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Analysis and Kinetics of the Sequencing Batch Reactors

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ABSTRACT

Fundamental analysis and kinetics of treatment reactors are major topics in environmental engineering literature. These fundamental topics in reactor processes are well known for the ideal batch reactor, the continuous stirred tank reactor, and the plug flow reactor. The sequencing batch reactors (SBRs) are relatively new in the field, but are widely used. Despite the wide application of sequencing batch reactors in the field, information is lacking on the fundamental analysis and kinetics, especially with comparison to the ideal batch reactor. This report presents analysis and kinetics of the sequencing batch reactors and compares the kinetics equations developed with those of the ideal batch reactor especially for zero-order, first-order, and second-order reactions. A significant result is that the SBRs' equations for the three reaction orders analyzed become the equations for the ideal batch reactor if the entire reactor volume of a sequencing batch reactor is decanted. The fundamental analysis and the kinetics presented will help enhance the understanding of the sequencing batch reactors and their use in waste treatment.

Key words: reactor-analysis; reaction-kinetics; reaction-order; sequencing batch reactors

INTRODUCTION

THE ANALYSIS AND KINETICS of the three ideal reactors: the ideal batch reactor, the continuous stirred tank reactor (mixed reactor), and the plug flow reactor (also known as tubular reactor) have been presented by Ray (1995), Reynolds and Richards (1996), Vesilind (1997), Metcalf and Eddy (2003), Davis and Masten (2004). Another reactor process that is now widely used in waste treatment and pollution control is the sequencing batch reactor (SBR). The SBR includes the aerobic sequencing batch reactor (aero-SBR or simply SBR) and the anaer-

obic sequencing batch reactor (anaerobic-SBR or ASBR). While there are research reports on modeling the performance of SBR biological treatment of wastewater (Nakawa and Tanaka, 1991; Ibrahim and Abasaeed, 1995; Bagley and Brodkorb, 1999; Ky *et al.*, 2001) including a report by the author (Ndon *et al.*, 2006), information is lacking on the fundamental analysis of the SBR process kinetics.

The difference between the ideal batch reactor and the sequencing batch reactor process is that only a predetermined volume of a sequencing batch reactor is emptied at the end of reaction, whereas the entire volume of the

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ideal batch reactor is emptied at the end of reaction. The SBR processes are widely used in the field and are widely studied by researchers. The author has previously reported on the performance of ASBR systems (Ndon and Dague, 1997) and on biochemical kinetics of the SBRs (Ndon *et al.*, 2006).

Unlike the continuous flow reactors, SBRs require no external clarifier as solids separation occurs within the system resulting in more efficient solids (chemical precipitate or microbial solids) separation from the reactor fluids. The more efficient solids separation results in longer solids retention time and the associated high process performance. Experience with SBRs show that they are simple to operate in comparison to continuous flow processes.

The operating principles of SBRs involve four sequential steps: feed, react, settle, and decant. The operational characteristics of a typical SBR process are illustrated in Fig. 1 along with some of the model variables. The feed step involves addition of substrate (waste) to the reactor. The volume of waste fed depends on a number of factors including the desired hydraulic retention time [the time required to treat waste volume that is equal to the volume of the reactor—hydraulic retention time (HRT)], substrate loading, and expected settling characteristics. The feed step is followed by the react step. The duration of the feed step is usually short in comparison to the duration of the react step. The react step is important in the destruction of pollutants. Continuous or intermittent mixing is used during the react step. The time required for the react step depends on required effluent quality, temperature, and waste characteristics. The react step is followed by the settling step and requires that mixing be shut off to allow solids to settle within the reactor. The required time for settling depends on temperature, solid concentration, and characteristics of solids (flocculent or dispersed) to be separated. After settling, a predetermined fraction of the reactor volume is decanted. The volume decanted is always equal to the volume fed so that the system HRT is maintained. The HRTs used affect the volume of waste treated per sequence.

It is important to note how system HRT is determined for a sequencing batch reactor process, as HRT has traditionally been associated with continuous flow reactors. For an SBR process, a partially emptied system, HRT is determine using the following procedure as an example; assume an SBR reactor that has a working volume of 6 liters with four cycles of operation each day (feeding step, react step, settling step, and decant step each occurring four times a day). In such an operation, each of the four cycles per day will be operated for 6 h. If 2 liters of waste are fed and decanted during each cycle, it will take three cycles to treat one full reactor volume (three cycles at 2 liters per cycle giving 6 liters). The three cycles will be completed in 18 h at 6 h per cycle. Therefore, the system HRT is 18 h.

Although the sequencing batch reactor processes are widely used in waste treatment and pollution control, information on the fundamental analysis and kinetics of the process is lacking, especially when compared to the ideal batch reactor process. The analysis and kinetics equations presented in this report for zero-order, first-order, and second-order kinetics show that the sequencing batch reactor equations become the ideal batch reactor equations when the entire volume of the sequencing batch reactor is decanted. This theoretical analysis is aimed at filling this void.

MODEL DEVELOPMENT

Mass balance for substrate: feed step

Substrate concentration and reactor fluid content in SBR systems are at their respective lowest value at the beginning of feeding. During feeding, the change in waste concentration and the change in reactor waste volume occur simultaneously. A mass balance on waste (accumulation rate = input rate − output rate − decay rate) gives:

$$\frac{d(VC)}{dt} = QC_o - 0 - Vr \tag{1}$$
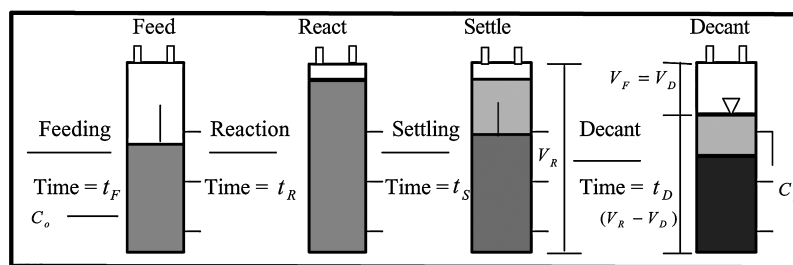
Figure 1.   Schematic of the four steps in seuencing batch reactor operation with some of the variables for kinetics modeling.

where Q is influent flow rate, t is time; C is waste concentration, r is waste conversion or decay rate, and other terms are as presented in Fig. 1. A list of variables used in this report are presented under nomenclature.

Applying the product rule of differentiation, Equation (1) gives:

$$V\frac{dC}{dt} + C\frac{dV}{dt} = QC_o - V_r$$

$$\text{or} \quad \frac{dC}{dt} + \frac{C}{V}\frac{dV}{dt} = \frac{Q}{V}C_o - r \quad (2)$$

Knowing that

$$\frac{dV}{dt} = Q,$$

Equation (2) becomes:

$$\frac{dC}{dt} + \frac{C}{V}Q = \frac{Q}{V}C_o - r$$

$$\text{or} \quad \frac{dC}{dt} = \frac{Q}{V}(C_o - C) - r \quad (3)$$

Observing that $V = V_o + V_t$ and that $V_t = Qt$, Equation (3) can be written in the form:

$$\frac{dC}{dt} = \frac{Q}{V_o + Qt}(C_o - C) - r \quad (4)$$

Mass balance for substrate: react step

For short feeding time in comparison to react time, we may assume that the reaction time controls waste decay. During the react step, flow into and out of the system are zero. Therefore, Equation (4) gives:

$$\frac{dC}{dt} = -r \quad \text{or} \quad dt = -\frac{dC}{r} \quad (5)$$

INTEGRATION LIMITS

Waste concentration in the reactor at any time during the feed step (C_F) can be represented by:

$$C_F = [V_o C_D + V_t C_o]\left(\frac{1}{V_o + V_t}\right)$$

$$= [(V_R - V_D)C_D + V_t C_o]\left(\frac{1}{V_o + V_t}\right) \quad (6)$$

where V_t is the volume of raw waste fed to the reactor at the time of interest, V_R is the total reactor volume, and V_o is the volume of waste in the reactor at the end of decant ($V_R - V_D$).

The lower integration limit for Equation (5) is the waste concentration at the beginning of react step. It is the waste concentration at the end of feed step (C_{EF}), and is given by:

$$C_{EF} = [V_D C_o + V_o C_D]\left(\frac{1}{V_R}\right)$$

$$= [V_D C_o + (V_R - V_D)C_D]\left(\frac{1}{V_R}\right) \quad (7)$$

The upper integration limit for Equation (5) is the waste concentration at the end of the react step or the decant concentration (C_D). With the integration limits, Equation (5) may be written as:

$$\int_0^{t_R} dt = -\int_{\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_R}C_D}^{C_D} \frac{dC}{r} \quad \text{or}$$

$$t_R = -\int_{\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_R}C_D}^{C_D} \frac{dC}{r} \quad (8)$$

ZERO-ORDER KINETICS

For zero order reaction kinetics, $r = K$. Therefore, Equation (8) becomes:

$$\int_0^{t_R} dt = -\int_{\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_R}C_D}^{C_D} \frac{dC}{k} \quad \text{or}$$

$$t_R = -\frac{1}{k}\int_{\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_R}C_D}^{C_D} dC \quad (9)$$

Integrating Equation (9), we have:

$$t_R = -\frac{1}{K}\left[C_D - \left(\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_D}C_D\right)\right]$$

$$= -\frac{1}{K}\left[C_D - \left(\frac{V_D}{V_R}C_o + \frac{V_R}{V_D}C_D - C_D\right)\right] \quad (10)$$

That is:

$$t_R = \frac{1}{K}\left[-C_D + \left(\frac{V_D}{V_R}C_o + \frac{V_R}{V_D}C_D - C_D\right)\right]$$

$$= \frac{1}{K}\left[-C_D + \frac{V_D}{V_R}C_o + \frac{V_R}{V_D}C_D - C_D\right] \quad (11)$$

That is:

$$t_R = \frac{1}{K}\left[\frac{V_D}{V_R}C_o + \frac{V_R}{V_D}C_D - 2C_D\right]$$

$$\text{or} \quad Kt_R = \left[\frac{V_D}{V_R}C_o + \frac{V_R}{V_D}C_D - 2C_D\right] \quad (12)$$

FIRST-ORDER KINETICS

For first order kinetics, $r = KC$. Therefore, Equation (8) becomes:

$$\int_0^{t_R} dt = -\int_{\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D}^{C_D} \frac{dC}{r} \quad \text{or}$$

$$t_R = -\int_{\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D}^{C_D} \frac{dC}{KC} \quad (13)$$

Integrating Equation (13), we have:

$$t_R = -\frac{1}{K}\left[\ln C_D - \ln\left(\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_D} C_D\right)\right]$$

$$= -\frac{1}{K}\left[\ln C_D - \ln\left(\frac{V_D}{V_R} C_o + \frac{V_R}{V_D} C_D - C_D\right)\right] \quad (14)$$

$$t_R = \frac{1}{K}\left[-\ln C_D + \ln\left(\frac{V_D}{V_R} C_o + \frac{V_R}{V_D} C_D - C_D\right)\right]$$

$$= \frac{1}{K} \ln\left[\left(\frac{\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D}{C_D}\right)\right] \quad (15)$$

$$t_R = \frac{1}{K} \ln\left[\frac{V_D}{V_R} \frac{C_o}{C_D} - \frac{V_D}{V_R} + 1\right]$$

$$\text{or } Kt_R = \ln\left[\frac{V_D}{V_R} \frac{C_o}{C_D} - \frac{V_D}{V_R} + 1\right] \quad (16)$$

SECOND-ORDER KINETICS

For a second order reaction kinetic, $r = KC^2$. Therefore, Equation (8) becomes:

$$\int_0^{t_R} dt = -\int_{\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D}^{C_D} \frac{dC}{KC^2}$$

$$\text{or } t_R = -\frac{1}{K} \int_{\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D}^{C_D} \frac{dC}{C^2} \quad (17)$$

Integrating Equation (17), we have:

$$t_R = \frac{1}{K}\left[\frac{1}{C_D} - \frac{1}{\left(\frac{V_D}{V_R} C_o + \frac{V_R - V_D}{V_R} C_D\right)}\right]$$

$$= \frac{1}{K}\left[\frac{1}{C_D} - \frac{V_R}{V_D C_o + (V_R - V_D)C_D}\right] \quad (18)$$

That is

$$t_R = \frac{1}{K}\left[\frac{[V_D C_o + (V_R - V_D)C_D] - V_R C_D}{C_D[V_D C_o + (V_R - V_D)C_D]}\right]$$

$$= \frac{1}{K}\left[\frac{V_D C_o + V_R C_D - V_D C_D - V_R C_D}{C_D[V_D C_o + (V_R - V_D)C_D]}\right] \quad (19)$$

$$t_R = \frac{1}{K}\left[\frac{V_D C_o - V_D C_D}{C_D[V_D C_o + (V_R - V_D)C_D]}\right]$$

$$\text{or } Kt_R = \left[\frac{V_D C_o - V_D C_D}{C_D[V_D C_o + (V_R - V_D)C_D]}\right] \quad (20)$$

SUMMARY OF THE SBRs' EQUATIONS AND COMPARISON TO IDEAL BATCH REACTOR EQUATIONS

A summary of materials balance terms (accumulation rate = input rate − output rate − decay rate) for the three ideal flow reactors (mixed flow reactor, plug flow reactor, and ideal batch reactor) along with the SBR process are presented in column 2 of Table 1. In column 2 of Table 1, the variable "V" in the equation for SBRs can be removed from the differential as a constant if the feeding of waste is instantaneous with input flow rate (Q) being zero. Under such conditions, the materials balance

Table 1. Kinetic expression for the sequencing batch reactors (aero-SBR and anaerobic-SBR or ASBR) and for the ideal reactors.

Reactor	Mass balance terms	Simplified mass balance expression
Batch	$V\dfrac{dC}{dt} = Vr$	$t = -\int_{C_o}^{C_t}\dfrac{dC}{r}$
Mixed flow	$V\dfrac{dC}{dt} = QC_o - QC_t + Vr$	$-\dfrac{V}{Q} = \dfrac{C_o - C_t}{r}$
Plug flow	$\Delta V\dfrac{dC}{dt} = (QC)\big|_x - (QC)\big|_{x+\Delta x} - \Delta Vr$	$-\dfrac{Q}{A}\dfrac{dC}{dx} - r = 0$
Aero-SBR and ASBR	$\dfrac{d(VC)}{dt} = QC_o + Vr$	$\int_0^{t_R} dt = -\int_{\frac{V_D}{V_R}C_o + \frac{V_R - V_D}{V_R}C_D}^{C_D}\dfrac{dC}{r}$

Table 2. Performance equations for the sequencing batch reactors (aero-SBR and anaerobic-SBR or ASBR) and for the ideal reactors.

Reactor	Performance equation for zero-order reaction	Performance equation for first-order reaction	Performance equation for second order-reaction
Batch	$Kt = C_o - C_t$	$Kt = \ln\dfrac{C_o}{C_t}$ or $Kt = -\ln\dfrac{C_t}{C_o}$	$Kt = \dfrac{1}{C_o}\left[\dfrac{C_o}{C_t} - 1\right]$ or $Kt = \dfrac{1}{C_t} - \dfrac{1}{C_o}$
Mixed flow	$K\theta = C_o - C_t$	$K\theta = \dfrac{C_o - C_t}{C_t}$	$K\theta = \dfrac{C_o - C_t}{C_t^2}$
Plug flow	$K\theta = \ln\dfrac{C_o}{C_t}$	$K\theta = \ln\dfrac{C_o}{C_t}$ or $K\theta = -\ln\dfrac{C_t}{C_o}$	$Kt = \dfrac{C_o - C_t}{C_o C_t}$ or $Kt = \dfrac{1}{C_t} - \dfrac{1}{C_o}$
Aero-SBR and ASBR	$Kt_R = \left[\dfrac{V_D C_o}{V_R} + \dfrac{V_R C_D}{V_D} - 2C_D\right]$ or $-Kt_R = \left[2C_D - \dfrac{V_D C_o}{V_R} - \dfrac{V_R C_D}{V_D}\right]$	$Kt_R = \ln\left[\dfrac{V_D}{V_R}\dfrac{C_o}{C_D} - \dfrac{V_D}{V_R} + 1\right]$ or $Kt_R = -\ln\left[\dfrac{V_R C_D}{V_D C_o - V_D C_D + V_R C_D}\right]$	$Kt_R = \left[\dfrac{V_D C_o - V_D C_D}{C_D[V_D C_o + (V_R - V_D)C_D]}\right]$ or $Kt_R = \dfrac{1}{C_D} - \dfrac{V_R}{V_D C_o + (V_R - V_D)C_D}$

Table 3. Equations for calculating the concentration of pollutions in the sequencing batch reactors (aero-SBR and anaerobic SBR or ASBR) and for the ideal Batch reactors.

Reactor	Zero-order equation	First-order equation	Second-order equation
Ideal Batch	$C_t = C_o - Kt$	$C_t = C_o e^{-Kt}$	$C_t = \dfrac{C_o}{1 + C_o Kt}$
Aero-SBR and ASBR	$C_t = \dfrac{\dfrac{V_D}{V_R}C_o - Kt}{2 - \dfrac{V_R}{V_D}}$	$C_t = \dfrac{1}{\dfrac{V_R}{V_D C_o}\left[\dfrac{V_D}{V_R} + e^{Kt} - 1\right]}$ or $\dfrac{1}{C_t} = \dfrac{V_R}{V_D C_o}\left[\dfrac{V_D}{V_R} + e^{Kt} - 1\right]$	$C_t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = (1 - x)Kt,\ x \neq 1$ $b = xC_o Kt + x$ $c = -xC_o$ (See Equations (21)–(28))

terms for the ideal batch reactor and for the SBRs become identical.

Column 3 of Table 1 presents simplified form of the materials balance expression for each of the reactors that have to be solved for various reaction orders. The simplified materials balance expression for mixed flow reactor and for plug flow reactor assume steady-state condition with respect to time, and $\theta = V/Q$ is the hydraulic retention. Comparing the lower integration limit for a SBR process [see Equation (7)] with the lower integration limit for the ideal batch reactor, we note that the lower integration limit for an SBR process is the same as that of the ideal batch reactor if the entire volume of the SBR process is emptied ($V_D = V_R$). Also, it is noted that the upper integration limit for the SBR process is the same as that of the ideal batch reactor (the reactor waste concentration at the end of treatment or decanted effluent) where C_D represents decanted effluent from an SBR at any time during the waste treatment.

Table 2 presents a summary of the performance equations for each of the reactors for zero-, first-, and second-

order reactions. A significant result in Table 2 is that the SBRs' equations for the three reaction orders analyzed become the equations for the ideal batch reactor if the entire reactor volume of an SBR is decanted ($V_D = V_R$), with the SBR effluent being C_D. In addition, the SBRs' equations summarized in Table 2 show that a plot of the right side of each of the equations as a function of time would yield a straight line where the value of the reaction rate constant (k) can be computed from the slope of the line.
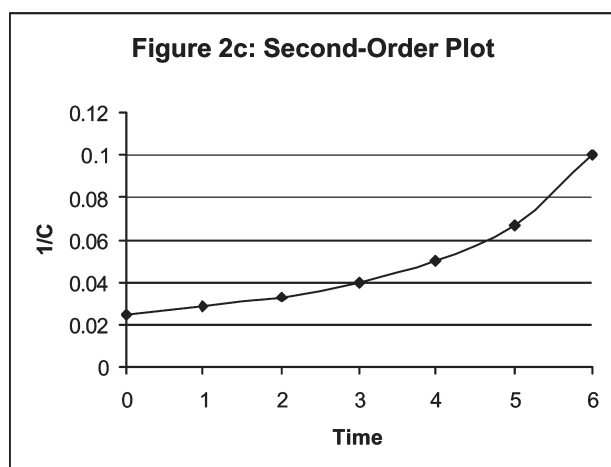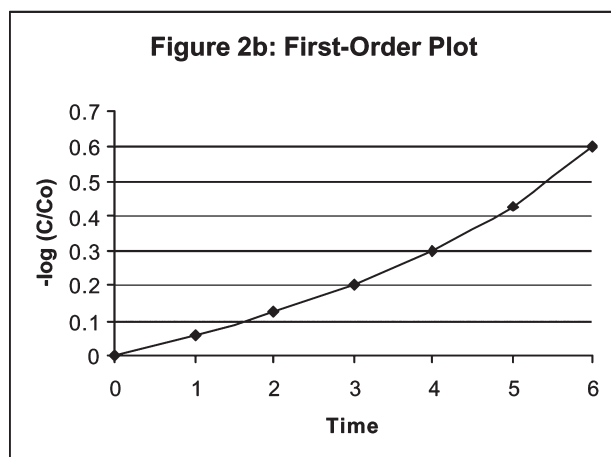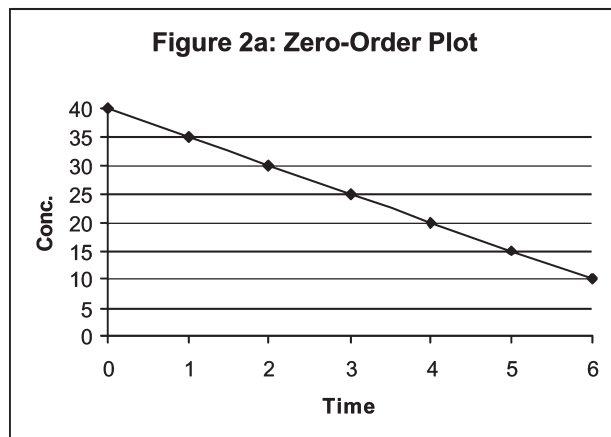
Equations for predicting the concentration of pollutants in the SBR systems under any of the three reaction orders can be obtained from Equations (12), (16), and (20), respectively. While the predicted concentration in the SBRs' systems (C_D) for zero- and for first-order reactions are easily obtained from Equations (12) and (16), the second-order equation [Equation (20)] results in a quadratic function given below as Equation (21):

$$(V_1 - V_D)C_D^2 Kt + [V_D C_o Kt + V_D]C_D - V_D C_o = 0 \quad (21)$$

From Equation (21), it is noted that for second-order reaction, $C_D = C_o$ at the start of treatment (time = 0). From

Table 4. Ideal batch reactor zero-, first-, and second order data for verifying the sequencing batch reactor equations.

Ideal batch reactor zero-order data		Ideal batch reactor first-order data		Ideal batch reactor second-order data	
Time, min	Conc, mg/L	Time, min	Conc, mg/L	Time, min	Conc, mg/L
0	40	0	28	0	48
1	35	1	24.3	1	6.22
2	30	2	21.2	2	3.32
3	25	5	13.9	3	2.27
4	20	10	6.9	5	1.39
5	15	20	17	10	0.704
6	10				

Figure 2a: Zero-Order Plot

Figure 2b: First-Order Plot

Figure 2c: Second-Order Plot

Figure 2. Verification of reaction-order for zero-order data.

Figure 3. Verification of reaction-order for first-order data.

quadratic formula, Equation (21) gives Equations (22) through (25):

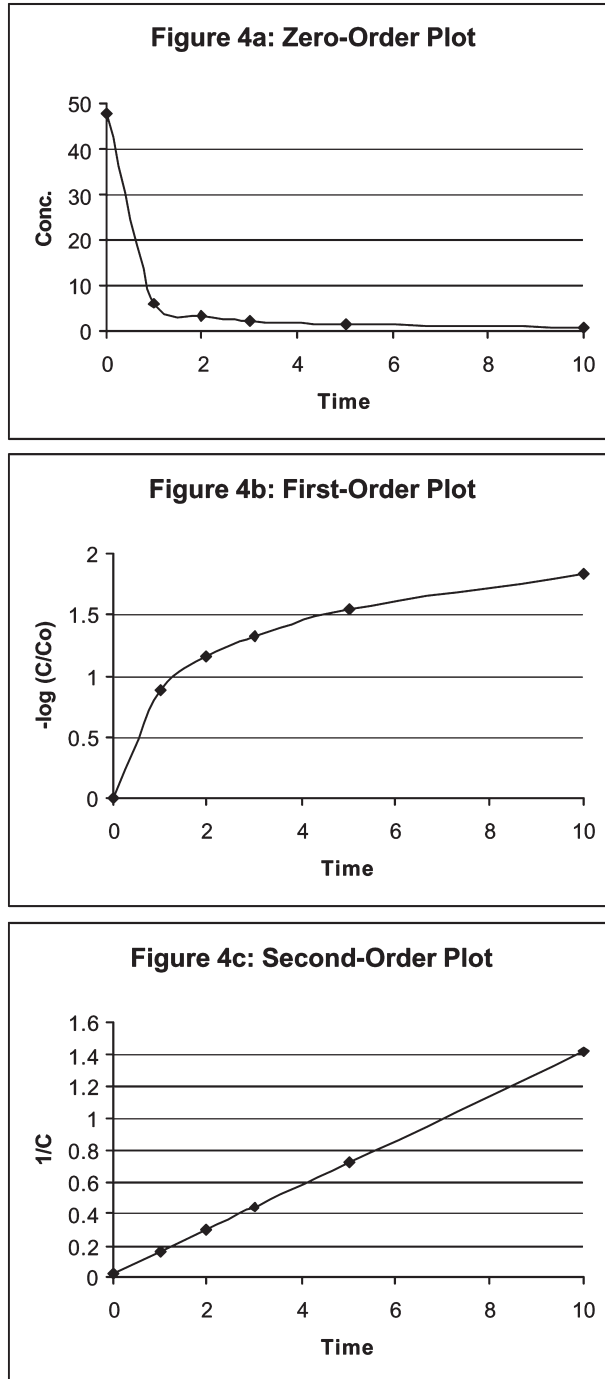$$C_D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (22)$$

$$a = (V_R - V_D)Kt \qquad (23)$$

$$b = V_D C_D Kt + V_D \qquad (24)$$

$$c = -V_D C_o \qquad (25)$$

Equation (21) should be used to determine the concentration at the start of reaction, as presented under Equation (21), since the quadratic formula is undefined

Figure 4. Verification of reaction-order for second-order data.

due to division by zero at time equal to zero. Defining the fractional volume of an SBR reactor decanted as x, Equations (23) through (25) can be written in the form of Equations (26) through (28), respectively:

$$a = (1 - x)Kt, \; x \neq 1 \qquad (26)$$

$$b = xC_o \, Kt + x \qquad (27)$$

$$c = -xC_o \qquad (28)$$

The equations for predicting the concentration of pollutants in the sequencing batch reactors for the three reaction orders are summarized in Table 3, along with the ideal batch reactor equations.

VERIFICATION OF THE SEQUENCING BATCH REACTOR EQUATIONS

To verify the sequencing batch reactor equations presented herein, known zero-, first-, and second-order ideal batch reactor data were obtained from the literature. The data for each of the three reaction orders are presented in Table 4. The zero-order data is based on information from Verilind (1997), while the first-order and the second-order data were obtained from Davis and Cornwell (1998). The reaction order of each of the ideal batch reactor data (Table 4) was confirmed by graphical analysis of the data. The graphical analysis involves plotting each of the data as a zero-, first-, and second-order data and noting the plot that is a straight line for each data. The plots are presented in Figures 2 through 4.

With the known reaction rate constant for each of the three ideal batch reactor data, the SBRs' equations derived in this report (see Table 3), were used to predict the expected concentration of the pollutant at various reaction times and various decanted SBR volume. The results for the zero-, first-, and second-order equations are presented in Tables 5, 6, and 7, respectively. As presented in Tables 5, 6, and 7, the expected concentration in the SBR system for each case is the same as the ideal batch data when the entire SBR volume is decanted ($V_D = V_R$) or when V_D is very close to V_R in the case of the second-order equation. This confirms the significant result of the SBRs' equations presented in this report that they become the ideal batch reactor equations if the entire reactor volume of a sequencing batch reactor is decanted (see Tables 2 and 3).

CONCLUSION

Kinetics equations are developed for the SBR processes for zero-, first-, and second-order reactions. The kinetics equations of the SBRs (aero-SBR and ASBR) become the ideal batch reactor equations if an entire SBR volume is decanted. Data from the literature used in verifying the equations show an accurate prediction of the ideal batch reactor data by the SBRs' equations, when the entire SBR volume is decanted. This theoretical research report provides information that is presently lack-

Table 5. Verification data for the zero-order sequencing batch reactor equation at various decanted volumes.

Time, min	Ideal batch reactor zero-order data K = 5/min	Calculated zero-order sequencing batch reactor concentrations				
		$V_D = 0.65V_R$	$V_D = 0.70V_R$	$V_D = 0.75V_R$	$V_D = 0.9V_R$	$V_D = V_R$
0	40	56.33	49.00	45.00	40.50	40
1	35	45.5	40.25	37.31	34.83	35
2	30	34.67	31.50	29.85	29.21	30
3	25	23.83	22.75	22.39	23.60	25
4	20	13.00	14.00	14.93	17.98	20
5	15	2.17	5.25	7.46	12.36	15
6	10	−8.67	−3.50	0.00	6.74	10

Table 6. Verification data for the first-order sequencing batch reactor equation at various decanted volumes.

Time, min	Ideal batch reactor first-order data K = 0.1401	Calculated first-order sequencing batch reactor concentrations				
		$V_D = 0.25V_R$	$V_D = 0.50V_R$	$V_D = 0.75V_R$	$V_D = 0.90V_R$	$V_D = V_R$
0	28	28	28	28.07	28.03	28
1	24.3	17.48	21.53	23.38	24.02	24.34
2	21.2	12.21	17.00	19.61	20.62	21.16
5	13.9	5.53	9.24	11.93	13.17	13.90
10	6.9	2.12	3.93	5.53	6.37	6.90
20	1.7	0.45	0.88	1.30	1.54	1.70

ing on the fundamental analysis and kinetics of the SBR processes.

NOMENCLATURE

C = substrate concentration, mg/L
C_o = influent substrate concentration, mg/L
C_F = substrate concentration in reactor during feed step, mg/L
C_{EF} = substrate concentration in reactor at end of feed step, mg/L
C_D = effluent (decanted) substrate concentration at any time during treatment, mg/L

K = reaction rate constant, zero-order reaction rate constant, concentration/time for zero-order; 1/time for first-order; and 1/(concentration)(time) for second-order
V_D = volume decanted per sequence, L
V_F = volume of feed per sequence, L
V_t = volume of substrate feed at time t during feed step (Qt), L
V_R = total reactor fluid volume, L
V_o = volume of fluid remaining after decant step $(V_R − V_D)$, L
V = volume of fluid in the reactor at any time during feeding $(V_o + V_t)$, L
Q = influent flow rate, volume/time

Table 7. Verification data for the second-order sequencing batch reactor equation at various decanted volumes.

Time, min	Ideal batch reactor second-order data K = 0.1401	Calculated zero-order sequencing batch reactor concentrations					Ideal batch second-order equation
		$V_D = 0.25V_R$	$V_D = 0.50V_R$	$V_D = 0.75V_R$	$V_D = 0.90V_R$	$V_D = 0.999V_R$	
0	48	48	48	48	48	48	48
1	6.22	4.70	5.39	5.72	5.85	5.92	5.92
2	3.32	2.72	2.98	3.09	3.13	3.15	3.15
3	2.27	1.93	2.06	2.12	2.14	2.15	2.15
5	1.39	1.22	1.28	1.30	1.31	1.31	1.31
10	0.704	0.64	0.66	0.66	0.66	0.67	0.67

r = substrate decay rate, mg/L · time
t_R = reaction time, t

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