# Mental Arithmetic across Three Language Groups 

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Luo, Tianyu, "Mental Arithmetic across Three Language Groups" (2012). Master's Theses. 4200.
DOI: https://doi.org/10.31979/etd.8zww-yde3
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# MENTAL ARITHMETIC ACROSS THREE LANGUAGE GROUPS 

## A Thesis

Presented to

The Faculty of the Department of Psychology

San José State University

In Partial Fulfillment
of the Requirements for the Degree

Master of Arts
by

Tianyu Luo

August 2012
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# MENTAL ARITHMETIC ACROSS THREE LANGUAGE GROUPS 

 byTianyu Luo

# APPROVED FOR THE DEPARTMENT OF PSYCHOLOGY 

## SAN JOSÉ STATE UNIVERSITY

August 2012

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#### Abstract

MENTAL ARITHMETIC ACROSS THREE LANGUAGE GROUPS by Tianyu Luo

Mental arithmetic performance was investigated among three language groups (English monolinguals, Chinese-English bilinguals, and Spanish-English bilinguals). Participants solved both small and large numerosity arithmetic problems in addition and multiplication and reported their solution strategies. All groups performed better in small problems than in large ones and better in addition than in multiplication, especially for a large size set. The results revealed all three groups performed equally well in solving problems correctly. Spanish-English bilinguals were equivalent to their English monolingual peers. However, Chinese-English bilinguals outperformed the other two groups in solution speed, especially when problems consisted of large numbers. No group differences were found in the frequency of using retrieval strategies to solve problems. Linguistic influence and other possible factors were discussed to explain the mental arithmetic advantage for Chinese-English bilinguals relative to other groups.


## ACKNOWLEDGEMENTS

It is a pleasure to thank the many people who made this thesis possible.

Above all, I would like to express my thanks to my advisor, Dr. Robert Cooper for his continuous help, support, and patience from the very beginning to this day. Not to mention his unsurpassed knowledge of psychology; his inspiration, advice, and encouragement has always been invaluable on both an academic and a personal level, for which I am extremely grateful.

I would like to show my gratitude to the other committee members Dr. Mildred Alvarez and Dr. Sean Laraway who helped me in completing the thesis and exchanged their interesting ideas from the original proposal to this final piece of work.

A special appreciation of mine goes to Dr. Clifton Oyamot, who kindly allowed me to use his E-prime software and his lab to collect and analyze experimental data and he is always ready to help whenever I had questions about the software.

I also wish to thank Alan Leventhal and his students who helped install the software onto all the computers in the lab, making the data collection possible and successful.

Additional thanks are due to all other great professors in Psychology department who have taught me and helped me enhance my academic knowledge so that I am able to complete this thesis work.

Last but not the least, I would like to thank my husband Feng and my parents for their undivided support and interest who inspired me and encouraged me to go my own way, without whom I would be unable to complete my project. I also want to thank my unborn baby Weiwei who not only completed my life but also motivated me to finish the thesis on time.

For any error or inadequacy that may remain in this work, the responsibility is entirely my own.

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## Introduction

The rise in cross-cultural research reflects not only the process of globalization, but also people's interest in understanding different cultures. Mathematics achievement has been an increasingly active area of study in cross-cultural comparison research since the 1980s (e.g., Stevenson, Chen, \& Lee, 1993; Stevenson, Lee, \& Stigler, 1986). East Asians have been found consistently to outperform their North American peers (e.g., Stevenson et al., 1990), even as early as in kindergarten (Siegler \& Mu, 2008), and throughout school ages (Stevenson et al., 1993; Stevenson et al., 1990; Stevenson et al., 1986) and young adulthood (Imbo \& LeFevre, 2009). The better mathematics performance of Asian children exists across many mathematical domains, such as number and operation, geometric knowledge, problem solving, and logical reasoning (Zhou, Peverly, Boehm, \& Lin, 2005). Zhou and his colleagues (2005) suggested that the differences are due to the quality of teaching practices, such as instructional strategies and family support, including parents' involvement in mathematical activities. Knowledge of simple arithmetic is a requirement of daily life, and mental arithmetic can help people calculate with speed, especially when computing tools are not available. Mental arithmetic (or so-called mental calculation) refers to the process of carrying out arithmetic operations without the aid of external tools, such as a calculator, computer, abacus, or pen and paper (Luo, Liu, He, Tao, \& Luo, 2009). It involves a
series of cognitive processes and consists of three main parts: encoding, operation, and response (Luo et al., 2009). Effective mental strategies are also characteristic of mental arithmetic (Campbell \& Xue, 2001; Grabner et al., 2009; Imbo, Vandierendonck, \& Rosseel, 2007). Excellent mental calculators can accomplish ten tasks of adding ten 10digit numbers within five minutes and ten tasks of multiplying two 8-digit numbers within nine minutes in competitive settings, such as the Mental Calculation World Cup (Mental Calculation World Cup, 2008). Although average people do not have the same amazing abilities as those world-class mental calculators, it is intriguing to explore how people conduct mental arithmetic as well as what factors are related to mental arithmetic.

Discovering the underlying mechanisms of mental arithmetic could help us to understand the cognitive development of mathematical concepts and might be beneficial for people, especially for children, to improve their basic arithmetic skills. Mental arithmetic is believed to promote the development of number sense (Maclellan, 2001). During mental arithmetic, children are forced to think and make use of basic knowledge of number facts and operations. They are also encouraged to apply effective strategies, meaningful shortcuts, and appropriate judgments of the size of numbers. Furthermore, the performance of mental arithmetic could be related to mathematical achievement and numerical-mathematical IQ (Grabner et al., 2007).

## Individual and Cross-Cultural Differences

There are group differences in mental arithmetic ability that exist across nations and language communities, in addition to individual differences. Several studies have found differences between performance in simple arithmetic between East Asians and North Americans (Campbell \& Xue, 2001; Imbo \& LeFevre, 2009). For example, Campbell and Xue (2001) found that Asian Chinese, who were studying in Canada, outperformed non-Asian Canadians for both simple and complex arithmetic.

In addition to culture, language also plays an important role in the mental arithmetic processes of bilinguals. Frenck-Mestre and Vaid (1993) found the presentation format of numbers (digit versus word) was related to bilinguals' mental arithmetic performance. Geary, Cormier, Goggin, Estrada, and Lunn (1993) found that strong bilinguals (English and Spanish speaking) were slightly slower when solving complex problems, compared to weak bilinguals and monolinguals. They proposed that bilingualism has an impact on the representation and processing of numerical information, and they suggested a working memory mechanism account for the findings.

Researchers have demonstrated that Chinese-English bilinguals outperformed North American English-speaking monolinguals and English-speaking monolinguals outperformed strong Spanish-English bilinguals. Based on these results, it is of interest to learn more about what role language (bilingual vs. monolingual) plays in the process of
mental arithmetic. Do bilinguals have advantages or disadvantages in mental arithmetic?

Do the effects of bilingualism depend on different kinds of languages and related cultures? In the current research, we focused on three languages (i.e., English, Chinese, and Spanish) and examined the differences among three groups (English monolinguals, Spanish-English bilinguals, and Chinese-English bilinguals) in mental arithmetic.

## Problem Features

In addition to language and culture, problem features influence the efficiency of the problem solving process. Some researches have discovered problem size effects in simple mental arithmetic (Imbo, Vandierendonck, \& Rosseel, 2007; Penner-Wilger, LethSteensen, \& LeFevre, 2002). In general, people solve problems faster when problems involve computations with small numbers (e.g., $2+3=$ ?) than with large numbers (e.g., $28+39=?$ ). Most previous studies on mental arithmetic focused on simple problems (e.g., single digit problems with integers between 2 and 9 , excluding tie problems such as $3+3=6)$ and defined small problems as problems with a product smaller than 25 , and large problems with products larger than 25 for addition and multiplication. In the present study, we tried to add new information to the literature by having larger problems (addition problems consisting of two two-digit integers, with a sum up to 94 ; multiplication problems consisting of one two-digit multiplicand and one one-digit
multiplier, with a product up to 392) and comparing them to small problems (addition problems with a sum up to 17 and multiplication problems with a product up to 72 ).

Solution speed and accuracy differ across operations as well. In general, people solve addition and multiplication problems faster and more accurately than solving subtraction and division problems (Campbell \& Xue, 2001). The results of comparing addition and multiplication vary across situations (Campbell \& Xue, 2001; Imbo et al., 2007). In the current study, we focused on only addition and multiplication problems, because subtraction and division are their inverse operations.

It is interesting to examine if there is an interaction between the two operations and the problem size effects as the numbers change from small to comparatively large. Solving large multiplication problems should take more time than solving addition problems involving the same numbers, especially when carrying is involved; however, when very small numbers (e.g., 2 and 3 ) are involved, multiplication problems may be solved faster than addition ones.

## Why There Are Such Differences?

Previous studies have revealed that both children and adults use varied strategies across different situations (Campbell \& Xue, 2001; Grabner et al., 2009; Imbo \& Vandierendonck, 2008). Two main types of strategies have been reported in the literature: retrieval strategies and procedural strategies. The retrieval strategies are
retrieving answers from long-term memory directly. The procedural strategies are using non-retrieval methods such as transformation (e.g., $7+4=7+3+1=10+1=11$ ) or counting (e.g., $7+2=7,8,9$ ).

A number of studies have found that strategy selection and efficiency are associated with problem size, operation type, and culture (Campbell \& Xue, 2001; Imbo \& Vandierendonck, 2008; Penner-Wilger el al., 2002). Specifically, compared to procedural strategies, retrieval strategies are more likely to be used in small problems rather than in large problems, in simple multiplications rather than in simple additions, and by Chinese rather than by North Americans.

Regarding performance and efficiency, retrieval strategies are associated with faster solution speed and more accuracy, whereas procedural strategies produce longer reaction times and more errors. Neuropsychological studies also confirmed the differential activation in brain regions when different strategies were applied (Grabner et al., 2009; Tang et al., 2006). For example, Grabner et al. (2009) found stronger activation of left angular gyrus when participants solved the problems that they reported to use fact retrieval as their problem solving strategy, while widespread activation in a fronto-parietal network was observed for the problems solved by procedural strategies. Moreover, Imbo, Vandierendonck, and Rosseel (2007) found that more skilled and highly practiced students used memory retrieval more often and executed their strategies more
efficiently than less skilled and practiced peers. Because practice and automaticity are given more attention in Asia, both at school and at home (Imbo \& LeFevre, 2009); we predicted that Chinese-English bilinguals would have higher percentage use of the retrieval strategy and higher efficiency levels (speed and accuracy) than English monolinguals and Spanish-English bilinguals.

In addition, less calculator use is found to be associated with better mental arithmetic performance (Campbell \& Xue, 2001; Imbo et al., 2007). For example, Campbell and Xue (2001) found that Chinese students used a calculator less often and performed better than did Canadians. Because Chinese-English bilinguals were instructed the same as Spanish-English bilinguals and English monolinguals at school, though they might not be encouraged to use calculators at home, it would be interesting to see whether or not Chinese-English bilinguals would use calculators less than their peers. Less calculator use might also be related with more confidence and less anxiety, especially when solving large problems.

## Overview of The Current Study

As noted previously, many cross culture studies on mathematical performance, especially on mental arithmetic, compared East Asians (e.g., Chinese and Japanese) and North Americans (e.g., Canadians and Anglo Americans). While comparing bilinguals and monolinguals, strong bilinguals had poorer mental arithmetic performance than
monolinguals. It would be interesting to examine the relationship between different language bilingualism and the process of mental arithmetic. Therefore, in the current study, mental arithmetic performance and the underlying procedures were investigated among three language-speaking groups: English speaking monolinguals, Spanish-English speaking bilinguals, and Chinese-English speaking bilinguals.

In addition to language effects, problem size effects and operation effects were examined. Group differences on strategy selection and calculator use were examined. The relationship between academic achievement (college grade point average) and mental arithmetic performance was also examined.

To summarize, the goals of the current study were to: (a) explore the differences in mental arithmetic performance among three language-based cultural groups, and (b) understand the sources of the differences as well as the underlying mechanism of number processing.

We hypothesized that: (a) Chinese-English speaking bilinguals would perform better in solving mental arithmetic problems than English speaking monolinguals and Spanish-English speaking bilinguals; (b) all groups would perform better in small problems than in large problems; and (c) Chinese-English bilinguals would have a higher percentage of using retrieval strategies.

## Method

## Design

The main task of the current study was mental arithmetic problem solving, and the design was a $3 \times 2 \times 2$ quasi-experimental design. The independent variables were language (three language groups: English monolinguals, Spanish-English bilinguals, and Chinese-English bilinguals), problem size (two levels: small, large), and operation (two levels: addition, multiplication). The dependent variables were reaction time and percent correct.

## Participants

The plan was to have 159 participants, with equal number coming from three language groups (53 in each group), recruited from San José State University (SJSU). According to power analysis by the G*Power software (Buchner, Erdfelder, \& Faul, 1997), the sample size would be 159 for a one-way ANOVA with medium effect size $\mathrm{f}=$ .25 , alpha $=.05$, power $=.80$, and three groups; if choosing large effect size $\mathrm{f}=0.4$, the required sample size would be 66 . However, we were only able to recruit 134 participants in the Spring of 2011. One out of the 134 did not fill out the questionnaire at all so we considered his/her experimental data to be invalid and did not include them in the analyses.

Of the remaining 133 participants, 89 qualified in one of the three specific language groups (i.e., English monolinguals, Spanish-English bilinguals, and ChineseEnglish bilinguals) set for the current study. We defined English monolingual participants as those who use only English to communicate with others at both school and home in their daily life. Chinese-English bilingual participants were those (a) who themselves or whose parents were originally from Chinese culture, including China, Hong Kong, Macau, and Taiwan; (b) whose first language was Chinese, either Mandarin or Cantonese; (c) who spoke Chinese at home; and (d) who learned English as a second language at a later time. Spanish-English bilingual participants were (a) Latinos, (b) whose first language was Spanish, (c) who spoke Spanish at home, and (d) who learned English as a second language at a later time.

We had 47 English monolinguals, 29 Spanish-English bilinguals, and 13 ChineseEnglish bilinguals who qualified. Some unqualified participants stated that English was their second language, but they had languages other than Spanish and Chinese as their first language (e.g., Vietnamese, Tagalog, Hindi, French, or Assyrian). Some unqualified participants had English as their first language and other languages as their second language (e.g., French, Dutch, Arabic, or Spanish). Though we specified the requirement on our sign-in sheet, we could not check whether or not they were qualified until they filled out the questionnaires. As our hypotheses were about specific language groups, we
eliminated the unqualified participant data from the analyses. Therefore, there were 89 participants for the final data analyses, 35 males and 50 females, and four who did not indicate their gender. There were 19 males out of 47 English monolinguals; 10 males out of 25 Spanish-English bilinguals; and six males out of 13 Chinese-English bilinguals. Their mean age was $20.14(S D=2.94)$ years.

Most of the participants were recruited from the Department of Psychology subject pool at SJSU. They received 1 hr of participation credit toward the participation requirement in their General Psychology course. Some were recruited from an undergraduate cognition class and they received participation credit to fulfill the course requirement. About 6 Spanish-English bilingual participants were recruited from El Círculo Hispánico (The Spanish Circle), a minority student organization with members interested in Hispanic culture and Spanish language. As they were volunteers, they were provided with free pizza and non-alcoholic drinks as a thank-you reward.

A signed consent form to permit the experimenter to look at college grade point averages (GPA) was collected before the experiment. The primary investigator was provided the GPAs as coded data so that identifiable participant information was hidden permanently.

## Procedures

Participants were tested in groups ranging from 1 to 8 participants in a quiet lab room. A single session lasted 20 to 45 min depending on each participant's problemsolving speed. At the beginning, the experimenter explained the purpose of the study and then asked all participants to sign the written informed consent form if they agreed to continue. Then the participants were given instructions about how to respond to each problem (choosing one correct answer from two) and how to report the strategy they used after each problem. They were told to choose strategies from the following categories: remember, transform, count, estimate, calculate, other, and error. Each strategy was defined in detail on a print copy of the strategy report instruction (Appendix A) provided to each participant as reference. Examples were also given along with the instruction to help participants better understand each strategy. Participants were instructed to respond to each arithmetic problem as quickly and accurately as possible.

Each participant received four blocks of problems: small addition, large addition, small multiplication, and large multiplication, in a randomly assigned order. When participants initiated each block, they would see an instruction message shown on the computer screen followed by five practice problems. Each problem ended with an equal sign and a question mark appeared at the center of the screen (e.g., $3+5=$ ?), and two answers appeared below the problem at the same time, one was correct and the other was
incorrect. The correct answer appeared randomly on the left or right, with half on each side. Participants had to press either the " 1 " or " 2 " key to choose one answer. Timing began when the stimuli appeared and ended when participants pressed the corresponding key. After each problem, participants saw a screen with two highlighted words, Strategy Choices, and seven words below, Remember, Transform, Count, Estimate, Calculate, Other, and Error. Participants had to press one key from the " 1 " " 2 " " 3 " " 4 " " 5 " " 6 " " 7 " number keys to choose one corresponding strategy after reflecting on the mental processes they used to solve that arithmetic problem.

At the beginning of each block, participants were given 5 practice trials and feedback (reaction time and correctness) was provided, but no feedback was offered for the following 28 experimental trials. A 60 s short break was provided after each block. All participants were required to begin the break, but they could start the next block whenever they were ready.

After completing the mental arithmetic problem solving tasks, participants were asked to complete a follow-up questionnaire that consisted of five parts, in order to collect self-report information about their demographic background, self-evaluation, language background/skills, calculator use, and math anxiety.

## Materials

Arithmetic tasks. There were 112 arithmetic problems presented in four blocks by problem type (small addition, large addition, small multiplication, and large multiplication). Each block consisted of 28 problems. Because tie problems that have two identical numbers may be calculated by either addition or multiplication, they were not included in the test. Because pair-wise problems have the same answers for both problems in a pair, one participant would only see one problem of a pair (e.g., $7+6$ or 6 +7 ). Two answers, a correct one and an incorrect one were provided for each problem. The problems were randomly selected by using a random number generator, and the correct answers appeared on the right for half time and appeared on the left for half time. To avoid short-cut strategy usage by some participants, the incorrect answer was created to have some similarity to the correct value. It might have the same integer as the correct one at the unit digit or at the tenth digit. For example, the candidate answers for the problem $15+69$ were 84 and 94 , not 84 and 14 .

Small addition. Each small addition problem consisted of two single-digit
integers from 2 to 9 , without tie problems (e.g., $2+2$ ), and pair-wise problems only appeared once (e.g., $2+4$ or $4+2$, but not both). There were 28 possible combinations. The sums ranged from 5 to 17 .

Large addition. Each large addition problem consisted of two double-digit integers, without tie problems (e.g., $22+22$ ), and pair-wise problems only appeared once (e.g., $23+14$ or $14+23$, but not both). Twenty-eight problems were selected using a random number generator. The sums ranged from 26 to 94 .

Small multiplication. Each small multiplication problem consisted of two singledigit integers from 2 to 9 , without tie problems (e.g., $2 \times 2$ ), and pair-wise problems only appeared once (e.g., $2 \times 4$ or $4 \times 2$, but not both). There were 28 possible combinations. The products ranged from 6 to 72 .

Large multiplication. Each large multiplication problem consisted of one twodigit multiplicand (less than 50 to avoid large products), and one one-digit multiplier with integers from 2 to 9 . Twenty-eight problems were selected using a random number generator. The products ranged from 38 to 392 .

Follow-up questionnaire. In order to collect further information about participants, a questionnaire consisting of five parts as described below were used, in the following order (see Appendix B).

Demographic information. Demographic variables including age, gender, major, and ethnicity of the participants were collected.

Self-evaluation. Two questions were designed to gather information about participants' self-evaluation, one concerned the current study, and the other concerned
overall self-confidence in math skills. The first question was "How well do you think you just did in the arithmetic tasks?" and participants responded on a 5-point Likert Scale from 1 = "very poorly" to $5=$ "very well". The second question was "Are you good at math?" and participants responded on a 5-point Likert Scale from $1=$ "very poor" to $5=$ "very good".

Language background/skill survey. Language background and skill of each participant was assessed by self-report. Participants were asked if they were monolinguals or bilinguals. If bilingual, they were asked what their first and second languages were, and if they were bilinguals and the second language was English, they were asked at what age they learned English and in which language they were taught to solve simple arithmetic problems. Participants were also asked which language they used to solve mental arithmetic problems in the experiment, which language they spoke most frequently out of class, and which language their parents spoke at home. In addition, participants rated their language skills in reading, writing, speaking, and listening of both first and second languages on a Likert scale from $1=$ "very poor or no ability" to $5=$ "excellent ability".

Calculator use survey. Experience with calculator use was assessed with eight questions. "How often did you use a calculator when solving arithmetic problems like 35 +68 ?" Four questions were about their experiences during elementary school, and the
other four were about their current college experiences. These questions were blocked according to the period to which they applied.

The Abbreviated Math Anxiety Scale (AMAS; Hopke, Mahadevan, Bare, \& Hunt, 2003). The AMAS is a 9-item scale and was designed to assess mathematics anxiety. There were two subscales; one was to assess learning math anxiety (LMA), and the other was to assess math evaluation anxiety (MEA). The questions began with one of two phrases: "Listening to a lecture in math class" for LMA, and "Thinking about an upcoming math test 1 day before" for LMA. The AMAS has an excellent internal consistency overall $(\alpha=.90)$, as well as for the LMA $(\alpha=.85)$ and MEA subscales $(\alpha=$ .88). Participants responded on a 5-point Likert scale, from $1=$ "low anxiety" to $5=$ "high anxiety", with the total score representing a summation of the nine items. The Internal consistency (Cronbach's Alpha) in the current study was 0.89 .

## Apparatus

The arithmetic problems appeared on the center of a computer monitor controlled by a desktop computer with a windows operation system. Stimuli were presented electronically using the E-Prime software program (Psychology Software Tools, Pittsburgh, PA). Reaction times were collected with high accuracy. Participants were seated about 70 cm from the video screen and responded to arithmetic problems and strategy choices by pressing corresponding keys of the keyboard.

## Results

In this section, we examine the mental arithmetic performance data within the 12 cells defined by the experimental design: three language groups by two problem types by two problem sizes. We also look into the follow-up questionnaire. First, the descriptive statistics of mean reaction times, mean percent correct, and the percentage of each strategy selected for each language group in each manipulated condition are presented. Next, the tests of our hypotheses from two mixed ANOVAs, one for mean reaction time, the other for mean percent correct are presented. Then group differences on strategy selection (mainly focused on Remember) are examined. Finally, some exploratory analyses on questionnaire data are reported, to see if any valuable information could be revealed.

## Descriptive Statistics

The mean reaction times (ms) and mean percent correct (\%) for each group in each condition are presented in Table 1.

Table 1
Mean Reaction Time and Mean Percent Correct for each Condition as a Function of Group

|  | English Monolingual $(n=47)$ | SpanishEnglish Bilingual $(n=29)$ | ChineseEnglish Bilingual ( $n=13$ ) | $\begin{gathered} \text { Total } \\ (N=89) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Mean RT (ms) |  |  |  |  |
| Small <br> Addition | 1807 (624) | 2271 (1192) | 1278 (355) | 1881 (883) |
| Small <br> Multiplication | 1809 (788) | 1943 (733) | 1274 (319) | 1774 (746) |
| Large <br> Addition | 3784 (1018) | 5155 (3152) | 2877 (990) | 4098 (2116) |
| Large <br> Multiplication | 6256 (2661) | 6475 (2330) | 4013 (1073) | 5999 (2505) |
| All Problems Combined | 3414 (1138) | 3961 (1141) | 2360 (1139) |  |
| Mean Percent Correct (\%) |  |  |  |  |
| Small Addition | 96.89 (4.25) | 95.20 (5.25) | 96.98 (4.08) | 96.35 (4.60) |
| Small <br> Multiplication | 95.59 (6.71) | 95.57 (3.90) | 96.43 (3.26) | 95.71 (5.47) |
| Large <br> Addition | 93.47 (7.04) | 93.84 (5.04) | 95.33 (5.72) | 93.86 (6.23) |
| Large <br> Multiplication | 83.82 (13.69) | 85.35 (11.63) | 85.71 (11.57) | 84.59(12.65) |
| All Problems Combined | 92.40 (4.80) | 92.50 (4.85) | 93.60 (0.50) |  |

Note. Values inside parentheses are standard deviations.

In general, participants spent less time on small problems $(M=1730, S D=84)$ than on large problems $(M=4760, S D=212)$, and they spent less time on addition problems $(M=2862, S D=163)$ than multiplication problems $(M=3628, S D=172)$. With respect to effects of group, Chinese-English bilinguals produced the fastest RT overall ( $M=2360, S D=316$ ), followed by English-only $(M=3414, S D=212)$, and Spanish-English bilinguals were the slowest ( $M=3961, S D=212$ ).

Interestingly, a consistent order of reaction times existed across four problem types among the three groups: Chinese-English (C-E) < English only (E) < SpanishEnglish (S-E). For each problem type, there were 6 possible orders among three groups (i.e., $\mathrm{C}-\mathrm{E}<\mathrm{E}<\mathrm{S}-\mathrm{E}, \mathrm{C}-\mathrm{E}<\mathrm{S}-\mathrm{E}<\mathrm{E}, \mathrm{E}<\mathrm{S}-\mathrm{E}<\mathrm{C}-\mathrm{E}, \mathrm{E}<\mathrm{C}-\mathrm{E}<\mathrm{S}-\mathrm{E}, \mathrm{S}-\mathrm{E}<\mathrm{E}<\mathrm{C}-\mathrm{E}$, \& S-$\mathrm{E}<\mathrm{C}-\mathrm{E}<\mathrm{E}$ ), so the probability of this particular pattern would occur across all problems was $(1 / 6)^{4}=1 / 1296$, which is less than 0.01 .

The mean percent correct for each group in each condition are also presented in Table 1. Small addition $(M=96.35, S D=4.60)$ and small multiplication $(M=95.71, S D$ $=5.47$ ) problems had the highest average percent correct, followed by large addition ( $M$ $=93.86, S D=6.23)$ and large multiplication $(M=84.59, S D=12.65)$.

The mean percentages of each strategy chosen when participants solved different types of problems are presented in Table 2.

Table 2

Mean Frequency of Strategy Selection for each Condition as a Function of Group

|  | English Monolingual ( $n=47$ ) | SpanishEnglish ( $n=29$ ) | Chinese- <br> English <br> ( $n=13$ ) | $\begin{aligned} & \text { Total } \\ & (N=89) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Small Addition |  |  |  |  |
| Remember | 71.35 (28.28) | 67.00 (28.04) | 79.12 (28.86) | 71.07 (27.71) |
| Transform | 4.49 (8.51) | 4.19 (8.76) | 1.11 (1.73) | 3.90 (8.01) |
| Count | 16.34 (24.16) | 22.54 (24.15) | 3.57 (9.09) | 16.49 (23.21) |
| Estimate | 1.75 (5.31) | 1.97 (4.54) | 3.02 (6.17) | 2.01 (5.17) |
| Calculate | 4.94 (13.29) | 2.09 (3.99) | 10.98 (27.26) | 4.90 (14.38) |
| Other | 0.08 (0.53) | 0.37 (1.12) | 1.10 (2.25) | 0.32 (1.17) |
| Error | 1.07 (2.09) | 1.85 (2.80) | 1.10 (2.25) | 1.33 (2.37) |
| Small Multiplication |  |  |  |  |
| Remember | 89.59 (18.26) | 88.42 (19.44) | 84.05 (25.20) | 88.40 (19.61) |
| Transform | 1.45 (2.54) | 1.48 (2.61) | 0.28 (1.00) | 1.29 (2.42) |
| Count | 0.76 (2.88) | 0.00 (0.00) | 0.00 (0.00) | 0.40 (2.12) |
| Estimate | 3.42 (11.17) | 2.83 (7.04) | 2.75 (6.86) | 3.13 (9.36) |
| Calculate | 3.04 (9.65) | 5.42(19.20) | 10.45 (25.02) | 4.90 (16.06) |
| Other | 0.15 (0.74) | 0.12 (0.67) | 0.00 (0.00) | 0.12 (0.65) |
| Error | 1.60 (3.23) | 1.73 (3.11) | 2.48 (2.25) | 1.77 (3.05) |
| Large Addition |  |  |  |  |
| Remember | 3.50 (6.16) | 10.10 (12.91) | 6.59 (11.09) | 6.10 (9.93) |
| Transform | 10.79 (21.87) | 7.64 (14.62) | 2.47 (6.07) | 8.55 (18.20) |
| Count | 19.45 (33.23) | 27.22 (28.54) | 1.65 (5.94) | 19.38 (30.13) |
| Estimate | 23.56 (26.20) | 18.96 (22.90) | 17.03 (17.20) | 21.11 (23.94) |
| Calculate | 32.60 (34.76) | 33.25 (34.49) | 70.33 (19.02) | 38.32 (35.17) |
| Other | 8.29 (25.68) | 1.11 (3.84) | 1.10 (2.25) | 4.90 (19.05) |
| Error | 1.83 (3.41) | 1.72 (2.63) | 0.83 (1.58) | 1.65 (2.95) |
| Large Multiplication |  |  |  |  |
| Remember | 10.64 (8.73) | 12.07 (12.86) | 15.38 (11.70) | 11.80 (10.67) |
| Transform | 10.11 (19.78) | 8.98 (12.17) | 4.40 (7.31) | 8.91 (16.21) |
| Count | 5.93 (19.14) | 15.14 (27.64) | 0.82 (2.13) | 8.19 (21.49) |
| Estimate | 25.46 (29.57) | 17.37 (17.59) | 21.43 (17.57) | 22.23 (24.72) |
| Calculate | 43.85 (35.31) | 43.48 (34.20) | 45.90 (26.43) | 44.03 (33.46) |
| Other | 2.58 (5.85) | 1.23 (2.57) | 9.62 (27.25) | 3.17 (11.35) |
| Error | 1.44 (2.43) | 1.72 (2.95) | 2.47 (3.67) | 1.69 (2.80) |

Note. Values inside parentheses are standard deviations.

Participants mainly solved Small Addition (71.07\%) and Small Multiplication ( $88.40 \%$ ) problems using a direct retrieval strategy (Remember). Interestingly, ChineseEnglish bilinguals had a little higher percentage of Calculate (over 10\%) than the other groups (from 2 to 5\%) in the Small Addition and Small Multiplication conditions, but a lower percentage of Count ( $3.57 \%$ vs. $16.34 \%-22.54 \%$ ) in the Small Addition condition.

Different from small problems, Remember was no longer the favorite strategy for large problems. Specifically, for Large Addition problems, the most frequent strategies used by all participants were Calculate (38.32\%), Estimate (21.11\%) and Count (19.38\%). Over 70\% of Chinese-English bilinguals used Calculate to solve Large Addition problems. For Large Multiplication problems, the most frequently used strategies were Calculate ( $44.03 \%$ ) and Estimate ( $22.23 \%$ ) and all three groups seemed to have a similar pattern in their strategy choices.

## Hypotheses Testing

The first hypothesis, based on the results of prior studies, was that ChineseEnglish bilinguals would perform better in solving mental arithmetic problems than English monolinguals and Spanish-English bilinguals. The second hypothesis was about problem features, in which it was predicted that participants would perform better in small problems than in large problems. The first and second hypotheses were examined
in two $3 \times 2 \times 2$ ANOVAs, one on reaction time and the other on percent correct. The third hypothesis was that Chinese-English bilinguals would have higher percentage of using retrieval strategies. This hypothesis was tested using four separate one-way ANOVAs by problem types.

Hypothesis 1 and 2. To analyze reaction time, a mixed ANOVA was performed with language group as a between-subject variable and problem size (small, large) and operation (addition, multiplication) as within-subject variables. Refer to Table 3 to see the analysis of variance for mean reaction times.

The results revealed a main effect of language group, $\mathrm{F}(2,86)=8.87, p<.001$ (see below planned comparison for more details), which supported the first hypothesis. The analysis also revealed that the problem size main effect was significant, indicating that participants solved small problems faster than large problems, $\mathrm{F}(1,86)=338.18, p<$ .001. So the second hypothesis was also supported.

In addition, the Operation main effect was significant; participants were faster on addition problems than on multiplication problems, $\mathrm{F}(1,86)=16.46, p<.001$. The group by problem size interaction effect was also significant, $\mathrm{F}(2,86)=5.78, p=.004$. An examination of the means revealed greater differences among language groups for large problems than for small problems. The problem size by operation interaction effect was also significant, $\mathrm{F}(1,86)=37.39, p<.001$. An examination of the means revealed
that there was a greater difference between Addition and Multiplication for large problems than for small problems.

Table 3

Analysis of Variance for Mean Reaction Times

| Effect | df | MS | F | $p$ |
| :---: | :---: | :--- | :--- | :--- |
| Between subjects |  |  |  |  |
| Group (G) | 2 | $46.12 \times 10^{6}$ | 8.87 | $<.001$ |
| Error | 86 | $5.20 \times 10^{6}$ |  |  |

Within subject: Problem Size (S)

| Size (S) | 1 | $622.80 \times 10^{6}$ | 338.18 | $<.001$ |
| :--- | :--- | :--- | :--- | :--- |
| G X S | 2 | $10.64 \times 10^{6}$ | 5.78 | .004 |
| Error | 86 | $1.84 \times 10^{6}$ |  |  |

Within subject: Operation (O)

| Operation (O) | 1 | $39.79 \times 10^{6}$ | 16.46 | $<.001$ |
| :--- | :--- | :--- | :--- | :--- |
| G X O | 2 | $5.77 \times 10^{6}$ | 2.35 | .098 |
| Error | 86 | $2.42 \times 10^{6}$ |  |  |

Within subject: Problem Size (S) x Operation (O)

| S X O | 1 | $52.11 \times 10^{6}$ | 37.39 | $<.001$ |
| :--- | :--- | :--- | :--- | :--- |
| G X S X O | 3 | $2.94 \times 10^{6}$ | 2.11 | .127 |
| Error | 86 | $1.39 \times 10^{6}$ |  |  |

Pairwise comparisons among the three language groups (Tukey HSD) (Stoline, 1981) were performed to examine the group difference. Cohen's $d$ was calculated to measure effect size. Results revealed that the Chinese-English bilingual group was significantly different from other groups. Specifically, Chinese-English bilinguals are $1054 \mathrm{~ms}(S E=357)$ faster than English monolinguals, $p=.01$, Cohen's $d=4.18$; and Chinese-English bilinguals are $1601 \mathrm{~ms}(S E=380)$ faster than Spanish-English bilinguals, $p<.001$, Cohen's $d=5.78$. However, there was no significant difference between the English monolinguals and the Spanish-English bilinguals. The results regarding reaction times confirmed our hypothesis that Chinese-English bilinguals were faster in solving mental arithmetic problems than English monolinguals and SpanishEnglish bilinguals.

A mixed ANOVA on mean percent correct was also performed with language group as a between-subject variable and problem size (small, large) and operation (addition, multiplication) as within-subject variables. Refer to Table 4 to see the analysis of variance for mean percent correct.

Table 4

| Effect | df | MS | F | $p$ |
| :---: | :---: | :---: | :---: | :---: |
| Between subjects |  |  |  |  |
| Group (G) | 2 | . 003 | . 29 | . 75 |
| Error | 86 | . 010 |  |  |
| Within subject: Problem Size (S) |  |  |  |  |
| Size (S) | 1 | . 29 | 64.13 | <. 001 |
| G X S | 2 | . 003 | . 72 | . 49 |
| Error | 86 | . 004 |  |  |
| Within subject: Operation (O) |  |  |  |  |
| Operation (O) | 1 | . 16 | 29.18 | $<.001$ |
| G X O | 2 | . 002 | . 32 | . 72 |
| Error | 86 | . 006 |  |  |
| Within subject: Problem Size (S) X Operation (O) |  |  |  |  |
| S X O | 1 | . 13 | 25.74 | <. 001 |
| G X S X O | 2 | . 0001 | . 02 | . 98 |
| Error | 86 | . 005 |  |  |

No group difference was found, $\mathrm{F}(2,86)=0.29, p=.75$. Therefore, our first hypothesis that Chinese-English bilinguals would have higher percent correct than the
other two language groups was not supported. However, the results revealed a significant problem size main effect, $\mathrm{F}(1,86)=64.13, p<.001$, which supported the second hypothesis that participants would perform better in small problems than in large problems. Specifically, the average percentage of correct answers on small problems was higher than on large ones ( $96.03 \%$ vs. $89.23 \%$ ).

Moreover, the operation main effect was significant, $\mathrm{F}(1,86)=29.18, \mathrm{p}<.001$. The average percentage of correct answers on addition problems was higher than multiplication problems ( $95.11 \%$ vs. $90.15 \%$ ). The interaction effect of problem size by operation was also significant, $F(1,86)=25.74, \mathrm{p}<.001$. Further examination revealed that there was a greater difference between addition and multiplication for large problems than for small problems.

Hypothesis 3. Group differences on strategy selection (Remember only) were examined using four one-way ANOVAs, one for each problem type. However, no significant difference was found. Specifically, for small addition problems, $\mathrm{F}(2,86)=$ $0.86, p=.43$; large addition, $\mathrm{F}(2,86)=4.28, p=.02$; small multiplication, $\mathrm{F}(2,86)=$ $0.40, p=.67$; large multiplication, $\mathrm{F}(2,86)=1.02, p=.36$. Pairwise comparisons (Tukey HSD) (Stoline, 1981) among the three language groups were performed and revealed Spanish-English bilinguals had a $6.60 \%(S E=2.26)$ higher percentage of Remember than English monolinguals, $p=.01$; but no differences between Chinese-English bilinguals
and the other two groups. Thus, the results did not show that Chinese-English bilinguals were more likely to choose Remember to solve the same kind of problems than others, so this hypothesis was not supported.

## Exploratory Analyses

The descriptive data of the follow-up questionnaire by three language groups are presented in Table 5.

We analyzed the follow-up questionnaire using several separate one-way ANOVAs, but found no significant group difference on AMAS $[\mathrm{F}(2,86)=2.58, p=.08]$, Task Expectation $[\mathrm{F}(2,86)=.16, p=.86]$, Math Skill Expectation $[\mathrm{F}(2,86)=.01, p=$ .99], Calculator Use [total, $\mathrm{F}(2,86)=2.22, p=.11]$, and $\operatorname{GPA}[\mathrm{F}(2,75)=.59, p=.56]$.

When looking at the descriptive data (Table 5), Chinese-English bilinguals seemed to use calculators less than their peers, both in elementary school and in college, especially when solving large addition problems in elementary school $[F(2,86)=3.09, p$ $=.05]$; however, the difference is not statistically significant.

Table 5
Descriptive Data of Follow-up Questionnaire by Groups

|  | English Monolingual $(n=47)$ | SpanishEnglish ( $n=29$ ) | $\begin{gathered} \hline \text { Chinese- } \\ \text { English } \\ (n=13) \\ \hline \end{gathered}$ | Total $(N=89)$ |
| :---: | :---: | :---: | :---: | :---: |
| AMAS (Math Anxiety) | 21.89 | 26.10 | 23.92 | 23.56 |
| Task Expectation | 3.96 | 4.07 | 4.00 | 4.00 |
| Math Skill Expectation | 3.72 | 3.72 | 3.69 | 3.72 |
| Calculator Use | 17.09 | 17.66 | 13.77 | 16.79 |
| SmAdd_Elementary | 1.66 | 1.72 | 1.38 | 1.64 |
| LgAdd_ Elementary | 2.45 | 2.41 | 1.62 | 2.31 |
| SmMulti_ Elementary | 1.91 | 2.03 | 1.46 | 1.89 |
| LgMulti_ Elementary | 2.89 | 2.93 | 2.23 | 2.81 |
| SmAdd_College | 1.30 | 1.45 | 1.31 | 1.35 |
| LgAdd_College | 2.21 | 2.45 | 2.15 | 2.28 |
| SmMulti_College | 1.66 | 1.52 | 1.31 | 1.56 |
| LgMulti_College | 3.00 | 3.14 | 2.31 | 2.94 |
| GPA | $\begin{gathered} 2.86 \\ (n=42) \end{gathered}$ | $\begin{gathered} 2.89 \\ (n=24) \end{gathered}$ | $\begin{gathered} 3.06 \\ (n=12) \end{gathered}$ | $\begin{gathered} 2.90 \\ (N=78) \end{gathered}$ |
| Language Background |  |  |  |  |
| Reading_1st | 4.30 | 3.97 | 2.15 | 3.88 |
| Writing_1st | 4.15 | 3.72 | 2.00 | 3.70 |
| Speaking_1st | 4.32 | 4.14 | 3.46 | 4.13 |
| Listening_1st | 4.47 | 4.66 | 4.08 | 4.47 |
| Reading_2nd | N/A | 4.28 | 3.62 | N/A |
| Writing_2nd | N/A | 3.79 | 3.46 | N/A |
| Speaking_2nd | N/A | 4.31 | 3.77 | N/A |
| Listening_2nd | N/A | 4.62 | 4.46 | N/A |

With regard to language background, first, not surprisingly, English monolinguals learned English at an earlier age ( $M=2.17$ years, $S D=.29$ ) than did Spanish-English bilinguals ( $M=5.42$ years, $S D=2.45$ ) and Chinese-English bilinguals ( $M=4.50$ years, $S D=1.65)$, and the main effect of group was significant $[\mathrm{F}(2,35)=3.19, p=.05]$. In addition, participants rated their first language skills differently, including reading $[\mathrm{F}(2$, $86)=26.51, p<.001]$, writing $[\mathrm{F}(2,86)=26.02, p<.001]$, speaking $[\mathrm{F}(2,86)=6.59, p=$ $.002]$, and listening $[F(2,86)=3.30, p=.42]$. In general, Chinese-English bilinguals rated their language skills lower than their peers, especially in reading, writing, and speaking; in contrast, Spanish-English bilinguals and English monolinguals had similar ratings regarding their first language. In rating their second language English, ChineseEnglish bilinguals rated their reading and speaking abilities a little lower than SpanishEnglish bilinguals, but rated similarly for writing and listening.

There was no main effect of order on arithmetic performance in our experiment: specifically, for small addition reaction time $(\mathrm{RT}), \mathrm{F}(3,85)=0.39, p=.76$; large addition RT, $\mathrm{F}(3,85)=0.71, p=.55$; small multiplication $\mathrm{RT}, \mathrm{F}(3,85)=1.98, p=.12$; large multiplication $\mathrm{RT}, \mathrm{F}(3,85)=1.40, p=.25$; small addition percent correct, $\mathrm{F}(3,85)=$ $0.51, p=.68$; large addition percent correct, $\mathrm{F}(3,85)=0.23, p=.88$; small multiplication percent correct, $\mathrm{F}(3,85)=2.35, p=.08$; large multiplication percent correct, $\mathrm{F}(3,85)=$
$1.00, p=.40 ;$ ). The results indicated that the order that we presented the four blocks of problems had no significant influence on the results.

The gender effect $(\mathrm{N}=85)$ on arithmetic performance (in terms of reaction time and percent correct) of different problem types was tested. Gender differences only existed in the percent correct of large multiplication problems $[\mathrm{F}(1,83)=7.68, p=.007]$; male students ( $M=88.78, S D=7.76$ ) had a higher average percent correct than females $(M=81.21, S D=14.77)$.

The Pearson correlations among the variables in the questionnaire and reaction times for different problem types were also investigated (see Table 6).

First, GPA had no correlation with any other variables, which is interesting and surprising. Second, the participants who had higher task expectation and math skill expectation scores, also had lower levels of math anxiety and used a calculator less often, and solved small problems (both addition and multiplication) faster as well. Moreover, the more frequently students used a calculator, the lower their self-expectations were, the higher levels of math anxiety they had, and the more they spent time solving arithmetic problems (except large multiplication ones). Finally, reaction times for the four different types of problems were positively correlated; meaning math performance was consistent within individuals across conditions.

Table 6

Correlations among Questionnaire Variables and Reaction Time of each Condition

|  |  | $\begin{aligned} & \text { Task } \\ & \text { Exp } \\ & (N=89) \end{aligned}$ | $\begin{aligned} & \text { Math } \\ & \text { Exp } \\ & (N=89) \end{aligned}$ | AMAS $(N=89)$ | $\begin{gathered} \text { GPA } \\ (N=78) \end{gathered}$ | Cal use $(N=89)$ | SmAdd RT ( $N=89$ ) | $\begin{aligned} & \text { LgAdd } \\ & \text { RT } \\ & (N=89) \end{aligned}$ | SmMul tRT $(N=89)$ | LgMult RT ( $N=89$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Task $r 1$ |  |  |  |  |  |  |  |  |  |  |
| Exp $p$ |  |  |  |  |  |  |  |  |  |  |
| Math |  | . 653 | 1 |  |  |  |  |  |  |  |
| Exp |  | $<.001$ | - |  |  |  |  |  |  |  |
| AM |  | -. 379 | -. 594 | 1 |  |  |  |  |  |  |
| AS | $p$ | $<.001$ | < . 001 | - |  |  |  |  |  |  |
|  | $r$ | . 116 | . 131 | . 030 | 1 |  |  |  |  |  |
| GPA |  |  |  |  |  |  |  |  |  |  |
|  | $p$ | . 311 | . 253 | . 793 | - |  |  |  |  |  |
| Cal | $r$ | -. 419 | -. 460 | . 543 | . 134 | 1 |  |  |  |  |
| use | $p$ | $<.001$ | $<.001$ | $<.001$ | . 243 | - |  |  |  |  |
| Sm | $r$ | -. 290 | -. 328 | . 391 | -. 094 | . 471 | 1 |  |  |  |
| Add |  |  |  |  |  |  |  |  |  |  |
| Lg | $r$ | -. 032 | -. 200 | . 298 | -. 029 | . 416 | . 826 | 1 |  |  |
| Add |  |  |  |  |  |  |  |  |  |  |
| Sm | $r$ | -. 291 | -. 268 | . 176 | -. 186 | . 331 | . 612 | . 549 | 1 |  |
| Mult 006 . 011 . 099 -002 $<001<001$ |  |  |  |  |  |  |  |  |  |  |
| Lg | $r$ | . 091 | . 104 | -. 080 | -. 120 | . 060 | . 291 | . 324 | . 552 | 1 |
| Mult RT | $p$ | . 395 | . 330 | . 454 | . 297 | . 577 | . 006 | . 002 | <. 001 | - |

## Discussion

In the current study, relationships among language groups (i.e., English monolinguals, Spanish-English bilinguals, \& Chinese-English bilinguals), problem features (i.e., problem size and operation), and mental arithmetic performance (in terms of reaction time and percent correct) were examined.

The results for problem size and operation provided face validity for the results that would be used for examining language effects. All groups performed better in small problems than in large ones, as large problems took longer to solve and were correctly solved less often than small problems. Multiplication was more difficult than addition, particularly for large set size. These results were consistent with past research (e.g., Campbell \& Xue, 2001) and our own intuition about problem difficulty. Although some problems were more difficult than others, the average percent correct was very high for all problem types (ranging from $84.59 \%$ to $96.35 \%$ ), suggesting that the participants were motivated to perform well. Thus, the findings discussed below are based on data in which there is reason to have strong confidence.

The language effects were more pronounced for large problems than for small problems. Because performance in the small problems was rapid and almost perfect (only about a $4 \%$ error rate), it was reasonable to interpret the lack of a language effect as a consequence of ceiling performance.

Our results on mental arithmetic performance among groups revealed that all three language groups did equally well on solving problems correctly, and SpanishEnglish bilinguals were equivalent to their English monolingual peers in terms of reaction time. However, as predicted and consistent with past research (Campbell \& Xue, 2001; Imbo \& LeFevre, 2009), Chinese-English bilinguals spent less time on solving problems than their English monolingual and Spanish-English bilingual peers, especially when the problems consisted of large numbers. Finally, Chinese-English bilinguals did not have a higher frequency of use of retrieval strategies than the other two groups.

## Cultural Differences

Why did Chinese-English bilinguals perform better than English monolinguals and Spanish-English bilinguals in our study? The reasons may include linguistic influence, parental value and involvement as well as motivation and practice.

The difficulty of the number naming system in some languages may account for the numeracy advantage of Chinese-English bilinguals over English monolinguals and Spanish-English bilinguals whose languages contain many irregularities (see Appendix C). In Chinese, the base-ten rule is transparently represented in the structure of the counting number words themselves. Numbers after ten follow a precise logic and a consistent pattern that the tens word precedes the units word. For example, "十一" (11) is formed as ten one, pronounced as "shi-yi" in Mandarin or "sahp-yat" in Cantonese, and
＂二十一＂（21）is formed as two ten one，pronounced as＂er－shi－yi＂in Mandarin or＂yih－ sahp－yat＂in Cantonese，and so on．In English，there are the special words＂eleven＂and ＂twelve，＂and then the rest of the teens decade has the unit word preceding the tens word， （e．g．，＂thirteen＂as three ten）．Starting with twenty the order is then reversed，（e．g．， ＂twenty－four＂with the tens word twenty preceding the unit word four）．Given these differences it is not surprising that Chinese－speaking children learn place value earlier than English－speaking children．

Similar to English，in Spanish there are the special words from 11 to 15 （i．e．， ＂once＂，＂doce＂，＂trece＂，＂catorce＂，and＂quince＂）．However，different from English，the rest of teens decade in Spanish has the unit word following the tens word＂dieci＂，（e．g．， ＂dieciséis＂（16）as ten six）．The patterns for numbers 21－29 are similar to those for teen numbers above 15 ，with the tens word＂veinti＂preceding the unit word，（e．g．，＂veintiuno＂ （21）as twenty－one）．It seems there are more consistent rules when forming counting numbers in Spanish，but still，the irregularities and variations make Spanish much more complicated than Chinese．

The brevity of the Chinese language for numbers may allow for a larger short－ term memory，which may speed up the number encoding and processing during mental arithmetic．Miller et al．（2000）found that U．S．kindergartners counted significantly more poorly than did their Chinese peers，and the differences were believed to be associated
with the differences in the structure of cardinal numbers in two languages. Ho and Fuson (1998) reported that Chinese-speaking 5-year-olds understood that teen numbers were composed of tens and ones while their English-speaking peers showed no evidence of understanding. Therefore, the internal logic in counting numbers in Chinese may result in these children's better or earlier understanding of the base-ten principle in numbers and then doing a better job in counting and calculating than children who speak English and who experience more irregularity in naming numbers. There was no literature found on how number naming in Spanish may influence Spanish-speakers learning of number concepts, but based on the comparison between English and Chinese we can infer that language plays a role here too.

Most previous studies on how languages affect math were conducted on children; in the current study, our participants were college students, so whether the linguistic influence still remains in adults could be a question. Much of the work on quantitative development seems to presume that the number concepts and number sense acquired at young ages affect overall math ability later on, including mental calculation. A possible scenario that would support this presumption is as follows: advantages in the language may make Chinese-speakers a little more likely to enjoy math, they then might spend more time and be more willing to practice math skills, this might lead to higher scores on
math assignments and tests，and this success could lead to higher motivation to do well in math which would continue the cycle．

The differences among languages exist not only in cardinal numbers，but also in the ordinal numbers（see Appendix D）．In Chinese，ordinal names are formed by adding the sequence prefix＂第＂（read as di in Mandarin or dai in Cantonese）to any cardinal number（e．g．，＂第二＂（2nd）as di／dai two）．However，naming rules are more complicated in English．There are special words＂first＂，＂second＂，and＂third＂，and then the suffix ＂th＂is added to cardinal numbers，sometimes in modified forms．For example，＂fourth＂ as four－th follows the general rule，while＂fifth＂as five－th is irregular．Also，the special words＂first＂，＂second＂，or＂third＂apply to all ordinal numbers above 20 whose units word is one，two or three（e．g．，＂twenty－first＂）．In Spanish，there are special words from 1 st to 10 th（i．e．，primero，segundo，tercero，cuarto，quinto，sexto，séptimo，octavo，noveno， décimo）．For larger numbers，it is common to simply use the cardinal number as the ordinal ones．In the formal usage for ordinal numbers above 10th，there are special words ＂undécimo＂（11th）and＂duodecimo＂（12th），and then the rest of teen decade has the tens word＂décimo＂preceding the units word，（e．g．，＂decimotercero＂（13th）as tenth third）． Similarly，＂vigésimo primero＂（21st）is formed with the tens word＂vigésimo＂（twentieth） and the units word＂primero＂（first）．

Miller et al. (2000) also found that acquisition of ordinal number names is significantly more difficult for English-speaking children than Chinese-speaking peers. Based on the linguistic analysis above, irregularity of the number naming system in English may make it difficult for children to learn numbers, especially to learn ordinal numbers. Miller found that only $30 \%$ of the English-speaking sample was able to count correctly with ordinal numbers up to 21 st, whereas more than $95 \%$ of the Chinesespeaking peers were able to do so. A big drop-off in successful counting rate between 19th and 21st occurred for English-speaking children, from nearly $70 \%$ to $30 \%$. The popular nonstandard ordinal rules produced by English-speaking children included ordinal + cardinal rule (e.g. twentieth-one), over-regularizing "-th"(e.g. twenty-oneth) and "teenth" (e.g. twenty-teenth). No literature in cross-cultural studies was found on ordinal number names in Spanish. Based on linguistic analyses and similar reasoning as mentioned above, we believe that learning both cardinal and ordinal number names is more difficult for Spanish speakers than Chinese speakers. Therefore, language, specifically, the difficulty level of the number naming system in different languages could cause differential achievements of different language speakers in mental arithmetic.

In addition to linguistic influence, a number of studies on cross-cultural differences on mathematics achievement (many between East Asia and the United States) have found that "culture" played an important role, such as instructional approaches and content in
school (e.g., Steven, Lee \& Stigler, 1986), parental attitude and involvement (e.g., Huntsinger, Jose, Liaw \& Ching, 1997), student's motivation (e.g., Chen \& Stevenson, 1995). In the current study, our participants were all recruited from the SJSU campus and most of them grew up in the US, so the impact of variations in schooling between groups is probably not great. Other cultural factors, however, could still be different and later influence their mathematics achievement.

Siegler and Mu (2008) attributed Chinese children's better performance in arithmetic compared to their American peers to their parents' greater involvement with teaching activities. The well-practiced activities, such as counting fingers, conveyed redundant information about numerical magnitudes, so they believed playing a numerical game could improve preschoolers' numerical sense. In the United States, it could be that Chinese-English bilinguals learned more numerical knowledge than their peers from their parents who were more likely to explicitly teach their children at home and use more drill and practice-oriented methods (Huntsinger \& Jose, 2009).

Imbo and LeFevre (2009) discovered practice and training were emphasized at schools in Asian countries; therefore, Chinese participants were highly practiced and automated both the execution of strategies (resulting in high efficiency or fast response) and the strategy selection process (resulting in low adaptivity levels when choosing among different strategies). In contrast, in European and North American schools,
exploration and flexibility were more highly favored, which could explain the higher adaptivity levels among Belgians and Canadians, and also their lower efficiency levels. In our study, the parents of Chinese-English bilinguals probably retain the cultural belief in the power of practice, so their children would have extra math practice at home.

Comparing Asian-American and Caucasian-American high school students, Chen and Stevenson (1995) found factors associated with the better achievement of the former included having positive attitudes about achievement and mathematics, having parents and peers who hold high standards, believing that the road to success is through effort, enrolling in more challenging courses, studying diligently, and facing less interference with their school work from jobs and informal peer interactions. Most of these factors could also explain the superior mental arithmetic performance of Chinese-English bilinguals in our study.

## Degree of Bilingualism

Our study found that Spanish-English bilinguals rated their language abilities on both first language (Spanish) and second language (English) higher than did ChineseEnglish bilinguals (the first language is Cantonese or Mandarin and the second language is English) on four aspects of language (i.e., reading, writing, speaking, and listening). Though they both rated their second language English better than their first language, Spanish-English bilinguals were more confident in their first language than Chinese-

English ones. This suggests that Spanish-English bilinguals may have higher levels of bilingualism than Chinese-English bilinguals overall. One reason could be the similarity of Spanish and English that makes it easy to transfer linguistic knowledge from one language to the other. The other reason could be the large number of people speaking Spanish in California and associated availability of bilingual services (e.g., bilingual legislated notices and official documents are required in Spanish and English) so that Spanish-English bilinguals have a better language-learning environment.

The degree of bilingualism might affect bilinguals' mental mathematic performance, especially when solving large problems. Geary et al. (1993) found that strong bilinguals were slower at executing the carry operation than weak bilinguals and monolinguals, although bilingualism had little effect on fact retrieval and encoding integers. Since large problems have more demands on working memory, less mental resource might be available for strong bilinguals to complete cognitive tasks.

## Grade Point Average (GPA)

In the current study, three language groups were found to have different mental arithmetic performance. However, there was no group difference in overall GPA, nor was there correlations between mental arithmetic performance and GPAs. Regarding this finding, there are two points worth mentioning.

First, nationwide assessments (i.e., National Assessment of Educational Progress (NAEP)) showed there are achievement gaps in math among races/ethnicities, such as a Caucasian-African American gap (White students perform better than Black ones) and a Caucasian-Hispanic gap (White students perform better than Hispanic ones). Also Asian Americans are often stereotyped as model students of academic achievement. In addition, according to the Office of Institutional Research at SJSU (oir.sjsu.edu), for the SJSU population from which the participants were drawn, academic performance of Asians and Whites is very similar, and substantially above that of Latino and African American students. Based on the above observation, we expected to see some difference in GPAs in our study, but no differences were found; all three groups have similar overall GPAs. Some possible explanations for the lack of GPA difference in groups are discussed below in a separate section.

Second, math does matter. Researchers have found that math capability is an important predictor of overall academic performance and can even affect future careers and earnings. From an early age, kindergarten number competence can predict later math outcome (Jordan, Kaplan, Ramineni, \& Locuniak, 2009). Later on, high school math performance and course completion (e.g., algebra I, which is typically taken in ninth grade and the grade not used to compute eligibility for college admission) can predict college enrollment and college-prep GPA (Cooper et al., 2002; Witkow \& Fuligni, 2011).

Witkow and Fuligni (2011) found that $75 \%$ of participants who had completed algebra I, geometry, and algebra II during high school were enrolled in a four-year college or university, whereas only $18.6 \%$ of those who did not complete these courses were enrolled in a four-year institution.

Rose and Betts (2001) discovered that taking a richer math curriculum in high school increased not only the probability of graduating from college, but also students' earnings 10 years after graduation from high school. Ethnic differences also exist in what math courses were or were not taken in high school. For example, $51 \%$ of the black and Hispanic students only took vocational math and pre-algebra courses and did not take any algebra/geometry or higher level courses, which was nearly double the rate for white students (27\%) and three times the rate for Asian students (17\%).

In our results, Chinese-English bilingual students performance in mental arithmetic was superior to both English monolinguals and Spanish-English bilingual students who in general did not differ from one another. The group differences might be partly due to their math class completion and/or performance before college.

We believe the mental arithmetic performance could reflect participants' basic number sense and even their overall math capability. However, no correlation between overall GPA and mental arithmetic performance was found, which conflicted with the findings reviewed above that math performance is an important predictor of college
performance. There are two possible explanations. First, math performance can predict academic achievement but mental arithmetic performance cannot. Mental arithmetic is only a small part of math skills, and the performance on one test can be affected by numerous other factors, such as weather, motivation, emotion, and previous experience on specific techniques. Unlike mental arithmetic, math performance in high school often reflects cumulative average grades over time, or at least calculated in a given academic term, so it is probably more reliable. Second, the GPA data in our study had insufficient power, and this lowered its predictability. We recruited 134 participants in total, but only 78 out of 89 qualified participants ( $87.64 \%$ ) had valid GPA reported. Additionally, because the participants were in different school years, the number and nature of courses one student took could be different from the other student, and one's GPA may not be comparable to the other.

## Calculator Use

Previous studies showed calculator use was not encouraged in East Asian cultures, and native Chinese rarely use a calculator to solve simple arithmetic problems at school (Campbell \& Xue, 2001). So we were curious about whether or not ChineseEnglish bilinguals would use calculators less often than Spanish- and English-speaking peers. No group difference was found, although Chinese-English bilinguals reported using a calculator a little less often than the other two groups (13 versus 17-18). Our
results are inconsistent with previous findings. This might be because most of our participants attended K-12 schools in the United States and received comparable education and instruction, so their attitudes and behaviors concerning calculator use were similar.

However, less calculator use was found to be associated with a higher level of confidence in math and a lower level of anxiety as well as better mental arithmetic performance, as we predicted.

## Strategy and Neuropsychological Evidence

There were both differences and similarities between our strategy results and previous findings. For example, previous research (Campbell \& Xue, 2001) showed that non-Asian Canadian (NAC) (72\%) reported less use of retrieval overall than both Chinese Canadian (CC) (87\%) and Asian Chinese (AC) (85\%). However, the current study failed to show Chinese-English bilinguals were more likely to choose Remember to solve the same kind of problems than the other two groups. Not only did we examine the overall retrieval use for all problems across different groups, but also we examined the retrieval rate for each problem in different problem types across language groups; no language effect was found. Additionally, our strategy results showed a lower rate of retrieval, compared to previous findings. For example, in Campbell \& Xue's (2001) study, AC reported $97 \%$ retrieval strategy usage for small addition and $100 \%$ for small
multiplication problems, but the Chinese-English bilinguals in our study only reported $79 \%$ and $84 \%$ for the same problem types. The retrieval rates reported by other groups in our study for the same types of problems (ranged from $67 \%$ to $90 \%$ ) were also lower than those reported by NAC in their study (ranged from $88 \%$ to $98 \%$ ).

However, the pattern of strategy reports was coherent and consistent with previous studies (e.g., Campbell \& Xue, 2001). Whereas retrieval (Remember) was the predominant strategy reported in small problems, procedures (e.g., Calculate, Estimate, Count) were reported more often for large problems. This was expected because small problems were encountered more frequently and were easier to memorize. Interestingly, for large addition problems, the 70\% Calculate selection reported by Chinese-English bilinguals in our study, were much higher than $33 \%$ reported by other groups; while less than $2 \%$ Count rate was reported by Chinese-English bilinguals, which were much lower than Spanish-English bilinguals (27\%) and English monolinguals (19\%).

Campbell \& Xue (2001) suggested that NAC's relatively poor simple arithmetic performance resulted both from less efficient retrieval skills and greater use of procedural strategies. Although our study did not show that Chinese-English bilinguals had better retrieval skills, we do believe that the retrieval strategy is associated with better arithmetic performance, and this has been confirmed by both behavioral data and neuropsychological data.

Recently, neuropsychological studies applied brain scan techniques, such as functional magnetic resonance imaging (fMRI), to assess the activation of certain brain regions, which helped localize the underlying mechanism of brain activities during the process of mental arithmetic. In line with behavioral data, researchers suggested arithmetic processing in the brain are shaped by cultures. Using fMRI, Tang el al. (2006) demonstrated a differential cortical representation of numbers between native Chinese and English speakers. They found that for a simple addition task, native English speakers largely employed a language process that relied on the left perisylvian cortices, while native Chinese speakers engaged a visuo-premotor association network. They suggested that faster processing due to the structure of Chinese language system for numbers could explain the lower activation of perisylvian areas in native Chinese speakers.

Furthermore, a number of fMRI studies showed that better performance in mental arithmetic was related to stronger activation of angular gyrus (e.g., Delazer et al., 2003; Grabner et al., 2007). Specifically, angular gyrus was associated with the retrieval of arithmetic facts from long-term memory (Grabner et al., 2009). For example, Grabner et al. (2007) found that the left angular gyrus was activated more strongly in small problems (i.e., single-digit multiplication) than in large problems (i.e., multi-digit multiplication), where three activation clusters located in the left inferior frontal gyrus and left thalamus occurred. They also found higher mathematical competence individuals (with higher
mathematical-numerical IQ and better performance in test conditions, but no difference in verbal and figural-spatial IQ, age, personality, and major) displayed stronger activation of the left angular gyrus and middle temporal gyrus when solving novel multiplication problems, compared to the lower math group. These findings indicated that the recruitment of the left angular gyrus might underlie individual differences of mathematical skills.

To examine training effects, Delazer et al. (2003) compared brain activation when solving trained or untrained arithmetic problems and they found trained problems activated greater activation of the left angular gyrus, whereas untrained problems were found to activate the intraparietal sulcus (IPS), suggesting a neural shift from the use of quantitative strategies to fact retrieval as a function of arithmetic training. In another study, Delazer et al. (2005) compared two types of training and they found learning by drill (learning the result of a problem) was related to greater activation of the angular gyrus, whereas problems learned by strategy elicited higher activation in frontal regions and in the precuneus.

Grabner et al. (2008) used trial-by-trail strategy self-report to directly test whether the angular gyrus mediates the retrieval of arithmetic facts during mental calculation. The fMRI data analysis revealed stronger activation of the left angular gyrus when participants reported using fact retrieval as their strategy to solve that problem. For the
problems that were solved by procedural strategies, such as counting and transformation, widespread activation in a fronto-parietal network was observed. Therefore, there was a link between angular gyrus and memory retrieval, and this link led to comparatively better performance in mental arithmetic.

It is exciting to see mental arithmetic studies were advanced with the aids of new technologies, and we believe that these methodologies could be productively used in cross-cultural and monolingualism/bilingualism studies. It would be particularly important to include assessments of the amount of drill and practice that is thought to influence the occurrence of retrieval strategies.

## Limitations

As we mentioned earlier, one third of total participants turned out to be unqualified even though they actually completed the whole experiment because this study had specific requirements about their language background and the unqualified did not belong to any designated language group. This factor reduced the sample size and lowered the power of our study.

Moreover, we relied on self-report to not only identify participants who met the requirements of our study, but also to run analyses on exploratory data we obtained from questionnaire. If some of the participants were not serious enough about the study, or
some held extremely different standards on certain questions/ratings, their response may not have been appropriate.

The language backgrounds of our participants were quite diverse, which were much more complicated than we anticipated. For example, we expected that the bilingual participants would not have been exposed to English before they learned basic mathematic rules, but, in fact, it was impossible to have strictly qualified participants since this study was conducted in the United States where English is the official language. Taking another example, we did not ask where bilingual participants were born, and if they emigrated from other countries, at what ages they came to the United States. Without asking such questions, we were unable to fully understand their language background.

From the after-experiment talk with participants, we learned most ChineseEnglish bilinguals were from immigrant families, and they were exposed to Cantonese/Mandarin only when they were little and they learned English later, but still, when they learned basic math remained unknown. There was one participant who completed her 16-year education in China and then came to the United States. The different experiences of learning math and language could have different impacts on their cognitive development process as well as cognitive skills.

The same problem existed in Spanish-English bilinguals. Some were originally from places where people only speak Spanish, some were born in Spanish-speaking families in California, and some were born in bilingual environments where parents/grandparents spoke both Spanish and English. A few participants even claimed themselves as simultaneous bilinguals because they were exposed to and learned both Spanish and English at the same time at early ages so they could not tell which one was their first language. Though we asked participants to rate their language skills in different aspects - reading, writing, speaking, and listening, it was still very difficult to evaluate and compare their overall levels of proficiency in different languages.

## Future Direction and Application

In light of current findings, several directions for future studies would be worthwhile undertaking. First, to assess the relationship of mental arithmetic and general academic performance, comparable GPAs should be considered (e.g., GE math), rather than overall GPA as we used in this study. Second, as we mentioned earlier, mental arithmetic is only a small part of overall math ability, so more work on the relationship between other aspects of mathematics and mental arithmetic, especially in adults, are needed. Lastly, investigation of cross-cultural difference in the connectivity and activation of the brain underlying mathematical processing will increase our understanding of how culture influences symbolic brain representations.

These findings and future investigations can further our understanding in numerical learning and intelligence, and may guide mathematical education to help students achieve their highest potentials.

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## Appendices

## Appendix A. Strategy Report Instruction

In this study, you are asked to indicate the strategy you used to solve each arithmetic problem immediately upon solving it. Studies have indicated that people are consciously or unconsciously using various strategies to solve arithmetic problems. Seven different strategies are proposed in the present experiment: Remember, Transform, Count, Estimate, Calculate, Other, and Error. Please refer to this instruction when you are uncertain about your strategy choice. Following are descriptions of each corresponding strategy:

1. Remember: You solve the problem by just remembering or knowing the answer directly from memory. Examples are "The answer 8 just jumps into my head when I see the problem $2 \times 4$," or "I just knew the answer 9 as I saw the problem $3+6$."
2. Transform: You solve the problem by referring to related operations or by deriving the answer from some known facts, such as rounding numbers to 10 . Examples are " $4+9$ equals 13 because it could be transformed into $4+6+3$," or " $29 \times 2=58$ because it could be transformed into $30 \times 2-1 \times 2=58$."
3. Count: You solve the problem by just counting a certain number of times to get the answer. Notice that here counting is defined as one by one strict counting. Examples
are "For the problem $25+3$, I counted 25, 26, 27, 28 in my head to get the answer 28 ," or "I got the answer by counting silently, such as $1,2,3,4,5,6,7,8,9,10$."
4. Estimate: You solve the problem by applying front end estimation, by evaluating the reasonableness of two provided answers, or making a sensible "guess", but you do not know the exact answer. Examples are "I know the answer to the problem $32+$ 25 is greater than 50 so I choose the answer 57 rather than 47 ," or "I know $5 \times 6$ equals 30 , so the unit digit of the answer to the problem $55 \times 6$ should be 0 , when I see the two answers 330 and 345, I think 345 must be wrong. "
5. Calculate: You solve the problem by applying standard algorithms, or you get the exact answer through actual step-by-step calculation. Examples are "When I solve the problem $24 \times 3$, I first calculate unit digit $4 \times 3=12$, then tenth digit $20 \times 3=60$, and add 12 and 60 , finally I get 72, ," or " $38+43=8+3+30+40=11+70=81 . "$
6. Other: You solve the problem by a strategy unlisted here, or you do not know what strategy you used to solve the problem, or you didn't solve the problem at all. Examples are "I do not know," "I cannot figure out what strategy I used," or "I just randomly guessed the answer."
7. Error: You press the wrong key as your response by accident, either you solve the problem correctly but press the wrong key or you just press the keyboard without solving the problem. Examples are "I do know the answer to the problem $3+2$ is 5 , and I
should press ' 1 ' key, but I actually pressed ' 2 ' key," or "Oops, I didn't see what's the problem yet."

## Appendix B. Follow-up Questionnaire

This questionnaire is to gather some background information about you, in order to investigate the relationship between language and mental arithmetic. Please put $X$ in the corresponding box or circle the number of the best choice. All data will be kept confidentially. Your honesty and your time and is highly appreciated. Thank you for your participation!

Student ID: $\qquad$ Major: $\qquad$ Age: $\qquad$

Gender: Male / Female

Ethnicity group:

1. Hispanic, Latino, or Spanish origin:

Mexican, Mexican American, Chicano

Puerto Rican

Cuban

Other (e.g., Colombian, Dominican, Nicaraguan)
2. White (non-Hispanic)
3. Asian/Pacific Islander:

Chinese (origin of mainland China, Taiwan, Hong Kong, and Macao)

Other Asian (e.g., Japanese, Korean, Hmong, Laotian, Thai, Pakistani,

[^0]4. African American
5. Native American
6. Other (please specify)

How well do you think you just did in the arithmetic tasks?

1. Very poorly
2. Poorly
3. Not sure
4. Well
5. Very well

How good or bad is your math skill?

1. Very bad
2. Bad
3. Not sure
4. Good
5. Very good

What language did you use to solve the arithmetic problems in the experiment?

1. English
2. Spanish
3. Mandarin
4. Cantonese
5. Other (please
specify)

What language do you speak most frequently outside of class?

1. English
2. Spanish
3. Mandarin
4. Cantonese
5. Other (please
specify) $\qquad$
What language do your parents speak to each other (or what language is used most at home)?
6. English
7. Spanish
8. Mandarin
9. Cantonese
10. Other (please
specify) $\qquad$

Are you bilingual (or multilingual)?

1. Yes
2. No

What is your first language (or the only language)?

1. English
2. Spanish
3. Mandarin
4. Cantonese
5. Other (please specify) $\qquad$

What is your second language?

1. English
2. Spanish
3. Mandarin
4. Cantonese
5. Other (please specify) $\qquad$

If your second language is English, please answer the following TWO questions:
What age did you learn English at? $\qquad$ years old.

In what language were you taught to solve simple arithmetic problems (e.g., $4+5=$ ?)

1. English
2. Spanish
3. Mandarin
4. Cantonese
5. Other (please specify) $\qquad$

Please rate your language skills, and put X in the corresponding cell.

| First language | 1. Very poor <br> or no ability | 2. Poor | 3. Medium | 4. Good | 5.Excellent |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Reading |  |  |  |  |  |
| Writing |  |  |  |  |  |
| Speaking |  |  |  |  |  |
| Listening | 1. Very poor <br> or no ability | 2. Poor | 3. Medium | 4. Good | 5. <br> Excellent |
| Second <br> language |  |  |  |  |  |


| Reading |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Writing |  |  |  |  |  |
| Speaking |  |  |  |  |  |
| Listening |  |  |  |  |  |

How often did you use a calculator in elementary school when solving arithmetic problems as ...

1) One-digit addition (e.g., $3+2=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
2) Two-digit addition (e.g., $35+18=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
3) One-digit multiplication (e.g., $5 \times 8=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
4) Two-digit multiplicand (less than 50) times one-digit multiplier (e.g., $32 \times 4=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always

How often did you use a calculator in college when solving arithmetic problems as ...

1) One-digit addition (e.g., $3+2=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
2) Two-digit addition (e.g., $35+18=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
3) One-digit multiplication (e.g., $5 \times 8=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always
4) Two-digit multiplicand (less than 50) times one-digit multiplier (e.g., $32 \times 4=$ ?)
1. Never
2. Rarely
3. Sometimes
4. Often
5. Always

Please rate your anxiety level in the following situations. Write the corresponding numbers in front of each sentence.

## 1. Low <br> 2. Low-medium <br> 3. Medium <br> 4. Medium-high <br> 5. High

$\qquad$ 1) Having to use the tables in the back of a math book.
$\qquad$ 2) Thinking about an upcoming math test 1 day before.
$\qquad$ 3) Watching a teacher work an algebraic equation on the blackboard.
$\qquad$ 4). Taking an examination in a math course.
$\qquad$ 5) Being given a homework assignment of many difficult problems that is due the next class meeting.
$\qquad$ 6) Listening to a lecture in math class.
$\qquad$ 7) Listening to another student explain a math formula.
$\qquad$ 8) Being given a "pop" quiz in math class.
$\qquad$ 9) Starting a new chapter in a math book.

## Appendix C．Counting Numbers in English，Spanish，and Chinese

| Number | English | Spanish | Chinese | Chinese <br> （Mandarin） | Chinese （Cantonese） |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | zero | cero | 零 | ling | lihng |
| 1 | one | uno | － | yi | yat |
| 2 | two | dos | 二 | er | yih |
| 3 | three | tres | 三 | san | saam |
| 4 | four | cuatro | 四 | si | sei |
| 5 | five | cinco | 五 | wu | ng |
| 6 | six | seis | 六 | liu | luk |
| 7 | seven | siete | 七 | qi | chat |
| 8 | eight | ocho | 八 | ba | baat |
| 9 | nine | nueve | 九 | jiu | gau |
| 10 | ten | diez | 十 | shi | sahp |
| 11 | eleven | on－ce | 十一 | shi－yi | sahp－yat |
| 12 | twelve | do－ce | 十二 | shi－er | sahp－yih |
| 13 | thir－teen | tre－ce | 十三 | shi－san | sahp－saam |
| 14 | four－teen | cator－ce | 十四 | shi－si | sahp－sei |
| 15 | fif－teen | quin－ce | 十五 | shi－wu | sahp－ng |
| 16 | six－teen | dieci－séis | 十六 | shi－liu | sahp－luk |
| 17 | seven－teen | dieci－siete | 十七 | shi－qi | sahp－chat |
| 18 | eigh－teen | dieci－ocho | 十八 | shi－ba | sahp－baat |
| 19 | nine－teen | dieci－nueve | 十九 | shi－jiu | sahp－gau |
| 20 | twenty | veinte | 二十 | er－shi | yih－sahp |
| 21 | twenty－one | veinti－uno | 二十一 | er－shi－yi | yih－sahp－yat |
| 22 | twenty－two | veinti－dós | 二十二 | er－shi－er | yih－sahp－yih |
| 23 | twenty－three | veinti－trés | 二十三 | er－shi－san | yih－sahp－saam |
| 24 | twenty－four | veinti－cuatro | 二十四 | er－shi－si | yih－sahp－sei |
| 25 | twenty－five | veinti－cinco | 二十五 | er－shi－wu | yih－sahp－ng |

## Appendix D．Ordinal Numbers in English，Spanish，and Chinese

| Number | English | Spanish | Chinese | Chinese （Mandarin） | Chinese （Cantonese） |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | first | primero | 第一 | di－yi | dai－yat |
| 2 | second | segundo | 第二 | di－er | dai－yih |
| 3 | third | tercero | 第三 | di－san | dai－saam |
| 4 | four－th | cuarto | 第四 | di－si | dai－sei |
| 5 | fif－th | quinto | 第五 | di－wu | dai－ng |
| 6 | six－th | sexto | 第六 | di－liu | dai－luhk |
| 7 | seven－th | séptimo | 第七 | di－qi | dai－chat |
| 8 | eigh－th | octavo | 第八 | di－ba | dai－baat |
| 9 | nin－th | noveno | 第九 | di－jiu | dai－gau |
| 10 | ten－th | décimo | 第十 | di－shi | dai－sahp |
| 11 | eleven－th | once／un－décimo | 第十一 | di－shi－yi | dai－sahp－yat |
| 12 | twelf－th | doce／duo－décimo | 第十二 | di－shi－er | dai－sahp－yih |
| 13 | thirteen－th | trece／decimo－ tercero | 第十三 | di－shi－san | dai－sahp－ saam |
| 14 | fourteenth | catorce／decimo－ cuarto | 第十四 | di－shi－si | dai－sahp－sei |
| 15 | fif－teen－th | quince／decimo－ quinto | 第十五 | di－shi－wu | dai－sahp－ng |

Note．In Spanish，cardinal numbers are used above 10th．


[^0]:    Cambodian)

