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TESTING YUKAWA-LIKE POTENTIALS FROM $f(R)$-GRAVITY IN ELLIPTICAL GALAXIES

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ABSTRACT

We present the first analysis of extended stellar kinematics of elliptical galaxies where a Yukawa-like correction to the Newtonian gravitational potential derived from $f(R)$-gravity is considered as an alternative to dark matter. In this framework, we model long-slit data and planetary nebula data out to 7 $R_{\rm eff}$ of three galaxies with either decreasing or flat dispersion profiles. We use the corrected Newtonian potential in a dispersion–kurtosis Jeans analysis to account for the mass–anisotropy degeneracy. We find that these modified potentials are able to fit nicely all three elliptical galaxies and the anisotropy distribution is consistent with that estimated if a dark halo is considered. The parameter which measures the "strength" of the Yukawa-like correction is, on average, smaller than the one found previously in spiral galaxies and correlates both with the scale length of the Yukawa-like term and the orbital anisotropy.

Key words: cosmology: theory – galaxies: elliptical and lenticular, cD – galaxies: general – galaxies: kinematics and dynamics

1. INTRODUCTION

The "concordance" ΛCDM cosmological model, which includes some unseen cold dark matter (DM) and a cosmological constant (Λ) acting as a repulsive form of dark energy (DE), has been remarkably successful in explaining the formation and evolution of cosmological structures at different scales (e.g., Springel et al. 2006).

However, at cosmological scales, the cosmological constant as a "vacuum state" of the gravitational field is about 120 orders of magnitude smaller than the value predicted by any quantum gravity theory (Weinberg 1989) and comparable to the matter density (coincidence problem), even if they evolved decoupled in the history of the universe.

In addition, looking at the galaxy scales there are a few critical issues yet to be solved, which are giving a hard time to the whole ΛCDM framework.

Since the discovery of the flat rotation curves of spiral systems, galaxies have been the most critical laboratory to investigate the gravitational effects of the DM halos, to be compared against the expectation of the cosmological simulations (Navarro et al. 1997, NFW hereafter; Burkert 1995; Navarro et al. 2010; Moore et al. 1999). Here, the ΛCDM model is not able to fully explain the shallow central density profile of spiral and dwarf galaxies (Gilmore et al. 2007; Salucci et al. 2007; Kuzio de Naray et al. 2008). Early-type galaxies (ETGs hereafter) have been proven only recently to be consistent with ΛCDM predictions (and WMAP5 cosmological parameters, e.g., Komatsu et al. 2009) from their centers (Tortora et al. 2009; Thomas et al. 2009; Napolitano et al. 2010) to their peripheries (Napolitano et al. 2011, N+11 hereafter), although there are also diverging results showing that ETGs in some cases have too high (Buote et al. 2007) or too low (e.g., Mandelbaum et al. 2008) concentrations.

This very uncertain context has been fertile soil for alternative approaches to the so-called missing mass. The basic approach is that the Newtonian Theory of Gravity, which has been tested only in the solar system, might be inaccurate on larger (galaxies and galaxy clusters) scales. The most popular theory investigated so far, the Modified Newtonian dynamics (MOND) proposed by Milgrom (1983), is based on phenomenological modifications of Newton dynamics in order to explain the flat rotation curves of spiral galaxies, and has passed a number of observational tests (Ferreira & Starkman 2009), including ETG kinematics (Milgrom & Sanders 2003; Tiret et al. 2007; Kroupa et al. 2010; Cardone et al. 2011; Richtler et al. 2011). Only lately has it been derived in a cosmological context (Bekenstein 2004).

A new approach, motivated from cosmology and quantum field theories on a curved spacetime, has been proposed to study the gravitational interaction: the Extended Theories of Gravity (Capozziello 2002; Capozziello & Faraoni 2011). In particular, the so-called $f(R)$-gravity seems to have passed different observational tests like spiral galaxies' rotation curves, X-ray emission of galaxy clusters, and cosmic acceleration (see, e.g., Capozziello et al. 2007a, C+07 hereafter; Capozziello et al. 2009, C+09 hereafter; Capozziello et al. 2008). This approach is based on a straightforward generalization of Einstein theory where the gravitational action (the Hilbert–Einstein action) is assumed to be linear in the Ricci curvature scalar R. In the case of $f(R)$-gravity, one assumes a generic function f of the Ricci scalar R (in particular analytic functions) and asks for a theory of gravity having suitable behaviors at small- and large-scale lengths.

As shown in Capozziello et al. (2009), analytic $f(R)$-models give rise, in general, to Yukawa-like corrections to the Newtonian potentials in the weak field limit approximation (see also Lubini et al. 2011). The correction introduces a new gravitational scale, besides the standard Schwarzschild one, depending on the dynamical structure of the self-gravitating system.

Here, we want to test these Yukawa-like gravitational potentials against a sample of elliptical galaxies. This approach has been proposed earlier, in a phenomenological scheme for antigravity, to model flat rotation curves of spiral galaxies (Sanders

1984), and recently, in $f(R)$ theories to model disk galaxies combined with NFW halos (see Cardone & Capozziello 2011). The test we are proposing at galaxy scales is crucial: reproducing kinematics and then dynamics of these very different classes of astrophysical systems in the realm of the same paradigm is needed to test these new gravitational theories as an alternative to DM which has not been definitely found out at a fundamental level.

The layout of the paper is the following. In Section 2, we sketch the main ingredients of $f(R)$-gravity deriving, in the weak field limit, the Yukawa-like corrected gravitational potential. Section 3 is devoted to the high-order Jeans analysis suitable for ellipticals. The dispersion–kurtosis fitting and the data sample are presented in Section 4. Discussion and conclusions are in Section 5.

2. POST- NEWTONIAN POTENTIALS FROM $F(R)$-GRAVITY

We are interested in testing a class of modified potentials which naturally arise in post-Newtonian approximation of $f(R)$-gravity for which no particular choice of the Lagrangian has been provided.

The starting point is a general gravity action of the form

$$\mathcal{A} = \int d^4 x \sqrt{-g} \left[f(R) + \mathcal{X} \mathcal{L}_m \right], \tag{1}$$

where $f(R)$ is an analytic function of the Ricci scalar, g is the determinant of the metric $g_{\mu\nu}$, $\mathcal{X} = 16\pi G/c^4$ is the gravitational coupling constant, and $\mathcal{L}_m$ describes the standard fluid-matter Lagrangian. Such an action is the straightforward generalization of the Hilbert–Einstein action obtained as soon as $f(R) = R$.

In Capozziello et al. (2009, and reference therein), it has been shown that if one solves the field equations in the weak field limit under the general assumption of an analytic Taylor expandable $f(R)$ function of the form

$$f(R) \simeq f_0 + f_1 R + f_2 R^2 + f_3 R^3 + \cdots \tag{2}$$

the following gravitational potential arises:

$$\Phi = -\left(\frac{GM}{f_1 r} + \frac{L\delta_1(t) e^{-\frac{r}{L}}}{6r} \right), \tag{3}$$

where $L \doteq -6f_2/f_1$, and f_1 and f_2 are the expansion coefficients obtained by Taylor expansion. We note that the L parameter is related to the effective mass $m = (-3/L^2)^{-1/2} = (2f_2/f_1)^{1/2}$ and can also be interpreted as an effective length.

From Equation (3), the standard Newton potential is recovered only in the particular case $f(R) = R$. Furthermore, the parameters f_1 and f_2 and the function δ_1 represent the deviations with respect to the standard Newton potential. On the solar system scale, it has been shown that Yukawa-like deviations from the pure Newtonian potential are not in contradiction with classical tests of general relativity (see, e.g., Capozziello & Tsujikawa 2008; Eingorn & Zhuk 2011), thanks to the so-called Chameleon mechanism (Khoury & Weltman 2004). In particular, f_1 and f_2 parameters are expected to allow the regular Newtonian potential, while at larger scales they can assume non-trivial values (e.g., $f_1 \neq 1$, $\delta_1(t) \neq 0$, $\xi \neq 1$, see Capozziello et al. 2007b, 2009).

Equation (3) can be recast as

$$\Phi(r) = -\frac{GM}{(1+\delta)r}(1 + \delta e^{-\frac{r}{L}}), \tag{4}$$

where the first term is the Newtonian-like part of the potential associated with baryonic point-like mass $M/(1 + \delta)$ (no DM), and the second term is a modification of the gravity including a "scale length," L associated with the above coefficient of the Taylor expansion. If $\delta = 0$, then the Newtonian potential is recovered. Comparing Equations (3) and (4), we obtain that $1 + \delta = f_1$, and δ is related to $\delta_1(t)$ through

$$\delta_1 = -\frac{6GM}{L^2} \frac{\delta}{1+\delta}, \tag{5}$$

where $6GM/L^2$ and δ_1 can be assumed to be quasi-constant. From Equation (5), it turns out that $L \propto \sqrt{-\delta/(1+\delta)}$. Due to the arbitrariness of $\delta_1(t)$, the actual value of the δ parameter can assume any value; however, in order to have a Yukawa potential with a non imaginary exponent (i.e., L must be real), it is required that $\xi < 0$ or $-1 < \delta < 0$. As a comparison, Sanders (1984) adopted the same potential as in Equation (4) under the assumption of anti-gravity generated by massive particles (of mass m_0) carrying the additional gravitational force. In this case a typical scale length would naturally arise ($L = h/m_0 c$ being a Compton length) and a $-1 < \delta < 0$ would provide a repulsive term to the Newtonian-like term, producing flat rotation curves at $r \gg L$ as observed in spiral galaxies. In particular, for a small sample of spiral systems Sanders (1984) found $-0.95 \lesssim \delta \lesssim -0.92$.

Here, we want to test the modified potential in elliptical galaxies as in Equation (4), and check whether it is able to provide a reasonable match to their kinematics and how the model parameters compare with the results obtained from spiral systems. We construct equilibrium models based on the solution of the radial Jeans equation (see Section 3) to interpret the kinematics of planetary nebulae (PNe; see Napolitano et al. 2002, 2005; Romanowsky et al. 2003; Coccato et al. 2009) which are the *only stellar-like tracers for galaxy dynamics available in ETGs out to* ∼5–10 *effective radii* ($R_{\rm eff}$).

We will use the inner long slit data and the extended PN kinematics for three galaxies which have published dynamical analyses within the DM halo framework: NGC 3379 (Douglas et al. 2007; De Lorenzi et al. 2009, DL+09 hereafter), NGC 4494 (Napolitano et al. 2009, N+09), and NGC 4374 (N+11). The decreasing velocity dispersion profiles of the first two galaxies have been modeled with an intermediate mass halo, $\log M_{\rm vir} \sim$ 12–12.2 $M_\odot$, with concentration $c_{\rm vir} = 6$–8 and a fair amount of radial anisotropy in the outer regions. For NGC 4374, having a rather flat dispersion profile, a more massive (adiabatically contracted) halo with $\log M_{\rm vir} \sim 13.4$ $M_\odot$ and $c_{\rm vir} \sim 7$ was required with a negligible amount of anisotropy in the outer regions. These models turned out to be in fair agreement with the expectation of a WMAP5 $c_{\rm vir}$–$M_{\rm vir}$ relation (N+11) and with a Kroupa (2001) initial mass function (IMF), making this sample particularly suitable for a comparison with an alternative theory of gravity with no-DM as we want to propose here.

Before we go on with detailed stellar dynamics, we show in Figure 1 the circular velocity of the modified potential as a function of the potential parameters L and δ for NGC 4494 and NGC 4374. As for the spiral galaxies, negative values of the δ parameter make the circular velocity more and more flat, also reproducing the typical dip (e.g., NGC 4374) of the circular velocity found for the DM models (dot-dashed curves) of the most massive systems. On the contrary, positive δ values cannot produce flat circular velocity curves (see Figure 1).
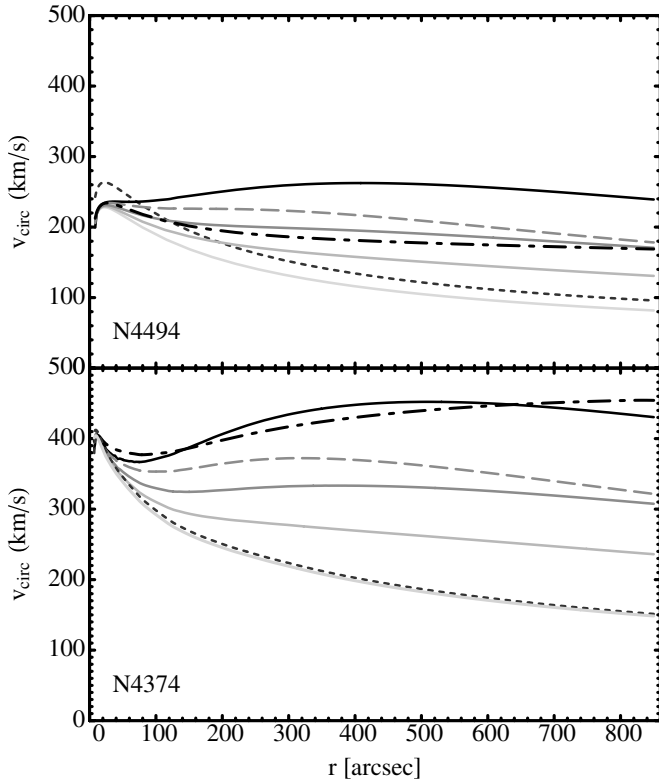
Figure 1. Circular velocity produced by the modified potential in Equation (4) for the two galaxies N4494 (top) and N4374 (bottom). In both cases the M/L_* has been fixed to some fiducial value (as expected from stellar population models and Kroupa 2001 IMF): $M/L_* = 4.3\Upsilon_{\odot,B}$ for NGC 4494 and $M/L_* = 5.5\Upsilon_{\odot,V}$ for NGC 4374. The potential parameters adopted are: $L = 250''$ and $\delta = 0, -0.65, -0.8, -0.9$ (lighter to darker solid lines) and $L = 180''$ and $\delta = -0.8$ (dashed lines). The dotted line is a case with a positive coefficient of the Yukawa-like term and $L = 5000''$ which illustrates that positive δ cannot produce flat circular velocity curves. Finally, some reference Navarro–Frenk–White (NFW) models are shown as dot-dashed lines.

2.1. A Consistency Check with Galaxy Scaling Relations

To conclude the inspection of the modified potential as in Equation (4), here we want to show that, beside flat rotation curves, this also naturally accounts for fundamental scaling relations of galaxies: the Tully–Fisher (TF) relation for spirals and Faber–Jackson (FJ) relation for ETGs.

Both relations connect the total mass M of galaxies with some characteristic velocity defining the kinetic energy of the systems (i.e., the maximum rotation velocity, v_{max}, for spirals and the central velocity dispersion, σ_0, for ETGs). In either case the kinematical quantities involved are proportional to the circular velocity of the systems through some "structure" constant, and thus the arguments below apply to galaxies in general.

Although the point-like version of the potential implies that the circular velocity v_c scales with mass as $M \sim v_{c,max}^2$ (as pointed out by Sanders 1986), if one derives the circular velocity for an extended galaxy this can be generalized as

$$v_c^2(r) = (GM_{tot}/r_*) \times f(r/r_*; \delta, L/r_*), \qquad (6)$$

where r_* is a characteristic radius (e.g., the disk length for spirals or the effective radius encircling half of the galaxy light for ETGs) and $f(r/r_*; \delta, L/r_*)$ is a generic function which includes the radial dependence of the enclosed mass and the above Yukawa-like term. This function is defined such as, for $\delta = 0$, it gives $v_c^2(r) = GM(r)/r$ as the usual Newtonian

expression. It is easy to show that if galaxies are homologous, then the maximum of v_c is reached at the same r/r_*, for a given δ and L/r_*, and this maximum can be written as

$$v_{c,max}^2 = KM_{tot}/r_*, \qquad (7)$$

where the constant K depends on the set of parameters $\{\delta, L, r_*\}$ adopted. In Equation (7), though, M_{tot} and r_* are linked by the size–mass relation which is generally written as $r_* \propto M_{tot}^\alpha$, from which Equation (7) can be written as

$$v_{c,max}^2 \propto M_{tot}^{1-\alpha}. \qquad (8)$$

The size-mass relation of spiral galaxies can be found in Persic et al. (1996, see also Thomas et al. 2009) to be $r_* \propto M_{tot}^{0.4}$, while it is $r_* \propto M_{tot}^{0.6}$ for ETGs (e.g., Shen et al. 2003; Napolitano et al. 2005). This would give a TF slope of 3.33 and FJ slope of 5, which are both in the range of the observed relations (see, e.g., McGaugh 2005; Nigoche-Netro et al. 2010, respectively) with the remaining discrepancy being mainly due to the conversion factor to the observed quantities and non homologies.

We finally remark that the TF relation has been found not to conflict with $f(R)$ potentials in Capozziello et al. (2006), although the potentials from $f(R) \propto R^n$ adopted there are just a series expansion of the Yukawa-like potential coming out from a more general polynomial $f(R)$ as in Equation (3).

3. HIGH-ORDER JEANS ANALYSIS

From the model point of view, the problem of fitting a modified potential as in Equation (4) (which is formally self-consistent since the source of the potential is the only mass of the dynamical tracers, i.e., stars) implies the same kind of degeneracies between the anisotropy parameter, $\beta = 1 - \sigma_\theta^2/\sigma_r^2$ (where σ_θ and σ_r are the azimuthal and radial dispersion components in spherical coordinates), and the non-Newtonian part of the potential (characterized by two parameters like typical dark halos) in a similar way to the classical mass–anisotropy degeneracy. We have shown (N+09, N+11) that these degeneracies can be alleviated via higher-order Jeans equations including in the dynamical models both the dispersion[6] (σ_p) and the kurtosis (κ) profiles of the tracers.

In the following, we will use the assumption of spherical symmetry since galaxies in the sample are all E0–E1, for which, if one excludes the singular chance that they are all flattened systems seen face-on (see discussion in Section 8.1. of Douglas et al. 2007), the spherical approximation is good at 10% (Kronawitter et al. 2000).[7] Under a spherical assumption, no-rotation, and $\beta = $ const (corresponding to the family of distribution functions $f(E, L) = f_0 L^{-2\beta}$; see Łokas 2002 and references therein),[8] the 2nd and 4th moment radial equations can be compactly written as

$$s(r) = r^{-2\beta} \int_r^\infty x^{2\beta} H(x) dx, \qquad (9)$$

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[6] For the slow-rotating models we use the velocity, $v_{rms} = \sqrt{v^2 + \sigma^2}$ as a measure of the velocity dispersion.

[7] The effect of non-spherical models is outside the scope of this paper, but details for NGC 3379 and NGC 4494 can be found in DL+09 and N+09.

[8] Here, there is the caveat that the solution of Jeans Equations does not ensure that the final distribution function is non negative and thus fully physical (see, e.g., An & Evans 2006).
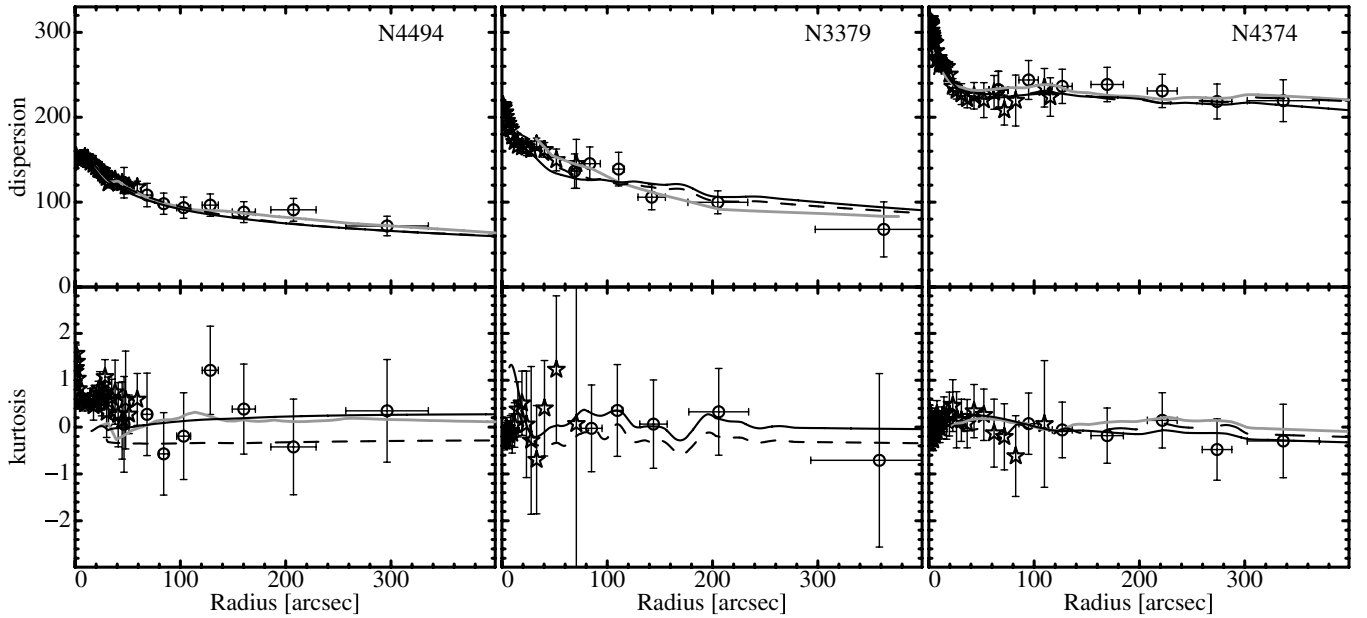
Figure 2. Dispersion in km s^{-1} (top) and kurtosis fit (bottom) of the galaxy sample for the different $f(R)$ parameter sets: the anisotropic solution (solid lines) is compared with the isotropic case (dashed line—for NGC 4374 and NGC 4494, this is almost indistinguishable from the anisotropic case). From left to right, NGC 4494, NGC 3379, and NGC 4374 are shown with DM models as gray lines from N+09, DL+09 (no kurtosis is provided), and N+11, respectively.

where $s(r) = \{\rho\sigma_r^2; \rho\overline{v_r^4}\}$, β is the anisotropy parameter, and

$$H(r) = \left\{\rho\frac{d\Phi}{dr}; 3\rho\frac{d\Phi}{dr}\overline{v_r^2}\right\},$$

respectively, for the dispersion and kurtosis equations, where the latter is $\kappa(r) = \overline{v_r^4}/\sigma_r^4$. In the same equations, $\Phi(r)$ is the spherical extended source version of the point-like potential as in Equation (4)[9] and $\rho(r)$ is the three-dimensional density of the tracer obtained by multiplying the deprojection of the stellar surface brightness profile, $j_\star(r)$, by some constant stellar mass-to-light ratio, $M/L_\star$.

This $M/L_\star = $ const might be a strong assumption to check further in a separate paper as it neglects the presence of stellar population gradients (see, e.g., Tortora et al. 2010). However, color (and M/L) gradients are generally stronger within R_{eff} (see, e.g., Tortora et al. 2010) and might mainly drive the best fit in the central regions, while they are possibly shallower outside (e.g., Tamura & Ohta 2003) where the $f(R)$ parameters should be better constrained. In the following, $j_\star(r)$ is derived by photometry presented in previous dynamical studies (i.e., DL+09, N+09, N+11 for NGC 3379, NGC 4494, and NGC 4374, respectively).

Equations (9) are the ones interested by the potential modification and include four free parameters to be best fitted: the $f(R)$ parameters $\{\delta, L\}$, the "dynamically inferred" stellar mass-to-light ratio $M/L_\star$, and the constant anisotropy β (see also Section 4). The solutions of Equations (9) on a regular grid in the parameter space are then projected to match the observed line-of-sight kinematical profile via ordinary Abel integrals (see N+09 for details).

As mentioned earlier, Equations (9) are written under the assumption of a constant β with radius, which provides an average global anisotropy distribution over all the galaxy. As seen in previous analyses (e.g., N+09, DL+09, and N+11), it is

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9 This is obtained assuming the onion shell approximation:
$\Phi(r) = \int_0^r \int_0^{2\pi} \int_0^\pi \phi(r) r^2 \sin\theta\, d\theta\, d\varphi\, dr$, see also Equation (18) of C+09.

Table 1
Model Parameters for the $f(R)$ Potential

Galaxy	Mag (band)	R_{eff}	$M/L_\star$	L	δ	β	χ^2/dof
NGC 3379	−19.8(B)	2.2	6 (7)	6	−0.75	0.5(<0.8)	14/25
NGC 4374	−21.3(V)	3.4	6 (6)	24	−0.88	0.01(0.01)	14/39
NGC 4494	−20.5(B)	6.1	3 (4)	20	−0.79	0.5(0.5)	18/43

Notes. Galaxy ID, total magnitude, effective radius, and model parameters for the unified solution. DM-based estimates for $M/L_\star$ and β (NGC 3379: DL+09; NGC 4374: N+11; NGC 4494: N+09) are shown in parentheses for comparison. $M/L_\star$ are in solar units, R_{eff} and L in kpc. Typical errors on $M/L_\star$ are of the order of 0.2 $M/L_\odot$ and on β of 0.2 (see also Figure 3). The small χ^2 values are mainly due to the large data error bars.

likely that this might not be a fair assumption, as β turns out to be constant somewhere in the outer regions, but strongly varying in the central radii. In this preliminary test we will skip this implementation of the models since we expect this to possibly improve the fit to the data in the central part only, where we do not expect the overall dynamics to be strongly ruled by the $f(R)$ potential, whose parameters are the main focus of this work. Furthermore, we have shown previously (see, e.g., N+09 and N+11) that the assumption of a constant or radial varying anisotropy did not strongly affect the determination of the other important parameter, the dynamically based stellar M/L. In the following we will take the $\beta = $ const as a fair estimate of the average galaxy anisotropy.

4. DISPERSION–KURTOSIS FITTING

In Figure 2, we show the dispersion and kurtosis profiles of the three galaxies with the $f(R)$ models superimposed (solid lines). The fitting procedure is based on the simultaneous χ^2 minimization of the dispersion and kurtosis profiles over a regular grid in the parameter space. The best-fit parameters are summarized in Table 1 together with some info from the galaxy sample.

Overall, the agreement of the model curves with the data is remarkably good and it is comparable with models obtained with DM modeling (gray lines in Figure 2).

In all cases, the $f(R)$ models allow us to accommodate a constant orbital anisotropy β which is very close to the estimates from the DM models (see, e.g., Table 1[10]). This is mainly guaranteed by the fit to the $\kappa(r)$ which does not respond much differently to the modified potential with respect to the DM models. Thus, an important result of the analysis is that the orbital anisotropy is fairly stable in relation to the change of the galaxy potential. In particular, the use of the kurtosis profiles has allowed us to solve the degeneracy of the models and favor the anisotropic solutions for NGC 3379 and NGC 4494 (NGC 4374 being almost isotropic everywhere). Although the isotropic solutions also provide a good fit for the dispersion profile only (see, e.g., the dashed lines in Figure 2), they do not correctly match the observed κ. This produces a significantly worse total χ^2/dof (NGC 3379: 45/26; NGC 4374: 35/40; NGC 4494: 27/44) with respect to the best fit in Table 1, although it is still close to $\chi^2/\text{dof} \sim 1$ mainly because of the large error bars.

Finally, the best-fit $M/L_\star$ in Table 1 are very similar to the values found for DM models (reported between brackets) in all cases, generally consistent with a Kroupa (2001) IMF.

Looking at the $f(R)$ parameters, in Figure 3 we show the marginalized confidence contours of the two main potential parameters for the three galaxies. As also reported in Table 1, the δ parameter has a mean value of $\delta = -0.81 \pm 0.07$ which is inconsistent with the one previously found for spiral galaxies (e.g., Sanders 1984, also shown in Figure 3). On the contrary, δ seems nicely correlated with the other potential parameter, L, as expected from Equation (5). In the same figure, the correlation is supported by the tentative fit into the $\delta-L$ plane (whether or not the spiral galaxy sample is included in the fit), although the sample is too small to drive any firm conclusion.

Interestingly, there seems to be a possible increasing trend of δ with the orbital anisotropy: this is also shown in Figure 3 where we have added the fiducial value obtained for the spiral sample (having assumed a reference $\beta = -1$ for fiducial tangential anisotropy for late-type systems, see, e.g., Battaglia et al. 2005). This evidence leaves room for an interpretation of δ and the physics of the galaxy collapse (e.g., the spherical infall model; Gunn & Gott 1972; Gunn 1977).

In fact, as discussed in Section 2, δ is linked to δ_1, which is an arbitrary function that comes out because the field equations in the post-Newtonian approximation depend only on the radial coordinate. From a physical point of view, such a function could be related to second-order effects related to anisotropies and non-homogeneities which could trigger the formation and the evolution of stellar systems. To take into account such a situation, one should perform the post-Newtonian limit of the theory, not only in the simple hypothesis of homogeneous spherical symmetry (Schwarzschild solution), but also considering more realistic situations such as Lemaître–Tolman–Bondi solutions (see, e.g., Herrera et al. 2010).

5. DISCUSSION AND CONCLUSIONS

There is a growing attention to alternative models to the ΛCDM paradigm as the latter is still suffering from some discrepancies at the galaxy scales and, most importantly, is
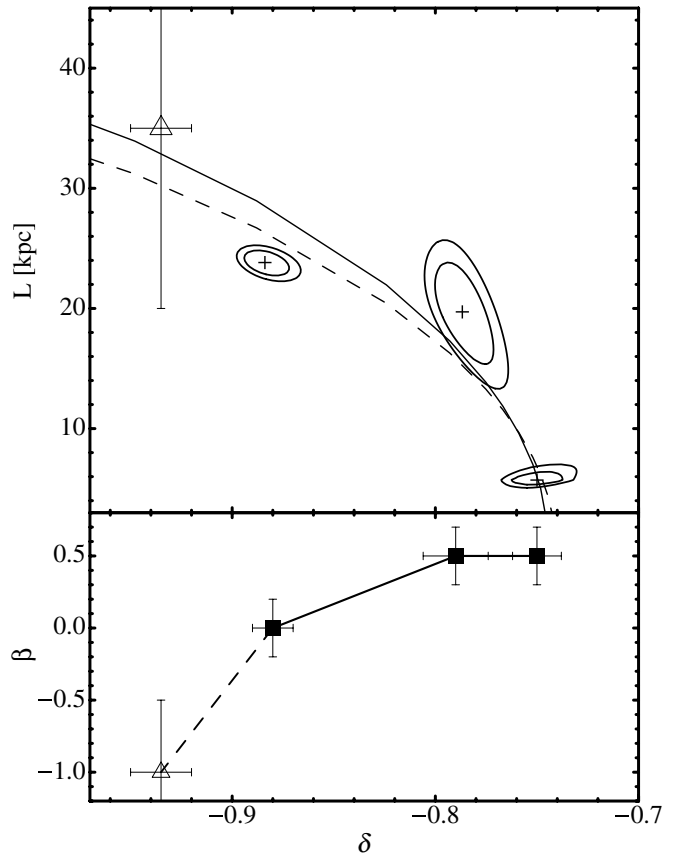
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[10] For NGC 4374 only to be nicely fitted at all radii we needed to include some radial anisotropy in the very central regions, following the $\beta(r)$ distribution adopted in N+11 (see Equation (5), with best fit $r_a = 22.5$ arcsec).

Figure 3. Top: 1σ and 2σ confidence levels in the $\delta - L$ space marginalized over $M/L_\star$ and β (see also Table 1). Spiral galaxy results from Sanders (1984) are shown as an empty triangle with error bars. The solid (dashed) curve shows the tentative best fit to the data, including (excluding) the spiral galaxies and assuming an $L \propto \sqrt{\delta/(1+\delta)}$ correlation as expected from Equation (5). Bottom: the anisotropy and the δ parameters turn out to be correlated for the elliptical sample (full squares). This correlation seems to also include the spiral sample cumulatively shown as the empty triangle (here we have assumed $\beta = -1.0 \pm 0.5$ as a fiducial value for spiral galaxies to draw a semi-quantitative trend across galaxy types).

based on the assumption of the existence of two ingredients (DM and DE) whose nature is still unknown.

Different attempts have tried to circumvent the problem by introducing modified dynamics, e.g., with the MOND theory (see Sanders & McGaugh 2002; Swaters et al. 2010; Cardone et al. 2011), but this still seems to be needing some DM, at least at cluster scales which might still be consistent with the primordial nucleosynthesis (e.g., via high energy neutrinos; Angus et al. 2010) and does not provide an explanation for the DE.

Lately, $f(R)$-gravity models have made their step out as a natural explanation for the two *dark* ingredients of the universe assuming that they are related to the fact that gravitational interaction could present further degrees of freedom whose dynamical effects emerge at large scales (Capozziello & Faraoni 2011). In this paper, we have checked the Yukawa-like modification to the Newtonian potential obtained as a post-Newtonian approximation of $f(R)$-gravity for which no particular choice of the Lagrangian has been provided, with the only assumption being that $f(R)$ is an analytic function.

We have used a combination of long-slit spectroscopy and PN kinematics out to $\sim 7\,R_{\text{eff}}$ in three systems (NGC 3379, NGC 4374, NGC 4494) for which ΛCDM models turned out to be fairly consistent with WMAP5 measurements (see N+11 for a discussion).

Due to the small galaxy sample, the spirit of this analysis has been to check whether (1) the modified potential introduced by the $f(R)$-gravity allowed a fit to the galaxy kinematics comparable to the DM models and (2) the three galaxies returned a parameter δ which is comparable with spiral galaxies (Sanders 1984).

We have found that the modified potentials allow us to nicely model the three galaxies with a distribution of the δ parameters which turned out to be inconsistent with the results found in spiral systems. We have shown some hints that δ might be correlated with the galaxy anisotropy, β, and the scale parameter, L, with elliptical and spiral galaxies following the same pattern.

This evidence can have interesting implications about the ability of the theory to make predictions on the internal structure of the gravitating systems after their spherical collapse (e.g., Gunn 1977) which have to be confirmed on a larger galaxy sample which we expect to do in the near future.

Despite some simplifications of the model adopted (e.g., constant M/L and anisotropy across the galaxy) and the degeneracies between the model parameters, the results are very encouraging. The fit to the data is very good in all cases and both the stellar M/L (with Kroupa IMF generally favored) and orbital anisotropy turn out to be similar to the one estimated if a dark halo is considered.

Getting a modified gravity to work self-consistently for all gravitating systems in general, and all galaxy families in particular, is a very non-trivial challenge that has foiled other theories (e.g., MOND).

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