January 1997

Analytical Models for Vehicle/Gap Distribution on Automated Highway Systems

Jacob Tsao
San Jose State University, jacob.tsao@sjsu.edu

Randolph W. Hall
University of Southern California

Indrajit Chatterjee
University of California - Berkeley

Analytical Models for Vehicle/Gap Distribution on Automated Highway Systems[1]

H.-S. JACOB TSAO

Institute of Transportation Studies, University of California, Berkeley, California 94720

RANDOLPH W. HALL

Department of Industrial and Systems Engineering, University of Southern California, Los Angeles, California 90089

INDRAJIT CHATTERJEE

Department of Industrial Engineering and Operations Research, University of California, Berkeley, California 94720

Highway congestion has in recent years become a pervasive problem for urban and suburban areas alike. The concept of Automated Highway Systems is based on the belief that integration of sensing, communication, and control technologies into vehicles and highways can lead to a large improvement in capacity and safety without requiring a significant amount of additional highway right-of-way. A fundamental determinant of Automated Highway Systems capacity is the vehicle-following rule, the rule that governs the behavior of vehicles traveling along a common lane (e.g., the spacing between any two longitudinally adjacent vehicles). Vehicle following affects the longitudinal capacity (achievable flow within a lane), the lateral capacity (achievable flow between lanes) and the conflicting relationship between the longitudinal flow and lateral capacity. The issues are investigated by developing probabilistic models for vehicle/platoon and gap distributions, for vehicles that travel in platoons, in slots, or as free-agents. Mathematical models are also developed to estimate the completion time of a lane change, which can be used as a surrogate for the lateral capacity. Numerical results for the three major vehicle-following rules and their comparison are also provided.

Highway congestion has in recent years become a pervasive problem for urban and suburban areas alike. The amounts of lost time, highway fatalities and injuries, and air pollution are no longer acceptable. The traditional approach to meeting the demand for automobile travel is to expand existing highways and/or build more highways. However, given the saturation of land dedicated to existing highways and the difficulty in acquiring private land for highway expansion and construction, this traditional approach is becoming prohibitive. The concept of Automated Highway Systems (AHS) is based on the belief that an appropriate integration of sensing, communication, and control technologies placed on the vehicle and on the highway can significantly decrease the average longitudinal spacing between vehicles and hence lead to a large improvement in capacity and safety without requiring a significant amount of additional right-of-way. Stemming from this belief are various conceptual scenarios for vehicle/highway automation. For a brief introduction to major AHS design options and issues, see TSAO, HALL, and SHLADOVER (1993). This article focuses on the fully automated AHS, i.e., those that

[1]Accepted by Mark S. Daskin.

0041-1655/97/3101-0018 $05.00

enable "hands-off" and "feet-off" driving on dedicated lanes.

AHS Capacity

An AHS consists of two major components: vehicle/highway automation technology and highway operating strategy. This article concentrates on the operating strategy and assumes the feasibility of the automation technology that supports it. The desirability of an AHS hinges on its performance. Crucial performance categories of AHS operation include safety, capacity, and comfort [TSAO, HALL, and SHLADOVER (1993); TSAO et al. (1993)]. This article focuses on AHS capacity.

The definition of capacity and its calculation for AHS with traffic needing no lane changes is relatively straightforward. In such a case, capacity of a lane can be defined and measured as the maximum achievable flow, subject to safety constraints, of vehicles per lane per hour. Note that this definition does not reflect at all the *lateral flow*—the flow of vehicles between lanes. When lane changes are required, this definition no longer suffices.

Consider an example AHS that consists of two automated lanes. Suppose that a vehicle can change lanes only if it encounters a sufficiently large gap in the destination lane. Also suppose that there is a nonzero speed differential between the two lanes. Then, when the longitudinal flow in one lane is maximized by packing the lane with vehicles, no vehicles can change into the lane. On the other hand, if there is no traffic at all in that lane, lane-changing incurs no waiting. This conflicting and non-linear relationship between the longitudinal flow and lateral capacity must be explicitly considered in predicting the capacity of an AHS. This article results from an attempt to study this relationship. We use the time required for a successful lane change (or *lane-change completion time* for short), without any interference by other lane change maneuvers, to represent lateral capacity. The exact definition of a lane-change maneuver may vary, depending on how the AHS is operated. Invariably, a lane-change maneuver is initiated as the vehicle decides to begin a sequence of steps that culminates in the lateral movement. A possible sequence consists of the following four steps: (i) using sensors or communication to determine if a sufficiently large gap in the destination lane is adjacent or nearby and approaching, (ii) establishing communication with nearby vehicles, if any, to express the desire to change lanes, (iii) negotiation with those vehicles for the use or creation of a gap in the destination lane and for a safe lateral movement from the origin lane, and (iv) the actual lateral movement. The lane change completion time is the difference between the time a vehicle initiates the lane-change maneuver (i.e., the beginning of step (i)) and the time it has completed the lateral movement into the gap (i.e., the end of step (iv)).

A Fundamental AHS Operating Rule: Vehicle-Following Rule

The vehicle-following rule governs the behavior of vehicles traveling along a common lane, particularly the spacing between any two longitudinally adjacent vehicles. Two basic vehicle-following rules are the *platooning rule* and the *free-agent rule*. (Both have several variations.) The platooning rule was first proposed and studied by SHLADOVER (1979) and has received renewed interest in the last few years. Under this rule, longitudinally adjacent vehicles either travel very close to, or very far from, each other. As a result, vehicles are organized in a clustered formation. Each cluster of vehicles is called a platoon. The large interplatoon spacing minimizes the probability of collisions between platoons and the short intraplatoon spacing ensures that any initial collision within a platoon will have a small relative speed and, presumably, low severity. Under the free-agent rule, vehicles move without any clustered formation and the minimum longitudinal spacing is significantly longer than typical intraplatoon spacings, but significantly shorter than typical interplatoon spacings. Relative to the platooning rule, the free-agent rule reduces the overall frequency of collisions, but potentially increases the frequency of severe ones. An AHS with the free-agent rule may be easier to operate. For an introduction to the platooning rule and its impact on AHS traffic control, refer to VARAIYA and SHLADOVER (1992) and VARAIYA (1993). TSAO and HALL (1993) developed a probabilistic model to study the probability and severity of a collision between two longitudinally adjacent vehicles after the front vehicle decelerates abruptly. They also applied the model to compare the safety of these two vehicle-following rules. Using computer simulation, HITCHCOCK (1994) reported a parametric study of the probability and severity of multiple collisions resulting from the abrupt deceleration by a vehicle in a platoon.

An operating strategy consists of a collection of operating rules, such as access, vehicle following, lane selection, lane change, merging, and egress rules. All of these rules have an effect on AHS capacity. Estimation of AHS capacity under platooning has received some attention in the literature. SHLADOVER (1979, 1991) estimated AHS capacity by scaling down the longitudinal capacity achievable without lane changes by 20%. Recognizing that vehicle's entry into and egress from a platoon are a primary cause of traffic stream disturbance, RAO, VARAIYA, and ESKAFI (1993) investigated different entry/exit strategies. The focus of their effort is the achievable longitudinal flow. With minimal interaction between traffic entering the automated lane and the traffic exiting it, exiting success rates were also simulated for different combinations of highway configuration and traffic demand. TSAO, HALL, and HONGOLA (1993) simulated AHS traffic and investigated the effect of platooning and lane barriers on the exit success rate, sustainable flow, and traffic stability. RAO and VARAIYA (1994) proposed a roadside controller design to optimize longitudinal flow along a stretch of automated highway. Each controller operates over a few-kilometer segment of automated highway and requires only simple information about traffic conditions in its vicinity and a small amount of information from the next controller downstream.

HALL (1995) used a workload model, where workload and highway capacity are defined in terms of a time–space product, to show that, under certain simplifying conditions, an excessive amount of lane changes or an excessive amount of workload (time–space) requirement per lane change may render the inner lanes under-utilized or even unused. His model covers the two major AHS vehicle-following rules as special cases. TSAO, HALL, and HONGOLA (1993) simulated AHS traffic under the free-agent vehicle-following rule. In a different direction, RAO and VARAIYA (1993) studied the achievable capacity and traffic stream stability when only a portion of vehicles on a highway use the Autonomous Intelligent Cruise Control technology–a partial automation technology.

Purpose of the Article

We develop probabilistic models for vehicle/gap distributions under two variations of the free-agent (vehicle-following) rule and for platoon-size and interplatoon-gap distributions under platooning. We also develop mathematical models to estimate the lane-change completion time. Numerical examples are given to illustrate the theory. These models can be used to estimate the lateral capacity of different operating scenarios. These analytical models, when coupled with models for lane assignment, can provide analytical capacity estimates for many AHS operating scenarios.

General Assumptions

No/Minimum Cooperation for Lane Change

We focus on the traffic on a single lane into which a vehicle in a neighboring lane desires to change lanes. For the free-agent rule, we assume that vehicles do not alter their speeds to facilitate lane changing by other vehicles. In other words, they do not cooperate for lane-changing. This is motivated by flow stability as well as technological simplicity. Therefore, the only way to complete a lane change is for the lane-change vehicle to encounter a sufficiently large gap.

The concept of platooning inherently assumes a certain degree of cooperation among neighboring vehicles. For example, when a vehicle in the middle of a platoon needs to change lanes, the platoon needs to isolate the lane-change vehicle from the rest of the platoon by creating a space in front and in rear of the vehicle for a safe lateral maneuver, which requires a speed change by the other vehicles in the platoon. To minimize the disturbance to the traffic on the destination lane, we require that a lane-changing vehicle join a platoon only at its front or rear end (so that the receiving platoon does not have to split and hence cause slow-down by the trailing vehicles).

Lane-Change Initiation Time Independent of Traffic Condition

For both free-agent and platooning rules, timing of lane change attempt is performed by the lane-change vehicle. Under both rules, an automated vehicle has limited ability to sense the traffic condition beyond its immediate vicinity on the destination lane. Therefore, timing of lane-change attempt is assumed to be independent of the traffic condition on the destination lane. (To simplify calculation of lane-change completion time for the examples to be given later, we will make minor assumptions about the exact position of the lane-change vehicle, relative to the adjacent vehicle or gap on the destination lane, at the initiation time of the lane-change attempt.)

Non-Zero Speed Differential

To increase the likelihood of encountering a sufficiently large gap, we also assume that the speed differential, δ, between the two lanes is non-zero. To simplify discussion, we assume a constant speed differential in space and time.

No Interference

Finally, we focus on a particular lane change and assume, for mathematical tractability, that no other lane changes interfere with it before its completion.

More precisely, the vehicle/gap distribution on the destination lane when the lane change was initiated remains the same until the completion of the lane change.

Organization of the Article

Section 1 considers two variations of the free-agent vehicle-following rule. After developing a vehicle/gap distribution model and a companion lane-change completion time model for each variation, it compares the performance of the two variations. Section 2 develops such models for the platooning vehicle-following rule and provides numerical results. Section 3 compares the performance of the two rules. Concluding remarks are given in Section 4.

1. GAP LENGTH DISTRIBUTION BETWEEN TWO FREE-AGENTS

We consider two variations of the free-agent rule and develop one vehicle/gap distribution for each. In both variations, a vehicle occupies a *slot* of length b consisting of (i) the maneuvering space of length h, including the space physically occupied by the vehicle and an additional space reserved to enable a lateral movement without affecting the traffic speed on either lane, and (ii) a supplemental spacing of length d half of which is "padded" onto each end of the maneuvering space for safety.

The motivation for allocating maneuvering space is that a vehicle can enter or exit a slot without disturbing either of the two traffic streams. More precisely, in such a model only the vehicle that changes lanes needs to change speed. We first describe the concept of maneuvering time τ, i.e., the time required for the lateral movement, and then use it to define the concept of maneuvering space.

If, during the lateral movement, the longitudinal acceleration/deceleration is constant and the lateral velocity is also constant, the maneuvering time is simply

$$\tau = \max\left\{\frac{\delta}{a}, \frac{w}{u}\right\}, \tag{1}$$

where δ = speed differential, a = constant acceleration/deceleration rate during lateral movement, w = lane width, u = constant lateral velocity during lateral movement. Note that these parameters can be adjusted so that the two quantities δ/a and w/u are equal, thus improving ride quality during the lateral movement. We assume in what follows that they are indeed equal. Also note that if (i) the magnitude of the acceleration rate of a vehicle changing from a slower lane into a faster lane is the same as the magnitude of the deceleration rate of a vehicle changing from a faster lane into a slower lane, and (ii) the lateral velocity is the same regardless of lane-changing direction, then the maneuvering time is independent of the direction of the lane change. We treat only the case where a vehicle on a faster lane tries to change into a neighboring slower lane. The opposite case is similar.

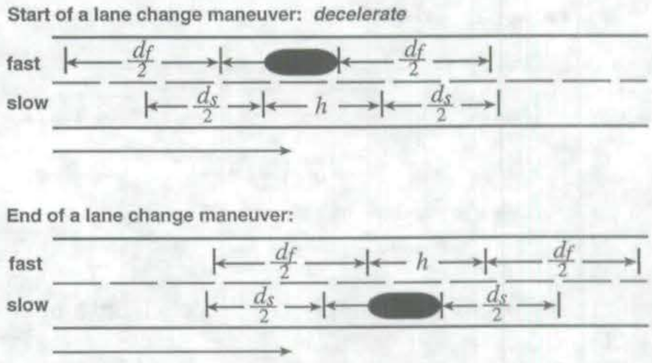

Fig. 1. Lane-changing with vehicle slots.

Slots of neighboring lanes should have a common maneuvering space. However, the safety spacing may depend on lane speed. We use Figure 1 to explain how vehicles make the lateral movement and what the maneuvering space means. One slot on each lane is shown and superscripts f and s indicate fast and slow lanes respectively. The maneuvering space is illustrated as the space between the two padded half safety spacings in a slot. When the lateral movement begins, the lane-changing vehicle should be at the front of the maneuvering space of the origin slot touching the front safety half spacing but it should be adjacent to the rear of maneuvering space of the destination slot with the rear bumper aligned with the rear end of maneuvering space of the destination slot. As the lateral movement progresses, the lane-changing vehicle slows down and drops back toward the rear end of the maneuvering space in the origin slot while catching up with the front end of the destination slot. When the lateral movement is completed, the lane-changing vehicle should be adjacent to the rear end of the maneuvering space in the origin slot but at the front end of the destination slot. This way, the lane-changing vehicle is never within a (longitudinal) safety spacing with respect to any vehicle in either lane throughout the lateral movement. The length of the maneuvering space is simply the vehicle length plus the distance, relative to the traffic on either lane, traveled by the lane-changing vehicle during the lateral movement. By the constant deceleration rate, the maneuvering time is δ/a. Since the average speed, with respect to the traffic on either lane, of the lane-changing vehicle is $\delta/2$, the length of the

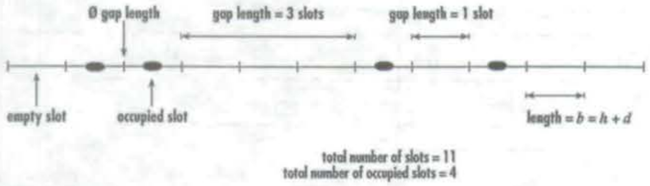

Fig. 2. Gaps in a slotted AHS.

maneuvering space is simply the vehicle length plus $\delta^2/2a$.

The first variation of the free-agent rule is based on the idea that a lane is partitioned into a number of moving slots and each slot is either occupied by a vehicle or empty. Therefore, the gap length can only be a non-negative integer multiple of the slot length. The second variation relaxes this assumption, and hence the corresponding technological requirements, and allows the gap length to be any non-negative real number.

1.1. Gap Length Distribution: Under the Slot Assumption

Consider a segment of an AHS lane that is partitioned into s slots of equal length b, where b is the sum of the supplemental spacing d for safety and the length h of the maneuvering space. A *gap* is defined to be the unoccupied space between two longitudinally adjacent occupied slots. Gap length is defined as the number of empty slots in the gap. There are v vehicles (occupied slots) in the lane, which are distributed in the s slots. In the absence of evidence favoring some distributions over others, we make the following assumption.

ASSUMPTION 1. *All (combinatorial) distributions of the v vehicles in the s slots are equally likely.*

This configuration is illustrated in Figure 2.

Independent Geometric Gap Length Distribution

Under this model, one can calculate the joint distribution of the $v - 1$ gap lengths as well as the marginal distributions. Furthermore, when the occupancy ratio, i.e., v/s, is kept constant while the length s of the segment is approaching infinity, it can be shown that (i) all the $v - 1$ gap lengths are independent and identically distributed, and (ii) the identical distribution is a Geometric Distribution with a success probability of v/s. Let L denote the gap length. The gap length probability function is simply

$$p(L = i) = \frac{v}{s}\left(1 - \frac{v}{s}\right)^i, \quad i = 0, 1, 2, \ldots . \qquad (2)$$

Lane-Change Completion Time

The probability function of the number of occupied slots passed by the lane-change vehicle, denoted by M, before encountering an empty slot is simply

$$p(M = i) = \left(1 - \frac{v}{s}\right)\left(\frac{v}{s}\right)^i, \quad \text{where} \quad i = 0, 1, 2, \ldots . \qquad (3)$$

When a vehicle initiates a lane change, its position relative to the neighboring slot in the destination lane may not allow a safe lane change even if the slot is empty. For simplicity, we assume that, at the time of initiating a lane-change attempt, the vehicle is properly positioned next to a slot in the destination lane so that it can move into the slot safely as long as the slot is empty.

The lane-change completion time T is the sum of the waiting time and τ, i.e., the maneuvering time. By the assumption of constant speed differential,

$$T = \frac{M \times b}{\delta} + \tau, \qquad (4)$$

and the probability function of T is

$$p\left(T = \frac{i \times b}{\delta} + \tau\right) = \left(1 - \frac{v}{s}\right)\left(\frac{v}{s}\right)^i, \quad i = 0, 1, 2, \ldots . \qquad (5)$$

Recall that the length of a maneuvering space is an increasing and quadratic function of the speed differential between the two lanes. An increase in speed differential implies a decrease in the number of slots. However, given the same vehicle/gap distribution, a higher speed differential implies a shorter time to encounter an empty slot for lane-changing. This trade-off will be illustrated in the following examples.

Examples

Consider an AHS with two automated lanes operating under the slot assumptions. We study, for many different combinations of parameter values, the vehicle/gap distribution on the slower lane and the lane-change completion time from the faster lane to the slower lane. A uniform vehicle length l is assumed and is set at $5m$. We set the safety spacing d between two longitudinally adjacent vehicles in the slower lane at $10m$. Let the speed of the slower lane be fixed at v = 100 km/hr (27.8 m/s). We consider 5 different δs: 1, 2, 3, 4, 5 m/s. The lane width and the lateral velocity during the lateral movement are fixed at $4m$ and $2m/s$ respectively. The deceleration rate a of the lane-changing vehicle

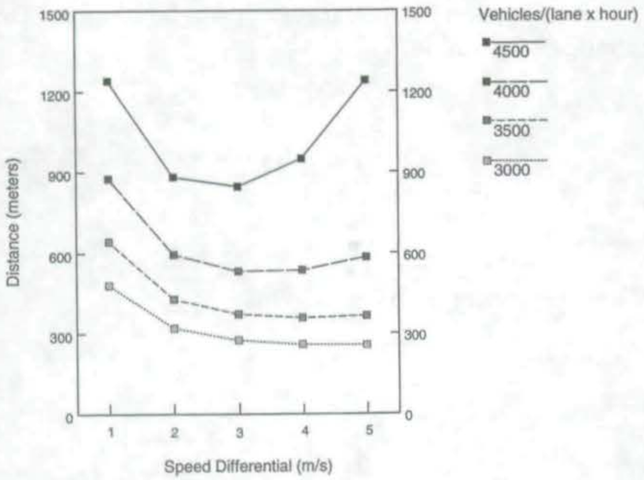

Fig. 3. Mean of lane-change completion distance for slotting.

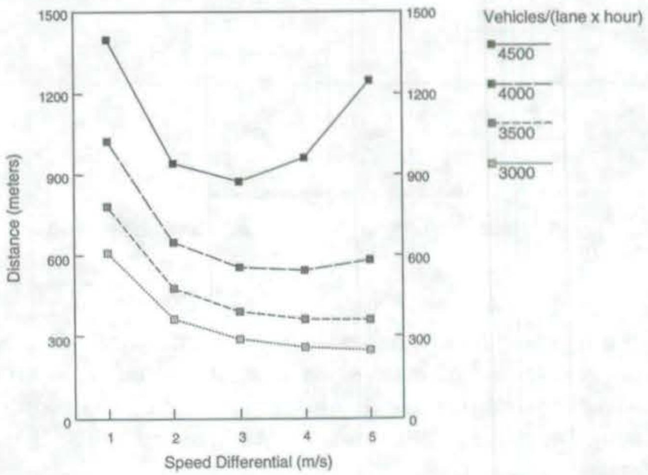

Fig. 4. Standard deviation of lane-change completion distance for slotting.

is initially set to a fixed value of $0.3g$. ($1g = 9.8$ m/s^2.) Four different traffic flows—3000, 3500, 4000, and 4500 vehicles/(lane × hour)—on the slower lane are considered.

At all the chosen parameter values, the time needed for the lateral movement w/u exceeds the time δ/a. To achieve a smoother ride, we assume a constant deceleration during the entire lateral movement. Therefore, the lateral movement and the speed change are completed at the same time. Note that the resulting constant deceleration rate is smaller than a. The slot length is therefore simply $l + d + (\delta/2) \times \tau$. The four flows, together with the speed and the slot length, determine four different vehicle/gap distributions. Based on the vehicle/gap distribution and the speed differential, we calculate the probability distribution of the distance traveled until a successful lane change. Note that in this and the examples given in the rest of this article, we convert the lane-change completion time to the corresponding distance for easier interpretation.

The results are shown in Figures 3 and 4, which contrast the mean and the standard deviation among the four different flows respectively. At higher flow levels, the lane-change completion distance grows fast with respect to flow. At flow levels of 3500–4500 and in the interval [2, 4], both the mean and the standard deviation of the distance increase by approximately 50% at every increment of 500 in flow. Note that the ratio of standard deviation over mean is close to 1, which indicates a large amount of variability.

These two figures also illustrate the trade-off between the decrease in the number of slots and the decrease in distance needed to encounter an empty slot, both resulting from an increase of δ. At the high flow level of 4500, the trade-off is clear and the "best" δs, in terms of both the mean and the standard deviation, are between 2 and 4 m/s. For the three lower flow levels, the lane-change completion distance appears insensitive to the change in δ between 2 and 5 m/s. For all four flow levels, a speed differential of 1 m/s leads to a much higher lane-change completion distance than the best speed differential. In general, the optimal speed differential declines as flow increases. This is because higher flows result from smaller slots, which necessitate smaller speed differentials.

1.2. Gap Length Distribution: Continuous Length

We consider a different variation of the free-agent rule in this subsection. Consider a segment of an AHS lane of length S with v vehicles each occupying a slot of length $b = d + h$, where d is the supplemental safety spacing and h is the length of the maneuvering space. However, in this model the whole highway segment as well as the unoccupied space between any two occupied slots are no longer partitioned into a sequence of adjacent slots. Consequently, the gap length is defined to be the actual distance between two longitudinally adjacent occupied slots. This variation relaxes the rigid moving-slot partitioning of the whole segment of the previous variation, whose implementation requires more sophisticated technologies on vehicles and on highways. These v vehicles are randomly distributed on the segment. In the absence of evidence favoring any particular pattern of distribution, we make the following assumption:

ASSUMPTION 2. *The v vehicles are at random positions on the lane in the following sense. After contracting the space occupied by a vehicle into a point,*

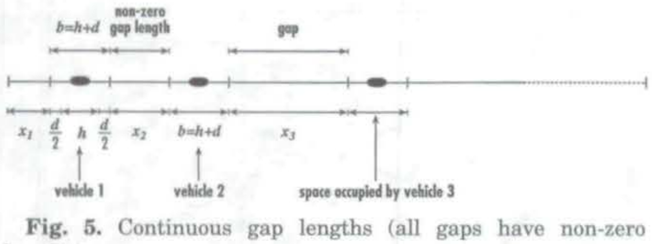

Fig. 5. Continuous gap lengths (all gaps have non-zero lengths).

the segment length becomes $S - vb$ and each vehicle is represented as a point on the contracted segment. Vehicle positions are a random sample of size v of a uniform random variable on the interval of $[0, S - vb]$.

The definition of a gap and some of the assumptions are illustrated in Figures 5 and 6.

Independent Exponential Gap Length Distribution

Based on Assumption 2, we can obtain the joint distribution of the $v - 1$ gap lengths. When v approaches infinity while $c \equiv v/S$ is kept constant, it can be shown that all the gap lengths are independent and identically distributed with an exponential distribution with a rate of $c_0 \equiv v/(S - vb)$. To illustrate the idea, calculate the gap length distribution between the ith and the $(i + 1)$th vehicles, from either end of the segment, as follows.

Denote the positions of the ith and the $(i + 1)$th vehicles as Y_i and Y_{i+1} respectively. Also, denote $S - vb$ by S_0. Then, the joint probability density function (p.d.f.) of the Y_i and Y_{i+1} can be found to be

$$f_{i,i+1}(Y_i = y_i,\ Y_{i+1} = y_{i+1}) = \frac{v!}{(i-1)!\,(v-i-1)!}\left(\frac{y_i}{S_0}\right)^{i-1}\left(1 - \frac{y_{i+1}}{S_0}\right)^{v-i-1}\left(\frac{1}{S_0}\right)^2, \quad \text{for} \quad 0 < y_i < y_{i+1} < S_0. \tag{6}$$

To find the distribution of the gap length $X \equiv Y_{i+1} - Y_i$, first obtain the joint p.d.f., $f^v_{X,Z}(x, z)$, of $X = x$ and $Z \equiv Y_i = z$, an auxiliary random variable for derivation convenience. (Note that the superscript of $f^v_{X,Z}$ is included to indicate its dependence on v.) By change of variable,

$$f^v_{X,Z}(x, z) = \frac{v!}{(i-1)!\,(v-i-1)!}\left(\frac{z}{S_0}\right)^{i-1}\left(1 - \frac{x+z}{S_0}\right)^{v-i-1}\left(\frac{1}{S_0}\right)^2, \quad \text{for} \quad 0 < z < x + z < S_0. \tag{7}$$

Now, replacing S_0 by v/c_0 gives

$$f^v_{X,Z}(x, z) = \frac{c_0^{i+1}}{(i-1)!}\left(\prod_{k=0}^{i}\frac{v-k}{v}\right)z^{i-1}\left(1 + \frac{-c_0(x+z)}{v}\right)^{v-i-1}, \quad \text{for} \quad 0 < z < x + z < S_0. \tag{8}$$

Denote the marginal distribution of X by $f^v_X(x)$. Then,

$$f^v_X(x) = \int_0^{(v/c_0)-x} f^v_{X,Z}(x, z)\, dz. \tag{9}$$

Although both the integrand and the range of the integral depend on v, it can be shown that

$$\lim_{v\to\infty} f^v_X(x) = \int_0^\infty [\lim_{v\to\infty} f^v_{X,Z}(x, z)]\, dz. \tag{10}$$

But,

$$\lim_{v\to\infty} f^v_{X,Z}(x, z) = \frac{c_0^{i+1}}{(i-1)!}\, z^{i-1} e^{-c_0(x+z)}, \quad \text{for} \quad z > 0 \quad \text{and} \quad x > 0. \tag{11}$$

Therefore, the probability density function $f(x)$ of the distribution of gap length X, as $v \to \infty$, is

$$\begin{aligned} f(x) &= \lim_{v\to\infty} f^v_X(x) \\ &= \int_0^\infty \frac{c_0^{i+1}}{(i-1)!}\, z^{i-1} e^{-c_0(x+z)}\, dz \\ &= \frac{c_0^{i+1}}{(i-1)!}\, e^{-c_0 x}\int_0^\infty z^{i-1} e^{-c_0 z}\, dz \\ &= c_0 e^{-c_0 x}. \end{aligned} \tag{12}$$

This demonstrates that the point process defined in Assumption 2 is a Poisson Process when the segment length tends to infinity and traffic density is kept constant. This can be viewed as the converse

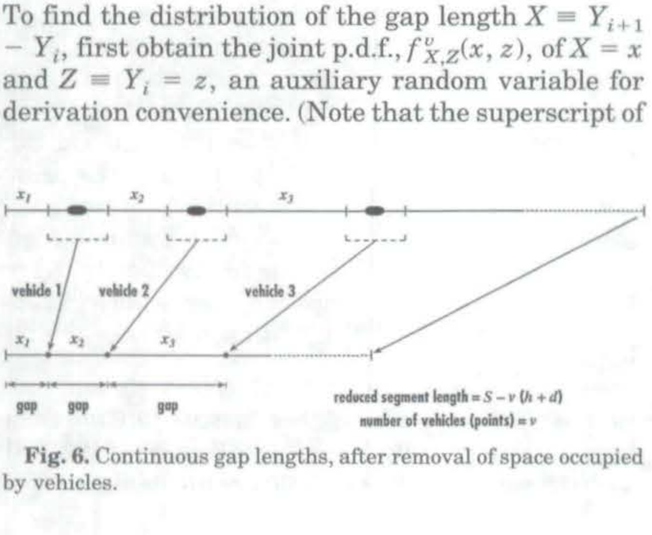

Fig. 6. Continuous gap lengths, after removal of space occupied by vehicles.

of the well-known fact that, given that n events ($n \geq 1$) of a Poisson process have occurred in time interval [0, t], the set of n arrival times has the same joint distribution as a set of n random variables that are independent and uniformly distributed on the interval.

Lane-Change Completion Time

To calculate the lane-change completion time, we assume that if at the initiation time of a lane-change attempt the vehicle is actually next to a sufficiently large gap in the destination lane, then it is properly positioned next to the gap so that it can move into the gap safely. In order for a lane change not to affect the speed of traffic on both lanes, the gap has to be no shorter than the slot length b. Note that the maneuvering space is needed as part of the space used by a vehicle while moving along a lane (i.e., not just during lane changing) to ensure that a lane change will not affect the speed of the origin lane during a lane change. Therefore, the probability that a gap is too short for a lane change is simply

$$q = 1 - e^{-c_0 b} \quad \text{and} \quad c_0 = \frac{v}{S - vb}. \tag{13}$$

The probability function of the number of gaps (too small for a safe lane change) passed by the lane-change vehicles, denoted by G, before the success is a geometric distribution, due to the memoryless property of the exponential distribution and the assumption that gap lengths are i.i.d.:

$$p(G = i) = (1 - q)q^i. \tag{14}$$

If a gap is too short, the lane-change vehicle has to pass that gap in search of a sufficiently large gap. The extra waiting time incurred depends on the actual length of the short gap. Given that the gap is shorter than b, the conditional probability density function of the gap length is simply

$$f(x|x \leq b) = \begin{cases} c_0 e^{-c_0 x}/(1 - e^{-c_0 b}) & \text{if} \quad 0 \leq x \leq b, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

The lane-change completion time is

$$T = \frac{Gb + \sum_{i=1}^{G} V_i}{\delta} + \tau, \tag{16}$$

where V_i, i = 1, 2, . . . , ∞, are independent and identically distributed with a probability density function of $f(x|x \leq b)$. Note that the range of the sum in (16) is a random variable. One can calculate the distribution of T by first conditioning on the outcome of G and then weighing the conditional distributions by that of G.

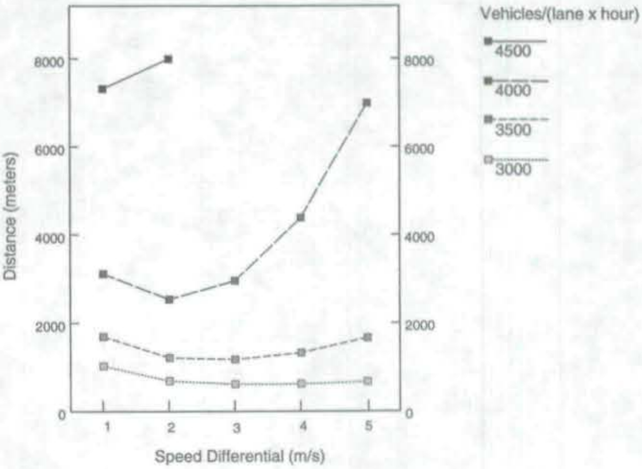

Fig. 7. Mean of lane-change completion distance for continuous gap lengths.

Examples

As in the examples given in Section 1.1, we study, for many different combinations of parameter values, the vehicle/gap distribution on the slower lane and the lane-change completion time from the faster lane to the slower lane. For fair comparisons to the slot scenario, we use the same set of parameter values for vehicle length, safety spacing, maneuvering space, speed of the slower lane, lane width, lateral velocity during lane changing, deceleration rate during lane changing, and four different traffic flows. We again consider 5 different δs: 1, 2, 3, 4, 5 m/s. The required gap length is again simply $l + d + (\delta/2) \times \tau$.

The four flows on the slower lane, together with the speed, vehicle length and safety spacing, determine the four different vehicle/gap distributions. Based on the vehicle/gap distribution and the speed differential we calculate the probability distribution of the distance traveled until a successful lane change.

The results are shown in Figures 7 and 8, which contrast the mean and the standard deviation among the four different flows respectively. These means and standard deviations are much higher than their slotting counterparts. In fact, they are at least approximately twice as high and are even 10 times higher in some cases. Note the difference in distance scale between the two sets of figures, e.g., Figure 3 and Figure 7. At the flow level of 4500, the lane-change completion distance is clearly excessive, especially at high δs. The rate of increase, in both mean and standard deviation, in lane-change completion distance is also higher than its slotting counterpart. Particularly, the rates of increase from 3500 to 4000 and from 4000 to 4500 are much higher than

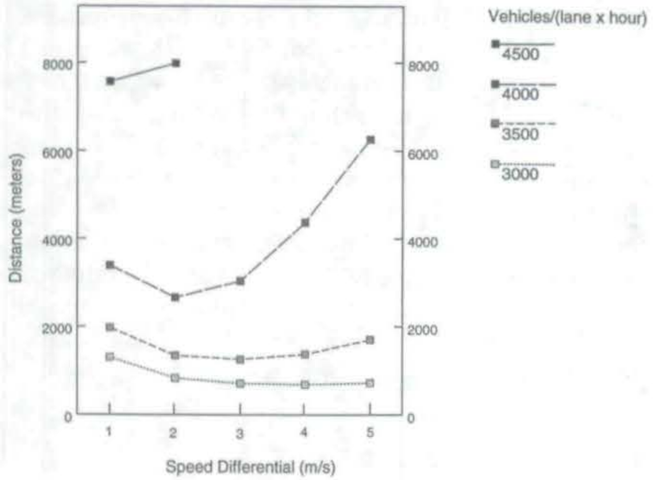

Fig. 8. Standard deviation of lane-change completion distance for continuous gap lengths.

doubling. Recall that the corresponding rates for the slotting scenario are approximately 50%. The ratio of standard deviation over mean is near 1. This indicates high variability, as in the case of slotting.

We close this subsection with the following remarks. To maximize the probability of successful exit and lane change, the vehicle-following rules do play an important role. For example, in a free-agent scenario, the gap between two vehicles should better be a multiple of the length of a *slot*, a space which a lane-changing vehicle in a neighboring lane can safely move into without affecting the traffic speed in either lane. Any gap shorter than the slot length will be a waste of space. Also, if the vehicles needing to change lane are randomly distributed in the AHS, then randomly distributed empty slots may shorten the time or distance requirement for a successful lane change.

2. MODELS FOR PLATOONING

IN STUDYING THE CAPACITY associated with any operating scenario with the platooning vehicle-following rule, the platoon size distribution is needed for a variety of reasons. For example, a possible lane-change strategy may be to allow a vehicle to join a platoon only at the front of the platoon and, in this case, the platoon length distribution is needed to estimate the time for a lane-change completion. More importantly, if a maximum size is imposed on platoons and the platoon next to the lane-changing vehicle is already full, then the waiting time depends on the probability that the platoon is already full.

A lane can be thought of as alternating cycles of platoon and gap. Assuming probabilistic independence among all cycle lengths and between the gap and platoon size distributions within a cycle, all one needs to know about the traffic in the target neighboring lane are the gap length distribution and the platoon size distribution.

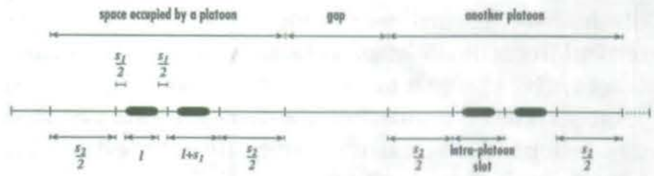

Fig. 9. Platoons and gaps.

2.1. Gap Length Distribution

Unlike the free-agent scenarios, vehicles in a platooning scenario cluster while moving down the lane and there are two types of spacings—intraplatoon spacing and interplatoon spacing. The former is between two adjacent vehicles in a platoon while the latter is between two longitudinally adjacent platoons. This key difference makes the approach described in the previous section unsuitable for the platooning scenarios and leads to a different approach to modeling the vehicle/gap distribution. Since the intraplatoon spacing is likely set to a small value based on safety considerations regardless of traffic density, it should have little impact on AHS capacity. Therefore, we concentrate on the length of interplatoon spacing.

A gap between two platoons is defined as follows and depicted in Figure 9. Each vehicle requires a length of $l + s_1$ within a platoon, including the vehicle length l and the safety spacing s_1 between two vehicles. In other words, each of the vehicles in a platoon occupies a slot of length $l + s_1$. For ease of discussion, assume that $s_1/2$ is allocated in the front and the other $s_1/2$ is allocated in the rear of the vehicle, regardless of whether there is an adjacent vehicle in its lane. Note that this intraplatoon slot is much shorter than the vehicle slot in either of the two free-agent scenarios discussed earlier. Particularly, s_1 is much shorter than d and no maneuvering space is included in the intraplatoon slot. In fact, the PATH Program of Institute of Transportation Studies at UC Berkeley is currently demonstrating a 4-car platoon with a $4m$ intraplatoon spacing and a $1m$ intraplatoon spacing has been targeted.

There is a minimum safety spacing s_2 required for any pair of adjacent platoons. Note that, as in the free-agent cases, this minimum safety spacing cannot be invaded by any vehicles during any lane-changing process. Assume that $s_2/2$ is allocated in the front and the other $s_2/2$ is allocated in the rear of the platoon, regardless of whether there is an adjacent platoon in its lane. Therefore, a platoon of size n *occupies* a length of $s_2 + n \times (l + s_1)$.

Given a fixed number of platoons per lane per unit distance of AHS, one can use the argument employed in Section 1.2 to justify the use of an exponential distribution for the gap length. However, given the number of vehicles per lane per unit distance of AHS, the number of platoons is uncertain. Therefore, given a traffic density of a lane, the use of an exponential distribution as the gap length distribution is theoretically unjustified. However, the gap length distribution can be modeled as a mixture of exponential distributions. When the average number of platoons per segment length is known and the variation of the number of platoons across different segments is small, the use of an exponential distribution may be an acceptable approximation.

2.2. Platoon Size Distribution

The dynamic nature of highway traffic adds the dimension of time to the definition of the platoon size distribution. The probability of a particular size may be interpreted as the long-term proportion of time any platoon has a particular size.

What Changes the Platoon Size

Before developing a model for the distribution, we first identify the important factors that affect the platoon size. We provide further detail of the platooning scenario as follows. Vehicles change lanes as individuals, not in platoons. When a vehicle is entering or departing from a platoon, no other vehicles in the same platoon may do so. In an AHS with multiple automated lanes, a vehicle, to reach a target lane, may have to pass through a number of intermediate lanes and in the process join and leave some platoons. We assume that such temporary stays with a platoon do not have a significant effect on the distribution of platoon size.

For safety reasons, two longitudinally adjacent platoons are not allowed to *merge* to become one. Also, one platoon is not allowed to *split* into two, unless for accommodating a lane-change maneuver. Therefore, the important factors affecting the size of a particular platoon are: (i) a vehicle, upon entering the AHS, *joins* the platoon and *stays* until its time for exiting the AHS, (ii) a vehicle, after an extended stay, *leaves* the platoon for exiting, (iii) a vehicle, for the purpose of balancing traffic flow in different lanes, *joins* the platoon, and (iv) a vehicle, for flow balancing, *leaves* the platoon. In short, only a lane change may result in a platoon size change and the size of a platoon can change only by 1.

We assume a known entry rate and consequently an identical departure rate, resulting from lane changes, to ensure a constant flow. As in Section 1, we consider only lane changes from the faster lane to the slower lane. A lane-changing vehicle can either enter a sufficiently large gap between two platoons or join a platoon in the destination lane, but only at the platoon's front end. Upon entering a sufficiently large gap between two platoons, the lane-changing vehicle effectively creates a single-vehicle platoon. A lane-change vehicle will join a platoon if the vehicle's longitudinal position, at the initiation of the lane-change, is within the space occupied by the platoon.

Maximum Platoon Size

The size of a platoon may be constrained for a variety of reasons, e.g., the need to be able to complete a cycle of message relay from the platoon leader to all its followers in the platoon and back. Such constraints may also be required for safety and capacity reasons. We will assume an upper limit on the platoon size.

A Special Feature of Size Evolution

The size of a platoon changes if and only if a new member vehicle arrives or an existing member vehicle departs. The arrival rate and the departure rate are the two principal determinants of the platoon size distribution. Suppose a vehicle will join a platoon if and only if it is adjacent to the space occupied by a platoon at the lane-change initiation time. Then the larger the platoon size, the more space it occupies and the higher the arrival rate. Also suppose each vehicle in a common lane has an identical probability of departing from the lane. Then the departure rate also increases with the platoon size. In our opinion, any credible model should definitely account for the dependence of size-change rates on the platoon size. It must also explicitly model the constraint of maximum platoon size.

Approach

Assuming that platoon sizes are identically distributed, we provide a dynamic treatment for calculating the platoon size distribution. In other words, instead of conducting a static combinatorial analysis for the vehicle distribution on a segment, as in deriving the gap distribution between two free-agents in Section 1, we concentrate on the size evolution of a platoon through time. We will model the evolution as a Continuous Time Markov Chain (ROSS, 1980).

By an argument similar to the one used in deriving the exponential gap-length distribution for the free-agent scenario in Section 1.2, we can justify the use of an exponential distribution to model the interarrival time and interdeparture time. Given the platoon size, the arrival and departure processes can be safely assumed to be independent. Therefore, we can use the Birth and Death Process, a special Continuous Time Markov Chain, to model platoon size evolution.

We first study the Markov Chain embedded in the Birth and Death Process. Let the size of the platoon be the state. The embedded Markov Chain clearly has only $n_{\max}$, the maximum allowable size, states. However, since a platoon disappears from a lane after a vehicle (as a single-vehicle platoon) changes lanes, we augment the state space of the embedded Markov Chain to include 0. Note that a complete specification of this augmented Markov Chain requires the knowledge of the unknown birth rate at state zero. However, it turns out that, to obtain the platoon size distribution, this birth rate is not required. This is because the platoon size distribution is the *conditional* distribution of states 1 through $n_{\max}$, *given* that the state is not 0.

The approach is, intuitively and simply put, that when a platoon has been created, we observe the size evolution. When a platoon has just vanished from the lane, we look elsewhere in the lane for a newly-formed single-vehicle platoon to continue the observation. We summarize the model in the following assumption.

ASSUMPTION 3. *The platoon size N behaves according to a Birth and Death Process in which (i) the arrival rate is proportional to the length of space occupied by the platoon (except size 0 and the maximum size) and (ii) the departure rate is proportional to the size of the platoon. The per-unit-distance-unit-time rate (number/(time × length)) of vehicles entering a lane is r_e. The per-vehicle-unit-time rate (number/(time × vehicle)) of vehicles leaving the lane is r_1. (Note that r_e/r_1 is traffic density, i.e., number of vehicles per unit length.)*

Solution

Let λ_i, $i = 0, 1, 2, \ldots, n_{\max}$, denote the birth rate, i.e., the exponential rate at which a new vehicle joins the platoon, when the platoon consists of i vehicles. Similarly, let μ_i, $i = 1, 2, \ldots, n_{\max}$, denote the death rate when the platoon has i vehicles. In terms of the rates defined in Assumption 3,

$$\lambda_i = r_e \times (i \times (l + s_1) + s_2), \quad \text{for } i = 1, 2, \ldots, n_{\max} - 1, \tag{17}$$

and

$$\lambda_{n_{\max}} = 0. \tag{18}$$

$$\mu_i = r_1 \times i, \quad \text{for } i = 1, 2, \ldots, n_{\max}, \tag{19}$$

and

$$\mu_0 = 0. \tag{20}$$

Note that λ_0 is unknown. The limiting probability p_i that the platoon is of size i is simply (see p. 211 of ROSS, 1980):

$$p_i = \frac{\prod_{k=0}^{i-1} \lambda_k}{\prod_{k=1}^{i} \mu_k} p_0, \quad i = 1, 2, \ldots, n_{\max}. \tag{21}$$

Since we are only interested in the conditional probabilities, denoted by p'_i, i = 1, 2, ..., $n_{\max}$, of positive platoon length given the existence of the platoon, regardless of the value of λ_0, we obtain the probability distribution of platoon size as follows:

$$p'_i = \frac{\prod_{k=1}^{i-1} \lambda_k}{\prod_{k=2}^{i} \mu_k} p'_1, \quad i = 2, \ldots, n_{\max}, \tag{22}$$

and

$$p'_1 = 1 \bigg/ \left[1 + \sum_{i=2}^{n_{\max}} \left(\frac{\prod_{k=1}^{i-1} \lambda_k}{\prod_{k=2}^{i} \mu_k} \right) \right]. \tag{23}$$

Examples

For fair comparison, we continue to use the common set of parameters shared by the two earlier sets of examples. The key parameters to the platoon size distribution is the traffic density and speed. We fix the speed at 100 km/hr (27.8 m/s or approximately 60 mph). We consider four different densities. The four densities and the corresponding approximate traffic flows are:

(i) 30 vehicles/kilometer (3000 vehicles per hour)
(ii) 35 vehicles/kilometer (3500 vehicles per hour)
(iii) 40 vehicles/kilometer (4000 vehicles per hour)
(iv) 45 vehicles/kilometer (4500 vehicles per hour)

Other Parameters

Consider a hypothetical scenario with

(a) a maximum platoon size of 10,
(b) a highway segment of length 1000 meters,
(c) four vehicles entering and leaving the segment per 60 seconds,
(d) an interplatoon safety spacing of 50 meters (including the maneuvering space),
(e) a common vehicle length of 5 meters, and
(f) an intraplatoon spacing of 1 meter.

Based on these parameters and the corresponding units, r_e and r_1 can be derived. Given any traffic density of k vehicles/kilometer, $r_e = 4/(1000 \times 60)$ whereas $r_1 = 4/(k \times 60)$.

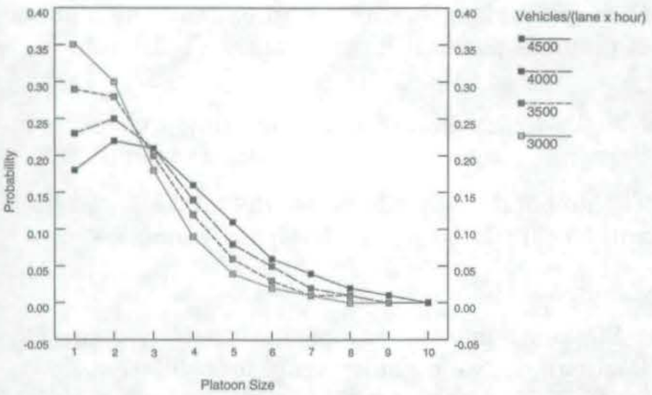

Fig. 10. Platoon size probability distribution.

Numerical Solution

Based on the common parameter values stated in (a) through (f) and the four different traffic densities, the four platoon size distributions are calculated and plotted in Figure 10. Note that, at all four different flows, the probability that a platoon is full is very small. The expected value and the standard deviation of platoon size distributions are plotted against the four different flow conditions in Figure 11.

The platoon size distribution can be used to estimate lane-change completion time. This distribution is useful not only for analytical estimation of AHS capacity but also for AHS traffic simulation. For example, it can be used in initializing the existing traffic on an AHS.

Before closing this section, we remark that the Birth and Death model cannot be used to find the distribution of the cluster size, i.e., the number of vehicles between two consecutive gaps, associated with the slot scenario described in Section 1.1. The main reason is that the platoon size can change only by 1 whereas a cluster of occupied slots may be broken into two clusters of various sizes.

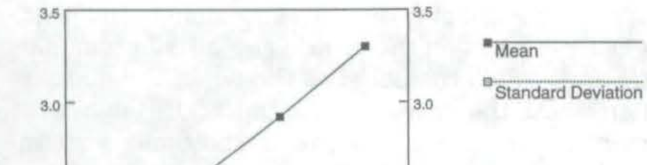

Fig. 11. Mean and standard deviation of platoon size distribution.

2.3. Lane-Change Completion Time

The gap and platoon size distributions can be used to estimate the time/distance until the completion of a lane change. The usage depends on the lane change rule. In those rules in which vehicles can join a platoon anywhere in the platoon with little preparation, e.g., a minor platoon splitting and the subsequent merging, the waiting time, and hence the time until lane-change completion, should be relatively short, if the platoon is not full. In this case, the primary use for the platoon size distribution is the probability that the platoon is full. Note that the preparation not only takes time but also increases the probability of interference between separate lane-change attempts. If preparation requires the platoon to perform a full split into two separate platoons and then subsequently join back into one, not only the lateral flow but also the stability of longitudinal flow suffer. The feasibility, in terms of safety and control technology, of a minor splitting followed by a subsequent minor merging for a lane change should be investigated.

Operating Rules

We consider a more realistic but restrictive rule where a lane-changing vehicle (from a faster lane to a slower lane) can join a platoon only at its front for safety and flow stability (so that the receiving platoon does not have to split and hence cause slowdown by the trailing vehicles). Under this rule, the lane-change completion time hinges upon the platoon size distribution.

The traffic on the destination lane consists of recurrent cycles of platoon and gap and any platoon/gap cycle is partitioned into 3 sections: (i) safety section: a section of length s_2, the minimum interplatoon safety spacing, (ii) platoon section: a section consisting of a sequence of vehicles each of which occupies a space of length $l + s_1$, and (iii) gap section: a section of empty space between the front of the platoon and the beginning of the next cycle, as depicted in Figure 12. If the lane-change attempt is initiated when the vehicle is adjacent to section 1 (safety section) or 2 (platoon section) in the destination lane, it will have to wait until it catches up with the front of the platoon. (If the lane-change attempt is initiated when the vehicle is in the safety section, allowing the vehicle to slow down to enter a gap or

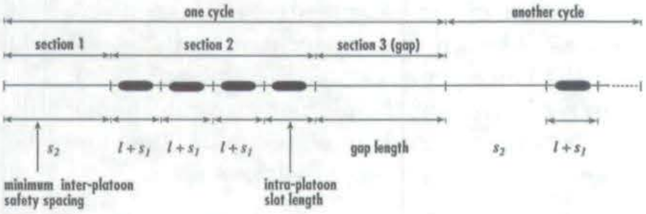

Fig. 12. Platoon/gap cycle.

join the platoon in the previous cycle may slow down the traffic in the origin lane.) Upon catching up, if the platoon is not full yet, the vehicle makes a lateral move and joins the platoon at its front end. Otherwise, it must continue traveling in search for a sufficiently large gap or a non-full platoon. If the attempt is initiated when the vehicle is adjacent to section 3 (gap section) and the gap is sufficiently large, then it makes the lane change with no waiting at all. But, if the gap is not large enough, it must wait further for the next cycle.

Modeling Approach

As discussed earlier, we make the following assumption.

ASSUMPTION 4. *The length of section 3, i.e., the gap length, has an exponential distribution.*

For simplicity, the position of the lane-change vehicle is represented as a point. We first find the probability that the vehicle, at the time of lane-change initiation, is adjacent to each of the 3 sections. Given the position, we calculate the conditional distribution of the total elapsed time. The unconditional elapsed time distribution is then obtained by weighting the three conditional distributions by the three position probabilities.

Denote the length of section i, $i = 1, 2, 3$, by L_i. Since this "cyclic" process can be modeled as an "extended" alternating process, the three probabilities are simply:

$$q_i = \frac{\mathrm{E}(L_i)}{\mathrm{E}(L_1) + \mathrm{E}(L_2) + E(L_3)}. \tag{24}$$

where

$$L_1 = s_2 \quad \text{with probability 1} \quad \text{and} \quad E(L_1) = s_2; \tag{25}$$

$$\mathrm{E}(L_2) = (l + s_1)\mathrm{E}(N) \quad \text{and} \quad \mathrm{E}(N) \equiv \sum_{i=1}^{n_{\max}} i p'_i. \tag{26}$$

Note that $\mathrm{E}(N)$ is the expected platoon size. To approximate $\mathrm{E}(L_3)$, we need to approximate the number of platoons per unit length. Given a traffic density c, i.e., the ratio of the number of vehicles over the segment length, we can approximate the number of platoons per unit length by $c/\mathrm{E}(N)$. Therefore,

$$\mathrm{E}(L_3) = \frac{\mathrm{E}(N)}{c} - s_2 - (l + s_1)\mathrm{E}(N). \tag{27}$$

The probability distribution of the total elapsed time will be calculated for the following examples.

Examples

We continue the examples given in Section 2.2. Particularly, we consider again four different flows: 3000, 3500, 4000, and 4500 vehicles/(lane × hour). We first discuss lane changes under these flow conditions. Under the four flow conditions, the probability that a platoon is of maximum size is minute. (See Figure 10.) This minute probability enables us to approximate the first two conditional lane-change completion times by the time the vehicle needs to catch up with the front of the platoon. If the lane-change attempt is initiated when the vehicle happens to be next to a gap and the gap is larger than the maneuvering space h, then the vehicle moves into the gap without delay and joins the platoon behind it. Under the four flow conditions, the probability that the gap is shorter than h is also small. Therefore, we approximate the conditional elapsed time by the maneuvering time τ. Note that when the gap is very large, the vehicle may be able to form a single-vehicle platoon. Otherwise, it can join the platoon if the platoon is not full yet. In either case, there is no waiting.

Note that in order not to slow down the traffic in the destination lane as well as the origin lane, a maneuvering space h is needed. However, by the nature of the platooning scenario, the maneuvering space is needed only during a lane change and hence has little effect on the vehicle/gap distribution, particularly when compared to the two free-agent scenarios. At the speed differential of 15 km/hr, the maneuvering space is approximately only 9 m, including the 5 m vehicle length.

By denoting, as before, the time required for the lateral movement itself by τ, we now calculate the three conditional probability distributions for the total time until lane-change completion. Denote the corresponding random variables by T_i, $i = 1, 2, 3$. Given that at the initiation time of a lane-change attempt the lane-change vehicle is next to a platoon, the probability p''_i that the platoon is of size i depends not only on p'_i but also on the length of the platoon section. This is because bigger platoons have higher probabilities of being next to the lane-change vehicle and the relative likelihood of a platoon of size i being next to the lane-change vehicle is propor-

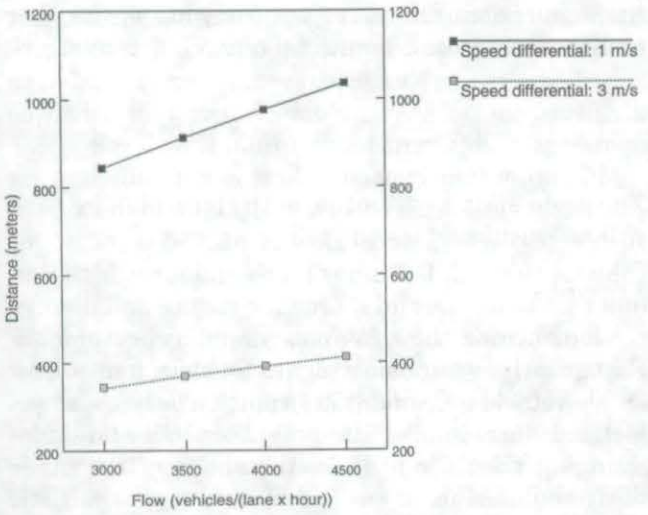

Fig. 13. Mean of lane-change completion distance for platooning.

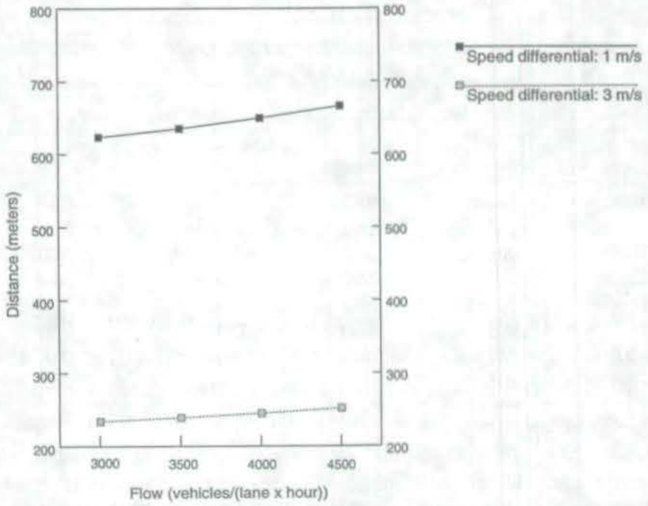

Fig. 14. Standard deviation of lane-change completion distance for platooning.

tional to the product of i and p'_i. More precisely,

$$p''_i = \frac{ip'_i}{\sum_{j=1}^{n_{\max}} jp'_j}, \quad i = 1, 2, \ldots, n_{\max}. \tag{28}$$

Therefore,

$$p\left(T_2 = \frac{U_{[0, i\times(l+s_1)]}}{\delta} + \tau\right) = p''_i, \quad i = 1, 2, \ldots, n_{\max} \tag{29}$$

and

$$T_1 = T'_2 + \frac{U_{[0,s_2]}}{\delta}, \tag{30}$$

where $U_{[0,s_2]}$ is the uniform random variable over the interval $[0, s_2]$, which is independent of T'_2, and

$$p\left(T'_2 = \frac{i(l+s_1)}{\delta} + \tau\right) = p'_i, \quad i = 1, 2, \ldots, n_{\max}. \tag{31}$$

Note that, unlike the distribution of T_2, the distribution of T'_2 involves p'_j, instead of p''_j. Finally,

$$T_3 = \tau. \tag{32}$$

Note that T_1 has a mixed probability distribution, i.e., a mixture of discrete probability distributions and absolutely continuous distributions. Further weighting the distributions of T_i according to q_i, $i = 1, 2, 3$, gives the unconditional distribution of total time until a successful lane change. This mixed distribution can be obtained numerically. The expected value and the standard deviation of lane-change completion distance at speed differentials of 1 m/s and 3 m/s are plotted against four different traffic flows in Figures 13 and 14 respectively.

Both the mean and the standard deviation of the lane change completion distance are quite linear with respect to the change in flow levels below 4500. In fact, they increase quite slowly with respect to the flow at these levels. Since, at these flow levels, a lane change can be most likely accomplished in one platoon-gap cycle, the increase in completion distance as flow level increases is attributable to the corresponding increase in platoon size. At the speed differential of 3 m/s, the lane-change completion distance is short. Since the intraplatoon spacing is independent of the speed differential, difference in speed differential has only a linear effect on the lane-change completion distance at the four different flow levels.

3. COMPARISON BETWEEN FREE-AGENT AND PLATOONING RULES

AT THE SPEED DIFFERENTIAL of 3 m/s (10.8 km/hr), we compare, for each of the four flows, the mean and standard deviation of the lane-change completion time associated with each of the three scenarios studied in this paper. Note that the speed differential of 3 m/s provides approximately the best lane-change completion time, in terms of mean and standard deviation, for the slot scenario at all four flow levels. The differences are contrasted in Table I.

From Table I it is clear that the continuous-gap-length free-agent scenario requires a much longer lane-change completion time than the slotting scenario at all four flow levels, whereas the slotting scenario requires longer, though not as much, such time than the platooning scenario for the three larger flow levels. The performance of the slotting scenario at a flow level of 4000 is better than that of

TABLE I

Comparison among three scenarios with a speed differential of 3 m/s. (Unit:meter)

Flow[1]	Moments	Scenario		
		Slot	Continuous gap length	Platooning
3000	Mean	275	620	346
	s.d.	295	717	234
3500	Mean	373	1179	370
	s.d.	396	1265	239
4000	Mean	534	2968	392
	s.d.	560	3040	245
4500	Mean	846	—	413
	s.d.	875	—	252

[1]Vehicles/(lane × hour).

its continuous-gap-length counterpart at the flow level of 3000. Note again the insensitivity of the lane-change completion time associated with the platooning scenario with respect to the four flow levels.

Although it is clear that, in terms of lateral capacity at the four different flow levels, both platooning and slotting are better than the continuous-gap-length free-agent scenario, the performance superiority comes at a price. Both slotting and platooning require additional technology support. The former likely requires infrastructure support while the latter requires more sophisticated vehicle technology and perhaps also some form of infrastructure support. This comparison of lateral capacity provides an important component of the overall performance evaluation of the vehicle-following rules.

The lane-change completion time distribution for platooning and even the platoon size distribution for flow levels higher than 4500 deserve further modeling attention, as do models incorporating possible clustering of vehicles under the two free-agent scenarios. All three scenarios provide for speed constancy by prohibiting disruption of speed by lane changes. Corresponding scenarios where local and temporary disruption of lane speeds due to lane changing is allowed, also deserve further research attention. Interaction among multiple lane-change maneuvers and the effect of cooperation among neighboring vehicles for lane-changing may be more easily studied through simulation.

4. CONCLUSION

In this article, we developed analytical models to study the vehicle/platoon and gap distributions on a lane for three AHS operating scenarios. Among the analytical models developed is the model for predicting the platoon size distribution. Although the concept of platooning has been fundamental to much of AHS operation, this is, to our knowledge, the first probabilistic model for the platoon size in the published literature. Numerical results are provided to illustrate the vehicle/gap distributions for all three scenarios at different longitudinal flow levels.

Maximum lane capacity alone is not sufficient for calculating the capacity of a multi-lane highway. On such a highway, lateral flow is needed to allow for vehicles to reach the inner lanes and for vehicles on inner lanes to exit and it tends to reduce and disturb the longitudinal flow. We considered a class of lane-changing rules under which a vehicle has microscopic vehicle movement information only about vehicles in its immediate neighborhood and lane-changing should not affect the speed of the other vehicles on either lane. Based on the probabilistic vehicle/gap models, this paper developed models for and compared the impact of longitudinal flow on the efficiency of such lane-changing under the vehicle-following rules mentioned above. Our numerical results show that the vehicle-following rule has a substantial effect on the efficiency of such lane-changing. The findings of this research can also be used to study the impact of longitudinal flow on traffic merging.

Most of the fundamental AHS concepts (e.g., platooning) are designed primarily to increase longitudinal flow. However, high longitudinal flow may actually hinder lateral flow. It may even decrease the lateral capacity to such a degree that the lateral capacity becomes the bottleneck of highway traffic flow. Since exiting vehicles at the desired off-ramps without sufficient lateral capacity will lead to traffic slowdown, the longitudinal flow suffers as a result. Therefore, the issue of how to optimize the longitudinal flow subject to the requirement of lateral flow is an important issue to be resolved. The findings of this research can be used further to study the interaction between the longitudinal capacity and the lateral capacity of an automated highway system.

ACKNOWLEDGMENT

FINANCIAL SUPPORT OF THIS RESEARCH by California Department of Transportation (Caltrans) is gratefully acknowledged. We would like to thank Professor Mark Daskin, an Associate Editor, and two anonymous referees for their careful reading of two earlier versions and many valuable suggestions.

REFERENCES

HALL, R. W., "Longitudinal and Lateral Throughput on an Idealized Highway," *Trans. Sci.* **29**, 118–127 (1995).

HITCHCOCK, A., "Intelligent Vehicle/Highway System Safety: Multiple Collisions in Automated Highway

Systems," presented at the 73rd Annual Meeting of Transportation Research Board, Jan. 1994, Washington, D.C.

RAO, B. S. Y. AND P. VARAIYA, "Flow Benefits of Autonomous Intelligent Cruise Control in Mixed Manual and Automated Traffic," *Trans. Res. Rec.* **1408,** 36–44 (1993).

RAO, B. S. Y. AND P. VARAIYA, "Roadside Intelligence for Flow Control in an IVHS," *Trans. Res. Part C,* **2,** 49–72 (1994).

RAO, B. S. Y., P. VARAIYA, AND F. ESKAFI, "Investigations into Achievable Capacity and Stream Stability with Coordinated Intelligent Vehicles," Trans. Res. Rec. **1408,** 27–35 (1993).

ROSS, S. M., *Introduction to Probability Models*, Academic Press, New York, 1980.

SHLADOVER, S., *Operation of Automated Guideway Transit Vehicles in Dynamically Reconfigured Trains and Platoons*, (Extended Summary, Vol. I & II), UMTA-MA-06-0085-79-1, UMTA-MA-06-0085-79-2, and UMTA-MA-06-0085-79-3, U.S. Department of Transportation, Urban Mass Transportation Administration, Washington, D.C., July, 1979.

SHLADOVER, S., "Potential Freeway Capacity Effects of Advanced Vehicle Control Systems," in *Proceedings of Second International Conference on Applications of Advanced Technologies in Transportation Engineering, Minneapolis, Minnesota, August 18–21*, 1991, pp. 213–217.

TSAO, H.-S. J., AND R. W. HALL, "A Probabilistic Model for AVCS Longitudinal Collision/Safety Analysis," *IVHS Journal*, **1,** 261–274 (1994).

TSAO, H.-S. J., R. W. HALL, AND S. E. SHLADOVER, "Design Options for Operating Automated Highway Systems," in *Proceedings of Vehicle Navigation & Information Systems Conference, Ottawa, Canada, Oct. 1993*, pp. 494–500.

TSAO, H.-S. J., R. W. HALL, AND B. E. HONGOLA, *Capacity of Automated Highway Systems: Effect of Platooning and Barriers*, PATH Research Report UCB-ITS-PRR-93-26. Institute of Transportation Studies, University of California, Berkeley, 1993.

TSAO, H.-S. J., R. W. HALL, S. E. SHLADOVER, T. A. PLOCHER, AND L. J. LEVITAN, *Human Factors Design of Automated Highway Systems: First Generation Scenarios*, FHWA Report No. FHWA-RD-93-123, Washington, D.C. (1993).

VARAIYA, P. AND S. E. SHLADOVER, *Sketch of an IVHS System Architecture*, PATH Research Report UCB-ITS-PRR-91-3, Institute of Transportation Studies, University of California, Berkeley, 1992.

VARAIYA, P., "Smart Cars on Smart Roads: Problems of Control," *IEEE Trans. Automatic Control*, **38,** 195–207 (1993).

(Received, March 1994; revised, December 1995; accepted, January 1996)