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Perceived internal depth in rotating and translating objects

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Abstract. Previous research has indicated that observers use differences between velocities and ratios of velocities to judge the depth within a moving object, although depth cannot in general be determined from these quantities. In four experiments we examined the relative effects of velocity difference and velocity ratio on judged depth within a transparent object that was rotating about a vertical axis and translating horizontally, examined the effects of the velocity difference for pure rotations and pure translations, and examined the effect of the velocity difference for objects that varied in simulated internal depth. Both the velocity difference and the velocity ratio affected judged depth, with difference having the larger effect. The effect of velocity difference was greater for pure rotations than for pure translations. Simulated depth did not affect judged depth unless there was a corresponding change in the projected width of the object. Observers appear to use the velocity difference, the velocity ratio, and the projected width of the object heuristically to judge internal object depth, rather than using image information from which relative depth could potentially be recovered.

1 Introduction

Ullman (1979) showed that it is theoretically possible to determine the relative depth within an object from as few as three views of four non-coplanar points when the object is shown rotating in parallel projection. Human observers, however, tend to make systematic errors in such judgments (eg Todd and Norman 1991; Norman and Todd 1993; Liter et al 1993). Braunstein and Andersen (1984) suggested that the perception of a 3-D shape in a structure-from-motion (SFM) display is based on several heuristics, such as the tendency to perceive a point moving with a sinusoidal velocity in the image as a point moving with constant speed along a circular path in depth (as observed by Johansson 1950). Todd and Bressan (1990) proposed that human observers do not recover metric depth from SFM displays because they use only information available in two views, that is, velocity information, and do not integrate over three or more views.

Proffitt et al (1992) and Liter et al (1993) have suggested specific heuristics that human observers may be using in judging the amount of depth in rotating objects. In particular, they suggested that observers use differences in velocity to judge object depth. Liter et al found a relationship between judged object depth and the maximum velocity difference between feature points in a display, computed after curl is removed. This result supports Todd and Bressan's position, since only two views are necessary to determine velocity differences.

It is certainly possible that other velocities in a display, in addition to the maximum and minimum, influence perceived depth. In the present study, however, we will describe our displays in terms of the maximum and minimum velocities only, both for simplicity and because the difference between the maximum and minimum velocity was sufficient to account for Liter et al's results. We will consider both the difference between these velocities and the ratio of these velocities. Alternatively, we could consider the difference and the mean or the ratio and the mean, as the relation between the difference and the ratio is determined by the mean (ie for any two numbers for which the ratio is defined, only two of the three measures—difference, ratio, and mean—can be varied independently).

There are both theoretical and empirical reasons for studying the effects of the velocity difference and the velocity ratio. In a parallel projection of a rotation, the difference between the velocities of the nearest and furthest feature points varies with object depth and rotation speed. In a perspective projection of a translation perpendicular to the line of sight, the ratio of the velocities of feature points varies with object depth and viewing distance (for details, see Liter and Braunstein 1998). Empirically, Domini and Braunstein (1998) found contributions both from the velocity ratio and from deformation, which is related to the velocity difference, to the judged distance between two points on a surface shown rotating in parallel projection, with the simulated distance between the two points held constant.

Proffitt et al (1992) and Caudek and Proffitt (1993) showed that the perceived depth within an object is scaled by projected width. Our purpose in the present study was to determine the relative effects of the velocity difference and the velocity ratio on the perceived internal depth of an object displayed in motion, when projected width is varied together with simulated depth and when simulated depth is varied without variations in projected width. In order to vary the velocity difference and velocity ratio independently, while keeping the simulated size and distance of the object constant, it was necessary to display objects that were both rotating and translating. To provide a context for the objects, and comparability to previous research (Sauer et al 2001), we embedded the objects in a scene consisting of a ground plane and a ceiling plane, with poles connecting the objects to these planes (see figure 1).

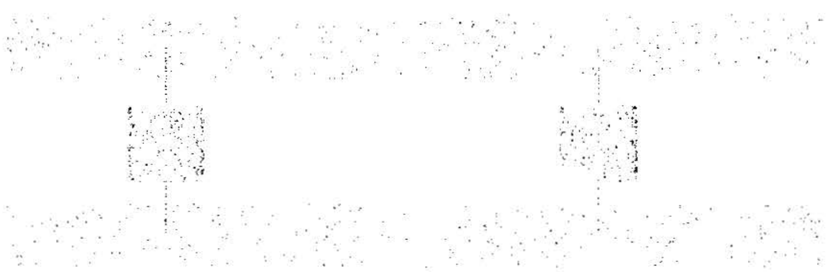

Figure 1. Two frames from a motion display that can be cross-fused to give a general impression of the stimuli. The actual stimuli were bright green dots on a dark background and were not stereoscopic.

In the first experiment, three levels of velocity difference and three levels of velocity ratio were combined factorially to provide an overall indication of the relative influence of these two variables. Because of the association of the velocity difference with rotation and of the velocity ratio with translation, in the second experiment we examined the influence of velocity difference with pure rotation and pure translation of the object. In this case, the velocity ratio could not be varied independently. In the third experiment the simulated object size was varied as well, to determine whether there is an effect of simulated size in addition to the effects of the velocity difference and velocity ratio. In the first three experiments the objects were cylinders, which have a constant relationship between width and depth, and therefore have a constant projected shape. To generalize our findings to objects other than surfaces of revolution, we included elliptical cylinders in the fourth experiment.

2 Experiment 1: Velocity difference versus velocity ratio

2.1 *Method*

2.1.1 *Observers.* Four paid observers with normal or corrected-to-normal visual acuity participated. None was familiar with the hypotheses of the experiment.

2.1.2 *Stimuli.* The stimuli were computer-generated displays of bright green dots moving horizontally on a dark background, simulating a ground plane and a ceiling plane with a pole connecting the planes and a cylinder mounted on the pole. Figure 1 shows two frames from a stimulus display which can be fused to provide a stereo view of the scene. The simulated distance from the eye to the pole was 113.7 cm. The simulated diameter of the cylinder was 9.3 cm.

2.1.3 *Design.* Two independent variables were manipulated: the difference between the velocity on the front and back of the cylinder and the ratio of these two velocities. The three levels of these variables, and the rotation and translation speeds used to produce these levels, are shown in table 1. The pairs of table entries are degrees of visual angle per second and degrees of angular rotation per second.

Table 1. Translation (deg s^{-1}) and rotation ($° s^{-1}$) amounts in experiment 1.

Velocity ratio	Velocity difference/deg s^{-1}		
	0.7	1.5	2.2
1.25	4.2, 4.5	8.4, 8.9	12.6, 13.4
1.50	2.3, 5.7	4.6, 11.4	7.0, 17.2
2.00	1.4, 6.3	2.8, 12.6	4.2, 19.1

2.1.4 *Apparatus.* The stimuli were displayed on a 21-inch (53 cm) Hewlett Packard 1321B X–Y display scope with a Tucker–Davis Technologies System II D/A converter controlled by a Pentium computer. Point-plotting resolution was approximately 16 000 by 16 000. Observers viewed the displays binocularly through a 19 cm diameter collimating lens at a distance of 89.5 cm. The lens magnified the image by approximately 19%. The velocities and velocity differences reported in this article take into account this magnification factor. A black viewing hood obscured the field of view outside the display window.

Response judgments were made on a separate 17-inch (43 cm) monitor situated to the left of the observer and oriented so that horizontal extent on the response monitor was parallel to the simulated depth direction in the display monitor.

2.1.5 *Procedure.* Observers were asked to adjust the length of a horizontal line on the response monitor, using a joystick, until it matched the perceived distance from the front to the back of the object. They were permitted to look back and forth between the stimulus and response displays until they were satisfied with their response. Observers participated in two sessions, each consisting of 10 randomly ordered repetitions of the 9 conditions. The first session was a practice session and the data from this session were not included in the analysis. Each session was divided into two blocks with a brief rest period between blocks.

2.2 *Results and discussion*

Figure 2 shows the results averaged across subjects and figure 3 shows the individual subject results. Judged depth was affected by both the velocity ratio and the velocity difference. Overall, the velocity difference had a greater effect on judged depth than the velocity ratio, although judged depth did not increase with difference for one observer at the highest difference level. An ANOVA showed significant main effects for

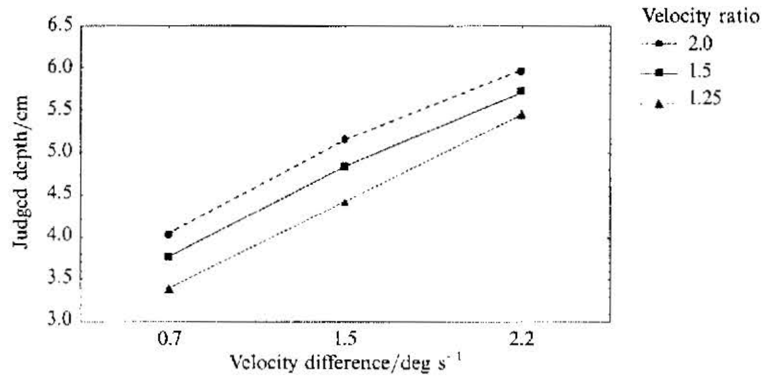

Figure 2. Judged depth as a function of velocity difference and velocity ratio averaged across observers in experiment 1.

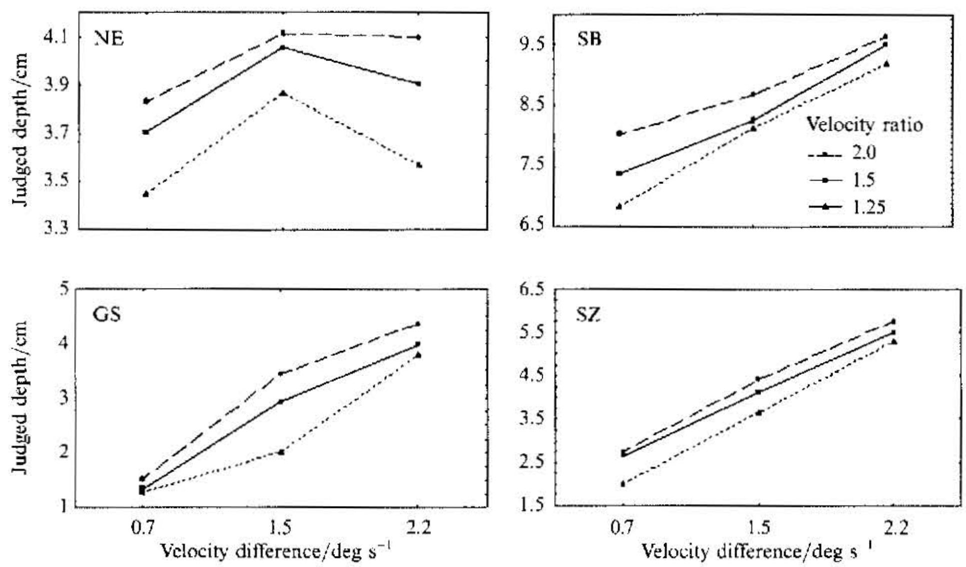

Figure 3. Judged depth as a function of velocity difference and velocity ratio for individual observers in experiment 1.

difference ($F_{2,6} = 9.7$, $p < 0.05$) and ratio ($F_{2,6} = 46.8$, $p < 0.01$), but no significant interaction ($F_{4,12} < 1$). The relative effect of two variables may of course depend on the range of levels selected. To address this possibility we replicated the experiment with a 3.0 velocity ratio replacing the 2.0 ratio. Judged depth was no greater with the 3.0 ratio than with the 2.0 ratio.

3 Experiment 2: Pure rotation and pure translation

The magnitude of the velocity difference has different implications for rotation and translation. For a constant velocity ratio, the velocity difference increases with increased object depth for rotations but not for translations perpendicular to the line of sight. For such translations, a change in the velocity difference without a change in the velocity ratio indicates a change in translation speed rather than a change in object depth. If observers are sensitive to this aspect of the geometry of object motion, we might expect less effect of the velocity difference on judged object depth for pure translations than for pure rotations, with the velocity ratio held constant.

3.1 *Method*

3.1.1 *Observers.* Four paid observers with normal or corrected-to-normal visual acuity participated. None was familiar with the hypotheses of the experiment. One observer (SB) had participated in experiment 1.

3.1.2 *Stimuli.* The stimuli were similar to those in experiment 1, except for the changes in rotation and translation magnitudes described in the next section.

3.1.3 *Design.* Two independent variables were manipulated: the type of motion (rotation, translation, or both) and the difference between the velocity on the front and back of the cylinder. For pure translation the velocity ratio was 1.09. For pure rotation it was −1.09. For combined rotation and translation it was 1.50. The rotation and translation speeds are shown in table 2.

Table 2. Translation (deg s^{-1}) and rotation ($^\circ s^{-1}$) amounts in experiment 2.

Motion	Velocity difference/deg s^{-1}		
	0.7	1.5	2.2
Rotation	0.0, 7.3	0.0, 14.5	0.0, 21.8
Translation	8.6, 0.0	17.2, 0.0	25.9, 0.0
Both	1.8, 5.7	3.6, 11.4	5.5, 17.2

3.1.4 *Apparatus and procedure.* The apparatus and procedure were the same as in experiment 1.

3.2 *Results and discussion*

Figure 4 shows the results averaged across subjects and figure 5 shows the individual subject results. An ANOVA found a significant main effect for velocity difference ($F_{2,6} = 5.43$, $p < 0.05$). The main effect of motion type was not significant ($F_{2,6} = 3.06$, $p > 0.05$). There was, however, a consistent ordering of translation and rotation judgments. Each of the four observers judged less depth for pure translation than for pure rotation at each level of velocity difference. The results for the mixed condition varied across observers. These results demonstrate the effectiveness of the velocity difference in determining judged depth both for pure rotation and for pure translation. They also suggest that there is at least an ordinal effect of type of motion (pure rotation versus pure translation) at each level of velocity difference.

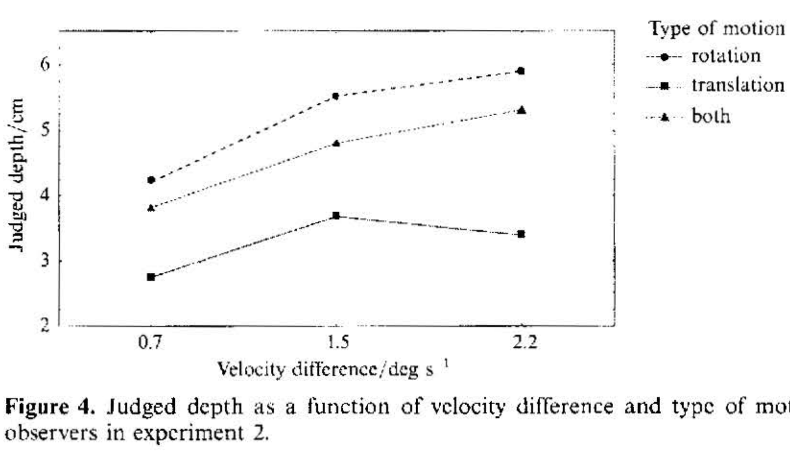

Figure 4. Judged depth as a function of velocity difference and type of motion averaged across observers in experiment 2.

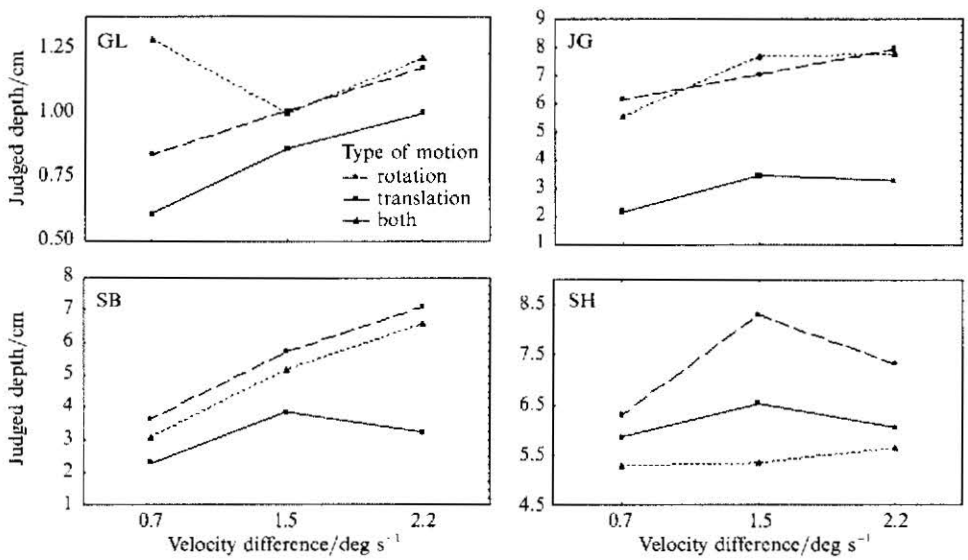

Figure 5. Judged depth as a function of velocity difference and type of motion for individual observers in experiment 2.

4 Experiment 3: Cylinder size

The results of the first two experiments show that judged depth varies with the velocity difference and velocity ratio when a constant object diameter is simulated. This raises the question whether a change in the simulated diameter would affect judged depth for constant levels of velocity difference and velocity ratio. In experiment 3 we examined judged depth at the same three levels of velocity difference studied in the two previous experiments, with a fixed velocity ratio. Object height was also varied as a control for any relationship that might exist between height, diameter, and judged depth. For one set of stimuli (rows 1–3 in table 3) the height was the same as the diameter. For a second, overlapping, set of stimuli (rows 3–5 in table 3) the height was fixed at the largest diameter.

Table 3. Translation (deg s^{-1}) and rotation ($^\circ$ s^{-1}) amounts in experiment 3.

Cylinder dimensions		Velocity difference/deg s^{-1}		
diameter/cm	height/cm	0.7	1.5	2.2
4.7	4.7	2.3, 13.0	4.7, 25.9	7.0, 39.0
7.0	7.0	2.3, 8.1	4.7, 16.2	7.0, 24.5
9.3	9.3	2.3, 5.7	4.6, 11.4	7.0, 17.2
4.7	9.3	2.3, 13.0	4.7, 25.9	7.0, 39.0
7.0	9.3	2.3, 8.1	4.7, 16.2	7.0, 24.5

4.1 *Method*

4.1.1 *Observers.* Four paid observers with normal or corrected-to-normal visual acuity participated. None was familiar with the hypotheses of the experiment. Three observers had participated in experiment 2.

4.1.2 *Stimuli.* The stimuli were similar to those in experiment 1, except for the changes in cylinder size described.

4.1.3 *Design.* Two independent variables were manipulated: the dimensions of the cylinder (diameter and height) and the difference between the velocity at the front and back of the cylinder. The velocity ratio was fixed at 1.50. The cylinder dimensions and rotation and translation speeds are shown in table 3.

4.1.4 *Apparatus and procedure.* The apparatus and procedure were the same as in experiment 1, except that each of the two sessions consisted of 10 replications of 15 conditions, and each session was divided into 3 blocks.

4.2 *Results and discussion*

Figure 6 shows the results averaged across subjects; figure 7 shows the individual subject results. An ANOVA showed a significant main effect for shape ($F_{4,12} = 8.42$, $p < 0.01$) and a significant interaction between shape and velocity difference ($F_{8,24} = 3.76$, $p < 0.01$). The main effect of velocity difference was not significant ($F_{2,6} = 4.86$, $p > 0.05$). The interaction between shape and velocity difference indicates that there was a smaller effect of velocity difference for the smallest diameter cylinder. The individual subject data indicate that one of the four subjects did not show an effect of

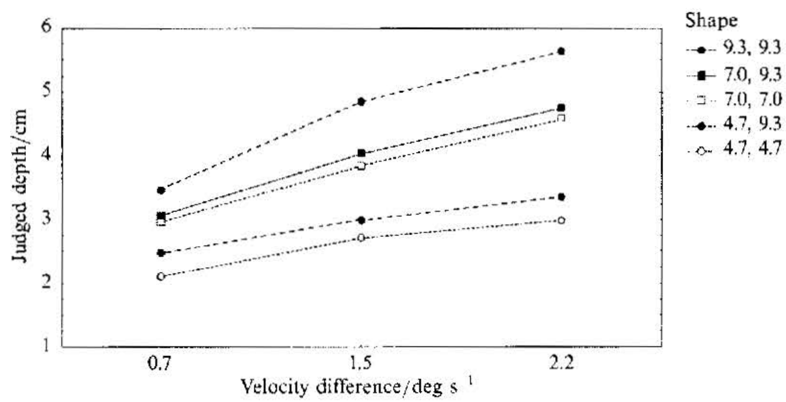

Figure 6. Judged object depth as a function of the velocity difference and the shape (diameter, height) of the object, averaged over observers in experiment 3.

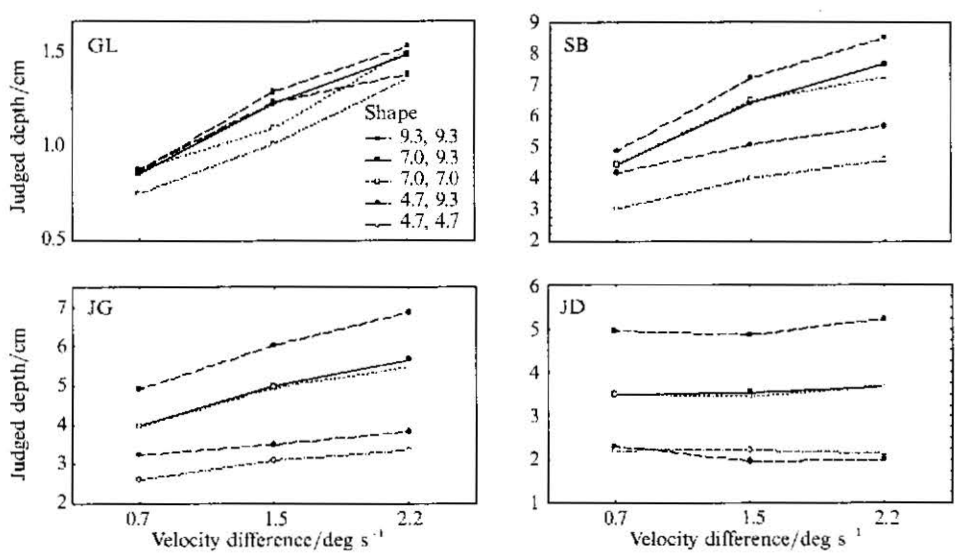

Figure 7. Judged object depth as a function of the velocity difference and the shape (diameter, height) of the object for individual observers in experiment 3.

velocity difference but showed a clear effect of shape. Figure 6 shows a trend towards slightly greater judged depth with the taller cylinders, but a posteriori comparisons (Tukey's HSD test) did not reveal any significant differences in the means for cylinders that were the same in diameter but different in height.

5 Experiment 4: Elliptical cylinders

In experiments 1 – 3 the rotating objects were surfaces of revolution. This had the advantage of providing a constant projection as the object rotated, avoiding cues to the object's depth available from changes in the projected contour (see, for example, Cortese and Andersen 1991; Norman and Todd 1994). The use of surfaces of revolution, however, has the disadvantage that observers could use the projected width to judge the object's depth. Our finding in experiment 1 that judged depth varies with the velocity difference and the velocity ratio, with projected width remaining constant, demonstrates that observers were not judging depth entirely by width. It is possible nevertheless that projected width influences judged depth. In experiment 3, larger diameter cylinders were judged to have greater depth, for constant values of velocity difference and velocity ratio. This difference in depth judgments could have been due either to differences in the projected width or to differences in the simulated depth, which are of course indistinguishable for surfaces of revolution rotating about their axis of symmetry. In experiment 4 we used elliptical cylinders to separate changes in projected width from changes in simulated depth. A rotating elliptical cylinder, however, would produce contour changes that would be related to the object's relative depth. To avoid these contour changes we superimposed a rectangular virtual mask on the rotating cylinder so that its projected contour was always the same. As a control for the possibility of the mask having unexpected effects on perceived depth, we conducted a second version of the experiment without the mask.

5.1 *Method*

5.1.1 *Observers.* Four paid observers with normal or corrected-to-normal visual acuity participated. None was familiar with the hypotheses of the experiment.

5.1.2 *Stimuli.* The stimuli were similar to those in experiment 1, except for the inclusion of elliptical cylinders (the dimensions are given below) and the use of a 5.5 cm wide by 7.3 cm high virtual aperture to conceal the edges of the projected cylinder in the first part of the experiment. (The width of the virtual aperture was based on the projection of the smallest simulated major axis of 7.0 cm at a simulated viewing distance to the center of the object of 113.7 cm, with the scene projected onto a plane 89.5 cm from the eye.) The rotation sequence was centered about the position in which the major axis of the cylinder was parallel to the line of sight.

5.1.3 *Design.* Three independent variables were manipulated: the velocity difference, the velocity ratio, and the major axis of the elliptical cylinder. The velocity difference and ratio were measured at the maximum displayed depth, that is, when the major axis of the cylinder was parallel to the line of sight. The rotation and translation speeds and major axes are shown in table 4. The minor axis was always 7.0 cm.

The experiment was conducted in two parts. In the first part a virtual mask concealed the edges of the projected cylinders that were not circular whenever the horizontal extent of the projection would have exceeded that of a circular cylinder. In the second part no mask was used and the projected contour changes were visible for objects that had unequal major and minor axes.

5.1.4 *Apparatus and procedure.* The apparatus was the same as in experiment 1. The procedure was the same as in experiment 1 except that each session consisted of 60 trials—6 practice trials plus 2 replications of the 27 stimulus conditions in a random

Table 4. Translation (deg s^{-1}) and rotation (° s^{-1}) amounts in experiment 4.

Major axis/cm	Velocity ratio	Velocity difference/deg s^{-1}		
		0.7	1.5	2.2
7.0	1.25	4.2, 6.9	8.4, 13.8	12.6, 20.7
	1.5	2.3, 8.1	4.7, 16.2	7.0, 24.5
	2	1.4, 8.7	2.8, 17.4	4.2, 26.4
9.3	1.25	4.2, 4.5	8.4, 8.9	12.6, 13.4
	1.5	2.3, 5.7	4.6, 11.4	7.0, 17.2
	2	1.4, 6.3	2.8, 12.6	4.2, 19.1
11.6	1.25	4.2, 3.0	8.4, 6.0	8.4, 8.9
	1.5	2.3, 4.3	4.6, 8.5	7.0, 12.8
	2	1.4, 4.9	2.8, 9.7	4.2, 14.7

order. Data were collected in five sessions (a total of 10 replications of each condition), preceded by either one or two practice sessions of the same length (depending on whether or not the observer had participated previously in an experiment with SFM stimuli).

5.2 *Results and discussion*

Figures 8 and 9 show the results for the two parts of the experiment. Although there was an overall increase in judged depth in the part with no mask, there are no noticeable differences in the pattern of results for the two parts, indicating that the visibility of the contour changes had little effect on the relation between the amount of simulated depth and the amount of judged depth. In both parts of the experiment there was no systematic effect of the simulated depth of the object on depth judgments. Although the maximum depth of the elliptical cylinder with the largest major axis was 65% greater than the one with the smallest major axis, depth judgments again depended on the velocity difference and velocity ratio, and not on the major axis (the simulated maximum depth). ANOVAs for each part of the experiment found significant main effects for the velocity difference ($F_{2,8} = 23.11, p < 0.01$, and $F_{2,4} = 161.15, p < 0.01$)

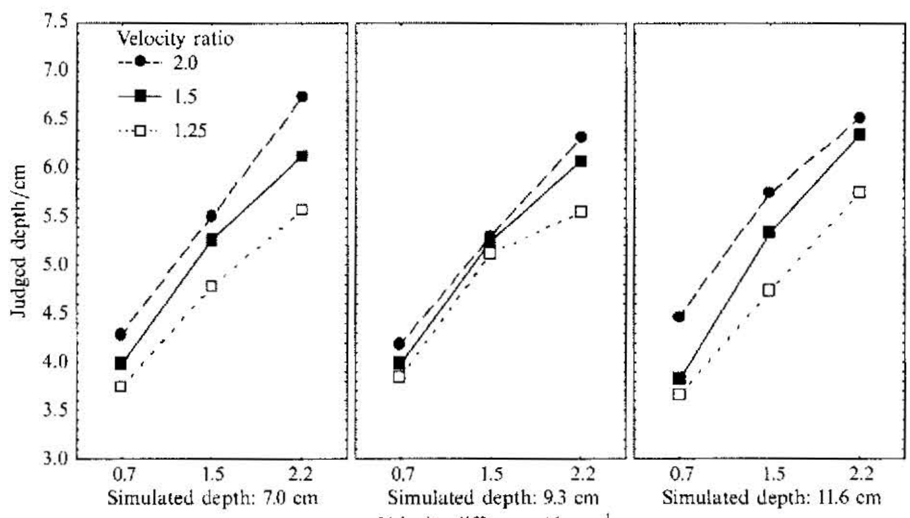

Figure 8. Judged depth as a function of velocity difference, velocity ratio, and simulated object depth, with contour changes hidden, in experiment 4, part 1.

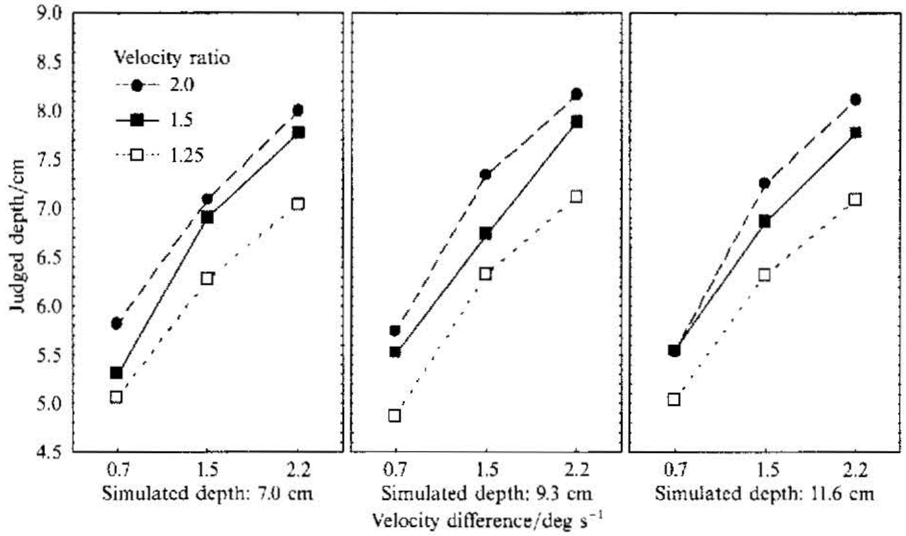

Figure 9. Judged depth as a function of velocity difference, velocity ratio, and simulated object depth, with contour changes visible, in experiment 4, part 2.

and velocity ratio ($F_{2,8} = 7.06$, $p < 0.05$, and $F_{2,4} = 96.87$, $p < 0.01$). The main effects of major axis were not significant ($F_{2,8} = 1.35$ and $F_{2,4} = 1.23$). There were no significant interactions.(1)

6 Conclusions

Judged depth in transparent flow fields is related primarily to the difference between the maximum and minimum velocity and the ratio of these velocities, if the projected size of the object is constant. The velocity difference appears to be the primary factor, but there may be some individual differences in the relative weighting of the difference and the ratio in determining judged depth. If the projected size varies, as a result of varying the diameter of the rotating cylinder in our experiments, the projected size also affects the judged distance from the front to the back of the object. For the largest size studied, a 3 : 1 increase in rotational velocity resulted in close to a 60% increase in judged depth. For the largest rotational velocity studied, a 2 : 1 increase in projected diameter resulted in a 90% increase in judged depth. This could suggest that observers were using veridical information about the object's actual depth and possibly weighting this more heavily than the information available from rotational velocity, which may not provide a veridical indication of object depth. Another possibility, however, is that observers were simply scaling their responses by the projected diameter of the object. Our experiment with elliptical cylinders supports the latter explanation: when deeper objects were simulated, for which the projected diameter did not change, or changed only slightly as a result of rotation, there was no measurable effect of simulated object depth.

These results are consistent with Todd and Bressan's (1990) proposal that observers use only the velocity field, and not information integrated over more than two views, in the perception of 3-D structure from motion. The results further demonstrate that the use of the velocity field, while it does not provide veridical depth judgments, is not arbitrary. Rather, there are specific perceptual heuristics underlying the use of this information. Observers' judgments of the relative depth in flow patterns representing

(1) Because of large individual differences in response scaling, these ANOVAs were conducted on transformed scores. The mean for a given observer in each condition was divided by that observer's standard deviation across condition means.

rotating and translating objects are based on the difference and ratio of the velocities of features spanning the depth being judged, and on the projected size of the object. The use of the velocity difference supports the conclusions of Liter et al (1993) for two-view and multiple-view displays with small numbers of dots. The use of the velocity difference, scaled by object size, is consistent with Proffitt et al's (1992) conclusions for stereokinetic and kinetic depth stimuli. The combined use of the velocity difference and the velocity ratio is consistent with Domini and Braunstein's (1998) finding that judged depth along a surface varies with deformation (related to the velocity difference), but also varies with the velocity ratio when deformation is constant.

The present results do not support the use by human observers of computation algorithms, that recover 3-D structure veridically from as few as three views of four non-coplanar points, eg Ullman's (1979) SFM theorem. Although such theoretical analyses are important for an understanding of the information potentially available about 3-D structure in a dynamic 2-D image, those analyses do not necessarily reveal the processes used by human observers. Human observers appear to use heuristic processes that may not produce the most accurate solutions that are theoretically possible (Braunstein 1976, 1994). These heuristic processes may reflect constraints that the process of evolution places on biological systems (Braunstein 1983). Heuristic processes should not be regarded as top-down, mentalistic or cognitive, but can be bottom-up, automatic, and biologically determined, and are consistent with the 'observers' proposed by Bennett et al (1991) and the smart mechanisms proposed by Runeson (1977). The use of the projected width of the object, velocity differences, and velocity ratios to judge the depth of a 3-D object is an example of such heuristic processes.

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