed.

IV. CONCLUSION

We analyzed the cooperative broadcasting over multiple hops of relays in a linear network. The dynamics of the single-shot transmission along with the continuous source transmission is analyzed for different ranges of pathloss exponent γ. For single-shot transmission, we found phase transitions in the network behavior for different ranges of γ. The decoding threshold has the same critical value for $\gamma = 1$ as with $\gamma = 2$ in 2-D networks. We also found a phase transition with a decoding threshold for $\gamma < 1$, while there exists no phase transition for $\gamma > 1$.

We also compared the power efficiency of this cooperative scheme with the classical multihop broadcast. It was found that multihop broadcast shows a more efficient behavior under unidirectional transmission, whereas cooperative broadcast proves to be more efficient for all ranges of γ if bidirectional transmission is employed.

For the continuous source transmission, for $\gamma < 1$, we found that interference can either blow up or approaches zero depending on the network parameters. We provided the condition for interference to approach zero and thus to guarantee successful propagation, which was also verified by simulation results. Similarly, for $\gamma = 1$, we showed that the interference can either blow up or saturate to a constant value. We provided the condition for bounded interference with $\gamma = 1$ under unidirectional transmission. We proved that the bidirectional transmission will be also successful given that the propagation with the same network parameters is successful under the unidirectional transmission. For $\gamma > 1$, we showed the bidirectional transmission and the unidirectional transmission exhibit the similar dynamics. We presented the condition for successful propagation which is the same for the unidirectional transmission and the bidirectional transmission.

Finally, we compared the power efficiency of this cooperative scheme with the classical multihop broadcast. First, we should note that multihop broadcast is interference limited for $\gamma \leq 1$, in the sense that transmission stops due to the accumulated amount of interference. Only for $\gamma > 1$, transmission can be successful under a certain condition. We compared the power efficiency of cooperative and noncooperative multihop broadcast when both satisfy their corresponding conditions. Like single-shot transmission, it was found that multihop broadcast shows more efficient behavior under the unidirectional transmission, whereas cooperative broadcast proves to be more efficient for all ranges of γ if the bidirectional transmission is employed.

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