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A Heuristic for Marketing-Production Decisions in Industrial Channels of Distribution

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10. The probability of observing payoff outcome on a reported high and indeed high value property is:
    $P(\hat{H}, ND, H) = P(\hat{H}, ND \mid H)P(H) = (1 - p_H)\eta$.
11. The probability of observing payoff outcome on a reported low but in fact high value property is:
    $P(\hat{L}, ND, H) = P(\hat{L}, ND \mid H)P(H) = 0$.
12. Lastly, the probability of observing payoff outcome on a reported low and indeed low value property is:
    $P(\hat{L}, ND, L) = P(\hat{L}, ND \mid L)P(L) = (1 - p_L)(1 - e)(1 - \eta)$.

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Eliashberg and Steinberg (1987) presented a model for firms in industrial distribution channels, which provides optimal pricing, processing, and inventory policies using an optimal control methodology. However, their model assumes that an interior solution exists for optimal control problem. In our paper, we demonstrate that applicability of optimal policies is parameter dependent—demand does not necessarily start at time 0 and terminate at time T, but depends upon model policies. We propose a heuristic which, when used with generalized optimal policies for channel firms, computes actual time horizons during which optimal policies will provide correct results. A numerical example illustrates the heuristic.

Keywords: Supply Chain Management, Marketing-Production Interface, Joint Decision Making, Distribution Channels, Pricing Policies, Production Policies, Inventory Policies

1 Introduction

Profits realized by manufacturing organizations are contingent on both the external environment in which they operates and the performance of its internal organizational functions. While manufacturing organizations operating in a supply chain environment may have little or no control over the uncertainty of environmental exigencies, they certainly can manage the interaction between their business functions. Marketing-production interactions and their result on organizational decision-making has been an important area of research in designing optimal supply chain policies (i.e. pricing, processing, and inventory policies) in recent years [1-9, 11, 14-16, 18]. In addition, several researchers [10, 12-13, 17] have proposed heuristics to arrive at optimal pricing and inventory policies in a distribution channel environment.

In a recent paper, Eliashberg and Steinberg [6]—henceforth referred to as 'ES87'—employ an optimal control theoretic approach to derive optimal pricing, processing, and inventory policies for both manufacturer and distributor in an industrial channel of distribution using a Stackelberg game theoretic model. In that article, they propose a novel approach to provide explicit policies for the manufacturer and distributor operating in a vertical distribution channel environment. This was one of the first papers to use an 'indirect adjoining' approach in their optimal control solution for this type of problem structure. The issue addressed in this paper deals with the specific assumption on the nature of the time horizon, from 0 to T, and its impact on the optimal policies. In their model, ES87 specify a quadratic formulation for the market potential term, $a_D(t)$. They say:

In order to capture the seasonality effect, we have chosen to model the market potential term, $a_D(t)$, through a quadratic formulation which provides interesting interpretations. That is

$$a_D(t) = -\alpha_1 t^2 + \alpha_2 t + \alpha_3, \quad 0 \le t \le T, \quad \text{where} \quad T = \alpha_2/\alpha_1 \quad \text{and } \alpha_1, \alpha_2, \alpha_3 > 0.$$

Here, α_3 represents the "nominal" size of the market potential before the season begins. The parameters α_1 and α_2 determine the timing $(\alpha_2/2\alpha_1)$ and the magnitude $([\alpha_3+(\alpha_2^2/4\alpha_1)])$ of the peak sales. It is straightforward to show that for larger values of α_1 will move the peak sooner and will lower its magnitude, whereas larger values of α_2 will have opposite effects. Finally, T is set equal to α_2/α_1 in order to encompass the season in its entirety [p. 988].

ES87 assume that the season starts at 0 and terminates at T. The length of the season, T, equals to α_2/α_1, which is the time the market potential drops back to its "nominal" size (α_3). Once the values of α_1, α_2, and α_3 are specified, the start and terminal times for the season are fixed and are not sensitive to any changes in the market-specific and firm-specific parameters such as b_D, K_D, h_D, etc.

Furthermore, in order to simplify their analysis, ES87 assume an interior solution while deriving the optimal policies of the distributor. They write:

In order to simplify the analysis below, we assume an interior solution. That is, $a_D/b_D > P_D^* > P_M$ and $0 < Q_D^*$ for $0 \le t \le T$ [p. 997].

The assumption of an interior solution ensures the length of the season to be [0,T]. This is a very restrictive assumption which can be violated very easily. In fact, the optimal policies in the numerical example provided in ES87 violate this assumption.

The violation of this assumption can occur when at least one of the constraints in the optimal control problem examined in ES87 is not being met. As an example, the demand for the product can drop to zero before the end of the season, presumably at T, even if there is positive demand potential. To resolve this issue, we introduce two new variables t_S and t_T, which represent the start and terminal values of the season. These values reflect the points in time at which the season "effectively starts and terminates." Note that ($t_S \ge 0$) and ($t_T \le T$). During this interval $[t_S, t_T]$, all constraints pertaining to both channel members are satisfied at the pre-specified parameter values (such as b_D, K_D, h_D, etc.).

The paper is organized as follows: In section 2, we provide the generalized optimal pricing, processing, and inventory policies for both distributor and manufacturer using variable start and terminal times of the season. A heuristic is proposed in section 3, which, when employed along with the generalized policies, will extract the time interval $[t_S, t_T]$, through an iterative process. In section 4, a numerical example is presented to illustrate the heuristic. The robustness of the heuristic is tested by treating the parameter for price sensitivity as a variable in section 5. Finally, in section 6, the implications of these results on channel members are discussed.

2 Generalized Optimal Policies

We re-solve the optimal control problem of ES87 under the assumption that an interior solution exists for the interval $[t_S, t_T]$, which is a subset of [0,T]. The generalized optimal policies for distributor and manufacturer are provided below:

Distributor's Policies:

Propositions 1, 2, and 3 of ES87 [p. 986], which allude to the nature of pricing, processing, and inventory policies of the distributor, also hold in the case of variable start and terminal time of the season. However, the condition in Proposition 4 of ES87 [p. 989], under which the distributor can smooth out his operation in contrast to when he should follow a stockless production policy, need to be revised as follow:

Proposition 4 *(revised).*

In general, if the distributor's inventory holding cost per unit is sufficiently low, price sensitivity is low, processing efficiency is low, and the seasonal demand is volatile, he can smooth out his operations. In particular, if:

(i) $h_D < (\alpha_2 - 2\alpha_1 t_S)/(3(b_D+K_D))$, *the distributor can smooth out his operations.*
(ii) $h_D \ge (\alpha_2 - 2\alpha_1 t_S)/(3(b_D+K_D))$, *the distributor should not smooth out his operations and act according to stockless production policy throughout the season.*

Corollary 5 *(revised).*

The optimal pricing, processing, and inventory policies for the distributor are:

$$Q_D^*(t) = \begin{cases} (K_D/(2(b_D+K_D)))(a_D(t_D^*) - h_D(b_D+K_D)(t_D^*-t) - b_D P_M) & t_S \le t \le t_D^* \\ (K_D/(2(b_D+K_D)))(a_D(t) - b_D P_M) & t_D^* \le t \le t_T, \end{cases} \tag{1}$$

$$P_D^*(t) = \begin{cases} (1/(2(b_D+K_D)))((b_D+K_D)(a_D(t)/b_D) + a_D(t_D^*) - h_D(b_D+K_D)(t_D^*-t) + K_D P_M) & t_S \le t \le t_D^* \\ (1/(2(b_D+K_D)))((2b_D+K_D)(a_D(t)/b_D) + K_D P_M) & t_D^* \le t \le t_T, \end{cases} \tag{2}$$

$$I_D^*(t) = \begin{cases} (\alpha_1/6)(t_D^*-t)^2(t-t_S) & t_S \le t \le t_D^* \\ 0 & t_D^* \le t \le t_T, \end{cases} \tag{3}$$

where,

$$t_D^{*}=(3/(4\alpha_1))(\alpha_2-(b_D+K_D)h_D-(2\alpha_1 t_S)/3). \tag{4}$$

Proof See Appendix A

Manufacturer's Policies:

Propositions 6, 7, and 8 of ES87 [p. 990-1], which allude to the nature of pricing, processing, and inventory policies of the manufacturer, also hold in the case of variable start and terminal time of the season.

Corollary 9 *(revised).*

The optimal pricing, processing, and inventory policies for the manufacturer are:

$$Q_M^{*}(t)=\begin{cases}(K_D/(2(b_D+K_D)))(a_D(t_M^{*})-h_M(K_M/K_D)(b_D+K_D)(t_M^{*}-t)-b_DP_M) & t_S\le t\le t_M^{*}\\ (K_D/(2(b_D+K_D)))(a_D(t)-b_DP_M) & t_M^{*}\le t\le t_T,\end{cases} \tag{5}$$

where,

$$t_M^{*}=(3/(4\alpha_1))(\alpha_2-(K_M/K_D)(b_D+K_D)h_M-(2\alpha_1 t_S)/3), \tag{6}$$

$$P_M^{*}=w_1[(1/(t_T-t_S))\int_{t_S}^{t_T}(a_D(t)/b_D)dt]+w_2C_M, \tag{7}$$

where,

$$w_1=(1+(2b_M/K_M))/(2+(2b_M/K_M)),\quad w_2=1/(2+(2b_M/K_M)), \tag{8}$$

and

$$b_M=b_DK_D/(2(b_D+K_D)), \tag{9}$$

$$I_M^{*}(t)=\begin{cases}(K_D/(2(b_D+K_D)))(\alpha_1/3)(t_M^{*}-t_D^{*})(t_M^{*}+t_D^{*}+2\alpha_1 t_S)(t-t_S)-(1/4)(K_Dh_D-K_Mh_M)(t^2-t_S^2) & t_S\le t\le t_D^{*}\\ (K_D/(2(b_D+K_D)))((\alpha_1/3)(t_M^{*}-t)^2(t-t_S)) & t_D^{*}\le t\le t_M^{*}\\ 0 & t_M^{*}\le t\le t_T.\end{cases} \tag{10}$$

Proof See Appendix B

Comparison with ES87

One distinct result is that both t_D and t_M computed in the generalized policies case are lower than those given in ES87 by the value $t_S/2$. Also, distributor's inventory policies $I_D(t)$, manufacturer's inventory policies $I_M(t)$, and the price charged by the manufacturer P_M, are different than that given in ES87 so as to reflect the effect of the variable start and terminal times. However, distributor's pricing policies $P_D(t)$, distributor's processing policies $Q_D(t)$, and manufacturer's processing policies $Q_M(t)$, are similar to those derived in ES87. The above equations contain the generalized pricing, processing, and inventory policies of channel members of which ES87 policies are a special case (when $t_S=0$ and $t_T=T$).

3 Derivation of Season's Start and Terminal Times

In the above section, we have provided the generalized optimal policies for both the distributor and the manufacturer operating in an industrial channel. However, the correct values of the start and terminal times (t_S and t_T) of the season are yet to be determined. It is only logical to initially assume the values of t_S and t_T to be 0 and T respectively, which corresponds to the full length of the season. In this case, the generalized policies degenerate to those provided by ES87. Nevertheless, these values of t_S and t_T may violate one (or more) of the constraints of the optimal control problems listed in Appendices A and B. In such a case, the values of t_S and t_T need to be updated so as to satisfy all the constraints, and establish the applicability of the optimal policies for members in the industrial distribution channel. This is achieved through an iterative process as specified in the heuristic which is provided below.

The Heuristic

Step 1: Read the parameter vector $\Theta'=(b_D,K_D,h_D,K_M,h_M,\alpha_1,\alpha_2,\alpha_3)$. Set $\{t_S^0=0,\ t_T^0=T$ $n=1\}$.

Step 2: Test if the condition $h_D<(\alpha_2-2\alpha_1 t_S)/(3(b_D+K_D))$, holds [refer *Proposition · (revised): condition (i)*]. If the condition is not satisfied, then go to Step 14.

Step 3: Use generalized policies of the manufacturer to obtain P_M^0 [refer Corollar 9].

Step 4: Individually solve the constraints equations [refer distributor an manufacturer problems in Appendices A and B]. for the inventory stocking perio (denoted by subscript 1). Obtain the boundary values of t from each equation.

Step 5: Select the *maximum* value of t from all roots of the constraints associate with the inventory stocking period. Call this t_S^n.

Step 6: If $t_S^n\le 0$ then $t_S^n=0$.

Step 7: Individually solve the constraints equations [refer distributor an manufacturer problems in Appendices A and B] for the stockless period (denoted b subscript 2). Obtain the boundary values of t from each equation.

Step 8: Select the *minimum* value of t from all roots of the constraints associate with the stockless period. Call this t_T^n.

Step 9: If $t_T^n\ge T$ then $t_T^n=T$.

Step 10: If $t_S^n=0$ **and** $t_T^n=T$. Go to Step 14.

Step 11: Compute the value of P_M^n from the following equation:

$$P_M^{n}=w_1[(1/(t_T-t_S))\int_{t_S}^{t_T}(a_D(t)/b_D)dt]+w_2C_M,$$

(refer equation 7). Here w_1, w_2, and b_M are computed from equations 8 and 9.

Step 12: Check if $|P_M^n-P_M^{n-1}|\le\varepsilon$ (here ε is a pre-specified infinitesimal value). If true, go to Step 14.

Step 13: Set n=n+1. Go to Step 2.

Step 14: Use the generalized optimal policies for both distributor and manufacturer (refer Corollaries 5 and 9), where $t_S=t_S^n$, $t_T=t_T^n$, and $P_M=P_M^n$. Go to Step 16.
Step 15: Stop. The channel members should follow the stockless policy [refer *Proposition 4 (revised): condition (ii)*].
Step 16: End.

The intuition behind the heuristic is as follows: For some specific parameter values of the problem, we compute the solutions for the optimal control problem including P_M based upon the demand interval [0,T]. We then check if one or more constraint equations are violated and if the assumption of interior solution is invalid. In such a case, we get a boundary solution, taking into account the binding constraint. As a result, we obtain the values of t_S and t_T which will satisfy all the constraints. Note that the length of the time interval $[t_S, t_T]$ is a subset of the original interval [0,T]. Since the manufacturer is the Stackelberg leader, (s)he would revise (increase) the value of P_M based on the new information on t_S and t_T. What follows is an iterative process of computing t_S, t_T and P_M till a point of convergence is reached. This provides us with the final (equilibrium) values of t_S, t_T and P_M which when used in the generalized optimal policies would, infact, result in true optimal profits for the channel members.

4 Numerical Example

To illustrate the heuristic, we use the example presented in ES87. Specifically,

$$h_D=1/20,\quad K_D=2,\quad a_D(t)=-t^2+6t+12,$$

$$h_M=1/30,\quad K_M=2,\quad [\alpha_1=1,\ \alpha_2=6,\ \alpha_3=12],$$

$$b_D=1,\qquad C_M=3\frac{9}{10}.$$

Here, $T=\alpha_2/\alpha_1=6$, $b_M=1/3$, $w_1=4/7$, and $w_2=3/7$ [p. 992-3].

The results obtained from ES87 policies and from the heuristic are tabulated in Table 1 for comparison purposes. In this example, the initial value of P_M is calculated to be 11.9571 for the interval [0, 6].

Following the heuristic, we find that the constraint, which requires the distributor's price to be less than what the market can bear at all times [refer equation (16); this constraint is similar to equation (2.10) in ES87, p. 988], is violated for the above parameter values. Hence, the correct start and terminal times of the "effective" season are not 0 and 6 as assumed by ES87. Therefore, the values of t_S and t_T need to be recalculated and P_M needs to be revised subsequently. The values of t_S^1 and t_T^1 (after the initial constraint validity check) are found to be 0.3788 and 6 respectively. Using these values in equation (7), P_M is calculated to be 12.1463. This P_M value, along with t_S^1 and t_T^1, is used to revise the optimal pricing, processing, and inventory constraint functions. Once again, a validity check on the revised constraint functions is conducted which provides $t_S^2=0.4370$ and $t_T^2=5.9755$. After subsequent iterations, the final values of t_S and t_T are computed to be 0.4495 and 5.9670. Also, the final value of P_M is 12.1970.

These results along with the resulting channel member profits are comparec ES87 results in Table 1. Notice that now the channel members effectively oper relatively shorter duration than assumed in ES87. In order to compensate fc market behavior, the price charged by the manufacturer (P_M) goes up which res lower demand. The end result is lower effective profits for both distributc manufacturer (hence for the entire channel) than originally estimated by ES87.

Table 1 *Comparison of original and heuristic-adjusted results for the numerical exampl ES87*

	Time Interval	Optimal Manufacturer's Price (P_M)	Optimal Distributor's Total Profits (Π_D^*)	Optimal Manufacturer's Total Profits (Π_M^*)	Opti Char Total I (Π_D^*+
I. ES87 Results (original)	[0,6]	11.9571	45.7230	84.3150	130.(
II. ES87 Results (heuristic-adjusted)	[0.4495, 5.9670]	12.1970	41.6194	82.0480	123.(

5 Variable Price Sensitivity Case

To further test the robustness of the heuristic, we preset the values of all parai to those used in numerical example provided in section 4, except one—say the sensitivity of the distributor (b_D), which is treated as a variable.

Table 2 *Constraint validity check using heuristic for various values of b_D (using t_S^0 an*

Equation#	Constraints	b_D=0.25	b_D=1.00	b_D= 2.00	b_D= 3.00	b_D= 4.00	b_D =
(Refer Appendices A & B)							
Inventory$							
(13)	$I_{D1}$$$	**0,4.4156**	**0,4.3875**	**0, 4.3500**	**0,4.3125**	**0,4.2750**	**0,4.2**
(13)	I_{D2}	-	-	-	-	-	-
(34)	I_{M1}	**0,4.4297**	**0,4.4062**	**0, 4.3750**	**0,4.3438**	**0,4.3125**	**0,4.1**
(34)	I_{M2}	**0,4.4438**	**0,4.4250**	**0, 4.4000**	**0,4.3750**	**0,4.3500**	**0,4.2**
(34)	I_{M3}	-	-	-	-	-	-
Inventory Stocking Period							
(Period I)							
(14)	Q_{D1}	**-76.1219f**	**-43.0638**	**-21.9375**	**-10.4891**	**-3.3063**	**1.62**

		$b_D=0.25$	$b_D=1.00$	$b_D=2.00$	$b_D=3.00$	$b_D=4.00$	$b_D=5.00$
(35)	Q_{M1}	-115.2900	-65.6973	-34.0000	-16.8197	-6.0375	1.3604
(15)	$P_{D1}-P_M$	-0.4656, 6.4781	-0.3435, 6.3935	-0.0446, 6.1446	0.3452, 5.8048	0.8260, 5.3740	1.4494, 4.8006
(16)	a_D/b_D-P_{D1}	-0.1804, 6.1679	0.3788, 5.5712	0.8079, 5.0921	1.0796, 4.7704	1.2725, 4.5275	1.4196, 4.3304
Stockless Period							
(Period 2)							
(14)	Q_{D2}	-0.3263, 6.3263f	-0.0071, 6.0071	0.3392, 5.6608	0.6712, 5.3288	1.0252, 4.9748	1.4356, 4.5644
(35)	Q_{M2}	-0.3263, 6.3263	-0.0071, 6.0071	0.3392, 5.6608	0.6712, 5.3288	1.0252, 4.9748	1.4356, 4.5644
(15)	$P_{D2}-P_M$	-0.3263, 6.3263	-0.0071, 6.0071	0.3392, 5.6608	0.6712, 5.3288	1.0252, 4.9748	1.4356, 4.5644
(16)	a_D/b_D-P_{D2}	-0.3263, 6.3263	-0.0071, 6.0071	0.3392, 5.6608	0.6712, 5.3288	1.0252, 4.9748	1.4356, 4.5644
Mfr. Profit Margin							
(30)	P_M-C_M	35.8421	8.0571	3.0600	1.2923	0.3750	-0.1894

$ The inventory constraints are used to arrive at t_D and t_M values and, therefore, do not influence the constraint validity check process.

$$ All the constraints have the right hand side as '≥ 0' except the last constraint (P_M-C_M) which has to be '> 0'.

f The values provided in bold face characters are the relevant roots of t which represent the boundary point of the constraint.

f The underlined values are the values of t that are most restrictive and belong to the most binding constraint.

Table 2 illustrates the procedure used by the heuristic to perform the constraint validity check for various values of b_D for the initial run. For example, when b_D=0.25, all the constraints listed in the first column of Table 2 are satisfied and, therefore, ES87 policies are valid from [0,6].

However, when b_D=1, constraint equation (16) is violated in the inventory stocking period (Period 1) which results in t_S^1=0.3788 (refer Table 2). After going through one iteration, we revise the optimal pricing, processing, and inventory constraint functions of both distributor and manufacturer and subsequently conduct a valid check on these constraints. This results in t_S^2=0.4370 and t_T^2=5.9755. The results from constraint validity check after one iteration are compiled in Table 3.

The heuristic computes the final values t_S, t_T and P_M^* through the iterative process. We find that the effective season is reduced on both ends to [0.4495,5.9670].

Table 3 *Constraint validity check using heuristic for various values of b_D (using t_S^1 and t_T*

Equation # *(Refer Appendices A & B)*	Constraints	b_D=0.25	b_D=1.00	b_D = 2.00	b_D = 3.00	b_D = 4.00	$b_D = 5.$
Inventory							
(13)	I_{D1}	0,4.4156	0.3788, 4.1981	0.8079, 3.9461	1.0796, 3.7727	1.2725, 3.6388	-
(13)	I_{D2}	-	-	-	-	-	-
(34)	I_{M1}	0,4.4297	0.3788, 4.2169	0.8079, 3.9711	1.0796, 3.8040	1.275, 3.6763	-
(34)	I_{M2}	0,4.4438	0.3788, 4.2356	0.8079, 3.9961	1.0796, 3.8352	1.2725, 3.7138	-
(34)	I_{M3}	-	-	-	-	-	-
Inventory Stocking Period							
(Period 1)							
(14)	Q_{D1}	-76.1219	-45.2571	-24.0316	-11.9531	-4.1628	-
(35)	Q_{M1}	-115.2900	-69.0346	-37.2421	-19.1505	-7.4814	-
(15)	$P_{D1}-P_M$	-0.4656, 6.4781	-0.3317, 6.3817	0.0175, 6.0825	0.4717, 5.6783	1.0528, 5.1472	-
(16)	a_D/b_D-P_{D1}	-0.1804, 6.1679	0.4370, 5.5130	1.0032, 4.8968	1.4113, 4.4387	1.7485, 4.0515	-
Stockless Period							
(Period 2)							
(14)	Q_{D2}	-0.3263, 6.3263	0.0245, 5.9755	0.4524, 5.5476	0.8720, 5.1280	1.3422, 4.6578	-
(35)	Q_{M2}	-0.3263, 6.3263	0.0245, 5.9755	0.4524, 5.5476	0.8720, 5.1280	1.3422, 4.6578	-
(15)	$P_{D2}-P_M$	-0.3263, 6.3263	0.0245, 5.9755	0.4524, 5.5476	0.8720, 5.1280	1.3422, 4.6578	-
(16)	a_D/b_D-P_{D2}	-0.3263, 6.3263	0.0245, 5.9755	0.4524, 5.5476	0.8720, 5.1280	1.3422, 4.6578	-
Mfr. Profit Margin							
(30)	P_M-C_M	35.8421	8.2463	3.3547	1.5905	0.6629	-

The initial and final values of t_S, t_T and P_M^*, along with the resulting profits of channel members are listed in Table 4.

For b_D=2, constraint equation (16) again proves to be most binding, now for b inventory stocking and stockless periods (periods 1 and 2) which gives us t_S^1=0.8C and t_T^1=5.6608 (refer Table 2). After one iteration, we get t_S^2=1.0032 and t_T^2=5.54

(refer Table 3). Finally, the effective length of the season is calculated to be [1.0570,5.5144] (refer Table 4).

Table 4 *Results using heuristic for various values of b_D.*

Values:	$b_D = 0.25$	$b_D = 1.00$	$b_D = 2.00$	$b_D = 3.00$	$b_D = 4.00$	$b_D = 5.00$
Initial:						
t_S^0	0	0	0	0	0	0
t_T^0	6	6	6	6	6	6
P_M^0	39.7421	11.9571	6.9600	5.1923	4.2750	3.7105
After one iteration:						
t_S^1	0	0.3788	0.8079	1.0796	1.2725	1.4494
t_T^1	6	6	5.6608	5.3288	4.9748	4.5644
After two iterations:						
t_S^2	0	0.4370	1.0032	1.4113	1.7485	-
t_T^2	6	5.9755	5.5476	5.1280	4.6578	-
Results:						
b_M	0.1111	0.3333	0.5000	0.6000	0.6667	-
t_S	0	0.4495	1.0570	1.5166	1.9286	-
t_T	6	5.9670	5.5144	5.0679	4.5560	-
t_D^*	4.4156	4.1627	3.8215	3.5542	3.3107	-
t_M^*	4.4438	4.2002	3.8715	3.6267	3.3857	-
P_M^*	39.7421	12.1970	7.3388	5.5746	4.6447	-
Π_D^*	388.0200	41.6194	6.7678	1.3402	0.1998	-
Π_M^*	730.6790	82.0480	14.3582	2.9622	0.4539	-
$\Pi_D^*+\Pi_M^*$	1218.6990	123.6674	21.1260	4.3024	0.6537	-

We arrive at similar results for b_D of 3 and 4 as constraint equation (16) again is violated for both periods. In contrast, for b_D=5, similar analysis cannot be done since constraint equation (30) [this constraint is similar to equation (3.5) in ES87, p. 990] is violated, which implies that the manufacturer has a negative profit margin. As a result, no solution exists for this (or a larger) value of distributor's price sensitivity, b_D.

6 Implications and Conclusions

One of the implications of only considering the season interval to be $[t_S,t_T]$ (instead of [0,T] as in ES87) to determine the channel member policies is that the resulting optimal profits of the channel members are lower than what were originally claimed by ES87. Also, from Table 4, note that as b_D increases, the 'effective' length of the season decreases which, in turn, causes P_M to increase from its initial value. Also observe that, with an increase in the value of b_D, the time for which the stockless policies are in effect gets diminished. This time interval is $[t_{\bar{D}}, t_T]$ for the distributor and $[t_M, t_T]$ for the manufacturer.

Although, not exhibited in Table 4, for certain parameter values, it is quite poss that the length of season dictates the employment/non-employment of stock policy by the manufacturer (in mathematical notations, the possibility that $t_M = t_T$ even the distributor. In such a scenario, both distributor's and manufactu inventory will become zero at same point at the end of season, i.e., $t_{\bar{D}} = t_M = t_T$. implies that, for certain parametric conditions, the channel members c potentially implement single part policies, rather than two-part policies as posite ES87.

In summary, in this paper, we have provided the generalized optimal policies ES87) for the channel members where the start and terminal times of a season considered variables. We have also proposed a heuristic which, when use conjunction with the generalized optimal policies, will compute the actual horizon during which the optimal policies will provide correct results. We demonstrated the appropriateness of this heuristic through a numerical exan similar to the one presented by ES87. The robustness of the heuristic subsequently tested by varying the distributor's price sensitivity, then computing effective season $[t_S,t_T]$ and the corresponding profits of the channel members. that this paper has similar implications to the research article by Eliashberg Steinberg [7].

7 References

1. Abad, P.L., "A Hierarchical Optimal Control Model for Coordination Functional Decisions In A Firm," European Journal Of Operational Resea 1987, 32, 62-75.
2. Bozarth, C.C. and Berry, W.L., "Measuring the Congruence Between Ma Requirements and Manufacturing: A Methodology and Illustration," Deci Sciences, 1997, 28, 121-150.
3. Chen, I.J., Calatone, R.J., and Chung, C.H., "The Marketing-Produc Interface and Manufacturing Flexibility," Omega, 1992, 20, 431-443.
4. Deane, R.H., McDougall, P.P., and Gargeya, V.B., "Manufacturing Marketing Interdependence in the New Venture Firm: An Empirical Stu Journal of Operations Management, 1991, 10, 329-343.
5. Desai, V.S., "Marketing-Production Decisions under Independent and Integr Channel Structures," Annals of Operations Research, 1992, 34, 275-306.
6. Eliashberg, J. and Steinberg, R., "Marketing-Production Decisions in Industrial Channel of Distribution," Management Science, 1987, 33, 981-10
7. Eliashberg, J., and Steinberg, R., "Competitive Strategies for Two Firms Asymmetric Production Cost Structures," Management Science, 1991, 37, 1 1473.
8. Freeland, J.R., "Coordination Strategies for Production and Marketing Functionally Decentralized Firm," AIIE Transactions, 1980, 12, 132.Jeuland, A.P. and Shugan, S.M., "Managing Channel Profits," Marke Science, 1983, 2, 239-272.
9. Khouja, M., Saydam, C. Vergara, F. E., and Rajagoplan, H. K., "Opt inventory policy under continuous unit cost decrease and risk of su

obsolescence," International Journal of Operations and Quantitative Management, 2006, 12(2), 89-106.

10. Kumar, K.R., Loomba, A.P.S., and Hadjinicola, G.C., "Marketing-Production Coordination in Channels of Distribution," European Journal of Operational Research, 2000, 126, 189-217.
11. Lee, W.J., and Kim, D., "Optimal and Heuristic Decision Strategies for Integrated Production and Marketing Planning," Decision Sciences, 1993, 24, 1203-13.
12. Lee, W.J, and Lee, K.C., "PROMISE: A Distributed DSS Approach to Coordinating Production and Marketing Decisions," Computers & Operations Research, 1999, 26, 901-20.
13. McGuire, T.W., and Staelin, R., "An Industry Equilibrium Analysis of Downstream Vertical Integration," Marketing Science, 1983, 2, 161-191.
14. Porteus, E.L., and Whang, S., "On Manufacturing/Marketing Incentives," Management Science, 1991, 37, 1166-1181.
15. Sogomonian, A.G., and Tang, C.S., "A Modeling Framework for Coordinating Promotion and Production Decisions Within A Firm," Management Science, 1992, 39, 191-203.
16. Wang, S.P., "On Inventory Replenishment with Non-Linear Increasing Demand," Computers & Operations Research, 2002, 29, 1819-25.
17. Weng, Z.K., "Channel Coordination and Quantity Discount," Management Science, 1995, 41(9), 1509-1522.

Appendix A

Proof of Corollary 5

The distributor's revised continuous profit maximization problem is formulated as:

$$\max_{P_D(t),Q_D(t)} \int_{t_S}^{t_T} \{(P_D(t)-P_M)(a_D(t)-b_DP_D(t))-(1/K_D)(Q_D(t))^2-h_DI_D(t)\}dt \qquad (11)$$

$$\text{s.t.} \quad \dot{I}_D(t)=Q_D(t)-a_D(t)+b_DP_D(t), \qquad (12)$$

$$I_D(t) \geq 0, \qquad (13)$$

$$Q_D(t) \geq 0, \qquad (14)$$

$$P_D(t) > P_M, \qquad (15)$$

$$P_D(t) < a_D(t)/b_D, \qquad (16)$$

$$I_D(t_S)=I_D(t_T)=0. \qquad (17)$$

The solution procedure of the revised problem is the same as the one presented in Appendix A of ES87 [p. 996-8]. For this reason, the nature of the optimal pricing and processing policies remain the same as ES87. In summary these policies are:

Unconstrained Segment

(Inventory is positive, $\lambda_D(t)=\lambda_D(t_S)+h_Dt$):

$$Q^*_D=Q^*_{D1}=K_D\lambda_D/2,$$

$$P^*_D=P^*_{D1}=(1/2)(\lambda_D+a_D/b_D+P_M),$$

$$\dot{I}_D =Q^*_{D1}-a_D+b_DP_{D1}^*.$$

On a Boundary Segment

(Inventory is zero, $\lambda_D(t)+\rho_D(t)=\Psi_D(t)=(a_D-b_DP_M)/(b_D+K_D)$):

$$Q^*_D=Q^*_{D2}=K_D\Psi_D/2=K_D(a_D-b_DP_M)/(2(b_D+K_D)),$$

$$P^*_D=P^*_{D2}=(1/2)(\Psi_D+a_D/b_D+P_M)$$

$$=((2b_D+K_D)a_D+b_DK_DP_M)/(2b_D(b_D+K_D)),$$

$$\dot{I}_D=I_D=0.$$

The determination of t^*_D, the time at which entry to the boundary occurs (the j at which the distributor moves from a stocking to a stockless inventory polic achieved through the simultaneous solution of the following two equations:

$$Q^*_{D1}(t^*_D)=Q^*_{D2}(t^*_D),$$

$$\int_{t_S}^{t^*_D} \dot{I}_D(t)dt=0.$$

The first equation ensures that at the boundary point, the production level c two processing policies is the same, where as, the second equation ensures th inventory is carried over in the stockless period after t^*_D. Note that equation differs from the respective one presented in ES87 since the lower limit of the int is set to t_S to accommodate a solution in the new interval $[t_S,t_T]$.

The solution of equations (25) through (26) results $\lambda_D(t_S)$ and t^*_D, which substituted in equations (18) through (24) yield the distributor's optimal pr processing and inventory policies as presented in corollary 5.

Proof of Proposition 4

For t^*_D to exist, it must lie to the right of the point at which Ψ_D reaches its maxi This point is $\alpha_2/(2\alpha_1)$. Therefore, $t^*_D > \alpha_2/(2\alpha_1)$. Substituting equation (4) giv necessary parametric condition under which the distributor can follow the op policies of the revised problem.

Appendix B

Proof of Corollary 9

After rearranging equation (1) the optimal quantity produced by the distributor can be written as:

$$Q^*_D(t)=\begin{cases} Q^*_{D1}(t)=a_{M1}(t)-b_M P_M & 0\le t\le t^*_D \\ Q^*_{D2}(t)=a_{M2}(t)-b_M P_M & t^*_D\le t\le T, \end{cases} \tag{27}$$

where $b_M=b_DK_D/(2(b_D+K_D))$, and

$$a_M(t)=\begin{cases} a_{M1}(t)=K_Dh_Dt/2+K_D(a_D(t^*_D) \\ \qquad -h_D(b_D+K_D)t^*_D)/(2(b_D+K_D)) & t_S\le t\le t^*_D, \\ a_{M2}(t)=K_Da_D(t)/(2(b_D+K_D)) & t^*_D\le t\le t_T. \end{cases} \tag{28}$$

Therefore the manufacturer's problem can be written as:

$$\max_{P_M} \pi_M(P_M) \tag{29}$$

$$\text{s.t.} \quad C_M<P_M<\min_t \; a_D(t)/b_D, \tag{30}$$

where,

$$\pi_M(P_M)=\max_{Q_M(t)} \int_{t_S}^{t_T} \{(P_M-C_M)(a_M(t)-b_MP_M)-(1/K_M)Q^2_M(t)-h_MI_M(t)\}\,dt \tag{31}$$

$$\text{s.t.} \quad \dot{I}_M(t)=Q_M(t)-Q^*_D(t), \tag{32}$$

$$Q^*_D(t)=\begin{cases} Q^*_{D1}(t)=a_{M1}(t)-b_M P_M & t_S\le t\le t^*_D \\ Q^*_{D2}(t)=a_{M2}(t)-b_M P_M & t^*_D\le t\le t_T, \end{cases} \tag{33}$$

$$I_M(t)\ge 0, \tag{34}$$

$$Q_M(t)\ge 0, \tag{35}$$

$$I_M(t_S)=I_M(t_T)=0. \tag{36}$$

Following a similar solution procedure as in Appendix B of ES87 [p. 998-9], and assuming an interior solution, we get the following manufacturer's optimal processing policies:

Unconstrained Segment

Inventory is positive, $\lambda_M(t)=\lambda_M(t_S)+h_M\, t$:

$$Q^*_M=Q^*_{M1}=K_M\,\lambda_M/2, \tag{37}$$

$$\dot{I}_M=Q^*_{M1}-Q^*_D.$$

Boundary Segment

Inventory is zero, $\lambda_M(t)+\rho_M(t)=\Psi_M(t)=(2/K_M)(a_{M2}-b_MP_M)$:

$$Q^*_M=Q^*_{M2}=K_M\,\Psi_M/2=a_{M2}-b_M\,P_M,$$

$$I_M=\dot{I}=0.$$

In order to determine the value of t^*_M, the time at which entry to the bou occurs where the inventory becoming zero, and the value of $\lambda_M(t_S)$, we need to the following two equations:

$$Q^*_{M1}(t^*_M)=Q^*_{M2}(t^*_M)$$

$$\int_{t_S}^{t^*_M} \dot{I}_M(t)\,dt=0$$

The obtained solution for t^*_M is shown in equation (6). Substituting yields equ (5) which is the manufacturer's optimal processing policies. Now, substit equations (37) through (40) into the objective function equation (31) results function that is quadratic in P_M. Maximizing over P_M will yield the revised [equation (7)], and equations (8) through (9). The manufacturer's revised inve policies are obtained by integrating equations (38).