1995

Actuarial cost methods in pension funding

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ACTUARIAL COST METHODS IN PENSION FUNDING

A Thesis
Presented to
The Faculty of the Department of Mathematics
San Jose State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

By
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December, 1995

UMI Number: 1377219

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ABSTRACT

ACTUARIAL COSTS METHODS IN PENSION FUNDING

by Irene Wai-Ling Chen

This thesis presents eight actuarial funding methods commonly used in the United States to fund pension plans. Some actuarial functions, commutation functions and life annuity functions needed for understanding these cost methods are introduced.

The thesis describes the construction of each cost method and further illustrates its concept by a numerical example. Criteria for desirable funding methods are discussed. This thesis also examines part of the Retirement Protection Act of 1994 and illustrates the problem that actuaries may encounter under this new law.

ACKNOWLEDGMENTS

I would like to thank Dr. Bem Cayco-Gajic who helped me on my FORTRAN programs for the computational work in my thesis. I would also like to thank Dr. Sin-Min Lee for his support to finish this thesis.

By far my greatest appreciation is to my thesis advisor, Dr. Ho Kuen Ng. He encouraged me to write this thesis and patiently helped me through the whole process. Without his guidance, I would not be able to finish this thesis.

Finally, I would like to thank my parents for all that they have done for me through the years and also my son, Albert Cheng, who gave me his love and understanding to finish this thesis.

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Chapter 1

Notation and Definition

The purpose of this chapter is to briefly introduce the actuarial functions, commutation functions and life annuity functions that we will use later in this paper. Before introducing these functions, some probability and survival functions need to be defined first.

Section 1.1: Basic Survival Functions

Let X, a continuous non-negative random variable, represent a newborn's age at death and F(x) denote the distribution function of X. Then the probability that a newborn will live at least x years is $P\{X \geq x\}=1-F(x)=S(x),\ x\geq 0$. We call S(x) the survival function of a newborn. Therefore, the probability that an x-year old will survive to age x+t , for t ≥ 0, can be expressed in terms of the survival function as

$$P\{X>x+t \mid X>x\} = \frac{P\{X > x+t \text{ and } X > x\}}{P\{X > x\}} = \frac{P\{X > x+t\}}{P\{X > x\}} = \frac{S(x+t)}{S(x)} .$$

Now let the symbol (x) denote a person at age x and T(x) denote the future lifetime of (x). Notice that T(x) is a continuous non-negative random variable. The survival function of (x) is

$$P\{T(x)>t\} = P\{(x) \text{ survives to age } x+t \mid (x) \text{ survives to age } x\} = \frac{S(x+t)}{S(x)} .$$

The survival function of (x) and the distribution function of T(x) are denoted by ${}_tp_x$ and ${}_tq_x$ respectively. That is

$$ {}_tp_x = P\{\text{ (x) will survive t more years}\} = P\{T(x)>t\} = \frac{S(x+t)}{S(x)} \tag{1.1.1} $$

and

$$ {}_tq_x = P\{\text{ (x) will die within t years}\} = P\{\ T(x) \le t\} $$

$$ = \frac{S(x)-S(x+t)}{S(x)} = 1 - {}_tp_x \tag{1.1.2} $$

for t≥0. If t=1, convention permits us to omit the prefix in the symbols defined in (1.1.1) and (1.1.2), and we have

$$ p_x = P\{\text{ (x) will survive 1 year}\} $$

$$ q_x = P\{\text{ (x) will die within 1 year}\}\ . \tag{1.1.3} $$

A mortality table which is constructed from past known experience, such as death records and census data, is usually needed to determine ${}_tp_x$ and ${}_tq_x$,. A mortality table contains two basic columns: age and the number living. The entries of these two columns are denoted by x and l_x respectively.

Let l_0 represent the number of newborns and L(x), a random variable, represent the number among the initial l_0 lives surviving to age x. For each of these l_0 lives, we associate an indicator random variable; that is

$$ I_j = \begin{cases} 0, & \text{if the jth life dies before reaching age x;} \\ 1, & \text{if the jth life survives to age x.} \end{cases} $$

Since

$$E[I_j] = 0 \times P\{I_j=0\} + 1 \times P\{I_j=1\} = P\{I_j=1\} = {}_xp_0 = S(x)$$

and $L(x) = \sum_{j=1}^{l_0} I_j$,we have

$$E[L(x)] = E[\sum_{j=1}^{l_0} I_j] = \sum_{j=1}^{l_0} E[I_j] = \sum_{j=1}^{l_0} S(x) = l_0 S(x) .$$

Let l_x denote E[L(x)]. This means that l_x represents the expected number of survivors to age x among the l_0 newborns. With $l_x = l_0 S(x)$, the notation ${}_np_x$ then also can be expressed in terms of l_x

$${}_np_x = \frac{S(x+n)}{S(x)} = \frac{l_{x+n}/l_0}{l_x/l_0} = \frac{l_{x+n}}{l_x} .$$

Sometimes a more refined mortality table is needed. As an example, applicants may not be offered insurance on a standard basis because of health conditions or other factors. The result of this selection process is that the group of standard insured lives may not be random. The mortality of such a group usually varies not only by age, but also by the duration since the policy is issued. Thus, a just insured life at age x is usually subject to a lower rate of mortality during the first year than another life at age x insured a year earlier. This variation in mortality experienced by insured groups of the same age will diminish after some years. We say that the groups reach an ultimate stage when there is no appreciable variation in the mortality experienced by the groups. A mortality table that shows the mortality variation by both age and duration is called a select mortality table. Additional column may be added into a select mortality table showing the corresponding number of survivors after the insured group has reached the ultimate stage. We then call it a select and ultimate mortality table [8].

Section 1.2: Composite Survival Functions

Since an actuarial cost method tries to fund for a "future" benefit, naturally, pension actuaries must first make some actuarial assumptions. These assumptions can be categorized into three different types: decrement, salary, and interest. For the decrement assumptions, they can be further divided into four kinds, namely, the mortality, withdrawal, disability, and retirement decrements [4].

A participant's age, sex, and occupation are the main factors related to mortality rates. The symbols $q_x^{(m)}$ and $p_x^{(m)}$ are used to denote the probability that (x) dies within one year and the probability that (x) lives to age x+1, respectively. The withdrawal decrement has significant relation with the participant's age and length of service. The probabilities that (x) withdraws within one year and that (x) does not withdraw for at least one year are denoted by $q_x^{(w)}$ and $p_x^{(w)}$ respectively. The corresponding symbols for the disability decrement are $q_x^{(d)}$ and $p_x^{(d)}$ respectively. Usually the above three types of decrement will either terminate or lower the cost of pension benefit. But they are often offset to some extent if the plan provides other forms of benefit, such as death benefit, disability benefit or vesting benefit. The last type of decrement is retirement; it is unlike the other decrement factors in that it initiates the pension payments. The probability that (x) retires within one year is denoted by $q_x^{(r)}$, and the probability that (x) does not retire for at least one year is $p_x^{(r)}$. With this notation, the probability of (x) surviving in service for one year can be defined as

$$p_x^{(T)} = 1 - q_x^{(T)} = 1 - [q_x^{(m)} + q_x^{(w)} + q_x^{(d)} + q_x^{(r)}]$$

where $q_x^{(T)}$ is the probability of (x) leaving service during the year. For simplicity, we will disregard the superscript $^{(T)}$ throughout the rest of the paper. After disregarding the superscript $^{(T)}$, the survival function p_x will represent the probability that an active participant aged x survives in service for at least one year and should not be confused with the p_x defined in (1.1.3). The probability that an active participant aged x survives at least n years is the product of successive one-year survival probabilities, that is

$$_n p_x = \prod_{t=0}^{n-1} p_{x+t} \ .$$

Other functions associated with survivorship are $l_x^{(T)}$, the total number of survivors in service at age x, and $d_x^{(T)}$, the total expected number of employees leaving active service during the year. So we have

$$d_x^{(T)} = l_x^{(T)} - l_{x+1}^{(T)} = l_x^{(T)}[1 - p_x^{(T)}] = l_x^{(T)} \cdot q_x^{(T)}$$

Again the superscript $^{(T)}$ will be disregarded. For example, the probability of an entrant aged 25 surviving in active service to age 65 is $_{65-25}p_{25} = \frac{l_{65}}{l_{25}}$.

All the above discussion only involved one person. Now a body of k lives with ages $x_1, x_2, \cdots, x_k$ will be considered. There are two statuses in the concept of group survival

[8]. The first one is the "joint-life" status $(x_1x_2 \cdots x_k)$ which continues in existence as long as all the lives (x_1), (x_2), $\cdots$, (x_k) survive, and fails upon the occurrence of the first death. The probability that the joint-life status $(x_1x_2 \cdots x_k)$ will survive for n years is denoted by ${}_n p_{x_1x_2\cdots x_k}$, and since it requires all lives to survive, we have

$${}_n p_{x_1x_2\cdots x_k} = {}_n p_{x_1} \cdot {}_n p_{x_2} \cdots {}_n p_{x_k} .$$

The second status is called the "last-survivor" status $(\overline{x_1x_2 \cdots x_k})$. This group continues to exist as long as at least one of (x_1), (x_2), $\cdots$, (x_k) is alive, and fails upon the occurrence of the last death. The probability of $(\overline{x_1x_2 \cdots x_k})$ will survive for n years is denoted by ${}_n p_{\overline{x_1x_2\cdots x_k}}$ which can be expressed as the complement of the probability that all the lives will die within n years. Thus,

$$\begin{aligned}
{}_n p_{\overline{x_1x_2\cdots x_k}} &= 1 - (1 - {}_n p_{x_1})(1 - {}_n p_{x_2}) \cdots (1 - {}_n p_{x_k}) \\
&= ({}_n p_{x_1} + {}_n p_{x_2} + \cdots + {}_n p_{x_k}) - ({}_n p_{x_1x_2} + {}_n p_{x_1x_3} + \cdots + {}_n p_{x_{k-1}x_k}) \\
&\quad + ({}_n p_{x_1x_2x_3} + {}_n p_{x_1x_2x_4} + \cdots + {}_n p_{x_{k-2}x_{k-1}x_k}) - ({}_n p_{x_1x_2x_3x_4} + \cdots + {}_n p_{x_{k-3}x_{k-2}x_{k-1}x_k}) \\
&\quad + \cdots + (-1)^{k+1} {}_n p_{x_1x_2\cdots x_k} .
\end{aligned}$$

Section 1.3: Benefit Function

There are two forms of obligations for an employer in establishing a pension plan: (1) an undertaking to provide benefits in accordance with a specified schedule, or (2) an undertaking to make contributions on a specified basis. We refer to the first approach as a defined benefit plan, and the second one as a defined contribution plan, or more specifically a money purchase plan [10].

A defined contribution plan is also called an individual account plan. The plan provides an individual account for each participant and defines how contributions are to be allocated to each participant's account. The benefit for each participant is solely based upon the amount contributed to his account and any expenses, investment earnings, and forfeitures allocated to his account. Under a defined contribution plan, the employer does not undertake to provide retirement benefits in accordance with any predetermined scale. Sometimes a projected target benefit at normal retirement age is determined for each participant at the establishment of the plan; and the amount needed to fund such target benefit is calculated as a level annual contribution which is allocated to the participant's account. This is called a target benefit plan. But still there is no guarantee that the projected target benefit will be there when the participant retires.

A defined benefit plan is one in which the benefits are established in advance by a formula. This benefit formula is used to determine the amount of benefits to be paid at retirement, vested termination, disability, or death. Note that a defined benefit plan treats employer contributions as the variable factor whereas a defined contribution plan treats the benefits as the variable factor. A defined benefit plan can provide either a fixed-dollar amount of pension or a varying amount of pension after retirement. The plans that provide fixed-dollar benefits are the dominant type which we will restrict our discussion to.

The benefit formulas can be classified into two types: (1) unit benefit, and (2) flat benefit. Under a unit benefit formula, an explicit unit of benefit is credited for each year of service. This unit of benefit may be expressed as a percentage of compensation or a specific dollar amount. When the unit of benefit credited during any particular year is based upon the participant's compensation during the same year, the formula is called a career average formula. But if the benefit credited per year is based on the compensation of a specified period, such as 5 or 10 years immediately before retirement, the benefit formula is called a final average formula. In contrast to a unit benefit formula, a flat benefit formula provides a benefit at retirement equal to a specific percentage of compensation or a flat dollar amount to participants generally without regard to years of service, if a certain minimum period is satisfied.

Generally defined contribution plans are easier for employers to administer and plan participants to understand. They ordinarily do not require actuarial computations which create an expense saving; furthermore, they usually have no unfunded accrued liability since the costs for all service rendered to date are fully funded. These are some of the advantages for defined contribution plans. On the other hand, defined benefit plans offer more flexibility in funding due to the amortization of the unfunded accrued liability. Another advantage of a defined benefit plan for participants is that the employer bears the investment risk while under a defined contribution plan the investment risk is directly reflected in each participant's account.

The discussion in this paper will be limited to defined benefit plans only. We assume that each participant retires at age y and let B(y) denote the annual benefit payment commencing at retirement. Thus, when a participant is hired at age w, his accrued benefit B(w) is zero; and when he retires at age y, his projected benefit reaches the maximum value B(y).

Section 1.4: Salary and Interest Functions

The retirement benefits provided by the required contributions to a pension plan are often functions of the compensations of its participants. So it is important to have a means of obtaining an estimate of a participant's future changes in salary. This is provided by a series of projected compensation factors called the "salary scale." A salary scale usually takes into account two elements: increases in average salary levels and merit increases. A salary scale should recognize the periodic upward shifts in the whole wage structure due to productivity gains in the economy and inflation. Separate salary scale functions may be used for male and female participants, for executives and other employees, or for salaried and hourly participants. The "salary scale function" used here is a general one and is used for all types of compensation.

We have a set of salary indices s_x such that if S_x denotes the current salary for an participant aged x, then his assumed salary at age x+g is $S_x \frac{s_{x+g}}{s_x}$. Note that g can take on either positive or negative values. Let w denote the participant's age at hire, then his assumed salary at age w is $S_w = S_x \cdot \frac{s_w}{s_x}$. Such a set of salary indices is called a salary scale.

The present value of a series of future contingent payments is a function of the rate of investment return, or interest rate, at which the payments are discounted. The funding interest rate is set at a level equal to the expected return on plan assets in future years. The higher the interest assumption is, the smaller the present value will be. Due to the long period of time between the accrual of a benefit and its payment, pension plan costs and liabilities are extremely sensitive to the interest assumption used. Therefore, the interest rate must be chosen with great care. The total interest rate can be regarded as the sum of a riskless (pure) rate of return, a premium for investment risk, and a premium for inflation.

For a given principal P, which would accumulate to an amount A after t years, at an effective level annual rate of interest i, the equation is:

$$A = P \times (1+i)^t.$$

If we want to accumulate to one dollar at the end of a year, the principal v, invested at the beginning of the year at an effective annual rate of interest i, should be

$$1 = v \times (1+i),$$

or $$v = \frac{1}{1+i}.$$

We call v the discount factor or the present value of one unit (dollar) payable one year from now. The present value of one unit (dollar) payable n years from now will then be v^n.

Section 1.5: Commutation and Annuity Functions

To simplify the construction and manipulation of actuarial formulas, commutation functions have been developed to express actuarial formulas in terms of convenient intermediate values.

Suppose that (x) will receive a payment of 1 at the end of n years if he survives and receive no payment if dies within n years. This is called an n-year pure endowment. The actuarial present value is $1 \times v^n \times {}_np_x$ where v is the discount factor and ${}_np_x$ is the probability that (x) will survive n years. The actuarial present value, $v^n{}_np_x$, is denoted by ${}_nE_x$. We now define a commutation function $D_x = v^x l_x$ and rewrite ${}_nE_x$ in terms of D_x; that is

$${}_nE_x = v^n{}_np_x = v^n \frac{l_{x+n}}{l_x} = \frac{v^{x+n} l_{x+n}}{v^x l_x} = \frac{D_{x+n}}{D_x}.$$

The summation of D_x is defined to be the commutation function N_x,

$$N_x = \sum_{u=x}^{\infty} D_u.$$

A life annuity is a series of level payments made at equal intervals of time. If the payments of a life annuity are made at the end of each time interval, then it is called a life annuity-immediate. Here we will only discuss life annuities-due since they play the more significant role in pension applications.

There are several life annuities-due. For convenience, we first assume that each payment amount is 1. First, a whole life annuity-due, payable to (x), is a series of annual payments payable at the beginning of each year while (x) is alive. Its actuarial present value is denoted by $\ddot{a}_x$ and may be expressed as the sum of a series of pure endowments:

$$\ddot{a}_x = 1 + v^1 p_x + v^2 {}_2p_x + v^3 {}_3p_x + \cdots + v^n {}_np_x + \cdots$$

$$= 1 + {}_1E_x + {}_2E_x + {}_3E_x + \cdots + {}_nE_x + \cdots$$

$$= \frac{D_x}{D_x} + \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+3}}{D_x} + \cdots + \frac{D_{x+n}}{D_x} + \cdots$$

$$= \frac{\sum_{u=x}^{\infty} D_u}{D_x} = \frac{N_x}{D_x} .$$

An n-year temporary life annuity-due is a life annuity-due where the payments stop at the nth payment or after (x) dies, whichever occurs first. The notation for the actuarial present value of an n-year temporary life annuity-due is $\ddot{a}_{x:\overline{n|}}$. It can also be expressed as the sum of a series of pure endowments:

$$\ddot{a}_{x:\overline{n|}} = 1 + {}_1E_x + {}_2E_x + {}_3E_x + \cdots + {}_{n-1}E_x$$

$$= \frac{D_x}{D_x} + \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \cdots + \frac{D_{x+n-1}}{D_x} = \frac{N_x - N_{x+n}}{D_x} .$$

The payment period of the above two life-annuities is equal to the interest conversion period, one year. Now let us consider the case when the annuities are payable more frequently than interest is convertible. Let m be the number of payment periods in one interest conversion period. In other words, the payments are made m times per year. The actuarial present value of a whole life annuity-due, payable m times a year in installments of $\frac{1}{m}$ at the beginning of each mth of a year while (x) survives, is denoted by $\ddot{a}_x^{(m)}$. In other words, the first payment of $\frac{1}{m}$ is at age x and second payment of $\frac{1}{m}$ is at age $x+\frac{1}{m}$, and so on. The actuarial present value can then be written as

$$\ddot{a}_x^{(m)} = \frac{1}{m}\sum_{t=0}^{\infty} {}_{\frac{t}{m}}E_x = \frac{1}{mD_x}\sum_{t=0}^{\infty} D_{x+\frac{t}{m}} \,. \tag{1.5.1}$$

Since $D_{x+\frac{t}{m}}$ is not defined when $x+\frac{t}{m}$ is fractional, we need to approximate $\ddot{a}_x^{(m)}$ by applying Woolhouse's summation formula [8] to the right-hand side of (1.5.1). We get

$$\frac{1}{mD_x}\sum_{t=0}^{\infty} D_{x+\frac{t}{m}} = \frac{1}{D_x}\left[\sum_{t=0}^{\infty} D_{x+t} - \frac{m-1}{2m}D_x + \frac{m^2-1}{12m^2}\frac{dD_x}{dx} - \cdots\right]. \tag{1.5.2}$$

The measurements of interest and mortality at an instantaneous moment of time are called the force of interest and the force of mortality respectively. We use δ to denote the force of interest, $\delta = \log_e(1+i)$ [9] . The force of mortality is denoted by μ_x and defined to be $\mu_x = \frac{-dl_x/dx}{l_x}$ [4] . Thus, it follows that the derivative of l_x is $-l_x\,\mu_x$ and the derivative of v^x is $v^x \log_e v = -v^x\delta$. Now taking the derivative of D_x with respect to x, we get

$$\frac{dD_x}{dx} = -v^x l_x \mu_x - v^x l_x \delta = -v^x l_x (\mu_x + \delta) = -D_x(\mu_x + \delta).$$

Substituting this result into (1.5.2), we have

$$\ddot{a}_x^{(m)} = \frac{1}{D_x}\Big[\sum_{t=0}^{\infty} D_{x+t} - \frac{m-1}{2m} D_x - \frac{m^2-1}{12m^2} D_x(\mu_x + \delta)$$

$$= \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x + \delta).$$

In practice, the approximation $\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}$ is regarded as sufficiently accurate. For the temporary life annuity-due payable m times a year, the actuarial present value can also be approximated by the formula $\ddot{a}_{x:\overline{n}|}^{(m)} \approx \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}(1 - {}_nE_x)$ [8].

Chapter 2

Basic Pension Cost Concepts

A pension plan is a program for providing regular payments to retired employees, usually for life. It involves the accumulation of funds in a systematic manner during the employees' working years. This systematic way to provide retirement benefit is called funding, or an actuarial cost method. The process of computing the actuarial values associated with a pension plan is called a valuation. The definitions of certain basic pension cost concepts which are essential for understanding the various actuarial cost methods are discussed in this chapter [1, 2, 17].

Present Value of Future Benefit

The first concept to be defined is the present value of future benefits (PVFB), where future benefits consist of future retirement, death, disability, vesting and other plan benefits. All these future benefits refer to benefits accrued to date plus benefits expected to be accrued throughout the remainder of each participant's working career. These future benefits will be used to determine a plan's annual costs and other liability measures. It is important to keep in mind that PVFB is based on plan provisions, participant's data, and actuarial assumptions, and does not depend on the cost methods. For simplicity, only the retirement benefit will be considered throughout this paper since other benefits can be handled in a similar way. The PVFB function for a participant currently at age x, having entered the plan at age w and assumed to retire at age y, is

$$PVFB_x = B(y)\ {}_{y-x}p_x\ v^{y-x}\ \ddot{a}_y \qquad (x \leq y)$$

where B(y) = the benefit at retirement payable annually as a life annuity. Note that annual payments are assumed here; however, payments are usually made monthly. After retirement ($x \geq y$) this function is equal to $PVFB_x = B(x)\ddot{a}_x$, usually with $B(x) = B(y)$.

Normal Cost

The normal cost (NC) is defined quite specifically by each actuarial cost method. The general objective of an actuarial cost method is to calculate the cost deemed to have accrued in one year. The portion of the actuarial present value of benefits assigned to that year is called the normal cost. In other words, the normal cost is designed to amortize the PVFB over the participant's working lifetime. It can be developed as a level dollar amount or as a level percentage of the participant's salary. The annual normal cost will be the only contribution required from the employer if 1) the normal cost is calculated from the inception of plan, and 2) all assumed interest and survival rates hold exactly, and 3) no service benefits before the inception of plan are credited, and 4) no amendments to the plan are made.

Actuarial Liability

The actuarial liability of the plan (AL) can be defined using two approaches, retrospectively or prospectively. From the retrospective point of view, the actuarial liability with no past service credit equals the cumulative normal costs of the plan which are increased by the valuation rate of interest, decreased by benefit and expense disbursements, and adjusted for actuarial gains and losses. In short, AL equals the accumulated value of past normal costs (AVPNC). So for a participant aged x, the associated AL is

$$AL_x = AVPNC_x = \sum_{t=w}^{x-1} NC_t (1+i)^{x-t} \frac{1}{{}_{x-t}p_t} \ .$$

The actuarial liability can also be defined prospectively. It then equals PVFB at that age less the present value of future normal costs (PVFNC) yet to be made. That is,

$$AL_x = PVFB_x - PVFNC_x$$

where $PVFNC_x = \sum_{t=x}^{y-1} NC_t \, {}_{t-x}p_x v^{t-x}$.

The term accrued liability and actuarial accrued liability are synonymous with actuarial liability. Theoretically, the plan's assets should be equal to the AL at all time. It is usually not the case in real life, when actual experience is not exactly in accord with assumptions, or past service benefits are granted. These will result in a difference between the accrued liability and the plan's assets (F). The difference, AL - F, is called the unfunded accrued liability (UAL). The portion of the actuarial liability offset by plan assets is referred to as the funded accrued liability.

Supplemental Cost Liability

If all actuarial assumptions are exactly realized, AL defined by both approaches will be equal. In some situation, there will be a difference between the two calculations of AL. A supplemental cost liability (SL) is then generated to balance the two different calculated ALs. When this occurs, the accrued liability is composed of two elements: the normal cost liability and the supplemental cost liability. The normal cost liability is the accumulated value of all past normal cost since the inception of the plan, AVPNC. The supplemental cost liability is the difference between the accrued liability and the normal cost liability.

There are several sources that will create a SL. A significant source of a SL is granting pension credits for service prior to the establishment of the plan or increasing the benefits attributable to service in prior years. Changing an actuarial assumption is another source of a SL. At the inception of the plan, AL and SL are equal. But by the end of the first plan year, AL will have increased by NC plus the interest at the valuation rate on the supplemental cost liability and less benefit payments, while SL will have increased only by the assumed interest and less benefit payment attributable to SL. These two values then will usually not be the same again.

The supplemental cost liability must be funded in accordance with various schedules allowed by law. The cost that is associated with the amortization of the SL is called the supplemental cost (SC). Additional layers of SL may be generated in later years if the benefits are retroactively liberalized, or new actuarial assumptions are adopted. Each layer of a SL has its own amortization schedule. The normal cost and supplemental cost together constitute the annual cost of the plan.

Actuarial Gains and Losses

Generally, plan contributions consist of a NC plus the amortization of UAL. For any year, the expected UAL is equal to the sum of the prior year's UAL and NC both increased by a year's interest and diminished by the previous year's contribution with interest. The year-to-year deviations of the plan's experience known as actuarial gains or losses are the difference between the expected UAL and actual UAL, assuming no changes in plan. So the actuarial gains and losses for the period t-1 to t is

$$\text{Gain}_t = \text{expected UAL}_t - \text{actual UAL}_t$$

$$= (\text{expected AL}_t - \text{expected F}_t) - \text{actual UAL}_t$$

where

$$\text{expected AL}_t = [\text{AL}_{t-1} + \text{NC}_{t-1}](1+i) - \text{P}_{t-1} \text{ with interest adjustment,}$$

and

expected F_t = $F_{t-1}(1+i)$ $+C_{t-1}$ with interest adjustment - P_{t-1} with interest adjustment.

C_{t-1} is the contribution for the year from t-1 to t and P_{t-1} is the amounts withdrawn to purchase pensions between t-1 and t.

If the experience of the plan is more favorable financially than what the assumptions would give, actuarial gains are generated. On the other hand, actuarial losses are generated when the experience is less favorable than what was assumed. For simplicity, the word "gains" will be used for both gains and losses in the rest of this paper. In other words, a negative gain represents an actuarial loss. An actuarial gain can be spread over the participants' remaining working lifetimes or reflected in the supplemental liability, which would then be spread over a certain amortization period.

Equation of Balance

Every actuarial cost method attributes PVFB to past and future years. The portion attributed to past years is AL, and PVFNC is the portion attributed to current and future years. NC is the portion of the PVFNC attributed to the current year. The relationship between PVFB, AL, and PVFNC is called the basic funding equation or equation of balance: PVFB = AL + PVFNC. Since UAL = AL - F, the equation of balance can also be written as PVFB = F + UAL + PVFNC. Every actuarial cost method should satisfy this equation.

Classifications

Actuarial cost methods may be classified in various ways, depending upon the particular characteristics of the methods. One of the most fundamental bases for classification is whether the method allocates the benefits of the plan or the actuarial present value of the projected normal retirement pension benefit to various plan years. A benefit-based method is used when only pension benefits currently earned are to be funded. It would derive the normal cost for each participant as the present value of the current year's earned benefit and AL as the present value of the benefits accrued by the beginning of a year. A cost-based method or projected benefit-based method allocates the actuarial present value of all benefits to various plan years without allocating the benefits themselves.

Another basis of classification is how the normal cost and actuarial liability are calculated. Under an aggregate funding method, the cost for a year is determined for all participants as a whole. But an individual funding method calculates a normal cost and an actuarial liability for each participant individually and sums to get the plan normal cost and actuarial liability for a year.

Another way to distinguish among methods is whether the method develops a supplemental cost liability which is usually related to past service benefits. If a method does not generate a supplemental cost liability, then the past service benefits and other retroactively granted benefits must be assigned in the form of normal costs.

The last way to distinguish among methods is how actuarial gains are dealt with. For an immediate gain method, a gain is explicitly calculated each year as the difference between the actual and expected unfunded actuarial liabilities, and separately amortized over a certain period of time. Under an immediate gain cost method, PVFB is first split between AL and PVFNC according to the equation of balance. Based on that, the NC and the actual UAL = AL - F are calculated. Then an actuarial gain which is the difference between the expected and actual UAL is determined. The other alternative is a spread gain method where gains are not directly computed; instead, they are spread over future working lifetimes of all participants and incorporated into the normal cost calculation. For spread gain methods, UAL is first defined to be the expected UAL, then the AL is set to be the sum of this UAL and F. After that, PVFB is split between AL and PVFNC; finally, the NC is determined based on the PVFNC. The analysis of actuarial gains could provide a periodic check on the appropriateness of actuarial assumptions.

The advantage of having a variety of cost methods available is that an actuary may choose the most suitable method to fit the needs of the plan sponsor. The eight most commonly used funding methods will be introduced in the following chapter.

Chapter 3

Actuarial Cost Methods

This chapter describes some of the actuarial funding methods commonly used in the United States. We will try to point out the requirements under United States laws, regulations and official guidance. Specifically, the rules under the Employee Retirement Income Security Act ("ERISA") [5], the Internal Revenue Code ("Code") [7], Federal Income Tax Regulations [6] and a couple of Revenue Procedures [12, 13] are taken into account. However, to illustrate the fundamental actuarial principles involved, we will ignore special and technical rules that are not actuarially related. Thus, for example, the requirements under Code sections 404, 412(c)(7), 412(l), 412(m), 415 and 416 are not considered.

There are a variety of cost methods. Among the wide variety of cost methods, eight of them that we will discuss in this chapter are commonly used by actuaries and approved by the Internal Revenue Service. For each of them, we will first introduce its principle and definition, then illustrate it by an example. For comparison purpose, we will use the same example for all the cost methods and evaluate over a 40-year period. The calculations for the first few years will be shown in the paper and other results will be given in a table. We assume that participant j, hired at age w_j, is to retire at age y_j with an annual pension $B(y_j)$ payable monthly until j dies. Then any proper funding method should have accumulated an amount of $B(y_j)\ddot{a}_{y_j}^{(12)}$ for the participant j when j reaches age

y_j. For any age x_j, $x_j < y_j$, j's accrued benefit is denoted by $B(x_j)$ and its present value is then equal to $B(x_j)\ddot{a}^{(12)}_{y_j}\frac{D_{y_j}}{D_{x_j}}$. Clearly, $B(w_j)$, accrued benefit at age w_j, is zero.

As mentioned at the beginning of Chapter 2, we only consider the retirement benefit. We assume there is no plan change unless specified and the valuation is at the beginning of the year. The closed-group technique is used and hence potential entrants after the date of valuation are ignored [3]. Normally the term "closed-group" refers to persons currently affiliated with the plan as an active participant, a terminated vested participant, a retired participant or a beneficiary. In our discussion, we assume there are no retirees, beneficiaries, and terminated vested participants.

Section 3.1: Unit Credit (UC)

Unit Credit is also called the Accrued Benefit Cost Method. It is the only benefit-based cost method we will introduce. Normal costs are calculated individually, and gains are recognized immediately. Like all proper funding methods, the UC cost method is built on the premise that it guarantees a sufficient amount will be there when an employee retires. In addition, it also requires the fund balance or asset on hand at any given time t will always be equal to the present value of accrued benefit if there are no past service credits. That is,

$$F_t = \sum_{A_t} B(x_j)\ddot{a}^{(12)}_{y_j}\frac{D_{y_j}}{D_{x_j}}$$

where A_t denotes the set of active participants at time t. Notice that at time t, participant j is at age x_j.

It is the second premise that makes the UC cost method a benefit-based cost method and distinguishes it from other cost methods. In order to accomplish this, a normal cost calculated at time t is defined to be the present value of the benefit accruing during the year, that is, the present value of the increase in accrued benefit between time t and t+1. Let $PVAB_t$ denote the present value of accrued benefit at time t and $PVAB_{t+1|t}$ denote the present value of accrued benefit at time t+1 but valued at time t. Also let ΔB^j be the increase in j's accrued benefit during the year. So at time t, the normal cost for the plan is

$$\begin{aligned} NC_t &= PVAB_{t+1|t} - PVAB_t \\ &= \sum_{A_t} \left(B(x_j + 1) - B(x_j) \right) \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \\ &= \sum_{A_t} \Delta B^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} . \end{aligned}$$

The AL is defined retrospectively as the present value of all benefits accrued to date, that is, $AL_t = PVAB_t$. For each participant j at age x_j, the accrued liability is $B(x_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}}$ and the accrued liability for the whole plan is the sum of all individual accrued liabilities: $AL_t = PVAB_t = \sum_{A_t} B(x_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}}$.

The unfunded accrued liability (UAL) measures the deviation of the actual fund balance (F) from its ideal value, the AL. The UAL_t at time t is then derived by subtracting the asset or fund balance F_t at time t from AL_t: $UAL_t = AL_t - F_t$. The fund balance or asset on hand is increased each year by the plan contribution and investment return, $i_f F_t$, where i_f is the actual rate of return on assets. Algebraically, $F_{t+1} = (1+i_f)F_t + (1+i_c)C_t$. The term $i_c C_t$ represents interest on the actual contributions at the actual rate i_c from the date they are actually made to year-end. The initial accrued liability is normally funded separately and amortized over a period of years, say n years. For minimum funding purpose, the law requires that n be 30. The subsequent increase in the accrued liability will be funded via the normal cost. After the initial accrued liability has been fully amortized, and if all actuarial assumptions have been exactly realized, there will be no UAL. When an amendment of the plan grants a benefit increase affecting benefits already earned, an additional accrued liability (SL) is then generated. The difference in the associated UALs, ΔUAL, will be amortized.

The excess of the actual unfunded accrued liability over the expected unfunded accrued liability is called the actuarial gain. The expected unfunded accrued liability is what the unfunded accrued liability would have been had all the actuarial assumptions been exactly realized. The expected UAL at time t is equal to the UAL at time t-1, reduced by any contribution in excess of the normal cost, and increased by the assumed interest rate. In short, the expected UAL_t is

$$\text{expected } UAL_t = (NC_{t-1} + UAL_{t-1})(1+i) - C_{t-1}(1+i)$$

where i is the assumed funding interest rate and C_{t-1} is assumed to be paid at the beginning of the year. Thus at time t, the actuarial gain is equal to the previous year's unfunded accrued liability plus the normal cost with interest for a year, minus contributions with interest, minus the actual unfunded accrued liability; or equivalently,

$$\begin{aligned} Gain_t &= \text{expected } UAL_t - \text{actual } UAL_t \\ &= [(NC_{t-1} + UAL_{t-1})(1+i) - C_{t-1}(1+i)] - UAL_t\,. \end{aligned} \tag{3.1.1}$$

Each year's actuarial gain is amortized over a certain period of years, say m years. For minimum funding purpose, the law requires that m be 5. Note that a negative $Gain_t$ is equivalent to an actuarial loss.

Except in the ideal situation where the fund balance is exactly equal to the accrued liability and all the assumptions are exactly realized, the normal cost will not be the whole cost of the plan. In general, the normal cost needs to be adjusted by some components, such as the amortization of the supplemental unfunded accrued liability if past service benefit is granted or the benefit is increased, or the amortization of the gain or loss if the plan has experienced good or bad fortune during the past years. Therefore the pension cost or contribution at time t is equal to the normal cost, plus amortization of the initial unfunded accrued liability and bases for subsequent plan changes, minus amortization of the gains:

$$C_t = NC_t + UAL_1/\ddot{a}_{\overline{n}|} + \Delta UAL_2/\ddot{a}_{\overline{n}|} + \ldots + \Delta UAL_t/\ddot{a}_{\overline{n}|}$$

$$-\text{Gain}_1/\ddot{a}_{\overline{m|}} - \text{Gain}_2/\ddot{a}_{\overline{m|}} - \ldots - \text{Gain}_t/\ddot{a}_{\overline{m|}} . \qquad (3.1.2)$$

Notice that some of the ΔUAL and Gain terms may equal zero. This would be the case if no amendment of the plan is made or assumptions are exactly realized. Furthermore, an amortization term will disappear once its base has been fully amortized. The immediate adjustment technique on the contribution for each year's gain makes the method an immediate gain method.

If the plan cost has an amortization base, the employer has a contribution range based on the longest and shortest amortization periods permitted under the law. This allows some flexibility in funding for the employer. The UC cost method is not suitable for small participant groups because the costs tend to rise more rapidly than covered payroll. We will demonstrate this by the following example.

To make the illustration simple, we consider a new plan with two participants aged 25 and 22 with zero and two years of past service respectively. The plan provides a flat monthly benefit of $100 for each year of service commencing at a participant's retirement age assumed to be 65. The post-retirement mortality assumption is based on the 1984 UP Mortality Table and no pre-retirement decrement is assumed. The funding interest rate is $i = 7\%$ and actual returns on assets and contributions are also 7%, that is $i_f = 7\%$ and $i_c = 7\%$, unless otherwise specified. The dollar amounts involved in all the examples are rounded to the nearest dollar. For all the examples in this chapter, we will assume a benefit increase at t =3 of $25 per month for each prospective year of service, and a decrease in both i_f and i_c to 5% for the period between t = 6 and t = 7.

Since the UC cost method is an individual cost method, the normal cost is the sum of individually calculated normal costs, NC_1^1 and NC_1^2. For the first year, we have

$$NC_1 = NC_1^1 + NC_1^2$$

$$= [B(26) - B(25)]\ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{25}} + [B(23) - B(22)]\ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{22}}$$

$$= 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{40} + 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{43} = \$1{,}272 .$$

The value for $\ddot{a}_{65}^{(12)}$ is approximately equal to 8.7358. The value of D_h / D_k in the example is equal to v^{h-k} because we have assumed no pre-retirement decrement. Since there is no asset on hand at the beginning of the plan, the initial unfunded accrued liability is equal to the accrued liability. Equivalently,

$$UAL_1 = AL_1 = PVAB_1$$

$$= B(25)\ \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{25}} + B(22)\ \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{22}}$$

$$= 0 + 2 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{43} = \$1{,}143.$$

With the UAL_1 amortized over 30 years, the contribution for the first year is

$$C_1 = NC_1 + UAL_1 \div \ddot{a}_{\overline{30|}} = 1272 + 86 = \$1{,}358 .$$

We assume this amount to be contributed on the valuation date.

The second year valuation is similar to the first except the fund balance is no longer zero; it is equal to

$F_2 = F_1(1.07) + C_1(1.07) = 0 + 1358 \cdot (1.07) = \$1,453$.

Since there is no benefit change in the plan, the normal cost for the second year is simply increased by one year of interest:

$NC_2 = NC_1(1.07) = \$1,361$.

Thus, the actual unfunded accrued liability is

$$UAL_2 = AL_2 - F_2$$

$$= 1200 \cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{26}} + 3 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{23}} - 1453$$

$$= 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{39} + 3600 \cdot \ddot{a}_{65}^{(12)} \cdot v^{42} - 1453$$

$$= 2584 - 1453 = \$1,131 .$$

Since i, i_f and i_c are the same here, the expected UAL_2 will coincide with the actual UAL_2 and there will be no actuarial gain. To verify this, we can calculate the expected UAL_2 and find that

$$\text{expected } UAL_2 = (NC_1 + UAL_1 - C_1)(1.07)$$

$$= (1272 + 1143 - 1358)(1.07) = \$1,131$$

which is the same as the actual UAL_2 . The second year's contribution is

$$C_2 = NC_2 + UAL_1 \div \ddot{a}_{\overline{30|}} = 1361 + 86 = \$1,447 .$$

The plan is amended to increase the monthly benefit from \$100 to \$125 for each prospective year of service beginning in the third year. Since the new benefit formula only applies prospectively, there is no additional amortization base due to the benefit change in the third year, that is the ΔUAL_3 = new UAL_3 - old UAL_3 = 0. With the benefit increase, the normal cost becomes

$$NC_3 = NC_3^1 + NC_3^2 = 12 \cdot 125 \cdot \ddot{a}_{65}^{(12)} \cdot v^{38} + 12 \cdot 125 \cdot \ddot{a}_{65}^{(12)} \cdot v^{41} = \$1,820 .$$

The contribution for the third year is

$$C_3 = NC_3 + UAL_1 \div \ddot{a}_{\overline{30|}} = 1820 + 86 = \$1,906 .$$

In year seven (t = 7), an actuarial loss is generated since the actual earning on asset during year 6 decreases from 7% to 5%. To determine the actuarial loss, we need to compare the expected and actual UAL_7 . The NC_t, C_t, F_t, and the UAL_t for $4 \leq t \leq 6$ are given in Table 3.1.1.

TABLE 3.1.1

Results for Selected Years under UC

Time (t)	NC_t	UAL_t	C_t	F_t
4	1,947	1,104	2,033	5,358
5	2,083	1,089	2,169	7,909
6	2,229	1,073	2,315	10,784

Then the expected UAL_7 and actual UAL_7 are equal to

$$\text{expected } UAL_7 = (NC_6 + UAL_6 - C_6) \cdot 1.07 = (2229 + 1073 - 2315) \cdot 1.07 = \$1,056$$

and

$$\text{actual } UAL_7 = PVAB_7 - F_7$$

$$= (2 \cdot 1200 + 4 \cdot 1500) \cdot \ddot{a}_{65}^{(12)} \cdot v^{34} + (4 \cdot 1200 + 4 \cdot 1500) \cdot \ddot{a}_{65}^{(12)} \cdot v^{37}$$

$$- (F_6 + C_6) \cdot 1.05 = \$1{,}318$$

respectively. Therefore, the $Gain_7$ can be obtained as \$1,056 - \$1,318 = \$ -262 . The amortization amount for this loss is equal to \$60 per year. Thus, the contribution for $t = 7$ is adjusted by one extra term:

$$C_7 = NC_7 + UAL_1 \div \ddot{a}_{\overline{30|}} - Gain_7 \div \ddot{a}_{\overline{5|}}$$

$$= 2385 + 86 + 60 = \$2{,}531$$

with $NC_7 = NC_6 \cdot 1.07 = 2229 \cdot 1.07 = \$2{,}385$. Table 3.1.2 shows the normal costs and the contributions as well as the net amortization charge for the entire valuation period. The contributions are show graphically in Figure 3.1.1.

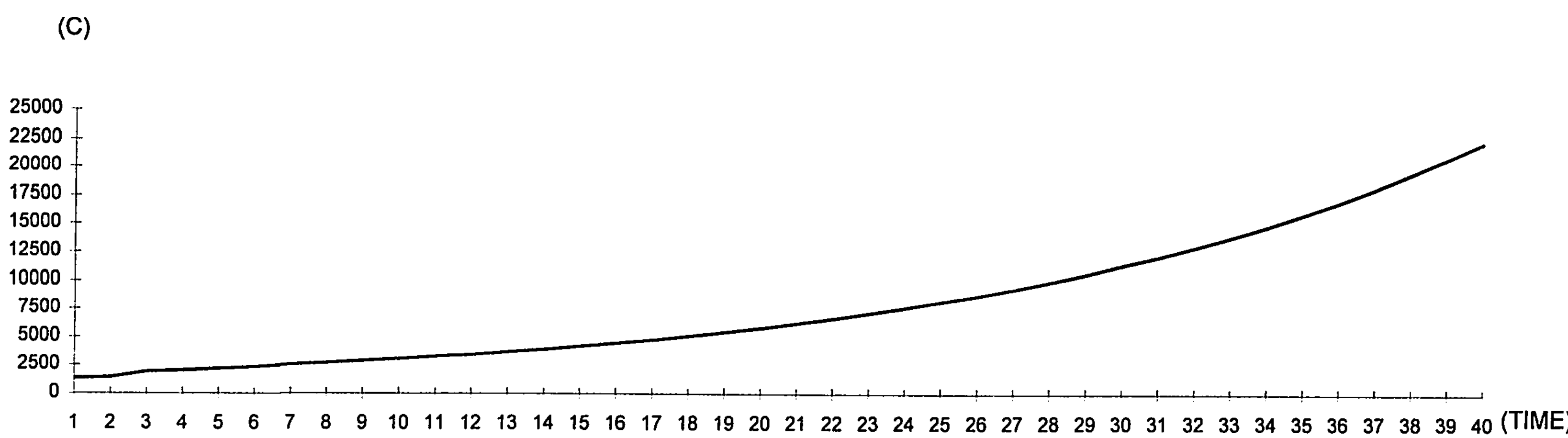
TABLE 3.1.2
NC and C under UC

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	1272	1361	1820	1947	2083	2229	2385	2552	2731	2922	3127	3345	3580	3830	4098	4385	4692	5021	5372	5748
NAC*	86	86	86	86	86	86	146	146	146	146	146	86	86	86	86	86	86	86	86	86
C	1358	1447	1906	2033	2169	2315	2531	2698	2877	3068	3273	3431	3666	3916	4184	4471	4778	5107	5458	5834

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	6150	6581	7042	7535	8062	8626	9230	9876	10568	11307	12099	12948	13852	14822	15859	16969	18157	19428	20788	22243
NAC*	86	86	86	86	86	86	86	86	86	86	0	0	0	0	0	0	0	0	0	0
C	6236	6667	7128	7621	8148	8712	9316	9962	10654	11393	12099	12948	13852	14822	15859	16969	18157	19428	20788	22243

*NAC -- Net Amortization Charge

FIGURE 3.1.1
Contributions under UC



Section 3.2: Individual Entry Age Normal (Individual EAN)

The second family of actuarial cost methods, Entry Age Normal, can be either individual or aggregate. They both are cost-based methods with immediate-gain recognition. Here we will introduce the Individual EAN cost method first, and the Aggregate EAN cost method will be introduced in a later section. To overcome the disadvantage of the UC cost method — that the normal cost tends to increase more rapidly than pay — this method defines the normal cost directly as a level dollar amount or a level percentage of pay from entry age w to assumed retirement age y which is sufficient to fund the projected benefit at retirement. The normal cost is defined in such a way that the present value of all future normal costs at age w is exactly equal to the present value of the future benefits at w. So for the level dollar approach, we have

$$B(y_j)\ddot{a}_{y_j}^{(12)}\frac{D_{y_j}}{D_{w_j}} = NC_t^j\frac{D_{w_j}}{D_{w_j}} + NC_t^j\frac{D_{w_j+1}}{D_{w_j}} + \cdots + NC_t^j\frac{D_{y_j-1}}{D_{w_j}}$$

$$= NC_t^j\frac{\sum_{k=w_j}^{y_j-1} D_k}{D_{w_j}} = NC_t^j\frac{N_{w_j} - N_{y_j}}{D_{w_j}} \qquad (3.2.1)$$

where NC_t^j is participant j's normal cost at time t. Solving (3.2.1) for NC_t^j gives

$$NC_t^j = B(y_j)\ddot{a}_{y_j}^{(12)}\frac{D_{y_j}}{N_{w_j} - N_{y_j}}, \qquad (3.2.2)$$

and the plan normal cost is simply the sum of all individually calculated normal costs,

$$NC_t = \sum_{A_t} NC_t^j \ .$$

Since the D_{w_j} terms in (3.2.2) has been canceled, it is not necessary to take the present values as of the participants' entry ages. Any other age can be used and it will not affect the value of NC_t^j. Note that, under the individual level dollar EAN cost method, the normal cost remains constant, $NC_{t+1} = NC_t$, for each participant during his entire career as long as the projected benefit $B(y_j)$ does not change.

The accrued liability under the individual level dollar EAN cost method can be defined retrospectively as the present value of prior normal costs, or prospectively as the present value of future benefits minus the present value of future normal costs. The accrued liability defined by the two approaches are identical. For the prospective definition,

$$
\begin{aligned}
AL_t &= PVFB_t - PVFNC_t \\
&= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - \sum_{A_t} NC_t{}^j \frac{N_{x_j} - N_{y_j}}{D_{x_j}} \\
&= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{w_j} - N_{y_j}} \cdot \frac{N_{x_j} - N_{y_j}}{D_{x_j}} \\
&= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \left(1 - \frac{N_{x_j} - N_{y_j}}{N_{w_j} - N_{y_j}} \right) \\
&= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \cdot \frac{N_{w_j} - N_{x_j}}{N_{w_j} - N_{y_j}} \\
&= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{w_j} - N_{y_j}} \cdot \frac{N_{w_j} - N_{x_j}}{D_{x_j}}
\end{aligned}
$$

$$=\sum_{A_t} NC_t^{\,j} \frac{N_{w_j} - N_{x_j}}{D_{x_j}}$$

which coincides with the retrospective definition.

As in the UC cost method, the unfunded accrued liability is also defined as $UAL_t = AL_t - F_t$. The immediate adjustment technique on contribution for the gain can be employed exactly as under the UC cost method. Namely,

$$\begin{aligned} Gain_t &= \text{expected } UAL_t - \text{actual } UAL_t \\ &= [(NC_{t-1} + UAL_{t-1})(1+i) - C_{t-1}(1+i)] - UAL_t\,. \end{aligned}$$

Each year's contribution is the normal cost plus amortization of the initial unfunded accrued liability and bases for subsequent plan changes, minus amortization of the gains.

With the same example as the UC cost method, the valuations for the first few years are as follows. The normal cost for the first year is the sum of each participant's individual normal cost:

$$\begin{aligned} NC_1 &= NC_1^1 + NC_1^2 \\ &= 40\cdot 1200\cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{N_{25} - N_{65}} + 45\cdot 1200\cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{N_{20} - N_{65}} \\ &= 40\cdot 1200\cdot \ddot{a}_{65}^{(12)}\cdot \frac{v^{40}}{\ddot{a}_{\overline{40|}}} + 45\cdot 1200\cdot \ddot{a}_{65}^{(12)}\cdot \frac{v^{45}}{\ddot{a}_{\overline{45|}}} \\ &= 1963 + 1543 = \$3{,}506\,. \end{aligned}$$

After the normal cost has been determined, we use the retrospective approach to calculate the accrued liability,

$$AL_1 = NC_1^1 \cdot \frac{N_{25} - N_{25}}{D_{25}} + NC_1^2 \cdot \frac{N_{20} - N_{22}}{D_{22}}$$

$$= 1963 \cdot 0 + 1543 \cdot \frac{\ddot{a}_{\overline{2|}}}{v^2} = \$3,417$$

which is also equal to UAL_1 since $F_1 = 0$. The first year's contribution is the sum of NC_1 and the amortization of the initial UAL:

$$C_1 = NC_1 + UAL_1 \div \ddot{a}_{\overline{30|}} = 3506 + 257 = \$3,763 .$$

Since there is no change in the plan and $i_f = i_c = i$ from t = 1 to t = 2, the normal cost NC_2 and the contribution C_2 are the same as NC_1 and C_1 respectively.

In the third year, a new normal cost for each individual participant must be determined since the benefit formula has been changed. For the new normal cost, we have

$$NC_3 = NC_3^1 + NC_3^2$$

$$= (40 \cdot 1200 + 38 \cdot 12 \cdot 25) \cdot \ddot{a}_{65}^{(12)} \cdot \frac{v^{40}}{\ddot{a}_{\overline{40|}}} + (45 \cdot 1200 + 41 \cdot 12 \cdot 25) \cdot \ddot{a}_{65}^{(12)} \cdot \frac{v^{45}}{\ddot{a}_{\overline{45|}}}$$

$$= 2430 + 1894 = \$4,324 .$$

This normal cost remains unchanged for the rest of valuation period since we assume no more plan amendment from now this point on. A new amortization base, ΔUAL_3, is generated to fund the benefit increase resulting from the new benefit formula. The ΔUAL_3 which is the difference in the new and old unfunded accrued liabilities is equal to

$$\Delta UAL_3 = \text{new } UAL_3 - \text{old } UAL_3 = \text{new } AL_3 - \text{old } AL_3$$

$$= (NC_3^1 - NC_2^1) \cdot \frac{N_{25} - N_{27}}{D_{27}} + (NC_3^2 - NC_2^2) \cdot \frac{N_{20} - N_{24}}{D_{24}}$$

$$= \$2,702 .$$

The i_f for t= 2 to t = 3 still remains at 7% which is equal to the assumed i. Therefore, there is no actuarial gain at t= 3. Thus, this year's contribution consists of three terms:

$$C_3 = NC_3 + UAL_1 \div \ddot{a}_{\overline{30|}} + \Delta UAL_3 \div \ddot{a}_{\overline{30|}} = 4324 + 257 + 203 = \$4,784 .$$

The contributions for the following years remain the same as C_3 until i_f decreases to 5% for the period between t = 6 and t = 7. In order to determine $Gain_7$, we need to have the values of the NC_t, UAL_t, C_t, and F_t for $3 \leq t \leq 6$. These values are given in Table 3.2.1.

TABLE 3.2.1

Results for Selected Years under Individual EAN

Time (t)	NC_t	UAL_t	C_t	F_t
3	4,324	6,044	4,784	8335
4	4,324	5,975	4,784	14,037
5	4,324	5,901	4,784	20,138
6	4,324	5,822	4,784	26,667

$Gain_7$ is generated from the difference between the expected and actual UAL_7 is

$$Gain_7 = (UAL_6 + NC_6 - C_6) \cdot 1.07 - (2430 \cdot \frac{\ddot{a}_{\overline{6|}}}{v^6} + 1894 \cdot \frac{\ddot{a}_{\overline{8|}}}{v^8}) + F_7 = \$ -629 .$$

Thus, the contribution for t = 7 consists of one extra term and is equal to

$$C_7 = NC_7 + UAL_1 \div \ddot{a}_{\overline{30}|} + \Delta UAL_3 \div \ddot{a}_{\overline{30}|} - Gain_7 \div \ddot{a}_{\overline{5}|}$$

$$= 4324 + 257 + 203 + 143 = \$4{,}927 .$$

The results for this illustrative plan simulated over 40 years are given in Table 3.2.2 and the graph of contributions is displayed in Figure 3.2.1. From Table 3.2.2, we can see that the contributions for t = 8, 9, 10, and 11 are the same as C_7, but C_{12} decreases back to \$4,784 after $Gain_7$ has been fully amortized. After 30 years, the initial UAL_1 has been fully amortized, C_{31} has only one amortization base, ΔUAL_3, and becomes $C_{31} = NC_{31} + \Delta UAL_3 \div \ddot{a}_{\overline{30}|} = 4324 + 203 = \$4{,}527$. ΔUAL3, which was created at t =3 is fully amortized 30 years later when t = 33, making $C_t = NC_t = \$4{,}324$ for $33 \leq t \leq 40$.

In the above discussion, the normal costs are in level dollar amounts. When the pension benefit is based on salary, the cost method is usually used with a salary-increase assumption. It may then be more appropriate to express the normal cost as a level percentage of pay for the projected benefit cost methods. As mentioned in Chapter 2, a set of salary indices, called the salary scale, gives the rate of salary increase based on a participant's age and length of employment. For simplicity, only the age factor is taken into account here. We then use s_{x_j} to denote the salary index for a participant aged x_j. Let us assume that at time t participant j, aged x_j, has a salary of S_t^j; then g years later, j's estimated salary would be $S_t^j \frac{s_{x_j+g}}{s_{x_j}}$. Before calculating the normal cost with a salary scale assumption, it is convenient to define two new commutation functions employing the salary indices: $\quad {}^{s}D_x = s_x D_x$ and ${}^{s}N_x = \sum_{z=x}^{\infty} {}^{s}D_z$.

Since we want the normal cost to be a level percentage of salary each year, so our goal is to find a constant fraction U_t^j for each participant such that the individual normal cost is this constant fraction U_t^j of the participant's salary each year, that is, $NC_t^j = U_t^j S_t^j$. If S_t^j is the salary at age x_j (time t) then $S_t^j \cdot \frac{s_{w_j}}{s_{x_j}}$ is the assumed salary at age of hire w_j. As with the level dollar derivation, we set the present value of future benefits at age w_j equal to the present value of future normal costs at age w_j. Note that the present value of future benefits $PVFB_{w_j} = B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}}$ remains unchanged; but now with the salary-increase assumption, the present value of future normal costs at age w_j is

$$PVFNC_{w_j} = U_t^j S_t^j \frac{s_{w_j}}{s_{x_j}} + U_t^j S_t^j \frac{s_{w_j+1}}{s_{x_j}} \cdot \frac{D_{w_j+1}}{D_{w_j}} + \cdots + U_t^j S_t^j \frac{s_{y_j-1}}{s_{x_j}} \cdot \frac{D_{y_j-1}}{D_{w_j}}$$

$$= U_t^j S_t^j \frac{s_{w_j}}{s_{x_j}} \sum_{z=w_j}^{y_j-1} \frac{s_z}{s_{w_j}} \frac{D_z}{D_{w_j}} \quad . \tag{3.2.3}$$

Using the new commutation functions, we can rewrite (3.2.3) as

$$U_t^j S_t^j \frac{s_{w_j}}{s_{x_j}} \sum_{z=w_j}^{y_j-1} \frac{s_z}{s_{w_j}} \frac{D_z}{D_{w_j}} = U_t^j S_t^j \frac{s_{w_j}}{s_{x_j}} \frac{\sum_{z=w_j}^{\infty} {}^{s}D_z - \sum_{z=y_j}^{\infty} {}^{s}D_z}{s_{w_j} D_{w_j}}$$

$$= U_t^j S_t^j \frac{s_{w_j}}{s_{x_j}} \frac{{}^sN_{w_j} - {}^sN_{y_j}}{{}^sD_{w_j}} = NC_t^j \frac{s_{w_j}}{s_{x_j}} \frac{{}^sN_{w_j} - {}^sN_{y_j}}{{}^sD_{w_j}},$$

and from which it follows that the individual normal cost at age x_j is

$$NC_t^j = B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \cdot \frac{{}^sD_{w_j}}{{}^sN_{w_j} - {}^sN_{y_j}} \cdot \frac{s_{x_j}}{s_{w_j}} . \tag{3.2.4}$$

Now equating $NC_t^j = U_t^j S_t^j$ and (3.2.4), we get

$$U_t^j = B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \cdot \frac{{}^sD_{w_j}}{{}^sN_{w_j} - {}^sN_{y_j}} \cdot \frac{s_{x_j}}{s_{w_j}} \cdot \frac{1}{S_t^j}$$

$$= B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \div S_t^j \frac{s_{w_j}}{s_{x_j}} \frac{{}^sN_{w_j} - {}^sN_{y_j}}{{}^sD_{w_j}}$$

$$= PVFB_{w_j} \div PVFS_{w_j} \tag{3.2.5}$$

where the $PVFS_{w_j}$ is the present value of future salary at age w_j; that is

$$PVFS_{w_j} = S_t^j \frac{s_{w_j}}{s_{x_j}} + S_t^j \frac{s_{w_j+1}}{s_{x_j}} \cdot \frac{D_{w_j+1}}{D_{w_j}} + \cdots + S_t^j \frac{s_{y_j-1}}{s_{x_j}} \cdot \frac{D_{y_j-1}}{D_{w_j}}$$

$$= S_t^j \frac{s_{w_j}}{s_{x_j}} \sum_{z=w_j}^{y_j-1} \frac{s_z}{s_{w_j}} \frac{D_z}{D_{w_j}} = S_t^j \frac{s_{w_j}}{s_{x_j}} \frac{{}^sN_{w_j} - {}^sN_{y_j}}{{}^sD_{w_j}}.$$

Therefore, each participant's U_t^j is determined by (3.2.5), $U_t^j = PVFB_{w_j} \div PVFS_{w_j}$, and this fraction should remain constant if there is no change in the plan and the salary-change assumption is exactly realized. Then the normal cost for the plan at time t is the sum of all individual normal costs, $NC_t = \sum_{A_t} NC_t^j = \sum_{A_t} U_t^j S_t^j$, and the present value of future normal cost is the sum of the present value of each participant's future salary ($PVFS_t^j$) multiplied by his constant fraction U_t^j.

$$PVFNC_t = \sum_{A_t} PVFS_t^j \cdot U_t^j = \sum_{A_t} S_t^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}} \cdot U_t^j$$

$$= \sum_{A_t} NC_t^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}} \; .$$

As mentioned previously, U_t^j should remain constant, i.e., $U_{t+1}^j = U_t^j$, if there is no change in the plan and actual salary changes coincide with the assumption; otherwise, a new normal cost fraction is computed. We can obtain the accrued liability by applying either the prospective or the retrospective definition. For the prospective definition, we have AL_t = $PVFB_t$ - $PVFNC_t$. The unfunded accrued liability and actuarial gain are calculated in a similar fashion; namely, set $UAL_t = AL_t - F_t$ and $Gain_t$ = expected UAL_t - actual $UAL_t = (NC_{t-1} + UAL_{t-1})(1+i) - C_{t-1}(1+i) - UAL_t$.

For both the level dollar and the level percentage of pay approaches, the Individual EAN cost method allows the employer some flexibility in funding due to the range of permissible amortization amounts allowed by law. It should be noted that federal regulations do not allow the level percentage of pay approach if the benefit is not related to salary.

TABLE 3.2.2
NC and C under Level Dollar Individual EAN

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3506	3506	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324
NAC*	257	257	460	460	460	460	603	603	603	603	603	460	460	460	460	460	460	460	460	460
C	3763	3763	4784	4784	4784	4784	4927	4927	4927	4927	4927	4784	4784	4784	4784	4784	4784	4784	4784	4784

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324	4324
NAC*	460	460	460	460	460	460	460	460	460	460	203	203	0	0	0	0	0	0	0	0
C	4784	4784	4784	4784	4784	4784	4784	4784	4784	4784	4527	4527	4324	4324	4324	4324	4324	4324	4324	4324

*NAC – Net Amortization Charge

FIGURE 3.2.1
Contributions under Individual EAN

Section 3.3: Aggregate Entry Age Normal (Aggregate EAN)

The aggregate version of the EAN cost method is very closely related to the Individual EAN cost method except that the normal costs are not individually calculated. Under the Aggregate EAN cost method, only a single normal cost rate is derived and applied to all the participants. This unit normal cost is redetermined at each valuation. As with the individual EAN cost method, both the level dollar and the level percentage of pay approaches can be applied to the Aggregate EAN cost method. Let us first focus on the level dollar approach. Instead of determining an individual normal cost for each participant, a unit normal cost is computed by equating the sum of the participants' present value of projected benefits with the sum of their present value of future normal costs. Traditionally, such present values are determined as of the entry ages of the participants. Algebraically, it is

$$\sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} = UNC_t \cdot \sum_{A_t} \frac{N_{w_j} - N_{y_j}}{D_{w_j}} \ . \qquad (3.3.1)$$

Solving equation (3.3.1) for UNC_t gives

$$UNC_t = \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \div \sum_{A_t} \frac{N_{w_j} - N_{y_j}}{D_{w_j}}$$

$$= \frac{\sum_{A_t} PVFB_{w_j}}{\sum_{A_t} PVFY_{w_j}} \qquad (3.3.2)$$

where $PVFY_{w_j} = \dfrac{N_{w_j} - N_{y_j}}{D_{w_j}}$ is the present value of participant j's future working years at age w_j. Then the plan normal cost is the unit normal cost UNC_t multiplied by the total number of participants at time t. Again, this UNC_t should remain constant if there is no change in the benefit and no change in the participant population; otherwise, a new UNC_t needs to be determined. It is impossible to simplify (3.3.2) any further because different participants usually have different entry ages and the D_{w_j} in both the denominator and the numerator cannot be canceled . Since the D_{w_j} terms cannot be canceled, the age at which present values are taken will have an effect on the unit normal cost. Examining our example, we first use (3.3.2) to determine the unit normal cost and have

$$UNC_{1(W)} = \sum_{A_1} PVFB_{w_j} \div \sum_{A_1} PVFY_{w_j}$$

$$= (40 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{40} + 45 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \cdot v^{45}) \div (\ddot{a}_{\overline{40}|} + \ddot{a}_{\overline{45}|}) = \$1{,}751 \ .$$

The prospective accrued liability

$$AL_1 = UAL_1 = PVFB_1 - PVFNC_1$$

$$= (40 \cdot 1200\ \ddot{a}_{65}^{(12)} \cdot v^{40} + 45 \cdot 1200\ \ddot{a}_{65}^{(12)} \cdot v^{43}) - UNC_{1(W)} \cdot (\ddot{a}_{\overline{40}|} + \ddot{a}_{\overline{43}|})$$

$$= 53717 - 1751 \cdot (14.26 + 14.45) = \$3{,}446$$

generates an amortization base with an amortization amount of \$259 over 30 years. Therefore, the contribution for the plan is $C_1 = NC_1 + UAL_1 / \ddot{a}_{\overline{30}|} = 2 \cdot 1751 + 259 = \$ 3{,}761$.

UNC_2 equals $UNC_{1(W)}$ since there is no benefit change at t = 2. UAL_2 is equal to

$$UAL_2 = AL_2 - F_2 = PVFB_2 - PVFNC_2 - F_2$$

$$= 57477 - 50043 - 4024 = \$3,410$$

where $PVFB_2 = PVFB_1 \cdot 1.07 = \$57,477$ and $PVFNC_2 = UNC_2 \cdot (\ddot{a}_{\overline{39|}} + \ddot{a}_{\overline{42|}}) = 1751 \cdot (14.19 + 14.39) = \$50,043$ and $F_2 = (F_1 + C_1) \cdot 1.07 = \$4,024$. The actual UAL_2 coincides with the expected UAL_2 which is $(UAL_1 + NC_1 - C_1) \cdot 1.07 = \$3,410$. Thus $Gain_2 = 0$.

In the third year, we need to determine a new UNC_3 as the monthly benefit is increased by $ 25,

$$UNC_3 = \sum_{A_3} PVFB_{w_j} \div \sum_{A_3} PVFY_{w_j}$$

$$= [(40 \cdot 1200 + 38 \cdot 12 \cdot 25)\, \ddot{a}_{65}^{(12)} \cdot v^{40} + (45 \cdot 1200 + 41 \cdot 12 \cdot 25)\, \ddot{a}_{65}^{(12)} \cdot v^{45}]$$

$$\div (\ddot{a}_{\overline{40|}} + \ddot{a}_{\overline{45|}})$$

$$= 62230 \div 28.82 = \$2,159 .$$

Note that this unit normal cost remains the same for the rest of the valuation period since there is no more plan amendment, that is $UNC_t = \$2,159$ for $3 \le t \le 40$. Next we update the old AL_3 and calculate the new AL_3, we get

$$\text{old } AL_3 = \text{old } PVFB_3 - \text{old } PVFNC_3 = PVFB_2 \cdot 1.07 - UNC_2 \cdot (\ddot{a}_{\overline{38|}} + \ddot{a}_{\overline{41|}})$$

$$= 57477 \cdot 1.07 - 1751 \cdot (14.12 + 14.33) = \$11,684,$$

and

$$\text{new } AL_3 = \text{new } PVFB_3 - \text{new } PVFNC_3$$

$$= [(40 \cdot 1200 + 38 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{38} + (45 \cdot 1200 + 41 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{41}] -$$

$$UNC_3 \cdot (\ddot{a}_{\overline{38|}} + \ddot{a}_{\overline{41|}})$$

$$= 75822 - 2159(14.12 + 14.33) = \$\ 14{,}398.$$

Then ΔUAL_3 can be obtained be taking the difference between the two ALs,

$$\Delta UAL_3 = \text{new } AL_3 - \text{old } AL_3 = 14398 - 11684 = \$2{,}714,$$

and the amortization amount for ΔUAL_3 is equal to \$ 204 over 30 years. Thus,

$$C_3 = NC_3 + UAL_1/\ddot{a}_{\overline{30|}} + \Delta UAL_3/\ddot{a}_{\overline{30|}} = 2\cdot 2159 + 259 + 204 = \$4{,}781\ .$$

With UAL_3 = new AL_3 - F_3 = 14398 - (4024 + 3761)1.07 = \$6,068, we calculate the following 3 years' UALs. The results are presented in Table 3.3.1 along with the Fs.

TABLE 3.3.1

Results for Selected Years under Aggregate EAN

Time (t)	**UAL_t**	**F_t**
4	5,997	14,029
5	5,921	20,127
6	5,840	26,652

From the data in the above table , we can compute the expected UAL_7 as

$$\text{expected } UAL_7 = (UAL_6 + NC_6 - C_6)\cdot 1.07 = (5840 + 4318 - 4781)\cdot 1.07 = \$5{,}753\ .$$

NC_6 and C_6 are the same as NC_3 and C_3 respectively. Since i_f decreases to 5% for the period from t = 6 to t =7, F_7 is equal to $(F_6 + C_6)\cdot 1.05$ =\$33,005 . The prospective UAL_7, that is

$$UAL_7 = PVFB_7 - PVFNC_7 - F_7$$

$$= [(40\cdot 1200 + 38\cdot 300)\ddot{a}_{65}^{(12)}\cdot v^{34} + (45\cdot 1200 + 41\cdot 300)\ddot{a}_{65}^{(12)}\cdot v^{37}]$$

$$- 2159\cdot(\ddot{a}_{\overline{34|}} + \ddot{a}_{\overline{37|}}) - F_7$$

$$= 99386 - 2159\cdot(13.75 + 14.04) - 33005 = \$6{,}382 ,$$

is then determined to generate $Gain_7 = 5753 - 6382 = \$ -629$. The contributions for the next 5 years are equal to $C_t = NC_t + UAL_1/\ddot{a}_{\overline{30|}} + \Delta UAL_3/\ddot{a}_{\overline{30|}} - Gain_7/\ddot{a}_{\overline{5|}} = 4318 + 259 + 204 + 143 = \$4{,}924$ for $7 \le t \le 11$. The contribution pattern is the same as under the Individual Aggregate EAN Cost Method and is demonstrated in Table 3.3.2 and Figure 3.3.1.

Recently, two actuaries at the Internal Revenue Service [16] discovered a problem with the Aggregate EAN cost method which has been used for many years. Complications arise from the adding together of present values that are not determined as of the same point in time. Since the entry ages for the participants are usually different, the sum of the present values in (3.3.2) will create an unexpected result which we will discuss in the next paragraph.

By defining the prospective AL as the present value of future benefits minus the present value of future normal costs, we have

$$AL_{t(P)} = PVFB_t - PVFNC_t$$

$$= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)}\frac{D_{y_j}}{D_{x_j}} - UNC_t \cdot \sum_{A_t}\frac{N_{x_j} - N_{y_j}}{D_{x_j}} , \qquad (3.3.3)$$

whereas the retrospective AL is equal to

$$AL_{t(R)} = UNC_t \cdot \sum_{A_t} \frac{N_{w_j} - N_{x_j}}{D_{x_j}} . \tag{3.3.4}$$

Recall under the Individual EAN cost method, both prospective and retrospective accrued liabilities are equal. This is not the case for the Aggregate EAN cost method. In order for the two accrued liabilities to be equal, the excess of the present value of future benefits over the present value of future normal costs must equal the present value of prior normal costs. Algebraically, this is

$$\sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - UNC_t \cdot \sum_{A_t} \frac{N_{x_j} - N_{y_j}}{D_{x_j}} = UNC_t \cdot \sum_{A_t} \frac{N_{w_j} - N_{x_j}}{D_{x_j}} . \tag{3.3.5}$$

Solving (3.3.5) for UNC_t, it gives

$$UNC_t = \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \div \sum_{A_t} \frac{N_{w_j} - N_{y_j}}{D_{x_j}} . \tag{3.3.6}$$

The UNC_t in (3.3.6) is the unit normal cost when present values are taken at the participants' attained ages, and is denoted by $UNC_{t(AA)}$. Therefore, we can conclude that the prospective and retrospective accrued liabilities will be equal only if the unit normal cost is determined as of the participants' attained ages.

For our numerical example, the unit normal cost for the first year determined as of the participants' attained ages is equal to

$$UNC_{1(AA)} = (40 \cdot 1200\ \ddot{a}_{65}^{(12)} \cdot v^{40} + 45 \cdot 1200\ \ddot{a}_{65}^{(12)} \cdot v^{43}) \div (\ddot{a}_{\overline{40|}} + \ddot{a}_{\overline{45|}} / v^2) = \$1,737$$

which is different from $UNC_{1(W)}$. The unit normal cost will still be level if it is determined as of the participants' attained ages. We can verify this by finding $UNC_{2(AA)}$, that is,

$$UNC_{2(AA)} = (40 \cdot 1200\, \ddot{a}_{65}^{(12)} \cdot v^{39} + 45 \cdot 1200\, \ddot{a}_{65}^{(12)} \cdot v^{42}) \div (\ddot{a}_{\overline{40}|}/v + \ddot{a}_{\overline{45}|}/v^3) = \$1,737$$

which is the same as $UNC_{1(AA)}$. To calculate the prospective AL_1, we replace UNC_1 in (3.3.3) by $UNC_{1(AA)} = \$1,737$ and have

$$AL_{1(P)} = PVFB_1 - PVFNC_1$$

$$= (40 \cdot 1200\, \ddot{a}_{65}^{(12)} \cdot v^{40} + 45 \cdot 1200\, \ddot{a}_{65}^{(12)} \cdot v^{43}) - UNC_{1(AA)} \cdot (\ddot{a}_{\overline{40}|} + \ddot{a}_{\overline{43}|})$$

$$= 53717 - 1737 \cdot (14.26 + 14.45) = 53717 - 49870 = \$3,847 .$$

For the retrospective AL_1, we substitute UNC_1 in (3.3.4) by \$1,737 and get

$$AL_{1(R)} = 1737 \cdot (0 + \ddot{a}_{\overline{2}|}/v^2) = \$3,847 .$$

The two accrued liabilities are equal. On the other hand, the two accrued liabilities do not coincide if we replace $UNC_{1(W)} = \$1,751$ in (3.3.3) and (3.3.4). Now the prospective AL_1 has the value of

$$AL_{1(P)} = 53717 - 1751 \cdot (14.26 + 14.45) = \$3,446,$$

and the retrospective AL_1 is

$$AL_{1(R)} = 1751 \cdot (0 + \ddot{a}_{\overline{2}|}/v^2) = \$3,878 .$$

which is larger than the prospective $AL_{1(P)}$. At any point in time, the present value of all normal costs generally will not equal the present value of future benefits. Thus, even if the plan has assets exactly equal to the retrospective accrued liability and also receive payments of the normal costs used in calculating that accrued liability, the plan could not be sure to have adequate assets to fund future benefits. By employing the prospective AL_t in the valuation process, the plan is guaranteed to have precisely the assets to pay future benefits if all assumptions are realized. Since the prospective accrued liability is defined with reference to the present value of benefits, it becomes the balancing item between the present value of future benefits and the present value of future normal costs. The paper [16] demonstrated mathematically the problem with the traditional application of the Aggregate EAN cost method. At this point, it is unclear how this problem will be resolved by the legislative or regulatory authorities.

The Aggregate version of the EAN cost method can also be done based on the level percentage of pay approach. A single constant fraction U_t is determined for the plan instead. Again, the traditional way is to determine all present values as of the entry age:

$$U_t = \frac{PVFB_w}{PVFS_w} = \frac{\sum_{A_t} PVFB_{w_j}}{\sum_{A_t} PVFS_{w_j}} .$$

The NC for the plan is equal to the product of U_t and the total payroll, $NC_t = U_t \cdot \sum_{A_t} S_t^j$.

To calculate the prospective accrued liability, we substitute the $PVFNC_t$ with

$$PVFNC_t = U_t \cdot \sum_{A_t} PVFS_t = U_t \cdot \sum_{A_t} S_t^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}}$$

in (3.3.3). For both approaches, the process of amortizing the UAL and the Gain is similar to that under the Individual EAN cost method.

TABLE 3.3.2
NC and C under Level Dollar Aggregate EAN

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3502	3502	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318
NAC*	259	259	463	463	463	463	606	606	606	606	606	463	463	463	463	463	463	463	463	463
C	3761	3761	4781	4781	4781	4781	4924	4924	4924	4924	4924	4781	4781	4781	4781	4781	4781	4781	4781	4781

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318	4318
NAC*	463	463	463	463	463	463	463	463	463	463	204	204	0	0	0	0	0	0	0	0
C	4781	4781	4781	4781	4781	4781	4781	4781	4781	4781	4522	4522	4318	4318	4318	4318	4318	4318	4318	4318

*NAC -- Net Amortization Charge

FIGURE 3.3.1
Contributions under Aggregate EAN

Section 3.4: Individual Level Premium (ILP)

The two previous cost methods, the UC and the EAN cost methods, are constructed on the premise that the desired amount of assets will be there to fund the participants' pension benefits at retirement. Even if there is an unfunded accrued liability (due to past service benefits), as long as it is amortized over a reasonable period, there will be no solvency problem. But in the short run, it is possible to have a negative fund balance while the present value of future benefits is still equal to the present value of future contributions. The ILP cost method solves this problem by not only accumulating the proper amount at retirement, but also guaranteeing solvency at all time. To accomplish this, it funds each participant's projected benefit with level premiums from his entry into the plan to retirement, and starts with a zero initial unfunded accrued liability.

Under the ILP cost method, upon entry into the plan, each participant's projected benefit is computed assuming benefit will not change in the future. It funds this projected benefit with level premiums, the normal cost NC_t^j, from attained age to retirement age. The NC_t^j is computed in the same way as under the Individual EAN cost method, with the only difference being that if participant j is hired at age w_j before the inception of the plan, we use the attained age x_j ($w_j < x_j$) at plan inception to be the entry age. For those who are hired after the effective date of the plan, we just use the age at hire ($w_j = x_j$).

Algebraically, the individual normal cost at the inception of the plan (t=1) is

$$NC_1^j = B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j} - N_{y_j}}$$

and $NC_1 = \sum_{A_1} NC_1^j$.

These steps are the same as under the EAN cost method. The difference comes in the second year. In order to simplify notation, we now write B_t^j instead of $B(y_j)$ at time t. For the second year (t =2), we re-estimate B_1^j and fund the increase in projected benefit $\Delta B_2^j = B_2^j - B_1^j$ by additional level premiums, called the incremental normal cost ΔNC_2^j, over a funding period which is one year shorter. Note the ΔB could be negative if there is a decrease in projected benefit. The second year individual normal cost is then the sum of the first year normal cost and the incremental normal cost, that is,

$$\begin{aligned} NC_2^j &= NC_1^j + \Delta NC_2^j \\ &= NC_1^j + \Delta B_2^j \cdot \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j+1} - N_{y_j}} . \end{aligned}$$

For each subsequent years, this process is repeated with an increment in normal cost for each participant.

By the way how normal cost is defined, the accrued liability is equal to zero at the effective date of the plan since the present value of future benefits is equal to the present value of future normal costs. Therefore, there is no initial past service liability. To verify this, we calculate the prospective accrued liability for participant j

$$AL_1^j = PVFB_{x_j} - PVFNC_{x_j}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - NC_1^j \frac{N_{x_j} - N_{y_j}}{D_{x_j}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j} - N_{y_j}} \cdot \frac{N_{x_j} - N_{y_j}}{D_{x_j}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} = 0\,,$$

and it follows that

$$AL_1 = \sum_{A_1} AL_1^j = 0\ .$$

Since AL_1 is zero, the unfunded accrued liability which is the difference between AL and fund asset will have to be zero. This is only true for the first year. Any time after that there can be a positive UAL_t, $t > 1$, if actuarial losses have occurred. Such losses will then be amortized as prescribed before. The AL for the second year is still equal to the present value of future benefits minus the present value of future normal costs:

$$AL_2^j = B_2^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - NC_2^j \frac{N_{x_j+1} - N_{y_j}}{D_{x_j+1}}$$

$$= \left(B_1^j + \Delta B_2^j\right) \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - (NC_1^j + \Delta B_2^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j+1} - N_{y_j}}) \cdot \frac{N_{x_j+1} - N_{y_j}}{D_{x_j+1}}$$

$$= \left(B_1^j + B_2^j - B_1^j\right) \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j} - N_{y_j}} \cdot \frac{N_{x_j+1} - N_{y_j}}{D_{x_j+1}}$$

$$- B_2^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} + B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} \left(1 - \frac{N_{x_j+1} - N_{y_j}}{N_{x_j} - N_{y_j}}\right)$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} \cdot \frac{D_{x_j}}{N_{x_j} - N_{y_j}}$$

$$= NC_1^j \frac{D_{x_j}}{D_{x_j+1}} . \tag{3.4.1}$$

For the third year (t =3), another incremental normal cost ΔNC_3^j is added to NC_3^j if there is a change in benefit:

$$NC_3^j = NC_1^j + \Delta NC_2^j + \Delta NC_3^j$$

where $\Delta NC_3^j = \Delta B_3^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{N_{x_j+2} - N_{y_j}}$. Then the accrued liability becomes

$$AL_3^j = \left(B_1^j + \Delta B_2^j + \Delta B_3^j\right) \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+2}} - \left(NC_1^j + \Delta NC_2^j + \Delta NC_3^j\right) \frac{N_{x_j+2} - N_{y_j}}{D_{x_j+2}}$$

$$= \left(AL_2^j + NC_2^j\right) \frac{D_{x_j+1}}{D_{x_j+2}} .$$

By induction, we can conclude that the individual actuarial accrued liability is equal to the sum of the previous year's accrued liability and normal cost accumulated with interest and mortality for one year. At any time $t \geq 1$,

$$AL_{t+1}^j = \left(AL_t^j + NC_t^j\right) \frac{D_{x_j+t-1}}{D_{x_j+t}} \tag{3.4.2}$$

and the accrued liability for the plan is equal to the sum of all individual accrued liabilities,

$$AL_{t+1} = \sum_{A_{t+1}} AL^{j}_{t+1} \ .$$

An important feature of the ILP cost method is that it puts gains and losses due to benefit changes into the normal cost because changes in the projected benefit are funded by the incremental normal cost.

The actuarial gain formula is similar to (3.1.1),

$$Gain_t = [(NC_{t-1} + UAL_{t-1})(1+i) - C_{t-1}(1+i)] - UAL_t \qquad (3.4.3)$$

which makes the ILP cost method an individual immediate gain recognition method. The cost of the plan for any year is equal to the normal cost minus amortization of the previous years' gains.

The ILP cost method can be used with a salary scale assumption similar to the EAN cost method. If we assume that j's salary changes from the year x_j to x_{j+1} by the factor $\frac{s_{x_j+1}}{s_{x_j}}$, then the first year's individual normal cost is computed as

$$NC^j_1 = B^j_1 \ddot{a}^{(12)}_{y_j} \frac{D_{y_j}}{D_{x_j}} \frac{{}^sD_{x_j}}{{}^sN_{x_j} - {}^sN_{y_j}} \qquad (3.4.4)$$

and the incremental normal cost computed in the second year is

$$\Delta NC^j_2 = \Delta B^j_2 \ddot{a}^{(12)}_{y_j} \frac{D_{y_j}}{D_{x_j+1}} \frac{{}^sD_{x_j+1}}{{}^sN_{x_j+1} - {}^sN_{y_j}} \ .$$

The normal cost for the second year is, therefore,

$$NC_2^j = NC_1^j \frac{s_{x_j+1}}{s_{x_j}} + \Delta NC_2^j . \tag{3.4.5}$$

Then the individual accrued liability can be determined as in (3.4.1) with NC_1^j substituted by (3.4.4) and NC_2^j by (3.4.5):

$$AL_2^j = B_2^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - NC_2^j \frac{{}^sN_{x_j+1} - {}^sN_{y_j}}{{}^sD_{x_j+1}}$$

$$= \left(B_1^j + \Delta B_2^j\right) \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - (NC_1^j \cdot \frac{s_{x_j+1}}{s_{x_j}} + \Delta B_2^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} \frac{{}^sD_{x_j+1}}{{}^sN_{x_j+1} - {}^sN_{y_j}})$$

$$\cdot \frac{{}^sN_{x_j+1} - {}^sN_{y_j}}{{}^sD_{x_j+1}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \frac{{}^sD_{x_j}}{{}^sN_{x_j} - {}^sN_{y_j}} \frac{s_{x_j+1}}{s_{x_j}} \cdot \frac{{}^sN_{x_j+1} - {}^sN_{y_j}}{{}^sD_{x_j+1}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} - B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} \frac{{}^sD_{x_j}}{{}^sN_{x_j} - {}^sN_{y_j}} \frac{s_{x_j+1}}{s_{x_j}} \cdot \left(\frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j+1}} - \frac{{}^sD_{x_j}}{{}^sD_{x_j+1}} \right)$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j} \cdot s_{x_j+1}}{{}^sN_{x_j} - {}^sN_{y_j}} \cdot \frac{{}^sD_{x_j}}{s_{x_j+1} D_{x_j+1}}$$

$$= B_1^j \ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j+1}} \cdot \frac{{}^sD_{x_j}}{{}^sN_{x_j} - {}^sN_{y_j}}$$

$$= NC^j \frac{D_{x_j}}{D_{x_j+}}$$

Note that (3.4.6) has the same form as (3.4.1), the one without a salary-increase assumption. By the same argument, the expressions for the accrued liability and the actuarial gain will be the same as (3.4.2) and (3.4.3), respectively.

For our example, the normal cost for the first year is

$$NC_1 = NC_1^1 + NC_1^2$$

$$= 40 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \cdot \frac{D_{65}}{N_{25} - N_{65}} + 45 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \cdot \frac{D_{65}}{N_{22} - N_{65}}$$

$$= 1963 + 1779 = \$3{,}742 .$$

The accrued liability is $AL_1 = 0$, and $UAL_1 = AL_1 - F_1 = 0$. So the first year's contribution $C_1 = NC_1 = \$\ 3{,}742$.

For the second year, we first update the individual accrued liability for each participant:

$$AL_2^1 = (AL_1^1 + NC_1^1)\frac{D_{25}}{D_{26}} = (0 + 1963)\cdot\frac{1}{v} = \$2{,}100$$

and

$$AL_2^2 = (AL_1^2 + NC_1^2)\frac{D_{22}}{D_{23}} = (0 + 1779)\cdot\frac{1}{v} = \$1{,}904 .$$

The accrued liability for the plan is the sum of these two individual accrued liabilities:

$$AL_2 = 2100 + 1904 = \$4{,}004 .$$

The unfunded accrued liability is then equal to

$$UAL_2 = AL_2 - F_2 = AL_2 - (F_1 + C_1)(1.07)$$

$$= 4004 - (0 + 3742)(1.07) = \$0 .$$

This result coincides with the expected unfunded accrued liability which is also zero,

$$\text{expected } UAL_2 = (NC_1 + UAL_1 - C_1)(1.07) = (3742 + 0 - 3742)(1.07) = \$0 .$$

Therefore, the contribution is again the same as the normal cost, $C_2 = NC_2 = \$3,742$.

The difference comes in the third year where there is a benefit increase. Since there is a $\Delta B_3^j = 12\times\$25\times$number of projective years, for j =1 and 2, an incremental normal cost ΔNC_3^j is generated for each participant. For each individual normal cost, we have

$$NC_3^1 = NC_2^1 + \Delta NC_3^1$$

$$= 1963 + 38\cdot 12\cdot 25\cdot \ddot{a}_{65}^{(12)}\cdot \frac{D_{65}}{N_{27} - N_{65}} = 1963 + 539 = \$2,502$$

and

$$NC_3^2 = NC_2^2 + \Delta NC_3^2$$

$$= 1779 + 41\cdot 12\cdot 25\cdot \ddot{a}_{65}^{(12)}\cdot \frac{D_{65}}{N_{24} - N_{65}} = 1779 + 469 = \$2,248 .$$

The normal cost is then $NC_3 = 2502 + 2248 = \$4,750$. AL_3 can be updated as

$$AL_3 = (AL_2 + NC_2)\cdot\frac{1}{v} = (4004 + 3742)(1.07) = \$8,288$$

which is the same as $F_3 = (F_2 + C_2)(1.07) = \$8,288$ where $F_2 = 3742\cdot 1.07 = \$4,004$. Since $NC_2 = C_2$ and $UAL_2 = 0$, the expected UAL_3 will be zero. $Gain_3$ = expected UAL_3 - UAL_3 will also be 0. The contribution is equal to $C_3 = NC_3 = \$4,750$.

Again during the year from $t = 6$ to $t = 7$, an actuarial loss occurs. In order to calculate this actuarial loss, we need to determine previous years' fund balances and the accrued liabilities. These values are shown in Table 3.4.1.

TABLE 3.4.1

Results for Selected Years under ILP

Time (t)	**NC_t**	**F_t**	**AL_t**
3	4,750	8,288	8,288
4	4,750	13,951	13,951
5	4,750	20,010	20,010
6	4,750	26,493	26,493
7	4,750	32,805	33,429

The actual $UAL_7 = AL_7 - F_7 = 33429 - 32805 = \$624$ and the expected $UAL_7 = 0$ since all previous $UAL_t = 0$ for $1 \leq t \leq 6$. $Gain_7$ then follows as \$-624 and generates a amortization amount of \$142 per year for the next 5 years. Therefore, C_t increases to \$4,892 for $7 \leq t \leq 11$. After $t = 11$, the normal costs and the contributions are the same again. The normal costs and contributions for the 40-year valuation period are given in Table 3.4.2, and Figure 3.4.1 shows the graph of all 40 contributions.

TABLE 3.4.2
NC and C under Level Dollar ILP

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	3742	3742	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750
NAC*	0	0	0	0	0	0	142	142	142	142	142	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C	3742	3742	4750	4750	4750	4750	4892	4892	4892	4892	4892	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750	4750

*NAC -- Net Amortization Charge

FIGURE 3.4.1
Contributions under ILP

Section 3.5: Frozen Initial Liability (FIL)

Note that under both the UC and the EAN cost methods, the plan cost is made up with three major components, the normal cost, the amortization of the initial unfunded accrued liability and bases due to subsequent plan changes, and the amortization of gains, whereas the plan cost for the ILP cost method consists only of two, namely the normal cost and the amortization of gains. Here we introduce a cost method, the FIL, which eliminates actuarial gains, and expresses the plan cost as the sum of the normal cost and the amortization of the initial unfunded accrued liability and bases for subsequent plan changes.

Under the FIL cost method, we assume the normal cost is a level dollar amount or a level percentage of pay from entry age to retirement age, and that dollar amount or percentage is the same for each active participant. The normal cost is calculated in the aggregate. For the level dollar approach, the calculations performed under the FIL cost method match those of the Aggregate EAN cost method in the first plan year. A first-year unit normal cost UNC_1 is computed as

$$UNC_1 = \frac{PVFB_w}{PVFY_w} = \frac{\sum_{A_1} PVFB_{w_j}}{\sum_{A_1} PVFY_{w_j}}$$

$$= \sum_{A_1} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \div \sum_{A_1} \frac{N_{w_j} - N_{y_j}}{D_{w_j}} .$$

Then normal cost for the plan at $t = 1$ is

$NC_1 = n_1 \cdot UNC_1$

where n_1 is the number of participants in A_1.

In order to determine the initial accrued liability which is the difference of the present value of future benefits and the present value of future normal costs, we need to first obtain the present value of future normal cost:

$$PVFNC_1 = \sum_{A_1} UNC_1 \frac{N_{x_j} - N_{y_j}}{D_{x_j}} = \frac{NC_1}{n_1} \sum_{A_1} \frac{N_{x_j} - N_{y_j}}{D_{x_j}} \quad . \qquad (3.5.1)$$

From (3.5.1), the initial unfunded accrued liability is

$$\begin{aligned} UAL_1 &= AL_1 - F_1 \\ &= PVFB_1 - PVFNC_1 \\ &= \sum_{A_1} B(y_j)\ddot{a}^{(12)}_{y_j} \frac{D_{y_j}}{D_{x_j}} - \sum_{A_1} UNC_1 \frac{N_{x_j} - N_{y_j}}{D_{x_j}} \end{aligned}$$

since the fund asset F_1 is zero at $t = 1$. This initial accrued liability is then assumed to be fixed or frozen and amortized over a period of n years where n is in a range specified by law. For minimum funding purpose, n = 30. Finally, the contribution for the first year is equal to the sum of the normal cost and the amortization of the initial unfunded accrued liability: $C_1 = NC_1 + UAL_1 / \ddot{a}_{\overline{n|}}$.

Up to this point all the calculations are identical to those under the Aggregate EAN cost method. The difference comes in during the second and subsequent years. The same formulas are applied, but in a different order. Note that the equation of balance, PVFNC = PVFB - AL = PVFB - UAL - F, is true regardless of the cost method used. So in later years ($t \geq 2$) if we want to calculate the present value of future normal cost $PVFNC_t$ first, we need to update the initial unfunded accrued liability and fund assets. The updated unfunded accrued liability increased with interest at the valuation rate is

$$UAL_t = (UAL_{t-1} + NC_{t-1})(1+i) - C_{t-1}(1+i) \tag{3.5.2}$$

and the updated amount of assets is

$$F_t = F_{t-1}(1+ i_f) + C_{t-1}(1+i_c)\ . \tag{3.5.3}$$

Note that UAL_t here is defined to be the same as the expected UAL_t; therefore, it forces the gain to be zero. With the updated UAL_t and F_t, the present value of future normal cost at time t then follows:

$$\begin{aligned} PVFNC_t &= PVFB_t - UAL_t - F_t \\ &= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - UAL_t - F_t\ . \end{aligned} \tag{3.5.4}$$

Now the unit normal cost can be obtained by dividing the present value of future normal costs by the present value of future working years for all participants $PVFY_t$:

$$UNC_t = \frac{PVFNC_t}{PVFY_t} = \frac{PVFNC_t}{\sum_{A_t} \frac{N_{x_j} - N_{y_j}}{D_{x_j}}}\ .$$

The normal cost for the plan is the product of this unit normal cost and the total number of participants in the plan,

$$NC_t = UNC_t \cdot n_t = \frac{PVFB_t - UAL_t - F_t}{PVFY_t} \cdot n_t\ . \tag{3.5.5}$$

The contribution C_t is equal to the sum of the new normal cost and the amortization of the initial unfunded accrued liability and bases for subsequent plan changes

$$C_t = NC_t + UAL_1 / \ddot{a}_{\overline{n}|} + \Delta UAL_2 / \ddot{a}_{\overline{n}|} + \cdots + \Delta UAL_t / \ddot{a}_{\overline{n}|} .$$

When a plan amendment occurs at time t, there will usually be either an increase or a decrease in the accrued liability based on the underlying method, the Aggregate EAN cost method in this case. Recall that the equation of balance should remain true at all times, that is,

$$PVFNC_t = PVFB_t - AL_t = PVFB_t - UAL_t - F_t . \qquad (3.5.6)$$

Note that UAL_t in (3.5.6) is funded by the amortization. So the change in the accrued liability (ΔUAL_t) based on the Aggregate EAN cost method should be reflected in UAL_t here. Therefore, UAL_t should be the expected old UAL_t under the FIL cost method and adjusted by ΔUAL_t. Algebraically,

$$\begin{aligned} UAL_t &= \text{old } UAL_t(FIL) + \Delta UAL_t \\ &= (UAL_{t-1} + NC_{t-1} - C_{t-1})(1+i) + \Delta UAL_t . \qquad (3.5.7) \end{aligned}$$

This ΔUAL_t is the difference between the new and old unfunded accrued liabilities under the Aggregate EAN cost method, that is,

$$\begin{aligned} \Delta UAL_t &= \text{new } UAL_t(EAN) - \text{old } UAL_t(EAN) \\ &= \text{new } AL_t(EAN) - \text{old } AL_t(EAN) . \qquad (3.5.8) \end{aligned}$$

Recall that under the Aggregate EAN cost method,

$$\text{new } UAL_t(EAN) = PVFB_t - PVFNC_t - F_t = PVFB_t - UNC_t \cdot \sum_{A_t} \frac{N_{x_j} - N_{y_j}}{D_{x_j}} - F_t$$

where both $PVFB_t$ and UNC_t are evaluated with the new plan provision. Note that for the old UAL under the Aggregate EAN cost method, we need to compute UNC_t using the old plan provision if the population has changed. After the adjusted UAL_t (3.5.7) has been determined, we then use the adjusted UAL_t in (3.5.5) to compute the normal cost. We will illustrate this procedure with our example at the end of this section.

As mentioned above, the term for amortization of gains is missing from the contribution equation because the actual and the expected UAL are identical under the FIL cost method. Since the normal cost is calculated in the aggregate and actuarial gains are pushed into the normal cost, it makes the FIL cost method an aggregate, spread gain cost based method.

When the benefits are based on salary and a salary-increase assumption is used, the normal cost may be expressed as a percentage, U_t, of salary. For the first plan year, the procedures of the level percentage Aggregate EAN cost method are adopted. Under the level percentage Aggregate EAN cost method, a normal cost rate U_1 is determined at the beginning and should remain unchanged if all actuarial assumptions are exactly realized.

$$U_1 = \frac{PVFB_w}{PVFS_w} = \frac{\sum_{A_1} PVFB_{w_j}}{\sum_{A_1} PVFS_{w_j}}$$

$$= \sum_{A_1} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{w_j}} \div \sum_{A_1} S_1^j \frac{s_{w_j}}{s_{x_j}} \cdot \frac{{}^sN_{w_j} - {}^sN_{y_j}}{{}^sD_{w_j}} .$$

Recall that $PVFB_{w_j}$ is the present value of participant j's future benefit as of entry age w_j and $PVFS_{w_j}$ is the present value as of w_j of j's future salary from entry age to retirement age y_j. The normal cost for the whole plan is then equal to the normal cost rate multiplied by the total salaries of all participants at t =1,

$$NC_1 = U_1 \cdot \sum_{A_1} S_1^j .$$

Again since the fund assets at t =1 is zero, the initial unfunded liability will simply be equal to the accrued liability:

$$\begin{aligned} UAL_1 &= AL_1 - F_1 \\ &= PVFB_1 - PVFNC_1 \\ &= \sum_{A_1} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - U_1 \sum_{A_1} S_1^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}} . \end{aligned}$$

For the second and subsequent years (t ≥2), we update UAL_t and F_t, and calculate $PVFNC_t$ in the same way as under the level dollar approach, namely the equations (3.5.2), (3.5.3), and (3.5.4). The normal cost rate can be obtained by dividing $PVFNC_t$ by the present value of future salaries for all participants, $PVFS_t$. Finally, the normal cost is determined by multiplying this normal cost rate by the total salaries of all participants:

$$NC_t = \frac{PVFNC_t}{PVFS_t} \cdot \sum_{A_t} S_t^j$$

$$= \frac{PVFB_t - UAL_t - F_t}{PVFS_t} \cdot \sum_{A_t} S_t^j$$

where $PVFS_t = \sum_{A_t} S_t^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}}$.

Now let us examine our example with the FIL cost method. The first year's valuation is identical to the one under the Aggregate EAN cost method. NC_1, UAL_1, and C_1 are then equal to

$$NC_1 = 2 \cdot UNC_1 = 2 \cdot 1751 = \$3{,}502 ,$$

$$UAL_1 = PVFB_1 - PVFNC_1 = \$3{,}446 ,$$

and $C_1 = 3502 + 3446 \div \ddot{a}_{\overline{30|}} = 3502 + 259 = \$3{,}761$ respectively. The difference is in the second year's valuation. For the second year's valuation, we first update UAL_2 and F_2:

$$UAL_2 = (3446 + 3502) \cdot 1.07 - 3761 \cdot 1.07 = \$3{,}410$$

and

$$F_2 = 3761 \cdot 1.07 = \$4{,}024 .$$

After that, we calculate the present value of future normal costs in order to derive the normal cost. We get

$$PVFNC_2 = PVFB_2 - UAL_2 - F_2 = PVFB_1 \cdot 1.07 - UAL_2 - F_2$$

$$= (28002 + 25715) \cdot 1.07 - 3410 - 4024 = \$50{,}043$$

and

$$NC_2 = 2 \cdot UNC_2 = 2 \cdot (PVFNC_2 \div PVFY_2) = 2 \cdot (50043 \div 28.58) = 2 \cdot 1751 = \$3{,}502$$

which is the same as NC_1. Therefore, $C_2 = \$3{,}761$, the same as C_1 .

The third year's valuation requires some extra work since the benefit formula is changed. We first need to determine the new UNC_3 under the Aggregate EAN cost method. From page 45, we have

$$\text{new } UNC_3 = \sum_{A_3} PVFB_{w_j} \div \sum_{A_3} PVFY_{w_j} = 62230 \div 28.82 = \$2{,}159 .$$

Then we can compute the accrued liability, new $AL_3(EAN)$,

$$\text{new } AL_3(EAN) = \text{new } PVFB_3 - \text{new } PVFNC_3$$

$$= 75822 - \text{new } UNC_3(\ddot{a}_{\overline{38|}} + \ddot{a}_{\overline{41|}}) = \$14{,}398$$

where

$$\text{new } PVFB_3 = (40 \cdot 1200 + 38 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{38} + (45 \cdot 1200 + 41 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{41}$$

$$= 39674 + 36148 = \$75{,}822 .$$

On the other hand, the old accrued liability under the Aggregate EAN cost method is

$$\text{old } AL_3(EAN) = \text{old } PVFB_3 - \text{old } PVFNC_3$$

$$= 61500 - \text{old } UNC_3(\ddot{a}_{\overline{38|}} + \ddot{a}_{\overline{41|}}) = \$11{,}684$$

where old $PVFB_3 = PVFB_2 \cdot 1.07 = \$61{,}500$ and old $UNC_3 = UNC_2 = \$1{,}751$. Comparing the new AL_3 with the old AL_3, we get

$$\Delta UAL_3 = \text{new } AL_3(EAN) - \text{old } AL_3(EAN) = 14398 - 11684 = \$2{,}714 .$$

Therefore, the adjusted UAL_3 is equal to

$$UAL_3 = \text{old } UAL_3(FIL) + \Delta UAL_3$$

$$= (UAL_2 + NC_2 - C_2)1.07 + \Delta UAL_3$$

$$= (3410 + 3502 - 3761)1.07 + 2714 = \$6{,}086 .$$

Then we can determine the normal cost as

$$NC_3 = \frac{\text{new } PVFB_3 - UAL_3 - F_3}{PVFY_3} \cdot n_t = \frac{75822 - 6086 - 8330}{(14.12 + 14.33)} \cdot 2 = \$4{,}317$$

where $F_3 = (4024 + 3761)1.07 = \$8{,}330$. The contribution then follows

$$C_3 = NC_3 + UAL_1 / \ddot{a}_{\overline{30|}} + \Delta UAL_3 / \ddot{a}_{\overline{30|}} = 4317 + 259 + 204 = \$4{,}780 .$$

The normal cost and contribution will remain the same for the next three years. When i_f decreases to 5% between t = 6 and t = 7, the normal cost increases to reflect the loss. The values needed for determining NC_7 are given in Table 3.5.1.

TABLE 3.5.1

Results for Selected Years under FIL

Time (t)	NC_t	UAL_t	C_t	F_t
4	4,317	6,017	4,780	14,028
5	4,317	5,943	4,780	20,125
6	4,317	5,864	4,780	26,648

The updated UAL_7 is $(5864 + 4317 - 4780)\cdot 1.07 = \$5{,}779$ and $F_7 = (F_6 + C_6)1.05 = \$32{,}999$.

So the normal cost for t =7 is

$$NC_7 = \frac{PVFB_7 - UAL_7 - F_7}{PVFY_7} \cdot 2 = \frac{99386 - 5779 - 32999}{(13.75 + 14.04)} \cdot 2 = \$4{,}362 .$$

And the contribution is

$$C_7 = NC_7 + UAL_1 / \ddot{a}_{\overline{30|}} + \Delta UAL_3 / \ddot{a}_{\overline{30|}} = 4362 + 259 + 204 = \$4,825 .$$

From Table 3.5.2, we can see that the normal costs are the same ($4,362) throughout the remaining valuation period since there are no more plan amendments. The contribution will also remain $4,825 until UAL_1 is fully amortized after 30 years. After that, the contribution has only one amortization base, namely ΔUAL_3, and

$$C_{31} = NC_{31} + \Delta UAL_3 / \ddot{a}_{\overline{30|}} = 4362 + 204 = \$4,566 .$$

When t =33, the normal cost and the contribution both equal $4,362 after ΔUAL_3 is fully amortized. Figure 3.5.1 presents a 40-year contribution graph for the example.

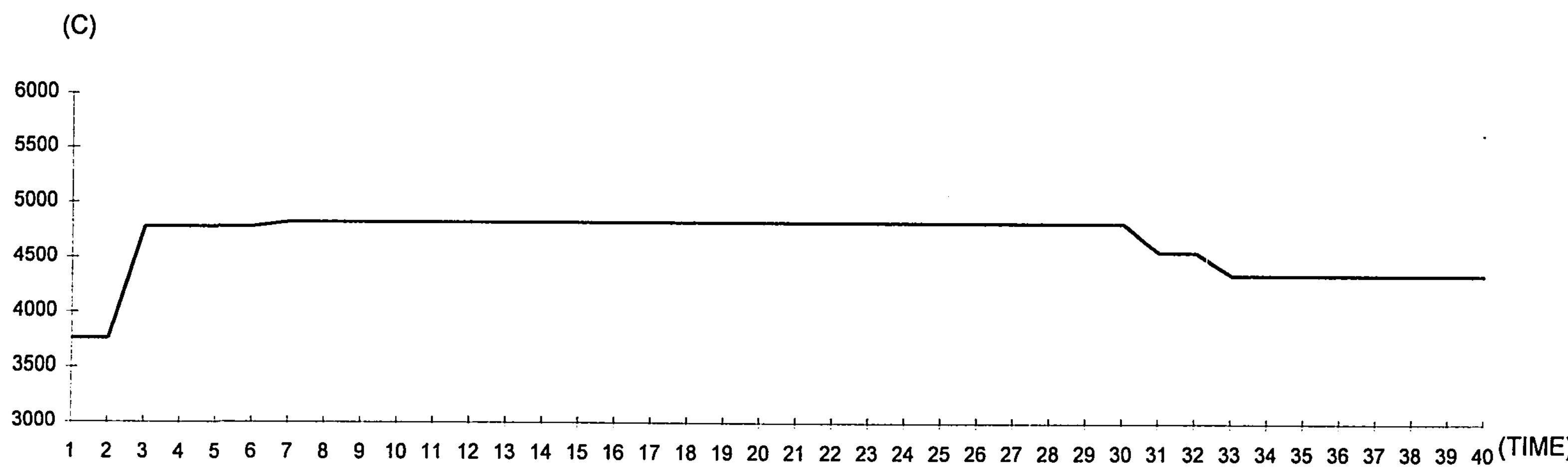
TABLE 3.5.2
NC and C under Level Dollar FIL

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3502	3502	4317	4317	4317	4317	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362
NAC*	259	259	463	463	463	463	463	463	463	463	463	463	463	463	463	463	463	463	463	463
C	3761	3761	4780	4780	4780	4780	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362	4362
NAC*	463	463	463	463	463	463	463	463	463	463	204	204	0	0	0	0	0	0	0	0
C	4825	4825	4825	4825	4825	4825	4825	4825	4825	4825	4566	4566	4362	4362	4362	4362	4362	4362	4362	4362

*NAC -- Net Amortization Charge

FIGURE 3.5.1
Contributions under FIL



Section 3.6: Aggregate

The ILP and the FIL cost methods have two characteristics; one eliminates the initial unfunded accrued liability and the other forces the gains to be zero. Now let us introduce the Aggregate cost method, which eliminates both the initial unfunded accrued liability and the gains. In order to accomplish that, the Aggregate cost method funds the total liabilities and gains over total service periods. The main characteristic of this cost method is that the unfunded accrued liability at any time, not only the initial unfunded accrued liability, will always equal zero. This is the same as saying that the accrued liability is always equal to the assets on hand, that is $AL_t = F_t$. From the equation of balance, we have

$$PVFB_t = AL_t + PVFNC_t = F_t + PVFNC_t.$$

Any effect of a plan amendment on benefits is reflected in the value of PVFB and any experience with regard to earnings and other assumptions is reflected in the value of the assets. Therefore, gains are automatically spread over future normal cost payments. Under the Aggregate cost method the plan contribution in any year is precisely equal to the normal cost, with no amortization of gains, the initial unfunded accrued liability, or bases due to subsequent plan changes.

Under the level dollar approach, we assume each participant has the same normal cost, the unit normal cost UNC_t. Thus, the plan normal cost is the product of this yet to be determined UNC_t and the total number of participants:

$$NC_t = \sum_{A_t} UNC_t = UNC_t \cdot n_t$$

where n_t stands for the number of participants in A_t. To derive the unit normal cost for year t, we need to calculate the present value of future normal cost $PVFNC_t$ and then divide it by the present value of future working years $PVFY_t = \sum_{A_t} \frac{N_{x_j} - N_{y_j}}{D_{x_j}}$. At t = 1, the present value of future normal cost is the same as the present value of future benefits since there is no fund asset at this point: $PVFNC_1 = PVFB_1$. But when $t \geq 2$, the present value of future normal cost is equal to the difference between the present value of future benefits and the fund assets:

$$PVFNC_t = PVFB_t - F_t$$

$$= \sum_{A_t} B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} - F_{t-1}(1+i_f) - C_{t-1}(1+i_c). \tag{3.6.1}$$

Dividing $PVFNC_t$ (for $t \geq 1$) by the present value of future working years, we can determine the unit normal cost as

$$UNC_t = \frac{PVFNC_t}{PVFY_t}. \tag{3.6.2}$$

Thus, the contribution of the year is simply the normal cost,

$$C_t = NC_t = UNC_t \cdot n_t.$$

The Aggregate cost method can also incorporate a salary scale assumption if benefits are based on salary. Instead of determining the unit normal cost, a normal cost rate, U_t, is determined for $t \geq 1$. This normal cost rate, which is a uniform level percentage of salary for all participants, is determined by substituting $PVFY_t$ in (3.6.2) with the present value of future salaries $PVFS_t$:

$$U_t = \frac{PVFB_t - F_t}{PVFS_t}$$

where $PVFS_t = \sum_{A_t} S_t^j \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}}$. Now the normal cost NC_t is simply the normal cost rate multiplied by the total payroll; that is, $NC_t = U_t \sum_{A_t} S_t^j$ which is also the cost of the plan.

We apply the Aggregate cost method to our example. The present value of future normal cost in the first year is

$$PVFNC_1 = PVFB_1$$

$$= 40 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{25}} + 45 \cdot 1200 \cdot \ddot{a}_{65}^{(12)} \frac{D_{65}}{D_{22}} = 28002 + 25715 = \$53,717 .$$

We divide $PVFNC_1$ by the present value of future working years to obtain the unit normal cost,

$$UNC_1 = PVFNC_1 \div PVFY_1 = 53717 \div \left(\frac{N_{25} - N_{65}}{D_{25}} + \frac{N_{22} - N_{65}}{D_{22}}\right)$$

$$= 53717 \div (\ddot{a}_{\overline{40|}} + \ddot{a}_{\overline{43|}}) = 53717 \div (14.26 + 14.45) = \$1,871 .$$

The normal cost and the contribution for the plan are then equal to

$$C_1 = NC_1 = 2 \cdot 1871 = \$3,742 .$$

The second year's valuation is similar to the first. Since there is no change in the plan and the experience coincides with the expected, the normal cost and the contribution remain the same. The fund asset at the beginning of the third year is accumulated to

$$F_3 = (F_2 + C_2)1.07 = [(F_1 + C_1)1.07 + C_2]1.07 = \$8,288 .$$

In the third year when there is a benefit increase, the present value of future benefit becomes

$$PVFB_3 = (40 \cdot 1200 + 38 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{38} + (45 \cdot 1200 + 41 \cdot 12 \cdot 25)\ddot{a}_{65}^{(12)} \cdot v^{41}$$

$$= 39674 + 36148 = \$75,822 .$$

It follows that the normal cost is equal to

$$NC_3 = 2 \cdot UNC_3 = 2 \cdot [(PVFB_3 - F_3) \div PVFY_3]$$

$$= 2 \cdot [(75822 - 8288) \div 28.45] = 2 \cdot 2374 = \$4,748 .$$

This normal cost remains unchanged for the next three years until i_f decreases to 5% during the sixth year. Table 3.6.1 gives the NCs and the Fs from t=4 to t=6.

TABLE 3.6.1

Results for Selected Years under Aggregate

Time (t)	NC_t	F_t
4	4,748	13,949
5	4,748	20,006
6	4,748	26,487

Since i_f decreases during the period from t =6 to t =7, an actuarial loss occurs. This loss is reflected in the value of $PVFNC_7$ and causes the normal cost to increase. We then have

$$NC_7 = 2 \cdot UNC_7 = 2 \cdot (PVFNC_7 \div PVFY_7) = 2 \cdot [(PVFB_7 - F_7) \div PVFY_7]$$

$$= 2 \cdot [(99386 - 32797) \div 27.79] = \$\,4792\,,$$

which remains level throughout the rest of the valuation period. Table 3.6.2 and Figure 3.6.1 demonstrate this result.

TABLE 3.6.2
NC and C under Level Dollar Aggregate

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3742	3742	4748	4748	4748	4748	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792
C	3742	3742	4748	4748	4748	4748	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792
C	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792	4792

FIGURE 3.6.1
Contributions under Aggregate

Section 3.7: Individual Aggregate

Under the Aggregate cost method, either a unit normal cost or a normal cost rate is determined for the whole plan and no explicit cost for each individual participant is defined. The Aggregate cost method can also be modified so that the normal cost for each participant is individually defined. We call this version of the Aggregate cost method the Individual Aggregate cost method or the Individual Spread Gain method [11]. In order to avoid carrying any amortization bases, we will force UAL to zero and spread the gains over future working years.

Since there is no asset on hand ($F_1 = 0$) in the first year valuation, the asset allocated to each individual participant is naturally zero. This forces the present value of future normal cost to equal the present value of future benefits for each participant:

$$PVFB_1^j = PVFNC_1^j = NC_1^j \cdot PVFY_1^j \tag{3.7.1}$$

From (3.7.1), each individual normal cost NC_1^j in the first year is calculated as

$$NC_1^j = \frac{PVFB_1^j}{PVFY_1^j}$$

with the individual present value of future working years $PVFY_1^j = \dfrac{N_{x_j} - N_{y_j}}{D_{x_j}}$. The contribution C_1 for the first year is equal to the sum of all individual normal costs:

$$C_1 = NC_1 = \sum_{A_1} NC_1^j .$$

In subsequent years ($t \geq 2$), the amount of fund assets is no longer zero and is equal to the amount in the previous year increased with interest for one year plus plan contribution and investment return: $F_t = F_{t-1}(1+i_f) + C_{t-1}(1+i_c)$. To determine each individual participant's normal cost, we need to allocate an asset share F_t^j to each participant. This is accomplished by finding a fund ratio for each participant and then multiplying this ratio to the current valuation assets. The individual fund ratio is defined to be the ratio of an individual participant's prior year normal cost and asset share over the sum of all participants'. Algebraically, each participant's current asset share is

$$F_t^j = \frac{NC_{t-1}^j + F_{t-1}^j}{\sum_{A_t}\left(NC_{t-1}^j + F_{t-1}^j\right)} \cdot F_t \, .$$

In the case of a new participant entering the plan at t, we will define this new entrant's normal cost and asset share for the previous year to be both zero. That is, if $j \notin A_{t-1}$, we define $NC_{t-1}^j = F_{t-1}^j = 0$. The individual normal cost can then be determined as

$$NC_t^j = \frac{PVFB_t^j - F_t^j}{PVFY_t^j} \, . \tag{3.7.2}$$

Since the normal cost for each participant is individually defined, and any benefit amendment and experience gains or losses are directly reflected in the normal cost, the Individual Aggregate cost method is classified to be an individual, cost based, spread gain funding method without any amortization base.

All the above derivations are done under the level dollar contribution approach. If the level percentage of pay approach is to be used, we replace $PVFY_t^j$ by $PVFS_t^j$, the present value of participant j's future salary, in the calculation of the individual normal cost. So the individual normal cost in (3.7.2) now becomes

$$NC_t^j = \frac{PVFB_t^j - F_t^j}{PVFS_t^j} \cdot S_t^j$$

where $PVFS_t^j = S_t^j \cdot \frac{{}^sN_{x_j} - {}^sN_{y_j}}{{}^sD_{x_j}}$ and $PVFB_t^j = B(y_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}}$. Finally the contribution for the plan is the sum of all individual normal costs: $C_t = \sum_{A_t} NC_t^j$.

Again examining our example, we first calculate the individual normal cost for each participant and sum to get the normal cost for the plan:

$$NC_1 = NC_1^1 + NC_1^2$$

$$= \frac{PVFB_1^1}{PVFY_1^1} + \frac{PVFB_1^2}{PVFY_1^2} = \frac{28002}{14.26} + \frac{25715}{14.45} = 1963 + 1780 = \$3,743 .$$

In the second year, the asset shares for the two participants are

$$F_2^1 = \frac{NC_1^1 + F_1^1}{NC_1 + F_1} \cdot F_2 = \frac{1963 + 0}{3743 + 0} \cdot 4005 = \$2,100$$

and

$$F_2^2 = \frac{NC_1^2 + F_1^2}{NC_1 + F_1} \cdot F_2 = \frac{1780 + 0}{3743 + 0} \cdot 4005 = \$1,905 .$$

Note that $F_2= (F_1 + C_1)(1.07) =(F_1 + NC_1)(1.07)= 3743\times1.07 =\$4,005$, and both F_1^1 and F_1^2 are equal to zero since $F_1 = 0$. Since there is no benefit change, the PVFB for each participant is increased by one year's interest, that is, $PVFB_2^1 = 28002\times1.07 = \$29,962$ and $PVFB_2^2 = 25715\times1.07 = \$27,515$. The individual normal cost can be determined as

$$NC_2^1 = \frac{PVFB_2^1 - F_2^1}{PVFY_2^1} = \frac{29962-2100}{14.19} = \$1,963$$

and

$$NC_2^2 = \frac{PVFB_2^2 - F_2^2}{PVFY_2^2} = \frac{27515-1905}{14.39} = \$1,780\ .$$

It follows that NC_2 is still \$3,743 .

In the third year, we need to determine a new normal cost since the benefit formula is changed. First we need to allocate an asset share to each participant:

$$F_3^1 = \frac{NC_2^1 + F_2^1}{NC_2 + F_2} \cdot F_3 = \frac{1963+2100}{3743+4005} \cdot 8290 = \$4,347$$

and

$$F_3^2 = \frac{NC_2^2 + F_2^2}{NC_2 + F_2} \cdot F_3 = \frac{1780+1905}{3743+4005} \cdot 8290 = \$3,943$$

where $F_3 = (F_2 + C_2)1.07 = (4005 + 3743)1.07 = \$8,290$. With F_3^1 and F_3^2 determined, the normal cost can be calculated as

$$NC_3 = NC_3^1 + NC_3^2 = \frac{PVFB_3^1 - F_3^1}{PVFY_3^1} + \frac{PVFB_3^2 - F_3^2}{PVFY_3^2}$$

$$= \frac{39674 - 4347}{14.12} + \frac{36148 - 3943}{14.33} = 2502 + 2247 = \$4,749 .$$

This normal cost remains level for the next 3 years since there is no change in the plan provision and $i_f = i_c = i$ for the period between t =3 to t =6. The individual normal costs and the individual asset shares, along with the contributions, for the next 3 years are presented in Table 3.7.1.

TABLE 3.7.1

Results for Selected Years under Individual Aggregate

Time (t)	NC_t^1	NC_t^2	F_t^1	F_t^2	C_t
4	2,502	2,247	7,330	6,622	4,749
5	2,502	2,247	10,520	9,490	4,749
6	2,502	2,247	13,933	12,559	4,749

In the seventh year, the normal cost increases to \$ 4,795 to reflect the actuarial loss due to the decrease in i_f. The calculations for determining NC_7 are as follows:

$$F_7 = (F_6 + C_6)1.05 = (26492 + 4749)1.05 = \$32,803 ,$$

$$F_7^1 = \frac{NC_6^1 + F_6^1}{NC_6 + F_6} \cdot F_7 = \frac{2502 + 13933}{4749 + 26492} \cdot 32803 = \$17,257 ,$$

$$F_7^2 = \frac{NC_6^2 + F_6^2}{NC_6 + F_6} \cdot F_7 = \frac{2247 + 12559}{4749 + 26492} \cdot 32803 = \$15,546 ,$$

and

$$NC_7 = NC_7^1 + NC_7^2 = \frac{PVFB_7^1 - F_7^1}{PVFY_7^1} + \frac{PVFB_7^2 - F_7^2}{PVFY_7^2}$$

$$= \frac{52004 - 17257}{13.75} + \frac{47382 - 15546}{14.04} = 2527 + 2268 = \$4,795 .$$

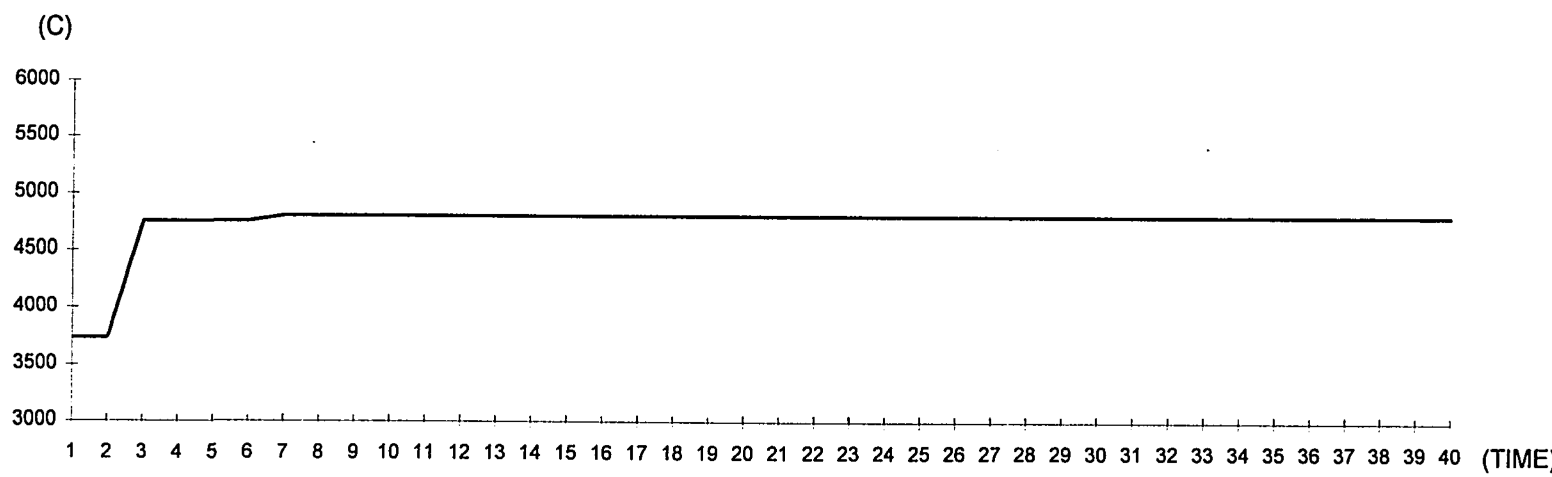
This normal cost remains level until the retirement of the first participant. This result is illustrated in Table 3.7.2 and Figure 3.7.1.

TABLE 3.7.2
NC and C under Level Dollar Individual Aggregate

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3743	3743	4749	4749	4749	4749	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795
C	3743	3743	4749	4749	4749	4749	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795
C	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795	4795

FIGURE 3.7.1
Contributions under Individual Aggregate



Section 3.8: Attained Age Normal (AAN)

Recall that the first year's calculation under the FIL cost method is done in the same way as the Aggregate EAN cost method. It is the different order for determining the normal cost in later years that distinguishes it from the Aggregate EAN cost method. A variation of the FIL cost method, called the Attained Age Normal cost method, uses the UC cost method to determine the first year's accrued liability.

Under the UC cost method, we do not need to compute the normal cost in order to get the accrued liability. The accrued liability in the first year is directly computed as the present value of accrued benefits on the valuation date. This gives

$$AL_1 = PVAB_1 = \sum_{A_1} B(x_j)\ddot{a}_{y_j}^{(12)} \frac{D_{y_j}}{D_{x_j}} = UAL_1$$

since the assets on hand is zero ($F_1 = 0$). With UAL_1 determined, the normal cost can be computed in the same way as under the FIL cost method. Using (3.5.5) for $t \geq 1$, we have

$$NC_t = \frac{PVFNC_t}{PVFY_t} \cdot n_t$$

$$= \frac{PVFB_t - UAL_t - F_t}{PVFY_t} \cdot n_t \ .$$

Again the experience gains are spread over the remaining working lifetime of all participants. The AAN cost method is an aggregate, cost based, spread gain method. The contribution for the plan is the normal cost plus the amortization of the initial unfunded accrued liability and any base due to subsequent plan changes:

$$C_t = NC_t + UAL_1/\ddot{a}_{\overline{n}|} + \Delta UAL_2/\ddot{a}_{\overline{n}|} + \cdots + \Delta UAL_t/\ddot{a}_{\overline{n}|}.$$

Similar to the FIL cost method, a ΔUAL_t is generated when there is an amendment in the plan. Recall from (3.5.8), ΔUAL_t is the difference between the new and old unfunded accrued liabilities under the Aggregate EAN cost method. The underlying cost method for the AAN cost method is the UC cost method. Therefore, ΔUAL_t here is equal to

$$\Delta UAL_t = \text{new } UAL_t(UC) - \text{old } UAL_t(UC) = \text{new } AL_t(UC) - \text{old } AL_t(UC).$$

We derive the new AL_t using the UC cost method with the new benefit formula and calculate the old AL_t under the UC cost method assuming no plan change. Then we can obtain the adjusted UAL_t by using (3.5.7):

$$\begin{aligned} UAL_t &= \text{old } UAL_t(AAN) + \Delta UAL_t \\ &= (UAL_{t-1} + NC_{t-1} - C_{t-1})(1+i) + \Delta UAL_t. \end{aligned} \tag{3.8.1}$$

Finally the normal cost will follow from (3.5.5).

Now we will illustrate this cost method by our example. In the first year, $AL_1 = UAL_1 = \$1,143$ is obtained from the UC cost method, and the normal cost for the first year is

$$NC_1 = 2 \cdot \frac{PVFB_1 - AL_1}{PVFY_1} = 2 \cdot 1831 = \$3,662.$$

The contribution is equal to $C_1 = NC_1 + UAL_1/\ddot{a}_{\overline{n}|} = 3662 + 86 = \$3,748$. In the second year, we first update UAL_2 and F_2,

$$UAL_2 = (UAL_1 + NC_1 - C_1) \cdot 1.07 = \$1,131$$

and $F_2 = (F_1 + C_1)\cdot 1.07 = 3748\cdot 1.07 = \$4{,}010$. The normal cost is then equal to

$$NC_2 = 2\cdot\frac{PVFB_2 - UAL_2 - F_2}{PVFY_2} = 2\cdot\frac{53717\cdot 1.07 - 1131 - 4010}{28.58} = 2\cdot 1831 = \$3{,}662\,,$$

which is the same as NC_1. It follows that $C_2 = C_1 = \$3{,}748$.

The change comes in the third year. Recall that under the UC cost method, the accrued liability is defined to be the PVAB, the present value of all benefits accrued to date. Since the new benefit formula only affects prospective accruals, the new AL and the old AL will be the same. It follows that

$$\Delta UAL_3 = \text{new } AL_3(UC) - \text{old } AL_3(UC) = 0,$$

and the adjusted UAL_3 in (3.8.1) is equal to the expected UAL_3,

$$UAL_3 = (UAL_2 + NC_2 - C_2)\cdot 1.07 = (1131 + 3662 - 3748)\cdot 1.07 = \$1{,}118\,.$$

Using (3.5.5), we get

$$NC_3 = \frac{PVFB_3 - UAL_3 - F_3}{PVFY_3}\cdot 2 = \frac{75822 - 1118 - 8301}{(14.12 + 14.33)}\cdot 2 = \$4{,}668$$

where $F_3 = (F_2 + C_2)\cdot 1.07 = (4010 + 3748)\cdot 1.07 = \$8{,}301$. The contribution for the third year is equal to the normal cost plus the amortization amount: $C_3 = NC_3 + UAL_1/\ddot{a}_{\overline{30}|} = 4668 + 86 = \$4{,}754$. Similar to the FIL cost method, the normal cost increases due to the decrease in i_f . Table 3.8.1 shows the normal costs, the unfunded accrued liabilities, and the contributions for the period between t = 4 to 6. The rest of the normal costs and the contributions, along with the net amortization bases, are given in Table 3.8.2.

TABLE 3.8.1

Results for Selected Years under AAN

Time (t)	NC_t	UAL_t	C_t	F_t
4	4,668	1,104	4,754	13,969
5	4,668	1,089	4,754	20,034
6	4,668	1,073	4,754	26,523

Updating the fund balance, we have $F_7 = (F_6 + C_6)\cdot 1.05 = (26523 + 4754)\cdot 1.05 = \$$ 32,841. Then the normal cost is equal to

$$NC_7 = \frac{PVFB_7 - UAL_7 - F_7}{PVFY_7}\cdot 2 = \frac{99386 - 1056 - 32841}{27.79}\cdot 2 = \$4{,}714\,,$$

with UAL_7 equal to the expected UAL_7. Adding the amortization amount, we get the contribution $C_7 = NC_7 + UAL_1/\ddot{a}_{\overline{30|}} = 4714 + 86 = \$4{,}800$, which remains level until t =30. From Table 3.8.2, we can see that after t =30, the initial UAL is fully amortized and the contribution and the normal cost are the same. The contribution graph is presented in Figure 3.8.1.

Like all the methods we have introduced, the AAN cost method can also be applied with a salary scale assumption and the formulas will be similar to those under the level percentage of salary FIL cost method except that the first year accrued liability is calculated under the UC cost method.

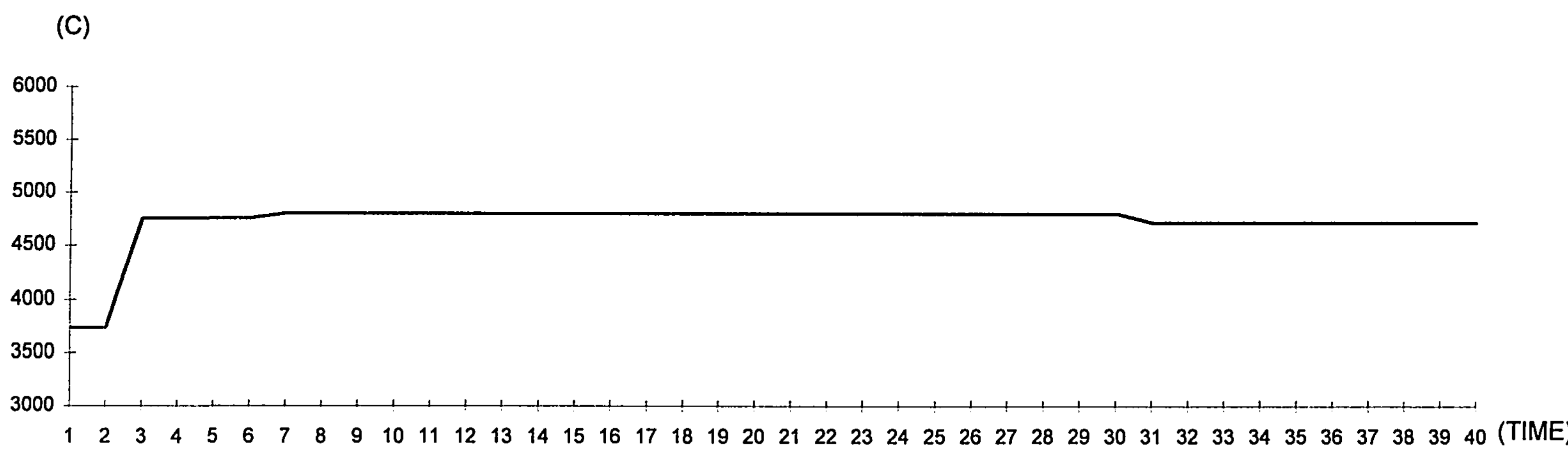
TABLE 3.8.2
NC and C under Level Dollar AAN

TIME	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NC	3662	3662	4668	4668	4668	4668	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714
NAC*	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86	86
C	3748	3748	4754	4754	4754	4754	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800

TIME	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
NC	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714
NAC*	86	86	86	86	86	86	86	86	86	86	0	0	0	0	0	0	0	0	0	0
C	4800	4800	4800	4800	4800	4800	4800	4800	4800	4800	4714	4714	4714	4714	4714	4714	4714	4714	4714	4714

*NAC -- Net Amortization Charge

FIGURE 3.8.1
Contributions under AAN



Chapter 4

Criteria for Desirable Methods

Among the wide variety of available actuarial cost methods, each method has its unique characteristics that provide some advantages or disadvantages over others. There are several criteria that help an actuary determine an appropriate actuarial cost method for a plan sponsor [15] .

Section 4.1: Adequacy of Funds

The first criterion is the adequacy of funds. An actuarial cost method is considered to have adequate funds if its assets can keep pace with the present value of accrued benefits, PVAB, at any point in time. In other words, if a plan is terminated at any time, there should be enough money to fund every participant his accrued benefits. Note that all the cost methods are constructed on the basis that an appropriate amount of assets will be on hand as each participant retires; but not all the cost methods guarantee sufficient funds will be there at all time.

The Unit Credit cost method is the only non-projected benefit method we have discussed. Since it calculates a specific benefit associated with each year of service and allocates the cost of that benefit accordingly, it obviously satisfies the adequate funding criterion if there is no past service liability or the initial unfunded liability has been fully amortized. None of the other cost methods guarantees this, since each year's contribution is based on the projected benefits at retirement and is not based on the benefit that the participants have currently accrued. The situation of insufficient funding generally occurs where the highest paid employee is oldest in a small participant group. For example, under the Aggregate cost method if the first retired person consumes a large share of the overall benefits promised, the assets may be insufficient to cover it.

Section 4.2: Consistency of Costs

The consistency of costs means that the annual cost developed by a cost method should be a level dollar amount or a level percentage of total compensation from year to year. If a plan produces fluctuating annual contributions due to the actuarial cost method used, there will certainly be dissatisfaction from the plan sponsor or accountant. There are several factors that will cause inconsistency in annual contributions. For example under the level percentage of pay Frozen Initial Liability cost method, the entry of new participants may result in the reduction of contribution since the total compensation base is increased while the addition in cost may be relatively small. Another factor that will reduce contribution considerably is the retirement of key people. The gains or losses caused by investment results also affect the consistency of the annual deposit.

The contribution for the Unit Credit cost method is generally not consistent from year to year. This is because the costs for accrued-benefit methods tend to increase with age due to increasing salaries and the shorter period before benefits are due. On the other hand, all other projected-benefit methods produce either a level dollar amount or a level percentage of payroll contribution from year to year when all the assumptions are exactly realized and there is no change in the plan. Realistically, actual experience does not always coincide with the expected and amendments in plan benefits do occur. Amortization bases are then generated and cause inconsistency in costs.

Section 4.3: Flexibility in Contributions

To a plan sponsor, it is a great advantage to have a range of annual contributions from which to choose. This flexibility allows the plan sponsor to have more control in response to company performance. The amortization of past service liability over different periods of time allowed by law is one source of flexibility. Additional flexibility may be obtained if gains and losses are also separately amortized.

Both the Aggregate and the Individual Aggregate cost methods produce only a single recommended contribution which does not give any flexibility to the plan sponsor. The contribution for the Frozen Initial Liability cost method consists of the amortization of the initial unfunded liability(UAL_1) and the normal cost. Therefore the method gives some flexibility only in this amortization period if there is no other additional liability generated in later years. The situation is similar in the Individual Level Premium cost method. Under the method, the changes in the unfunded accrued liability due to plan experience, the gains or losses, are separately accounted for and amortized. So the method does satisfy the flexibility of funding criterion.

The Entry Age Normal cost method tends to produce a large unfunded accrued liability that results in a big range based on the number of years over which it is amortized. In the case when the unfunded accrued liability is not large, not much flexibility will be produced regardless of the duration over which it is amortized. Both the past service liability and the actuarial gains are calculated separately under the Unit Credit cost method. The amortization of these costs over various periods as permitted by law provides flexibility in funding.

Section 4.4: Robustness of Plans

An actuarial cost method is considered to have robustness if it can automatically adjust for unanticipated experience to guarantee sufficient funds for any participant at retirement. The robustness of an actuarial cost method is the next criterion. It is not unusual for actual plan experience to differ considerably from the assumptions used by the plan actuary. So it is important for a cost method to have the ability to handle gains and losses from experience. Note that robustness does not guarantee sufficient funds at each point in time. All the actuarial cost methods we have discussed in Chapter 3 have the ability to handle gains or losses; therefore, they all satisfy this criterion. It is obvious that an actuarial cost method generally will not have both consistency and robustness at the same time.

Section 4.5: Simplicity of Methods

A cost method should be easy to explain to plan sponsors, and should not involve cumbersome techniques, and should not require maintenance of extensive census data.

Generally, individual methods require more extensive recodkeeping than aggregate methods, and are not suitable for large participant groups. On the other hand, individual methods give an explicit cost for each individual participant. Some plan sponsors likely would want to know the exact cost of benefits for each participant.

With the exception of an explicit cost for each participant, the Aggregate cost method possesses all the above attractive characteristics, namely, easily explained to plan sponsors, no cumbersome techniques, and no extensive recodkeeping, whereas the Individual Aggregate cost method is more difficult to apply because plan assets have to be allocated to each participant annually. The Individual Level Premium cost method carries an extra burden of having to determine an incremental normal cost each year after the first. The recordkeeping might become very massive when the participant group is large. From the view point of simplicity of concept and ease of explanation, the Unit Credit cost method should be preferred by small-plan actuaries. But the Unit Credit cost method may not be suitable for a small participant group where an aging work force with little turnover causes increasing normal cost burdens on the plan sponsor.

The concept of the other cost methods, such as the Aggregate Entry Age Normal, Frozen Initial Liability, and Attained Age Normal cost methods, might not be as obvious as the Unit Credit cost method and may not be easily explained to the plan sponsor. They are suitable for large participant groups since the costs are calculated in the aggregate and no extensive recordkeeping is required.

Section 4.6: Deductibility of Funds

The deductibility of funds is the next criterion to be considered. Annual retirement plan cost is usually deductible by plan sponsors as a business expense for federal income tax purposes. A cost method is considered ineffective if its cost does not fall within deductibility limits as legislated in the law and interpreted by the Internal Revenue Service. In general, contributions are deductible if they are computed using an acceptable cost method and reasonable actuarial assumptions. Any contribution exceeding the funding limitation is not deductible during the valuation year. All the cost methods introduced in the previous chapter are considered reasonable funding methods under the law. This will be discussed in the next section. If reasonable actuarial assumptions are used, the contributions derived from these methods are deductible.

Section 4.7: Acceptability under the Law

The last but not least criterion is the acceptability to the Internal Revenue Service. An acceptable method is one that satisfies the requirements under the law, regulations, and other official guidance.

Internal Revenue Code sections 412(b) and (c), along with Employee Retirement Income Security Act ("ERISA") sections 301 and 302, prescribe rules for a reasonable funding method. The rules require each plan to establish and maintain a funding standard account. Such account should be charged with the sum of the normal cost and the amounts necessary to amortize the unfunded past service liability, increases in unfunded accrued liability arising from plan amendments, experience losses, and losses resulting from changes in actuarial assumptions. The account should also be credited by the annual contribution and the amortization of the decreases in unfunded accrued liability arising from plan amendments, experience gains, and gains resulting from changes in actuarial assumptions.

The rules are further explained by Federal Regulations section 1.412(c)(3)-1 on reasonable funding methods. The regulations are summarized as follows:

(1) The present value of future benefits must equal the sum of the present value of future normal costs, the net sum of the unamortized portions of amortizable bases, and the plan assets;

(2) Normal cost must be expressed as a level dollar amount, or a level percentage of pay, or an amount equal to the present value of benefits accruing for a particular plan year;

(3) All liabilities of the plan for benefits must be taken into account; and no experience gains or losses are produced if each actuarial assumption is exactly realized;

(4) Plan population shall include participants who are currently employed, former participants who have either terminated service or retired, and all other individuals currently entitled to benefits under the plan;

(5) Use of a salary scale is generally acceptable regardless of the funding method which is used; and the salary reflected in projected benefits must be the expected salary used to determine benefits;

(6) The allocation of assets and liabilities among participants must be reasonable;

(7) The method is prohibited to anticipate future benefit changes and future participants who are not employed on the plan valuation date;

(8) When normal cost is expressed as an amount equal to the present value of benefits accruing for a particular plan year, the projected benefits must be allocated between past years and future years in determining a plan's normal cost and accrued liability; and this allocation must be in proportion to the applicable rates of benefit accrual under the plan except in the case of a career average pay plan;

(9) An ancillary benefit is a benefit that is paid as a result of a specified event which occurs not later than a participant's separation from service, and is detrimental to the participant's health; ancillary benefit costs must be the term costs or computed under the same method used to compute retirement benefit costs.

In addition Revenue Procedure 80-50 provided official guidance for certain defined benefit pension plans to change funding methods for plan years commencing on or after July 1, 1979 and before January 1, 1986. This procedure was later modified by Revenue Procedure 81-29 which was effective for changes in funding methods made with respect to plan years beginning on or after January 1, 1980 and before January 1, 1986. Although these two Revenue Procedures have expired, they gave an indication of what specific methods the Internal Revenue Service deemed to be reasonable.

All the cost methods we have discussed in the previous chapter are "legal" in the sense of complying with the Internal Revenue Code and the Employee Retirement Income

Security Act. There is a well-known method called "pay-as-you-go." [10] The method, instead of recognizing the cost of the plan during the working lifetime of the participants, simply pays at the time benefit payments are due. In the first several years, no one will likely be receiving a pension and the contribution amount will be very small. So the cost will only rise moderately. As time goes on, the cost will rise steeply because more participants are retiring on bigger and bigger pensions, and retirees may not be dying at a sufficiently fast rate to offset new additions to the pension rolls. The cost may be at a level that is too high for the employer to bear. The "pay-as-you-go" method is the simplest and most straightforward method of pension funding, but the budgeting problem and the security of funding make the method unacceptable to the Internal Revenue Service for private companies. Some governmental plans, Social Security in particular, are still using variations of this method.

There is another "illegal" method called terminal funding. Under this method, only those who are about to retire will be funded. The normal cost for any time period becomes the present value of future benefits, summed over all individuals retiring within that period. As each employee reaches retirement, the employer usually will purchase an immediate annuity to provide the benefit to which the employee is entitled. The amounts required for such purchase usually come out of the employer's operating revenue since no assets have been set aside in advance. The annual contribution will tend to increase each year until a stable population is achieved. This method is now almost completely obsolete.

Chapter 5

Retirement Protection Act of 1994

In December of 1994, President Clinton signed into law the Uruguay Round Agreements Act, which were trade agreements after negotiation under the General Agreement of Tariffs and Trade. Part of this legislation is known as the Retirement Protection Act of 1994. Among its many provisions, it stipulates the actuarial assumptions that a defined benefit plan must use in determining lump sum distributions. Specifically, the maximum distributable lump sum allowable under Internal Revenue Code section 415 must be computed based on the 1983 Group Annuity Mortality Table (as clarified in Revenue Ruling 95-6) [14] and an interest rate equal to the annual rate of 30-year Treasury securities for the month before distribution.

As mentioned in Chapter 4, one desirable characteristic of pension funding is the stability of cost. We will illustrate that the new legislation will most likely disrupt such stability.

To make the illustration simple, we consider a plan established when the sole participant is aged 55. It provides a flat monthly benefit of $10,000 commencing at age 65. The individual aggregate method is used. Since a lump sum at retirement is contemplated, the post-retirement mortality assumption is based on the 1983 Group Annuity Mortality Table. As is customary in small plans, no pre-retirement decrement will be assumed. The valuation interest rate is set at the rate of 30-year Treasury securities (Appendix A) for the month of valuation. In reality, the actuary of the plan may try some smoothing procedure to avoid extreme fluctuations. But given that interest rates do fluctuate, smoothing procedures may not necessarily achieve their objectives. Our model does not incorporate any smoothing technique. We also assume that the return on assets is exactly given by these Treasury rates. Finally, we will compare the asset amount on attainment of age 65 with the present value of benefit at age 65.

In the previous examples in Chapter 3, a fixed funding interest rate was used throughout the entire valuation process since we assumed no modification in actuarial assumptions. Now we apply a new funding interest rate for each valuation instead of a fixed one. For each year's valuation, we employ the rate of 30-year Treasury securities for the first month of that year as the new funding interest rate. The result of this process is that the funding interest rate changes according to the variation of the Treasury rates. Therefore, with each new assumed funding interest rate, news set of commutation functions and life annuity values are computed. The normal cost is then generated with these new functions and values. For example, the interest rate for January of 1978 was 8.18%. This gives $\ddot{a}_{65}^{(12)} \approx 9.0831$; and the normal cost rounded to the nearest dollar is approximately $68,959. For the following year's valuation, i is replaced by 8.94%, the interest rate for January of 1979, which generates a new $\ddot{a}_{65}^{(12)} \approx 8.6333$. The normal cost for 1979 is then calculated, using this new life annuity value, to be $61,774. The result of the ten-year valuations is illustrated in Table 5.1 at page 105.

In addition to the modification of the funding interest rate, the fund asset is also being updated in a different manner. In the previous examples, the fund asset is updated at the end of each year by a single rate i_f, the annual return on assets. Now the fund asset is being updated at the end of each year using the previous twelve months' interest rates. If i_k denotes the kth month's annual interest rate in a year, where k=1, 2, ..., 12, the fund asset at the end of year t, for t≥1, will be equal to

$$Asset_t = (Asset_{t-1} + NC_{t-1})(1+i_1)^{1/12}(1+i_2)^{1/12} \cdots (1+i_{12})^{1/12}$$

instead of

$$Asset_t = (Asset_{t-1} + NC_{t-1})(1+i_f)$$

Note that the fund asset at the beginning of the first year is denoted by $Asset_0$ which is equal to \$0.

To illustrate this modification, let us assume 1978 is the first year of valuation and we will do the valuation for ten years. The interest rate for February of 1978 is 8.25%, and for March is 8.23%, and the interest rates for the rest of the year can be found in Appendix A. Then the fund asset at the end of 1978 is

$$\begin{aligned}Asset_1 = 68{,}959 \cdot &(1+0.0818)^{1/12}(1+0.0825)^{1/12}(1+0.0823)^{1/12}(1+0.0834)^{1/12}\\ &(1+0.0843)^{1/12}(1+0.085)^{1/12}(1+0.0865)^{1/12}(1+0.0847)^{1/12}\\ &(1+0.0847)^{1/12}(1+0.0867)^{1/12}(1+0.0875)^{1/12}(1+0.0888)^{1/12} = \$74{,}924.\end{aligned}$$

The fund asset for each year during the January 1978 to December 1987 valuation period can be found in Table 5.1.

Theoretically the fund asset on hand at retirement should be the same as the present value of benefit at retirement. The present value of benefit at the retirement is equal to

$$PVB = \ddot{a}_{65}^{(12)} \cdot 120{,}000 = \$1{,}023{,}930$$

where $\ddot{a}_{65}^{(12)}$ = 8.5328 is derived using i =9.12%, the interest rate for December 1987. Notice that i is the last month's interest rate of the entire valuation period. Here the difference between the fund assets at the end of 1987 and the present value of benefit is $Asset_{10}$ - PVB = \$1,164,580 - \$1,023,930 = \$140,650. It is clear that the plan is significantly overfunded at the end of the ten-year period. Recall from Chapter 4 that the adequacy of funds is one of the criteria for desirable funding methods. Here the method seems to satisfy this criterion. Later, we will show that the method fails to satisfy the criterion in some cases.

Both the mean and the standard deviation for the normal costs are computed at the end of the ten-year valuation process. The mean for the normal costs is \$73,862 with a standard deviation of \$70,737. The large standard deviation indicates a wide range of normal costs are generated and indicates that there is no consistency in the normal costs. The graph in Figure 5.1 demonstrates the fluctuation of the normal costs for the ten- year period.

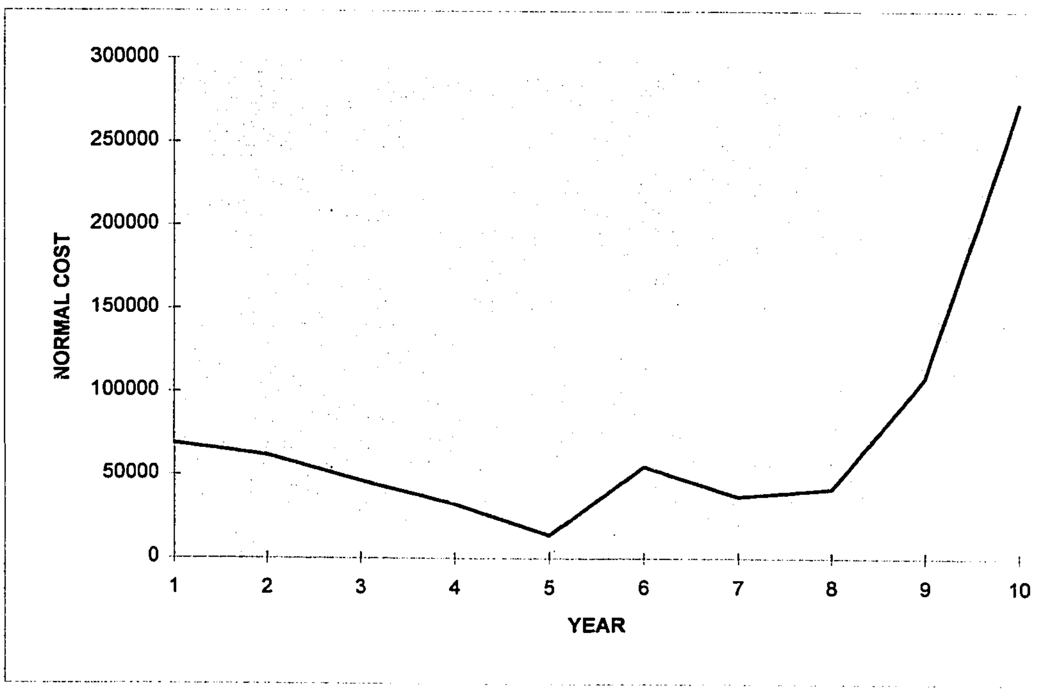
In order to investigate the adequacy of funds and the consistency of costs, we repeat the ten-year valuation process 14 more times. For each time, the valuation period is six months after the previous valuation period. For example, the second valuation period runs from July 1978 to June 1988 and the third period is from January 1979 to December 1988. All the results are given in Appendix B, along with the graph of the normal costs for each period. The normal costs graphs in all the figures fluctuate dramatically corresponding to the shift of interest rates.

TABLE 5.1
Results for Valuation from 1/78 ~ 12/87

VALUATION PERIOD :	1/78-12/87									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/78-12/78	1/79-12/79	1/80-12/80	1/81-12/81	1/82-12/82	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87
INTEREST RATE i (n)	8.18	8.94	10.60	12.14	14.22	10.63	11.75	11.45	9.40	7.39
LIFE ANNUITY FACTOR	9.0831	8.6332	7.7830	7.1263	6.3942	7.7692	7.2822	7.4068	8.3806	9.5986
NORMAL COST *	68959	61774	46374	32497	13935	54910	36861	41525	108296	273486
ASSET **	74924	149399	217886	284032	335969	434568	529838	632998	799088	1164580
PVB =		1023930		EVALUATED AT 12/87 WITH i = 9.12%						
ASSET-PVB =		140650								
NORMAL COST MEAN =		73862								
STANDARD DEVIATION =		70737								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.1
The Normal Costs for 1/78 ~ 12/87



Let i(n), n =1,2,···,10, denote the funding interest rate for the nth year in each valuation period. From Table 5.1, we notice that i(10) = 7.39%, the interest rate for January 1987, is much lower than the interest rate i=9.12%, the interest rate for December 1987, which is used to calculate the PVB at the end of the valuation period. Because of the low i(10) and high interest rate at the end of the valuation period, the plan tends to generate a large normal cost and a small PVB. This results in the big difference of $Asset_{10}$ - PVB. Among the fifteen valuation periods, the first and the last periods have the largest difference in absolute value between i and i(10) and produce the biggest positive value of $Asset_{10}$ - PVB. On the other hand, the seventh valuation period (see Table 5.7) has the smallest difference of $|i - i(10)|$ and produces the smallest value of $|Asset_{10} - PVB|$ = \$2,010. From our observation, we can conclude that usually the larger $|i - i(10)|$ is, the larger the value of $|Asset_{10} - PVB|$ will be. From Table 5.1, 5.2, 5.3, 5.6, 5.14, and 5.15, we notice that i is greater than i (10) and this results in a positive value of $Asset_{10}$- PVB. Although this means that there are sufficient assets to fund the retirement benefits when they become due, a significant overfunding problem occurs in some cases. For example, the first year's $Asset_{10}$ - PVB=\$140,650 is much greater than the second year's $Asset_{10}$- PVB=\$28,210. All fifteen values of the difference between the fund asset and the present value of benefit at retirement are shown in Table 5.16.

Observing Table 5.16, we find that the difference $Asset_{10}$ - PVB takes on a negative value in the fourth, fifth and eighth to thirteenth valuation periods. In these cases, the plan does not accumulate enough asset to fund the participant's retirement benefits. This usually occurs when i is smaller than i(10). Table 5.4, 5.5, and 5.8 to 5.13 illustrate this result. Therefore, the method fails to satisfy the criterion for adequacy of funds. The graph of $Asset_{10}$ -PVB is given in Figure 5.16.

The funding method is not the main culprit here. The main reason for the inconsistency of costs is the highly variable interest rates. No matter which funding method is used, it is impossible to have consistency in costs given the unstable interest rates.

Even if the plan sponsor can tolerate the fluctuating costs, the potential overfunding or underfunding causes concern. As the examples show, it is possible for the value of assets at retirement to differ from the present value of benefits by a wide margin. This threatens the retirement security of the plan participant, contrary to the intent of ERISA. The main problem here is the moving target which the actuary is supposed to fund to. Since there is little reason to expect that the 30-year Treasury rate will become stable, the actuary is given an impossible task. It seems that the only way to alleviate the problem is a legislative change in the law.

Bibliography

1. Abegglen, T. Actuarial Cost Methods. Transcripts of 1989 Enrolled Actuaries Meeting, (1989): 1749-1781.

2. Allen, J. D. and Robert A. Alps. Actuarial Cost Methods. Transcripts of 1993 Enrolled Actuaries Meeting, (1993): 1181-1211.

3. Anderson, A. W. Pension Mathematics for Actuaries, 2nd ed. Massachusetts: Windsor Press, 1990.

4. Bowers, N. L. et al. Actuarial Mathematics. Chicago: Society of Actuaries, 1986.

5. Employee Retirement Income Security Act, Sections 301 and 302.

6. Federal Regulations 1.412(c)(3)-1. Reasonable Funding Methods.

7. Internal Revenue Code section 412(b) and 412(c). Funding Standard Account and Special Rules.

8. Jordan, C. W. Life Contingencies, 2nd ed. Chicago: Society of Actuaries, 1967.

9. Kellison, S. G. The Theory of Interest. Homewood: Richard D. Irwin, Inc, 1970.

10. McGill, D. M. Fundamentals of Private Pensions, 5th ed. Homewood: Richard D. Irwin, Inc, 1984.

11. Ng, H. K. Funding pension Benefits: The Individual Spread Gain Method. UMAP Journal, Vol. 8.1, (1987): 63-77.

12. Revenue Procedure 80-50, 1980-2 CB 816.

13. Revenue Procedure 81-29, 1981-2 CB 550.

14. Revenue Rulings 95-6, 1995-1 IRB 153.

15. Shapiro, A. F. Modified Cost Methods for Small Pension Plans. Transactions of the Society of Actuaries 35, (1983): 11-36.

16. Sypher, E and Paulette Tino. Entry Age Normal Cost Method (with Aggregate Normal Cost). Photocopied.

17. Williamson, W. J. Actuarial Cost Methods. Transcripts of 1991 Enrolled Actuaries Meeting, (1991): 1177-1215.

APPENDIX A

TABLE A
30-YEAR U.S. TREASURY SECURITY RATES (%)

	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
1977		7.75	7.80	7.73	7.80	7.64	7.64	7.68	7.64	7.77	7.85	7.94
1978	8.18	8.25	8.23	8.34	8.43	8.50	8.65	8.47	8.47	8.67	8.75	8.88
1979	8.94	9.00	9.03	9.08	9.19	8.92	8.93	8.98	9.17	9.85	10.30	10.12
1980	10.60	12.13	12.34	11.40	10.36	9.81	10.24	11.00	11.34	11.59	12.37	12.40
1981	12.14	12.80	12.69	13.20	13.60	12.96	13.59	14.17	14.67	14.68	13.35	13.45
1982	14.22	14.22	13.53	13.37	13.24	13.92	13.55	12.77	12.07	11.17	10.54	10.54
1983	10.63	10.88	10.63	10.48	10.53	10.93	11.40	11.82	11.63	11.58	11.75	11.88
1984	11.75	11.95	12.38	12.65	13.43	13.44	13.21	12.54	12.29	11.98	11.56	11.52
1985	11.45	11.47	11.81	11.47	11.05	10.45	10.50	10.56	10.61	10.50	10.06	9.54
1986	9.40	8.93	7.96	7.39	7.52	7.57	7.27	7.33	7.62	7.70	7.52	7.37
1987	7.39	7.54	7.55	8.25	8.78	8.57	8.64	8.97	9.59	9.61	8.95	9.12
1988	8.83	8.43	8.63	8.95	9.23	9.00	9.14	9.32	9.06	8.89	9.02	9.01
1989	8.93	9.01	9.17	9.03	8.83	8.27	8.08	8.12	8.15	8.00	7.90	7.90
1990	8.26	8.50	8.56	8.76	8.73	8.46	8.50	8.86	9.03	8.86	8.54	8.24
1991	8.27	8.03	8.29	8.21	8.27	8.47	8.45	8.14	7.95	7.93	7.92	7.70
1992	7.58	7.85	7.97	7.96	7.89	7.84	7.60	7.39	7.34	7.53	7.61	7.44
1993	7.34	7.09	6.82	6.85	6.92	6.81	6.63	6.32	6.00	5.94	6.21	6.25
1994	6.29	6.49	6.91	7.27	7.41	7.40	7.58	7.49	7.71	7.94	8.08	7.87
1995	7.85											

Appendix B

TABLE 5.2
Results for Valuation from 7/78 to 6/88

VALUATION PERIOD :	7/78-6/88									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/78-6/79	7/79-6/80	7/80-6/81	7/81-6/82	7/82-6/83	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88
INTEREST RATE i(n)	8.65	8.93	10.24	13.59	13.55	11.40	13.21	10.50	7.27	8.64
LIFE ANNUITY FACTOR	8.7999	8.6389	7.9536	6.5997	6.6132	7.4279	6.7301	7.8297	9.6816	8.8058
NORMAL COST *	65049	62477	49810	21004	21182	46498	20715	73969.1	201034	86771
ASSET **	70906	147157	220977	275535	330016	422208	494884	621214	885888	1060150
PVB =	1031940	EVALUATED AT 6/88 WITH i =9.00%								
ASSET-PVB =	28210									
NORMAL COST MEAN =	64851									
STANDARD DEVIATION =	50454									

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.2
Normal Costs for 7/78 to 6/88

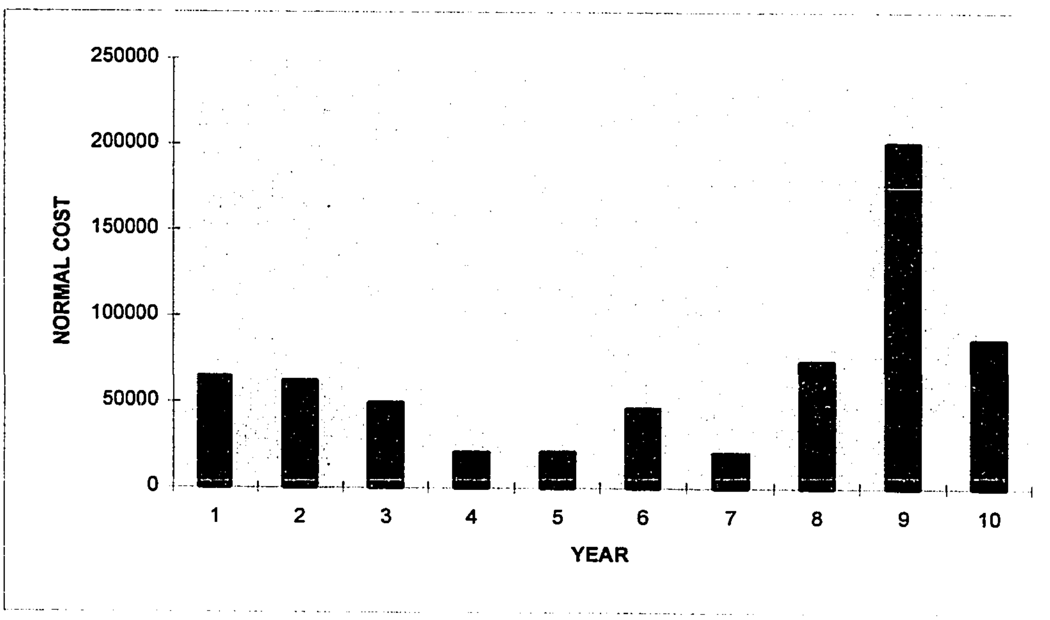


TABLE 5.3
Results for Valuation from 1/79 to 12/88

VALUATION PERIOD :	1/79-12/88									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/79-12/79	1/80-12/80	1/81-12/81	1/82-12/82	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88
INTEREST RATE i(n)	8.94	10.60	12.14	14.22	10.63	11.75	11.45	9.40	7.39	8.83
LIFE ANNUITY FACTOR	8.6332	7.7830	7.1262	6.3942	7.7691	7.2822	7.4067	8.3806	9.5986	8.6958
NORMAL COST *	62773	49605	37951	22658	56265	41796	45542	91718	178015	54380
ASSET **	68605	131563	192295	242368	332011	420121	515896	654986	904452	1044730

PVB =	1031270	EVALUATED AT 12/88 WITH i = 9.01%
ASSET-PVB =	13460	
NORMAL COST MEAN =	64070	
STANDARD DEVIATION =	41654	

* normal cost is deposited at the beginning of the year

** asset is as of the end of the year

FIGURE 5.3
Normal Costs for 1/79 to 12/88

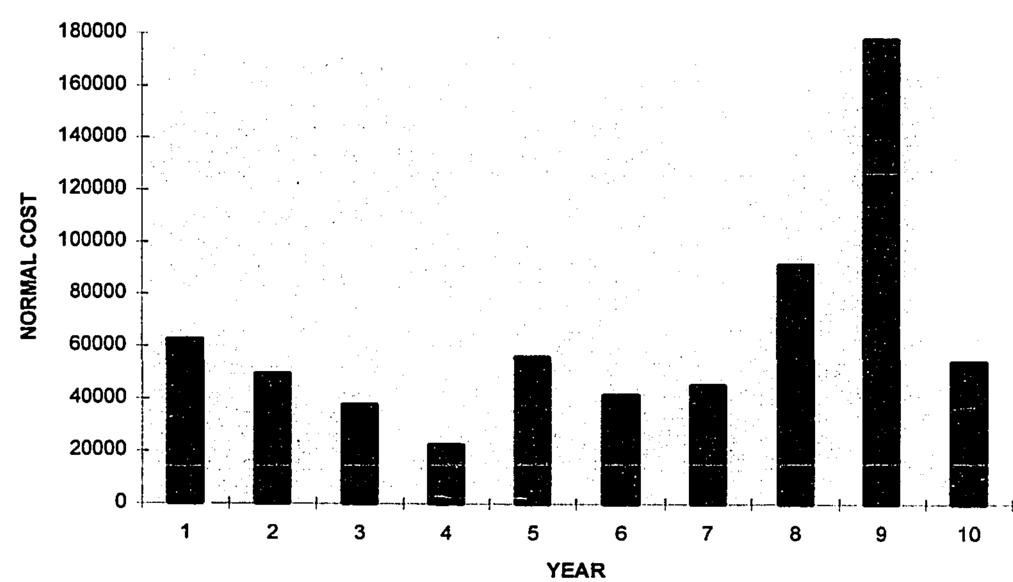


TABLE 5.4
Results for Valuation from 7/79 to 6/89

VALUATION PERIOD :		7/79-6/89								
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/79-6/80	7/80-6/81	7/81-6/82	7/82-6/83	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89
INTEREST RATE i(n)	8.93	10.24	13.59	13.55	11.40	13.21	10.50	7.27	8.64	9.14
LIFE ANNUITY FACTOR	8.6389	7.9536	6.5997	6.6132	7.4279	6.7301	7.8297	9.6816	8.8058	8.5217
NORMAL COST *	62850	52090	27986	28167	48739	28093	68662	156308	96107	57550
ASSET **	69340	136233	186990	239302	322999	392280	503370	710735	879419	1021040
PVB =		1083310		EVALUATED AT 6/89 WITH i = 8.27%						
ASSET-PVB =		-62270								
NORMAL COST MEAN =		62655								
STANDARD DEVIATION =		37206								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.4
Normal Costs for 7/79 to 6/89

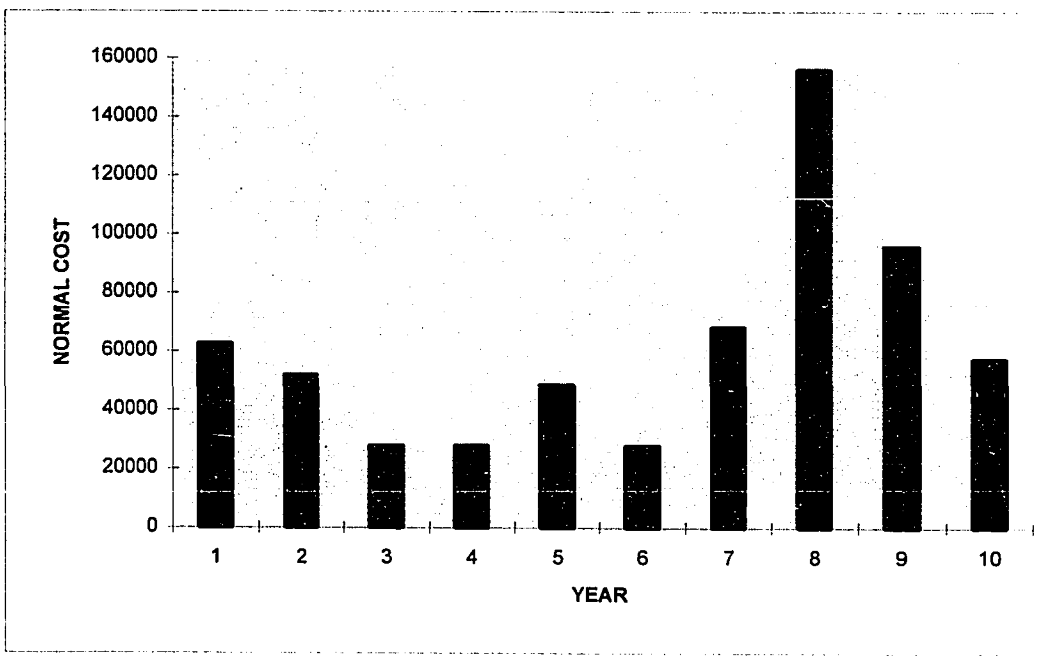


TABLE 5.5
Results for Valuation from 1/80 to 12/89

VALUATION PERIOD :	1/80-12/89									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/80-12/80	1/81-12/81	1/82-12/82	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88	1/89-12/89
INTEREST RATE i(n)	10.60	12.14	14.22	10.63	11.75	11.45	9.40	7.39	8.83	8.93
LIFE ANNUITY FACTOR	7.7830	7.1262	6.3942	7.7691	7.2822	7.4067	8.3806	9.5986	8.6958	8.6389
NORMAL COST *	51481	41665	29015	56932	45018	48155	83499	142742	78138	69948
ASSET **	57296	112260	159294	240394	320774	408727	530602	731101	881738	1032090

PVB =	1111190	EVALUATED AT 12/89 WITH i = 7.90%
ASSET-PVB =	-79100	
NORMAL COST MEAN =	64660	
STANDARD DEVIATION =	30581	

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.5
Normal Costs for 1/80 to 12/89

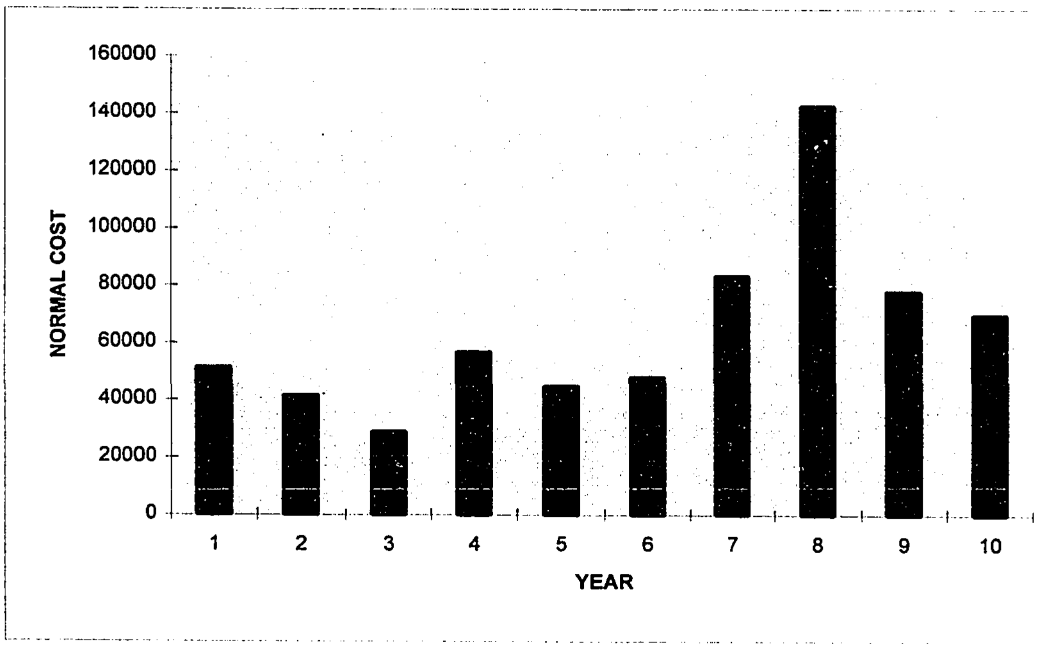


TABLE 5.6
Results for Valuation from 7/80 to 6/90

VALUATION PERIOD :	7/80-6/90									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/80-6/81	7/81-6/82	7/82-6/83	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89	7/89-6/90
INTEREST RATE i(n)	10.24	13.59	13.55	11.40	13.21	10.50	7.27	8.64	9.14	8.08
LIFE ANNUITY FACTOR	7.9536	6.5997	6.6132	7.4279	6.7301	7.8297	9.6816	8.8058	8.5217	9.1455
NORMAL COST *	53702	33544	33722	50627	33691	66111	133195	91591	71359	151132
ASSET **	60248	106798	156290	232030	296893	396417	570603	721760	864286	1099540

PVB = 1069490 EVALUATED AT 6/90 WITH i = 8.46%

ASSET-PVB = 30050

NORMAL COST MEAN = 71867

STANDARD DEVIATION = 39533

* normal cost is deposited at the beginning of the year

** asset is as of the end of the year

FIGURE 5.6
Normal Costs for 7/80 to 6/90

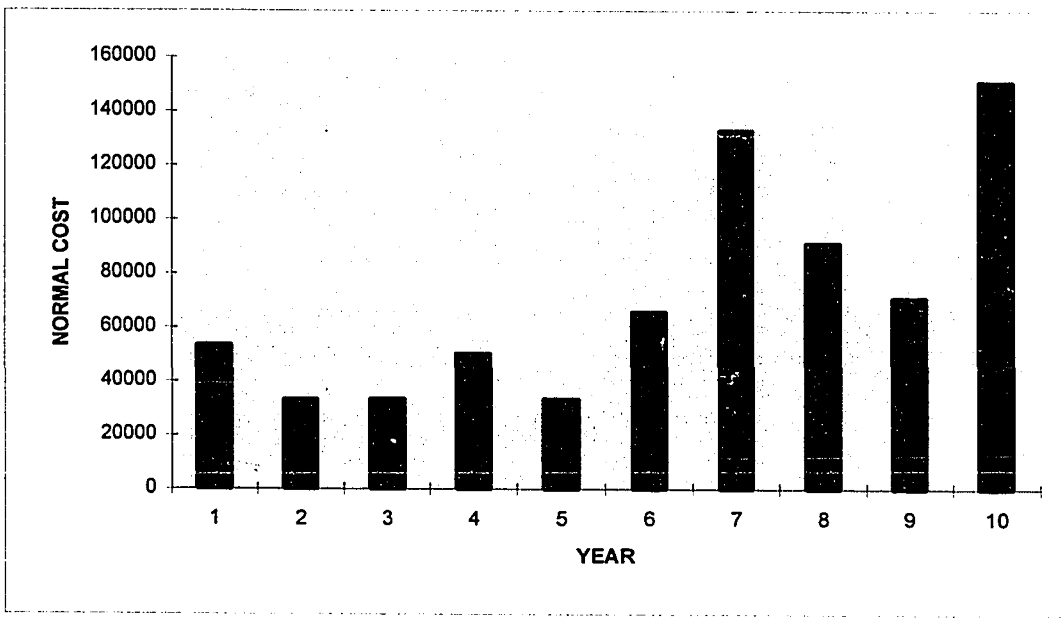


TABLE 5.7
Results for Valuation from 1/81 to 12/90

VALUATION PERIOD :	1/81-12/90									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/81-12/81	1/82-12/82	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88	1/89-12/89	1/90-12/90
INTEREST RATE i(n)	12.14	14.22	10.63	11.75	11.45	9.40	7.39	8.83	8.93	8.62
LIFE ANNUITY FACTOR	7.1262	6.3942	7.7691	7.2822	7.4067	8.3806	9.5986	8.6958	8.6389	9.0337
NORMAL COST *	43161	32638	56074	46082	48766	77257	122498	78101	73842	127375
ASSET **	48962	92008	164633	236822	316395	424343	593746	732038	873961	1087530
PVB =		1085520		EVALUATED AT 12/90 WITH i = 8.24%						
ASSET-PVB =		2010								
NORMAL COST MEAN =		70579								
STANDARD DEVIATION =		30839								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.7
Normal Costs for 1/81 to 12/90

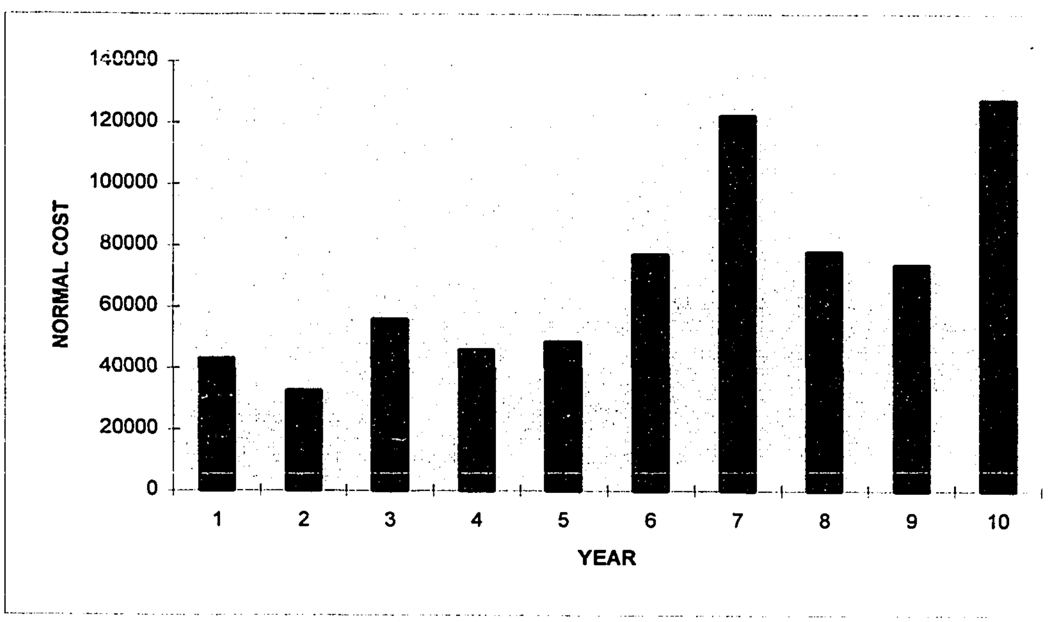


TABLE 5.8
Results for Valuation from 7/81 to 6/91

VALUATION PERIOD :	7/81-6/91									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/81-6/82	7/82-6/83	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89	7/89-6/90	7/90-6/91
INTEREST RATE i(n)	13.59	13.55	11.40	13.21	10.50	7.27	8.64	9.14	8.08	8.50
LIFE ANNUITY FACTOR	6.5998	6.6132	7.4280	6.7302	7.8297	9.6817	8.8058	8.5217	9.1455	8.8885
NORMAL COST *	36782	36953	50979	36862	63508	117692	85741	71810	114122	80061
ASSET **	41882	87683	155490	214918	304053	454387	588714	719793	883695	1066270
PVB =		1068770		EVALUATED AT 6/91 WITH i = 8.47%						
ASSET-PVB =		-2500								
NORMAL COST MEAN =		69451								
STANDARD DEVIATION =		28754								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.8
Normal Costs for 7/81 to 6/91

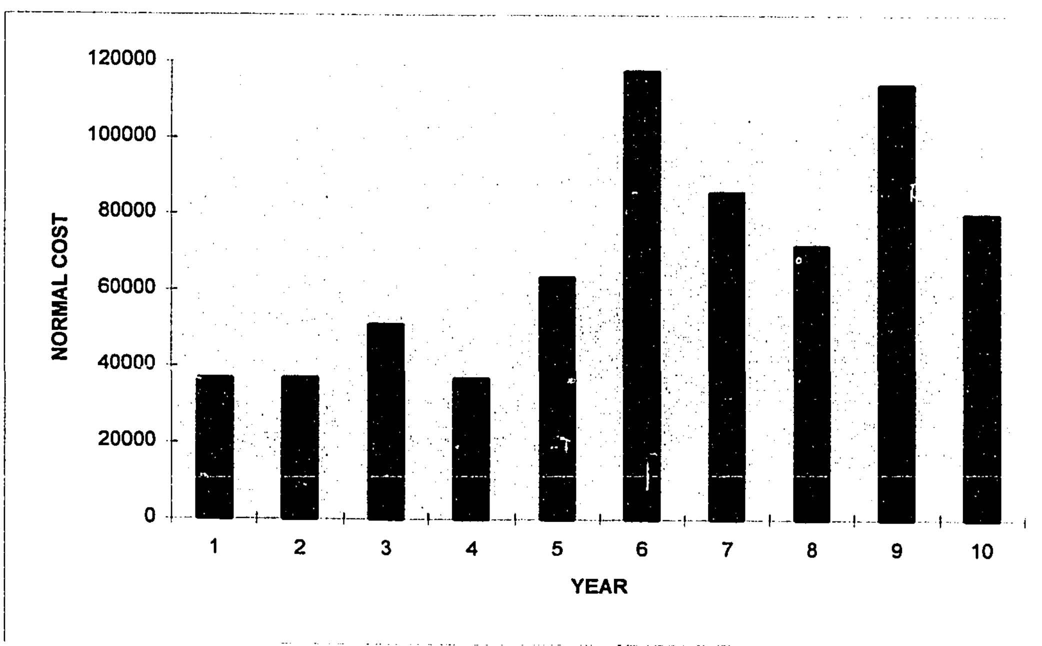


TABLE 5.9
Results for Valuation from 1/82 to 12/91

VALUATION PERIOD :	1/82-12/91									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/82-12/82	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88/12/88	1/89-12/89	1/90-12/90	1/91-12/91
INTEREST RATE i(n)	14.22	10.63	11.75	11.45	9.40	7.39	8.83	8.93	8.26	8.27
LIFE ANNUITY FACTOR	6.3942	7.7692	7.2822	7.4068	8.3807	9.5987	8.6958	8.6389	9.0337	9.0276
NORMAL COST *	34370	54198	45707	48034	71721	108250	74285	71370	99505	95833
ASSET **	38753	103340	167514	238800	334731	480977	605007	733519	904731	1081970
PVB =		1126810		EVALUATED AT 12/91 WITH i = 7.70%						
ASSET-PVB =		-44840								
NORMAL COST MEAN =		70327								
STANDARD DEVIATION =		23696								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.9
Normal Costs for 1/82 to 12/91

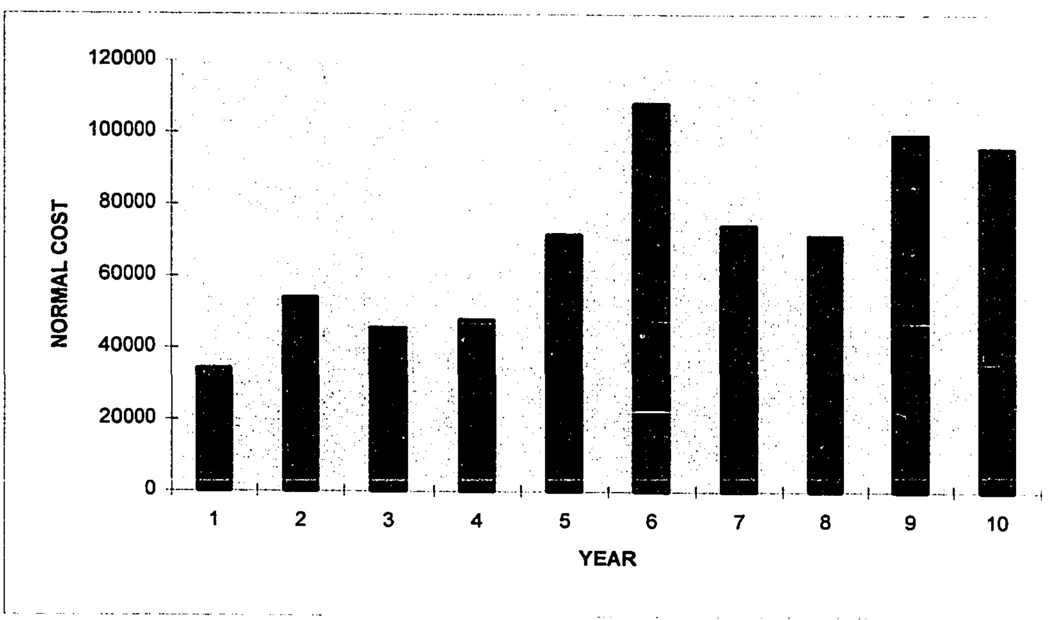


TABLE 5.10
Results for Valuation from 7/82 to 6/92

VALUATION PERIOD :	7/82-6/92									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/82-6/83	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89	7/89-6/90	7/90-6/91	7/91-6/92
INTEREST RATE i(n)	13.55	11.40	13.21	10.50	7.27	8.64	9.14	8.08	8.50	8.45
LIFE ANNUITY FACTOR	6.6132	7.4280	6.7301	7.8297	9.6817	8.8058	8.5217	9.1455	8.8885	8.9184
NORMAL COST *	36942	48780	36809	59212	104522	78559	67886	97313	79343	83406
ASSET **	41088	100774	153723	232534	363144	481435	598611	753579	903418	1065090

PVB =	1115830	EVALUATED AT 6/92 WITH i = 7.84%
ASSET-PVB =	-50740	
NORMAL COST MEAN =	69277	
STANDARD DEVIATION =	22447	

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.10
Normal Costs for 7/82 to 6/92

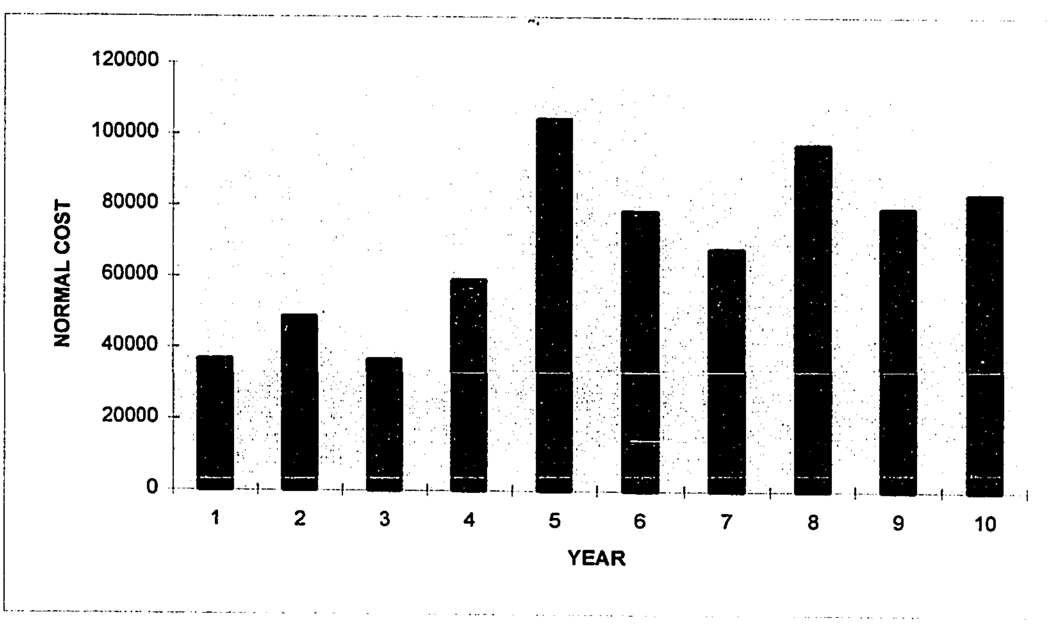


TABLE 5.11
Results for Valuation from 1/83 to 12/92

VALUATION PERIOD :	1/83-12/92									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/83-12/83	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88	1/89-12/89	1/90-12/90	1/91-12/91	1/92-12/92
INTEREST RATE i(n)	10.63	11.75	11.45	9.40	7.39	8.83	8.93	8.26	8.27	7.58
LIFE ANNUITY FACTOR	7.7692	7.2822	7.4068	8.3807	9.5987	8.6958	8.6389	9.0337	9.0276	9.4698
NORMAL COST *	51301	43999	46035	66155	96693	69185	66963	86398	84719	141604
ASSET **	57035	113552	176802	261899	389350	499615	614444	761171	914708	1137290
PVB =		1147730		EVALUATED AT 12/92 WITH i =7.44%						
ASSET-PVB =		-10440								
NORMAL COST MEAN =		75305								
STANDARD DEVIATION =		27692								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.11
Normal Costs for 1/83 to 12/92

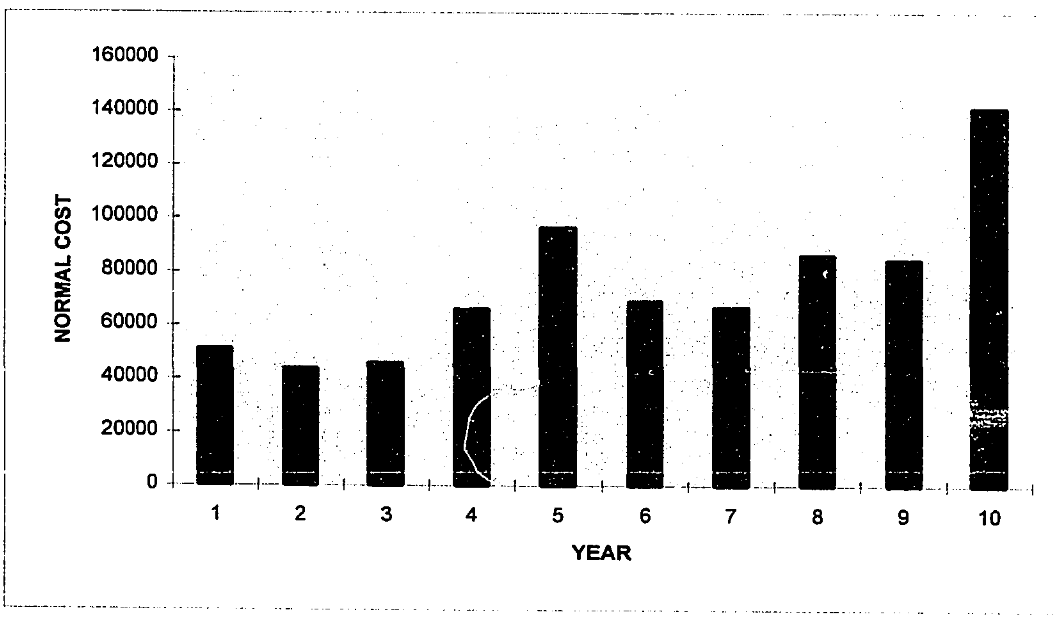


TABLE 5.12
Results for Valuation from 7/83 to 6/93

VALUATION PERIOD :	7/83-6/93									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/83-6/84	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89	7/89-6/90	7/90-6/91	7/91-6/92	7/92-6/93
INTEREST RATE i(n)	11.40	13.21	10.50	7.27	8.64	9.14	8.08	8.50	8.45	7.60
LIFE ANNUITY FACTOR	7.4280	6.7302	7.8297	9.6817	8.8058	8.5217	9.1455	8.8885	8.9184	9.4565
NORMAL COST *	46936	36735	55766	94449	72642	64003	86730	74278	76416	148570
ASSET **	52632	99851	169941	284853	389653	494363	629234	763056	906054	1130850

PVB =	1201470	EVALUATED AT 6/93 WITH i = 6.81%
ASSET-PVB =	-70620	
NORMAL COST MEAN =	75653	
STANDARD DEVIATION =	29453	

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.12
Normal Costs for 7/83 to 6/93

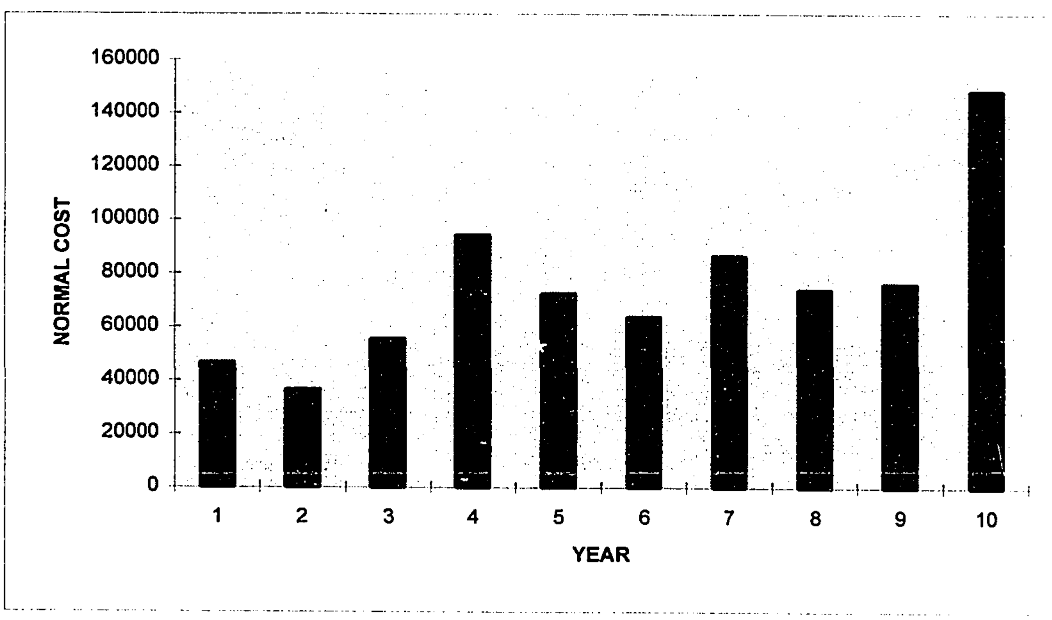


TABLE 5.13
Results for Valuation from 1/84 to 12/93

VALUATION PERIOD :	1/84-12/93									
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/84-12/84	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88	1/89-12/89	1/90-12/90	1/91-12/91	1/92-12/92	1/93-12/93
INTEREST RATE i(n)	11.75	11.45	9.40	7.39	8.83	8.93	8.26	8.27	7.58	7.34
LIFE ANNUITY FACTOR	7.2822	7.4068	8.3807	9.5987	8.6958	8.6389	9.0337	9.0276	9.4698	9.6331
NORMAL COST *	45102	46886	64035	89972	67027	65243	80145	79126	109308	129158
ASSET **	50691	108103	185558	299164	398998	503460	633842	770972	947766	1147970
PVB =		1253180		EVALUATED AT12/93 WITH i = 6.25%						
ASSET-PVB =		-105210								
NORMAL COST MEAN =		77600								
STANDARD DEVIATION =		25047								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.13
Normal Costs for 1/84 to 12/93

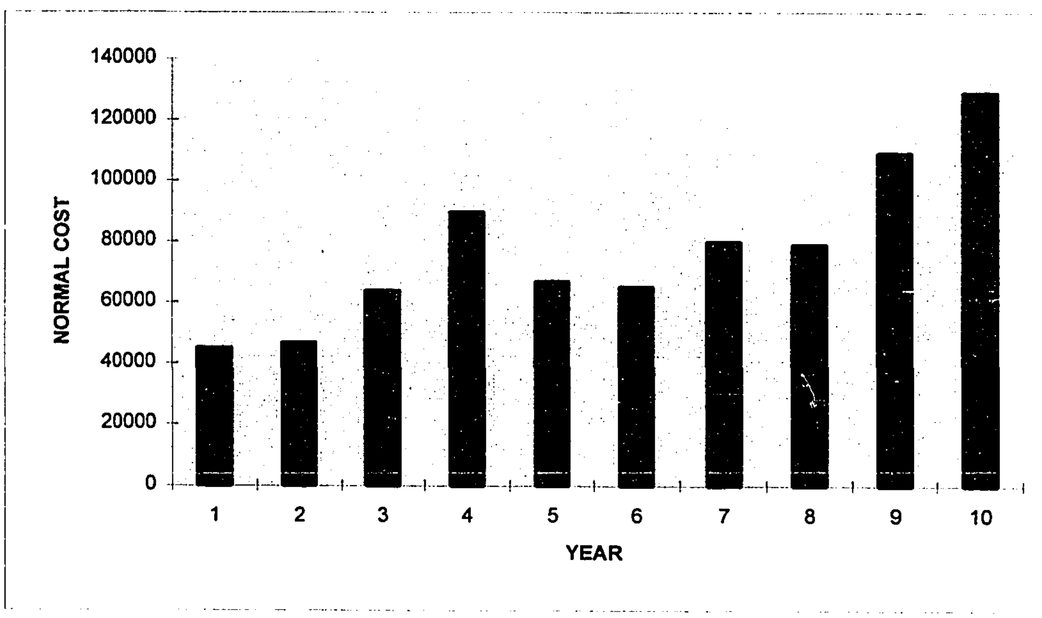


TABLE 5.14
Results for Valuation from 7/84 to 6/94

VALUATION PERIOD :		7/84-6/94								
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	7/84-6/85	7/85-6/86	7/86-6/87	7/87-6/88	7/88-6/89	7/89-6/90	7/90-6/91	7/91-6/92	7/92-6/93	7/93-6/94
INTEREST RATE i(n)	13.21	10.50	7.27	8.64	9.14	8.08	8.50	8.45	7.60	6.63
LIFE ANNUITY FACTOR	6.7302	7.8297	9.6817	8.8058	8.5217	9.1455	8.8885	8.9184	9.4565	10.1472
NORMAL COST *	38336	54446	87783	69123	61920	80418	70835	72313	110335	200933
ASSET **	42834	106234	209033	303176	397856	517897	638561	767257	941024	1217240
PVB =		1151010		EVALUATED AT 6/94 WITH i = 7.40%						
ASSET-PVB =		66230								
NORMAL COST MEAN =		84644								
STANDARD DEVIATION =		42857								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.14
Normal Costs for 7/84 to 6/94

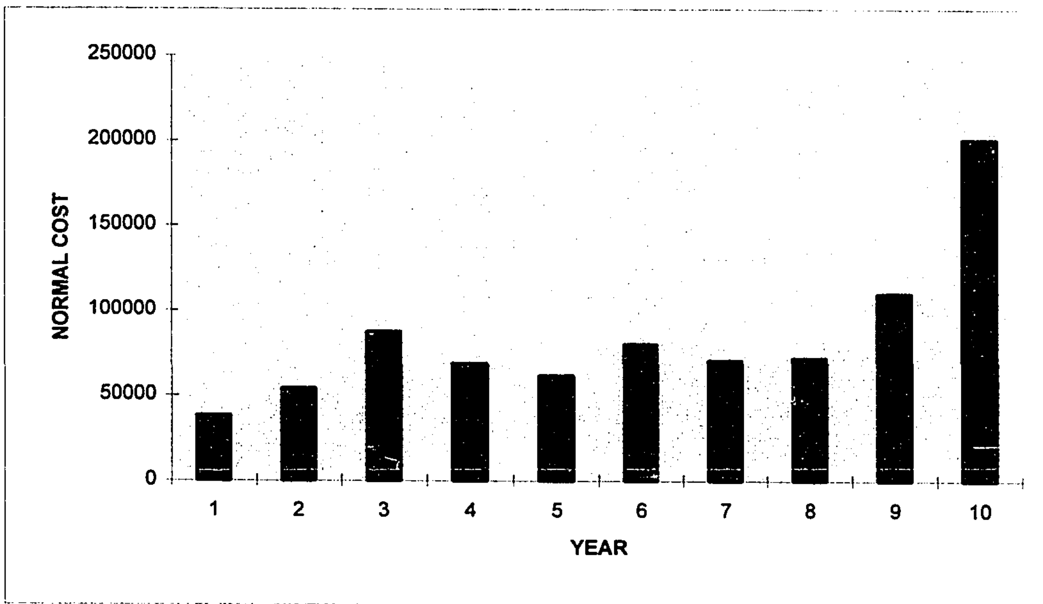


TABLE 5.15
Results for Valuation from 1/85 to 12/94

VALUATION PERIOD :		1/85-12/94								
YEAR (n)	1	2	3	4	5	6	7	8	9	10
VALUATION YEAR	1/85-12/85	1/86-12/86	1/87-12/87	1/88-12/88	1/89-12/89	1/90-12/90	1/91-12/91	1/92-12/92	1/93-12/93	1/94-12/94
INTEREST RATE i(n)	11.45	9.40	7.39	8.83	8.93	8.26	8.27	7.58	7.34	6.29
LIFE ANNUITY FACTOR	7.4068	8.3807	9.5987	8.6958	8.6389	9.0337	9.0276	9.4698	9.6331	10.4113
NORMAL COST *	46668	61413	83779	64214	62733	74802	74108	95122	105765	210979
ASSET **	51703	121934	223359	313337	407840	524188	646971	798985	964440	1262030
PVB =		1113510		EVALUATED AT 12/94 WITH i =7.87%						
ASSET-PVB =		148520								
NORMAL COST MEAN =		87958								
STANDARD DEVIATION =		44153								

* normal cost is deposited at the beginning of the year
** asset is as of the end of the year

FIGURE 5.15
Normal Costs for 1/85 to 12/94

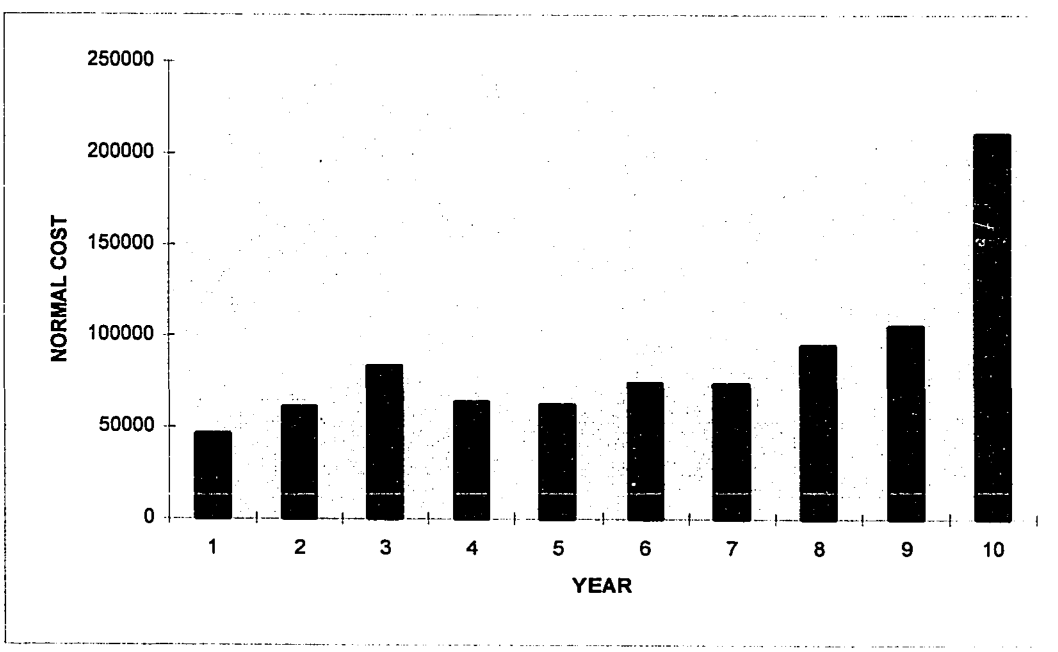


TABLE 5.16
Difference between Asset and Present Value of Benefit

Valuation Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Valuation Years	1/78-12/87	7/78-6/88	1/79-12/88	7/79-6/89	1/80-12/89	7/80-6/90	1/81-12/90	7/81-6/91	1/82-12/91	7/82-6/92	1/83-12/92	7/83-6/93	1/84-12/93	7/84-6/94	1/85-12/94
Asset-PVB	140650	28210	13460	-62270	-79100	30050	2010	-2500	-44840	-50740	-10440	-73620	-105210	66230	148520

FIGURE 5.16
Difference between Asset and Present Value of Benefit