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Inductive reasoning in the algebra classroom

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INDUCTIVE REASONING IN THE ALGEBRA CLASSROOM

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ABSTRACT

INDUCTIVE REASONING IN THE ALGEBRA CLASSROOM

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This study compared the effects of two teaching methods for an 8th grade algebra unit on linear functions. The experimental method involved designing and implementing activities based on inductive reasoning, multiple representations, and guided discovery, whereas the control method involved traditional teaching. The goal was to improve students' achievement in linear functions. The researcher identified three mathematical truths relevant to linear functions and two representational translation abilities, using them as measures of the students' achievement on a unit exam. All 29 students in the study mastered the concepts to varying degrees. The results of the comparison were only slightly in favor of the experimental group. The experimental group appeared to participate and engage more fully in mathematical sense-making with the activity-based instruction than they had in prior units with traditional instruction. Thus, this method appears to hold promise of future success, particularly with revisions in the activities and preparatory instruction.

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" *And my success (in my task) can only come from Allah.*

In Him I trust and unto Him I turn." (Quran 11:88)

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I would like to dedicate this thesis to the loving memory of my father,

Mohammed Mahmoud Ibrahim,

who always encouraged me to pursue graduate studies and

believed that I can do it!

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Chapter 1

Introduction

As a math teacher, the researcher has observed over the last ten years, the movement to make algebra accessible for all students in high school. And in the last few years, algebra has replaced pre-algebra for the students' last year of middle school. As more students and younger ones are required to pass algebra, teachers are being faced with the challenges of making this abstract material more comprehensible to them.

To help the students better understand algebraic concepts, researchers and mathematics educators introduced the pattern-based approach to teaching algebra. This method starts by introducing a pattern, then students work on a verbal description of the pattern and its *n*th term, and then they derive the algebraic expression for the pattern (Stacey & MacGregor, 2001). This method uses inductive reasoning, the discovery method, and multiple representations. These elements of the pattern-based approach to teaching algebra form the main components in the design of this research study: a teaching experiment in an 8th grade algebra class.

Students' Difficulty with Algebra

Studies indicate that algebra is a major stumbling block for students in secondary school (Herscovics, 1989). Many people, even those who did well in mathematics and got good grades, recall that "algebra was the point at which mathematics stopped having much connection with the real world" (Wagner & Parker, 1993, p. 119). There are three major areas where students experience difficulty in their first introduction to algebra through linear functions: (1) generalizing to the *n*th term, (2) relational understanding of the concepts, and (3) algebraic notation (Johari, 2003; Kuchemann, 1981; Pegg & Redden, 1990; Stacey & MacGregor, 2001).

Generalization

One area where students experience difficulty is generalization of patterns. They are usually successful with generalizations of linear patterns in tabular and visual form, but they have major difficulty in generalizing to the *n*th term and using symbols to express patterns (Becker & Rivera, 2004). This skill is an essential part of inductive reasoning. Students do not get enough practice during their school years in inductive reasoning, and therefore have difficulty applying it in different situations. O'Daffer and Thornquist (1993) observed that "students often make inductive inferences from limited samples" (p. 47). This leads them to incorrect conclusions. To improve the students' generalization skills, the activities planned for this study make extensive use of inductive reasoning.

Relational Understanding

Students exhibit a difficulty and/or lack of making connections of the different algebraic concepts. Sometimes algebra students learn, memorize, and repeat procedures without fully understanding the mathematical truths underlying them. Crawford and Scott (2000) remarked, "We have observed that many students can calculate slopes and write equations for lines without understanding the concept of slope" (p. 116). Relational understanding involves knowing what to do in a certain situation and the reasons why. Karsenty (2002) notes that understanding the link between different representations of one situation can be regarded as relational understanding. Working with slopes and other concepts of algebra in various representations, namely graphs, tables, and equations, should improve students' comprehension of those concepts. To address this lack of relational understanding, one of the main components of the proposed research is using multiple representations.

Algebraic Notation

Algebraic notation can cause difficulty and confusion for students. For instance, literal symbols are used in many different ways as variables, unknowns, constants, arguments, parameters, generalized numbers, names, placeholders, or indeterminates. A novice learner cannot differentiate between those uses, and this causes confusion, and misunderstandings of concepts. Kuchemann (1981) classified six categories of children's interpretation of letters: letter evaluated, letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalized number, and letter used as a variable. Very few students are able to reach this last category. "Most students will need extensive experience in interpreting relationships among quantities in a variety of problem contexts before they can work meaningfully with variables and symbolic expressions" (National Council of Teachers of Mathematics (NCTM), 2000, p. 225).

Research Goals

This study sought to identify and describe the comparative effects of two different teaching methods for a unit on linear functions in an 8^{th} grade algebra classroom. The experimental method involved designing and using learning activities based on multiple representations, inductive reasoning, and guided discovery, whereas the control method involved traditional teaching strategies. The goal for both groups was to improve the students' comprehension of and achievement in linear functions.

In particular, the researcher's goal was for students to achieve comprehension of the following mathematical truths, as these seem to be significant indicators of understanding of linear function concepts:

> *Mathematical Truth 1:* Given a linear relationship, it has a unique slope and a unique intercept; and we can find them from the equation, from a table of values, from a graph or from a verbal description that contains sufficient information.
>
> *Mathematical Truth 2:* Given a linear relationship, its ordered pairs can be expressed using an equation, a table of values, a graph, or a verbal description. Specifically:
>
> - Points on the graph satisfy the equation
> - Points on a table have constant rise over run
>
> *Mathematical Truth 3:* A linear relationship has a unique equation in standard form and a unique equation in slope-intercept form. A standard form equation can only be derived from another equation of the line.

In addition, the researcher aimed for students to achieve the ability to make the following representation translations, which would seem to be particularly salient in the understanding of linear functions.

Translation Ability 1: From any representation to a graph

Translation Ability 2: From any representation to an equation

Previous Research

This section briefly reports the findings of previous research about the three main design factors of this study: (1) inductive reasoning, (2) multiple representations, and (3) discovery learning.

Inductive Reasoning

Working with generalization is an essential part of algebra. As early as 1934, a Mathematical Association report (as cited in Pegg & Redden, 1990) stated strongly that any approach taken to instruct algebra should bring out the major reasons for learning the subject, namely, that "it affords a compact symbolism … (which) compels, or at least conduces towards an accurate analysis. But more than that – a point of the highest importance – the symbolization involves, or brings to light, generalization" (p. 6). Stacey and MacGregor (2001) also reported that as educators and researchers are identifying a definition and shared understanding of algebra and algebraic thinking, mathematics educators have come to consider working with generality as one of algebra's characteristics.

Using patterns and generalizing are essential skills that the algebra student needs to acquire. The first NCTM algebra standard for grades 6-8 addresses patterns, relations and functions. It advises that:

> The study of patterns and relationships in the middle grades should focus on patterns that relate to linear functions, which arise when there is a constant rate of change. Students should solve problems in which they use tables, graphs, words, and symbolic expressions to represent and examine functions and patterns of change. (NCTM, 2000, p. 223)

This standard stresses the importance of students' learning of patterns and representations of linear functions.

To be able to generalize and use patterns, the algebra student needs to be skilled in inductive reasoning. According to Klauer (1989), fostering inductive reasoning skills using inductive teaching methods has been successful. Algebra is an example of a subject matter for which a substantial portion is governed by regularities, i. e. rules and laws. Klauer infers that "there is a direct transfer effect of a training to reason inductively on acquiring subject matter which demands inductive reasoning" (Klauer, 1999, p. 138).

Acquiring inductive reasoning skills has far reaching effects on the learners. Inductive reasoning plays a key role in learning and transfer (Csapo, 1997). Induction can be used to acquire new knowledge as well as to make the acquired knowledge more readily applicable in new contexts. Inductive reasoning ability affects to a high degree problem solving, concept formation, critical thinking and creativity. The researcher proposes that training students using inductive reasoning along with multiple representations can be beneficial in learning algebra.

Multiple Representations

Translating among different representations should be a part of algebra instruction. "The power of algebra lies in its capacity to develop and communicate insight by representing situations in alternative ways" (Thornton, 2001, p. 392). To achieve conceptual competence of linear functions, students need to know much more than procedures; they need to develop understanding of the connections between representations (Chiu, Kessel, Moschkovich, and Munoz-Nunez, 2001). To improve algebra instruction, Choike (2000) suggests that the teacher emphasizes multiple representations of mathematics: verbal, numerical, visual and algebraic. The students need to learn to translate back and forth between any two representations. As the students make connections among the different representations, their understanding of the concepts deepens.

Guided Discovery

Algebra instruction needs to be modified as well. The teacher needs to involve the students in guided explorations to help them learn by discovery (Choike, 2000). Guided-explorations "actively involve students in this process of organizing their arithmetic to find answers to questions" (p. 559). "Guided discovery is just one of many successful teaching methods" (Gerver & Sgroi, 2003, p. 12). Mathematics was developed through discovery, so it makes sense to teach it using discovery (Gerver & Sgroi). To help students to "discover" teachers need to prepare lessons that gradually guide the students to learn new skills. The benefit of this method is the satisfaction from and the appreciation for the discovery (Gerver & Sgroi, 2003).

In order to improve algebra instruction in the classroom, innovative methods need to be employed that empower the students and motivate them. Algebraic concepts need to be practiced in multiple representations. The learning activities employed in this study were based on principles known to have been successful in other educational settings. They relied on guided discovery, inductive reasoning, and multiple representations.

Relevance of this Research

To address the students' difficulty with algebra, the current available research mainly offers activities that are not directly related to the topics in the curriculum and are time consuming. It becomes a challenge for the teacher to have time to be able to accommodate more objectives in the limited time available and the long curriculum that is expected to be covered.

The activities that this study proposes focus explicitly on the concepts of linear functions. The activities guide the students to find the slope and vertical intercept of a line, given its graph, a table of values, the equation or a verbal description. In addition, the students will learn to write the equation of a line when given sufficient information in more than one form. They will also learn to graph a line on a coordinate plane when given its equation or the slope and any point. The activities are designed to prepare the students to translate from any representation to another.

This experimental teaching method was expected to help the students overcome some of the above stated difficulties. The activities were designed to train the students to translate among different representations. This addressed the relational understanding of the students and helped to ensure their comprehension of the underlying mathematical truths. The inductive reasoning component of the learning activities addressed the students' difficulties with patterns and generalizations. The activities were also designed to allow the students to "discover" mathematical truths. This method of instruction was designed to actively engage the students, and ensure that they become actively involved in the learning process. The algebraic notation was presented gradually as the need arose.

Research Questions

1. How do the two teaching methods (symbolic versus patterns-and-functions) affect 8th grade algebra students' understanding of the three mathematical truths related to linear functions? Does one method appear to be "better" than the other? If so, in what ways? Was any mathematical truth better understood by the students than the others? Was there any significant difference between the students' performance on the quizzes and the unit exam?

2. How do the two teaching methods (symbolic versus patterns-and-functions) affect students' abilities to translate among various representations of linear situations? Does one method appear to be "better" than the other? If so, in what ways?

Did the students master one representation more than the other? How do students' representational abilities relate to their understanding of the three mathematical truths? Is there a strong correlation?

Chapter 2

Literature Review

This chapter includes five main sections. The first one presents definitions of algebra and discusses its significance. The second section describes the impediments to learning algebra. The third section describes three approaches used to teach algebra, namely, the symbolic approach, the patterns approach, and the functions approach. The fourth section presents suggestions to improve learning of algebra: multiple instructional strategies, oral and written communication, guided discovery learning, and conceptual understanding. The fifth section discusses inductive reasoning skills and their effect on learning, intelligence and algebra.

Algebra

Definitions of Algebra

There are several different definitions of the term "algebra," each focusing on a different aspect of this area of mathematics. One classical definition of algebra was given by Wagner and Parker (1993): "Algebra is a language for describing actions on, and relationships among, quantities" (p. 122). Therefore, one of the most obvious functions of algebraic notation is to demonstrate the general properties of numbers and operations on numbers (Stacey & MacGregor, 2001). At higher levels, symbols continue to be used to express general properties of functions, sets, etc. Consequently, mathematics educators consider working with generality as one of the characteristics of algebraic thinking (Stacey & MacGregor). This definition of algebra limits the scope of this strand of mathematics, and makes it appear that the aim of studying algebra is for learning its properties.

The researcher has always thought of and used algebra as a tool for problem solving. This definition has support from research as well. Choike defined algebra as, "the process of organizing the arithmetic needed to find an answer to a question involving quantities that are not yet known" (2000, p. 559). This definition expands the role of algebra and suggests that the goal of using it is to find answers to problems.

Significance of Learning Algebra

Studies indicate that there is a strong correlation between mathematical skills and success in college, regardless of the major (Johari, 2003). Algebra is considered by educators and policymakers as an important gatekeeper course for college preparation and the world of work (Choike, 2000). Concepts in algebra are the basis for many other sciences, and that is the reason that learning algebra is very important. Schliemann and Carraher (2002) conducted a study of the uses of everyday mathematics, and concluded that, "Although people can learn meaningful mathematical ideas in mundane, nonacademic situations, they nonetheless need symbolic systems and representations they are not likely to acquire out of school" (p. 254). Consequently, policymakers require the students to pass algebra in order to be able to graduate from high school. Meanwhile, many students have difficulty with mathematics in general and algebra in particular.

Impediments to Learning Algebra

Studying algebra is completely different than earlier mathematics classes because of the high level of abstraction that is used. Studies point out that students have difficulty working with variables in general and translating from verbal to symbolic representations (Johari, 2003). This lack of understanding of algebra leads students to memorize algebraic rules and procedures which make them unable to apply these rules to problem solving. This section presents three sources of impediments to learning algebra: the subject of mathematics itself, students, and teaching methods (Wagner & Parker,1993).

Impediments Stemming from Algebra

Several aspects of algebra, itself, pose obstacles for many students' understanding. First, the syntax of algebra is a problem for some students (Stacey & MacGregor, 2001). This is caused by the difficulty and confusion that can arise from algebraic notation of symbols and expressions. Literal symbols are used in many different ways as variables, unknowns, constants, arguments, parameters, generalized numbers, names, placeholders, or indeterminates. A novice learner cannot differentiate between those uses, and this causes misunderstandings and confusion of concepts. Wagner and Parker remarked that, "there is evidence that the students' early impressions about variables may impede their construction of a sufficiently general concept" (p. 123). Variable understanding is one of the key elements of successful problem solving (Johari, 2003).

Second, algebraic expressions, which combine numerals, letters, and symbols, provide an economy in notation, but at the expense of possible confusion (Wagner & Parker, 1993). For instance, "many students assume that $-x$ is a negative number" (Matz, 1982). There are actually three uses of the '-' sign: the subtraction operation, the integer sign, and the opposite of an expression. There is a need to alert students to these different uses of one symbol. Moreover, some rules that apply to numerals are different when applied to letters. For example, concatenating numerals, like 37, implies the addition of 30 and 7. However, concatenating letters, like $3a$, implies the multiplication of 3 and a. It takes students a while to internalize these conventions that are in conflict with each other (Chalouh & Herscovics, 1988).

Third, the level of abstraction in algebra can cause difficulties for students. Algebra contains many properties that are highly abstract, such as the field properties of the associative and distributive law (Wagner & Parker, 1993). Misunderstandings of these properties lead to one of the most common errors in simplifying expressions: students would simplify $4+3n$ as $7n$. A major concept of algebra is the variable, but very few students are able to reach the high degree of understanding required to interpret a letter as a variable (Kuchemann, 1981). An important skill in algebra is to write algebraic equations from tables of values or situations, and it is well known that students experience difficulty mastering this skill (Stacey & MacGregor, 2001).

Impediments Stemming from Students

Students create many of their own impediments to learning algebra. For example, they create their own incorrect strategies. "Analyses of students responses to paper-andpencil tasks have shown that students' errors are not random and have suggested that students' errors are associated with erroneous strategies" (Chiu et al., 2001). Students also learn how to manipulate symbols without understanding the underlying concepts. "Research shows that students can work with variables without fully understanding the power and flexibility of literal symbols" (Wagner & Parker, 1993, p. 122). Crawford and Scott (2000) observed that "many students can calculate slope and write equations for lines without understanding the concept of slope" (p. 116).

Another impediment that students create for themselves is a tendency to overgeneralize the rules they learn. A classical example is "the freshmen theorem": $(a+b)^2 = a^2 + b^2$. The students make this mistake by applying the power of a product property: $(ab)^2 = a^2b^2$ to the addition case, where it does not apply. The students make this and similar mistakes because they compare the new situation on the basis of superficial characteristics to a familiar property (Wagner & Parker, 1993). No matter how much effort the teacher exerts, instruction alone cannot cause structural changes. The learner is the one who can alter the mental structure in his mind on his own (Herscovics, 1989). It is difficult to significantly change a learner's existing cognitive structures.

Impediments Stemming from Teaching Strategies

Many impediments to learning algebra are related to teaching techniques that can lead to incomplete mental constructs (Wagner & Parker, 1993). One cause of this is considering an overly narrow range of simple, special cases of a given concept. For instance, when students learn multiplication in second or third grade, they are familiar only with whole numbers. Therefore, multiplication of whole numbers always results in an answer that is larger than both factors. However, once they start learning fractions, and they multiply by a number between zero and one, the product in not larger than the original number anymore (Tall, 1989). Students need to be aware of exceptions to different rules, and the reason for those exceptions. And this is where the role of the teacher is very important. Wagner and Parker (1993) note that:

> Cognitive conflict is not necessarily bad for students; in fact, it is an important stimulus to learning. It is our responsibility as teachers to be aware of possible sources of conflict and alert students to differences, as well as similarities, among the various phenomena they study. (p. 122)

To address some of those issues with algebra, researchers and teachers looked for different means of teaching this subject.

Methods of Teaching Algebra

The NCTM Principles and Standards for School Mathematics (2000) provide a well-rounded approach to teaching and learning algebra. The standards that need to be emphasized for grades 6-8 relating to algebra are the following: algebra, reasoning and proof, and representation. According to the algebra standard, "students in the middle grades should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations" (p. 223).

The specific components of the algebra standard are the following (p. 222):

- Understand patterns, relations and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various concepts.

In the case of reasoning and proof, the standards document recommends that students have various experiences with the following skills (p. 262):

- Examine patterns and structures to detect regularities.
- Formulate generalizations and conjectures about observed regularities.
- Evaluate conjectures.
- Construct and evaluate mathematical arguments.

The representation standard specifies the following skills for the students to master (p. 280):

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Thornton (2001) identified three approaches of algebra instruction: the symbolic approach, the patterns approach, and the functions approach. In the following few paragraphs, a description of each approach is presented along with the research related to it.

The Symbolic Approach

The symbolic approach to algebra instruction is the most traditional and formal one, focusing on manipulation of algebraic expressions. The students learn that letters stand for unknowns, and variables are defined as letters that represent numbers. The students are also introduced to the rules for using the letters (Stacey & MacGregor, 2001) and to formal procedures for simplifying symbolic expressions. In this approach, strategies are more stressed than conception.

This approach is not sufficient for the students to grasp algebraic thinking. "Although the majority of students appear to be able to carry out routine exercises, we are increasingly concerned that many students appear to perform rules in isolation- divorced from reality and without much understanding" (Pegg & Redden, 1990, p. 386). This finding demonstrates that this method emphasizes procedural knowledge, rather than conceptual knowledge.

It is necessary that methods used in the classroom address the way students learn. Kuchemann (1981) concluded from the research on the meaning that students give to letters that "children's methods and their levels of understanding need to be taken far more into account, however difficult this may be in practice" (p. 118). This has not always been the case. "Curriculum designers are often more concerned with how students ought to think instead of how they really do think" (Stacey & MacGregor, 2001, p. 152). To address some of those issues, the patterns approach was introduced.

The Patterns Approach

In the patterns approach, the students are asked to generalize relationships (Thornton, 2001). The letters are introduced as pattern generalizers instead of specific unknown numbers. This approach emphasizes algebraic thinking (Stacey & MacGregor, 2001) and is very well aligned with NCTM algebra standards (2000) for grades 6-8. "A major goal in the middle grades is to develop students' facility with using patterns and functions to represent, model, and analyze a variety of phenomena and relationships in mathematics problems or in the real world" (p. 227).

To introduce this method to first year algebra students, the teacher uses the following steps (Pegg & Redden, 1990). First, the students start examining a number pattern, usually geometric. Then the students describe the pattern in words. This step is of major importance because the students who "could clearly articulate their patterns tend to have greater success at writing the correct rules in symbolic forms" (Becker & Rivera, 2004, p. 3). As students write descriptions of patterns, they start learning about abbreviations and algebra conventions. Students are encouraged to come up with alternative descriptions for the same pattern, which leads them to discover rules of algebra in a spontaneous manner.

A major step in the patterns approach is generalization. Becker and Rivera (2004) identified three types of generalizations based on similarity: The first type is the numerical which is used by most students. Trial and error is the strategy of the students who use this type. The second type of generalization is the figural. The students who use this type use perceptual similarity and focus on relationships among numbers. The third type is the pragmatic, which is the most advanced. The students who use this type combine both numerical and figural strategies. They are usually able to translate among representations with ease and examine sequences of numbers in terms of properties and relationships.

Stacey & MacGregor (2001) established several difficulties that students experience when generating algebraic rules from patterns and tables. Students see many patterns and cannot identify the ones that are more relevant than others. Students can see the rule of a pattern but are not able to write it as an equation. Many students have trouble verbalizing their understanding of the pattern. Some students use one rule for simple calculations and another for larger values of the variables. Teachers need to address those issues to help the students with generalizations. Inductive reasoning training can be helpful.

The Functions Approach

The functions approach to teaching algebra "emphasizes a realistic application of graphs and encourages students to represent situations in words, symbols, graphs, and tables" (Thornton, 2001, p. 391). In this approach, the students generate and interpret graphs. In order to do that, they need to explore many representations.

Multiple representations are vehicles for learning and communicating. "Representation is central to the study of mathematics. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations" (NCTM, 2000, p. 280). Representations are used to understand a mathematical structure by setting up a correspondence between it and a better-understood structure (Cuoco, 2001).

The ability to translate among representations is a sign of relational understanding, which leads to maintenance of the content for long periods of time (Karsenty, 2002). "The four representations (words, symbols, graphs, and tables) lead to twelve possible transitions between representations … all twelve of the possible transitions add to students' understanding of the nature of functions and relationships" (Thornton, 2001, p. 391). Thornton (2001) remarks:

> The ability to represent a relationship graphically, in a table, as a function, and in words, and the thinking required to convert directly from one representation to another in every possible permutation is to promote the development of different insights into the situation being studied. (p. 391)

The use of representation in the classroom supports learners of different styles. Preston and Garner (2003) noticed that students tend to choose their favorite representation, and not what is most useful for them to solve a problem. They also remarked that students would choose one representation to solve problems, and another to communicate their solutions to their classmates. To help students be familiar with all representations, Preston and Garner advise teachers to focus on a certain form and require its use.

Suggestions to Improve Learning Algebra

There are many articles that have been written on ways to improve algebra instruction (Becker and Rivera, 2004; Chiu et al., 2001; Choike, 2000; Crawford and Scott, 2000; Kuchemann, 1981; Pegg and Redden, 1990; Stacey and MacGregor, 2001, Thornton, 2001). Four major suggestions were common: to use multiple instructional strategies in the classroom, to offer opportunities for oral and written communication, to plan guided discovery lessons, and to emphasize conceptual understanding.

Multiple Instructional Strategies

It is obvious that students would benefit from instruction that combines the three approaches that were explained earlier, namely using symbols, patterns, and functions (Thornton, 2001). None of the above approaches by itself is sufficient to lead to deep understanding or to generate powerful algebraic reasoning. Each of these alternative methods leads to new insights into algebraic relationships. Hence, multiple instructional strategies would help the students to develop algebraic concepts. In addition, students have different learning styles, and instruction in the class needs to address the three modes of learning: visual, verbal and symbolic (Crawford & Scott, 2000).

Oral and Written Communication

Teachers need to provide opportunity for the development of effective communication between students and their peers and between students and their teacher (Pegg & Redden, 1990). Research shows that the students who could clearly articulate a description of a pattern verbally also were able to write its rule in symbolic form (Stacey & MacGregor, 2001). Pegg and Redden allocate a major focus of their instruction on clarifying language, both oral and written. "Students were encouraged to talk among themselves about the questions, to record their answers in English sentences written out in full, and finally, to share their answers with the class" (p. 388).

There is a need for teachers to promote the development of language specifically related to mathematics. "The teachers should provide models of language structures that can be used for talking about patterns and relationships and for developing and refining ideas" (Stacey & MacGregor, 2001, p. 148). "To learn algebra, students need to be able to recognize and articulate the processes of arithmetic and the structures of relationships between numbers" (p. 148).

To assess students' internalization of concepts, the teacher needs to provide opportunities for them to communicate their understanding orally and in writing (Crawford & Scott, 2000). To assess students' understanding of patterns, Pegg and Redden (1990) required students to make up a pattern, describe it in words and give it to other students to try to figure it out. If the other students could figure out the pattern, this was a sign that the description was clear.

Discovery Learning in the Math Classroom

There are both philosophical and research-based reasons to encourage discovery learning, that is, to encourage students to discover some of the generalizations of algebra themselves. In an algebra class, students are introduced to many rules, laws and formulas. They spend a considerable amount of time acquiring skills on the use of these rules. There is a benefit for the students to come up with the rules themselves instead of just using the results. Zheng (2002) explains:

> In a sense, whenever teachers teach a rule or formula, they are teaching generalization. However, the method used to teach the rule or formula may fall into two very different kinds of instruction. One way teaches the results of a generalization, and the other teaches how to generalize. The former may emphasize remembering through repeated practice, whereas the latter can result in a better understanding of the discipline of mathematics and can empower the students in problem solving (p. 492).

In Gerver and Sgroi's opinion, "discovery is an inherent part of the way mathematics is developed and should be a part of the way that mathematics is taught in school" (2003, p. 6).

To accomplish this, teachers need to prepare lessons that lead the students on a discovery trail to learn new mathematics. Such a lesson usually includes a storyline that engages the students as well. In this kind of lesson the students sequentially uncover layers of mathematical knowledge and at the same time acquire new skills. The benefit of this method is the satisfaction of discovery (a.k.a. the "Aha" component). The students feel that they are "archaeologists on a mathematical dig" (Gerver & Sgroi, p. 6).

Conceptual Understanding

Teaching algebra needs to focus on concepts rather than strategies. The goal of instruction should be on developing connected conceptual understanding rather than procedural knowledge (Chiu et al., 2001). To fully understand a concept, students need to examine both examples and nonexamples (Crawford & Scott, 2000). For instance, when learning slopes, students can examine the difference between constant rate of change in a linear case and the rate of change that is not constant like the percent increase or decrease. Obviously the graphs of both situations will be different as well as the patterns in the tables of values.

When teachers emphasize conceptual understanding, their main focus is on the big ideas rather than the small details of procedural proficiency. "Algebra essentially involves just a few conceptual themes, or 'big ideas'" (Choike, 2000, p. 556). Planning instruction that focuses on a few important ideas, according to Choike, accomplishes two instructional goals: it helps all students understand how a new topic that is introduced is related to other previously learned materials, and it gives a framework for teaching that depends on conceptual understanding rather than following the textbook one section after another.

To ensure conceptual understanding, the students' methods and their levels of understanding need to be taken into account when planning lessons (Kuchemann, 1981). For instance, in regard to variables, the teacher needs, "to base the teaching of children on the meanings of the letters that these children readily understand" (p. 119). The goal, of course, is to expand their knowledge to interpret a letter as a variable. In regard to generalizing patterns, "teachers need to recognize that their students see many patterns. They need to be given time to discuss why some particular patterns and relationships are more helpful than others they have seen. They have also to be able to pick what is significant and what isn't" (Stacey & MacGregor, 2001, p. 146).

One way to reach conceptual competence in the area of linear functions is for students to work with them using multiple representations. "Conceptual competence in the domain of linear functions includes much more than knowing procedures; it involves understanding the connections between representations (e. g., the graphical and algebraic representations)" (Chiu et al., 2001, p. 220). The goal of the instruction should be on facilitating the development of deep connections among various representations of linear functions, including tables, equations, and graphs (Chiu et al., 2001).

To provide another avenue to conceptual understanding, applications should play a major role. Examining real-world applications provides meaning to the theory being studied and "contributes to an application-based curriculum that builds understanding of the concepts and insight into the importance of mathematics" (Crawford & Scott, 2000, p. 116). Choike (2000) advises of molding the lessons around the interests of students, whenever possible. Teachers can change the setting of an application to make it relevant to the students while using the same information.

Training in Inductive Reasoning Skills

This section reports on research on inductive reasoning skills. It contains five parts. It starts by providing definitions of inductive reasoning. Then it presents the research findings on the effects of improving inductive reasoning skills on learning and intelligence. The third section examines the relation between inductive reasoning and algebra. The fourth section considers the role of inductive reasoning in the guided discovery method. Finally, an example of the use of inductive reasoning and guided discovery is presented in teaching geometry.

Definitions of Inductive Reasoning

Inductive reasoning is a process of mathematical reasoning in which one uses a limited number of instances of a set to form generalizations about other members (O'Daffer, 1993) and find a description that applies to all of them (Tomic, 1995). Inductive reasoning was also described as a rule-finding process that is achieved by searching for similarities as well as differences between the elements being examined (Roth-Van Der Werf, Resing, and Slenders, 2002). In defining inductive reasoning, Polya describes an inductive attitude that "aims at adapting our beliefs to our experience as efficiently as possible" (1954a, p. 7). This attitude makes it possible to make generalizations from observations, and vice versa, given any generalization, find the most concrete example.

Effects of Inductive Reasoning on Learning and Intelligence

Tomic (1995) has attributed several aspects of learning and intelligence to inductive reasoning. By applying inductive reasoning, one is able to make inferences, and this increases the knowledge we possess. Induction also enables us to make predictions about new possibilities and to anticipate results. An element of inductive reasoning that is often used is the principle of analogy. It is used to perform such routine tasks as seriation, classification, and spelling. Induction is also of importance for the acquisition of new knowledge and skills. It builds on the prior knowledge to solve problems encountered in a new domain.

Training students to reason inductively should also improve learning in academic settings. The process of inductive reasoning relies mainly on generalizations, which are critical in the development of children and in human learning in general (Roth-Van Der Werf et al., 2002). Csapo (1997) concluded from research that inductive reasoning plays a key role in learning and transfer. Through improving inductive reasoning, learning in general can be enhanced. Induction can be used to acquire new knowledge as well as to make the acquired knowledge more readily applicable in new contexts. Inductive reasoning ability affects to a high degree problem solving, concept formation, critical thinking, and creativity. When Csapo compared inductive reasoning to academic achievement, there was a high correlation between inductive reasoning and applied science knowledge, and significant (although weaker) correlation between inductive reasoning and school grades.

Reading the literature about inductive reasoning, it becomes very evident that there is a strong relationship between intelligence and inductive reasoning. Inductive reasoning, or one of its components, has often been identified with general intelligence (Csapo, 1997). Klauer et al. (2002) asserts that:

> There is agreement among researchers that inductive reasoning constitutes a central aspect of intellectual functioning. Ever since Spearman (1923), there has been no doubt about the close relationship between inductive reasoning and intelligence. (p. 2)

Klauer et al. further conclude that inductive training does indeed improve intellectual competence as well as intellectual performance.

Inductive Reasoning & Algebra

Algebra is a good application for inductive reasoning training. According to Klauer (1989), fostering inductive reasoning skills using inductive teaching methods has been successful. Klauer et al. assert that "inducing adequate comparison processes in learners would improve the learners' abilities of inductive reasoning" (2002, p. 3). Algebra is an example of a subject matter in which a substantial proportion is governed by regularities, i. e. rules and laws. Klauer infers that "there is a direct transfer effect of a training to reason inductively on acquiring subject matter which demands inductive reasoning. For these kinds of subject matter a definitely higher transfer effect should be expected because of the joined direct and indirect transfer" (Klauer, 1999, p. 138).

Inductive Reasoning in the Guided-Discovery Method

Inductive reasoning plays a major role in mathematical discovery (Polya, 1954b). The discovery method usually includes a step that requires making a *conjecture* using inductive reasoning. Polya defined a conjecture as "a simple description of the facts within the limits of our experimental material, and a certain hope that this description may apply beyond the limits of our experimental material" (1954a, p. 68). Polya described the method of obtaining a conjecture as follows:

> We collected relevant observations, examined and compared them, noticed fragmentary regularities, hesitated, blundered, and eventually succeeded in combining the scattered details into an apparently meaningful whole. (p. 68)

Adaptations from a Geometry Book Using The Inductive Approach

The researcher has applied the discovery method while teaching geometry using Michael Serra's textbook. This research is an adaptation of this model to algebra. Michael Serra's textbook is composed entirely of investigations, most of which are inductive, that lead students to make conjectures. This text is very different from other geometry texts. There are no theorems in the textbook; instead the author has incomplete conjectures that are left for the students to complete by filling in blanks upon completing an investigation. At the beginning of the course, the students learn basic constructions, and use them for the investigations.

Learning in such a class is interesting and engaging for the students. The activities presented in the text make the students responsible for the learning. For example to learn the relationship between the external angle and the remote interior angles, the students perform the following steps. First, the students draw a triangle and extend every segment on one side beyond the vertex. In the next step, the students measure all six resulting angles then tabulate the results including sums of every two interior angles. The last step involves looking for patterns and making a conjecture. In this kind of class, the students are in charge.

The role of the teacher changes when teaching in such an investigative, guided discovery curriculum. The teacher needs to make sure the students are on task, follow the directions correctly, and finally make the correct conjecture. It is important for the teacher also to provide examples on how to use this conjecture. Sometimes it is not clear for the students how to apply the new knowledge they discovered. The teacher in this case has to do a lot of work preparing for the class, but the students do all the work during class.

Conclusion

Research has shown there are several innovative approaches to teaching and modes of learning that empower students, motivate them to learn, and lead to increased understanding of algebra. The researcher capitalized on these results by designing and implementing an experimental unit on linear functions in an 8^{th} grade algebra class that utilized the functions and patterns approaches to teaching and the guided discovery with inductive reasoning mode of learning. The hope was to improve students' inductive reasoning skills, and in turn their performance in algebra and in school in general.

Chapter 3

Research Method

The research method chapter consists of five sections. The first section explains the research design of this study. The second section introduces the inductive reasoning activities that were designed by the researcher, outlining in detail the overall structure for the inductive reasoning activities. It also delineates the overall structure of the experimental group class sessions. The third section describes the sampling procedures and the measures taken for the protection of the human subjects. Section four includes the measures used to assess the results of the experiment, namely, the data-collection procedures, the assessment item types, and the data analysis methods. The last section of the chapter is a time line of the research.

Research Design

This study involved an experiment comparing two groups of 8^{th} grade algebra I students from a private school. A class of 29 students was divided into an experimental group and a control group. Each group studied the same material: a 17-day unit on linear functions. The experimental group received instruction based on a combination of the patterns and functions approaches to teaching algebra. In particular, this group engaged in inductive reasoning activities. The control group received instruction based on the symbolic approach to teaching.

Two instructors team-taught the intact class of 29 students for the 12 weeks leading up to the experimental unit on linear functions. Both instructors, one of whom is the author of this research, were always present in the classroom, and they alternated in teaching the intact class. Therefore, the students were familiar with both instructors and their styles of teaching. At the time of the training, the students were divided into two groups. Each group was given instruction in a different classroom by one of the teachers. (The students of this class were used to being divided into levels during Arabic language instruction. Therefore, this procedure was very common to them.) The researcher of this study taught the experimental group, and the other teacher taught the control group.

The unit that was covered for this research was an introduction to linear functions. The students were introduced to the concepts of slope, horizontal, and vertical intercepts of a line. They learned to calculate slope given two points or a graph of a line. Graphing and writing linear equations was the ultimate goal of this unit. Students were introduced to the standard and the slope-intercept forms of linear equations. They learned to graph lines given two points, intercepts, slope and a point, or any linear equation. The students also examined many linear situations in verbal form. (See Appendix A for the unit plan.)

The instruction for the patterns-and-functions approach group was mainly discovery learning where students worked in small groups of two or three members. The activities led the students to learn the concepts and provided opportunities for application. The students in this group had a chance to practice and develop their inductive reasoning skills. They also were required to explain their findings in writing. They had practiced those skills only occasionally prior to the experimental unit.

The symbolic approach group mainly received direct instruction, and the students worked individually to practice the taught material. The teacher presented each new lesson by introducing the concepts, going through examples to demonstrate the skills, and giving the students time for independent applications. The students in this group were accustomed to this type of instruction.

Homework was assigned daily to both groups. Similar homework was assigned whenever possible. It was hard to expect the two groups to cover the material at the same speed. However, the types of questions that the students practiced at home were very similar.

Inductive Reasoning Activities

This section explains in detail the inductive reasoning activities designed by the researcher for the experimental group. It contains three parts: the first part gives an overview of the design of all the activities. The second part is an example of how the activities were designed to reach Mathematical Truth 1. This is presented as an illustration for all the activities. The last part of this section is an overview of the plan of each class session for the experimental group and a description of how the activities were used.

Overall Structure for Inductive Reasoning Activities

The experimental group completed a collection of activities based on inductive reasoning and guided discovery. Each activity included any prerequisite skill or knowledge that the students needed to successfully reach the objective of the activity, namely, each activity contained tables, graphs and essay questions. It has been established by many researchers that students benefit from multiple instructional strategies to develop algebraic concepts and that instruction in the class needs to address the three modes of learning: visual, verbal, and symbolic (Thornton, 2001; Chiu et al., 2001; Crawford and Scott, 2000; Choike, 2000). The tables in the activities provided numerical representations of the different concepts and were used to help students organize the information to guide them to generalize and reach the desired results. The graphs provided the visual representation of the different concepts. The essay questions led students to make desired conjectures related to the mathematical truths. (See Appendix B for a copy of all the activities.)

The activities were designed to guide the students to translate from one representation to another and concurrently to introduce concepts of linear functions. Research suggests that the teacher needs to emphasize multiple representations of mathematics "verbally in words, numerically in tables, visually in graphs and algebraically in symbols" (Choike, 2000, p. 558). Consequently, linear situations in the activities were presented as graphs, tables of values, equations or through verbal descriptions. One of the objectives of each activity was to reach another representation or at least parts of it. For instance, the sub-goal of finding the equation of a line is to find the slope of the line or its vertical intercept.

Every activity contains some examples that are worked out for the students to demonstrate to them the work they need to do. This method was chosen to give the students some independence to work on their own. This also minimized the need for the teacher to give instruction to the whole class, and maximized the time the students worked on the activities in their groups.

A few activities were designed to teach the students some basic skills and not necessarily to generalize and "discover." For instance, one prerequisite skill that the students needed to know was to find points on a line, when given the equation of the line. There is no discovery in this process. The students needed to learn that once they decided on one coordinate, they needed to substitute it in the equation to find the other coordinate. In these activities, examples were used to demonstrate to the students the skill. The examples were followed by some exercises for the students to apply the skills they learned.

Activities That Focused on Mathematical Truth 1

Mathematical Truth 1 states: Given a linear relationship, it has a unique slope and a unique intercept; and we can find them from the equation, from a table of values, from a graph or from a verbal description that contains sufficient information. For students to achieve understanding of this mathematical truth, they worked on six activities for eight days. The objective was for them to be able to determine the slope and the *x*- and *y*-intercepts if they were given a graph, a table of values, a verbal situation or an equation. Table 1 lists the activities that were used to achieve comprehension of each algebraic concept in a different representation.

Table 1

Activities for Mathematical Truth 1

Algebraic concepts	Graph	Table of Values	Verbal description	Equation
Slope	Activity I (parts 1 and 2) Activity III (part 3)	Activity II (parts 1, 2, 3, 4)	Activity III (part 3) Activity IV (part 4) Activity VI (part 6)	Activity VI (parts 2, 3, 4)
x- and y-intercepts	Activity III (parts 1, 2) Activity III (part 3)	Activity IV (parts 1, 2, 3)	Activity III (part 3) Activity IV (part 4) Activity VI (part 6)	Activity VI (part 5)

During the first week, the students were introduced to the definition of the slope and the intercepts. They learned to compute them if they were given a graph or a table of values. During the second week of instruction, students were ready to examine verbal situations and equations. They relied on their prior knowledge of graphs and the definitions to determine the slope of a situation and the intercepts. They investigated linear equations and discovered that the slope is the coefficient of x and that the vertical intercept is the constant if the equation is written in the form $y = mx + b$.

The activities presented to the students a general, but highly structured, situation at the start and then progressively less structured situations that were designed to help them refine their findings until they reached the correct conclusion. For example, in Activity II, when the students were calculating the slope, they used the quotient of the rise to the run of two points. The first example was a table of values of points that were one unit apart, horizontally, from one another. The conclusions were that all the runs, rises and slopes were the same for all pairs of points. The runs calculated for this table of values were equal to one and the slopes were the same as the rises. The second set of points had points that were equally spaced from one another but the run was not one. The conclusions continued to be that the runs, rises and slopes were the same for all pairs of points. But the runs were no longer one and the rises were no longer equal to the slopes. The third set of points consisted of random points from a line. In this example the students were expected to decide that the only consistent conclusion was that the slope is the same regardless of the pairs of points chosen. As another example, when the students discovered that the slope of the line is the coefficient of x in the equation in Activity VI (part 2), they were then presented with equations that were not written in the slope-intercept form in part 3 of the same activity. This forced them to decide that this conclusion works only for the linear equations that are written in slope-intercept form.

The activities were designed to help students make the connections between the different representations and allow them the option to use their preferred representation. When the students computed the slopes by finding the rise and run in Activity II, they also plotted the points on the coordinate plane and drew the rise and run. They noticed that the triangles formed by the rise and run of the same line were all similar triangles. In many activities the students were provided with a graph as an aid for completing the activity.

The students also were made aware of the advantages of some representations over others. For instance, when the students calculated the intercepts from a graph in Activity VI, their answer was an estimate because the intercept was not an integer. On the other hand, when they calculated the intercepts algebraically, their answer was exact. Having all this information on the same activity sheet helped the students to differentiate between the uses of the different representations.

To find the slope and intercepts from a verbal situation is always a challenge for students. The activities that addressed this objective were prepared in a way to guide students to be able to reach the end result. For Activity III (part 3), a graph was provided along with the word description to provide information in solving the problem. For Activity IV (part 4), a graph was provided for the students to plot points to aid them to estimate the answer and then calculate it algebraically. For Activity VI (part 6), the students were given an equation in the verbal situation and were required to find and explain the meaning of the slope and vertical intercept of this situation. To aid them, they were required to answer a few questions on certain instances of the situation.

While addressing Mathematical Truth 1, the students achieved other objectives as well. Given a table of values, the students learned to find the intercepts in Activity IV (parts 2 and 3) and in the same way learned to find other points on the same line using the slope definition. Before using linear equations, the students learned in Activity V (parts 1 and 2) to verify that a point satisfied an equation, and consequently that it was on the graph of the line. They also learned to find points that satisfied the equation in Activity VI (part 1). Students also practiced rewriting any linear equation to and from the slope-intercept form in Activity VI (part 4) to be able to determine the slope and the vertical intercept.

To achieve Mathematical Truths 2 and 3, the students went through similar activities. The number of activities was fewer than for Mathematical Truth 1 as it laid the foundation for the rest of the concepts in the unit. One main objective of this unit was for students to be able to translate from one representation to another. The activities provided multiple opportunities for this practice. Few lessons were taught using the direct instruction and demonstration method. These lessons covered writing equations in slope-intercept form and graphing linear equations.

Overall Structure of the Experimental Group Class Session

The students worked in small groups on the activities during class. Each group consisted of two or three students. The teacher assigned the students to the groups and changed the members every week. When the students worked in small groups, they had a chance to share their findings and discuss the conclusions. Even though the students were encouraged to share ideas, each student was required to respond to the essay questions on the activity sheet in their own words. The students worked individually on the homework assignments and quizzes. This gave them a chance to demonstrate what they learned and what concepts they had difficulty with.

Each class period for the experimental group lasted for forty-five minutes and consisted of three main parts. The teacher started the class with a review of the homework and discussion of the findings. This part lasted about ten to fifteen minutes. This part was necessary to ensure the students' understanding of the previous day's concepts. The main part of the lesson was the day's set of activities. The teacher introduced it and the students worked on this part for about twenty to thirty minutes. The last part of the period was a whole group discussion of the findings. This part was necessary to get the whole class in-sync with the conclusions. The final remarks took the last ten minutes of class.

The main part of each class was the students' work on the inductive reasoning activities. Before they got started, the researcher made sure that the students were familiar with the directions for the activity and they knew what they were supposed to do. Then the students worked in their small groups following the directions provided for each activity. The teacher circulated in the class, assisting the students, listening to their conversations, and making sure they are on the right track with the aim of the activity.

When all the students were done with the activity and completed the conjecture part, the teacher involved the students in a class discussion about their results. Students were given chance to explain their findings and justify their reasoning. The teacher made sure that all the students had a good understanding of the final result. At this time the teacher demonstrated to the students how to implement the result that they reached using a few examples. The students also practiced the skills independently.

The homework assigned for this group consisted of two types. The first type, which was assigned more often at the beginning of the experiment, was to continue investigations in some activities. The teacher made sure the students were familiar with the directions for the activity, and usually these activities were a continuation of work that was started in class. The second type of homework assignment involved practicing skills and concepts that were based on the activities. These assignments consisted of exercises from the students' textbook, and were similar to the assignments of the control group and the types of questions presented on the quizzes and unit exam.

Sample Selection and Procedures for Human Subject Protection

Sampling Procedures

The class of students that participated in this research consisted of 8th graders from a private school who, like many of their counterparts in California public schools, were covering a first year algebra curriculum. The students of this class received a full year of pre-algebra instruction in 7th grade, and completed it successfully. They regularly received encouragement and assistance from their parents when it came to math. Their parents tended to value math education, and placed an emphasis on success in the subject.

The students in this class were very diverse ethnically, culturally and economically. The majority of the students were bilingual who learned English as a second language, which was a characteristic of many of the students in the California public schools as of the year of this research study. The majority of the students had a fairly good command of the English language and they expected to enroll in the public system for high school.

The students were divided into two heterogeneous groups. Each student was paired with another student from the same gender and with similar math level. Then one member was randomly assigned to one group, and the other member was assigned to the other group. This method of grouping was preferred over random assignment to ensure heterogeneity within groups and comparability between groups. Groups were comparable on the following measures: course grade for the first trimester in the 2004-05 school year, standardized Terra Nova test scores (math subtest) administered in the spring of 2004, and gender.

Protection of Human Subjects

The standard protocols for protection of human subjects were observed. In particular, permission to use students' data for this study was sought from students' parents and the students, themselves. Students' identities were connected with the data during the collection period, but no information that could identify any participant is included in this report. (See Appendix C for the permission forms, and Appendix D for the letter granting permission to use human subjects)

Measures

This section consists of four parts: data collection procedures, assessment item types, assessment item scoring, and data analysis. The first part explains the different data-collection measures used and the materials assessed in each measure. A table is included in this section with the objectives of the unit and the assessment items on each data-collection measure that address these objectives. The second section describes the different assessment item types used on the data-collection measures. It includes a table that displays each assessment item type and the number of assessment items that use this type of assessment. The third section describes the method used to score the assessment items. This section includes the two rubrics used to score the assessment items on the data-collections measures and a table that presents for each item on the data-collection measure the rubric used and the assessment item type. The fourth section describes the data analysis, which is the method used to address the research questions. This section includes a table that determines the questions addressing each of the three mathematical truths, and a table that determines the questions addressing both representational translation abilities that were considered for this study.

Data-Collection Procedures

The students were assessed twice during the training and once upon the completion of the unit. Quizzes were given on days 6 and 13 of the 17-day unit to assess the students' understanding of the material, and to guide the teachers to modify instruction. A final comprehensive unit exam was given at the end of the unit. The data from the unit exam constitutes the most critical comparison that was used for this study. The tests' questions were predominantly free-responses including a few True/False questions that required students to explain their answers. Students were expected to show all their work, and they were awarded partial credit for every correct step toward the solution. These data-collection measures were given to both groups of students in the same format on the same days (See Appendix E for a copy of both quizzes and unit exam).

The first quiz (administered on the 6th day of the unit) covered the concepts of slopes and intercepts. The second quiz (administered on the 13th day of the unit) assessed the students' abilities to write equations in slope-intercept and standard forms and to graph a linear equation given in any form. The unit exam was administered on the last (17th) day of the unit and was comprehensive. It covered all the concepts assessed in the first and second quizzes, and in addition, the students were required to write equations of parallel and perpendicular lines.

Seven objectives were determined at the beginning of this unit along with three mathematical truths and two representational translation abilities. (Note that several objectives are drawn from California Mathematics Content Standards (California State Board of Education, 2004). The following is a listing of all unit objectives, mathematical truths and representational translation abilities.

Unit Objectives:

A. Find the slope of a line given coordinates of two points on the line.

B. Determine the x- and y-intercepts of linear graphs from their equations. (6. 0)

C. Graph a line given any linear equation. (6. 0)

D. Verify that a point lies on a line, given an equation of the line. Derive linear equations by using the [slope-intercept and standard] formula. (7. 0)

E. Understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Find the equation of a line parallel or perpendicular to a given line that passes through a given point. (8. 0)

F. Students solve multistep problems, including word problems, involving linear equations in one variable and provide justification for each step. (5. 0)

G. Understand linear functions and their properties (e. g., slope). Solve linear equations, and represent solutions on graphs. Create and interpret graphs of linear equations.

Mathematical Truths:

Mathematical Truth 1: Given a linear relationship, it has a unique slope and a unique intercept; and we can find them from the equation, from a table of values, from a graph or from a verbal description that contains sufficient information.

Mathematical Truth 2: Given a linear relationship, its ordered pairs can be expressed using an equation, a table of values, a graph, or a verbal description. Specifically:

- Points on the graph satisfy the equation
- Points on a table have constant rise over run

Mathematical Truth 3: A linear relationship has a unique equation in standard form and a unique equation in slope-intercept. A standard form equation can only be derived from another equation of the line.

Representational Translation Ability:

Translation Ability 1: From any representation to a graph

Translation Ability 2: From any representation to an equation

Table 2, on the next page, displays all the assessment items that addressed the objectives, as well as the mathematical truths and the representational translation abilities.

Assessment Item Types

The quizzes and unit exam consisted of three main types of questions. (1) Linear function skills: most of the questions tested the skills needed to write a linear equation, or graph a line, given sufficient information. (2) True/ False situations: the second most used type of questions is the true/false situation. These questions presented a situation where two people are discussing a concept from the linear function unit. One person is presenting a statement, possibly true or false, and the other is disagreeing with him/her. Students were asked to identify the person saying the correct statement and explain their answers. (3) Essay questions/explanations: the last type of question asked students to describe a particular concept or explain a certain answer in their own words. (See Appendix E for a copy of Quiz 1, Quiz 2 and Unit Exam) Table 3 details the type of questions included in each assessment.

Table 2

Relation of Assessment Items to Unit Objectives, Mathematical Truth and Representation

Assessment Item	Unit Objective	Mathematical Truth	Representation
	Quiz 1		
1	A	1, 2	
2, 3, 7b, 7c, 8b, 8c	B	1	
4	G	1, 2	
5, 6	G	1	
7a, 8a	A	1, 2	graph
9	A	1	
	Quiz 2		
1	G	3	
2	D	2, 3	graph, equation
3	G	2	
4	G	1	
5	C	2	graph
6	D	2	
7	D	3	

Table 2 (*continued*)

Relation of Assessment Items to Unit Objectives, Mathematical Truth and Representation

Assessment Items	Unit objective	Mathematical Truth	Representation
		Unit Exam	
1	C	2	graph
2	C	1	graph
3	E	1	
4, 5	A	1	
6, 7	B	1	
8, 9	D	3	equation
10, 12	G	3	
11a	D	2, 3	equation
11b, 11c	D	2	
11d, 11e	G	1	
13a	F, G	2	graph
13b	F, G	1	
13c	D, G	3	equation
13d	G	2	

Table 3

Number of Assessment Items for Each Assessment Type

Type of Assessment Item	Quiz 1	Quiz 2	Unit exam
Linear functions skills	7	3	15
True/ False situations	3	2	3
Essay questions/ explanations	3	4	4

Assessment Item Scoring

The assessment items were scored using a three-point rubric (Rubric A) and a five-point rubric (Rubric B). The following is a detailed description of both rubrics. Rubric B was adapted from the Arizona Mathematics Rubric used by the Arizona Department of Education (Arizona Mathematics Rubric, 2004). Rubric A was written by the researcher.

Rubric A (3-point)

2 A 2 represents complete explanation/answer for the problem.

1 A 1 represents a partial explanation/answer that is missing an important part.

0 A 0 represents no explanation/answer or completely incorrect explanation/answer.

Rubric B (5-point)

4 A 4 represents an effective solution. It shows complete understanding of the problem, thoroughly addresses all points relevant to the solution, shows logical reasoning and valid conclusions, communicates effectively and clearly and includes adequate and correct computations and/or setup.

3 A 3 response contains minor flaws. Although it shows an understanding of the problem, communicates adequately, and generally reaches reasonable conclusions, it shows minor flaws in reasoning and/or computation or neglects to address some aspect of the problem.

2 A 2 response shows gaps in understanding and/or execution. It shows one or some combination of the following flaws: an incomplete understanding of the problem, faulty reasoning, weak conclusions, unclear communication, or poor understanding of relevant mathematical procedures or concepts.

1 A 1 shows some effort beyond restating the problem or copying given data. It shows some combination of the following flaws: little understanding of the problem, failure to address most aspects of the problem, major flaws in reasoning that lead to invalid conclusions, or a lack of understanding of relevant mathematical procedures or concepts.

0 Response shows no mathematical understanding of the problem or the student has failed to respond to the item.

The questions that were assessed using Rubric A were the true/false situations and the essay questions. Some linear functions skills were also assessed using this rubric if the question entailed only a one- or two-step solution. Most linear functions skills questions were assessed using Rubric B. Table 4 identifies the rubric used and the assessment type for each question on the quizzes and unit exam.

Since rubrics were used to score the assessment items, it was necessary to assess the inter-rater reliability of these scoring rubrics. The questions on the quizzes were independently assessed using the same rubric by both teachers. The scores were compared, and the rubrics adjusted accordingly. For quiz 2, the inter-rater reliability was 0. 88. For the unit exam, the inter-rater reliability was 0. 97.

Table 4

Rubric Used and Assessment Type for the Items on Quizzes and Unit Exam

Assessment type	Rubric	Assessment Item
Quiz 1 (max 34 pts)		
Essay question	A	1, 2, 3
Linear Function skills	A	7b, 8b, 9
Linear Function skills	B	7a, 7c, 8a, 8c
True/ False situation	A	4, 5, 6
Quiz 2 (max 30 pts)		
Essay question	B	1
Linear Function skills	A	7
Linear Function skills	B	2, 5
Linear Function skills, essay questions	A, A	6a, 6b, 6c
True/ False situation	A	3, 4

Table 4 (*continued*)

Rubric Used and Assessment Type for the Items on Quizzes and Unit Exam

Assessment type	Rubric	Assessment Item
Unit Exam (max 62 pts)		
Essay question	A	12d, 12e
Essay question	B	14b, 14d
Linear Function skills	A	5, 9a, 9b, 10a, 10b, 12b, 12c, 14c
Linear Function skills	A, A	8
Linear Function skills	B	1, 2, 4, 6, 7, 12a
True/ False situation	A	3, 11, 13, 14a

Data Analysis

This section identifies the method used to analyze the data obtained from the unit exam and the quizzes. This section includes five parts: first, analysis of data that address research question 1 relating to mathematical truths. The second part explains the analysis of the data that was collected to address research question 2 relating to representational translation ability. The last three parts identify methods that were used to address both questions, namely pair comparison, assessment item comparison and journal reflections.

Mathematical truths. To address research question 1; scores for each student were computed from the unit exam. For each mathematical truth, questions on the unit exam were identified that addressed that particular mathematical truth (see Table 2). An overall scaled score for a student on that mathematical truth was obtained by calculating his or her overall percent (to the nearest whole percent) on the unit exam items addressing that mathematical truth. To obtain a single overall average score for a student on the three mathematical truths, a weighted average of the individual scaled scores was obtained, weighting each mathematical truth according to the percent of the overall raw assessment score on items devoted to that truth. (Note weightings for the three mathematical truths were 45%, 26% and 29% respectively).

For the sake of comparison, the researcher identified other means to compare the students' understanding of the mathematical truths. First, the number of students who scored in each 10-point range on the total mathematical truth score were counted. Moreover, comparable quiz scores were computed and compared to unit exam scores. Finally, students' responses on the unit exam were analyzed qualitatively to assess their understanding. These data were analyzed, using exploratory data analysis, to compare the two groups' acquisition of the mathematical truths over the course of the experiment.

Representational translation abilities. To address question 2, scores for each student were computed from the unit exam. For each representational translation ability, questions on the unit exam were identified that addressed that particular representation (see Table 2). Similar to the computation of scores for the mathematical truths, individual scaled scores for each translation ability were computed for each student. To obtain a single overall average score for a student on both translation abilities, a weighted average of the individual scaled scores was obtained, weighting each translation ability according to the percent of the overall raw assessment score on representation items devoted to that ability. (Note weightings for the two translation abilities were 42% and 58% respectively.) These scores were analyzed, using exploratory data analysis, to compare the two group's acquisition of these translational abilities over the course of the experiment.

Pairs comparison. Since the students originally were divided into pairs, and one member of each pair was assigned to a different group, the pairs' performance on the unit exam was also compared. For the total mathematical truths and the total representations, the difference between the experimental group member scaled score and that of the control group member was computed. If the result was positive, it indicated that the experimental group member outperformed his/her pair partner, and a negative result indicated that the control group member outperformed his/her partner.

Assessment item comparison. The researcher also computed averages of both groups on each assessment item on the unit exam. These averages were compared to determine if one group performed better than the other. This information was used to address questions 1 and 2.

Journal reflections. To provide additional context, the researcher kept a journal that was updated once a week on the observations and progress of the students in the experimental group. While no such journal was kept by the teacher of the control group, the researcher's journal does offer some insight, although not necessarily the most objective, into the dynamics of the experimental class activities.

Time Line

Preparatory instruction and experimental instruction took place during the fall semester of 2004. The experiment lasted for four weeks including time allotted for quizzes and the unit exam. A rough time line is given below. A detailed unit plan is in Appendix A.

Preparatory Instruction – Intact class with team teaching		12 weeks
Experimental Instruction – Experimental and Control Groups		17 days
Instruction	Days 1 – 5	
Quiz 1	Day 6	
Instruction	Day 6 – 12	
Quiz 2	Day 13	
Instruction	Days 13 – 15	
Review	Day 16	
Unit Exam	Day 17	

Chapter 4

Research Findings

This chapter consists of three main sections presenting the findings for the linear function experiment conducted with 8^{th} grade algebra students. The introductory section discusses general procedures for processing and analyzing the research data. The second section presents the findings related to the mathematical truth questions. The third section presents the findings related to the representational translation ability questions.

Data Preparation

The 29 students that participated in the experiment were paired according to their Terra Nova standardized math scores and according to their achievement in the algebra class during the trimester before the experiment. The students were also paired according to gender. There were 13 pairs in total, nine pairs of girls and four pairs of boys. Three students, two boys and a girl, were not paired because there was no match for them. When the results were examined comparing the pairs of students, these three students were excluded. When the results compared both groups, control and experimental, and whenever averages were calculated, all students were accounted for.

For the quizzes, the scaled scores for mathematical truths and representational translation abilities were calculated only if there were sufficiently many questions to assess them. For quiz 1, the only scaled scores calculated were for Mathematical Truth 1 and 2. For quiz 2, scaled scores for Mathematical Truth 2 and 3 were calculated. No scaled scores for representations were calculated for the quizzes. For the unit exam, all the mathematical truths and the representations were assessed sufficiently and scaled scores were calculated for all of them.

The researcher used Excel (XP) to analyze the results of the experiment. All graphical displays were prepared using Excel as well.

Mathematical Truths

The first question that was addressed by this research involves the three mathematical truths that were identified at the beginning of the study. The researcher wanted to know the effect of the two teaching methods (symbolic versus patterns-and-functions) on the students, and whether one of the methods appeared to produce better results. The researcher also wanted to find out if the students mastered one mathematical truth more than the others. Finally, a comparison was made of the students' mastery of the mathematical truths on the assessments that took place during the unit (quizzes) and at the end of the unit (unit exam).

How do the two teaching methods (symbolic versus patterns-and-functions) affect 8th grade algebra students' understanding of the three mathematical truths related to linear functions?

The average of all the students on all the mathematical truths was 69% (see Table 5). The groups' average ranged from 59% (control group average on Mathematical Truth 3) to 75% (experimental group average on Mathematical Truth 1). One-tailed (unequal variances) t-tests were run on each mathematical truth and on the overall scores. All tests indicated no significant differences between the groups at the 0.05 level.

Table 5

Mathematical Truths Results on the Unit Exam [a]

Mathematical Truth	Minimum	Average	(s. d.)	Maximum
		Control Group		
MT1	39	73	(18. 6)	100
MT2	19	65	(26. 2)	100
MT3	28	59	(20. 1)	100
all MT's	36	67	(18. 8)	100
		Experimental Group		
MT1	36	75	(22. 2)	100
MT2	31	74	(22. 9)	100
MT3	6	60	(30. 7)	100
all MT's	34	70	(23. 4)	98
		All Students		
MT1	36	74	(20. 0)	100
MT2	19	69	(24. 6)	100
MT3	6	59	(25. 3)	100
all MT's	34	69	(20. 8)	100

[a] See Appendix F for individual students' scaled scores.

Table 6 lists the number of students who scored in each 10-point range starting from 30 to 100. The control group has a more even distribution with two to three students in each range, except that there is only one student with an average in the 30s. On the other hand, there are five students from the experimental group who scored above 90, and the rest are distributed by one or two in each range.

Table 6

Distribution of Students' Mathematical Truth Total Scores

Average Range	Control	Experimental
Above 90	2	5
80 - 89	2	1
70 - 79	3	1
60 - 69	2	2
50 - 59	3	1
40 - 49	2	2
30 - 39	1	2

To provide some context to these figures, it helps to see a few student responses to specific exam items. For example, many students demonstrated good understanding of the concept of slopes in relation to parallel lines (question 3 on the unit exam). The average of all the students on this question was 93%. This question is a partial assessment of Mathematical Truth 1. The average of the control group (97%) was a bit higher than the average of the experimental group (89%). Table 7 contains samples of students' responses.

Table 7

Sample Student Responses to Question 3[a] on the Unit Exam

- *The slope of a line is kind of like the 'steepness' of the line, and all parallel lines have the same 'steepness', so all parallel lines have the same slope (experimental group).*
- *Parallel lines have the same slope because with the same slope they will be going in the same direction without ever touching (control group).*
- *Parallel lines must have the same rate of change, otherwise, they will intersect and that isn't parallel (experimental group)*

[a] Question 3: Samia and Ahmad were discussing parallel lines. Samia said that parallel lines have the same slope, but Ahmad said they have the same *y*-intercept. Who is right? Explain.

On the other hand, not many students demonstrated good understanding of the concept of slopes in relation to slope-intercept form and equations of vertical lines (question 12 on the unit exam). The average of all students on this question was 62%. The average of the experimental group (71%) was higher than the average of the control group (53%). This question is a partial assessment of mathematical truth 3. Table 8 contains samples of students' responses.

Table 8

Sample Student Responses to Question 12[a] on the Unit Exam

Correct Responses

- *In a vertical line, the slope is undefined, because the denominator is zero (0), so it is not possible to divide (control group)*
- *There is no definite slope of a vertical line (experimental group).*
- *A vertical line has no slope (experimental group).*

Incorrect Responses

- *If there is no slope you could just put m=0 (control group).*
- *There is no equation because slope = 0 (experimental group).*

[a] Question 12: Adel is frustrated because he cannot write the equation of a vertical line in slope-intercept form. Amal told him that there is no such equation. He disagrees with her. Who is right? Explain.

Does one method appear to be "better" than the other? If so, in what ways?

The averages of the experimental group students were higher on all the mathematical truths, but the difference was not very significant (see Table 5). The largest difference between both groups was on the questions relating to Mathematical Truth 2, with a difference of 9%. For Mathematical Truth 3, the difference between both groups was 1%. However, if we eliminate the lowest score in the experimental group, an outlier of 6%, the average becomes 64%, which is 5% higher than the control group.

When examining the 22 questions that made up the unit exam, the experimental group appeared to perform at a slightly higher level (see Table 9). The experimental group's average was higher than the control group on 14 out of the 22 questions addressing the mathematical truths. The difference ranged from 1% to 22%. The highest difference was on question 11b covering mathematical truth 2. The control group's average was higher on the remaining eight questions covering mainly Mathematical Truths 1 and 3. The difference ranged from 7% to 15%. The highest difference in favor of the control group was on question 5 covering Mathematical Truth 1.

Table 9

Control and Experimental Group Averages on Unit Exam Questions

Questions	Control	Experimental	all	Mathematical Truth	Rep.
1	80	96	88	2	Graph
2	73	86	79	1	Graph
3	97	89	93	1	
4	90	91	91	1	
5	90	75	83	1	
6	72	88	79	1	
7	68	61	65	1	
8a	70	71	71	3	Equation

Table 9 (*continued*)

Averages of Control and Experimental Group Averages on Unit Exam Questions

Questions	Control	Experimental	all	Mathematical Truth	Rep.
8b	33	46	40	3	Equation
9a	53	68	60	3	Equation
9b	43	32	38	3	Equation
10	57	64	60	3	
11a	68	55	62	2, 3	Equation
11b	60	82	71	2	
11c	60	75	67	2	
11d	73	61	67	1	
11e	60	50	55	1	
12	53	71	62	3	
13a	53	71	62	2	Graph
13b	50	64	57	1	
13c	83	75	79	3	Equation
13d	50	57	53	2	

The students in the experimental group demonstrated a clear understanding of the mathematical truths in class discussion. They came up with more than was required of them in terms of discoveries. For example, when working on intercepts, they noticed that if a line passes through the origin, then the origin is both the x-intercept and the y-

intercept. They also questioned what happened if the line was parallel to one of the axes and concluded that such a line will not have an intercept with that axis.

Was any mathematical truth better understood by the students than the others?

The highest average on the unit exam of all students was on Mathematical Truth 1 (74%) followed by Mathematical Truth 2 (69%), then Mathematical Truth 3 (59%). The experimental group's averages for Mathematical Truths 1 and 2 were very close, with a difference of one percentage point. The students scored the lowest on the questions pertaining to Mathematical Truth 3. These questions tested the students' skills of writing linear equations in different forms.

The students scored the lowest on unit exam questions 8b (40%) and 9b (38%) , where they were required to write equations of lines in standard form (see Table 9). These questions addressed Mathematical Truth 3. Students from both groups scored lowest on these questions. On the other hand, the students scored the highest on questions 3 (93%) and 4 (91%). These questions covered Mathematical Truth 1. These results confirm that Mathematical Truth 1 was the most understood of the mathematical truths.

Figures 1, 2, and 3 give visual displays of students' comparative understanding of each mathematical truth. Each figure compares the students' scores on two mathematical truths at a time in a scatter plot. By overlaying an imaginary line $y = x$ on each scatter plot, it's possible to identify how many students performed better on each mathematical truth being measured. Comparing the students' individual scores on

Mathematical Truth 1 to their scores on Mathematical Truths 2 and 3, the students performed the same or better on Mathematical Truth 1. Comparing the students' individual scores on Mathematical Truths 2 and 3, the students performed the same or better on Mathematical Truth 2. In conclusion, the students performed the highest on Mathematical Truth 1, followed by Mathematical Truth 2, then finally Mathematical Truth 3.

Figure 1. Comparison of Mathematical Truth 1 and 2.

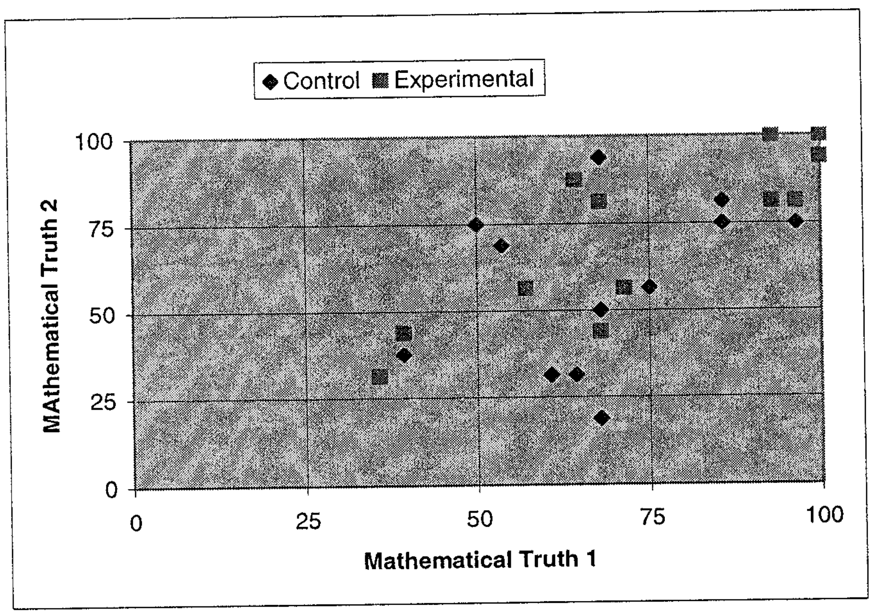


Figure 2. Comparison of Mathematical Truth 1 and 3.

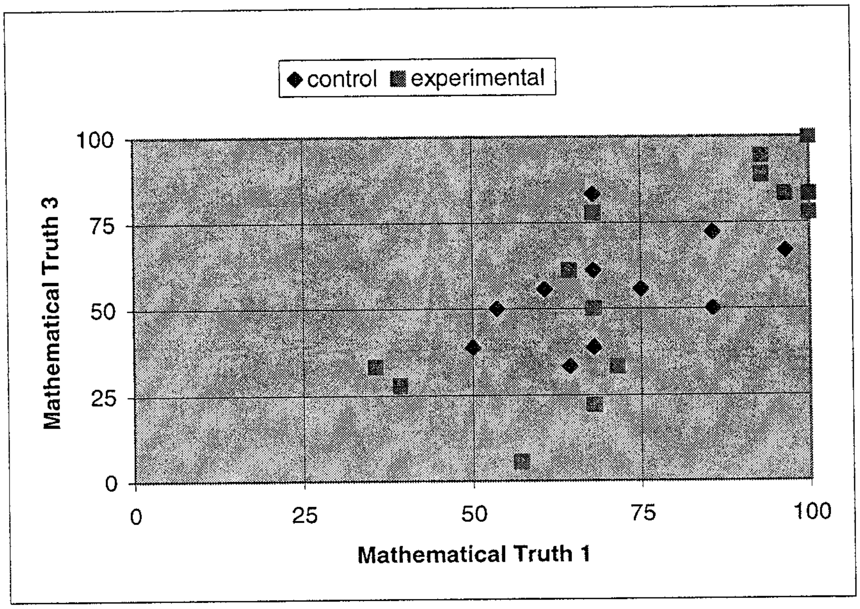
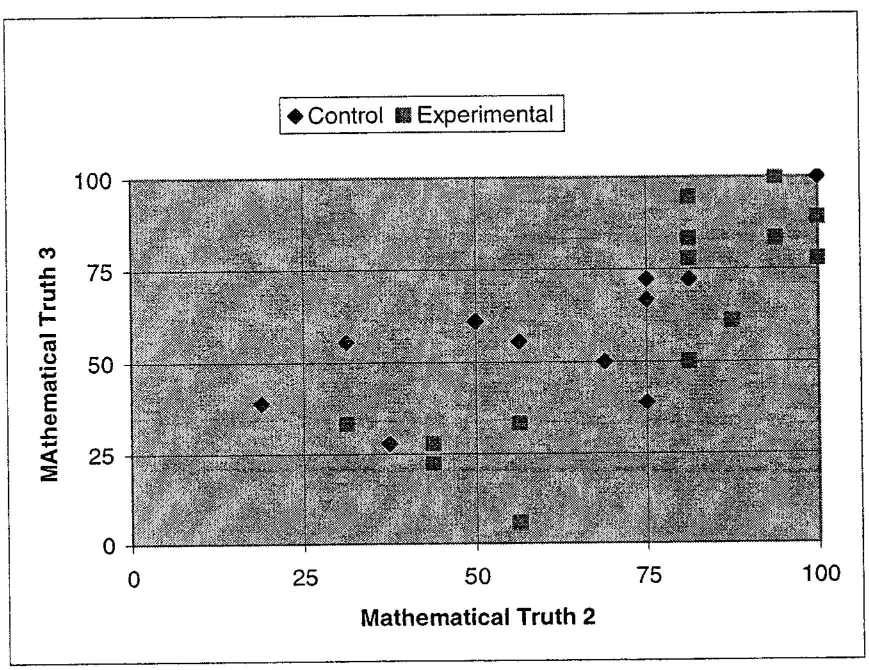


Figure 3. Comparison of Mathematical Truth 2 and 3.



Was there any significant difference between the students' performance on the quizzes and the unit exam?

On quiz 1, the control group's average (80%) was slightly higher than the experimental group's average (79%). (See Table 10). Both groups' averages dropped between the first and second quiz, and again the control group's average (73%) was higher than the experimental group's average (70%). For the unit exam, the experimental group maintained its average (70%), but the control group dropped to a 67%.

Both groups started out with very close percentages on the three mathematical truths, but the control group dropped more than the experimental group throughout the unit (see Table 10). The experimental group was able to maintain its average on Mathematical Truth 1 between the first quiz (76%) and the unit exam (75%). However, the control group dropped from 78% to 73%. In regard to Mathematical Truth 2, the control group dropped 10% between the first and second quiz and another 10% between the second quiz and the unit exam. On the other hand, the experimental group dropped 14% on Mathematical Truth 2 between the first and second quiz, but it was able to regain 3% and ended up with a higher percentage than the control group. Both groups dropped about 10% on Mathematical Truth 3.

Table 10

Averages on Quizzes and Unit Exam

Groups	MT1	MT2	MT3	All MTs
		Quiz 1		
Control	78	86		80
Experimental	76	85		79
All	77	85		79
		Quiz 2		
Control		75	70	73
Experimental		71	69	70
All		73	69	72
		Unit Exam		
Control	73	65	59	67
Experimental	75	74	60	70
All	74	69	59	69

Representational Translation Ability

The second question that this research addressed was on the effect of the two teaching methods on students' representational translation ability. Two representations were targeted in this study, namely graphs and equations. The researcher wanted to find out if one teaching method helped the students to translate better, and whether the students mastered one representation more than the other. The researcher also studied the relation between the students' scores on the mathematical truths and the representations, if any.

How do the two teaching methods (symbolic versus patterns-and-functions) affect students' abilities to translate among various representations of linear situations?

The average of all the students on the representations was 67% (see Table 11). The group averages ranged from 58% (experimental group on equations) to 87% (experimental group on graphs). Two students from the experimental group scored 7% on questions about equations. These scores are outliers, and when they are excluded from the average, it becomes 66%, versus 60% for the corresponding control group average.

Does one method appear to be "better" than the other? If so, in what ways?

Translation to graphs. Students in the experimental group performed much better than their classmates in the control group on the questions assessing graphs. The unit exam included three questions on graphs (1, 2, 13a), and the experimental group's average was higher than the control group's average on each of the three questions (see

Table 9). Overall, the difference between the groups was 15% (see Table 11). Moreover, a one-tailed t-test on the graphing scores (unequal variances) indicated a significant difference between the groups (p-value = 0.03). T-tests of the equation scores and overall were not significant.

Table 11

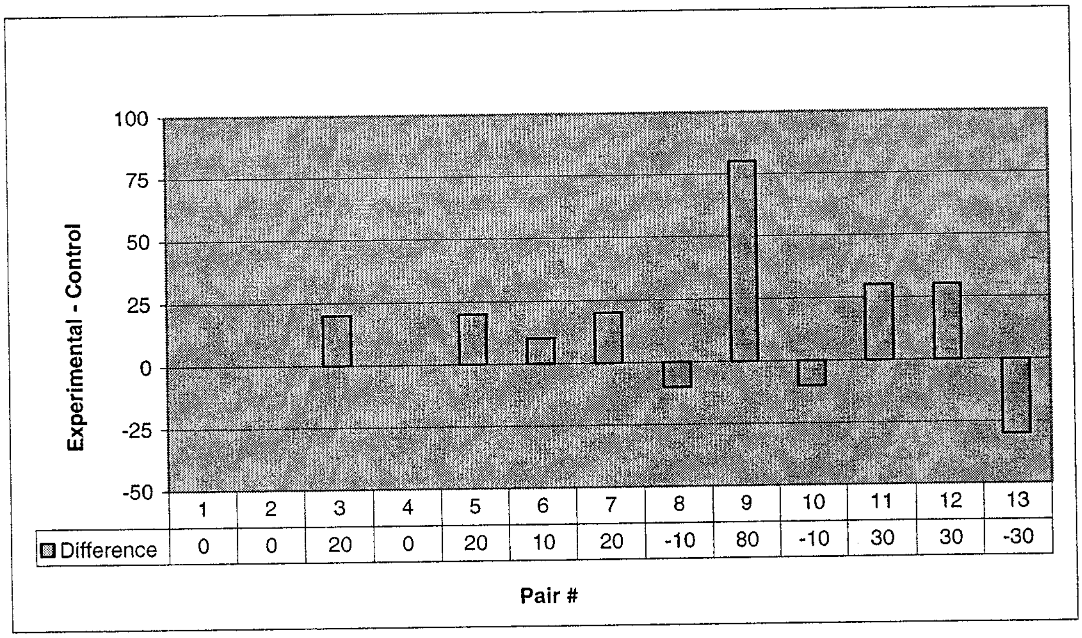
Averages for Representational Translation Ability on the Unit Exam [a]

Representations	Minimum	Average	(s. d)	Maximum
		Control Group		
Graph	10	72	(25. 4)	100
Equation	21	60	(24. 1)	100
All	38	65	(20. 5)	100
		Experimental Group		
Graph	40	87	(16. 4)	100
Equation	7	58	(33. 8)	100
All	33	70	(22. 5)	96
		All Students		
Graph	10	79	(22. 5)	100
Equation	7	59	(28. 7)	100
All	33	67	(21. 3)	100

[a] See Appendix F for individual students' scaled scores.

In comparing students pairwise, the experimental group students generally outperformed or were equivalent to their counterparts in the control group. Figure 4 compares the scaled scores of pairs of students on their ability to graph. A positive bar represents that the experimental group member outperformed his or her counterpart from the control group. A negative bar represents that the control group member outperformed his or her counterpart from the experimental group. There are seven positive bars compared to three negative bars, and three pairs' differences were zero.

Figure 4. Translation to a Graph (Pairwise Comparison).

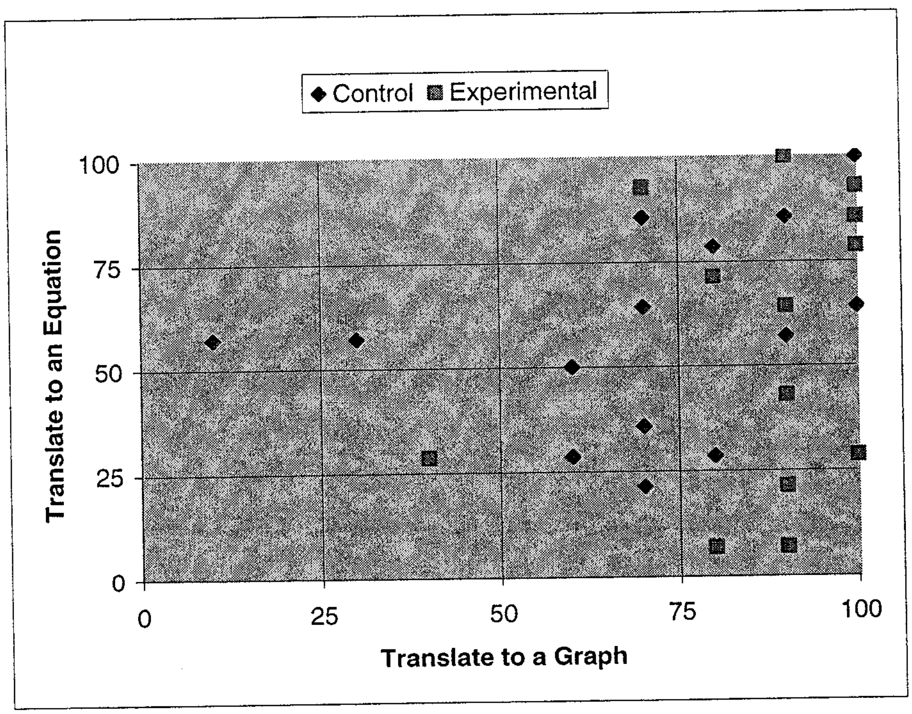


Translation to equations. Students in the control group performed slightly better on the questions assessing equations. The unit exam included six questions on equations (8a, 8b, 9a, 9b, 11a, 13c). The experimental group scored higher on the first three questions, and the control group scored higher on the other three questions. On questions 8a and 9a, the students were provided graphs as an option to help them write the equations. Overall, the average of the control group was higher than the average of the experimental group by a difference of 2%.

Did the students master one representation more than the other?

All the students mastered translation to graphs much better than writing equations. Overall, students' average on the graph representation was 79% which is much higher than their average on the equation representation (59%) (see Table 11). The experimental group's average on graph representation was 87%, which is the highest of all the averages. Figure 5 compares each student's score on the ability to translate to a graph to his or her score on the ability to translate to an equation. All the experimental group students, except one, scored very high on translation to a graph, even if their ability to write equations was low. Two students from the control group scored higher on equations than on graphs, which was unusual.

Figure 5. Comparison of Translational Abilities.



The experimental group students appeared to be comfortable using the graph representation. Questions 8 and 9 on the unit exam required the students to write equations. Graphs were provided as an option for the students to use to help them write the equations. Out of the fourteen students in the experimental group five students used graphs for both questions, eight students used a graph on only one question, and only one did not use the graphs. On the other hand, out of the fifteen students in the control group, two used graphs for both questions, eight used a graph on only one question, and five did not use the graphs at all.

How do students' representational abilities relate to their understanding of the three mathematical truths? Is there a strong correlation?

There is a strong positive correlation (0. 96) between the students' understanding of the mathematical truths and their representational translation ability. Figure 6 is a scatter plot comparing the students' averages on all the mathematical truths to their averages on all the representations. Students in the control group as well as the students in the experimental group showed the same trend of performing at the same level on the mathematical truths and the representations.

All the questions addressing the representations also address the mathematical truths. Therefore, the high correlation is somewhat expected because of the questions overlapping. However, Figure 7 compares the representations to the questions about mathematical truths excluding any questions about representations. Figure 7 also demonstrates a strong positive correlation (0. 88), especially for the experimental group members.

Figure 6. Representations versus Mathematical Truths (questions overlap)

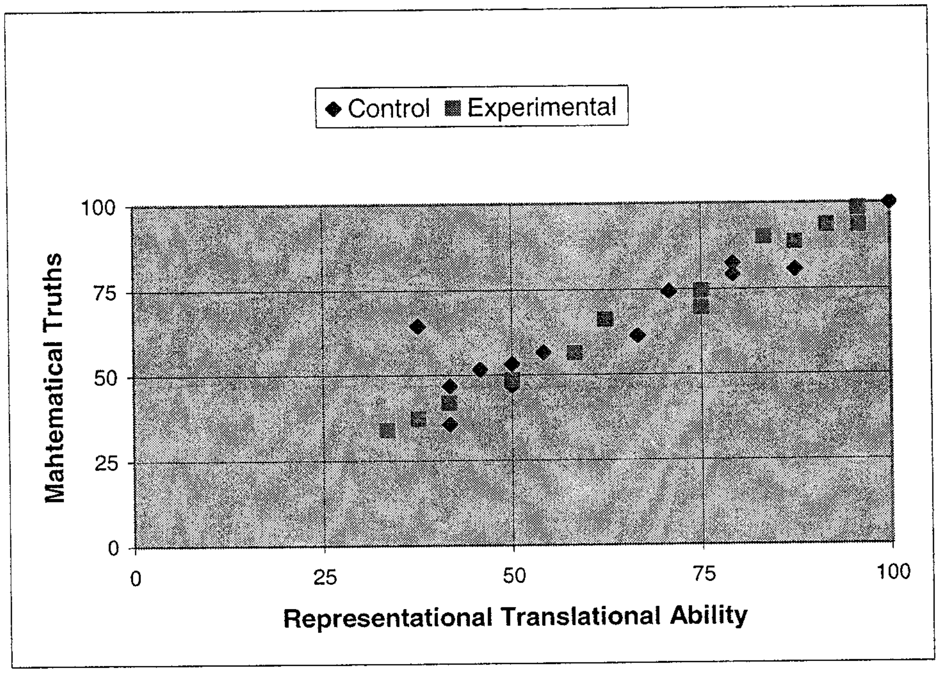
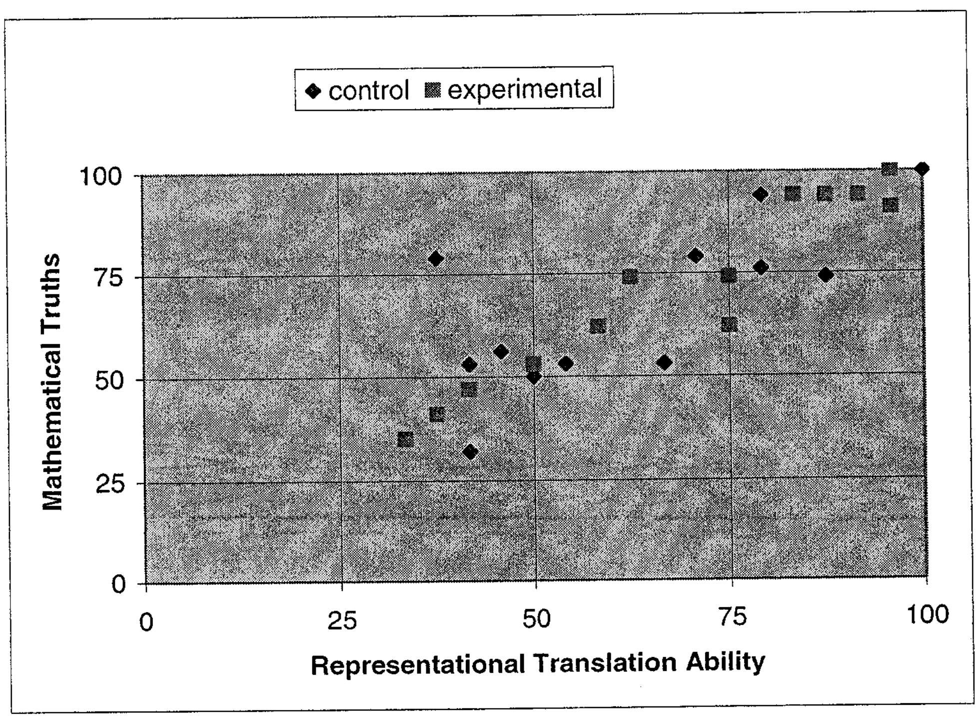


Figure 7. Representations versus Mathematical Truth (no questions overlap)



Conclusion

All the 29 students that participated in the experiment mastered the concepts of the linear function unit in varying degrees. It is clear that there was a very strong correlation between the students' understanding of the mathematical truths and their ability to translate to different representations. The results of the comparison of the two groups are in favor of the experimental group. The students mastered the skills to translate to graphs much better than the ability to translate to equations. This was especially true for the experimental group students. All the students mastered the Mathematical Truth 1 more than the others, and Mathematical Truth 3 the least. The experimental group mastered the skills on Mathematical Truth 2 much better than the control group. The differences between the groups were not significant using usual measurements of group comparisons, except for translation to graphs, but there are subtle differences worthy of further reflection and investigation when the data are examined more closely.

Chapter 5

Discussion

The results of this research indicate that the students in the experimental group did not do worse than their counterparts in the control group. They achieved similar or slightly better results. In this chapter, the main features of this study are discussed examining what worked and how it can be improved. This chapter includes five sections: the first section discusses the design elements of this study, namely the inductive reasoning activities, the use of multiple representations, and discovery learning. The second section reviews the classroom dynamics that were influenced by this study: group work and teacher role. The third and fourth sections present reflections of the researcher on the implementation of this experiment and its results. The last section discusses future direction for research in the field of linear functions.

Design Elements

This research relied on three main design elements: the inductive reasoning activities, multiple representation, and discovery learning. In the following section each design element is analyzed.

Inductive Reasoning Activities

The inductive reasoning activities were inspired by the book by Michael Serra (2002) on teaching geometry with an inductive approach. This book comprises a whole year course using this teaching method. Students using this book get a lot of training applying inductive reasoning strategies. By contrast, the linear function unit in this study was the first time that the experimental group students were introduced to the inductive reasoning approach. They did not have any prior experience of learning in this fashion and they needed more practice. It would be beneficial to have the students be more familiar with some inductive reasoning activities prior to starting this unit. This way they would derive more benefit from them.

The experimental students had difficulty with generalization. They could not always identify the correct pattern because they usually looked for superficial similarities. This confirms research findings of others (Chiu et al., 2001; Crawford and Scott, 2000; Herscovics, 1989; Wagner and Parker, 1993). For example, while working on Activity VI (part 3), the students needed to identify in which form of the linear equation the slope is the coefficient of x. The students could not come up with the correct generalization in their small groups. The teacher copied the table on the board and as a group the class looked for consistent patterns that worked all the time, until they arrived at the correct conclusion.

Some students spontaneously continued using similar tables to the ones presented in the activities while doing other exercises. For example, to find the slope a table was set to identify the two points to be used, their rise, run, and then the slope. The students set up tables like that for their homework on their own. This method minimized their mistakes in identifying which coordinate to use for y_1, y_2, x_1, and x_2. Using tables also made finding new points with missing coordinates much easier.

Multiple Representations

When the students in the experimental group learned different representations, they were empowered by being given choices for finding solutions. For example, they liked having the knowledge and ability to find the slope of a line graphically by plotting points on a coordinate plane or algebraically by using the definition. The students were almost graphing on a daily basis throughout the 17-day linear function unit. The worksheets for the activities were prepared with graphs to facilitate graphing. It is no wonder that the highest result achieved by these students was on their ability to translate to the graph representation from any other one.

There were other representations that were addressed in this unit; however they were not measured because they were not as significant as graphs and equations. The ordered pairs were the main numerical representations in this unit. Students learned to find ordered pairs given a linear equation, a graph, a verbal description, or at least two ordered pairs. The students also solved linear situations that were presented verbally as word problems. These were usually the hardest representations for the students to navigate.

Discovery Learning

The students in the experimental group enjoyed the alternative learning environment using discovery learning and were actively involved in the learning process. From the first week of the experiment, the students were more involved in the class activities than they had been during the previous 12 weeks of instruction. Moreover, they tended to stay on task, often coming up with more than was required of them in terms of discoveries.

Some students "discovered" their own versions of mathematical truths (theorems). For example, when studying intercepts, they noticed that if a line passes through the origin then the origin is both the x-intercept and the y-intercept. They also questioned what happened if the line was parallel to one of the axes and concluded that such a line will not have an intercept with that axis.

Some of them "discovered" mathematical procedures on their own. For example, when writing a linear equation in slope-intercept form ($y = mx + b$), the typical procedure is to first find m then calculate b. The teacher guided them to the definition of the slope, which was the method used to find more points on a given line. One student "discovered" that it is possible to use any point for x and y and since m is known to plug all the three in the equation and find b.

Classroom Dynamics

To gain maximum benefits of the above mentioned design elements, the experimental group students worked in small groups under the direction of the researcher, who facilitated their learning.

Group Work

The researcher divided the students of the experimental group into teams of two or three members. The members in the teams rotated three times during the 17-day unit after each assessment. The groups were designed to be heterogeneous, so one strong math student was a part of each group. The students in the experimental group enjoyed working in small groups and being allowed to work with their classmates. The regular algebra class does not allow for a lot of group work. The students needed more guidelines on working in teams since they were not used to it. The students were very comfortable working independently. They were not helping each other as much as the activities required. They did not have enough discussions and sharing of the information on the worksheets. Some students copied off other students' answers without fully understanding the concepts.

During the experiment, the students were divided into two groups of 14 (experimental) and 15 (control) students to compare the two teaching methods. The teacher of each group was able to give more attention to the students in the smaller groups. After the training, more than one student, from both the experimental and control groups, inquired about the possibility of having two groups for a longer period of time. It is advisable to plan math classes to include smaller number of students. This allows the teacher to be able to follow-up on the progress of each student and prepare activities that accommodate his/her needs.

Teacher Role

Teaching the experimental group required more preparation outside of class and facilitation of the learning during class. Since the activities were prepared in a way for the students to work on their own, the teacher had to get all the materials ready before class. The agenda for each class had to be set carefully to include the planned activities and their timing. There was much less direct instruction during class time. The role of the teacher was more of a facilitator making sure the students were on the right track with the activities. The closure of the lesson was very important because the tie-in to the objective had to be made very clear.

Reflections on Implementation

To prepare the activities for this unit, the researcher spent many hours and gained a much deeper, more connected understanding of the concepts than she had had before. The result was two well-prepared unit plans for the control and the experimental groups. This kind of preparation cannot be done for all the units all the time. However, it is beneficial to be attempted at least for the units that contain content that is difficult for the students to understand, yet crucial for their mathematical knowledge.

To accommodate all the activities, the students spent 17 days on the linear function unit, instead of the usual 12 days allocated to such a unit. The more time the students spent learning contents the more they gained understanding of the concepts and mastery of the skills taught. The five extra days allowed the students more chance to comprehend the theory and practice the operations. It is advisable to limit the units taught to the students and give them more time to absorb the material.

Some revisions are needed to get maximum benefits from the activities. As the students worked on the activities, it was clear that sometimes the directions or questions were not very clear. As the activities are revised, it is conceivable that students will be able to achieve more benefits from them, and consequently gain better understanding of linear functions.

Reflections on the Results

Translation to Graphs and Equations

The experimental group mastered the translation to graphs much better than their counterparts in the control group. This is not surprising since most of the activities for the experimental group contained coordinate planes to provide opportunity for the students to graph. The use of graphs was integrated throughout the unit with all the other concepts. This was not the case for the control group who learned graphing as a lesson on its own.

It is not surprising that all the students mastered translation to graphs more than translation to equations. As a teacher, the researcher has always observed this trend. This was especially true for the experimental group. Allowing them more chance to graph throughout the unit has proven successful. However, their ability to translate to equations did not improve in the same magnitude. The activities used for this research were not as successful in regard to translating to equations as it was to graphs. More research is needed on activities that can help the students better understand linear equations and writing them. It must be possible to improve students' abilities to translate to equations provided they have opportunity to gain understanding of them in a suitable way.

Correlation Between Mathematical Truths and Representations

The results show a strong positive correlation between understanding of mathematical truths and representation translation ability. This suggests that the students who learned to translate between the different representations, namely numerical, graphical, symbolic, and verbal, also had better understanding of the mathematical truths and mastery of the skills related to linear functions. Hence, students appeared to gain the two abilities simultaneously. It is quite conceivable that one of them affects the other, especially in light of other researchers' results about the positive effects of students gaining representation translation abilities (Karsenty, 2002; Preston and Garner, 2003; Thornton, 2001). This could be a topic for future research.

Future Directions for Research

An obvious sequel to this study would be another one that avoids the pitfalls that were encountered here, namely activities revision and better student preparation. The parts of the activities that need revision are the questions that lead to the generalizations and word problems. Students need preparation before starting the experiment in two main areas. First, they need to be familiar with learning using the inductive approach. The teacher can introduce similar activities in the period preceding the experiment with different content. Consequently, at the time of the study, the main focus will be the new concepts to be studied. Second, students need preparation in working in a collaborative group work setting. Again this needs to be introduced before the experiment to ensure all the students get the maximum benefit of the learning in this environment.

Another area of research is to devise a similar training that focuses on increasing the students' achievement in writing equations. Linear equations are the first type of equations that students learn, yet they are usually quite challenging for them. Later they work with quadratic equations and higher order ones. It could be that students need a lot more opportunity to write equations and get information out of them. Methods need to be devised that facilitate understanding and writing equations. This is an area that needs further research.

Other research is needed in other units that algebra teachers determine the students have difficulty with, especially if this content is a factor for students' success in more advanced math classes. After the units are identified, similar activities need to be devised and tested for their benefits in helping the students' comprehension and achievement. The first-year algebra course provides foundations for the students' subsequent math classes in high school and college. It is necessary to identify the areas that will ensure that the foundation will be solid.

Two issues that seemed to influence this study were forming smaller classes of students and allocating more time to study the concepts. Further studies in these areas would be beneficial to determine the real effect of these factors, if any.

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Appendix A

Linear Function Unit Plan

Course: Algebra I
Unit No: 6 out of 13
Title: Analyzing Linear Equations
Goals: Understand patterns, relations, and functions. Represent and analyze mathematical situations and structures using algebraic symbols. Use mathematical models to represent and understand quantitative relationships. Analyze change in various contexts.

Weighted Objectives:

15% A. Find the slope of a line given coordinates of two points on the line

15% B. Determine the x- and y-intercepts of linear graphs from their equations (6.0)

Given a linear relationship, it has a unique slope and a unique intercept; and we can find them from the equation, from a table of values, from a graph or from a verbal description that contains sufficient information.(Mathematical Truth 1)

10% C. Graph a line given any linear equation (6.0)

20% D. Verify that a point lies on a line, given an equation of the line. Derive linear equations by using the [slope-intercept and standard] formula. (7.0)

A linear relationship has a unique equation in standard form and a unique equation in slope-intercept. A standard form equation can only be derived from another equation of the line. (Mathematical Truth 3)

5% E. Understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Find the equation of a line parallel or perpendicular to a given line that passes through a given point. (8.0)

15% F. Students solve multistep problems, including word problems, involving linear equations in one variable and provide justification for each step (5.0)

20% G. Understand linear functions and their properties (e.g., slope). Solve linear equations, and represent solutions on graphs. Create and interpret graphs of linear equations.

Given a linear relationship, its ordered pairs can be expressed using an equation, a table of values, a graph, or a verbal description. Specifically:

- *Points on the graph satisfy the equation*
- *Points on a table have constant rise over run (mathematical Truth 2)*

Estimated Number of Class Periods: 17 days
Textbook Page References: Sections 6.1, 6.4 – 6.6
Unless otherwise indicated, page numbers referred to in the overall lesson plan are from the course textbook: Algebra By Glencoe.

Quiz 1: slopes and intercepts
Quiz 2: writing and graphing linear equations
Unit Exam: Comprehensive

Unit Text and Activity References

Obj	Text pp.	Examples	Exercises	Activities
A	p. 325 – 329	Lesson 6.1 # 1, 2, 3	p. 329 # 8, 10, 16-28 (even), 30-39	I, II, VI
B	p. 346	Lesson 6.4 # 1	p. 350 # 8, 9, 22-27, 40-45 p. 353 # 1-4, 5-7 (find intercepts), 9, 10	III, IV, VI
C	p. 356 – 359	Lesson 6.5 # 2	p. 359 # 9, 10, 17-20, 23, 25-29, 32, 33 graph y=5, y=-1, x=7, x=-2	
D	p. 347 – 349	Lesson 6.4 # 2, 3, 4	p. 350 # 12-16, 34-45 p. 770 # 19-24	V
E	p. 362 – 365	Lesson 6.6 # 1, 2, 3, 4, 5	p. 367 # 18-23, 24-46 (even), 43 – 49 (odd)	VIII, IX
F	Throughout the chapter	Lesson 6. 4 # 5	p. 350 # 18, 56 p. 366 # 17, 47, 49 p. 378 # 66	III (part 3), IV (part 4), VI (part 6)
G	Throughout the chapter	Lesson 6.1 # 4 Lesson 6.5 # 4	p. 359 # 13, 37, 38 p. 378 # 67	V

Activity I: Given graph, find slope
Activity II: Given table of values, find slope
Activity III: Given graph, find x- and y-intercepts
Activity IV: Given table of values, find x- and y-intercepts and points on the line
Activity V: Graph and equation of a line
Activity VI: Given equation of a line, find points, slope and y-intercept
Activity VII: Equations in standard form
Activity VIII: Parallel lines
Activity IX: Perpendicular lines

Experimental Group Plan

Monday	Tuesday	Wednesday	Thursday	Friday
Nov 29 Activity I- 1, 2	**30** Activity II- 2, 3	**Dec 1** Discuss slope Activity III- 1, 2	**2** Activity IV- 1, 2	**3** Graph a line given a point and the slope
6 No Class	**7** No Class	**8** **Quiz 1** Activity V	**9** Activity VI- 2, 3	**10** Activity VI- 5, 6
13 Lesson 6.4 Slope-intercept form of linear eq	**14** Activity VII Standard form	**15** Lesson 6.5 Graphing linear equations	**16** Reading Day No class	**17** Vertical and Horizontal lines
20 **Quiz 2** Activity VIII – 1, 2	**21** Finish Activity VIII	**22** Activity IX – 1,2	**23** Review	**24** **Unit Exam**

Assignments (Experimental Group):

Homework 1: Activity II- 1
Homework 2: Activity II- 4
Homework 3: slope and intercepts p. 329 # 15-26, p. 350 # 7, 19, 20, 21 (a, b)
Homework 4: Activity IV- 2, 3 and p. 330 # 30-38
Homework 5: p. 769 # 2-20 (even)
Homework 6: Activity VI- 1
Homework 7: Activity VI- 4
Homework 8: p. 350 # 12-14 and p. 351 # 34-39
Homework 9: p. 350 # 19-21 p. 351 # 28-33, 40-45, 49, 52 (slope-intercept form)
Homework 10: p. 770 # 7-12 and 19-24
Homework 11: p. 359 # 14-16, 20, 21, 23, 24, 26, 27, 28
Homework 12: p. 769 (lesson 6.1 # 4, 5), (lesson 6.2 # 4, 10), (lesson 6.4 #4, 6), graph $y=5$, $y=-1$, $x=7$ and $x=-2$.
Homework 13: Activity VIII- 3, 4
Homework 14: p. 367 # 18-32, 43, 45
Homework 15: p. 367 # 33-42, 44, 46 and p.771 #1-6
Homework 16: p. 792 # 1-24

Daily Lesson Plan (Experimental Group)

Day 1

Objectives:

- Intro to the unit
- Given graph, find rise, run, slope
- Find definition of slope

Activity I (parts 1, 2)

Homework: A1 Activity II (part 1)

Day 2

Objectives:

- Given table of values, find slope
- Understand property of slope

Activity II (parts 2, 3)

Homework: A2 Activity II (part 4)

Day 3

Objectives:

- Introduce x- and y-intercepts
- Given graph, find x- and y-intercepts

Activity III (parts 1, 2)

Homework: A3 p. 329 # 15-26, p. 350 # 7, 19, 20, 21 (a, b)

Day 4

Objectives:

- Given table of values, find x- and y-intercepts
- Given tables of values, find other points on line

Activity IV (part 1)

Homework: A4 Activity IV (parts 2, 3) and text p. 330 # 30-38

Day 5

Objectives:

- Graph a line given a point and the slope
- Review and prep for quiz

Homework: A5 text p. 769 # 2-20 (lesson 6-1)

Day 6 – Quiz 1

Objectives:

- Given a linear equation and a point, determine if the point satisfies the equation

Activity V (parts 1, 2)

Homework: A6 Activity VI (part 1)

Day 7
Objectives:
- Given equation of a line, find points on the line
- Given equation of a line, find slope

Activity VI (parts 2, 3)
Homework: A7 Activity VI (part 4) and p. 350 # 12-14 and 34-37 (find slope)

Day 8
Objectives:
- Given equation of a line, find y-intercept
- Solve problems involving linear situations

Activity VI (parts 5, 6)
Homework: A8 p. 350 # 12-14 and p. 351 # 34-39

Day 9
Objectives:
- introduce the slope-intercept form of a linear equation: $y = mx + b$
- find the equation for the following situations (graphically and algebraically):
  - given the slope and the y-intercept
  - given the y-intercept and another point
  - given any two points

Homework: A9 P. 350 # 19-21, p. 351 # 28-33, 40-45, 49, 52 (slope-intercept form)

Day 10
Objectives:
- introduce the standard form of a linear equation: $Ax + By = C$
- change any linear equation to standard form

Activity VII
Homework: A10 p. 770 # 7-12, 19-24

Day 11
Objectives:
- given a slope-intercept form, graph the line
- given a standard form
  - find intercepts and graph
  - find two points and graph
  - change to slope-intercept and graph

Homework: A11 p. 359 # 14-16, 20, 21, 23, 24, 26, 27, 28

Day 12
Objectives:
- Determine slope of horizontal lines
- Graph any horizontal line
- Write equation of a horizontal line
- Determine slope of vertical lines
- Graph any vertical line
- Write equation of a vertical line

Homework: A12 p. 769 (lesson 6.1 # 4, 5), (lesson 6.2 # 4, 10, 11), (lesson # 6.4 # 4, 6)
Graph y = 5, y = -1, x = 7 and x = -2
For review p. 376 # 25-30, 34, 35, 42, 44, 45

Day 13 – Quiz 2
Objectives:
- Determine slopes of parallel lines
- Write equation of a parallel line graphically

Activity VIII (part 1, 2)
Homework: A13 Activity VIII (part 3, 4)

Day 14
Objectives:
- Write equation of a parallel line algebraically

Homework: A14 p. 367 # 18-32, 43, 45

Day 15
Objectives:
- Determine slopes of perpendicular lines
- Write equation of a perpendicular

Activity IX (parts 1, 2, 3, 4)
Homework: A15 p. 367 # 33-42, 44, and p.771 (lesson 6-6) #1-6

Day 16
Objectives:
- review and prepare for unit exam

Homework: A16 p. 792 # 1-24

Day 17 – Unit Exam

Control Group Plan

Monday	Tuesday	Wednesday	Thursday	Friday
Nov 29 Define and graph slope Calculate slope	**30** +ve & -ve slope find pts given slope and a pt	**Dec 1** Problem solving with slope	**2** define x- and y-intercepts find intercepts from eq and pts	**3** review for quiz
6 No Class	**7** No Class	**8** **Quiz 1** Introduce slope-int form of eq	**9** given linear eq, determine slope and y-int	**10** Given 2 pts, write linear eq
13 more practice writing eq in slope-int and standard forms	**14** graph line given pts, slope & pt, slope-int eq	**15** graph line given any equation	**16** Reading Day No Class	**17** Vertical & horizontal lines
20 **Quiz 2** Parallel lines	**21** Parallel lines	**22** Perpendicular lines	**23** Applications Review	**24** **Unit Exam**

Assignments (Control Group):

Homework 1: p. 329 # 8, 10, 16- 28 (even)
Homework 2: p. 330 # 30-39 (all)
Homework 3: p.330 # 40-43
Homework 4: p. 350 # 8, 9, 22-27, 40-45 (find slope and y-int)
Homework 5: p. 353 # 1-4, 9, find y-int for # 5-7, 10
Homework 6: p. 350 # 10, 11, 28-33
Homework 7: p. 350 # 12-14, 34-39
Homework 8: p. 350 # 15, 16, 40-45
Homework 9: p. 770 # 19-24
Homework 10: p. 359 # 9, 10, 17-20
Homework 11: p. 360 # 23, 25-29, 32, 33
Homework 12: p. 769 (lesson 6.1 # 4, 5), (lesson 6.2 # 4, 10), (lesson 6.4 #4, 6), graph $y = 5$, $y = -1$, $x = 7$ and $x = -2$.
Homework 13: p. 367 # 18-20 (all)
Homework 14: p. 367 # 24-32 (even), 43, 45
Homework 15: p. 367 # 21-23 (all), 34-46 (even), 47, 49
Homework 16: p. 792 # 1-24

Daily Lesson Plan (Control Group)

Day 1

Objectives:

- define and graph slope
- find slope, given two points
- find slope, given graph of a line

Examples: p. 326 # 1, 2

Homework: A1 p. 329 # 8, 10, 16-28 (even)

Day 2

Objectives:

- understand when is slope +ve, -ve, zero, or undefined
- given slope and a point, find other points on the line

Examples: p. 327 # 3

Homework: A2 p. 330 # 30-39 (all)

Day 3

Objectives:

- solve problems involving slope

Examples: p. 328 #4

Homework: A3 p. 330 # 40-43

Day 4

Objectives:

- define x- and y-intercepts of a line
- find x- and y-intercepts, given equation of a line
- find x- and y-intercepts, given slope and a point or 2 points

Examples: p. 346 #1

Homework: p. 350 # 8, 9, 22-27, 40-45 (find slope and y-intercept)

Day 5

Objectives:

- review for quiz (slope and intercepts)

Homework: p. 353 # 1-4, 9, find y-intercept for # 5-7, 10

Day 6 – Quiz 1

Objectives:

- introduce slope-intercept form of a linear equation $y = mx + b$
- given slope and y-intercept, write linear equations in slope-intercept

Examples: p. 347 # 2

Homework: p. 350 # 10, 11, 28-33

Day 7

Objectives:

- given linear equation in slope-intercept or standard form, determine slope and *y*-intercept
- review standard form
- review solve for *y*

Examples: p. 348 # 3

Homework: p. 350 # 12-14, 34-39

Day 8

Objectives:

- given the *y*-intercept and a point, write equation in slope-intercept form
- given any two points, write equation in slope-intercept form
- write an equation in slope-intercept form in standard form

Examples: p. 348 # 4 (method2)

Homework: p. 350 # 15, 16, 40-45

Day 9

Objectives:

- more practice writing equations
- find *y*-intercept from slope definition and from slope-intercept equation

Homework: p. 770 # 19-24

Day 10

Objectives:

- Graph line, given two points
- Graph line, given slope and any point
- Graph line, given slope and *y*-intercept

Examples: p. 357 # 2, 3

Homework: p. 359 # 9, 10, 17-20

Day 11

Objectives:

- Graph line, given equation in any form
- Graph line, given intercepts

Examples: p. 357 # 2, 3

Homework: p. 360 # 23, 25-29, 32, 33

Day 12
Objectives:
- Equations of vertical and horizontal lines
- Graphs of vertical and horizontal lines

Examples: graph $y=3$ and $x=4$
Homework: p. 769 (lesson 6.1 # 4, 5), (lesson 6.2 # 4, 10), (lesson 6.4 #4, 6), graph $y=5$, $y=-1$, $x=7$ and $x=-2$.

Day 13 – Quiz 2
Objectives:
- Determine if two lines are parallel

Examples: p. 363 # 1
Homework: p. 367 # 18-20 (all)

Day 14
Objectives:
- Write equation of a line parallel to a given line

Examples: p. 363 # 2, 3
Homework: p. 367 # 24-32 (even), 43, 45

Day 15
Objectives:
- Determine if two lines are perpendicular
- Write equation of a line perpendicular to a given line

Examples: p. 364 # 4, 5
Homework: p. 367 # 21-23 (all), 34-46 (even), 47, 49

Day 16
Objectives:
- Solve problems involving linear situations
- Review and prepare for unit exam

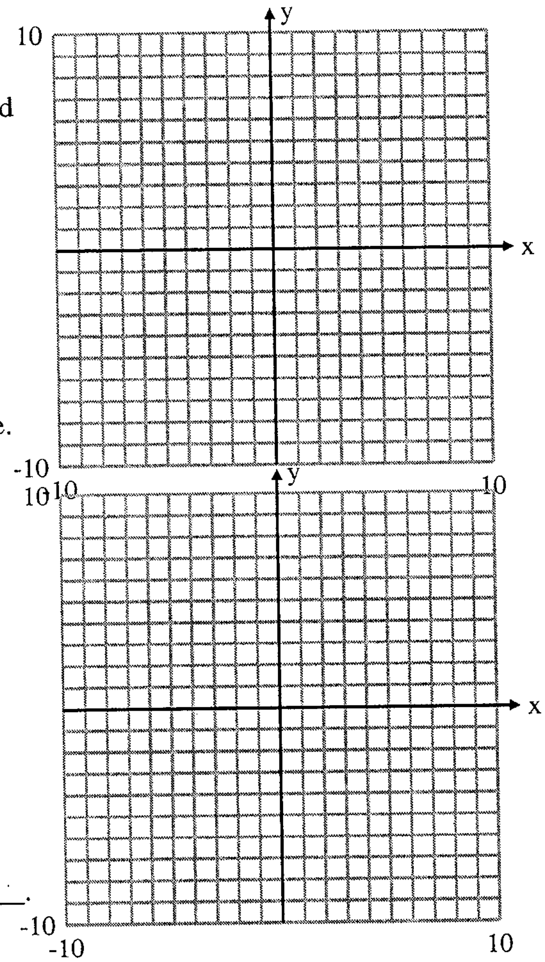
Homework: p. 792 # 1-24

Day 17 – Unit Exam

Appendix B

Inductive Reasoning Activities

Activity I: Part 1 (Given graph, find slope)



1. On the coordinate plane, plot the points (-1, 4) and (1, 1).
2. You will need to move from the first point to the second according to the following conditions:
   a. follow gridlines
   b. move only to the right, and up and down
3. Answer the following questions about the move:
   a. How many units to the right did you move? ______ This number is the **run** of your move.
   b. Did you move up or down? __________ The "up" movement is positive and the "down" movement is negative.
   c. How many units did you move vertically? _______ This number with its sign is the **rise** of your move.

Examples:
Plot the following points then find the rise and run of each pair.

a. (-3, 5) and (2, 10)

   The rise is ________, and the run is ________.

b. (-5, 0) and (7, 3)

   The rise is ________, and the run is ________.

Using the same points, fill the following table:

Point 1	Point 2	Difference between y-coordinates	Difference between x-coordinates	Rise	Run
(-1, 4)	(1, 1)	1 – 4 = -3	1 – (-1) = 2		
(-3, 5)	(2, 10)				
(-5, 0)	(7, 3)				

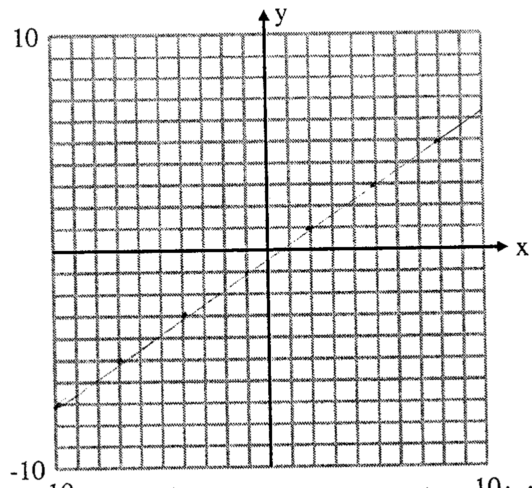
Give a definition of the **rise**:

____

Give a definition of the **run**:

____

Activity I: Part 2
Examine the graph of the line on the coordinate plane. Locate six points on the line and write down their coordinates:



A (,) B (,)

C (,) D (,)

E (,) F (,)

Connect Point 1 to Point 2 on the coordinate plane according to the conditions of the last activity. Find rise and run for each pair of points, fill in the following table, then answer the questions:

Point 1	Point 2	Rise	Run	Rise/Run
A (,)	B (,)			
B (,)	C (,)			
C (,)	D (,)			
D (,)	E (,)			
E (,)	F (,)			

What are your observations about the triangles formed by the rise and run in the graph?

____

____

What are your observations about the quotient of the rise by run?

____

____

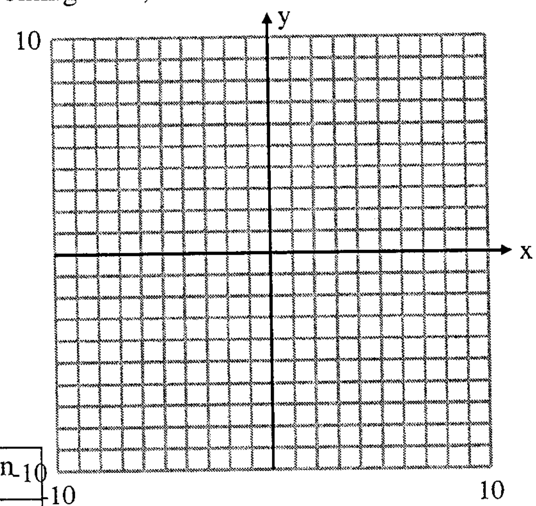
The quotient rise by run is called the slope. Give a definition of the slope.

____

Activity II: Part 1 (Given table of values, find slope)

For each of the following tables of values,

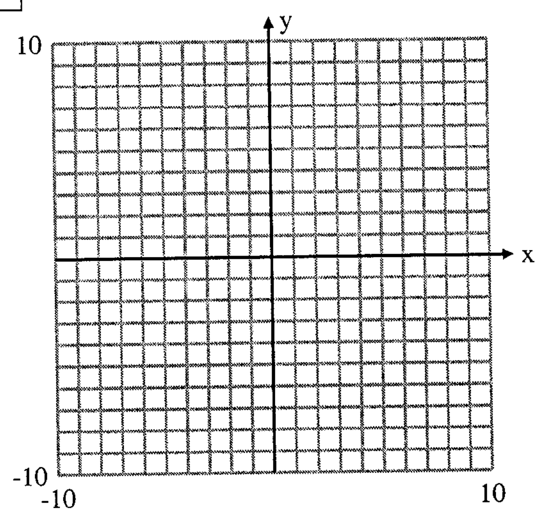
1) plot the points on the graph, connect the line joining them, draw rise and run
2) fill the rise and run table



a)

x	y
1	-1
2	2
3	5
4	8
5	11

1st pt	2nd pt	rise	run	rise/run
(1,-1)	(2, 2)	2 – (-1) = 3	2 – 1 =1	3/1= 3
(2, 2)	(3,5)			
(3,5)	(4,8)			
(4,8)	(5,11)			



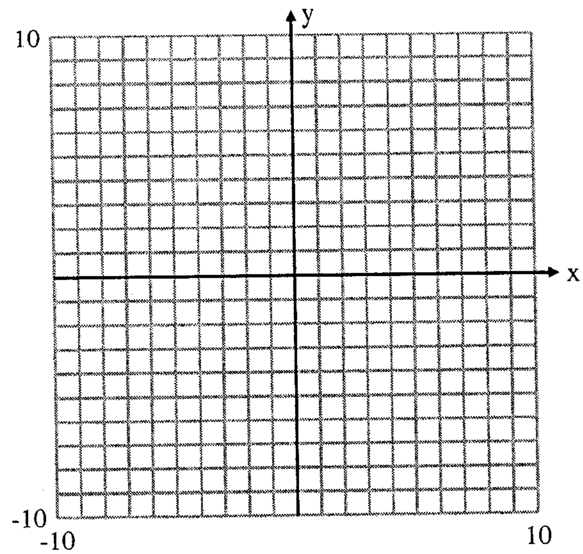
b)

x	y
3	5
4	3
5	1
6	-1
7	-3

1st pt	2nd pt	rise	run	rise/run
(3, 5)	(4, 3)	3 – 5 = -2	4 – 3 = 1	-2/1= -2
(4, 3)	(5, 1)			
(5, 1)	(6,-1)			
(6, -1)	(7,-3)			

c)

x	y
-3	1
-2	2
-1	3
0	4
1	5



1^{st} pt	2^{nd} pt	rise	run	rise/run
(-3, 1)	(-2, 2)	2 – 1 =1	-2 – (-3)=1	1/1 = 1
(-2, 2)	(-1, 3)			
(-1, 3)	(0, 4)			
(0, 4)	(1, 5)			

What are your observations about the rise from the tables? From the graphs?

____

What are your observations about the run from the tables? From the graphs?

____

What are your observations about the quotient of rise/run from the tables? From the graphs?

____

Do you think you have enough information to generalize to all lines? Explain.

____

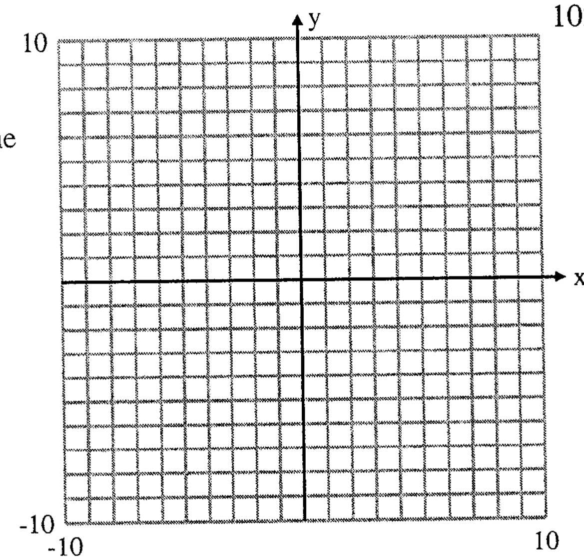
____

____

Activity II: Part 2

For each of the following tables of values,

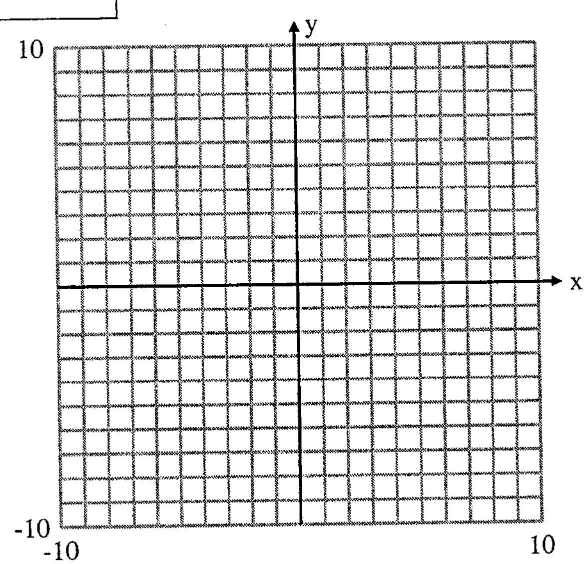
1) plot the points on the graph, connect the line joining them, draw rise and run
2) fill the rise and run table



a)

x	y
0	-1
2	2
4	5
6	8
8	11

1^{st} pt	2^{nd} pt	rise	run	rise/run
(0, -1)	(2, 2)	2 – (-1) = 3	2 – 0 = 2	3/2 = 3/2
(2, 2)	(4, 5)			
(4, 5)	(6, 8)			
(6, 8)	(8, 11)			



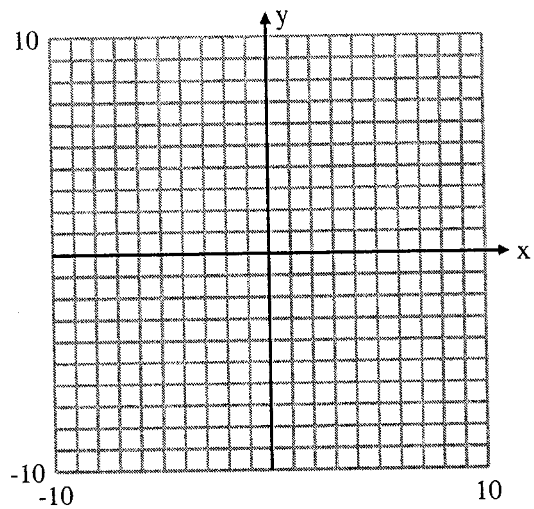
b)

x	y
-5	1
-1	3
3	5
7	7
11	9

1^{st} pt	2^{nd} pt	rise	run	rise/run
(-5, 1)	(-1, 3)	3 – 1 = 2	-1 – (-5) = 4	2/4 = 1/2
(-1, 3)	(3, 5)			
(3, 5)	(7, 7)			
(7, 7)	(11, 9)			

c)

x	y
0	9
2	5
4	1
6	-3
8	-7



1st pt	2nd pt	rise	run	rise/run
(0, 9)	(2, 5)	5 – 9 = -4	2 – 0 = 2	-4/2= -2
(2, 5)	(4, 1)			
(4, 1)	(6, -3)			
(6, -3)	(8, -7)			

What are your observations about the rise from the tables? From the graphs?

What are your observations about the run from the tables? From the graphs?

What are your observations about the quotient of rise/run from the tables? From the graphs?

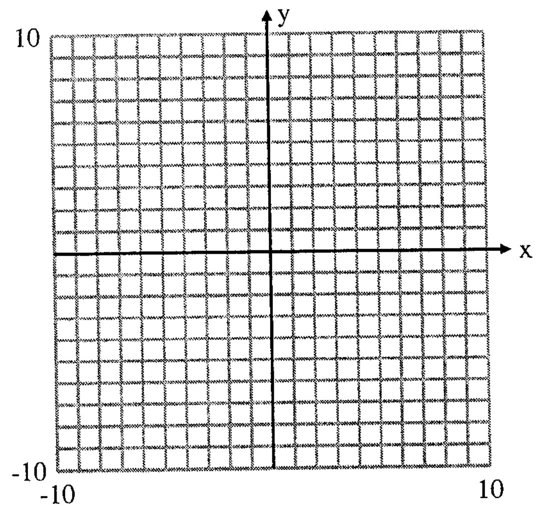
Did your observations change after this activity? Explain.

Do you think you have enough information to generalize to all lines? Explain.

Activity II: Part 3

For each of the following tables of values,

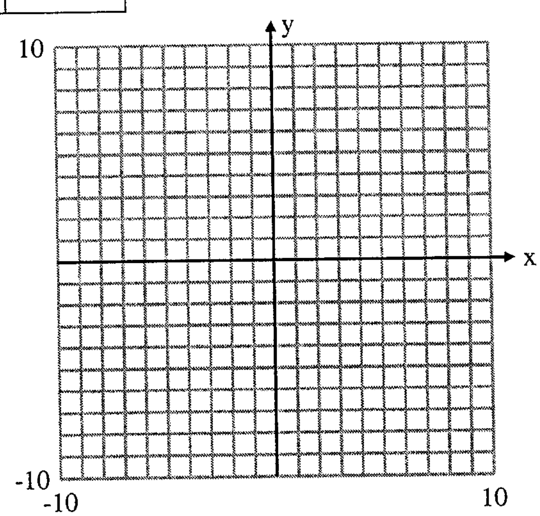
1) plot the points on the graph, connect the line joining them, draw rise and run
2) fill the rise and run table



a)

x	y
-4	-7
-1	-1
0	1
3	7
5	11

1^{st} pt	2^{nd} pt	rise	run	rise/run
(-4, -7)	(-1, -1)	-1 – (-7) = 6	-1 – (-4) = 3	6/3 = 2
(-1, -1)	(0, 1)			
(0, 1)	(3, 7)			
(3, 7)	(5, 11)			



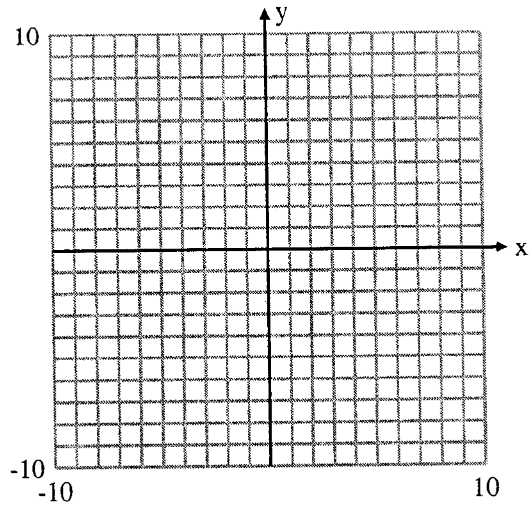
b)

x	y
-2	4
0	2
1	1
5	-3
11	-9

1^{st} pt	2^{nd} pt	rise	run	rise/run
(-2, 4)	(0, 2)	2 – 4= -2	0 – (-2) = 2	-2/2 = -1
(0, 2)	(1, 1)			
(1, 1)	(5, -3)			
(5, -3)	(11, -9)			

c)

x	y
-3	10
-2	7
1	-2
3	-8
4	-11



1^{st} pt	2^{nd} pt	rise	run	rise/run
(-3,10)	(-2, 7)	7 – 10= -3	-2 – (-3) = 1	-3/1 = -3
(-2, 7)	(1, -2)			
(1, -2)	(3, -8)			
(3, -8)	(4, -11)			

What are your observations about the rise from the tables? From the graphs?

____

What are your observations about the run from the tables? From the graphs?

____

What are your observations about the quotient of rise/run from the tables? From the graphs?

____

Did your observations change after this activity? Explain.

____

____

Do you think you have enough information to generalize to all lines? Explain.

____

____

Activity II: Part 4
Given any two points from a line in parts 1, 2 and 3, fill the table below.

	pt 1	pt 2	rise	run	rise/run	Previously calculated rise/run
1 a)	(1, -1)	(5,11)				
1 b)	(4, 3)	(6,-1)				
1 c)	(-2,2)	(1,5)				
2 a)	(6, 8)	(2, 2)				
2 b)	(-1,3)	(11,9)				
2 c)	(2, 5)	(8,-7)				
3 a)	(3,7)	(-4, -7)				
3 b)	(1,1)	(-2, 4)				
3 c)	(-2, 7)	(4, -11)				

Examine the last two columns of the table. How do they compare to each other?

____

____

What are your observations about the quotient of rise/run from the tables? From the graphs?

____

____

Did your observations change after this activity? Explain.

____

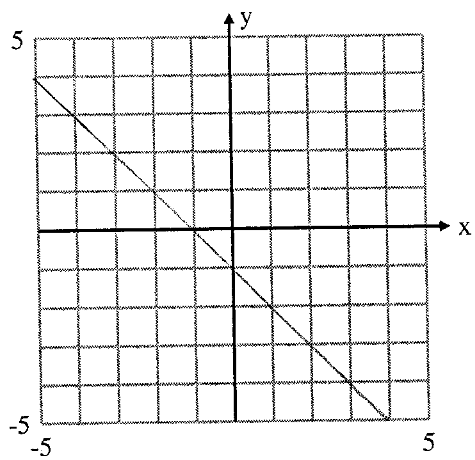
____

Do you think you have enough information to generalize to all lines? Explain.

____

____

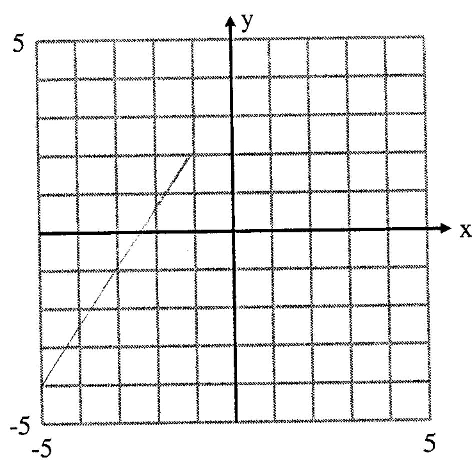
Activity III: Part 1(Given graph, find intercepts)



The y-intercept of a line is the point that intersects the y-axis. Observe the lines in the graphs, and determine the y-intercept in each one. Write the coordinates of the y-intercept.

y-intercept: ______________________________

If you are given part of a line that does not intersect the y-axis, you can use your knowledge of rise and run to extend the line and locate the y-intercept.



Exercise 1:

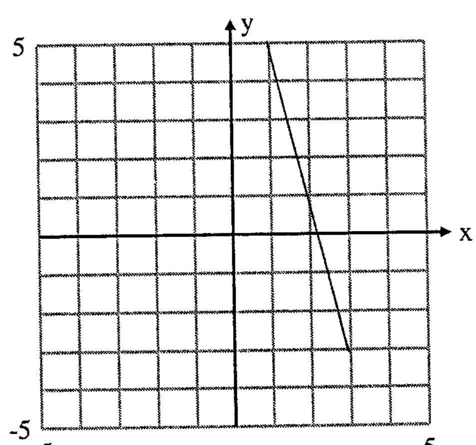
Rise: ____________ Run: ________

y-intercept: ______________________________

Explain your method:

____

____



Exercise 2:

Rise: ____________ Run: ________

y-intercept: ______________________________

Explain your method:

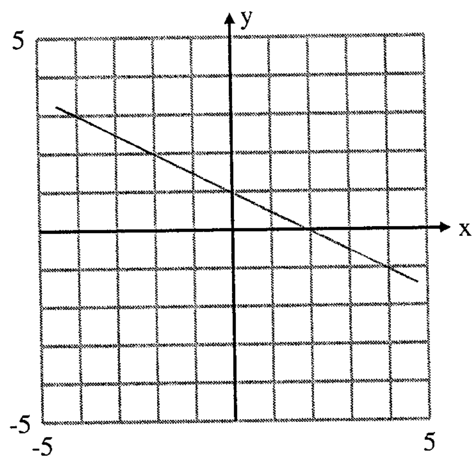
____

____

How many y-intercepts can a line have? Explain.

____

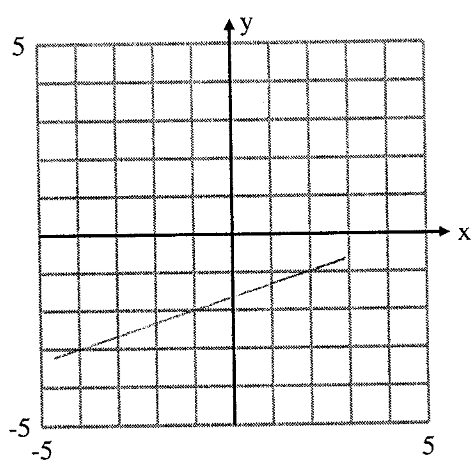
Activity III: Part 2



The x-intercept of a line is the point that intersects the x-axis. Observe the lines in the graphs, and determine the x-intercept in each one. Write the coordinates of the x-intercept.

x-intercept:: ______________________________

If you are given part of a line that does not intersect the x-axis, you can use your knowledge of rise and run to extend the line and locate the x-intercept.



Exercise 1:

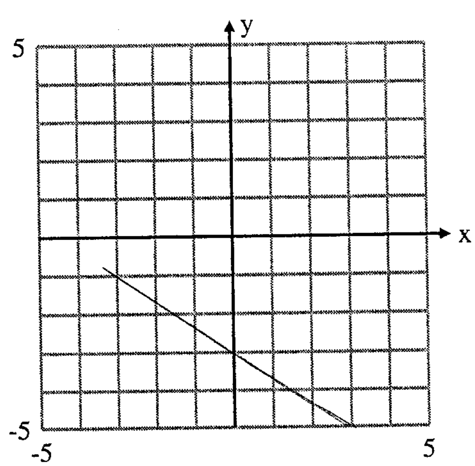
Rise: ____________ Run: ________

x-intercept: ______________________________

Explain your method:

____

____



Exercise 2:

Rise: ____________ Run: ________

x-intercept: ______________________________

Explain your method:

____

____

How many x-intercepts can a line have? Explain.

____

Activity III: Part 3

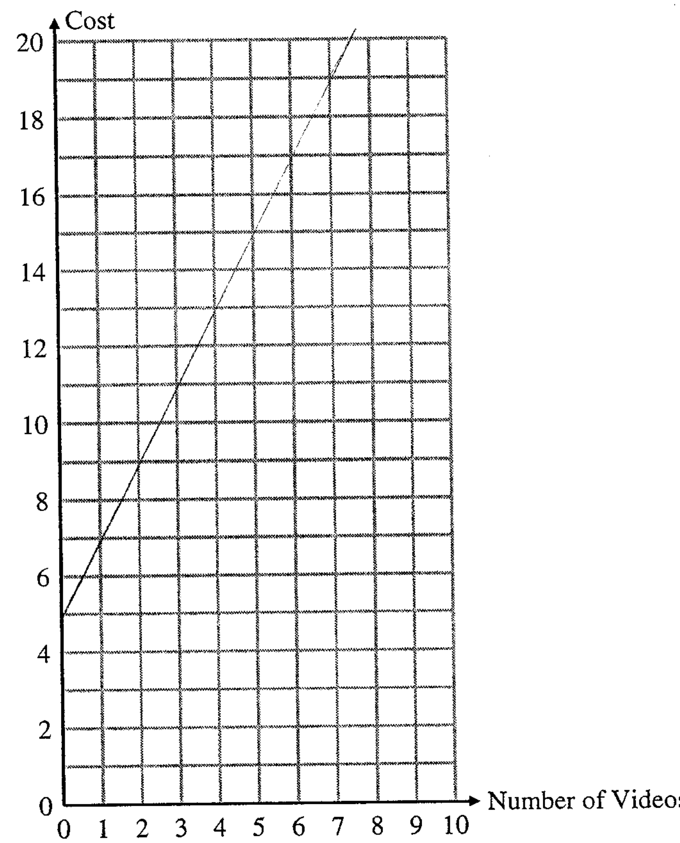
Video Rental

The graph at the right represents the cost of renting videos at BuckBuster Video Store. To rent videos, you need to be a member of BuckBuster for an initial yearly cost. There is a cost every time the member rents a video.

Your task is to answer the following questions:

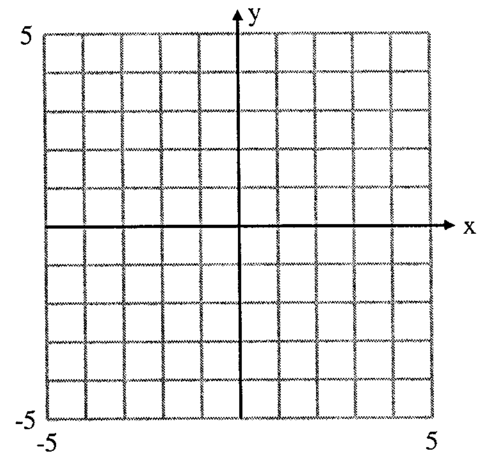
- What is the initial yearly cost?

- What is the cost for renting one video for any member?



Activity IV: Part 1 (Given table of values, find *x*- and *y*- intercepts and other points)

x	y
-1	-4
3	4
☐	0
0	☐



Steps for estimating the intercepts on the graph:

1. Plot the given points
2. Connect the unique line joining the points
3. Estimate the point that intersects the y-axis, the y-intercept is ____________.
4. Estimate the point that intersects the x-axis, the x-intercept is ____________.

Steps for finding the x- and y-intercepts from the table of values, algebraically:

1. use the given points to find the unique rise/run of the line.
2. use the rise /run to find the intercepts

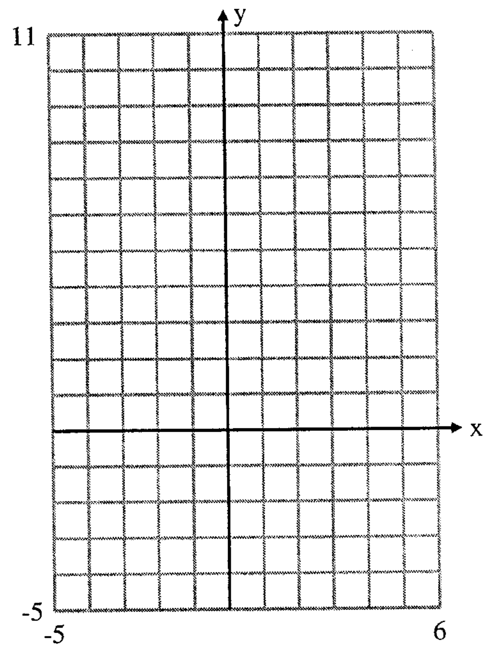
Steps	**Point 1**	**Point 2**	**rise**	**run**	**rise/run**
Find slope	(-1,-4)	(3,4)	4 – (-4) = 8	3 – (-1) = 4	8/4 = 2
x-intercept	(a, 0)	(-1, -4)	-4 – 0 = -4	-1 – a	$\frac{-4}{-1-a}=2$ $-4=2(-1-a)$ $-4=-2-2a$ $-2=-2a$ $1=a$
y-intercept	(0, b)	(3, 4)	4 – b	3 – 0 = 3	$\frac{4-b}{3}=2$ $4-b=6$ $-b=2$ $b=-2$

The x-intercept is (1,0) and the y-intercept is (0, -2).

Exercise:

Find the intercepts of the line containing the following points:

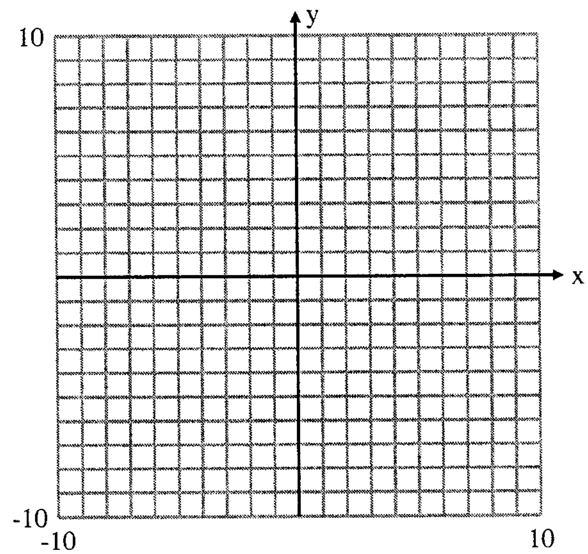
x	y
-2	11
3	-4
a	0
0	b



Steps	Point 1	Point 2	rise	run	rise/run
Find slope					
x-intercept ________					
y-intercept ________					

Activity IV: Part 2

x	y
-4	□
-2	4
0	□
1	-8
2	□
□	-3
□	-10



Steps for estimating the points on the graph:

1. Plot the given points
2. Connect the unique line joining those points
3. Explain how you would estimate the missing coordinate of (-4, □)

____

____

Describe your method of finding the y-coordinate of (2 , □):

____

____

Describe your method of finding the x-coordinate of (□, -3):

____

____

Describe your method of finding the x-coordinate of (□, -10):

____

____

Activity IV: Part 3

x	y
-4	□
-2	4
0	□
1	-8
2	□
□	-3
□	-10

Steps for finding the missing coordinate of any point from the table of values:

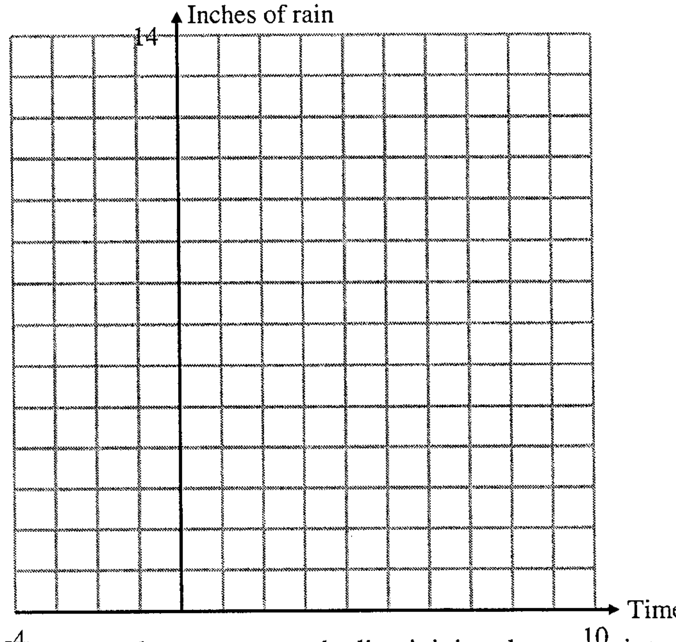
1. use the known points to find the unique rise/run of the line.
2. use the rise /run to find the missing coordinates of the points

Point 1	Point 2	rise	run	rise/run
(-2,4)	(1,-8)	-8 – 4 = -12	1 – (-2) = 3	-12/3 = -4
(-4, b)	(1,-8)	-8 – b	1 – (-4) = 5	$\frac{-8-b}{5}=-4$ $-8-b=-20$ $-b=-12$ $b=12$
(2, b)				
(a, -3)	(-2,4)	4 – (-3) = 7	-2 – a	$\frac{7}{-2-a}=-4$ $7=-4(-2-a)$ $7=8+4a$ $-1=4a$ $a=-\frac{1}{4}$
(a, -10)				

Activity IV: Part 4

Oh! So much rain

Three hours before noon, the number of inches of rain that had fallen was equal to 7 inches. Rainfall total increased hourly at a constant rate. When it was 9:00, It had rained a total of 11 inches.



1. Plot the given points in the graph and, use a ruler, to connect the line joining the two points.
2. From the graph, estimate the following points:

   a) At noon ____

   b) At 5:00 ____

   c) 10 inches of rain __________________________________

3. Calculate the rise/run from the two given points.

Use your answer to calculate:

<u>Amount of rain at noon</u>	<u>Amount of rain at 5:00</u>	<u>The time when it rained 10 inches</u>

Activity V: Part 1 (Graph and equation of line)
Given a linear equation and a point, determine if the point satisfy the equation.

Example:
Equation: $2x - y = 7$
Point (2, -1)

Substitute the x- and y- coordinates and check if the equation is true or not.
$2(2) - (-1) = 7$
$4 + 1 = 7$
False

Conclusion: The point (2, -1) does not satisfy the equation.

Exercise 1:
Equation: $2x + y = 5$
Point (1, 3)

Substitute:

Conclusion: ______________________________

Exercise 2:
Equation: $y = -3x + 2$
Point (1, 4)

Substitute:

Conclusion: ______________________________

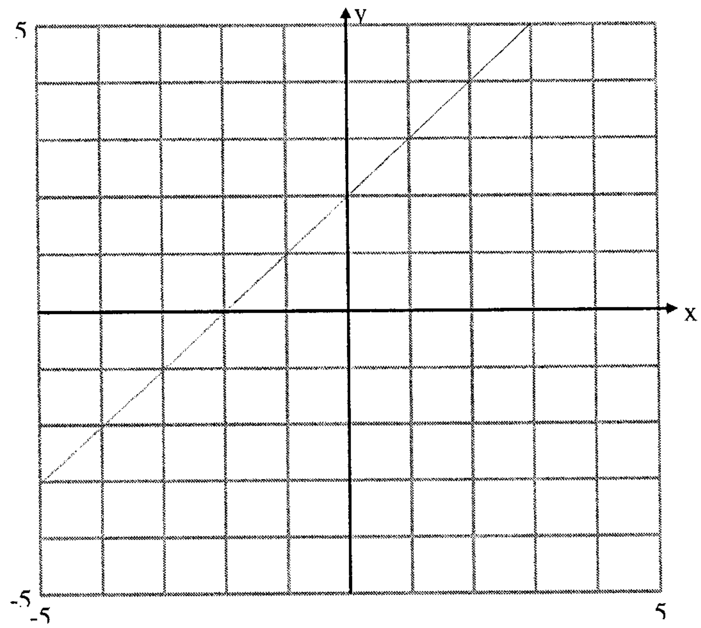
Exercise 3:
Equation: $-x + 3y = 11$
Point (2, -1)

Substitute:

Conclusion: ______________________________

Activity V: Part 2

The line that is plotted on the coordinate plane has the equation $y = x + 2$.



Fill the following table referring to the above graph:

Points	Is this point on the line in the graph?	Check whether the point satisfy the equation: $y = x + 2$	Equation satisfied?
(1,2)			
(0,1)			
(1,0)			
(1,3)			
(-2,0)			
(-1,3)			
(4,2)			
(2,4)			

Compare the second and last columns, what do you observe?

____

____

____

Activity VI: Part 1 (Given equation, find points, slope and intercepts)
Given equation of a line, find coordinates of some points on the line.

Exercise 1:

x-coordinate	$y = 3x+1$	Point
2	y = 3(2) +1 y = 6+1 y = 7	(2, 7)
-3		(-3,)
5		

y-coordinate	$y = 3x+1$	Point
10	10 = 3x +1 9 = 3x x = 3	(3, 10)
-2		(, -2)
4		

Exercise 2:

x-coordinate	$x+2y=5$	Point
1	1 + 2y = 5 2y = 4 y = 2	(1, 2)
-3		(-3,)
7		

y-coordinate	$x+2y=5$	Point
4	x + 2(4) = 5 x + 8 = 5 x = -3	(4, -3)
7		(, 7)
0		

Activity VI: Part 2
Review: coefficient of a variable is ____

Fill in the following table, then answer the questions that follow.

Equation	Coef. of x	Pt 1	Pt 2	Rise	Run	Rise/Run
y = 3x + 4	3	(1, 7)	(-2, -2)	-2 – 7= -9	-2 – 1 =-3	-9/-3= 3
y = 2x – 1						
y = -x + 5						
y = x						
y = -5x +7						

Examine the column of the coefficient of *x* and rise/run column. How do they compare?

____

____

What can you conclude from this observation about the slope (rise/run) of a line?

____

____

What is the slope of the line that has the following equation: $y = -2x + 7$? Explain how did you get your answer?

____

____

Activity VI: Part 3
Fill in the following table, then answer the questions at the bottom of the page.

Equation	Coef. of x	Pt 1	Pt 2	Rise	Run	Rise/Run
x + y = 5	1	(1, 4)	(3, 2)	2 – 4 = -2	3 – 1 = 2	-2/2 = -1
x = -2y						
y = 3x + 1						
x + 2y = 9						
y = 7						

Examine the column of the coefficients of *x* and rise/run column. For some equations, they are the same. What do you notice about those equations?

____

____

What can you conclude from this observation about the slope (rise/run) of a line?

____

____

Activity VI: Part 4
Given any form of a linear equation, find the slope.

Example: $x + y = 5$

Method # 1:
Find point 1: ______________

Find point 2: ______________

Calculate slope:

Method #2:
Write equation in the form y = ______

Solve for y: $x + y = 5$

$y = -x + 5$

Slope is coefficient of $x = -1$

Exercise 1: $x = -2y$

Method #1:
Find point 1: ______________

Find point 2: ______________

Calculate slope:

Method #2:
Write equation of the form y = ______

Solve for y: $x = -2y$

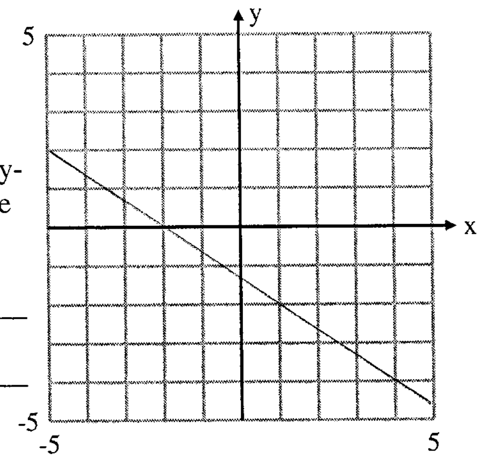
Which method do you prefer to use to find the slope? Explain why?

____

____

____

Exercise 2: Find the slope of the line: $x + 2y = 9$ using your preferred method.



Activity VI: Part 5

The y-intercept of a line is the point that intersects the y-axis. Use the graph to determine the x-coordinate of the y-intercept of any line. Explain.

____

____

Fill the table then answer the following questions:

Equation	Coefficient of x in equation	Constant in equation	Find y, when x=0	y-intercept
$y=3x-1$	3	-1	$y=3x-1$ $y=3(0)-1$ $y=0-1$ $y=-1$	(0, -1)
$y=2x+5$				
$y=7$				
$y=2x$				

Examine the column of the constant in the equation and y-intercept column. How do they compare?

____

____

What can you conclude from this observation about the y-intercept of a line?

____

____

What is the y-intercept of the line that has the following equation: $y=-2x+7$? ____________

Activity VI: Part 6

Selling cookies

Farid is making money selling boxes of cookies. He will sell cookies to raise funds for his Chess club. The equation that represents his income from the sales is $y = 0.5x + 4$, where x represents the number of boxes of cookies that he sells, and y represents his income.

Given the equation $y = 0.5x + 4$, find the following:

How much will Farid make if he sells 10 boxes of cookies?

How many boxes does he need to sell to make $20?

How much will he make if he sells 25 boxes?

How many boxes does he need to sell to make $50?

What is the slope of the equation $y = 0.5x + 4$? Explain what it represents in this problem.

What is the y-intercept of the equation $y = 0.5x + 4$? Explain what it represents in this problem.

Activity VII
Linear Equations in Standard Form
The following columns represent examples of linear equations written in standard form and non-examples that are not written in standard form.

Examples	Non-examples
	$y=3x-2$
$2x+3y=10$	$\frac{1}{2}x+y=8$
$x-7y=-3$	$-4x+2y=11$
$3x+y=0$	$y-4=3(x-7)$
$x=5$	$x=2y-6$
$y=-13$	

List the characteristics of a linear equation from the above examples:

____

____

____

____

According to your definition, determine which group the following equations belong to:

$y=-2x+5$	standard form	not standard form
$\frac{2}{5}x-3y=1$	standard form	not standard form
$3x-y=15$	standard form	not standard form
$-x+2y=9$	standard form	not standard form

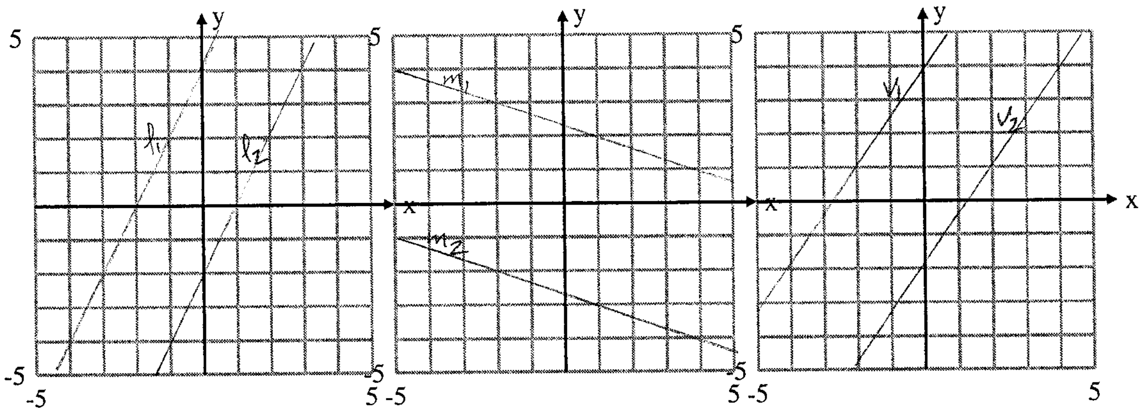
Write a linear equation in standard form and another that is not in standard form:

Standard form: ______________________________________

Not in standard form: ______________________________________

Change each equation that is not in standard form to standard form.

Activity VIII: Part 1 (Parallel lines)



Every coordinate plane contains two parallel lines. Find the slopes and intercepts of the pairs of parallel lines. Then answer the questions.

Lines	l_1	l_2	m_1	m_2	v_1	v_2
Slope						
Intercept						

l_1 and l_2 are parallel lines. What do you notice about their slopes?

____

What do you notice about their intercepts?

____

Is your findings consistent for m_1 and m_2? Explain.

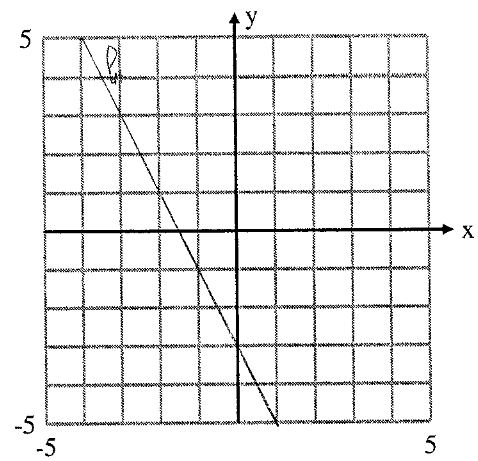
____

Is your findings consistent for v_1 and v_2 ? Explain.

____

Activity VIII:Part 2

1. Find the equation of the line l_1.



2. Graph a line that is parallel to the line l_1. Call this line l_2.

3. Explain how you graphed the line l_2 .

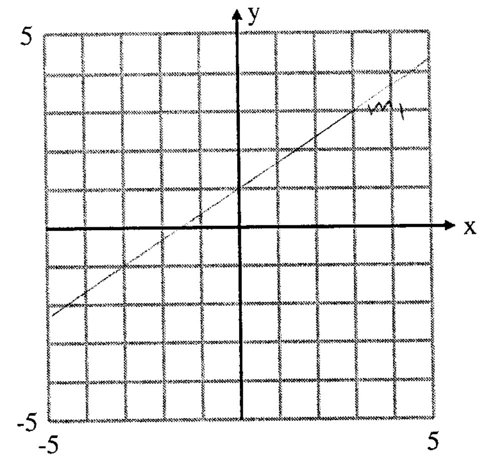
4. Find the equation of l_2.

5. Explain your method in finding this equation.

6. Describe the similarities and differences between the equations of l_1 and l_2.

Activity VIII:Part 3

1. Find the equation of the line m_1.



2. Graph a line that is parallel to the line m_1 and has intercept -4. Call this line m_2.

3. Explain how you graphed the line m_2 .

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4. Find the equation of m_2.

5. Explain your method in finding this equation.

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6. Describe the similarities and differences between the equations of m_1 and m_2.

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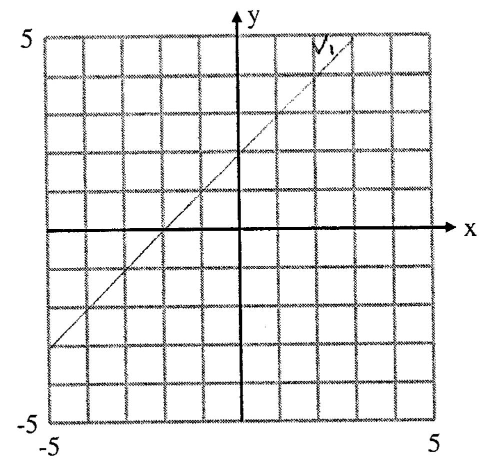
Activity VIII: Part 4

1. Find the equation of the line v_1.



2. Graph a line that is parallel to the line v_1 and passes through the point (3,2). Call this line v_2.

3. Explain how you graphed the line v_2.

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4. Find the equation of v_2.

5. Explain your method in finding this equation.

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6. Describe the similarities and differences between the equations of v_1 and v_2.

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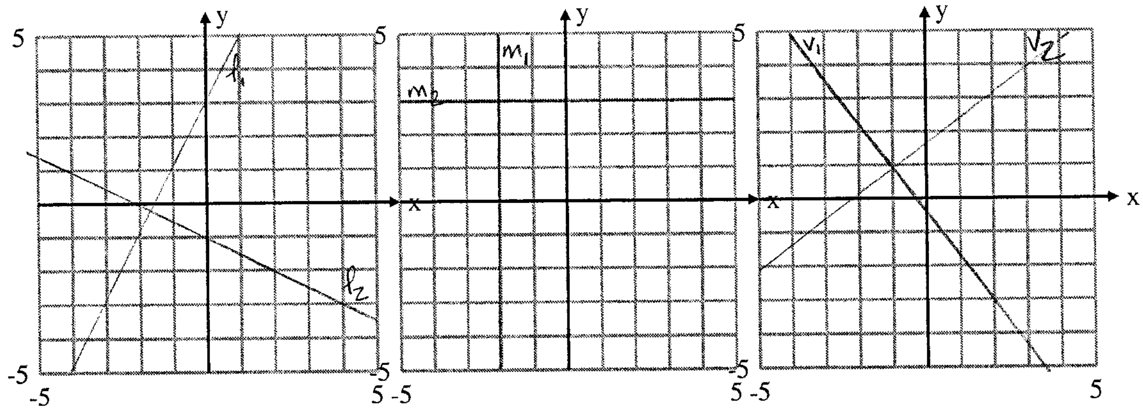
Activity IX: Part 1



Every coordinate plane contains two perpendicular lines. Find the slopes of the pairs of perpendicular lines. Then answer the questions.

Lines	l_1	l_2	m_1	m_2	v_1	v_2
Slope						

l_1 and l_2 are perpendicular lines. What do you notice about the signs of their slopes?

____

What do you notice about the magnitude of their slopes?

____

Is your findings consistent for m_1 and m_2? Explain.

____

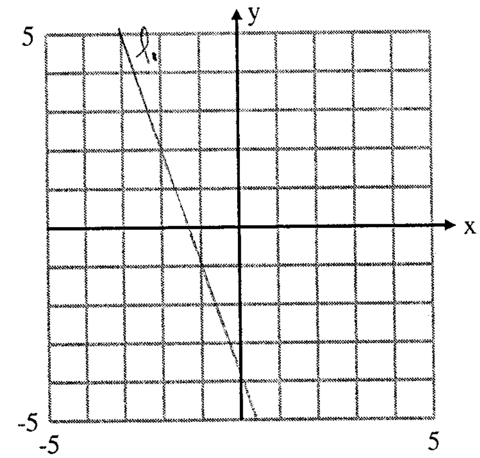
Is your findings consistent for v_1 and v_2 ? Explain.

____

If a line has slope $\frac{2}{5}$, then any perpendicular line will have slope ______________

Activity IX: Part 2

7. Find the slope of the line l_1.



8. l_2 is a line that is perpendicular to the line l_1. What is the slope of l_2?

9. Graph l_2. Explain how you graphed the line l_2.

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10. Find the equation of l_2.

11. Explain your method in finding this equation.

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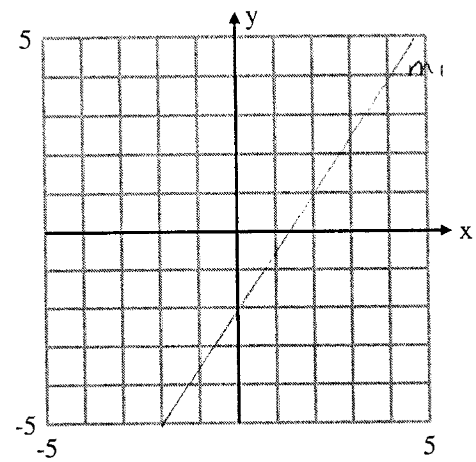

Activity IX: Part 3

7. Find the slope of the line m_1.

8. m_2 is a line that is perpendicular to the line m_1. What is the slope of m_2?

9. Graph a line that is perpendicular to the line m_1 and has intercept 2.

10. Explain how you graphed the line m_2 .

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11. Find the equation of m_2.

12. Explain your method in finding this equation.

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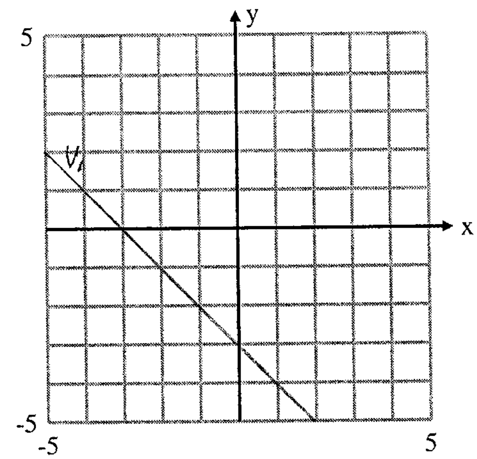
Activity IX: Part 4

7. Find the slope of the line v_1.



8. v_2 is a line that is perpendicular to v_1. What is the slope of v_2?

9. Graph a line that is perpendicular to the line v_1 and passes through the point (-1,3).

10. Explain how you graphed the line v_2 .

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11. Find the equation of v_2.

12. Explain your method in finding this equation.

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Appendix C

Consent Forms

San José State
UNIVERSITY

Department of Mathematics

One Washington Square
San José, CA 95192-0103
Voice: 408-924-5100
Fax: 408-924-5080
E-mail: info@math.sjsu.edu
www.math.sjsu.edu

Agreement to Participate in Research

Responsible Investigator: Nihad M. Mourad
Title of Protocol: Inductive Reasoning in the Algebra Classroom

1. Your child or ward has been asked to participate in a research study investigating the effect of teaching algebra using inductive reasoning.

2. Your child or ward will be asked to be part of a control or experimental group learning linear functions in the classroom, starting 11/29/04 until 12/24/04 during algebra class at Granada Islamic School. Both groups will receive equally beneficial instructional methods.

3. Students will be randomly assigned to one of the groups. Because the class size is small, the students will be expected to participate more fully. This might cause the students some discomfort. There is a risk that the difference in instruction could result in a difference in understanding and mastery of the material in this unit. Although this is not anticipated, should this arise, students in the group that performs less well will be given additional instructional time after school to mitigate the difference.

4. The teacher/student ratio will be much smaller during this experiment. This will allow the teachers to give more attention to the needs of all students.

5. All students will partake in the training as described above. If the parents decline their child's participation in this research, the student data will not be included in the analysis.

6. Although the results of this study may be published, no information that could identify your child or ward, your family, or you will be included.

7. Questions about this research may be addressed to Nihad M. Mourad at (408) 980-1161. Complaints about the research may be presented to Eloise Hamann, Ph. D., Interim Department Chair, Math Dept/ SJSU, (408)924-5100. Questions about research subjects' rights, or research-related injury may be presented to Pamela Stacks, Ph.D., Interim Associate Vice President, Graduate Studies and Research, at (408)924-2427.

Initials _______

8. No service of any kind, to which a you and/or your child or ward are otherwise entitled, will be lost or jeopardized if you choose to "not participate" in the study.

9. Your consent for your child or ward to participate is being given voluntarily. You may refuse to allow his or her participation in the entire study or in any part of the study. If you allow his or her participation, you are free to withdraw your child or ward from the study at any time, without any negative effect on your relations with Granada Islamic School, San Jose State University or with any other participating institutions or agencies.

10. At the time that you sign this consent form, you will receive a copy of it for your records, signed and dated by the investigator.

- **The signature of a parent or legal guardian on this document indicates:**
  - **a) approval for the child or ward to participate in the study,**
  - **b) that the child is freely willing to participate, and**
  - **c) that the child is permitted to decline to participate, in all or part of the study, at any point.**

- **The signature of a researcher on this document indicates agreement to include the above named subject in the research and attestation that the subject's parent or guardian has been fully informed of the subject's rights.**

____
Name of Child or Ward

____________________________ ____________
Parent or Guardian Signature Date

____
Relationship to Child or Ward

____
Full Mailing Address

[signature] 11/24/04
Investigator's Signature Date

San José State
UNIVERSITY

Department of Mathematics

One Washington Square
San José, CA 95192-0103
Voice: 408-924-5100
Fax: 408-924-5080
E-mail: info@math.sjsu.edu
www.math.sjsu.edu

Agreement to Participate in Research

Responsible Investigator: Nihad M. Mourad
Title of Project: Inductive Reasoning in the Algebra Classroom

You are being asked to participate in a research study investigating the effects of two different teaching methods in an algebra class.

You will be in one of two groups learning linear functions, for a four-week period during your algebra class at Granada Islamic School. Both groups will receive equally beneficial instructional methods.

You will be randomly assigned to one of the groups. Because the class size is small, you will be expected to participate more fully. This might cause you some discomfort. There is a risk that the difference in instruction could result in a difference in understanding and mastery of the material in this unit. Although this is not anticipated, you have the right to request additional instructional time after school, if you feel that your group received less beneficial instruction.

The teacher/student ratio will be much smaller during this experiment. This will allow the teachers to give more attention to the needs of all students.

Although the results of this study may be published, no information that could identify you will be included.

The signature on this document indicates:
a) approval to participate in the study,
b) that you are freely willing to participate, and
c) that you are permitted to decline to participate, in all or part of the study, at any point.

The signature of the researcher on this document indicates agreement to include the below named subject in the research and attestation that the subject has been fully informed of the subject's rights.

____
Name of Child

__________________________ ____________
Signature Date

__________________________ ____________
Investigator's Signature Date

Appendix D

Letter of Approval

San José State
UNIVERSITY

Office of the Academic Vice President
Academic Vice President Graduate Studies and Research
One Washington Square
San José, CA 95192-0025
Voice: 408-283-7500
Fax: 408-924-2477
E-mail: gradstudies@sjsu.edu
http://www.sjsu.edu

To: Nihad M. Mourad
3861 Eastwood Circle
Santa Clara, CA 95054

From: Pam Stacks, Pam Stacks
Interim AVP, Graduate Studies & Research

Date: February 4, 2005

The Human Subjects-Institutional Review Board has approved your request to use human subjects in the study entitled:

"Inductive Reasoning in the Algebra Classroom."

This approval is contingent upon the subjects participating in your research project being appropriately protected from risk. This includes the protection of the anonymity of the subjects' identity when they participate in your research project, and with regard to all data that may be collected from the subjects. The approval includes continued monitoring of your research by the Board to assure that the subjects are being adequately and properly protected from such risks. If at any time a subject becomes injured or complains of injury, you must notify Pam Stacks, Ph.D. immediately. Injury includes but is not limited to bodily harm, psychological trauma, and release of potentially damaging personal information. This approval for the human subjects portion of your project is in effect for one year, and data collection beyond February 4, 2006 requires an extension request.

Please also be advised that all subjects need to be fully informed and aware that their participation in your research project is voluntary, and that he or she may withdraw from the project at any time. Further, a subject's participation, refusal to participate, or withdrawal will not affect any services that the subject is receiving or will receive at the institution in which the research is being conducted.

If you have any questions, please contact me at (408) 924-2480.

cc: Trisha Bergthold

Appendix E

Quizzes and Unit Exam

Name: ______________________________ Date: ___________________________

Ch. 6 Algebra Quiz 1 Score: ___________________________

Define the following terms in your own words

1. Slope of a line ____

____

2. x-intercept of a line ____

____

3. y-intercept of a line ____

____

4. Sarah and Mona are studying slope and disagreeing. Sarah says that if you choose any two points on the line, you can determine the slope. But Mona says the slope changes depending on the points you choose. Who is right? Explain.

____

____

5. Farid and Adam have different ideas about x-intercepts. Farid says that it is possible for a line to have two different x-intercepts, but Adam says that the x-intercept is unique. Who is right? Explain.

____

____

____

6. Mariam and Ahmad are discussing y-intercepts. Mariam says that it is possible for a line <u>not</u> to have a y-intercept, but Ahmad insists that every line has a y-intercept. Who is right? Explain.

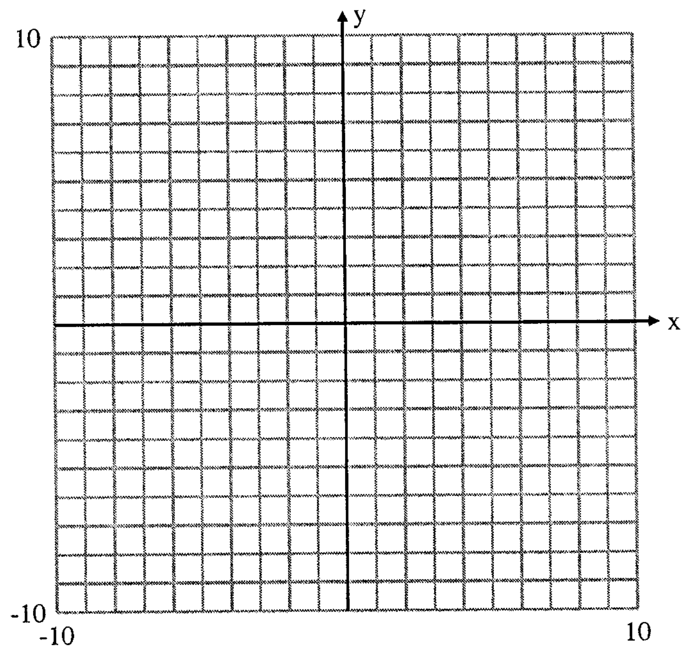
____

____

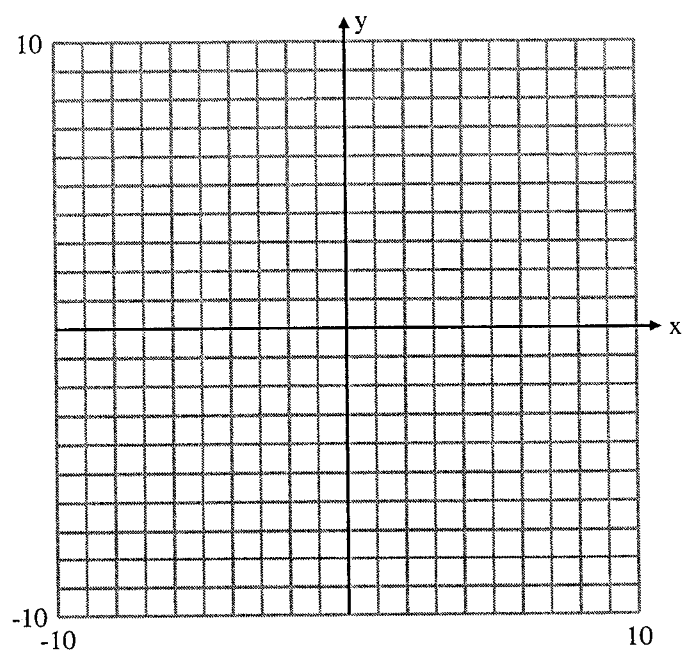
The Following pairs of points lie on line *p* and *q*.

a. Determine the slope of the line, and graph it.
b. Estimate the x- and y- intercepts from the graph.
c. Calculate x- and y- intercepts algebraically.

7. (2, 5) and (3, 3)



8. (2, -5) and (4, 3)



9. Determine the value of *r* so that the line passing through (6, 3) and (*r*, 2) has a slope of $\frac{1}{2}$.

Name: ______________________________ Date: ____________________________

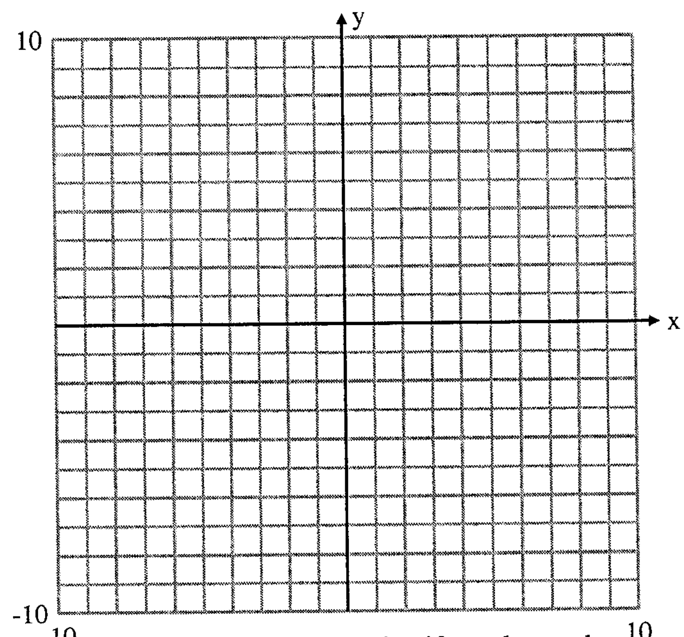
Ch. 6 Algebra Quiz 2 Score: ____________________________

1. Write the slope-intercept form of linear equation. Explain what each variable stands for:

____

____

2. Write the slope-intercept and standard forms of the equation of the line that passes through (-3, 2) and (7, -3). Use the coordinate plane to plot the points and draw the graph of the line.



3. Umar and Adam have different ideas about graphing lines. Umar says that if you know the slope of a line you can graph the line, but Adam says that you need the slope and another point to graph . Who is right? Explain.

____

____

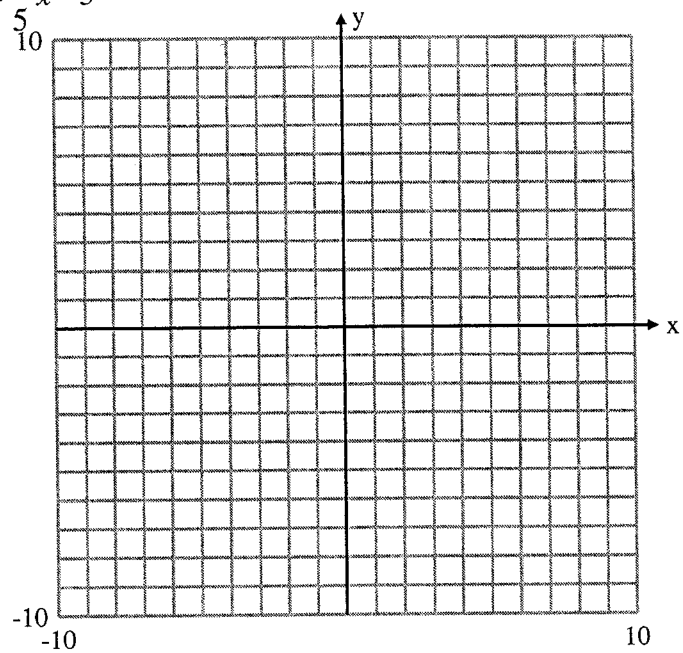
____

4. Nida and Sarah are discussing the following equation $3x+2y=7$. Nida says that the slope of this line is 3, but Sarah insists that it is not. Who is right? Explain.

____

____

5. Graph the following linear equation: $y = \frac{2}{5}x - 3$



6. Is the point (3, -2) on the line $2x + y = -4$? Explain.

Find a point A that lies on this line $2x + y = -4$. Explain.

Find a point B that does not lie on the line $2x + y = -4$. Explain.

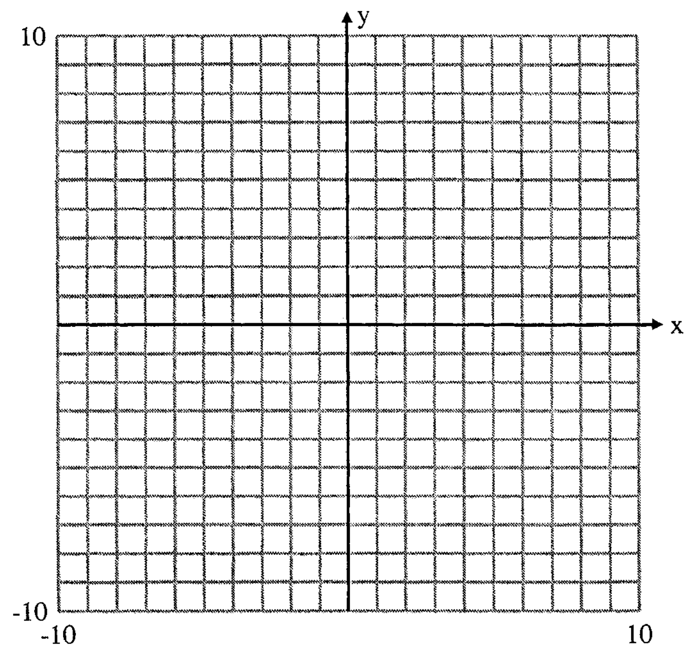
7. Is the equation $y = 2x - 8$ in standard form? If not, write it in standard form.

Name: ____________________ Date: ____________________

Algebra Test: Ch. 6
Analyzing Linear Equations
Show all work. Score: ____________________

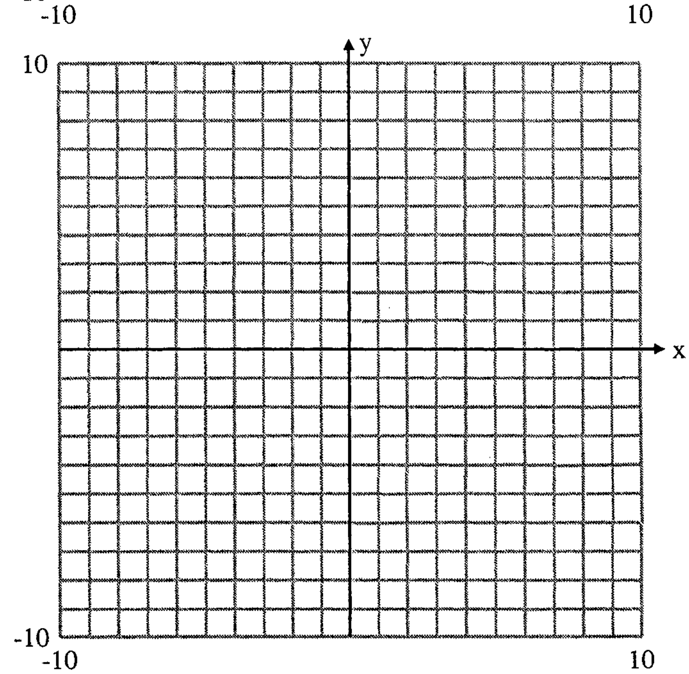
Graph the following lines:

1) The line has x-intercept = -3, and y-intercept = 7.



2) The line that has the equation:

$4x - 3y = -9$



3) Samia and Ahmad were discussing parallel lines. Samia said that parallel lines have the same slope, but Ahmad said that they have the same y-intercept. Who is right? Explain.

4) Determine the slope of the line passing through $(-2,-6)$ and $(7,-6)$. Determine if this line is increasing, decreasing, or constant?

5) Determine the value of r so that the line through $(-4,8)$ and $(r,-6)$ has a slope of $\frac{2}{3}$.

6) Find the x- and y-intercepts of the following line: $4x-2y=12$.

7) Find the x- and y-intercepts of the following line: $x=-3$.

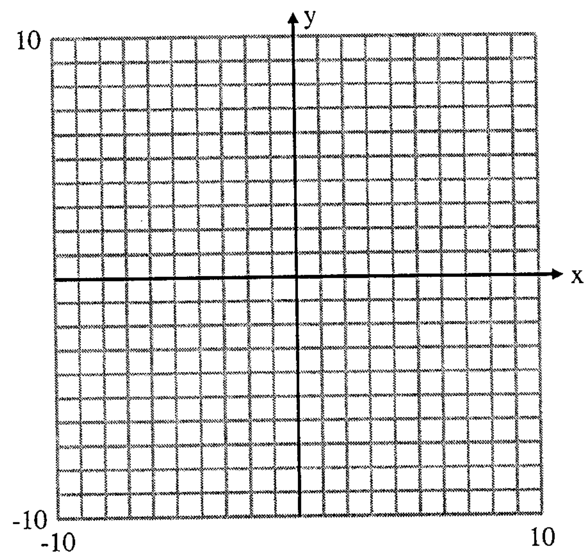
For each of the lines described below,

(a) Write the equation of the line in <u>slope-intercept</u> form.

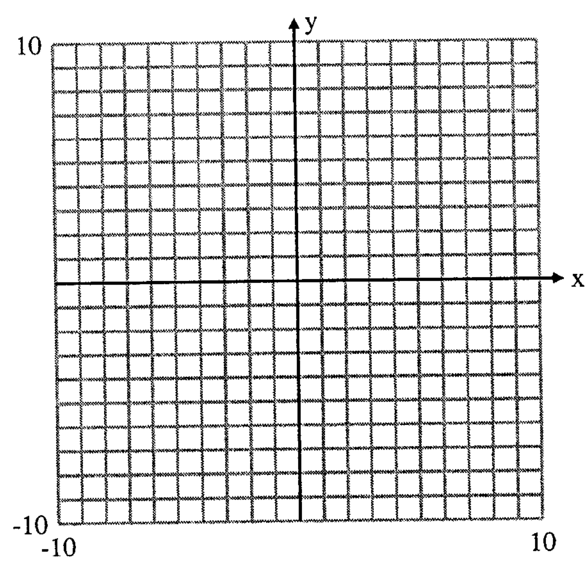
(b) Write the equation of the line in <u>standard form.</u>

(A graph is not required, but feel free to use the coordinate plane.)

8) is parallel to the x-axis and passes through $(4,2)$.



9) is perpendicular to the graph of $4x - y = 12$ and passes through $(8,2)$.



10) Dalia planned to find the standard form of the linear equation before finding the slope-intercept form. Mona told her that this is not possible. Who is right? Explain.

11) In a certain lake, a 1-year old bluegill fish is 3 inches long, while a 4-year-old bluegill is 6.6 inches long.

a. Assuming the growth rate can be approximated by a linear equation, write an equation in slope-intercept form for the length (y) of a bluegill fish in inches after x years.

b. Use this equation to estimate the length of a 10-year-old bluegill.

c. About how old is a 9-inch-long bluegill?

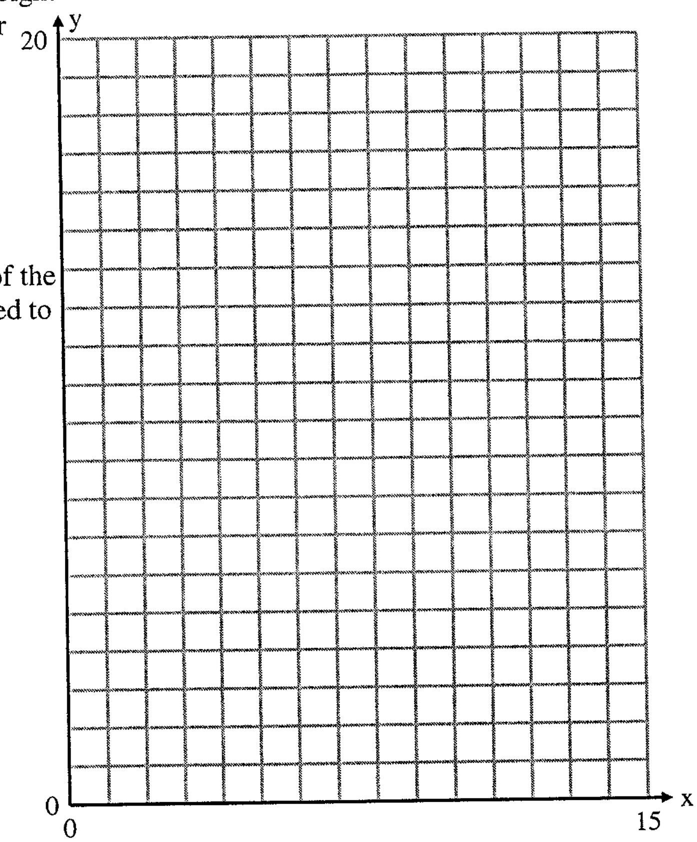
d. What does the slope mean in this situation?

e. What does the y-intercept mean in this situation?

12) Adel is frustrated because he cannot write the equation of a vertical line in slope-intercept form. Amal told him that there is no such equation. He disagrees with her. Who is right? Explain.

13) The Sheikh family keeps their cement fish pond filled to a height of 20 in. At the end of the summer, they drain the pond. The height of the water drops at a rate of 1.5 in. per minute.

a. Model the situation with a graph.



b. Find the slope and the y-intercept of the line and explain how they are related to the original situation.

c. Model the situation with an equation.

d. How long will it take to empty the pond? Where does this information appear in your graph?

Appendix F

Individual Students Scaled Scores

Table F1

Mathematical Truths Scaled Scores on Unit Exam – Control Group

Subject	MT1	MT2	MT3	Total
1	100.0	100.0	100.0	100.0
2	85.7	75.0	72.2	79.0
3	67.9	93.8	83.3	79.1
4	100.0	100.0	77.8	93.6
5	64.3	31.3	33.3	46.8
6	85.7	81.3	72.2	80.6
7	50.0	75.0	38.9	53.2
8	85.7	81.3	50.0	74.2
9	75.0	56.3	55.6	64.5
10	96.4	75.0	66.7	82.3
11	67.9	18.8	38.9	46.8
12	53.6	68.8	50.0	56.5
13	67.9	50.0	61.1	61.3
	60.7	31.3	55.6	51.6
	39.3	37.5	27.8	35.5

Table F2

Mathematical Truths Scaled Scores on Unit Exam – Experimental Group

Subject	MT1	MT2	MT3	Total
1	92.9	100.0	88.9	93.6
2	92.9	81.3	94.4	90.3
3	96.4	81.3	83.3	88.7
4	100.0	93.8	83.3	93.6
5	100.0	100.0	77.8	93.6
6	71.4	56.3	33.3	56.4
7	67.9	81.3	50.0	66.2
8	67.9	81.3	77.8	74.2
9	57.1	56.3	5.6	41.9
10	100.0	93.8	100.0	98.4
11	64.3	87.5	61.1	69.4
12	67.9	43.8	22.2	48.4
13	35.7	31.3	33.3	33.9
	39.3	43.8	27.8	37.1

Table F3

Representational Translation Ability Scaled Scores on Unit Exam – Control Group

Subject	Graph	Equation	All Reps
1	100.0	100.0	100.0
2	70.0	85.7	79.2
3	80.0	78.6	79.2
4	100.0	85.7	91.7
5	80.0	28.6	50.0
6	90.0	85.7	87.5
7	70.0	35.7	50.0
8	90.0	57.1	70.8
9	10.0	57.1	37.5
10	100.0	64.3	79.2
11	60.0	28.6	41.7
12	60.0	50.0	54.2
13	70.0	64.3	66.7
	30.0	57.1	45.8
	70.0	21.4	41.7

Table F4

Representational Translation Ability Scaled Scores on Unit Exam – Experimental Group

Subject	Graph	Equation	All Reps
1	100.0	92.9	95.9
2	70.0	92.9	83.4
3	100.0	78.6	87.5
4	100.0	85.7	91.7
5	100.0	85.7	91.7
6	100.0	28.6	58.4
7	90.0	42.9	62.5
8	80.0	71.4	75.0
9	90.0	7.1	41.6
10	90.0	100.0	95.8
11	90.0	64.3	75.0
12	90.0	21.4	50.0
13	40.0	28.6	33.4
	80.0	7.1	37.5