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# Time-delay compensation in a Boiling Water Reactor Feedwater-Vessel level Control system

Damon T. Genetti  
*San Jose State University*

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**Time-delay compensation in a boiling water reactor  
feedwater-vessel level control system**

Genetti, Damon Tyrone, M.S.

San Jose State University, 1994

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Ann Arbor, MI 48106



**TIME-DELAY COMPENSATION IN A  
BOILING WATER REACTOR FEEDWATER-VESSEL LEVEL  
CONTROL SYSTEM**

**A Thesis**

**Presented to**

**The Faculty of the Department of Mechanical Engineering  
San Jose State University**

**In Partial Fulfillment**

**of the Requirement for the Degree**

**Master of Science**

**by**

**Damon T. Genetti**

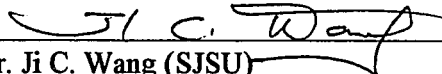
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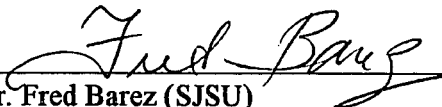
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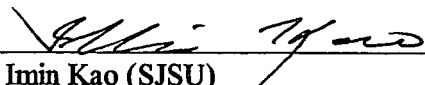
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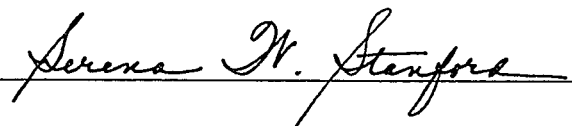
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## **Abstract**

### **TIME-DELAY COMPENSATION IN A BOILING WATER REACTOR FEEDWATER-VESSEL LEVEL CONTROL SYSTEM**

by Damon T. Genetti

Two classical methods of time delay compensation for continuous control systems and a Fuzzy Logic control method, are applied to models of a Boiling Water Reactor Feedwater-Vessel Control system and simulation results are presented. The objective is to examine the ability of these methods to stabilize otherwise unstable systems. These systems are continuous system models of discrete systems. The two classical methods examined are the Smith predictor and Watanabe & Ito Process Model Control. Simulations show that the Smith predictor provides excellent reference input responses, but the disturbance input response results in an error in finite time. The Watanabe & Ito Process Model Control system is shown to be impractical for high order systems. A combination of Integral and Fuzzy Logic control is shown to be a good alternative to classical delay compensation techniques. Potential testing of these methods on a Feedwater and Recirculation System Simulator is also discussed.

## **Acknowledgment**

I would like to thank the following people:

L. H. Youngborg of GENE for suggesting the topic of this study,

Dr. Ji Wang for guidance in this study,

Sui Cheng of GENE for allowing the use of the Fuzzy Logic controller he created  
and for his help in implementing it,

Edward Schmidt of GENE and SJSU students Mark Shands, Joe Brenner and  
Chris Lang for their help in understanding the GE Feedwater and Recirculation  
System Simulator,

Dr. Fred Barez and Dr. Imin Kao for reviewing this thesis.

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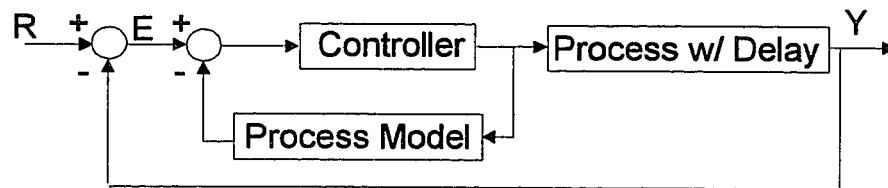
## **1.0 Introduction**

The original Primary non-safety process control systems of Boiling Water Reactors (BWRs) built by General Electric are analog systems that have been carefully designed and tuned to obtain a specified performance. Several of the BWR owners have retrofitted these systems with microprocessor-based devices. In some cases the entire control system has been replaced. The original analog design basis should be maintained when replacing the analog components with digital components. With this, the digital devices should provide the same performance as the analog devices. In most cases the retrofitted systems are providing acceptable performance. However, in a few cases, digital systems which were designed to the original analog design bases have provided unstable behavior. The instability of these systems is thought to be due in part to the cycle times of the of the digital devices [1].

The inherent discrete nature of digital systems impose physical limitations which are not shared by continuous, analog systems. The cycle time or sampling time period of a digital system is governed by operational and computational cycles, such as the clock speed of a microprocessor and the computation of a control algorithm. These cycle times act as a time delay in the system which can lead to instabilities. The adverse affects of a cycle time or time delay on the stability and performance of a digital control system were investigated by Wang [2] using simplified models of a BWR Feedwater Vessel-Level Control System. In that study a single sampling time period was used and considered to be a statistical mean of all the accumulated cycle times resulting from each digital device in the control system. The digital systems were modeled using discrete system techniques and continuous system techniques with the addition of a time delay to model the sampling time. It was shown that as the sampling

time period is increased the cyclic behavior of the system increases and eventually the system becomes unstable. Potential solutions to this problem were not discussed.

In this thesis two of the simplified BWR Feedwater Level Control systems presented by Wang are used to investigate the performance of two delay compensation methods and a Fuzzy Logic controller. The two delay compensation methods are a predictor method developed by Smith [3] known as the Smith predictor and a process-model control system developed by Watanabe and Ito [4]. Both compensation methods are for continuous linear systems with a time delay. The systems to which these methods are intended to be applied are single input, single output, unity feedback systems with a conventional controller and a time delay in the process. Both methods use mathematical models of the process in a minor feedback loop about the controller as seen in the figure below.



The Fuzzy Logic controller does not require a minor feedback loop, but it does require the error signal, an error rate signal ( $de/dt$ ) and an integrator in parallel. By using a continuous system with a time delay to model the digital system, the performance of these methods as potential solutions to the stability problem caused by the sampling time can be examined. The two Feedwater Level control systems to be used are the Three Element and Simplified One Element Flow-Level Control systems. Figures 1.1 and 1.2<sup>1</sup> show these systems as continuous systems with time delays. In the simplified system the dynamic compensator, feedwater flow sensor and steam flow sensor are not

<sup>1</sup>Note: numbered figures can be found at the end of their respective sections.

used. These systems have been modified to provide unity feedback systems which can be seen in the following sections.

All of the simulations have been performed using MATRIX<sub>X</sub><sup>2</sup> with SystemBuild software. MATLAB<sup>3</sup> software has also been used for some calculations.

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<sup>3</sup>MATLAB is a trademark of The Mathworks, Incorporated.

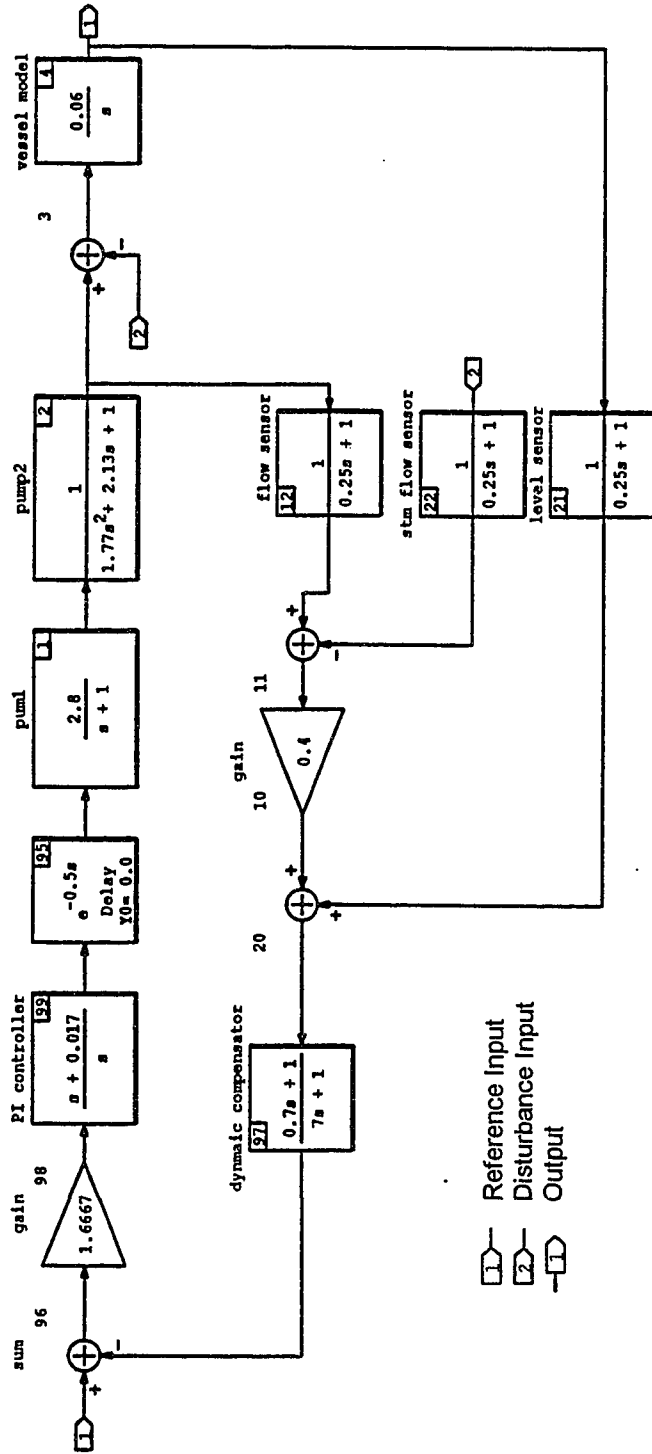


Figure 1.1 Three Element Flow-Level Control System

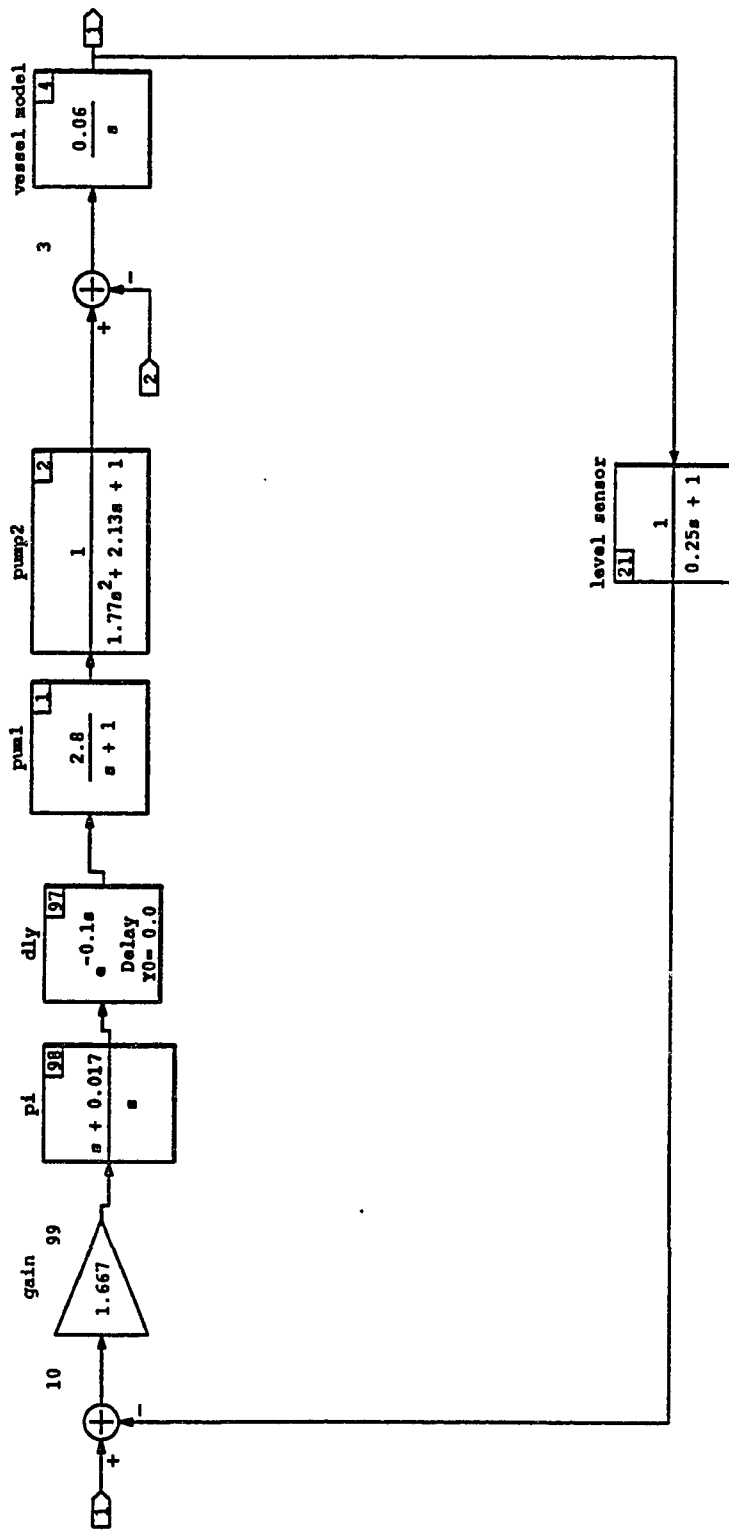
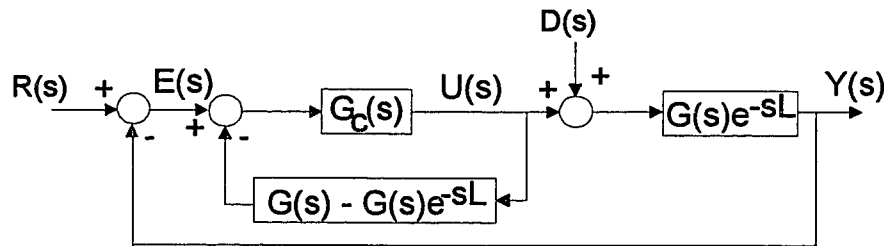


Figure 1.2 Simplified One Element Flow-Level Control System



## 2.0 Smith Predictor

The first process-model control system to be examined is the Smith predictor control system [3]. A block diagram of the basic system is shown in following figure, where  $G(s)e^{-sL}$  is the process with delay,  $G_c(s)$  is a conventional controller and  $G(s)-G(s)e^{-sL}$  is the predictor process model. The intent of this system is to remove the time delay from the controller design problem for a process with a time delay.



The transfer function of the inner loop is

$$\frac{U(s)}{E(s)} = \frac{G_c(s)}{1 + G_c(s)G(s) - G_c(s)G(s)e^{-sL}} \quad (1)$$

The overall transfer function of this system for a reference input is

$$T_r(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)e^{-sL}}{1 + G_c(s)G(s)} \quad (2)$$

The transfer function of the system without the predictor (minor feedback loop) is

$$T_r(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)e^{-sL}}{1 + G_c(s)G(s)e^{-sL}} \quad (3)$$

Note that the characteristic equation of the closed loop system with the predictor ( $1 + G_c(s)G(s) = 0$ ) does not contain a time delay and thus the delay does not affect the poles of the system. Since the poles of the system are not affected by the time delay, conventional controller design techniques can be used. The response of this system,

$y(t)$  to a reference input,  $r(t)$  will be delayed by an amount equal to the delay time,  $L$ . The reference input response curve of the system with delay will be virtually identical to that of the system without the delay except for a time shift equal to  $L$ .

The transfer function due to a disturbance input,  $d(t)$  alone is

$$T_d(s) = \frac{Y(s)}{D(s)} = \frac{[1 + G_c(s)G(s) - G_c(s)G(s)e^{-sL}]G(s)e^{-sL}}{1 + G_c(s)G(s)} \quad (4)$$

The Smith predictor does not handle a disturbance input as well as the reference input. The response of the system to a disturbance  $d(t)$  will be stable for systems with a time delay, but the responses to a disturbance will take longer to approach zero as the delay time is increased [4]. Thus the disturbance response may become unacceptably slow. This effect is caused by the  $(e^{-sL})^2$  term in the numerator of  $T_d(s)$ .

## 2.1 Feedwater Control System With Smith Predictor

The Smith predictor was first applied to a modified version of the Simplified One Element System which is shown in Figure 2.1. The output has been taken after the level sensor to provide a unity feedback system. Figure 2.2 shows the modified one element system with the Smith predictor. The pump dynamics, vessel model and level sensor have been combined into one block labeled process. Figure 2.3 shows the response of the modified one element system to a step reference input with and without the Smith predictor for various delay times. The gains of the proportional integral controller have been held constant at  $k_i=0.017$  and  $k_p=1.0$ . As the delay time is increased, the uncompensated system response becomes increasingly oscillatory and eventually unstable while the response of the system with the Smith predictor remains unchanged except for an initial delay equal to the system time delay.

The Smith predictor was next applied to the Modified Three Element system. With the controller gains held constant at  $k_i=0.017$  and  $k_p=1.0$ . Figures 2.4 and 2.5 show the Modified Three Element system and the Modified Three Element system with the Smith predictor. In this model the disturbance to the system, or disturbance input, would be a fluctuation in the steam flow. The steam flow sensor is not included in the Smith predictor because a disturbance in the steam flow would be unpredictable. Figure 2.6 shows the response of the Modified Three Element system to a step reference input and a step disturbance for various delay time values. As the delay time is increased the oscillation of the response increases. At a delay time of 3 seconds the system is approaching the verge of instability. Figure 2.7 shows the response of the system with the Smith predictor. Responses are plotted for delay times of 0 seconds (no delay), 0.1, 0.5, 1.0, 3.0 and 5.0 seconds. The response to a step reference input does not change except for an initial delay equal to the system delay time as seen with the one element system. This initial delay can be best seen in the plots for delay times of 3.0 and 5.0 seconds.

The shape of the disturbance response is also somewhat consistent. Like the step reference response, the rise time appears to be delayed by an amount equal to the system delay, but there is no initial delay. The response is immediate and rises on a consistent slope to the maximum peak, after which the responses have the same shape. In the general Smith predictor system the delay element is in between the disturbance input and the system output because the delay is a physical part of the process. In the Modified Three Element system the delay is intended to model the effects of a discrete system sampling time. The delay element is not between the disturbance input and the system output, thus the system immediately responds to the disturbance input. Steady state error analysis shows that the response to a step disturbance will have zero steady

state error as time approaches infinity. Within a practical finite time, the disturbance response contains a substantial error which increases as the delay time is increased. This can be seen in Figure 2.7 for delay times of  $L=3$  seconds and  $L=5$  seconds. Table 2.1 summarizes the results for the Three Element system with the Smith predictor.

Table 2.1 Three Element System With Smith Predictor

| Delay (sec) | Step Reference Input |                 | Step Disturbance |                   |
|-------------|----------------------|-----------------|------------------|-------------------|
|             | Max. Peak            | Rise Time (sec) | Max. Peak        | Error at 200 sec. |
| 0.0         | 1.3                  | 6.9             | - 0.260          | 0                 |
| 0.1         | 1.3                  | 7.0             | - 0.268          | 0.01              |
| 0.5         | 1.3                  | 7.4             | - 0.291          | 0.03              |
| 1.0         | 1.3                  | 7.9             | - 0.323          | 0.06              |
| 3.0         | 1.3                  | 9.9             | - 0.440          | 0.18              |
| 5.0         | 1.3                  | 12.9            | - 0.554          | 0.30              |

The seven roots of the characteristic equation for the Three Element Smith system are:

-3.992309  
 -0.921270 + 0.262358i  
 -0.921270 - 0.262358i  
 -0.169016 + 0.347346i  
 -0.169016 - 0.347346i  
 -0.155259  
 -0.018108

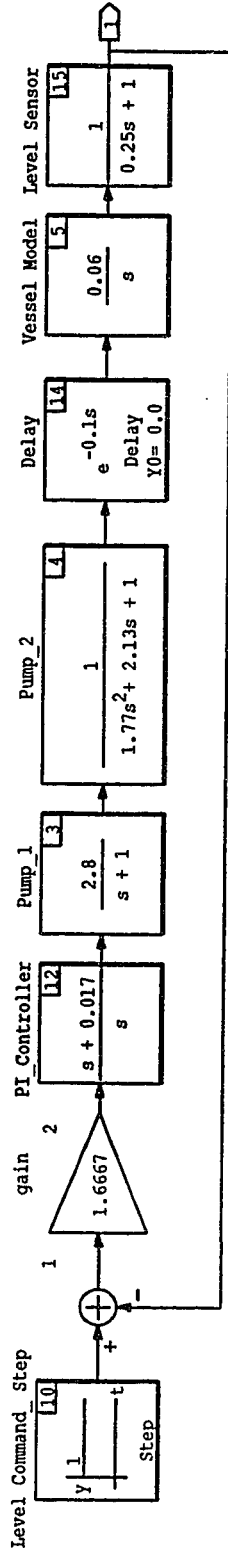


Figure 2.1 Modified One Element Flow-Level Control System

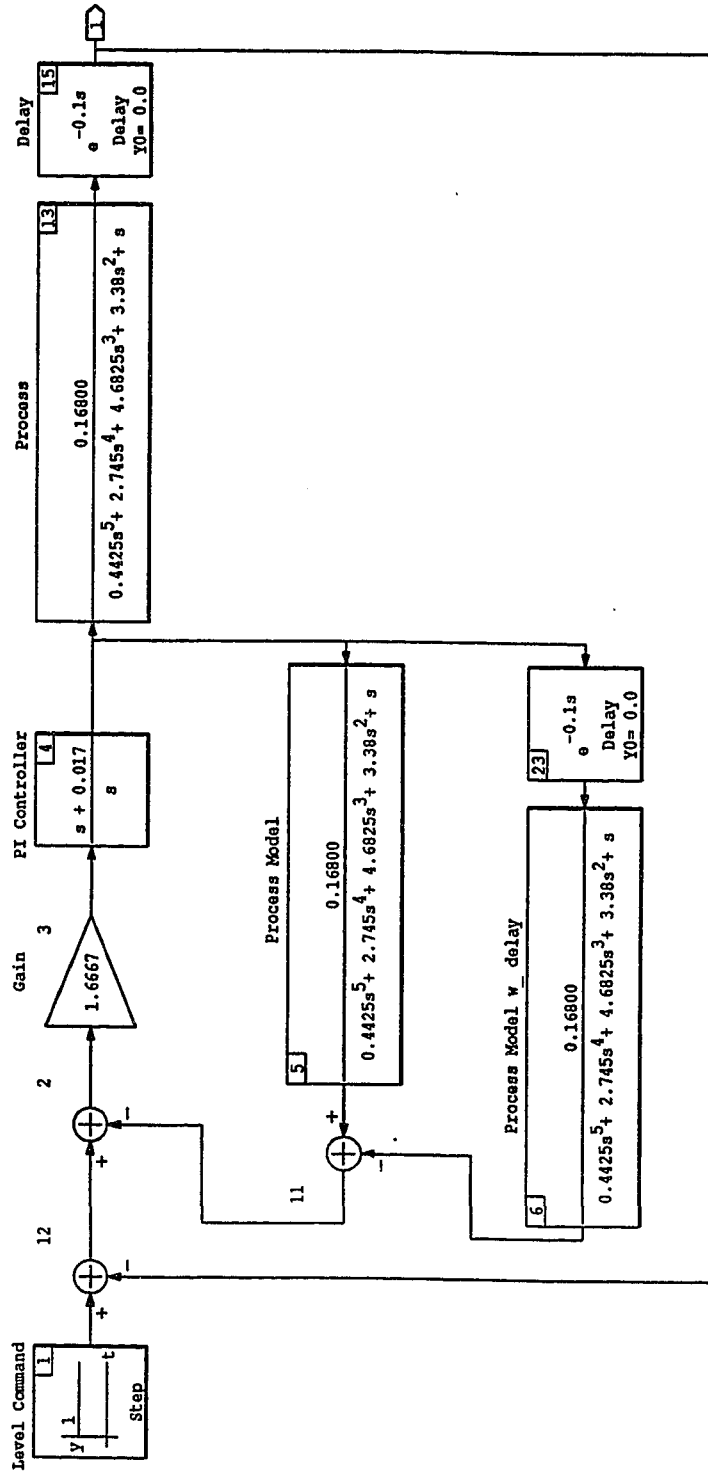


Figure 2.2 Modified One Element System With Smith Predictor

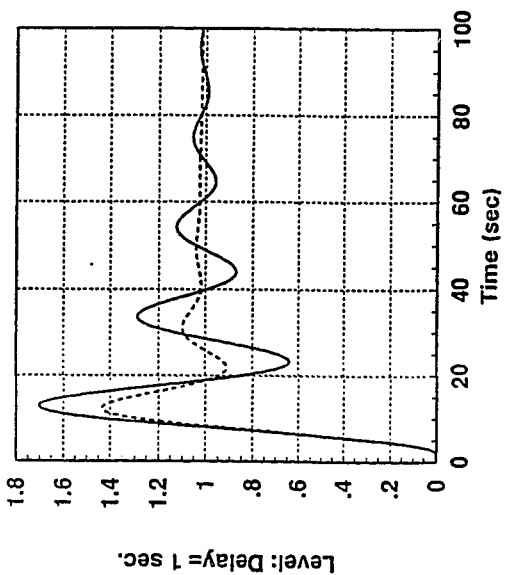
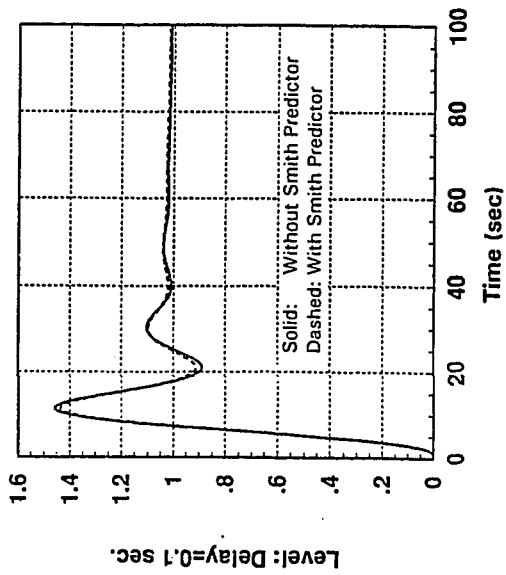
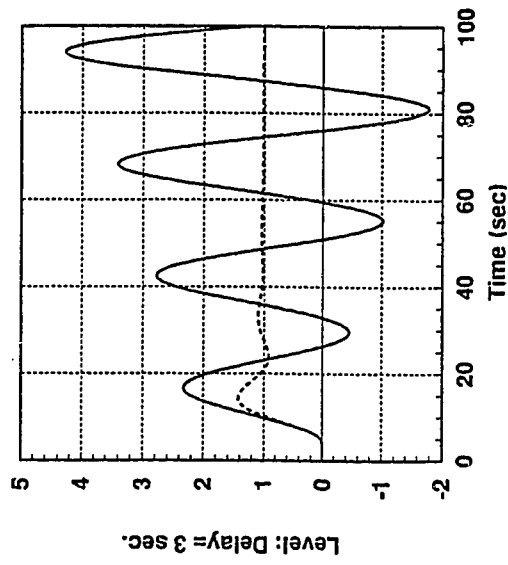
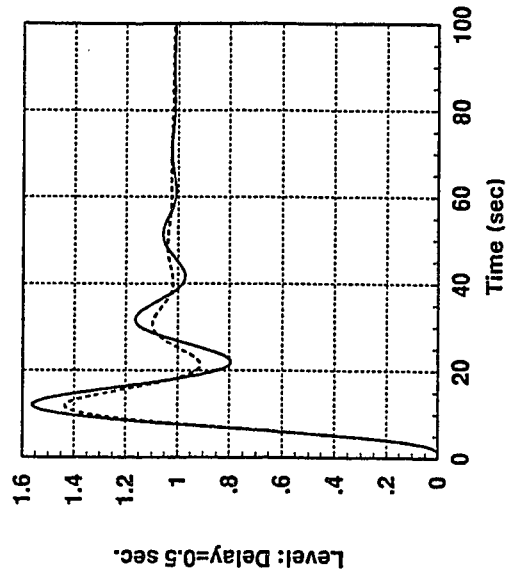


Figure 2.3 Step Reference Response of One Element System with and without Smith Predictor

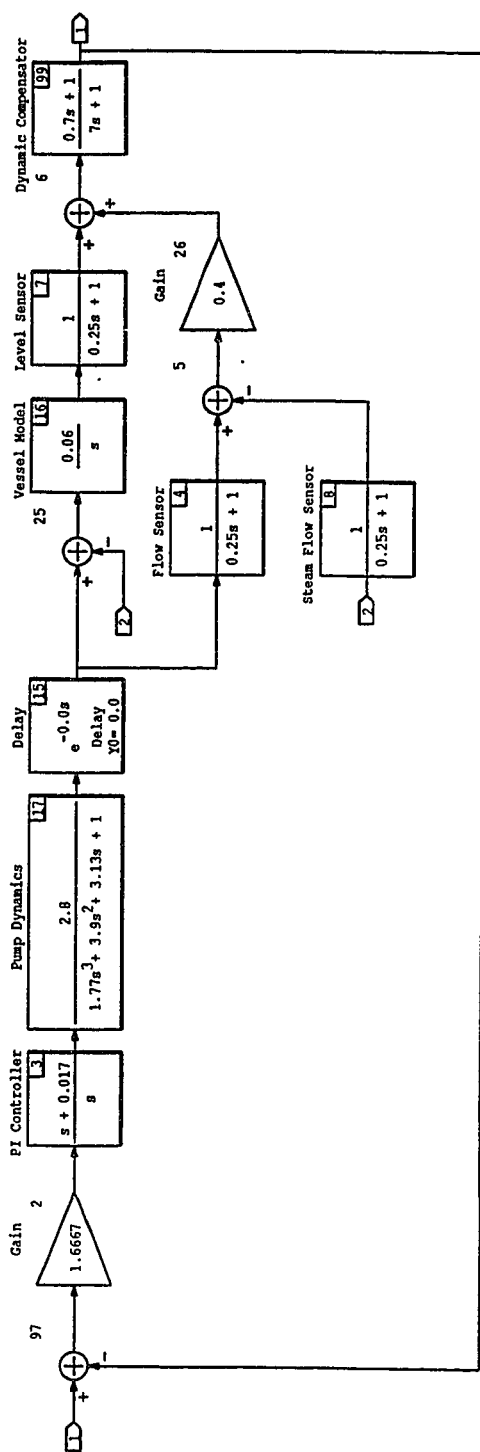


Figure 2.4 Modified Three Element Control System



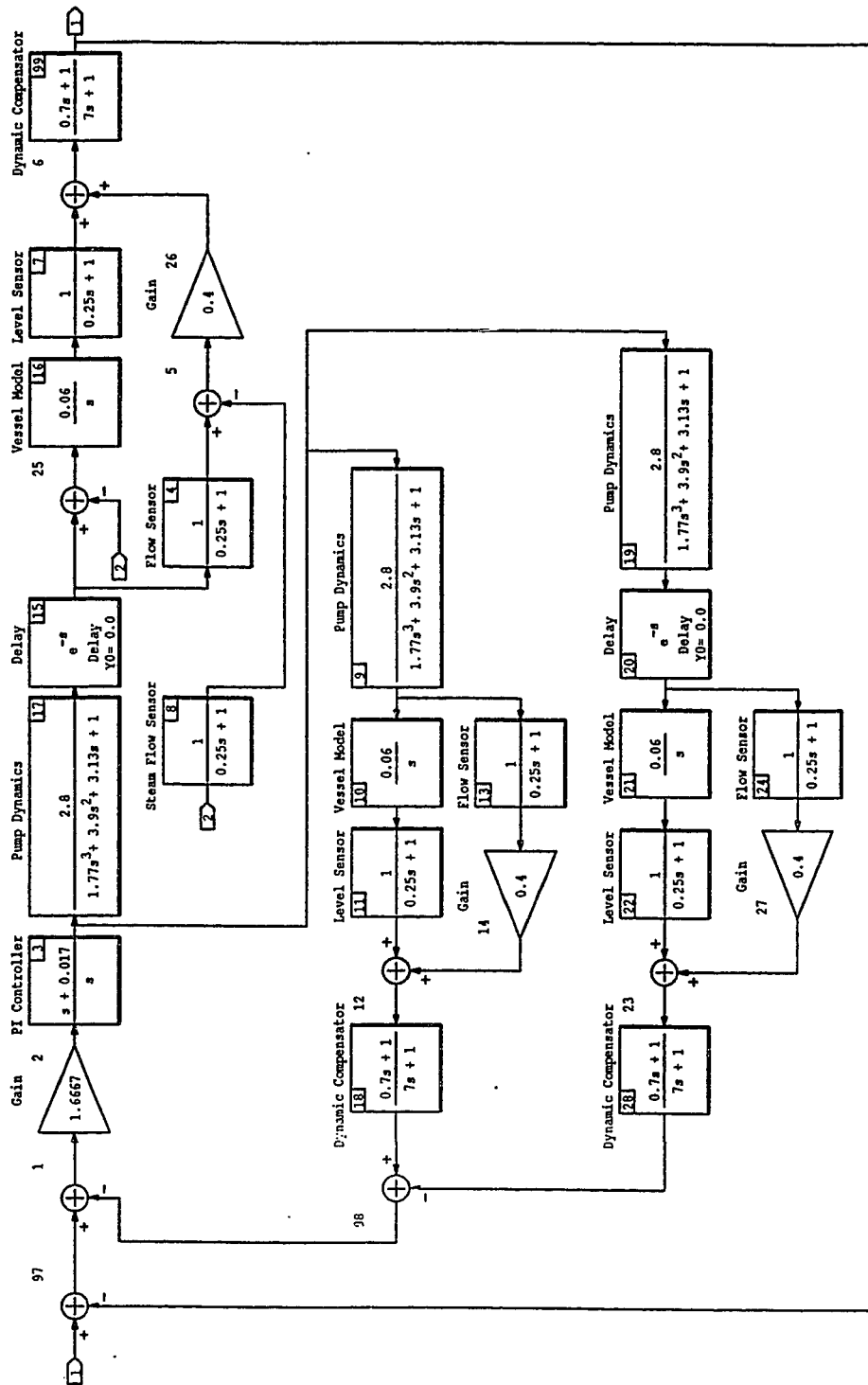


Figure 2.5 Modified Three Element System with Smith Predictor

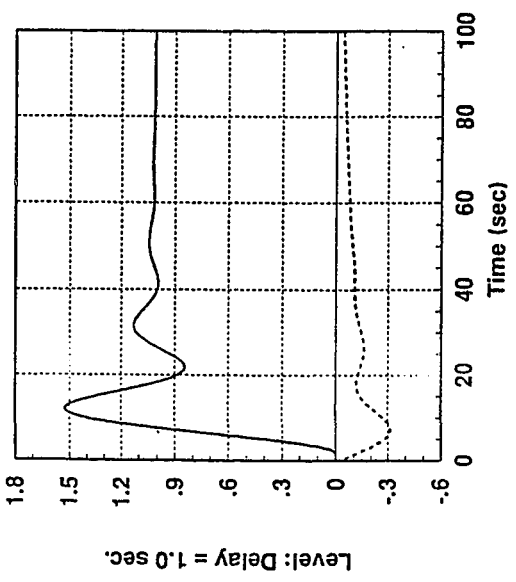
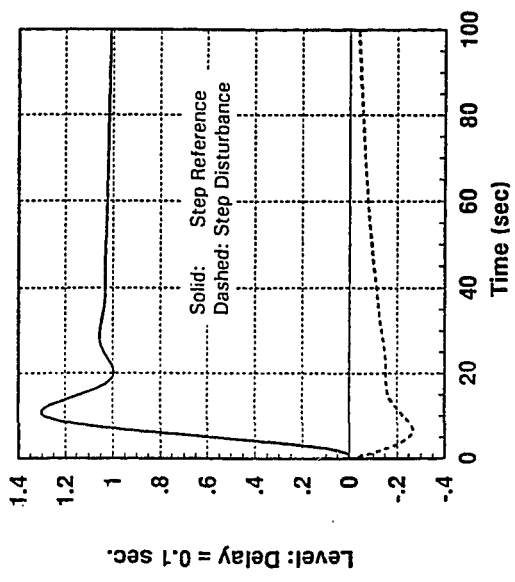
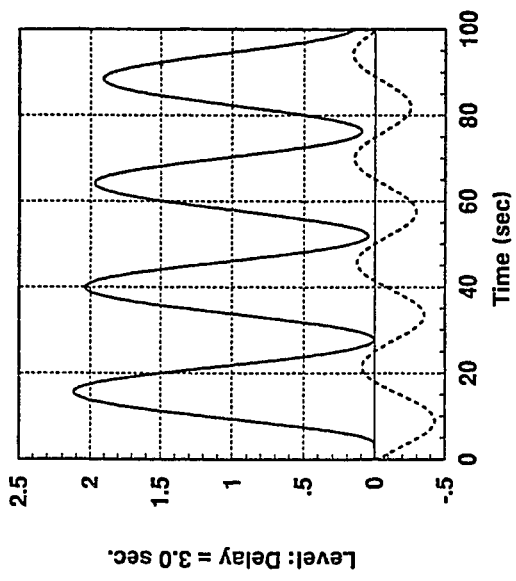
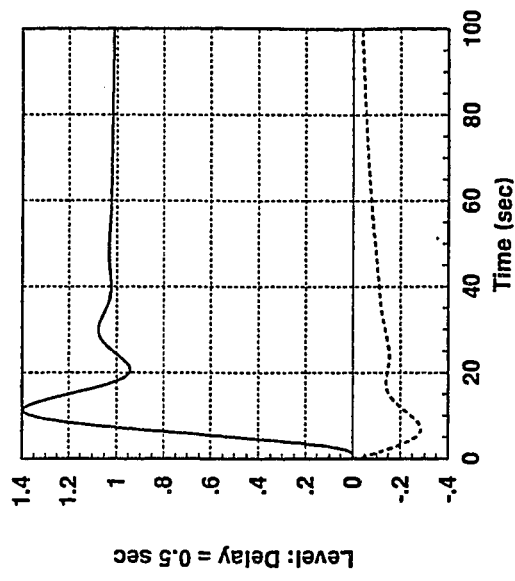


Figure 2.6 Step Reference & Disturbance Responses of Modified Three Element System

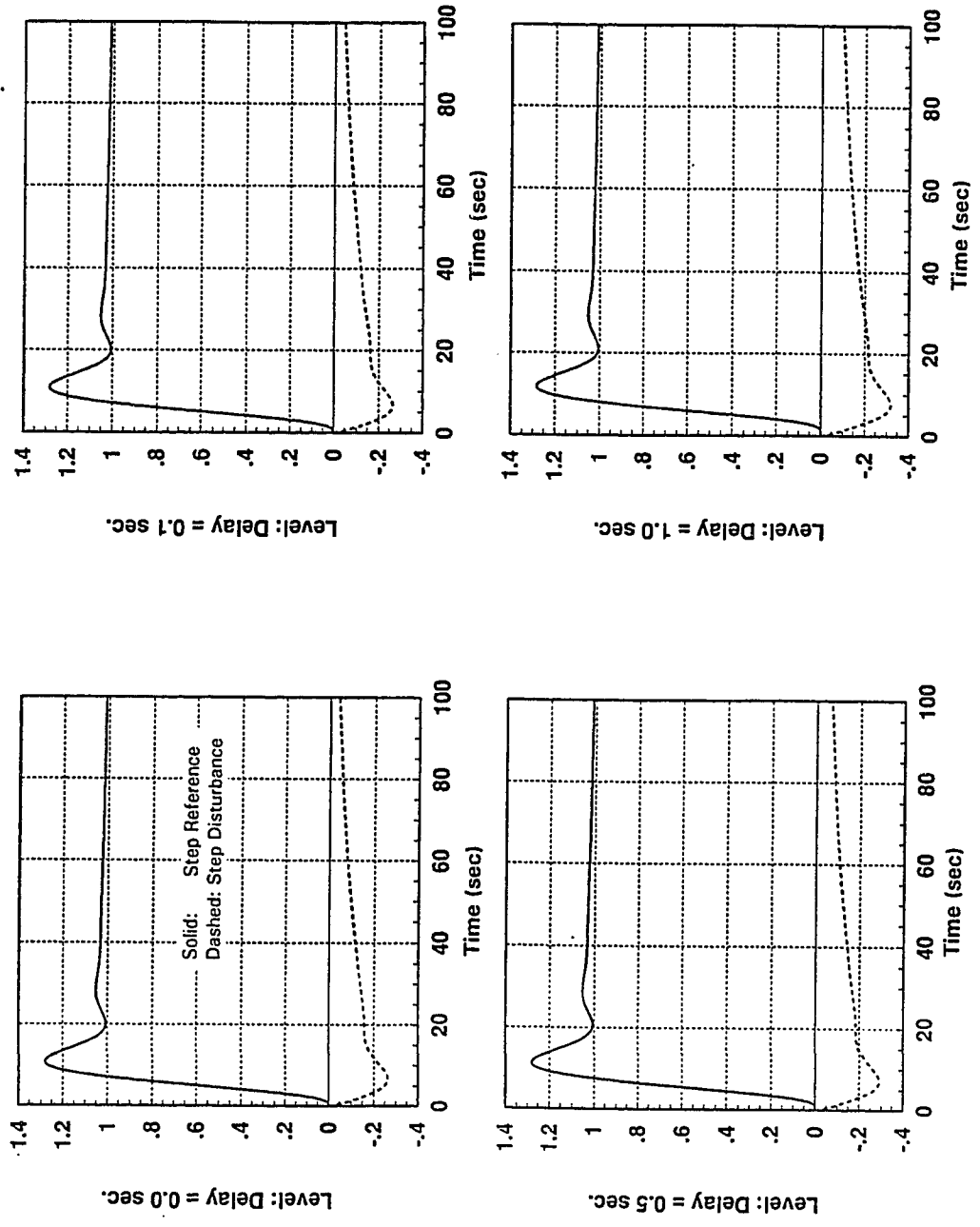


Figure 2.7 Step Reference & Disturbance Responses of Modified Three Element System with Smith Predictor

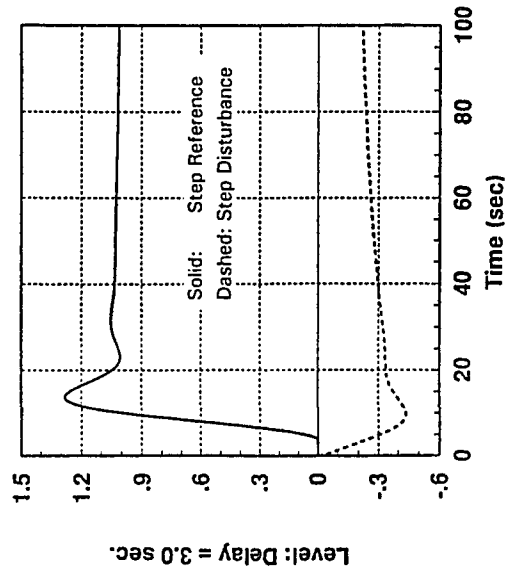
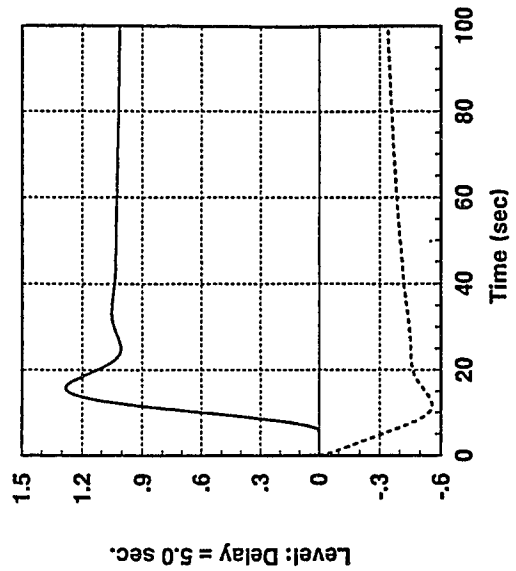
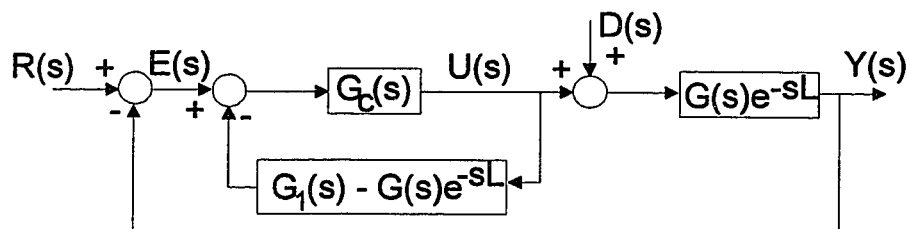


Figure 2.7 Step Reference & Disturbance Responses of Modified Three Element System with Smith Predictor (Continued)

### 3.0 Watanabe & Ito Process-Model Control

Watanabe and Ito have proposed a process model-control system which is intended to overcome the disturbance response error problem encountered with the Smith predictor. Their control method is a modification of the Smith predictor method. The resulting block diagram can be seen in the following figure [4].



The block  $G(s)e^{-sL}$  is the process with delay,  $G_c(s)$  is the controller which must contain an integrator and  $G_1(s)$  is a modified process model. The process without the delay can be modeled in state space form as  $G(s) = C(sI - A)^{-1}B + D$ . The modified process model is defined as  $G_1(s) = C_1(sI - A)^{-1}B + D_1$  where the matrices  $C_1$  and  $D_1$  are

$$C_1 = Ce^{-AL}$$

$$D_1 = -\int_0^L Ce^{-A\tau} B d\tau \quad (5 \& 6)$$

The transfer function for this system is

$$T_r(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)e^{-sL}}{1 + G_c(s)G_1(s)}. \quad (7)$$

The resulting characteristic equation is  $1 + G_c(s)G_1(s) = 0$  which does not contain a delay element,  $e^{-sL}$ . Since  $G_1$  is a function of the delay time,  $L$ , the system poles are not independent of the delay time as with the Smith predictor. With this system the controller gain values have to be changed as the delay time is changed to maintain the same system performance. In the next section this will be shown to be difficult for

higher order systems. With the following conditions the system can have stable poles in the left hand plane [4]. The controller,  $G_c(s)$  must contain an integrator. The matrices A and B must be controllable. Matrices  $C_1$  and D must contain constant values and the size of  $C_1$  must be  $(n \times 1)$  where n is the order of the system. Matrices  $C_1$  and A must be observable and

$$\text{rank} \begin{bmatrix} A & B \\ C_1 & 0 \end{bmatrix} = n + 1 \quad (8)$$

Watanabe and Ito present the following conditions for responses with zero steady state error. To obtain zero steady state error to a step reference input the following condition must be met. Letting the initial conditions of the system and the disturbance response be zero, zero steady state error is obtained if and only if

$$\lim_{s \rightarrow 0} \frac{G(s)}{G_1(s)} = 1 \quad (9)$$

The transfer function for a disturbance input is

$$T_d(s) = \frac{Y(s)}{D(s)} = \frac{G(s)e^{-sL} + G_c(s)G(s)e^{-sL}(G_1(s) - G(s)e^{-sL})}{1 + G_c(s)G_1(s)} \quad (10)$$

To obtain zero steady state error to a step disturbance in finite time, the poles of  $G_1(s) - G(s)e^{-sL}$  must cancel with its zeros thus reducing the disturbance transfer function to

$$T_d(s) = \frac{Y(s)}{D(s)} = \frac{G(s)e^{-sL}}{1 + G_c(s)G_1(s)} \quad (11)$$

Watanabe and Ito show that these conditions are true for their system when the conditions of rank, controllability and observability stated earlier are met. Note that the  $(e^{-sL})^2$  term found in the numerator of the disturbance transfer function for the Smith predictor system is removed in this system.

### 3.1 Feedwater Control System With Watanabe and Ito Controller

The Watanabe and Ito process model control system was applied to the Modified One Element and Modified Three Element feedwater control systems. The same proportional integral controller,  $G_c(s)$ , from the Smith predictor model was used in each case (gains  $k_i=0.017$  and  $k_p=1$ ). The system models are shown in Figures 3.1 and 3.3. The matrices A, B, C and D for the One and Three Element systems are can be found in the appendix.

Stable step reference input responses were obtained for the One Element system for delay values of 0.1, 0.5 and 1.0 seconds. For the Three Element system stable responses to step reference and step disturbance inputs were obtained for delay values of 0.1, 0.5 and 1.0 seconds. Both systems provided unstable responses at a delay value of 3 seconds. See Figures 3.2 and 3.4. The Three Element system poles for delay times of 0.1, 0.5, 1.0 and 3.0 seconds are listed in Table 3.1 ( $k_p=1$  and  $k_i=0.017$ ).

Table 3.1 Poles of Three Element System with Watanabe & Ito Controller

| L=0.1 sec        | L=0.5 sec        | L=1.0 sec        | L=3.0 sec              |
|------------------|------------------|------------------|------------------------|
| -4.0000          | -4.0000          | -4.0000          | -4.0000                |
| -3.9931          | -3.9658          | -3.7518          | -1.0295                |
| -0.8343 +0.2021i | -0.8940 +0.2798i | -0.9622 +0.3693i | -0.5925 +0.4723i       |
| -0.8343 -0.2021i | -0.8940 -0.2798i | -0.9622 -0.3693i | -0.5925 -0.4723i       |
| -0.2486 +0.1899i | -0.1995 +0.2120i | -0.1548 +0.2216i | -0.1450                |
| -0.2486 -0.1899i | -0.1995 -0.2120i | -0.1548 -0.2216i | <b>0.0003 +0.0553i</b> |
| -0.1682          | -0.1643          | -0.1612          | <b>0.0003 -0.0553i</b> |
| -0.0190          | -0.0190          | -0.0190          | -0.0187                |

The PI controller gains were kept constant to test the ability of the system to compensate for the delay without retuning the PI controller. As stated in the previous section the system poles in the Watanabe and Ito system are dependent on the delay time. In comparing the results of the Watanabe and Ito system to those of the uncompensated system (no predictor), there is little difference in the results. Compare Figure 2.3 to Figure 3.2 and Figure 2.6 to Figure 3.4. To obtain any benefit from the application of the Watanabe and Ito system, the PI controller gains must be retuned.

Recall that the characteristic equation of this system is  $1 + G_c(s)G_1(s) = 0$  where  $G_1(s)$  is a function of the process dynamics and the delay time. Watanabe and Ito state that  $G_c(s)$  can be tuned to obtain a set of pre-determined poles in the right hand plane [4]. The following derivation shows that this is only practical for low order systems. For a general system with a Proportional Integral controller let

$$G_c(s)G_1(s) = \frac{a_0 k_p s^n + (a_0 k_i + a_1 k_p) s^{n-1} + (a_1 k_p + a_2 k_p) s^{n-2} + \dots + a_n k_i}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_{n-1} s} \quad (12)$$

and thus the characteristic equation would be

$$\begin{aligned} (b_0 + a_0 k_p) s^n + (b_1 + a_0 k_i + a_1 k_p) s^{n-1} + (b_2 + a_1 k_i + a_2 k_p) s^{n-2} + \dots \\ + (b_{n-1} + a_{n-2} k_i + a_{n-1} k_p) s + a_n k_i = 0 \end{aligned} \quad (13)$$

For a given set of desired poles, the desired characteristic equation may be represented by

$$c_0 s^n + c_1 s^{n-1} + c_2 s^{n-2} + \dots + c_n = 0 \quad (14)$$

Equating like terms from the two equations



$$\begin{aligned}
c_0 &= b_0 + a_0 k_p \\
c_1 &= b_1 + a_0 k_i + a_1 k_p \\
c_2 &= b_2 + a_1 k_i + a_n k_p \\
&\vdots \\
c_{n-1} &= b_{n-1} + a_{n-1} k_i + a_n k_p \\
c_n &= a_n k_i
\end{aligned} \tag{15}$$

With all of the coefficients, a, b, and c being non zero, and  $k_i$  and  $k_p$  being the only unknowns, the above equations would only be valid for a first order system such that

$$\begin{aligned}
c_0 &= b_0 + a_0 k_p \\
c_1 &= a_1 k_i
\end{aligned} \tag{16}$$

Thus, in the case of higher order systems with Proportional Integral control, a set of pre-determined poles can not be obtained. A higher order system would require a more complicated controller.

An iterative method must be used to find values of  $k_i$  and  $k_p$  which provide a stable system with a delay time of  $L=3$  seconds. Through the use of a Routh Table created in a spreadsheet program, a small range of values of  $k_i$  and  $k_p$  which provide a stable system have been found. An example is presented in the appendix. For values of  $k_i \approx 0.005$  and  $0.05 \leq k_p \leq 0.5$  a stable system has been obtained, but the resulting step reference input responses are very oscillatory. The step reference input response using  $k_i = 0.005$  and  $k_p = 0.3$  is shown in Figure 3.5. The roots of the system with  $L=3$  seconds,  $k_i = 0.005$  and  $k_p = 1.0, 0.5, 0.3, 0.1, 0.05$  and  $0.01$  are contained in Table 3.2. Note that the positions of the poles only vary slightly. The dominant poles, those closest to the origin, are plotted in Figure 3.6. It is very difficult to control eight poles with only the two gains,  $k_i$  and  $k_p$ . Though stable poles have been found by iteration,

an acceptable response may not be obtainable. More advanced state space methods are required for higher order systems.

Table 3.2 Roots of Three Element System with Retuned Watanabe & Ito Controller  
( $L=3$ ,  $k_i=0.005$ )

| $k_p=1.0$                   | $k_p=0.5$             | $k_p=0.3$                   |
|-----------------------------|-----------------------|-----------------------------|
| -4.000000                   | -4.000000             | -4.000000                   |
| -1.029365                   | -1.029622             | -1.029974                   |
| -0.592430 + 0.472079i       | -0.592571 + 0.472376i | -0.592781 + 0.472778i       |
| -0.592430 - 0.472079i       | -0.592571 - 0.472376i | -0.592781 - 0.472778i       |
| -0.144900                   | -0.145063             | -0.145351                   |
| <b>0.003432 + 0.057488i</b> | -0.002223 + 0.055841i | -0.008390 + 0.051044i       |
| <b>0.003432 - 0.057488i</b> | -0.002223 - 0.055841i | -0.008390 - 0.051044i       |
| -0.005089                   | -0.010751             | -0.020754                   |
| $k_p=0.1$                   | $k_p=0.05$            | $k_p=0.01$                  |
| -4.000000                   | -4.000000             | -4.000000                   |
| -1.031950                   | -1.035770             | -1.366880                   |
| -0.594237 + 0.47484i        | -0.597991 + 0.477603i | -0.608953 + 0.457534i       |
| -0.594237 - 0.47484i        | -0.597991 - 0.477603i | -0.608953 - 0.457534i       |
| -0.156727                   | -0.299935             | -0.967264                   |
| -0.126962                   | -0.142266             | -0.143400                   |
| -0.004188 + 0.034123i       | -0.000524 + 0.032434i | <b>0.002230 + 0.031507i</b> |
| -0.004188 - 0.034123i       | -0.000524 - 0.032434i | <b>0.002230 - 0.031507i</b> |

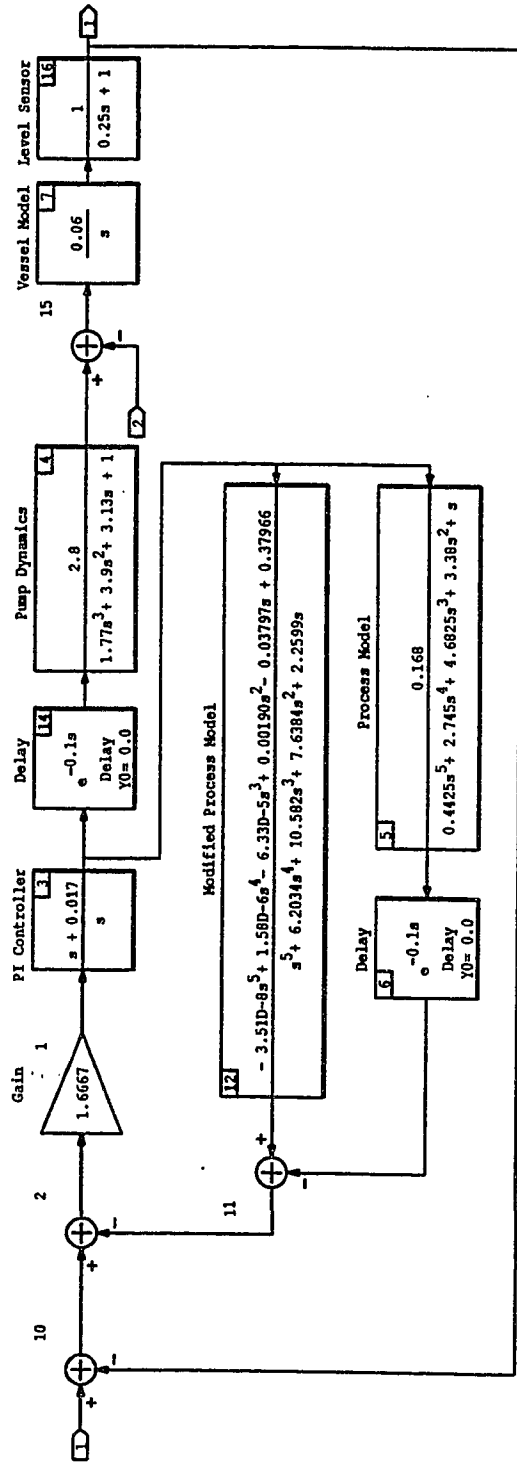
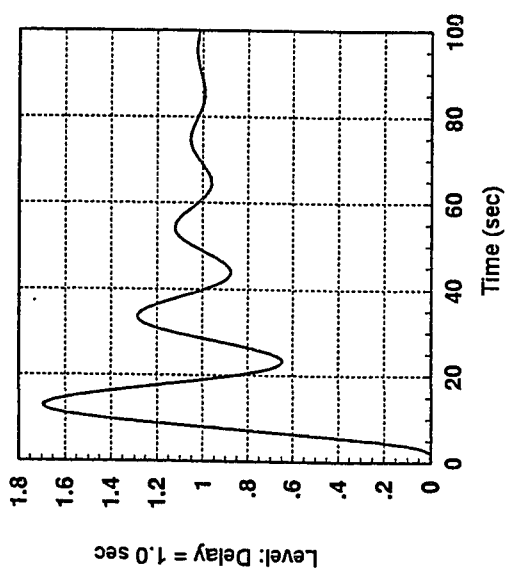
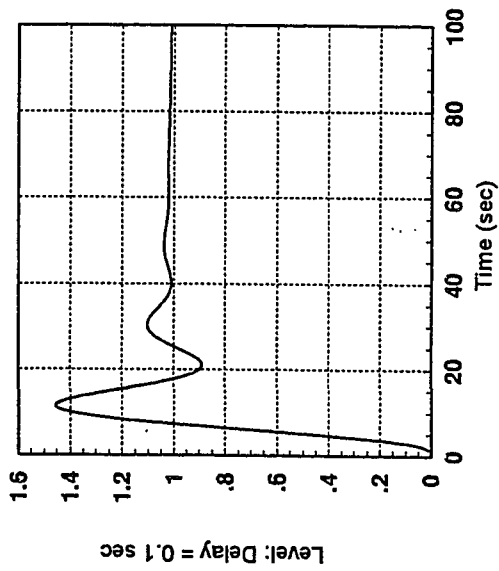
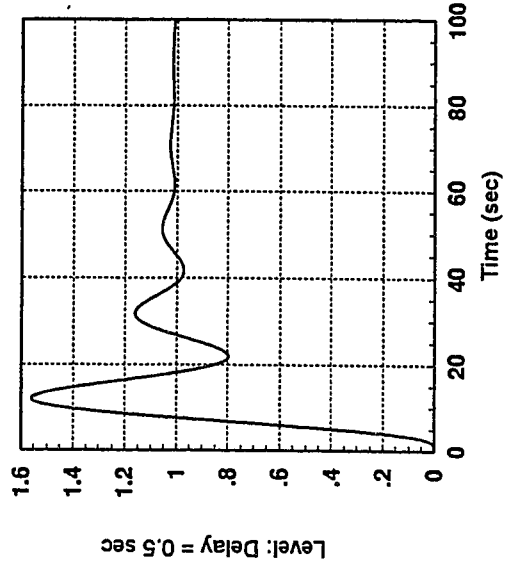


Figure 3.1 Modified One Element System with Watanabe & Ito Controller



System is unstable for: Delay,  $L=3.0$  sec  
 Integral Gain,  $k_i=0.017$   
 Proportional Gain,  $k_p=1.0$

Figure 3.2 Step Reference Response of Modified One Element System with Watanabe & Ito Controller

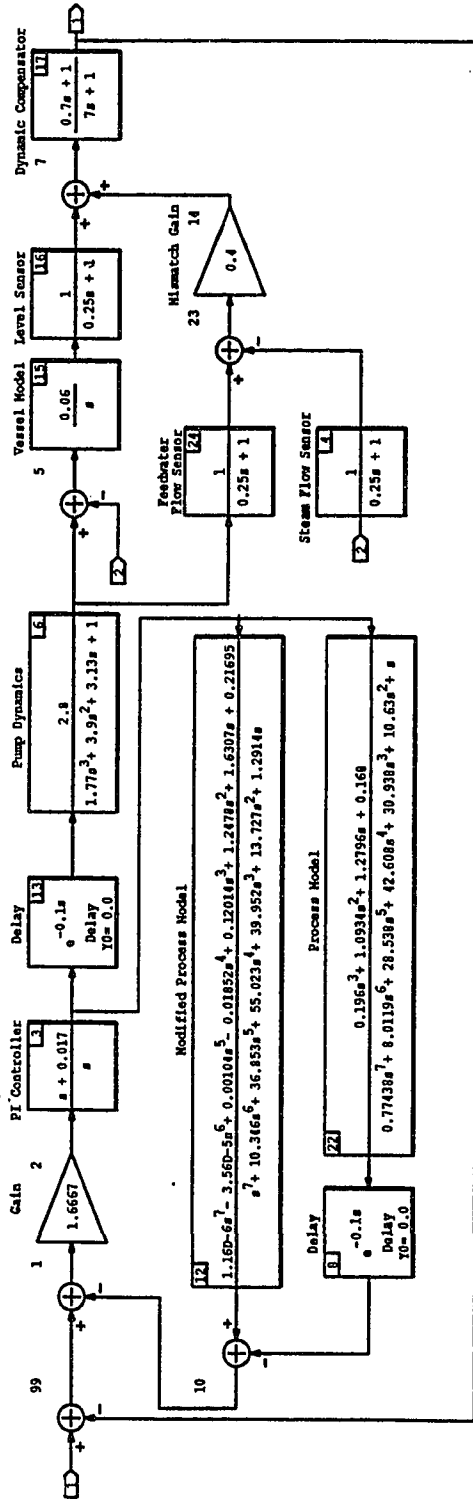
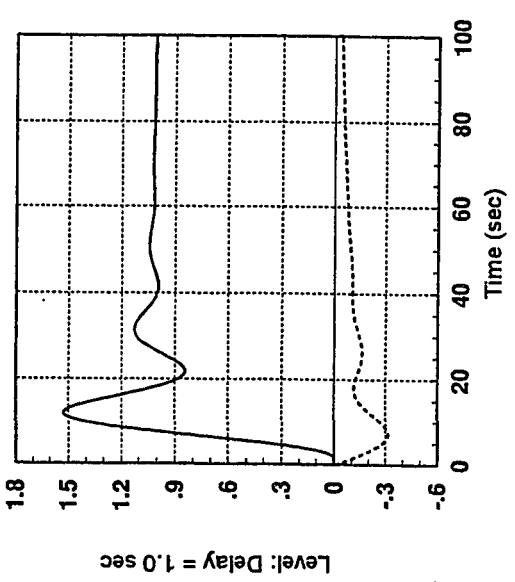
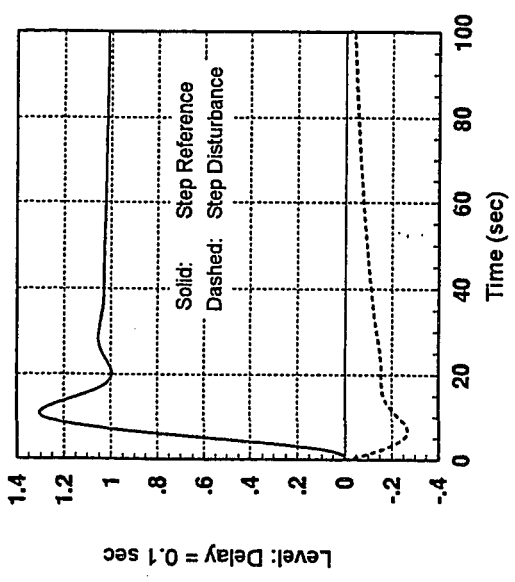
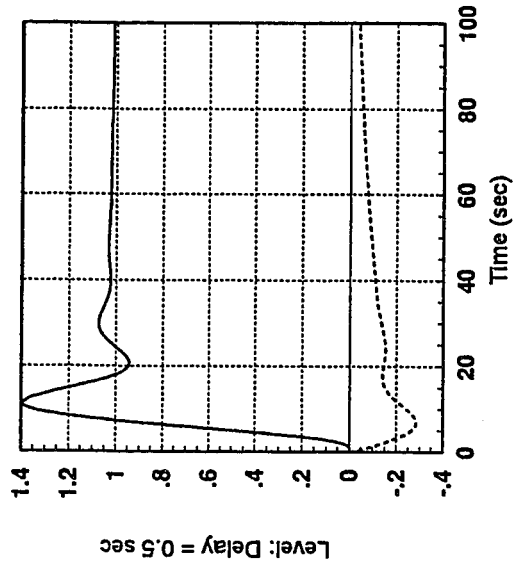


Figure 3.3 Modified Three Element System with Watanabe & Ito Controller



System is unstable for: Delay,  $L=3.0$  sec  
Integral Gain,  $k_i=0.017$   
Proportional Gain,  $k_p=1.0$

Figure 3.4 Step Responses of Modified Three Element System with Watanabe & Ito Controller

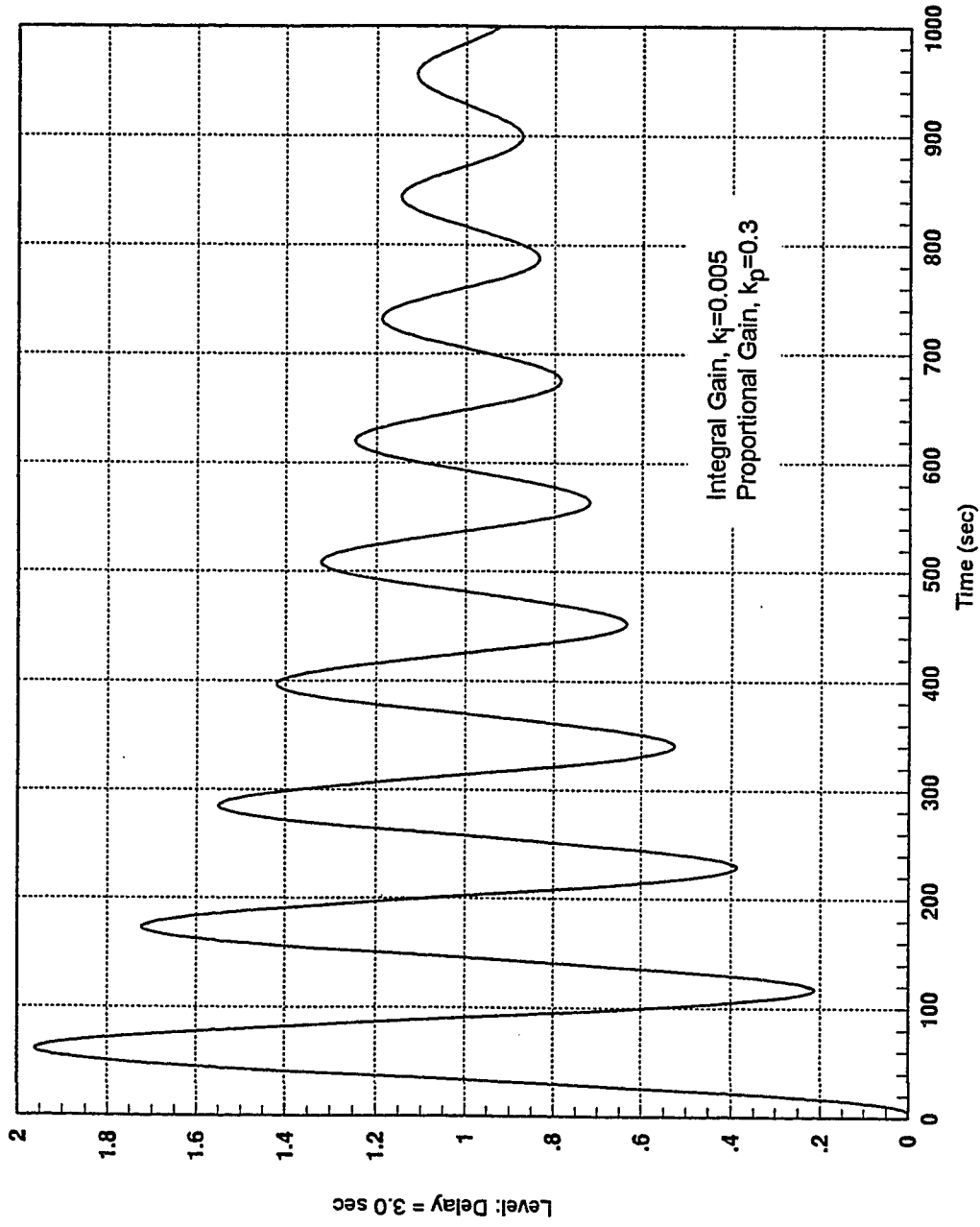


Figure 3.5 Step Reference Response of Modified Three Element System with Watanabe & Ito Controller

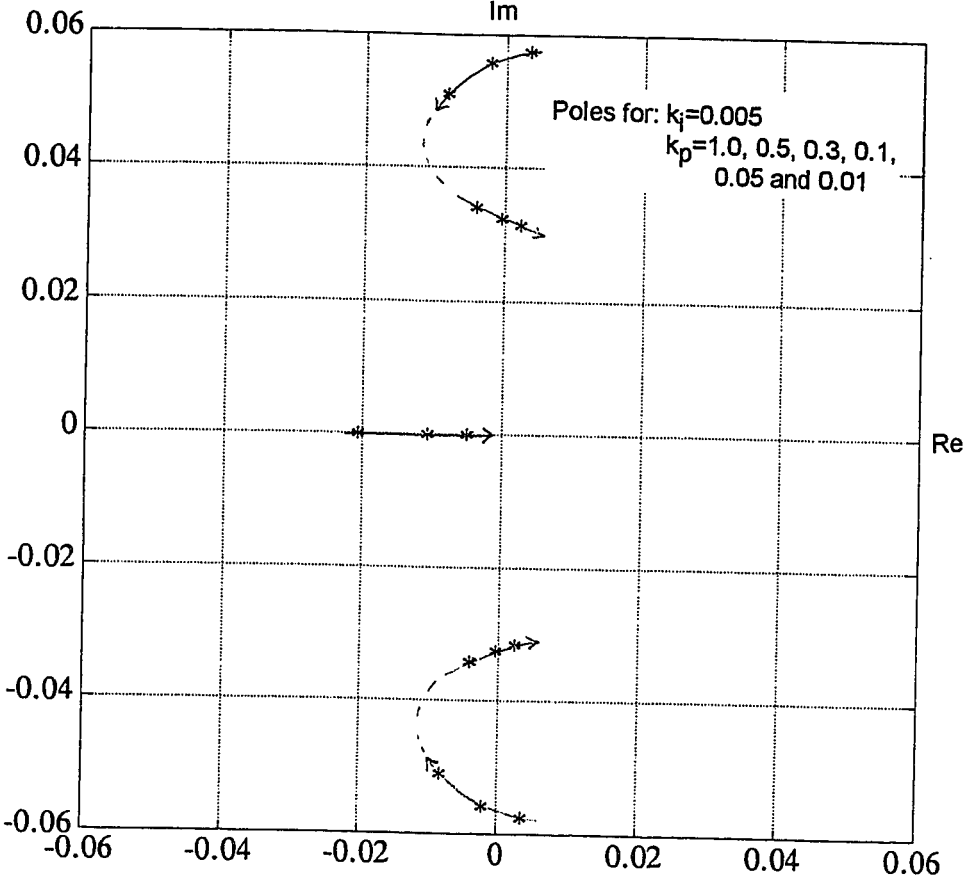


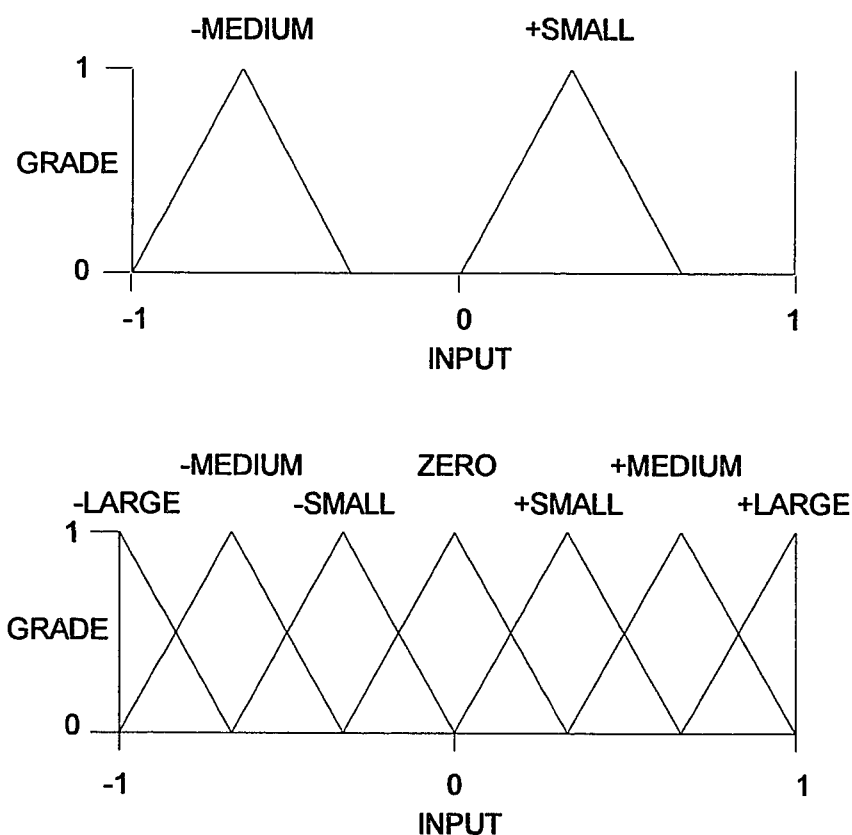
Figure 3.6 Dominant Roots of Modified Three Element System with Watanabe & Ito Controller



## 4.0 Fuzzy Logic Control

### 4.1 Basics of Fuzzy Logic

Fuzzy Logic is a control logic in which the values of the input and output of the controller can be described by a discrete set of values such as large, medium, small and zero. With these loosely defined verbal descriptions, rules of action can easily created and understood on an individual basis. The value descriptions are assigned to functions known as membership functions (see the following figure). A membership function, such as the positive small membership function, defines how much an input may be considered to be positive small. The rating is in terms of a grade with values between 0 and 1. The lower figure shows a common range of membership functions [5].



The membership functions overlap such that a given input value may intersect more than one membership function. When the input value intersects more than membership function the action taken is a combination of the distinct actions assigned to the individual membership functions. The action resulting from the few discrete conditions, used for the development of the process logic, are smoothed into a more continuous result. The Fuzzy Logic controller uses two input values and one output. The two inputs to the controller are a typical input, such as an error signal, and the time rate of change of that input, such as the derivative of the error ( $e$  and  $de/dt$ ). Rules of action or conditional statements can be used to obtain a desired action to particular combinations of the two inputs. For example, if the error is near zero and the error rate is large, a large action may be desired to resist overshooting a desired zero value for the error. Alternatively, if the error is near zero and the error rate is small, only a small action may be needed to obtain a zero error value.

The steps in the Fuzzy Logic process are:

1. Normalization (optional)
2. Fuzzification
3. Minimization
4. Maximization (optional)
5. Defuzzification
6. Unit Conversion (optional)

Steps 2 through 5 are the most basic Fuzzy Logic steps. Steps 1, 4 and 6 are optional.

### Fuzzification

The fuzzification process is the comparison of the input values to each of the input membership functions. From the intersections of the input value and the input membership functions, grade values are found for each membership function. The grade is the participation value or percentage for each of the input membership functions. This is done for both the error input and the error rate input. In general the two inputs and the output can all have differently defined membership functions. In this explanation of the Fuzzy Logic controller and in the controller tested, the inputs and output use the same membership functions.

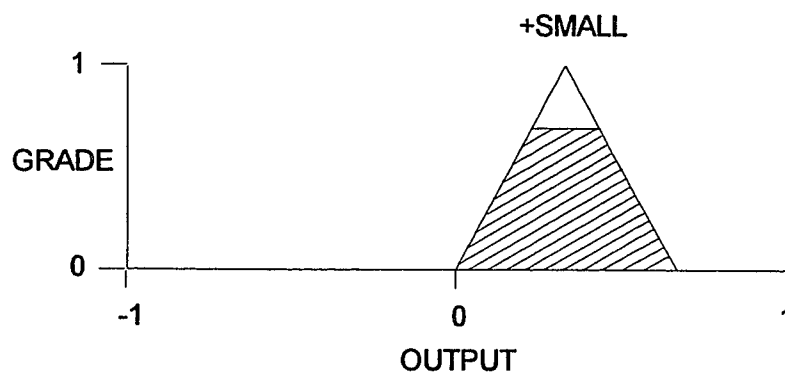
### Minimization

In the minimization operation each of the conditional statements is evaluated. The grade assigned to the resulting action is the minimum of the grades of the input conditions tested. This best explained with an example. A possible rule may be stated as "if the error is ZERO and the error rate is POSITIVE SMALL then the control output is NEGATIVE SMALL." If the error has a ZERO grade of 0.8 and the error rate has a POSITIVE SMALL grade of 0.5, the result of this rule would be a control output of NEGATIVE SMALL with a grade of 0.5. Similarly the rest of the rules would be evaluated. If a rule input has a grade of zero then the output of that rule would have a grade of zero. A rule may not test both inputs and thus there may be only one grade value.

### Maximization

The maximization operation is the combining of all of outputs from each of the rules into one output profile. In the fuzzification process, the grade is determined by the intersection of the input value and the corresponding membership function. For the output

membership functions the grade is used to find the area under the curve created by the intersection of the membership function and the grade line as seen in the following figure.



The maximization process is the combination of the curves resulting from each rule. This step is not mandatory. The operation of combining the effects of all of the rules can also be performed in the defuzzification process. In general explanations of the Fuzzy process this operation is usually left out. The Fuzzy controller created by Cheng [6] uses the maximization operation.

### Defuzzification

The defuzzification process is the calculation of the final output value based on a center of gravity calculation. With the use of the maximization process, defuzzification calculates the center of gravity of the area under the single output profile created in the maximization process. Without the maximization process the individual rule outputs can be used with a discrete, point mass type center of gravity calculation. The resulting center of gravity is the defuzzified output.

### Normalization of Inputs / Unit Conversion of Output

In the Fuzzy controller created by Cheng the two inputs and the output all share the same membership functions. This was done for simplicity. The input (or output) value

ranges from -1 to 1. Gains on the controller inputs and output are used to normalize the inputs and output with a maximum values chosen by the designer. The choice of these maximum values would be based on what the designer considers to be a large error or a large error rate. Limiters are used on the inputs to limit the normalized inputs to values from -1 to 1. It is possible to individually define the membership functions for the inputs and output using non-normalized values. Through the use of the normalization and unit conversion processes the controller can be more easily tuned and applied to different systems.

A Fuzzy Logic controller can be tuned in various ways. The number and shape of the membership functions can be changed. The conditional statements can be redefined. The range of the inputs and outputs can be changed. With normalization and unit conversion the inputs and output can be easily changed. Without normalization and unit conversion the membership functions must be redefined to change the input and output ranges.

More complicated membership functions, a larger number of membership functions and more complicated rules can increase the calculation cycle time of the Fuzzy controller program. Adequate results are usually obtained with seven membership functions, one at zero, three in the positive direction and three in the negative direction [5]. Complicated bell shape membership functions can be used, but triangular membership functions are simpler to implement and tend to produce similar or superior performance [5]. Similarly shaped membership functions with a 50 percent overlap usually provide good results [5]. The cycle time of a Fuzzy Logic controller can be further reduced by producing a Fuzzy surface or table of outputs for given sets of inputs. Testing would be required to determine the number of points and the type of interpolation routines necessary for acceptable performance.

## 4.2 Fuzzy Logic Controller

The Fuzzy Logic controller implemented in this thesis was created by Cheng for a model of a GE Simplified Boiling Water Reactor feedwater system [6]. Figure 4.1 shows the Fuzzy Logic controller as it appears in the simulation software. The inputs are the error signal and the error signal rate. The error signal rate is obtained by feeding the error signal through a derivative function. The output is the speed signal to the feedwater pump. The gains GERROR and GRATE along with the limiters perform the normalization of the error signal and error rate signal. Unit conversion of the output is performed by the gain GOUT.

The same normalized membership functions are used for the two inputs and the output. They are defined in Table 4.1 and the positive functions are plotted in Figure 4.2. The functions are exponential in nature giving them distinct peaks like triangular functions. Unlike triangular functions, there will always be a non zero grade returned for any point in the input/output range (-1 to 1). A non zero output grade will be obtained from each conditional statement.

Table 4.1 Fuzzy Membership Functions

| Membership Function | Expression                |
|---------------------|---------------------------|
| Large Positive      | $1 - \exp(-0.1/ 1-x )$    |
| Medium Positive     | $1 - \exp(-0.1/ 0.6-x )$  |
| Small Positive      | $1 - \exp(-0.1/ 0.3-x )$  |
| Zero                | $\exp(-10x)$              |
| Small Negative      | $1 - \exp(-0.1/ -0.3-x )$ |
| Medium Negative     | $1 - \exp(-0.1/ -0.7-x )$ |
| Large Negative      | $1 - \exp(-0.1/ -1-x )$   |

The conditional statements are summarized in table 4.2. For each rule in the table, an "X" in both the error input and the error rate input columns is an "and" operation. If more than one "X" is found in an input column there is an "or" operation. For example, rule 4 states that if the error is -small and the error rate is +small, zero or -small, the output is +small. When the "or" operation is used the "or" option with the maximum grade is used in the minimization operation. Note that in the medium and large ranges the error rate is not taken into account. Figure 4.3 shows the internal structure of the Fuzzy Controller block. The fuzzification and minimization operations are performed in each rule block.

Table 4.2 Fuzzy Logic Controller Conditional Statements

| Rules    | 1      | 2       | 3       | 4      | 5    | 6      | 7       | 8       | 9       |        |
|----------|--------|---------|---------|--------|------|--------|---------|---------|---------|--------|
| Inputs   | e      | de/dt   | e       | de/dt  | e    | de/dt  | e       | de/dt   | e       | de/dt  |
| Large +  |        |         |         |        |      |        |         | X       |         |        |
| Medium + |        |         |         |        |      | X      |         | X       |         |        |
| Small +  |        |         | X       |        |      | X      | X       |         |         |        |
| Zero     |        |         | X       |        | X    | X      | X       |         |         |        |
| Small-   |        |         | X       | X      |      | X      |         |         |         |        |
| Medium-  |        | X       |         |        |      |        |         |         |         |        |
| Large-   | X      |         |         |        |      |        |         |         |         |        |
| Output   | Large+ | Medium+ | Medium+ | Small+ | Zero | Small- | Medium- | Medium- | Medium- | Large- |



### 4.3 Feedwater System with Fuzzy Logic Controller

The Fuzzy Logic controller created by Cheng has been applied to the Three Element control system as shown in Figure 4.4. The simulations performed by Cheng using a model of the GE Simplified Boiling Water Reactor did not include disturbance input responses. It was found that disturbance inputs produced responses with steady state error when the Fuzzy Logic controller was used alone. An integrator was added in parallel with the Fuzzy Logic controller to remove the steady state error from the disturbance input responses. The membership functions and conditional statements of the Fuzzy Logic controller have not been changed. Three simulation cases are presented here where the input normalization and the output unit conversions have been varied. See Table 4.3.

Table 4.3 Test Cases for Fuzzy Logic Controller

| Case | Error      |        | Error Rate |            | Output     |                 |
|------|------------|--------|------------|------------|------------|-----------------|
|      | Gain value | Inches | Gain value | Inches/sec | Gain value | % of Rated Flow |
| 1    | 0.8        | 1.25   | 0.25       | 4.0        | 1.1        | 5.0             |
| 2    | 0.8        | 1.25   | 0.25       | 4.0        | 2.1        | 10.0            |
| 3    | 0.2        | 5.0    | 0.1        | 10.0       | 5.0        | 23.0            |

In the simulations performed by Cheng, the inputs to the Fuzzy Logic controller were normalized to 1.2 inches for the error signal and 4 inches/second for the error rate signal. The maximum output of the controller was 3 percent of rated feedwater flow. Using these values with the three element BWR feedwater control system and an integrator gain of  $k_i=0.017$  produced slow responses. Increasing the output of the controller to 5 percent of rated feedwater flow (case 1) produced the responses as seen in Figure 4.5. Delay values

of  $L=0, 0.1, 0.5, 1.0, 3.0$  and  $5.0$  seconds were used. The system responses to step reference and step disturbance inputs become oscillatory at a delay value of  $L=5$  seconds. In case 2 the output of the controller was increased to 10 percent of rated feedwater flow. See Figure 4.6. The resulting responses are similar to the original system with a Proportional Integral controller. The responses to step reference and step disturbance inputs become oscillatory at a delay value of  $L=3$  seconds. In case 3 the range of the error signal input was increased to 5 inches and the error rate signal was increased to 10 inches per second. The output range was increased to 23 percent of rated feedwater flow. The responses are similar to those of case 1, but with faster rise times and less overshoot. See Figure 4.7. The responses become oscillatory at a delay value of  $L=5$  seconds. The system in case 3 is more dependent on the rules in the range of +small to -small where the error rate is taken into account in the calculation of the controller output. The selection of acceptable normalization values requires advanced knowledge of the system performance and all possible input conditions. Modification of the membership functions and the conditional statements, taking the delay into account, may also improve the systems response. In general, responses shown here are superior to those provided by the Smith predictor and the Watanabe and Ito Process-Model Control system.

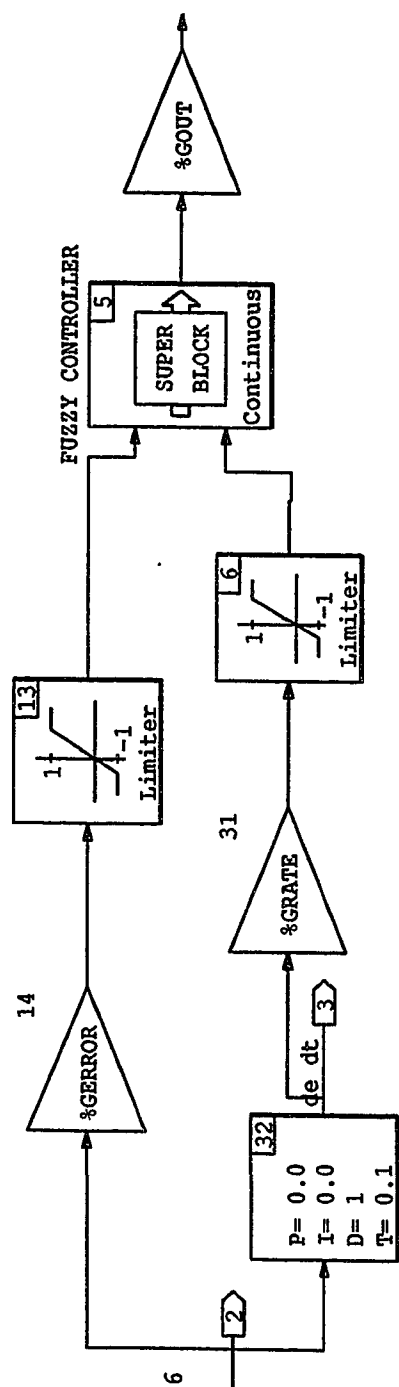
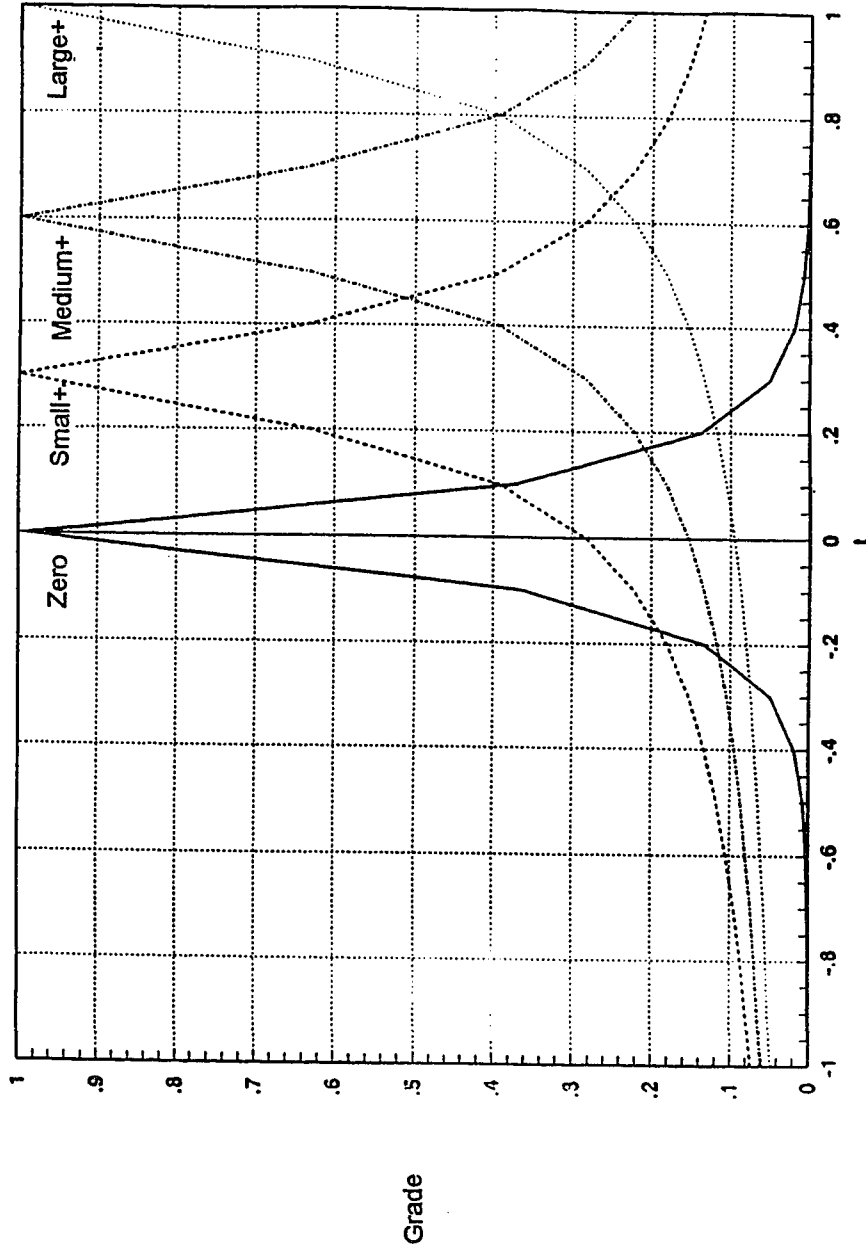


Figure 4.1 Fuzzy Logic Controller Block Diagram



Range of Normalized Values  
(Small-, Medium-, and Large- not shown)

Figure 4.2 Fuzzy Logic Membership Functions

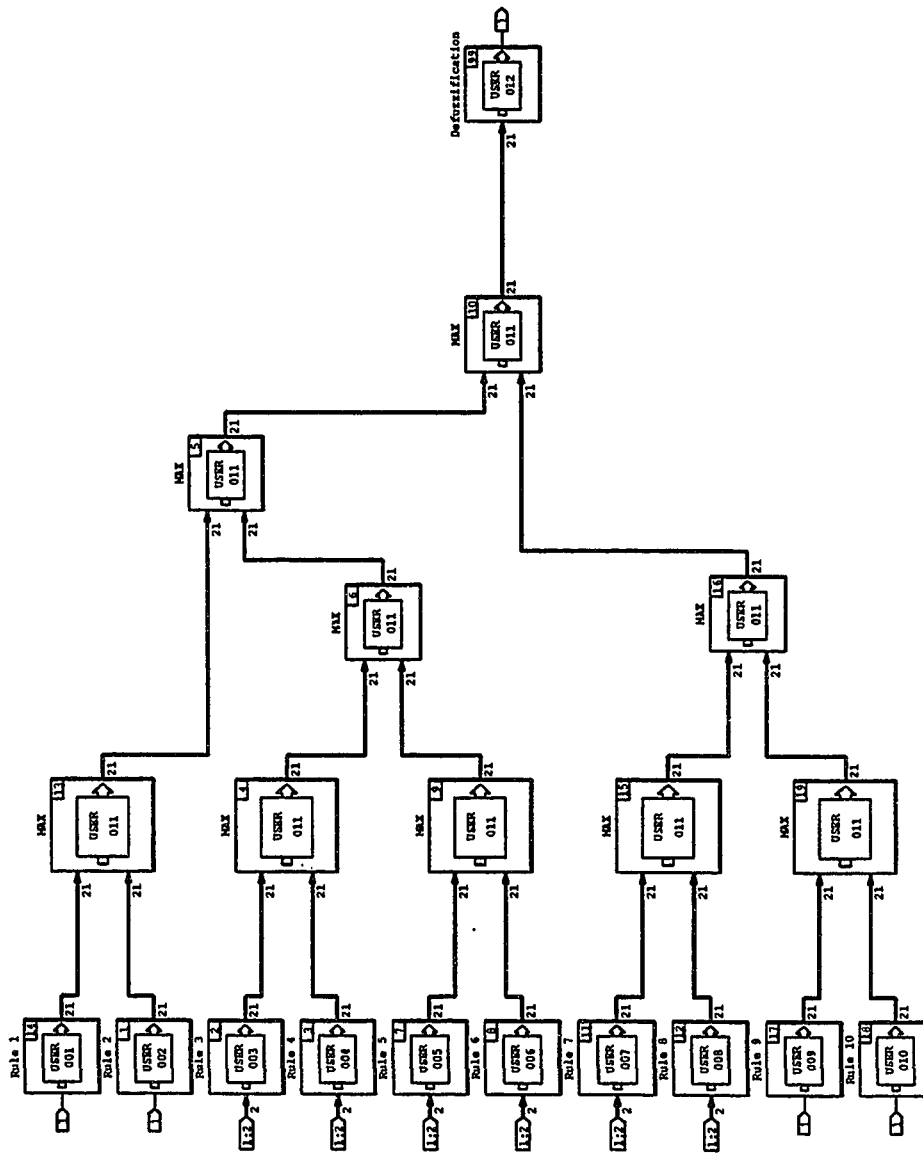


Figure 4.3 Fuzzy Logic Controller Process Diagram

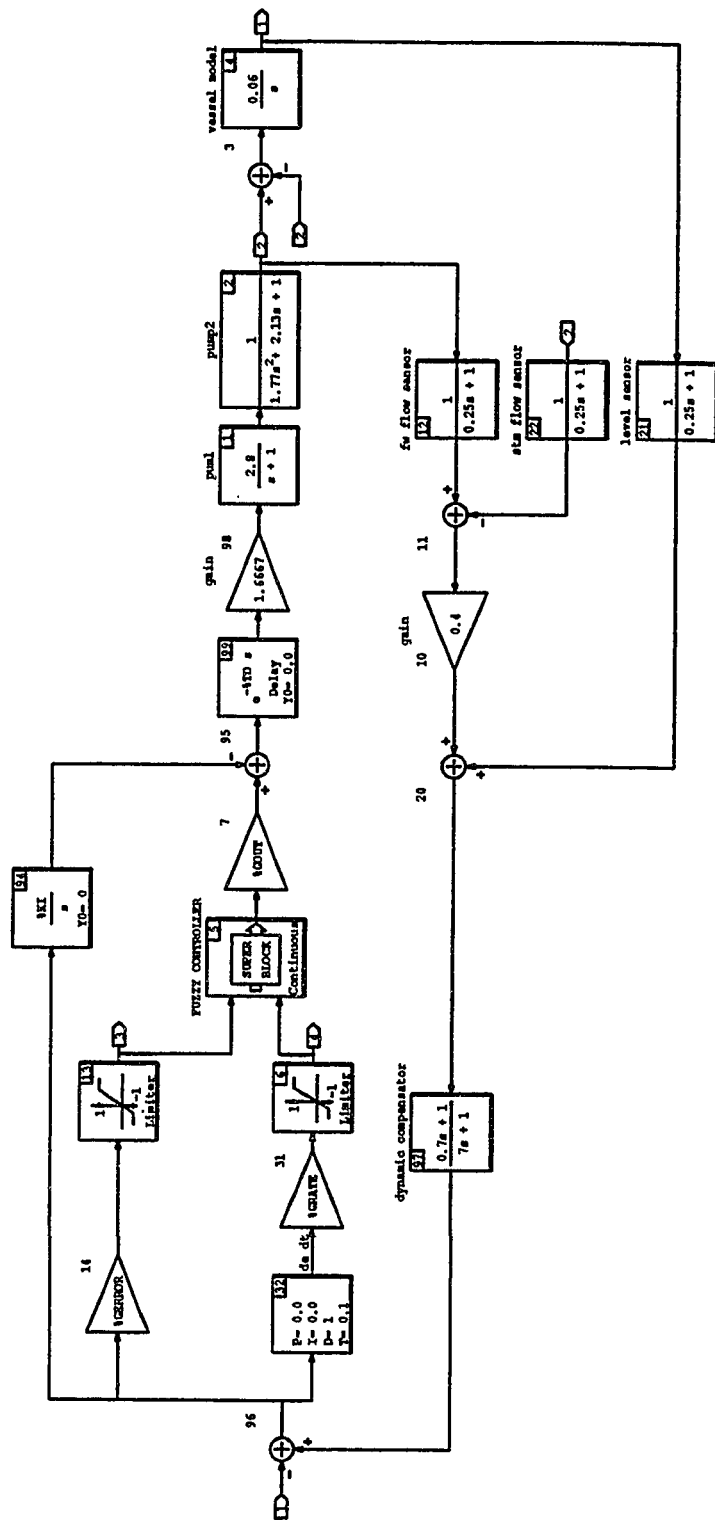


Figure 4.4 Three Element System with Fuzzy Logic Controller

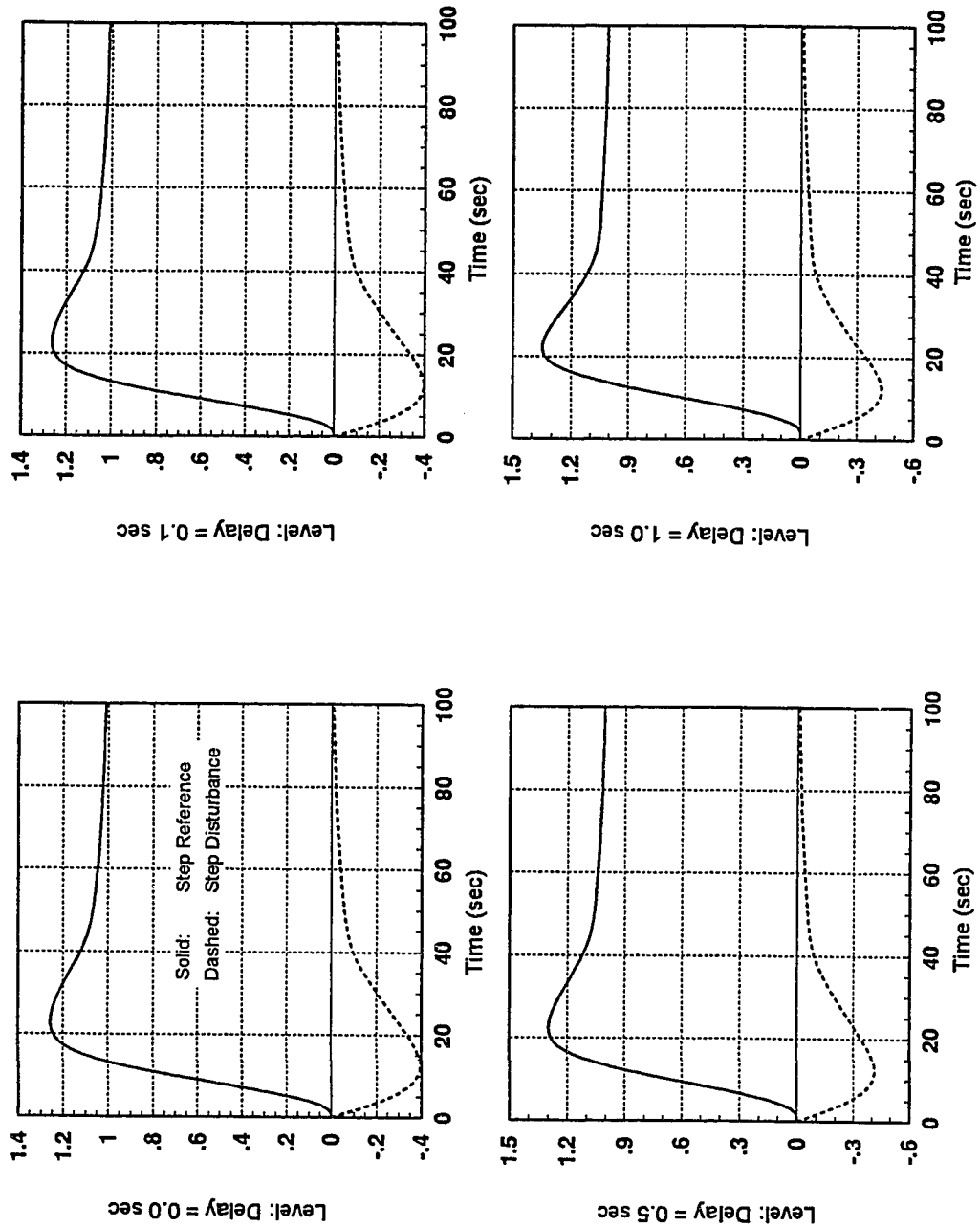


Figure 4.5 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 1 4

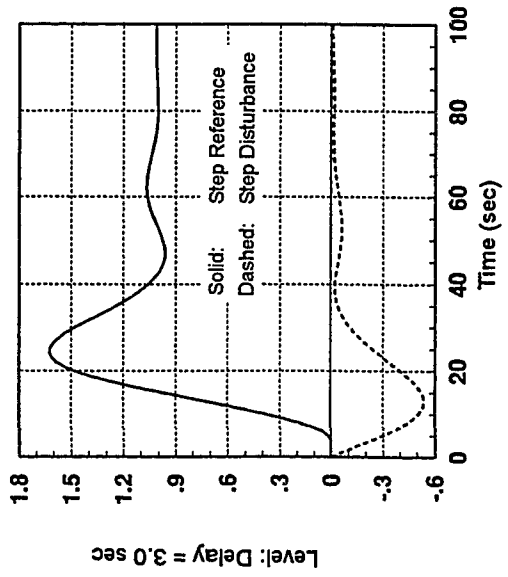
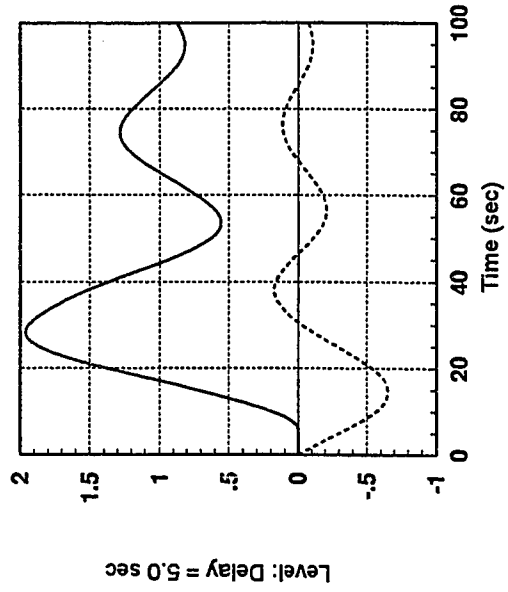


Figure 4.5 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 1 (Continued)



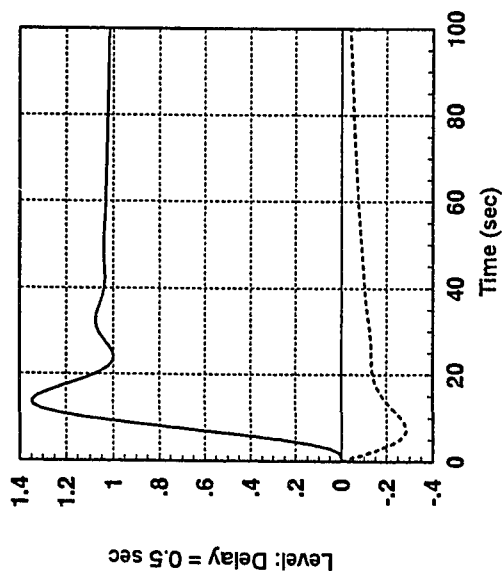
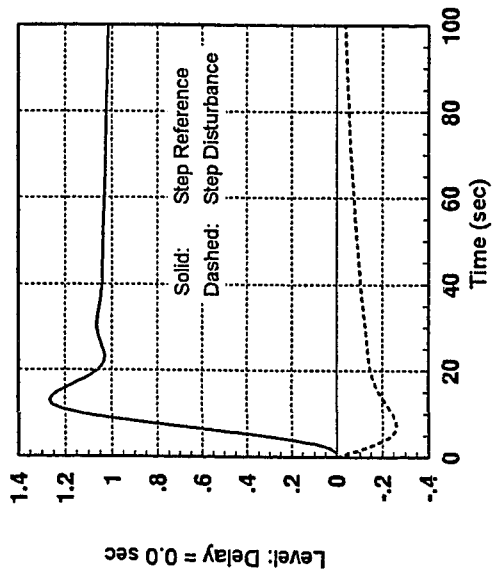
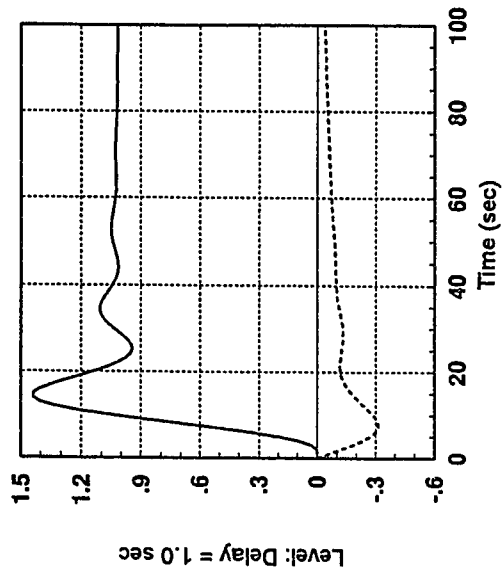
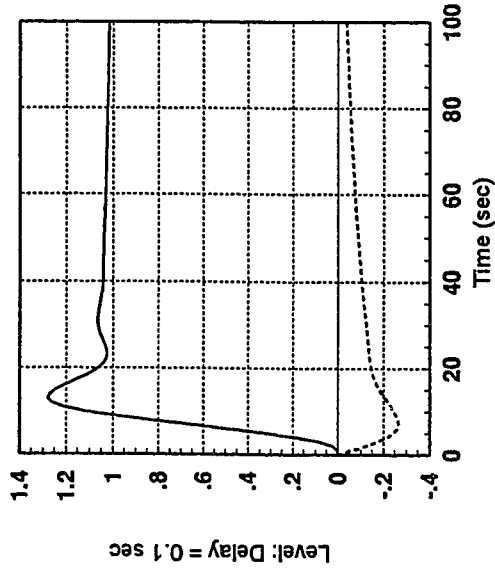


Figure 4.6 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 2 46

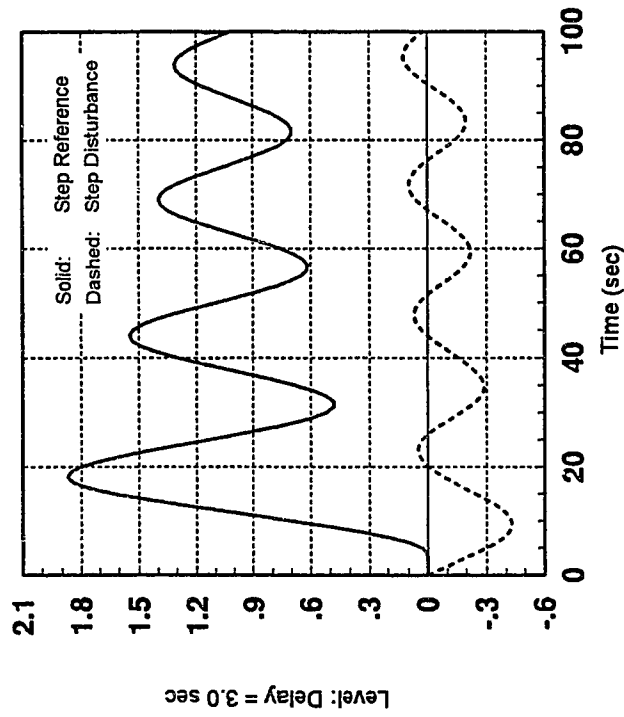


Figure 4.6 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 2 (Continued)

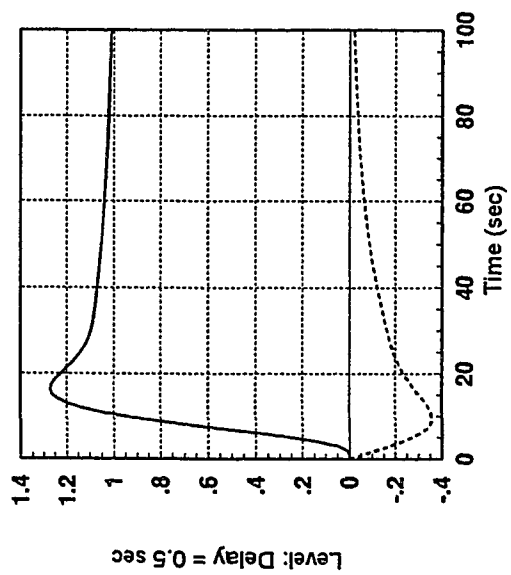
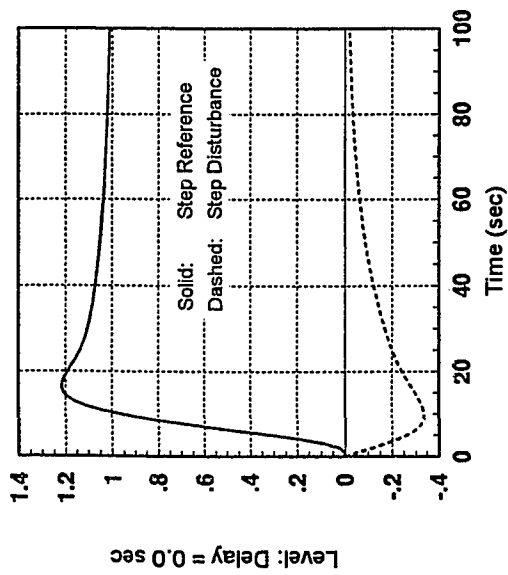
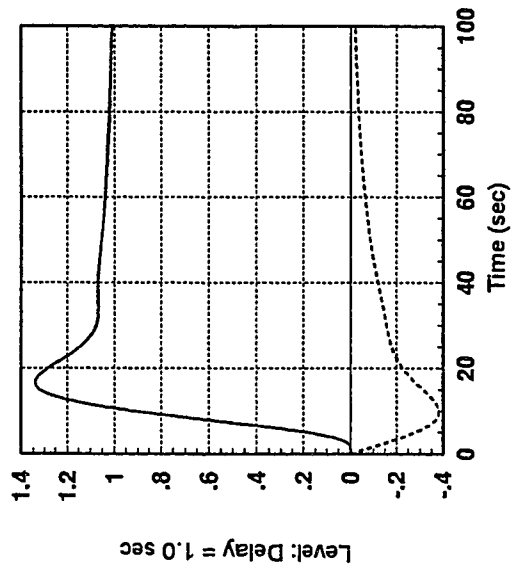
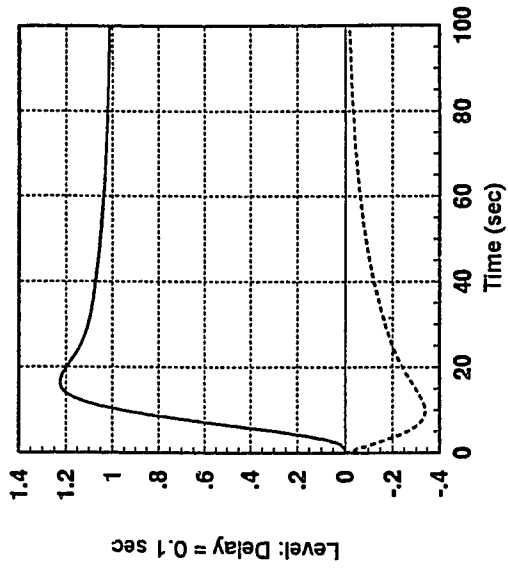


Figure 4.7 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 3 48

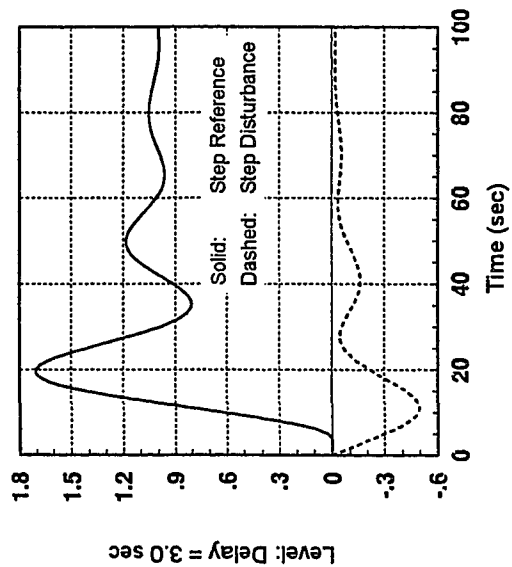
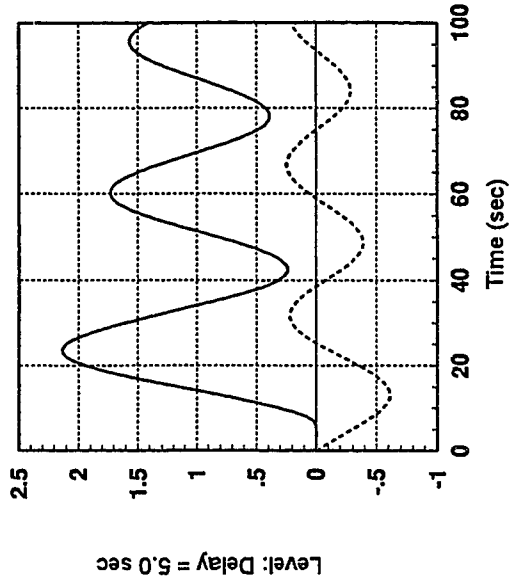
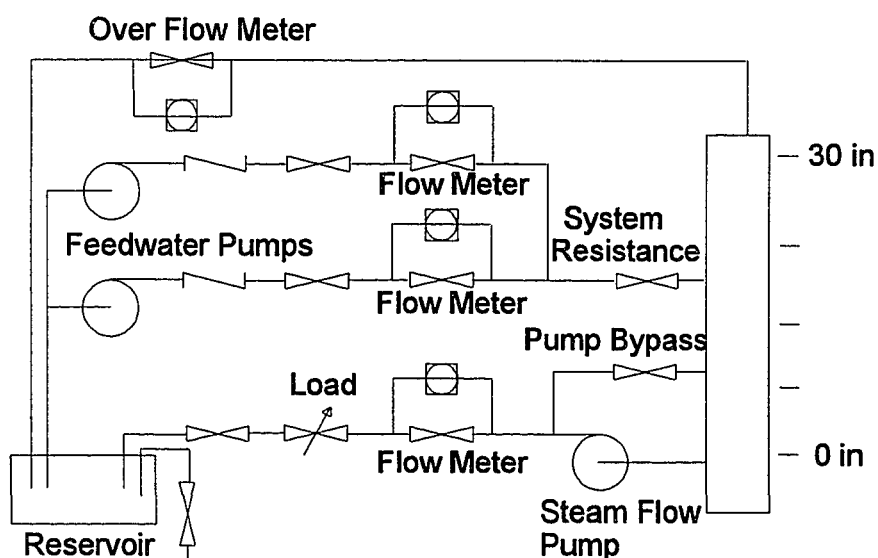


Figure 4.7 Step Reference & Disturbance Responses of Three Element System with Fuzzy Controller, Case 3 (Continued)

### **5.0 GENE Feedwater and Recirculation System Simulator**

The General Electric Nuclear Energy (GENE) Feedwater and Recirculation System Simulator developed by L. H. Youngborg consists of two main components, the control unit and the dynamic system unit. These two units are contained in two separate standard electronic equipment cabinets, approximately two feet wide, two feet deep and 6 feet tall. The control unit contains two GE Fanuc Programmable Controllers model 90-70, a switching unit, and a user interface consisting of a video display and a keyboard. The switching unit is used to change the control of the system from one computer to the other for redundancy testing. The dynamic system unit consists of a mechanical fluid system, electronic sensors, amplifiers, and GENIUS input/output data acquisition and control blocks.



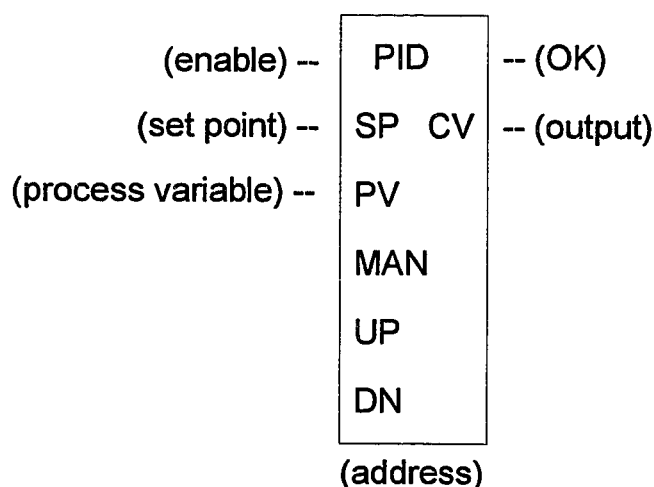
A basic schematic diagram of the mechanical system is can be seen in the figure above. Two independently controlled pumps simulate the feedwater pumps of a reactor. With two pumps, an event such as a pump trip can be simulated. The flow through the pumps

is measured by measuring the pressure differential across valves in the piping which leads the vessel model. The vessel model has a water level range of 30 inches which is about half the normal operating water level range in a reactor. The vessel is always at atmospheric pressure and under normal conditions water is removed from the vessel model by a pump or by the force of gravity through a line which bypasses the pump. The steam flow out of the vessel can be simulated by using either the pump or the gravity fed line. There is also an overflow line leading from the top of the vessel model to the reservoir. The load on the normal flow out of the vessel can be adjusted with the use a hand adjustable valve. This valve can be used to manually simulate a disturbance in the steam flow. The flow out of the vessel model is measured as the pressure differential across a valve in the line. There are electronically controlled shutoff valves in the two feedwater pump lines and the lines out of the vessel. Other valves are used to simulate the flow resistance in a reactor.

Two types of GENIUS input/output blocks are used, the Analog 24Volt, 0.5Amp. block which handles inputs and outputs with varying values and the Source 24Volt, 0.5Amp. block which handles discrete, on or off, inputs and outputs. The source blocks are used for on/off, open/closed, yes/no type conditions and operations such as opening valves and checking for an overflow condition. The analog blocks are used for controlling the pumps, measuring water flow and measuring the vessel water level. The analog blocks contain A/D and D/A converters for communicating with the control computer.

The GE Fanuc Programmable Controller uses a Programmable Logic Controller (PLC) language. This type of control language is typically used for controlling distinct events (on/off, yes/no open/close). It can be used to simulate the safety related control system that will open and close valves, start and stop pumps, and scram the reactor if necessary.

Process control for processes such as the control of the reactor water level would be performed by program blocks in the PLC ladder logic program. An example is the PID control block that is commonly found in PLC packages. These PID function blocks are written into the PLC software by the manufacturer of the controller. The end user can set some parameters such as the proportional, integral and derivative gains, but the programming itself is usually proprietary and can not be modified by the end user [7]. In the GE Feedwater and Recirculation System Simulator the water level in the vessel model is controlled by a built-in PID block. The following figure is a graphical representation of the PID function block as it would be seen in the PLC program [8].



The block is activated through the enable input. The set point input (SP) would be the desired water level and the output signal, control variable (CV) would be pump speed signal. The process variable (PV) would be the feedback signal from the sensors. The OK output is associated with internal error checking for the PID function. It is energized when the block is activated and there are no errors. The up (UP) and down (DN) inputs are used when the manual mode input (MAN) is activated to adjust the water level manually. The simulator is programmed to run in one element and three element modes.

Similar to the systems presented in the beginning of this thesis, the one element mode uses a level feedback signal and the three element mode uses a feedback signal consisting of vessel level and flow in and out of the vessel. The mode used depends upon the operating conditions of the simulator. Activation of the three element mode requires that certain flow rates are achieved beyond the requirements of the one element mode.

To implement any of the control algorithms studied in this paper, a PLC program block which can incorporate a user provided subroutine or program in a programming language such as C, FORTRAN or Basic, must be available. The PID controller in the current simulator PLC program could be easily replaced by a program block with the same inputs and outputs as the PID block. Unfortunately the model of the GE Fanuc controller and subsequent software currently installed in the simulator does not support this type of user created program. Newer models do have a program block which can incorporate a user created program in C. The control computer would have to be upgraded before any of the delay compensation methods studied in this thesis could be implemented on the simulator.

The simulator was tuned to provide performance similar to an actual reactor. This tuning involved the adjustment of valves in various locations and even modifications to the pump impeller blades. The general dynamic equations of the system, or transfer functions of the system have not been derived. This would have to be done before a control algorithm such as the Smith predictor, which requires the dynamic equations, could be implemented. The Smith predictor could be implemented by performing a bilinear transformation to the digital, z-transform form and then incorporating it into a C program to be accessed by the PLC program. A control method such as Fuzzy Logic which is based on the system performance and not the dynamic equations would be easier to implement as a C program block.



## **6.0 Conclusion**

The Smith predictor method appears to have potential as a means time delay compensation. The response to a step disturbance input slowly approaches a zero steady state error condition. This problem increases as the delay time is increased, eventually producing an unacceptable error for practical finite time ranges. The Smith predictor is best suited for small delay times. Further study of the application of the Smith predictor to the Feedwater-Vessel Level Control System should include simulations using discrete models rather than the continuous model with a time delay.

The process-model control system proposed by Watanabe and Ito is intended to overcome the disturbance response error problem encountered by the Smith predictor. Unfortunately it is not practical for high order systems such as the Feedwater control system. As the order of a dynamic system is increased, the order of the controller must be increased to obtain any real benefit from this compensation method. When Watanabe and Ito proposed this method of process-model control, it was applied to a first order servo mechanism process.

The Fuzzy Logic controller appears to be a good alternative to classical control techniques. Unlike the Smith predictor and Watanabe and Ito processes, the Fuzzy Logic controller does not depend on known dynamic equations and delay times. Its system performance based logic can provide good control without deriving the dynamic equations of a system. The Fuzzy Logic method may also be best for systems with a varying delay/cycle time. A good understanding of the performance of a system is required for the implementation of a Fuzzy Logic controller. Testing should encompass the entire range of the controller's inputs and output making sure that any possible condition is tested.

Other alternative methods not studied here are the optimal regulator problem with time delays as investigated by Fuller [9], Klienman [10] and Mee [11]. In a case where the continuous system with a delay is modeling a discretized form of the system, the best alternative may be the retuning of the controller gains using discrete system techniques.

The Fuzzy Logic controller can be easily implemented on the GE Feedwater and Recirculation System Simulator. The control computer must be updated with a more current model which includes a C program block in the PLC programming. The implementation of a classical delay compensation technique, such as the Smith predictor, will require the derivation of the system dynamics.

## 7.0 References

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## **8.0 Appendix**

### **8.1 Feedwater Vessel-Level Control System**

The transfer functions for the feedwater system components are:

**Pump**

$$G_p(s) = \frac{2.8}{s+1} * \frac{1}{1.77s^2 + 2.13s + 1}$$

**Vessel Model**

$$G_{vm}(s) = \frac{0.06}{s}$$

**Level, Feedwater Flow and Steam Flow sensors**

$$G_s(s) = \frac{1}{0.25s + 1}$$

**Dynamic compensator to filter out high frequency variations**

$$G_{dc}(s) = \frac{0.7s + 1}{7s + 1}$$

The mismatch gain for combining flow and level sensor signals is 0.4. The gain before the PI controller is a unit conversion, 100% rated level/60 inches = 1.6667. For block diagrams of the One and Three Element systems, refer to Figures 1.1, 1.2, 2.1 and 2.4.

Rearranging the One and Three Element systems for unity feedback (Figures 2.1 and 2.4) the process portions of the models can be described in state space system matrix form as:

$$S = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

The transfer functions of the processes for the One and Three Element systems have been converted to state space controller canonical form. The process system matrix for the

**Modified One Element system is**

$$\begin{bmatrix} -6.2033 & -10.5819 & -7.6384 & -2.2598 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} \begin{bmatrix} 1.0000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0.3796 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

**The process system matrix of the Modified Three Element system is**

$$\begin{bmatrix} -10.3462 & -36.8531 & -55.0225 & -39.9515 & -13.7271 & -1.2913 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0.2531 & 1.4119 & 1.6524 & 0.2169 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

## 8.2 Obtaining Matrices $C_1$ and $D_1$ for the Watanabe & Ito Controller

Recall that the process control system proposed by Watanabe and Ito requires the calculation of

$$C_1 = Ce^{-AL}$$

$$D_1 = -\int_0^L Ce^{-A\tau} B d\tau$$

where the matrices A, B, C and D are the components of the process system matrix as shown in the previous section. Using the programs `MATRIXx` or `MATLAB`, the matrix  $C_1$  can be calculated using the matrix exponential function, `expm`. A typical command line entry might be

$$c1 = \text{expm}(-a * l)$$

where  $a$  is the matrix A, and  $l$  is the delay time L in seconds.

To obtain the matrix  $D_1$ , a function which converts a continuous system equation to a discrete system equation is used. This type of function converts from the Laplace domain to the  $z$  domain including a zero-order hold on the input.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \xrightarrow{\text{Discretize}} \begin{bmatrix} P & Q \\ C & D \end{bmatrix}$$

Taking P, Q, C and D as the components of the discretized process system matrix and letting T be the sampling time, the matrix Q is defined as

$$Q = \int_0^T e^{A\tau} d\tau \cdot B$$

Typical command line entries might be

$$[p,q] = \text{c2d}(a,b,t)$$

**MATLAB**

$$\langle s, ns \rangle = \text{discretize}([a,b;c,d], ns, t)$$

**MATRIX<sub>x</sub>**

where  $s=[p \ q; \ c \ d]$ , the discretized system matrix and  $ns$  is the number of states. By replacing  $a$  in the command line with  $-a$  and replacing the sampling time  $t$  with the delay time, the computed matrix  $q$  would be

$$Q = \int_0^L e^{-A\tau} d\tau \cdot B$$

The matrix  $D_1$  can then be obtained from the equation,  $D_1=C*(-Q)$ .

### 8.3 Routh Table

The following spreadsheet was used to find ranges of the proportional, integral controller gains,  $k_p$  and  $k_i$ , which provide stable responses for the modified three element system with a Watanabe & Ito controller and a delay time of 3 seconds. The first Routh table shows that gain values of  $k_i=0.017$  and  $k_p=1.0$  provides an unstable system.

|          |                |          |          |          |          |          |
|----------|----------------|----------|----------|----------|----------|----------|
| G =      | 1.666667       |          |          |          |          |          |
| G * ki = | 0.028333       |          | ki =     | 0.017    |          |          |
| G * kp = | 1.666667       |          | kp =     | 1        |          |          |
|          |                |          |          |          |          |          |
|          | s <sup>8</sup> | 312.8577 | 3634.985 | 1131.987 | 4.59007  | 0.006146 |
|          | s <sup>7</sup> | 1992.688 | 2954.931 | 127.3146 | 0.389879 |          |
|          | s <sup>6</sup> | 3171.053 | 1111.998 | 4.528858 | 0.006146 |          |
|          | s <sup>5</sup> | 2256.152 | 124.4686 | 0.386017 |          |          |
|          | s <sup>4</sup> | 937.0559 | 3.986306 | 0.006146 |          |          |
|          | s <sup>3</sup> | 114.8708 | 0.37122  |          |          |          |
|          | s <sup>2</sup> | 0.958085 | 0.006146 |          |          |          |
|          | s <sup>1</sup> | -0.3656  |          |          |          |          |
|          | s <sup>0</sup> | 0.006146 |          |          |          |          |
|          |                |          |          |          |          |          |
|          |                |          |          |          |          |          |
|          |                |          |          |          |          |          |
|          |                |          |          |          |          |          |
|          |                |          |          |          |          |          |
| G =      | 1.666667       |          |          |          |          |          |
| G * ki = | 0.008333       |          | ki =     | 0.005    |          |          |
| G * kp = | 0.833333       |          | kp =     | 0.5      |          |          |
|          |                |          |          |          |          |          |
|          | s <sup>8</sup> | 156.9288 | 1829     | 576.0317 | 2.605283 | 0.001808 |
|          | s <sup>7</sup> | 1000.426 | 1492.501 | 66.8677  | 0.189097 |          |
|          | s <sup>6</sup> | 1594.883 | 565.5427 | 2.575621 | 0.001808 |          |
|          | s <sup>5</sup> | 1137.752 | 65.25209 | 0.187963 |          |          |
|          | s <sup>4</sup> | 474.0734 | 2.312138 | 0.001808 |          |          |
|          | s <sup>3</sup> | 59.70307 | 0.183625 |          |          |          |
|          | s <sup>2</sup> | 0.85406  | 0.001808 |          |          |          |
|          | s <sup>1</sup> | 0.057272 |          |          |          |          |
|          | s <sup>0</sup> | 0.001808 |          |          |          |          |