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Design and analysis of multistage cooperative broadcast with amplify and forward relays

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DESIGN AND ANALYSIS OF MULTISTAGE COOPERATIVE BROADCAST WITH AMPLIFY AND FORWARD RELAYS

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DESIGN AND ANALYSIS OF MULTISTAGE COOPERATIVE BROADCAST WITH AMPLIFY AND FORWARD RELAYS

by

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ABSTRACT

DESIGN AND ANALYSIS OF MULTISTAGE COOPERATIVE BROADCAST WITH AMPLIFY AND FORWARD RELAYS

by Bhargava Yammanuru

Cooperative communication achieves spatial diversity by having the transceivers in an *ad-hoc* network pool their resources at the physical layer and cooperatively transmit their information. For this to be possible without adding a large overhead, we need low overhead-distributed protocols. This thesis proposes one such distributed scheme for wireless ad-hoc networks.

In this work, we study the propagation of the signal in a cooperative network where a single source message is retransmitted by multiple stages (levels) of relays. Relays are assumed to have limited computational abilities and hence adopt the amplify-and-forward scheme. At each node, cooperative diversity is obtained by combining the signals from the multiple levels of relays (in different time slots) using a matched filter. The network is distributed in the sense that the levels are not predetermined and are formed based on the decisions made independently at each node. The retransmission criterion is based on the signal-to-noise ratio (*SNR*) of the signal after the matched filtering operation. If the received *SNR* is greater than the *SNR threshold* then the signal is retransmitted. The parameter *SNR threshold* plays a critical role in determining the broadcast rate.

We provide the expressions for the received signal at each node as the message is forwarded in the network. We study the channel and noise statistics for a specific realization of a network. We also recursively characterize the effective channel, and accumulated noise. We study the effects of noise accumulation, the number of levels used in the signal combination and the decoding and retransmission threshold on the number of nodes that successfully receive the message.

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CHAPTER 1

INTRODUCTION

1.1 Wireless environment

Wireless communication is evolving rapidly. Especially in the last decade tremendous progress has been made. The proliferation of laptops and smart phones exemplify the importance of wireless communication. Increased demands for high data rates and the advances in very-large-scale integration (VLSI) technology have made wireless communication an active research field. The design of a wireless communication system is extremely challenging. Due to multiple reflections from various objects, especially in urban areas where there is no direct line-of-sight path between the transmitter and receiver, the signal travels along different paths with different path lengths. This results in several versions of the same signal that differ in amplitude, phase, and delay. The interaction between these versions results in multi-path fading at a specific location. Consequently, the power of the received signal fluctuates randomly in space, time, and frequency.

The randomness in its behavior makes the modeling of a wireless channel very difficult. Typically, the wireless channels are modeled statistically using the measurements made for a specific communication system. Propagation models can be classified into the following two categories.

- *Large scale propagation models:* These models estimate the mean signal strength for large distances between the transmitter and receiver. When the receiver moves away from the transmitter over larger distances, the average received signal strength decreases. This average is predicted by the large scale propagation models. The variation in signal strength is due to path loss and shadowing. Path loss is caused by the dissipation of transmitted power with distance. Shadowing is caused by the obstacles between the transmitter and receiver that might absorb the signal.

- *Small scale propagation models:* These models characterize the rapid fluctuations of the received signal strength over small distances. The received signal is a combination of signals coming from different directions. Because of random phases of the individual signals, the resultant signal varies widely in amplitude and phase. The main factors influencing small scale fading are multi-path propagation of signals, the relative motion between the transmitter and receiver resulting in a doppler effect, and the transmission bandwidth of the signal.

Large scale propagation models and fading models have been studied by Rappaport (2002) [1].

1.2 Wireless networks

The wireless channel is an important resource that is used by many users for different purposes. This resource needs to be carefully used. Because many users are using the channel, we have to device protocols and network configurations that properly handle the interference between them. The protocols should adopt strict scheduling algorithms to allocate the channel to users over time. In this section, we briefly describe a few network configurations.

1.2.1 Network configurations

- *Point-to-point communication channel:* In point-to-point communication channels, there is one source trying to communicate with a destination, as shown in Fig. 1.1. This is been the most heavily researched link over the years. Many problems such as inter-symbol interference (ISI) and capacity achieving codes have been addressed for this link [2,3]. Though this channel looks simple in the sense that it has only one source and a receiver, it poses several challenges to the designer such as the time varying nature of the wireless channel and multi-path signal propagation. The fading channels have been studied by Biglieri et al. (1998) [4].

Figure 1.1. A point-to-point communication channel

- *Broadcast channel:* In a broadcast channel, there is one source and multiple receiving nodes, as shown in Fig. 1.2. The broadcast channel has been studied by Cover (1991) [5]. A simple example of the broadcast channel is the TV station. The TV station transmits the same information to many receivers. The channel poses different issues to the designer. For example consider a TV station. With *high-definition TV* (HDTV) becoming popular, the designer has encoding issues owing to a variety of receivers. Some TV's are equipped to handle HDTV information and others are not. The information has to be encoded in such a fashion that both receivers should be able to decode the information. There are several other issues in a wireless broadcast channel.

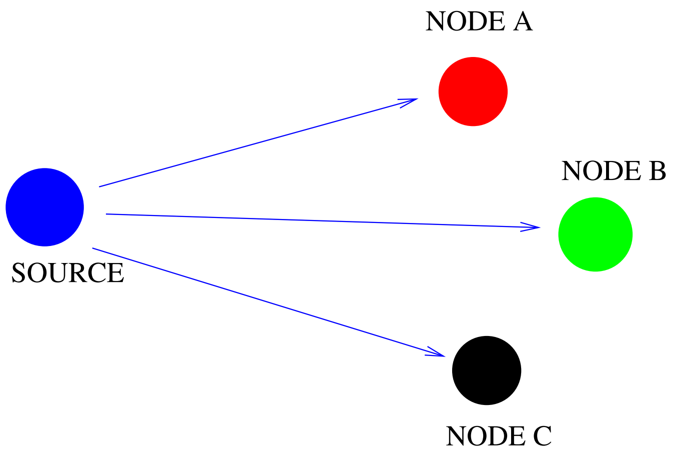

Figure 1.2. A broadcast channel

The source might have different information for different receivers. This setup is different from that of the previous example where the TV station transmits the same information to all the receivers. In such cases, the source can employ simple mechanisms, such as time sharing, to send the information to the receivers alternately or can use complex superposition coding schemes to obtain a higher information rate [5].

- *Multiple access channel (MAC):* In the multiple-access-channel model, there are several senders and a single receiver communicating. Classical MAC schemes such as time division multiple access (TDMA), frequency division multiple access (FDMA), and code division multiple access (CDMA) can be utilized in order to remove or reduce the interference among senders. A MAC channel with two senders transmitting information to one receiver over a channel with two inputs and one output is shown in Fig. 1.3.

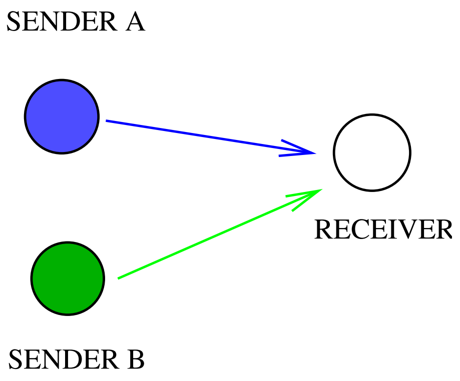

Figure 1.3. A multiple access channel with 2 senders and 1 receiver

- *Relay channel:* In a relay channel, there is a single source and a single destination with a number of intermediate nodes relaying the message from the source to the destination. Fig. 1.4 shows a relay channel with a single relay.

  The relay transmits a processed version of the signal it receives from the source. The destination either utilizes the relayed message (multi-hop) or combines the signals received from both the source and the relay node. In the latter case, it can be seen that even if the nodes have only a single antenna,

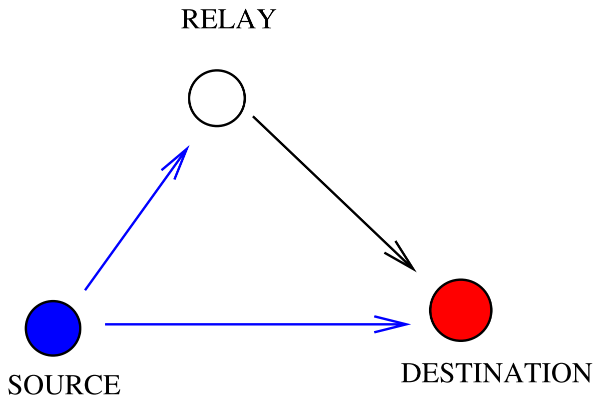

Figure 1.4. A relay channel

the relay can be used to provide diversity. A relay channel can also be viewed as a combination of a broadcast channel (from the source to the relay and destination) and a multiple access channel (from the source and the relay to the destination).

1.3 Wireless communication with multiple antennas

In order to combat the effects of the rapidly varying fading channel, the use of multiple antennas at the transmitter and receiver ends was suggested in the pioneering works of Winters, Foschini, and Telatar [6–8].

For a multiple-input and multiple-output (MIMO) system with M_T transmitting and M_R receiving antennas, the discrete-time model is represented as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{y}$ is the $M_R \times 1$ dimensional received signal vector, $\mathbf{x}$ is the $M_T \times 1$ dimensional transmitted signal vector, and $\mathbf{n}$ is the $M_R \times 1$ dimensional noise vector. The channel at any point of time is given by a $M_R \times M_T$ matrix, $\mathbf{H}$. In Eqn.(1.1), h_{ij} represents the channel gain between the j^{th} transmitting antenna and i^{th} receiving antenna.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M_T} \\ h_{21} & h_{22} & \dots & h_{2M_T} \\ . & . & \dots & . \\ . & . & \dots & . \\ h_{M_R1} & h_{M_R2} & \dots & h_{M_RM_T} \end{bmatrix} \tag{1.1}$$

The multiple antennas also help the designer in exploiting the spatial domain. This spatial domain can be used to obtain the spatial diversity gain and/or spatial multiplexing gain.

- *Spatial diversity gain:* The multiple antennas at the transmitter and receiver help in providing multiple copies of the message at the receiver. The signal experiences multiple independently faded links and hence with high probability we have at least one link which does not have deep fading. This improves the quality of the signal at the receiver. The number of independent copies of the signal at the receiver is often termed as diversity order.

- *Spatial multiplexing gain:* With multiple antennas we can even transmit multiple independent data streams. Given appropriate channel conditions the receiver is able to decode all the data streams. This does not even use additional bandwidth, hence can boost the speeds at which the data can be transmitted with minimum costs.

The maximum multiplexing gain over a zero-mean white Gaussian noise MIMO channel is $r_{max} = min(M_T, M_R)$ and the maximum diversity gain is given by $d_{max} = M_R M_T$. There is usually a trade-off between diversity gain and multiplexing gain. It is shown in [9] that for a zero-mean white Gaussian channel, the trade of is given by

$$d(r) = (M_R - r)(M_T - r), \quad 0 \leq r \leq min(M_T, M_R), \tag{1.2}$$

where $d(r)$ is the diversity gain expressed as a function of the multiplexing gain, r.

The benefits of the MIMO system over the single-input single-output (SISO) system is that for fixed probability of error, the transmission rate can be increased by r_{max} bps/Hz for every 3 dB increase in SNR, while for a single antenna system it is only 1 bps/Hz. Similarly, in the high SNR region, compared to the 2^{-1} decrease in the probability of error for a fixed rate of transmission in a SISO system, in the MIMO system it decreases by $2^{-M_R M_T}$. The performance of the MIMO systems is analyzed in detail in [10].

1.4 Cooperative communication

Even though the MIMO systems are highly beneficial, they impose severe constraints on the hardware. It is not practical to have multiple antennas on small devices. Hence cooperative communication was proposed as an alternative [11–13]. The idea behind this was to create a *virtual multi-antenna system* where different devices cooperate with each other mimicking the multi-antenna system. The work on the relay channel in [14] was one of the motivations for cooperative communication.

1.4.1 Cooperative diversity protocols

We now present a few cooperative diversity protocols due to the work in [14, 15]. The message from the source propagates to the destination with the help of the relays. The relays process and forward the message using any of the following methods.

- *Amplify and Forward:* The node retransmits the received signal after scaling the power level according to a fixed constraint. This method is very simple to implement and does not require complex hardware. However the main drawback is that the noise is also amplified along with the signal and forwarded to the destination.

- *Decode and Forward:* The node first decodes the message, re-encodes it, and then retransmits the message. Error propagation can limit the performance of this method. If the node decodes the message incorrectly then this lowers the probability with which the destination can decode it correctly. Hence the performance is limited by the source-relay link. If the channel between the source and the relay is good, decode and forward scheme performs better.
- *Compress and Forward:* The node forwards a compressed version of the received signal. For optimal compression the Wyner-Ziv Coding can be used. The major draw back of this method is its complexity.

There are other methods such as selective relaying, incremental relaying, relaying with feedback etc. Because of high attenuation in wireless channels it is difficult to achieve sufficient electrical isolation between the transmitter and receiver, it is assumed that all the nodes are *half-duplex*, *i.e.,* they cannot transmit and receive at the same time.

It has to be noted that the nodes retransmit the processed version of the signal in the methods described. However the nodes can have their own independent messages. This is the case in a multi-user network where the nodes act as "*partners*", [11, 12]. The nodes pool their resources such as bandwidth, power to help each other transmit their messages to their respective destinations. In such scenarios coded cooperation, proposed in [11, 13], can be used. As the number of nodes increase it becomes extremely challenging to design protocols for the relay networks. Using peer-to-peer communication protocols in a cooperative network introduce a huge overhead. This overhead becomes increasingly significant as the network grows and hence will negate all the gains achieved due to cooperation. Hence we require distributed protocols which avoid the node-to-node connection and reduce the overhead.

1.5 Cooperative broadcast

In this section a brief account of cooperative broadcast is provided. In a cooperative broadcast, the goal of the network is to distribute a message of a source (or multiple sources) to everybody in the network via retransmission by multiple relays. First we present the advantage of cooperative broadcast over a multi-hop broadcast by providing a simple example in the next section. Then, cooperative broadcasting techniques are discussed in detail in the following sections.

1.5.1 Multi-hop broadcast vs cooperative broadcast

Consider an automatic fire monitoring system with 4 relay nodes as shown in the Fig. 1.5. In case of fire, suppose that we require all the nodes to be notified to take specific actions. We now look at how multi-hop broadcast method and cooperative broadcast behave in this situation.

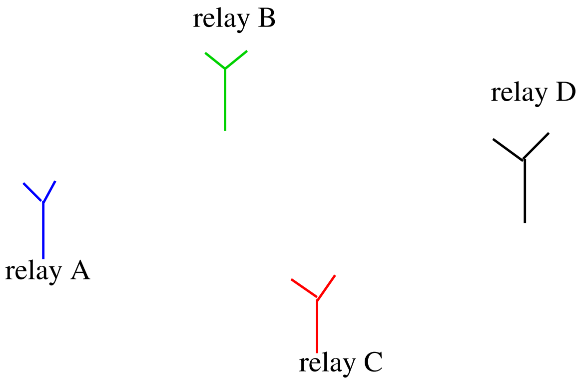

Figure 1.5. Network of 4 relays, in a automatic fire monitoring system

First we consider multi-hop broadcast technique. In this method the message is relayed to the nodes as shown in Fig. 1.6 The message hops from node to node. If one of the relay links is poor, say the link between the relays A and B. Then the other relays, C and D, do not receive the message. Hence one bad link in the network stops the propagation of the message.

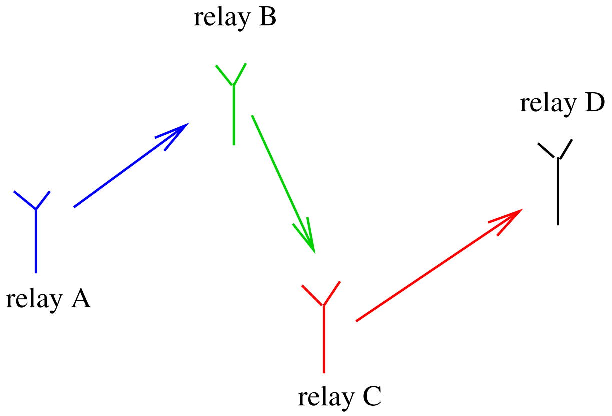

Figure 1.6. Multi-hop broadcast

Now consider the cooperative broadcast method. Here we take the advantage of the broadcast nature of the wireless channel, *i.e.,* when a message is transmitted all the nodes can 'listen' to the message. The nodes that receive the message retransmit it. This retransmission benefits the nodes which experienced bad links. Consider the previous scenario where the link between the relays A and B was poor resulting in disrupting the flow of the message in the network. Fig. 1.7 shows the propagation of message in a cooperative broadcast network.

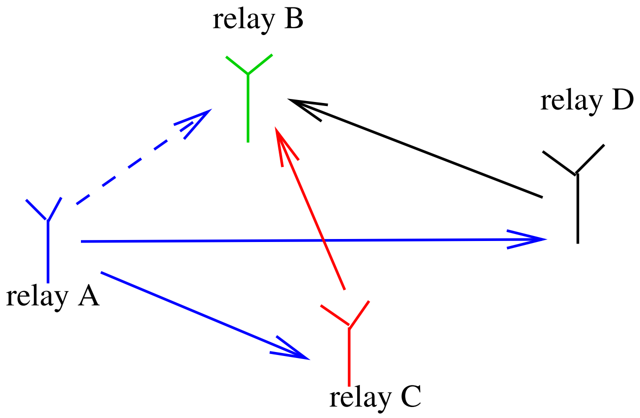

Figure 1.7. Cooperative broadcast (dashed line indicates a bad link)

The node A transmits the message and all the nodes receive it. Because of the bad channel between the relay A and B, the node B can not get the message. However because nodes C and D have received the message and they retransmit it node B still has a chance to receive the message. Hence cooperative broadcast helps in propagation of the message. This method benefits from the diversity obtained due to the signal experiencing different channels without having multiple antennas to get this diversity.

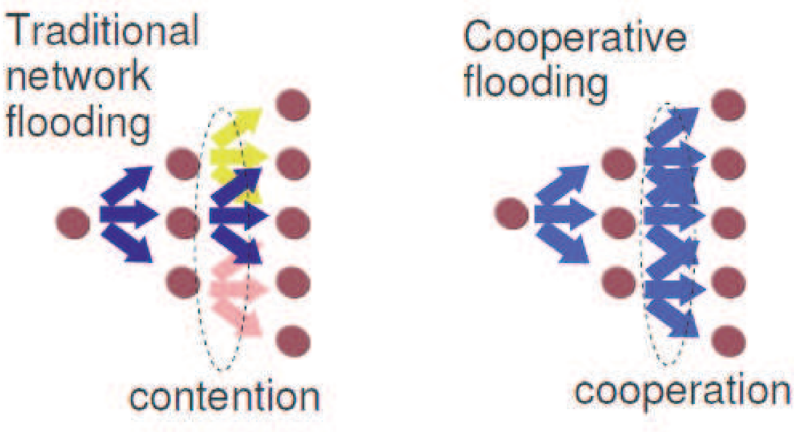

Figure 1.8. Non-cooperative vs cooperative broadcast

Cooperative broadcasting also removes the overhead of the MAC layer protocols in the networks. When all the nodes are trying to transmit the same message, leading to intentional collisions, there is no need for protocols that avoid collisions. Thus there is no contention for the channel access. This is shown in Fig. 1.8 (adopted from [16]). Also managing cooperative networks with large numbers of nodes becomes extremely difficult using centralized controlling. Therefore we need decentralized algorithms, where the nodes make their own decisions.

1.5.2 Cooperative broadcast techniques

In this section, we study a few cooperative broadcast techniques previously studied in the literature.

- *Opportunistic large array networks (OLA):* In [17], the use of cooperative transmission to send a message to a far of receiver was proposed. In this scheme the nodes act as repeaters that echo the signal received from the source (called "*leader*," in the paper). The connectivity and the scalability of the ad hoc network is studied. The information is forwarded with the help of receivers capable of tracking the *signature waveforms*, without making use of the channel information. The relays can either be regenerative (can decode-and-forward) and non-regenerative (only amplify-and-forward). This method provides signal diversity through cooperation. OLA is physical layer algorithm which helps in removing the overhead of the routing and the MAC layer.

- *Accumulative multi-cast:* The authors in [18] provide an energy efficient method using cooperative broadcast for time-invariant AWGN channels, where the nodes decode the message based on the transmissions from nodes that are *reliable*. A node becomes reliable by combining these accumulated signals. Once a node becomes reliable it retransmits the signal. The order of transmission is specified by the *reliability schedule*, which is determined based on a heuristic algorithm proposed. The problem is addressed for both distributed and centralized networks.
- *Cooperative broadcast using decode and forward relays:* The work in [16, 19] analyzes a multistage cooperative broadcast network using decode-and-forward (DF) technique. It provides insights into the effects of the parameters of the network, like the decoding threshold and the transmission power at the nodes, on the number of nodes reached by the cooperative broadcast. It is shown that if the decoding threshold is lower than a *critical value*, the whole network receives the message, otherwise only a part of the network receives it. The effect of network parameters is analyzed for both the *wideband and the narrowband* networks. In [16], the problem of allocating optimal power in a dense cooperative broadcast network is studied. The 'scheduling algorithm' decides when a node has to transmit. An optimal scheduling algorithm for dense networks is also proposed. However it is shown that finding the optimal scheduling algorithm for general cooperative broadcast is an NP-complete

problem.

1.6 Dissertation outline

In Chapter 2, we study a multi-stage cooperative broadcast scheme using amplify and forward technique and analyze the signal propagation, effective channel and noise models. In Chapter 3, we evaluate the performance of the proposed scheme and compare it s performance with other schemes. Finally, in Chapter 4, we present the conclusions of this work.

CHAPTER 2

DESIGN OF MULTISTAGE COOPERATIVE BROADCAST WITH AMPLIFY AND FORWARD RELAYS

2.1 Organization

This chapter is organized as follows. The system model employed is specified in the next section. In Section 2.3, we derive the transmitted and received signal structures which lead to models for effective channel and accumulated noise. In Section 2.4, we discuss the details for retransmission and decoding criteria. In Section 2.5, we derive statistics for the effective channel and noise.

Notation: We adopt the following notations. The lower case letter denotes a scalar, the bold lower case letter denotes a vector, bold upper case letter denotes a matrix. $\mathbf{I}_n$ denotes an $n \times n$ identity matrix. $\mathbf{0}_{m\times n}$ denotes a $m \times n$ dimensional matrix of zeros. $\mathcal{N}_c(0, \sigma^2)$ denotes the complex Gaussian distribution with zero-mean and σ^2 variance. $\mathbb{E}\{X\}$ denotes the expected value of X.

2.2 System model

In the considered set-up, a single source transmits its message and the relays retransmit the message in multiple levels using amplify-and-forward strategy. The goal of the network is to distribute the source message to the entire network. The choice of relays' retransmission method strongly depends on the channel conditions, network setup, performance metrics, and also complexity constraints [15, 20, 21]. AF is considered to be simple (when compared with DF), and could outperform DF under certain cases [20]. In this work, we are interested in using multilevel AF relays for cooperative broadcasting. Note that in cooperative broadcast, the goal is to distribute source message to the entire network.

Amplify-and-forward (AF) relays are utilized in [22, 23] to help the transmission from a source to a destination. The authors analyze the capacity for large number of relays divided into fixed number of levels. [22] addresses the effect of increasing number of nodes with fixed number of levels on noise amplification in a multistage multi-hop relay network. Here the nodes can transmit/receive only in the time slots allotted to them. In [23] they analyze the capacity of a large relay network when the source and relay nodes can transmit/receive only in time slots allotted to them but the destination nodes can listen all the time as shown in Fig. 2.1 (adopted from [23]). When the source(s) transmit(s) a message, the first level relays listen to the signal and retransmit a scaled version of the received signal (classical AF). Similarly in the k^{th} time slot, the k^{th} level relays amplify and forward the signal received in the previous time slot. In [22], the destination nodes detects the message from the signals received from the last level relays. In [23], the destination nodes attempts to decode the signal by combining signals received in all time slots. The results are obtained for the case when the number of nodes per level tends to *infinity*. It is shown in [23] that the capacity of a multistage orthogonal amplify-and-forward relay network increases linearly as the number of nodes goes to infinity.

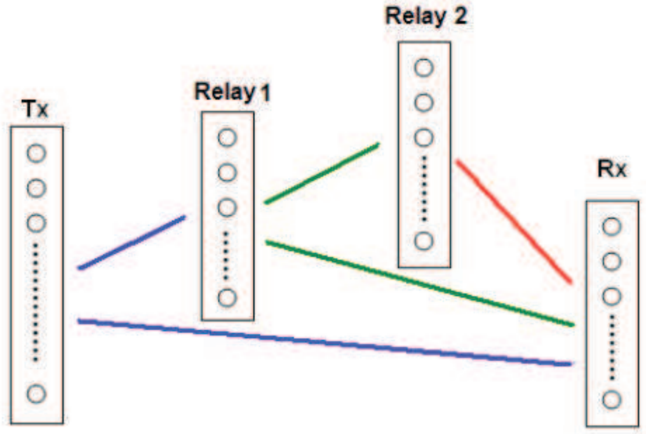

Figure 2.1. Orthogonal amplify and forward relay network

We employ a completely distributed system, wherein the levels are formed on the fly based on local decisions. There is no central controlling system which

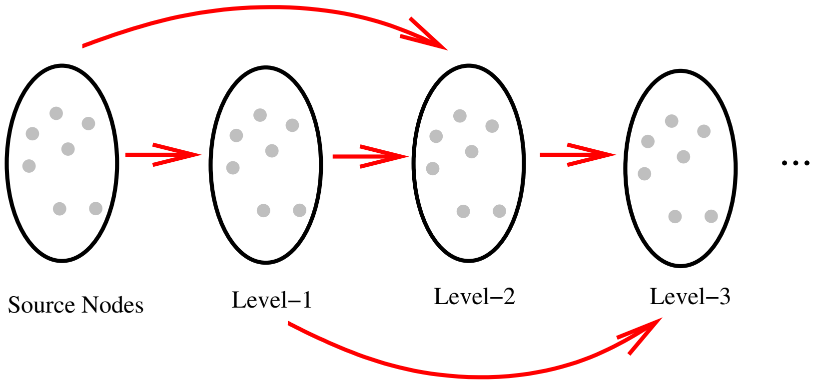

Figure 2.2. Multilevel AF broadcast with memory $M = 2$

dictates the time intervals in which the nodes can transmit. We also introduce memory into the nodes so that they remember the M of the most recent received signal. The nodes that satisfy a threshold criterion ($> \tau_1$) on the receive SNR are allowed to retransmit (See Fig. 2.2). The receive SNR is obtained by whitening and combining the receptions from M previously transmitted levels via matched filtering (See Fig. 2.3). We are interested in the maximum rate that can used to broadcast a network, in which the nodes act as cooperative relays, so as to maximize the number of nodes receiving the message. Nodes are assumed to be able to decode the message if their receive SNR exceeds another threshold value τ_2. The τ_2 is assumed to be greater than τ_1, and hence relays allow the flow of the signal even if they can not decode it. They continue to accumulate the signal after retransmission. The dynamics of the network as a function of *retransmission threshold* τ_1 and *decoding threshold* τ_2 are also provided.

Consider a slotted transmission. At each time slot a group of nodes (levels) transmits the message. We assume the relays are only capable of simple processing, hence after whitening and matched filtering, amplify-and-forward the message (see Fig. 2.3). The relays are assumed to have the channel state information (CSI) at the receiver needed for matched filtering.

Each node belongs to a level, that is if k'th node belongs to l'th level, then k'th

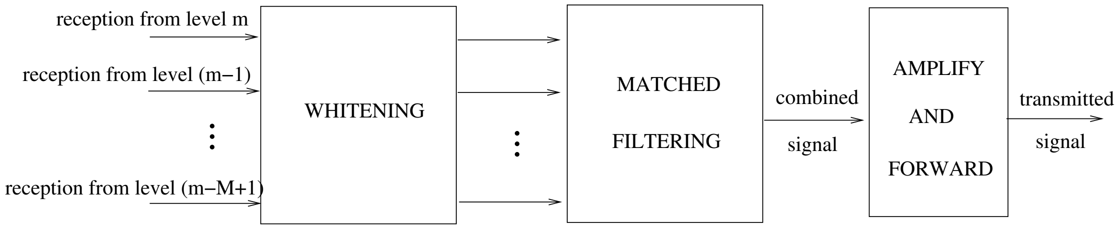

Figure 2.3. Relay processing during m'th time slot

node transmits at l'th time instant. Let $S = \{1, \ldots, N\}$ denote the set of nodes. Let

$$\ell : S \to \{1, \ldots, L\}$$

denote the level-mapping. That is k'th node belongs to level $\ell(k)$, where $\ell(\cdot)$ is a function. In this scheme the leveling is random (it is a function of channel and network realizations), and based only on the local decisions. Each node accumulates the received signal from M previous levels until it satisfies a retransmission criterion. The nodes continue accumulating signals until they are able to decode the message. We assume the relays are half-duplex, i.e. they can not receive and transmit at the same time slot.

2.3 Effective channel and noise models

Let $\ell(k)$-dimensional vector $\mathbf{r}_k$ denote the received vector right before retransmission at the k'th node. We can rewrite $\mathbf{r}_k$ as

$$\mathbf{r}_k = [r_k[0], r_k[1], \ldots r_k[\ell(k)-1]]^T,$$

where $r_k[m]$ denotes the received symbol at the k'th node due to transmission of the nodes in level-m. Let t_i denote the transmitted signal by the i'th node, then

$$r_k[m] = \sum_{i \in S_m} t_i h_{ki} + w_k[m] \quad \forall\, m \geq 0 \tag{2.1}$$

where $S_m = \{i \in S : \ell(i) = m\}$ denotes the set of nodes that belongs to level-m, h_{ki} denotes the channel gain between i'th and k'th nodes, and $w_k[m]$ denotes the received additive white Gaussian noise, with distribution $\mathcal{N}_c(0, N_0)$. For each node β_i denotes the power scaling, such that the transmitted signal $\mathbb{E}\{|t_i|^2\} = P_i$. Let N_m denote the number of nodes in level-m. We can rewrite the index set for level-m nodes as

$$S_m := \{s_{m1}, s_{m2}, \ldots, s_{mN_m}\}.$$

We now describe the transmitted and received signal at each time-slot. At time slot 0, the transmitted signal by source nodes is

$$t_m = x_0\sqrt{P_m},\ m \in S_0,$$

where $\mathbb{E}\{|x_0|^2\} = 1$. Note that all the source nodes are transmitting the same message (mimicking the multiple antenna system). We can rewrite Eqn.(2.1), for $m = 0$, as

$$\mathbf{r}_k^{(0)} = [r_k[0]] = \mathbf{h}_k^{(0)} x_0 + \mathbf{A}_k^{(0)} \mathbf{w}_k^{(0)},$$

where $\mathbf{w}_k^{(0)} = [w_k[0]]$ and

$$\mathbf{h}_k^{(0)} = \sum_{m \in S_0} h_{km}\sqrt{P_m}, \quad \mathbf{A}_k^{(0)} = 1. \tag{2.2}$$

At time slot 1, the i'th node that belongs to the first level transmits the signal t_i for all $i \in S_1$

$$t_i \;=\; \sqrt{\beta_i}\frac{\mathbf{h}_i^{(0)*}}{|\mathbf{h}_i^{(0)}|} r_i[0] = \sqrt{\beta_i}\frac{\sum_{m \in S_0} h_{im}^* \sqrt{P_m}}{|\sum_{m \in S_0} h_{im}\sqrt{P_m}|} r_i[0]. \tag{2.3}$$

where $\beta_i = \frac{P_i}{|\mathbf{h}_i^{(0)}|^2 + N_0}, \quad i \in S_1$. After the transmission of level-1 nodes, the received signal vector (due to transmission of level-0 and level-1) at a node $k \in S_2$ is

$\mathbf{r}_k^{(1)} = [r_k[0]\ r_k[1]]^T$. We can write $\mathbf{r}_k^{(1)}$, for all $k \in S_2$ as

$$\mathbf{r}_k^{(1)} = \begin{bmatrix} r_k[0] \\ r_k[1] \end{bmatrix} = \mathbf{h}_k^{(1)} x_0 + \mathbf{A}_k^{(1)} \mathbf{w}_k^{(1)} \tag{2.4}$$

where

$$\mathbf{h}_k^{(1)} = \begin{bmatrix} \sum_{m \in S_0} h_{km} \sqrt{P_m} \\ \sum_{i \in S_1} |\sum_{m \in S_0} h_{im} \sqrt{P_m}| h_{ki} \sqrt{\beta_i} \end{bmatrix} \tag{2.5}$$

and $\mathbf{A}_k^{(1)} := [\ \mathbf{I}_2 \mid \mathbf{B}_k^{(1)}\ ]$ where

$$\mathbf{B}_k^{(1)} := \begin{bmatrix} 0 & \dots & 0 \\ \mathbf{a}_k^{(s_{11})} & \dots & \mathbf{a}_k^{(s_{1N_1})} \end{bmatrix}, \tag{2.6}$$

with

$$\mathbf{a}_k^{(s_{1i})} = \frac{\sum_{m \in S_0} \sqrt{P_m} h_{s_{1i}m}^* \sqrt{\beta_{s_{1i}}} h_{ks_{1i}}}{|\sum_{m \in S_0} \sqrt{P_m} h_{s_{1i}m}|}, \quad i = 1 \dots N_1. \tag{2.7}$$

The noise vector $\mathbf{w}_k^{(1)}$ is given by:

$$\mathbf{w}_k^{(1)} = [w_k[0]\ \ w_k[1]\ \ \mathbf{w}_{S_1}^T]^T,$$

with $\mathbf{w}_{S_1}$ defined as

$$\mathbf{w}_{S_1} := [w_{s_{11}}[0]\ \ \dots\ \ w_{s_{1N_1}}[0]]^T. \tag{2.8}$$

Here $w_{s_{1i}}[0]$ denotes the receiver noise at node s_{1i} belonging to level-1 during time-slot 0.

At time slot 2, the i'th node that belongs to second level, S_2, transmits the signal t_i ($\forall i \in S_2$)

$$t_i = \frac{\mathbf{h}_i^{(1)H} (\mathbf{A}_i^{(1)} \mathbf{A}_i^{(1)H})^{-1} \mathbf{r}_i^{(1)}}{\sqrt{\mathbf{h}_i^{(1)H} (\mathbf{A}_i^{(1)} \mathbf{A}_i^{(1)H})^{-1} \mathbf{h}_i^{(1)}}} \sqrt{\beta_i} \tag{2.9}$$

Using Eqn.(2.1) and Eqn.(2.9), the received signal $\mathbf{r}_k^{(2)}$, $\forall k \in S_3$, can be written as

$$\mathbf{r}_k^{(2)} = \begin{bmatrix} r_k[0] \\ r_k[1] \\ r_k[2] \end{bmatrix} = \mathbf{h}_k^{(2)} x_0 + \mathbf{A}_k^{(2)} \mathbf{w}_k^{(2)} \tag{2.10}$$

where

$$\mathbf{h}_k^{(2)} = \begin{bmatrix} \sum_{i \in S_0} h_{ki} \sqrt{P_i} \\ \sum_{i \in S_1} \sqrt{\mathbf{h}_i^{(0)H} \left(\mathbf{A}_i^{(0)} \mathbf{A}_i^{(0)H} \right)^{-1} \mathbf{h}_i^{(0)}} h_{ki} \sqrt{\beta_i} \\ \sum_{i \in S_2} \sqrt{\mathbf{h}_i^{(1)H} \left(\mathbf{A}_i^{(1)} \mathbf{A}_i^{(1)H} \right)^{-1} \mathbf{h}_i^{(1)}} h_{ki} \sqrt{\beta_i} \end{bmatrix}, \tag{2.11}$$

$$\mathbf{A}_k^{(2)} = [\ \mathbf{I}_3 \mid \mathbf{B}_k^{(2)}\ ] \tag{2.12}$$

where

$$\mathbf{B}_k^{(2)} := \left[\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & 0 & \mathbf{B}_k^{(1)} \\ \mathbf{a}_k^{(s_{21})} & \dots & \mathbf{a}_k^{(s_{2N_2})} & \mathbf{g}_k^{(2)} \end{array} \right] \tag{2.13}$$

where

$$\mathbf{a}_k^{(s_{2i})} := h_{k(s_{2i})} \sqrt{\beta_{(s_{2i})}} \frac{\mathbf{h}_{(s_{2i})}^{(1)H} (\mathbf{A}_{(s_{2i})}^{(1)} \mathbf{A}_{(s_{2i})}^{(1)H})^{-1}}{\sqrt{\mathbf{h}_{(s_{2i})}^{(1)H} (\mathbf{A}_{(s_{2i})}^{(1)} \mathbf{A}_{(s_{2i})}^{(1)H})^{-1} \mathbf{h}_{(s_{2i})}^{(1)}}}, \tag{2.14}$$

and

$$\mathbf{g}_k^{(2)} := \sum_{i \in S_2} h_{ki} \sqrt{\beta_i} \frac{\mathbf{h}_i^{(1)H} (\mathbf{A}_i^{(1)} \mathbf{A}_i^{(1)H})^{-1} \mathbf{B}_i^{(1)}}{\sqrt{\mathbf{h}_i^{(1)H} (\mathbf{A}_i^{(1)} \mathbf{A}_i^{(1)H})^{-1} \mathbf{h}_i^{(1)}}}. \tag{2.15}$$

The noise vector can be written as

$$\mathbf{w}_k^{(2)} = [w_k[0]\ w_k[1]\ w_k[2]\ \mathbf{w}_{S_2}^T\ \mathbf{w}_{S_1}^T]^T, \tag{2.16}$$

where

$$\mathbf{w}_{S_2} := [w_{s_{21}}[0]\quad w_{s_{21}}[1]\quad \dots\quad w_{s_{2N_2}}[0]\quad w_{s_{2N_2}}[1]]^T \tag{2.17}$$

and $\mathbf{w}_{S_1}$ is given in Eqn. (2.8).

We can generalize the above derivations and obtain a recursive formulation when the node has enough memory to accumulate signals from M slots until its turn for transmission. If memory is full then the oldest signal received is flushed out and the new copy of the signal is stored at the end of the array. The recursive formulation, **for** $m \geq 2$**,** is given as follows:

$$\mathbf{r}_k^{(m)} = \mathbf{h}_k^{(m)} x_0 + \mathbf{A}_k^{(m)} \mathbf{w}_k^{(m)}. \tag{2.18}$$

The effective channel vector can be written as

$$\mathbf{h}_k^{(m)} = \begin{bmatrix} \mathbf{h}_k^{(m-1)} \\ \\ \\ h_k[m] \end{bmatrix}, \tag{2.19}$$

where

$$h_k[m] = \sum_{i \in S_m} \sqrt{\mathcal{P}_i^{(m-1)}}\, h_{ki} \sqrt{\beta_i} \tag{2.20}$$

$$\mathcal{P}_i^{(j)} = \mathbf{h}_i^{(j)H} \left(\mathbf{A}_i^{(j)} \mathbf{A}_i^{(j)H} \right)^{-1} \mathbf{h}_i^{(j)}. \tag{2.21}$$

Define $\mathbf{U}_i^{(j)} = \mathbf{h}_i^{(j)H} (\mathbf{A}_i^{(j)} \mathbf{A}_i^{(j)H})^{-1}$ and

$$\mathbf{V}^{(m)} = \begin{cases} \mathbf{I}_{(m)} & \text{if } m < M \\ \left[\mathbf{0}_{(M-1)\times 1} \mid \mathbf{I}_{(M-1)} \right] & \text{if } m \geq M \end{cases} \tag{2.22}$$

Note that the dimension of the matrix $\mathbf{h}_k^{(m)}$ is $D(m) \times 1$, where

$$D(m) = \begin{cases} m+1 & \text{if } m < M \\ M & \text{if } m \geq M \end{cases} \tag{2.23}$$

Then we can write $\mathbf{A}_k^{(m)}$ as

$$\mathbf{A}_k^{(m)} = [\mathbf{I}_{D(m)} \mid \mathbf{B}_k^{(m)}], \tag{2.24}$$

where

$$\mathbf{B}_k^{(m)} = \left[\begin{array}{ccc|c} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{V}^{(m)}\mathbf{B}_k^{(m-1)} \\ \mathbf{a}_k^{(s_{m1})} & \dots & \mathbf{a}_k^{(s_{mN_m})} & \mathbf{g}_k^{(m)} \end{array}\right] \tag{2.25}$$

$$\mathbf{a}_k^{(s_{mi})} = h_{k(s_{mi})}\sqrt{\beta_{(s_{mi})}}\frac{\mathbf{U}_{s_{mi}}^{(m-1)}}{\sqrt{\mathcal{P}_{s_{mi}}^{(m-1)}}}, \tag{2.26}$$

$$\mathbf{g}_k^{(m)} = \sum_{i \in S_m} \frac{h_{ki}\sqrt{\beta_i}\mathbf{U}_i^{(m-1)}\mathbf{B}_i^{(m-1)}}{\sqrt{\mathcal{P}_i^{(m-1)}}}, \tag{2.27}$$

$$\beta_i = \frac{P_i}{\mathcal{P}_i^{(m-1)} + N_0}$$

For $M = 1$ (the maximum number of levels accumulated), it should be noted that $\mathbf{h}_k^{(m)} = h_k[m]$, with similar changes made to the matrices $\mathbf{A}_k^{(m)}$ and $\mathbf{B}_k^{(m)}$.

Eqns. 2.18, 2.19, and 2.24 determine recursive relations for effective channel and effective accumulated noise. The initial conditions are given in Eqn. (2.2).

2.4 Retransmission and decoding criterion

In the proposed scheme, we use the SNR threshold criterion to decide whether the node will transmit or not. The node transmits if the SNR of the accumulated signal is greater than or equal to a predefined threshold (See Fig. 2.3). We assume that the SNR is perfectly estimated at all the nodes. The SNR, $\gamma_k[m]$ can be found as:

$$\gamma_k[m] = \frac{\mathbf{h}_k^{(m)H}\left(\mathbf{A}_k^{(m)}\mathbf{A}_k^{(m)H}\right)^{-1}\mathbf{h}_k^{(m)}}{N_0} = \frac{\mathcal{P}_k^{(m)}}{N_0}, \tag{2.28}$$

where $\mathbf{h}_k$ and $\mathbf{A}_k$ is obtained via the recursive formulation obtained in the previous section and $\mathcal{P}_k^{(m)}$, is given by Eqn. (2.21). A node retransmits in the m^{th}-level ($\ell(k) = m$) if

$$\{\gamma_k[m] \geq \tau_1\} \cap \{\gamma_k[n] < \tau_1 \, \forall n < m\}. \tag{2.29}$$

We will call τ_1 the retransmission threshold. The nodes are assumed to able to estimate their receive SNR and each node is assumed to retransmit only once.

Even if nodes use AF to retransmit the message, they will not be able to decode it. Hence, in the proposed scheme the nodes continue accumulation of the signal from M previously transmitted levels until they are able to decode the message.

The successful reception (node is able to decode the message) is assumed if the receive SNR exceeds a threshold τ_2.

$$\{\tilde{\gamma}_k[m] \geq \tau_2\}. \tag{2.30}$$

The $\tilde{\gamma}_k[m]$ can be obtained similar to $\gamma[m]$. However, note that due to half-duplex constraint, the k'th relay can not accumulate the transmitted signals in level $\ell[k]$. We consider that the range of τ_2 as $\tau_2 \geq \tau_1$.

2.5 Channel and noise statistics

This section provides a recursive formulation for the channel and noise statistics. The statistics are derived for a given network configuration *i.e.,* node locations and for given level sets $S_1, S_2 \ldots$. We define this event as $E = \{d_{km}, \forall k, m, \text{ and } S_1, S_2, \ldots\}$. Here d_{km} denotes the distance between k'th and m'th node.

First we derive the initial conditions for channel and noise statistics using Eqn.

(2.2). We assume the channel coefficients between the k'th and m'th relay is $h_{km} \sim \mathcal{N}_c(0, d_{km}^{-\alpha})$ for a given network realization. In addition, h_{km} are assumed to be independent $\forall k, m$. Here α denotes the pathloss exponent.

$$\begin{aligned} \mathbb{E}\{\mathbf{h}_k^{(0)}|E\} &= 0,\ \mathbb{E}\{\mathbf{h}_k^{(0)}\mathbf{h}_k^{(0)H}|E\} = \sum_{i\in S_0}\frac{P_i}{d_{ki}^{\alpha}} \\ \mathbb{E}\{\mathbf{A}_k^{(0)}|E\} &= 1,\ \mathbb{E}\{\mathbf{B}_k^{(0)}|E\} = 0 \end{aligned} \tag{2.31}$$

Since the channel coefficient h_{km} for between any pair of nodes has zero mean, the effective channel vector has also zero mean: $\mathbb{E}\{\mathbf{h}_k^{(m)}|E\} = 0$.

The covariance matrix for $\mathbf{h}_k^{(m)}$ can be derived using recursive formulation (2.19)

$$\mathbf{K}_k^{(m)} := \mathbb{E}\{\mathbf{h}_k^{(m)}\mathbf{h}_k^{(m)H}|E\} = \begin{bmatrix} \mathbf{K}_k^{(m-1)} & 0 \\ 0 & c_k^{(m)} \end{bmatrix}, \tag{2.32}$$

where $\mathbf{K}_k^{(m-1)} = \mathbb{E}\{\mathbf{h}_k^{(m-1)}\mathbf{h}_k^{(m-1)H}|E\}$ and

$$c_k^{(m)} = \mathbb{E}\{|h_k[m]|^2|E\} = \sum_{i\in S_m}\frac{P_i}{d_{ki}^{\alpha}}\mathbb{E}\left\{\frac{\mathcal{P}_i^{(m-1)}}{\mathcal{P}_i^{(m-1)} + N_0} \mid E\right\},$$

where $\mathcal{P}_i^{(m-1)}$ is given by Eqn. (2.21). The off-diagonal entries in the channel covariance vanishes to zero since the channel coefficients between pairs of nodes are independent and zero-mean. In addition, the covariance matrix for the noise can be obtained as

$$\begin{aligned} \mathbf{C}_k^{(m)} &:= N_0\mathbb{E}\{\mathbf{A}_k^{(m)}\mathbf{A}_k^{(m)H}|E\} \\ &= N_0\left(\mathbf{I}_{D(m)} + \mathbb{E}\{\mathbf{B}_k^{(m)}\mathbf{B}_k^{(m)H}\}\right) \end{aligned}$$

where $\mathbf{B}_k^{(m)}$ is given in (2.25). By using the recursive formulation for $\mathbf{B}_k^{(m)}$, we obtain

$$\mathbf{C}_k^{(m)} = N_0 \begin{bmatrix} \mathbf{C}_{11} & 0 \\ 0 & f_k^{(m)} \end{bmatrix},$$

where $\mathbf{C}_{11} = \mathbf{I}_{D(m)-1} + \mathbf{V}^{(m)} \mathbb{E}\{\mathbf{B}_k^{(m-1)} \mathbf{B}_k^{(m-1)H}\} \mathbf{V}^{(m)H}$ and

$f_k^{(m)} = 1 + \mathbb{E}\left\{\sum_{i=1}^{N_m} \mathbf{a}_k^{(s_{mi})} \mathbf{a}_k^{(s_{mi})H} + \mathbf{g}_k^{(m)} \mathbf{g}_k^{(m)H} \mid E\right\}$, $\mathbf{V}^{(m)}$ is given in Eqn. (2.22), $\mathbf{a}_k^{(s_{mi})}$ is given in Eqn. (2.26) and $\mathbf{g}_k^{(m)}$ is given in Eqn. (2.27).

Notice that $\mathbf{V}^{(m)} \mathbf{V}^{(m)H} = \mathbf{I}_{D(m)-1}$. Hence, $\mathbf{C}_k^{m}$ could be expressed recursively as

$$\mathbf{C}_k^{(m)} = \begin{bmatrix} \mathbf{V}^{(m)} \mathbf{C}_k^{(m-1)} \mathbf{V}^{(m)H} & 0 \\ 0 & N_0 f_k^{(m)} \end{bmatrix}. \tag{2.33}$$

Note that $\mathbf{C}_k^{(0)} = N_0$.

2.6 Summary

In this chapter we recursively formulated the channel and noise models in a multistage cooperative broadcast network using AF relays. These expressions allow us to simulate the performance of the proposed scheme. Using these recursive expressions we can analyze the system performance in the asymptotic regime.

CHAPTER 3

PERFORMANCE ANALYSIS OF MULTISTAGE COOPERATIVE BROADCAST WITH AMPLIFY AND FORWARD RELAYS

3.1 Organization

In this chapter we evaluate the performance of the scheme proposed in Chapter 2 and also compare its performance with other schemes. In Section 3.2 we first study the performance of the scheme proposed in Chapter 2 through simulations. In section 3.3 we present two other combination schemes and compare the performance of the original scheme with the newly proposed schemes.

Notation: We adopt the following notations. The lower case letter denotes a scalar, the bold lower case letter denotes a vector, bold upper case letter denotes a matrix. $\mathbf{I}_n$ denotes an $n \times n$ identity matrix. $\mathbf{0}_{m\times n}$ denotes a $m \times n$ dimensional matrix of zeros. $\mathcal{N}_c(0, \sigma^2)$ denotes the complex Gaussian distribution with zero-mean and σ^2 variance. $\mathbb{E}\{X\}$ denotes the expected value of X.

3.2 Effect of decoding and retransmission thresholds on the performance

An ad-hoc network of uniformly distributed nodes in a circular region, with power distributed uniformly among all nodes, is considered for the simulations. A Rayleigh flat fading channel and a path loss exponent $\alpha = 2$ are used to model the channel between any pair of nodes. The pathloss model used is shown in Fig. 3.1. The levels are formed as described in the previous sections. Monte Carlo methods were used for the simulations. The achievable rate for the proposed scheme is calculated as the average of $\frac{1}{T} \times log_2(1 + \tau_2)$ where T is the number of slots the message is forwarded. T is random and depends on the SNR threshold, power density and number of nodes. This section gives a brief account of different

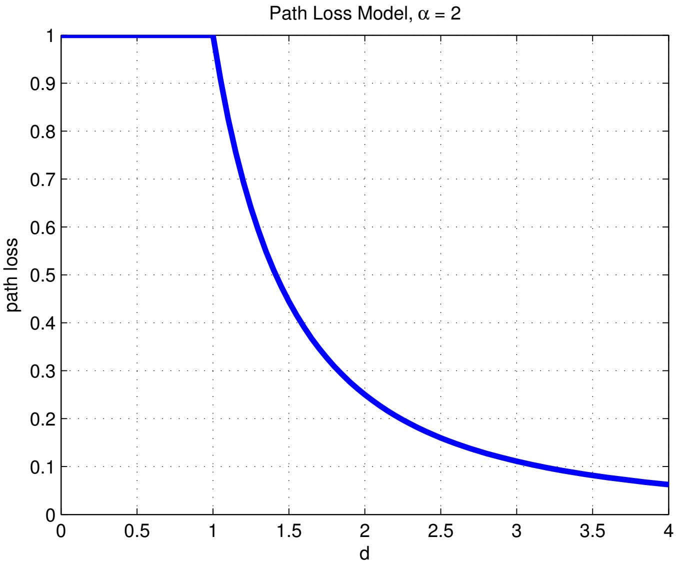

Figure 3.1. Path loss model, $\alpha = 2$

simulations done.

First the case with $\tau_1 = \tau_2 = \tau$ is considered. Each relay first receives a copy of the signal transmitted by the source. Then the relays which have the $SNR \geq \tau$ transmit. The rest of the nodes save the copy of the signal and continue listening and accumulating. This process is followed by each relay till it satisfies the retransmission criterion in (2.29). The message is assumed to be successfully received if the SNR of the combined signal is greater than equal to τ and the corresponding node is deemed successful.

Fig. 3.2 shows the total number of successful nodes as a function of the threshold, τ, for different number of accumulation levels, M.

Fig. 3.3 shows the broadcast rate as a function of τ. Fig. 3.4 shows the broadcast rate as function of τ, in the low threshold regime. Fig. 3.5 shows the average number of slots the message was forwarded, $\bar{T}$, as a function of τ. Fig. 3.6 shows the number of successful nodes versus the broadcast rate. The network parameters are power per unit area, $\bar{P} = 13\ watts$, total number of nodes, $N = 100$ in a radius, $r = 8\ m$. As can be seen in Fig. 3.2, the number of successful nodes

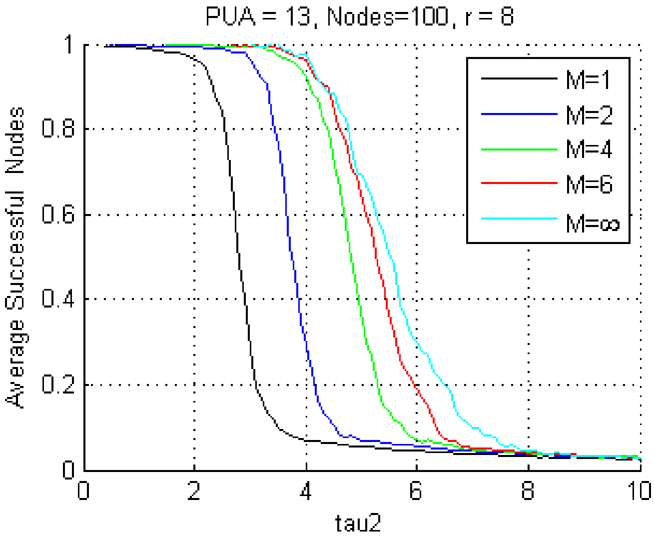

Figure 3.2. Fraction of successful nodes vs. τ for different M

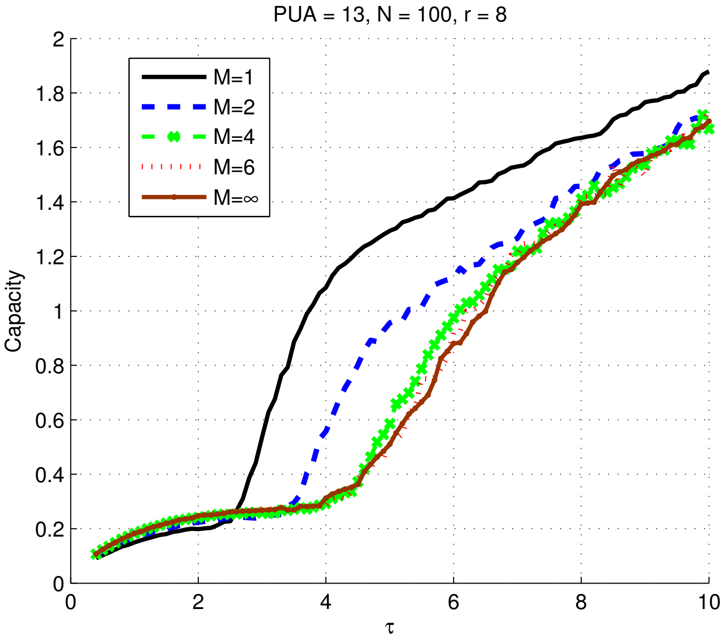

Figure 3.3. Capacity vs. τ for different M

increases with the increase of M.

Fig. 3.5 shows that this increase in total number of transmissions is obtained in more number of slots leading to a lower capacity in high threshold regions, reflected in Fig. 3.3 and Fig. 3.6. The combination of previous level signals gives a better performance in terms of both total number of transmissions and capacity in the low threshold regions as shown in Fig. 3.4.

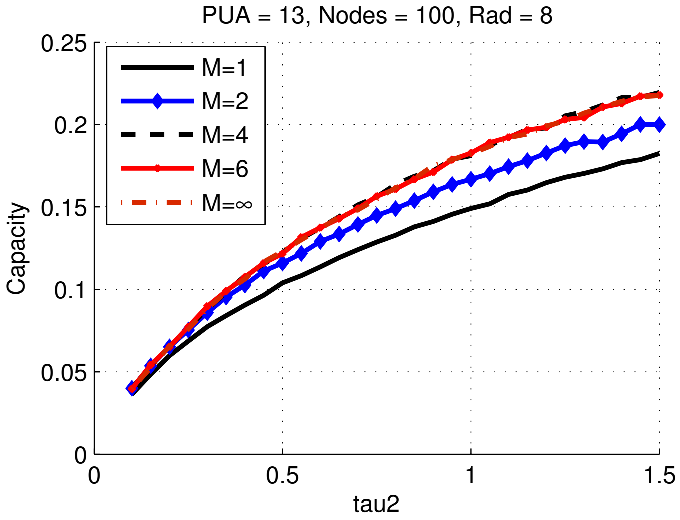

Figure 3.4. Zoomed version of capacity vs. τ in low τ region

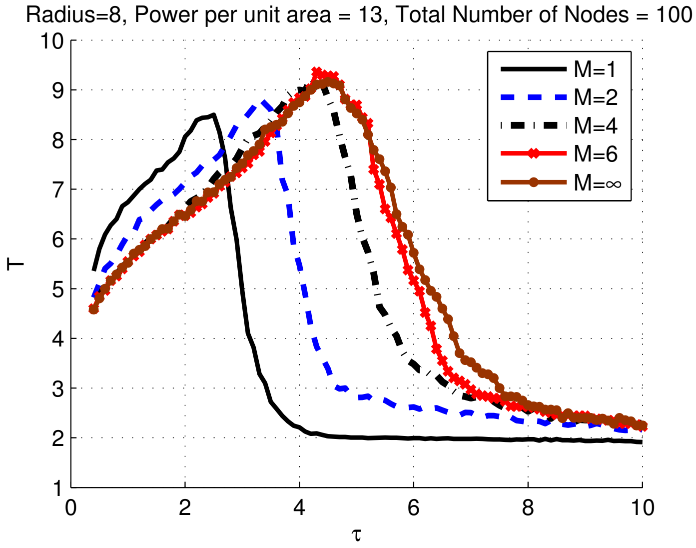

Figure 3.5. Number of slots the message is forwarded vs. τ

The next simulation is performed to observe the effect of radius. Fig. 3.7 shows the number of successful nodes as a function of broadcast rate for radius, $r = 2\ m$, and power per unit area, $\bar{P} = 100\ watts$ with the same number of nodes $N = 100$. The interesting thing to notice is that the number of nodes successfully receiving the message is decreasing abruptly even for the low SNR thresholds for the higher values of M. And hence the broadcast rate is lower for higher values of M. This abrupt transition resembles the phase transition effect observed in [19] for high density networks.

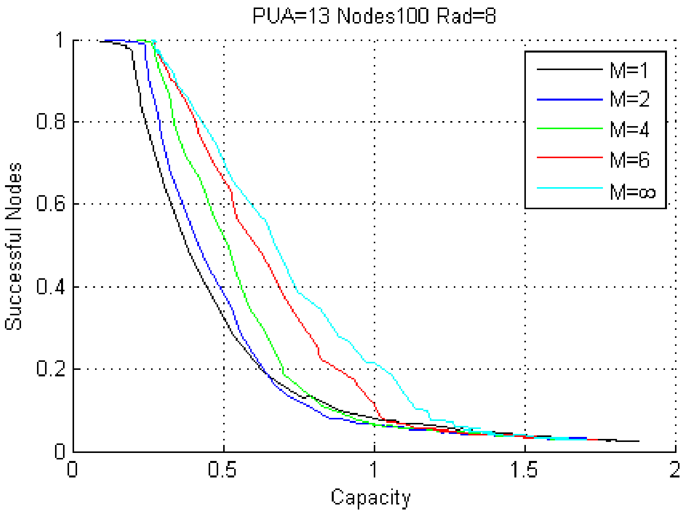

Figure 3.6. Fraction of successful nodes vs. capacity with $r = 8$

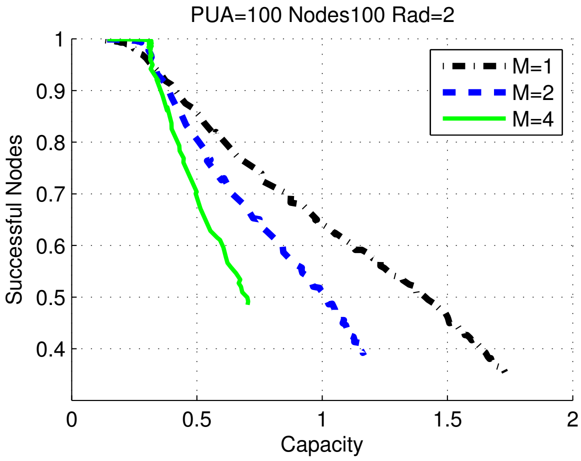

Figure 3.7. Fraction of successful nodes vs. Capacity for $r = 2$

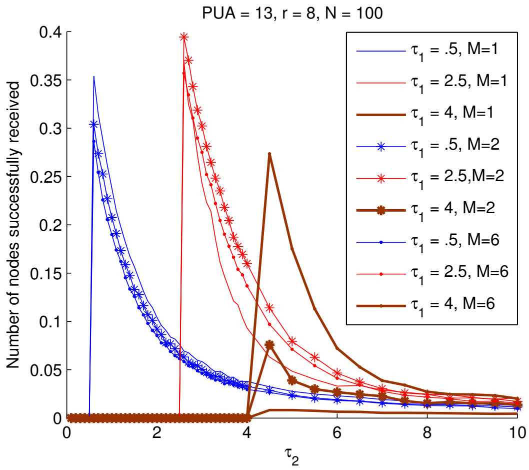

Figure 3.8. Fraction of successful nodes vs. τ_2

The setup considered next is the case for different values of τ_1 and τ_2. Fig. 3.8 shows the simulation results of a network with the number of nodes $N = 100$, $\bar{P} = 13$ *watts* and radius $r = 8$ *m*. As can be seen, for lower values of τ_2, $M = 1$, outperforms the cases where the nodes accumulate signals from the previous levels. But the results change when a higher value of τ_2 is selected. The number of nodes that successfully receive the message increases with M in the high τ_2 region. Notice that for each τ_2, the nodes do not successfully decode till $\tau_2 \geq \tau_1$.

3.3 Comparison of performance with other schemes

In this section we first propose two new schemes. In Section 3.3.1 we will study the importance of whitening before combining the signal. We will see that without whitening the performance of the system is drastically reduced. In Section 3.4 we will study an optimum signal combination scheme. We will also compare the performance of these schemes with the original scheme.

3.3.1 Signal combination without whitening

In this section we study the effects of combining the signals without whitening. A similar system model presented in Section 2.2 is used. The signals are combined by the matched filtering operation (see Fig. 3.9). Note that the signal processing at each node is same as in Fig. 2.3 except that the whitening block is removed.

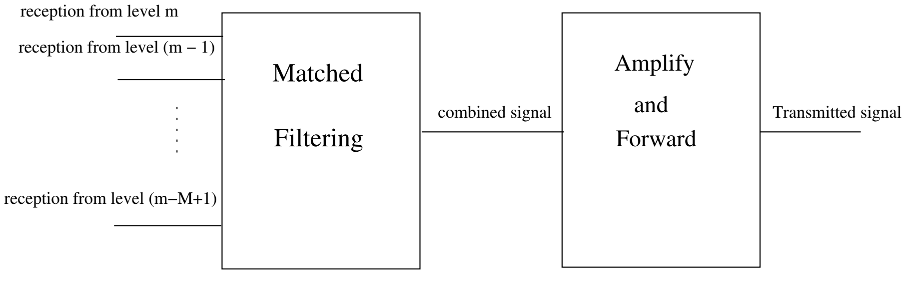

Figure 3.9. Relay processing during m'th time slot, without whitening

3.3.2 Effective channel and noise models

We now study the signal propagation when the signals are combined without whitening and the resulting channel model. Following a similar procedure used in Section 2.3 we find the signal that the level-1 nodes retransmit is given by

$$t_i \;=\; \sqrt{\beta_i}\frac{\mathbf{h}_i^{(0)*}}{|\mathbf{h}_i^{(0)}|}r_i[0] = \sqrt{\beta_i}\frac{\sum_{m\in S_0} h_{im}^{*}\sqrt{P_m}}{|\sum_{m\in S_0} h_{im}\sqrt{P_m}|}r_i[0]. \tag{3.1}$$

where $\beta_i = \frac{P_i}{|\mathbf{h}_i^{(0)}|^2+N_0}, \quad i \in S_1$.

After the transmission of level-1 nodes, the received signal vector at a node $k \in S_2$ is $\mathbf{r}_k^{(1)} = [r_k[0]\ r_k[1]]^T$. We can write $\mathbf{r}_k^{(1)}$, for all $k \in S_2$ as

$$\mathbf{r}_k^{(1)} = \begin{bmatrix} r_k[0] \\ r_k[1] \end{bmatrix} = \mathbf{h}_k^{(1)}x_0 + \mathbf{A}_k^{(1)}\mathbf{w}_k^{(1)} \tag{3.2}$$

where

$$\mathbf{h}_k^{(1)} = \begin{bmatrix} \sum_{m\in S_0} h_{km}\sqrt{P_m} \\ \sum_{i\in S_1}|\sum_{m\in S_0} h_{im}\sqrt{P_m}|h_{ki}\sqrt{\beta_i} \end{bmatrix} \tag{3.3}$$

and $\mathbf{A}_k^{(1)} := [\ \mathbf{I}_2 \mid \mathbf{B}_k^{(1)}\ ]$ where

$$\mathbf{B}_k^{(1)} := \begin{bmatrix} 0 & \dots & 0 \\ \mathbf{a}_k^{(s_{11})} & \dots & \mathbf{a}_k^{(s_{1N_1})} \end{bmatrix}, \tag{3.4}$$

with

$$\mathbf{a}_k^{(s_{1i})} = \frac{\sum_{m\in S_0}\sqrt{P_m}h_{s_{1i}m}^{*}\sqrt{\beta_{s_{1i}}}h_{ks_{1i}}}{|\sum_{m\in S_0}\sqrt{P_m}h_{s_{1i}m}|}, \quad i = 1\dots N_1. \tag{3.5}$$

The noise vector $\mathbf{w}_k^{(1)}$ is given by:

$$\mathbf{w}_k^{(1)} = [w_k[0]\ \ w_k[1]\ \ \mathbf{w}_{S_1}^T]^T,$$

with $\mathbf{w}_{S_1}$ defined as

$$\mathbf{w}_{S_1} := [w_{s_{11}}[0] \quad \ldots \quad w_{s_{1N_1}}[0]]^T. \tag{3.6}$$

Here $w_{s_{1i}}[0]$ denotes the receiver noise at node s_{1i} belonging to level-1 during time-slot 0.

In the time slot 2, the i'th node in second level, S_2, transmits the signal t_i $(\forall i \in S_2)$

$$t_i = \sqrt{\beta_i}\frac{\mathbf{h}_i^{(1)H}}{\sqrt{\mathbf{h}_i^{(1)H}\mathbf{h}_i^{(1)}}}\mathbf{r}_i^{(1)} \tag{3.7}$$

The rest of the nodes receive, $\mathbf{r}_k^{(2)}$, $\forall k \in S_3$, which is given by,

$$\mathbf{r}_k^{(2)} = \begin{bmatrix} r_k[0] \\ r_k[1] \\ r_k[2] \end{bmatrix} = \mathbf{h}_k^{(2)}x_0 + \mathbf{A}_k^{(2)}\mathbf{w}_k^{(2)} \tag{3.8}$$

where

$$\mathbf{h}_k^{(2)} = \begin{bmatrix} \sum_{i\in S_0} h_{ki}\sqrt{P_i} \\ \sum_{i\in S_1} |\sum_{m\in S_0} h_{im}\sqrt{P_m}| h_{ki}\sqrt{\beta_i} \\ \sum_{i\in S_2} h_{ki}\sqrt{\beta_i}\sqrt{\mathbf{h}_i^{(1)H}\mathbf{h}_i^{(1)}} \end{bmatrix}, \tag{3.9}$$

$$\mathbf{A}_k^{(2)} = [\ \mathbf{I}_3 \mid \mathbf{B}_k^{(2)}\ ] \tag{3.10}$$

where

$$\mathbf{B}_k^{(2)} := \left[\begin{array}{ccc|c} 0 & 0 & 0 & \\ 0 & 0 & 0 & \mathbf{B}_k^{(1)} \\ \mathbf{a}_k^{(s_{21})} & \ldots & \mathbf{a}_k^{(s_{2N_2})} & \mathbf{g}_k^{(2)} \end{array}\right] \tag{3.11}$$

where

$$\mathbf{a}_k^{(s_{2i})} := h_{k(s_{2i})}\sqrt{\beta_{(s_{2i})}}\frac{\mathbf{h}_{(s_{2i})}^{(1)H}}{\sqrt{\mathbf{h}_{(s_{2i})}^{(1)H}\mathbf{h}_{(s_{2i})}^{(1)}}}, \tag{3.12}$$

and

$$\mathbf{g}_k^{(2)} := \sum_{i\in S_2} h_{ki}\sqrt{\beta_i}\frac{\mathbf{h}_i^{(1)H}\mathbf{B}_i^{(1)}}{\sqrt{\mathbf{h}_i^{(1)H}\mathbf{h}_i^{(1)}}}. \tag{3.13}$$

The noise vector can be written as

$$\mathbf{w}_k^{(2)} = [w_k[0] \ w_k[1] \ w_k[2] \ \mathbf{w}_{S_2}^T \ \mathbf{w}_{S_1}^T]^T, \tag{3.14}$$

where

$$\mathbf{w}_{S_2} := [w_{s_{21}}[0] \quad w_{s_{21}}[1] \quad \ldots \quad w_{s_{2N_2}}[0] \quad w_{s_{2N_2}}[1]]^T \tag{3.15}$$

and $\mathbf{w}_{S_1}$ is given in Eqn. (3.6).

The recursive formulation, for $m \geq 2$, is given as follows:

$$\mathbf{r}_k^{(m)} = \mathbf{h}_k^{(m)} x_0 + \mathbf{A}_k^{(m)} \mathbf{w}_k^{(m)}. \tag{3.16}$$

The effective channel vector can be written as

$$\mathbf{h}_k^{(m)} = \begin{bmatrix} \mathbf{h}_k^{(m-1)} \\ \\ h_k[m] \end{bmatrix}, \tag{3.17}$$

where

$$h_k[m] = \sum_{i \in S_m} \sqrt{\mathcal{P}_i^{(m-1)}} \ h_{ki} \sqrt{\beta_i} \tag{3.18}$$

$$\mathcal{P}_i^{(j)} = \mathbf{h}_i^{(j)H} \mathbf{h}_i^{(j)}. \tag{3.19}$$

Define

$$\mathbf{V}^{(m)} = \begin{cases} \mathbf{I}_{(m)} & \text{if } m < M \\ \left[\mathbf{0}_{(M-1)\times 1} \mid \mathbf{I}_{(M-1)}\right] & \text{if } m \geq M \end{cases} \tag{3.20}$$

We can write $\mathbf{A}_k^{(m)}$ as

$$\mathbf{A}_k^{(m)} = [\mathbf{I}_{D(m)} \mid \mathbf{B}_k^{(m)}], \tag{3.21}$$

where $D(m)$ is given by Eqn. (2.23).

$$\mathbf{B}_k^{(m)} = \left[\begin{array}{cccc|c} \mathbf{0} & \mathbf{0} & & \mathbf{0} & \mathbf{V}^{(m)}\mathbf{B}_k^{(m-1)} \\ \mathbf{a}_k^{(s_{m1})} & \dots & & \mathbf{a}_k^{(s_{mN_m})} & \mathbf{g}_k^{(m)} \end{array}\right] \tag{3.22}$$

$$\mathbf{a}_k^{(s_{mi})} = h_{k(s_{mi})}\sqrt{\beta_{(s_{mi})}}\frac{\mathbf{h}_{s_{mi}}^{(m-1)}}{\sqrt{\mathcal{P}_{s_{mi}}^{(m-1)}}}, \tag{3.23}$$

$$\mathbf{g}_k^{(m)} = \sum_{i\in S_m} h_{ki}\sqrt{\beta_i}\frac{\mathbf{h}_i^{(m-1)}\mathbf{B}_i^{(m-1)}}{\sqrt{\mathcal{P}_i^{(m-1)}}}, \tag{3.24}$$

$$\beta_i = \frac{P_i}{\mathcal{P}_i^{(m-1)} + N_0}$$

3.3.3 Retransmission and decoding criterion

We use the SNR threshold criterion proposed in Section 2.4 to decide whether the node will transmit or not. The node transmits if the SNR of the accumulated signal is greater than or equal to the retransmission threshold (See Fig. 3.9). We assume that the SNR is perfectly estimated at all the nodes. The SNR, $\gamma_k[m]$ can be found as:

$$\gamma_k[m] = \frac{\mathbf{h}_k^{(m)H}\mathbf{h}_k^{(m)}}{\mathcal{P}_{(noise)}} = \frac{\mathcal{P}_k^{(m)}}{\mathcal{P}_{(noise)}}, \tag{3.25}$$

$\mathcal{P}_{(noise)}$ can be calculated as

$$\begin{aligned} \mathcal{P}_{(noise)} &= \mathbb{E}\left\{\left(\frac{\mathbf{h}_k^{(m)H}\mathbf{A}_k^{(m)}\mathbf{w}_k^{(m)}}{\sqrt{\mathcal{P}_k^{(m)}}}\right)\left(\frac{\mathbf{h}_k^{(m)H}\mathbf{A}_k^{(m)}\mathbf{w}_k^{(m)}}{\sqrt{\mathcal{P}_k^{(m)}}}\right)^H\right\} & (3.26) \\ &= \frac{\mathbf{h}_k^{(m)H}\left(\mathbf{A}_k^{(m)}\mathbf{A}_k^{(m)H}\right)\mathbf{h}_k^{(m)}}{\mathcal{P}_k^{(m)}}N_0 & (3.27) \end{aligned}$$

where $\mathbf{h}_k$ and $\mathbf{A}_k$ is obtained via the recursive formulation obtained in the previous section and $\mathcal{P}_k^{(m)}$, is given by Eqn. (3.19). A node retransmits in the m^{th}-level

$(\ell(k) = m)$ if

$$\{\gamma_k[m] \geq \tau_1\} \cap \{\gamma_k[n] < \tau_1 \, \forall n < m\}, \tag{3.28}$$

where τ_1 is the retransmission threshold and τ_2 is the decoding threshold. The node is able to decode if $\tilde{\gamma}_k[m] \geq \tau_2$. The $\tilde{\gamma}_k[m]$ can be obtained similar to $\gamma[m]$.

3.4 Optimum signal combination

In this section we study an optimum combination technique which combines signals which give the maximum SNR. The processing at each node is shown in Fig. 3.10.

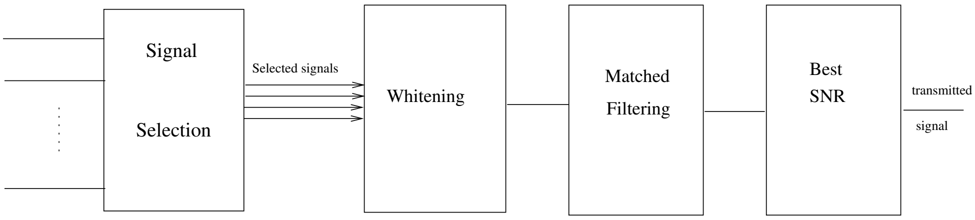

Figure 3.10. Optimum relay processing during m'th time slot

The "signal selection" block selects different combinations of the M signals in the memory. The selected signals are whitened and are processed by a matched filter. The "best SNR" block stores the SNR and the "label" of all the combinations and outputs a combined signal which has the best SNR. This technique is optimum in the sense that the node transmits the signal which has the maximum SNR. This maximum SNR should also satisfy the retransmission criterion.

We now compare the combination methods proposed in Chapter 2 with the methods studied in Sections 3.3.1 and 3.4. Each node receive a copy of the signal from the previous level. The SNR at each node is calculated based on the kind of processing being used at the node according to Eqn. (2.28) or Eqn. (3.25).

We consider a setup similar to the one described in Section 3.2. The nodes are assumed to combine signals from 4 previous levels, *i.e.,* the nodes have a memory of

$M = 4$.

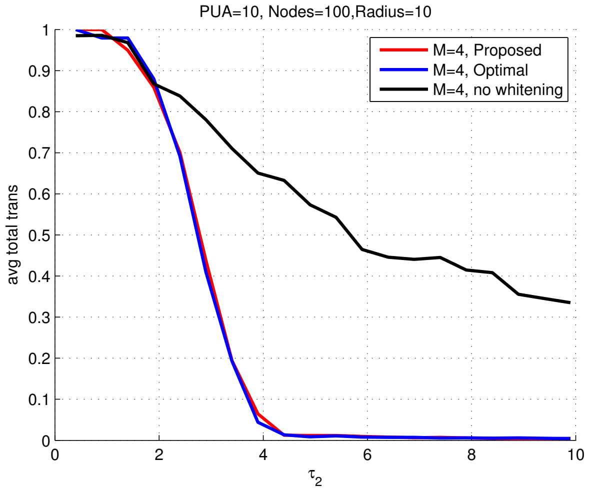

Figure 3.11. Comparison of schemes: fraction of total successful nodes vs τ_2

In Fig. 3.11 the fraction of total number of successful nodes in each scheme is compared. Fig. 3.12 shows the comparison of average number of slots the message propagates.

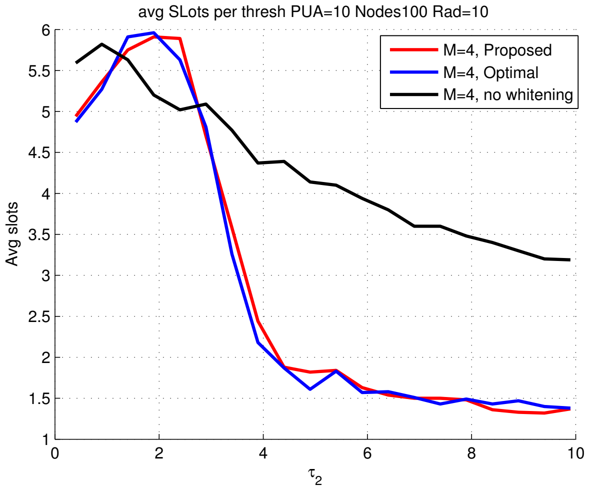

Figure 3.12. Comparison of schemes: average number of slots vs τ_2

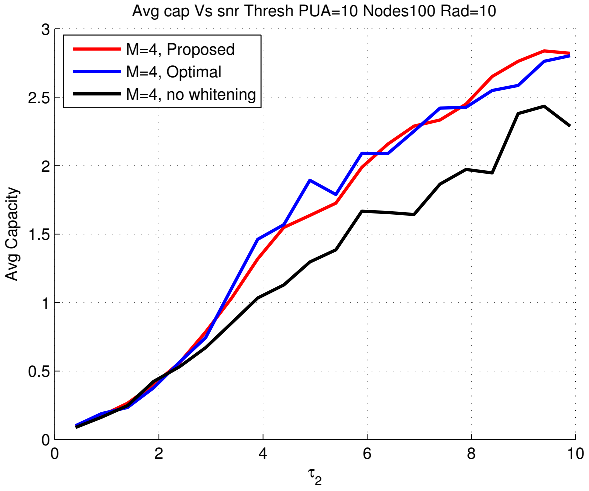

Figure 3.13. Comparison of schemes: broadcast rate vs τ_2

We see in Figures 3.11, 3.12, and 3.13 that the plots for the optimum signal combination studied in Section 3.4 and the processing technique studied in Chapter 2 exactly overlap. Hence by combining all the M signals in memory we obtain a signal with maximum SNR. We can also observe that the performance is reduced at higher SNR‘s when the signal combination without whitening is adopted. Thus the proposed scheme transmits the signal with maximum SNR and in that sense it is optimal.

CHAPTER 4

CONCLUSION

In this thesis we proposed a cooperative broadcast scheme which uses amplify and forward relays. We studied different combination schemes through recursive formulation and simulations.

In Chapter 2, the effective channel and the accumulated noise are recursively characterized. For a specific network realization and for a particular level division, the conditional channel statistics were derived. These recursive formulations are useful in simulating the performance of this scheme. However finding the theoretical limitations on the achievable broadcast rate still remains an open problem. The recursive formulations obtained in this chapter can be used to theoretically analyze the performance of the scheme in the asymptotic regime.

In Chapter 3, we studied different types of signal processing at the nodes. We analyzed the performance of the scheme proposed in Chapter 2 and compared it with two other combination schemes. An interesting observation is that at high SNR regions the total number of successful nodes increases with increase in M. However this increase also leads to decrease in the rate of transmission. Another observation is that the to obtain a combined signal with maximum SNR, the nodes have to combine all the M signals in the memory. Also seen in this chapter is the importance of whitening if the signals before matched filtering. We have seen that without whitening the performance of the system decreases drastically.

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