On some estimates and the dead core of solution in some nonlinear parabolic problems

Tokumori Nanbu

(Recieved November 6, 1998)

Abstract

We do research on some estimates and the dead core of solution in some nonlinear parabolic problems .

1 Introduction

In this note we are concerned with the behaviour of solution $u(x,t)$ of the problem (1.1)-(1.3):

$$u_t - \Delta_x(u^m) = -\lambda u^p \quad \text{in} \quad Q = \Omega \times R^+, \tag{1.1}$$

$$u(x,t) = \chi(x) \quad \text{on} \quad \mathit{\Gamma} = \partial\Omega \times R^+, \tag{1.2}$$

$$u(x,0) = u_0(x) \quad \text{in} \quad \Omega. \tag{1.3}$$

Here Ω is a bounded, arcwise connected domain in $R^N(N > 1)$ whose boundary $\partial\Omega$ is of class C^3, $R^+ = (0,+\infty)$, $\lambda \in (0,+\infty)$, Δ_x denotes the N-dimensional Laplace operator, $m > 1$ and $p > 0$ are constants, $\chi(x)$ is a nonnegative continuous function on Γ and $u_0(x)$ is a nonnegative continuous function in $\overline{\Omega}$ with $u_0(x) = \chi(x)$ on $\partial\Omega$ and $\mathrm{Max}_{\overline{\Omega}} u_0(x) = 1$.

We shall give the notation of weak solution of the problem (1.1)-(1.3).

Definition 1.1. A function $u \in C([0,T] : L^1(\Omega)) \cap L^\infty(Q_T)$ is called a weak solution of the problem (1.1)-(1.3) if it satisfies

$$\int_\Omega u(x,T)\sigma(x,T)dx - \int\int_{Q_T} [u\sigma_t + u^m \Delta_x \sigma]dxdt + \int_0^T \int_{\partial\Omega} (\chi)^m \frac{\partial\sigma}{\partial n} dS = \int_\Omega u_0\sigma(x,0)dx + \int\int_{Q_T} -\lambda u^p \sigma dxdt \tag{1.4}$$

for all $\sigma \in C^2(\overline{Q}_T)$ with $\sigma = 0$ on $\partial\Omega \times (0,T)$. Here we put $Q_T = \Omega \times (0,T)$, $\frac{\partial}{\partial n}$ stands for the outer normal derivative at $\partial\Omega$.

By an upper solution of the problem (1.1)-(1.3), we mean any $\overline{u}(x,t) \in C([0,T]:L^1(\Omega)) \cap L^\infty(Q_T)$ which satisfies (1.4) with the inequality $\geq$ for any positive σ as above. Similarly $\underline{u}(x,t) \in C([0,T]:L^1(\Omega)) \cap L^\infty(Q_T)$ is called a lower solution of the problem (1.1)-(1.3) the inequality sign is reversed.

The local existence and uniqueness of solutions of the problem (1.1)-(1.3) have been studied by several authors (see [1], [5] and [8]).

When $\chi(x) = 0$ in (1.2), it is well known that the solution $u(x,t)$ of the problem (1.1)-(1.3) converges to zero as $t \to \infty$ (Cf. [6]).

In ([2]) and ([4]) the following steady state problem has been studied:

$$(1.5) \qquad -\Delta_x \phi^m = -\lambda \phi^p \quad \text{in} \quad \Omega,$$

$$(1.6) \qquad \phi(x) = \chi(x) \quad on \quad \partial\Omega.$$

Here λ is a positive constant. Let $\phi(x;\lambda)$ be a solution of the problem (1.5)-(1.6). They have shown that if $\chi(x) > 0$ in $\overline{\Omega}$, either Ω or $\lambda(> 0)$ is large enough, and $p < m$, then the solution $\phi(x;\lambda)$ of the problem (1.5)-(1.6) has a dead core $D(\phi(x;\lambda))$, that is , the set $D(\phi(x;\lambda)) = \{x \in \Omega : \phi(x;\lambda) = 0\} \neq \emptyset$.

Definition 1.2. Suppose that a solution $\phi(x;\lambda)$ of the problem (1.5)-(1.6) has a dead core. Let $x_0 \in \Omega$ and $p < m$. Define

$$(1.7) \qquad \lambda_0 = \inf_{\lambda} \{\phi(x_0;\lambda) = 0\}, \quad \lambda^* = \inf_{x_0} \lambda_0.$$

Definition 1.3. Let $u(x,t)$ be the solution of the problem (1.1)-(1.3). For any $t(\geq 0)$ we set $D(u(t)) = \{x \in \Omega : u(x,t) = 0\}$. The set $D(u(t))$ is called by a time-dependent dead core of solution $u(x,t)$.

For the solution $u(x,t)$ of the problem (1.1)-(1.3), we briefly mention on the existence of the time T such that $D(u(t))(\neq \emptyset)$ $(t \geq T)$.

Let $z(t)$ be a solution of the problem:

$$(1.8) \qquad z_t = -\lambda z^p \ (\ 0 < t \ ), \quad z(0) = 1.$$

Then the solution of the problem (1.8) is given by

$$(1.9) \qquad z(t) = \{1 + \lambda(p-1)t\}_+^{\frac{-1}{(p-1)}} \quad if \ p \neq 1$$

and

$$(1.10) \qquad z(t) = e^{-\lambda t} \quad if \ p = 1.$$

Here $\{f\}_+ = \max(f, 0)$.

In [2] Bandle, Nanbu and Stakgold have proved the following result:

Theorem 1.1. (Cf. [2]. Theorem 4.1.) Let $u(x,t)$ be the solution of the problem (1.1)-(1.3) in the case $0 < p < m$. For fixed $x_0 \in \Omega$, choose λ $(> \lambda_0)$ where

λ_0 is defined in (1.7). Then
(a) If $0 < p < 1$, then $u(x_0, t) = 0$ if $t \geq \frac{1}{\lambda - \lambda_0}$.
(b) If $p \geq 1$ and $\min_{\overline{\Omega}} u_0(x) > 0$, then $u(x_0, t) > 0$ for all t.

We know some results on the dead core of the solutions of the problem (1.5)-(1.6).

Theorem 1.2.(Cf.[2], [4].)
(i) Let $\phi_1(x; \lambda)$ be the solution of (1.5) satisfing $\phi_1 = \chi_1(x)$ on $\partial\Omega$ and $\phi_2(x; \lambda)$ the solution of (1.5) satisfing $\phi_2 = \chi_2(x)$ on $\partial\Omega$. If $\chi_1(x) \leq \chi_2(x)$ on $\partial\Omega$, then $\phi_1(x; \lambda) \leq \phi_2(x; \lambda)$ in Ω and $D(\phi_2(x; \lambda)) \subset D(\phi_1(x; \lambda))$.
(ii) Let $\phi_1(x; \lambda_1)$ be the solution of (1.5) for $\lambda = \lambda_1$ satisfing $\phi_1 = \chi(x)$ on $\partial\Omega$ and $\phi_2(x; \lambda_2)$ the solution of (1.5) for $\lambda = \lambda_2$ satisfing $\phi_2 = \chi(x)$ on $\partial\Omega$. If $\lambda_1 < \lambda_2$, then $\phi_2(x; \lambda_2) \leq \phi_1(x; \lambda_1)$ in Ω and $D(\phi_1(x; \lambda_1)) \subset D(\phi_2(x; \lambda_2))$.

There are some results on the convergence of solution $u(x, t)$ of the problem (1.1)-(1.3) .

At first we are interested in Ricci's result for $u_t - \Delta_x u = -u^q$. Ricci([11]) has proved the following result:

Theorem 1.3. (Cf.[11].)
Let $0 < q < 1$ and $u(x, t)$ be a solution of the problem

$$u_t - \Delta_x u = -u^q \quad \text{in} \quad Q = \Omega \times R^+, \tag{1.11}$$

$$u(x, t) = 1 \quad \text{on} \quad \Gamma = \partial\Omega \times R^+, \tag{1.12}$$

$$u(x, 0) = 1 \quad \text{in} \quad \Omega. \tag{1.13}$$

Let $\phi(x; \lambda)$ $(\lambda > 0)$ be a solution of the problem

$$-\Delta_x \phi = -\lambda \phi^q \quad \text{in} \quad \Omega, \phi(\partial\Omega) = 1. \tag{1.14}$$

Then there exist positive constants K, K_1 and K_2 such that for any positive λ and μ

$$\|\phi(x; \lambda) - \phi(x; \mu)\|_{C^0(\overline{\Omega})} \leq K|\lambda - \mu|, \tag{i}$$

and

$$\|u(x, t) - \phi(x; 1)\|_{L^\infty(\Omega)} \leq K_1 e^{-K_2 t}. \tag{ii}$$

This estimate (ii) holds whether $D(\phi(x; \lambda)) \neq \emptyset$ or not. Moreover if $D(\phi(x; \lambda)) \neq \emptyset$, there exists a constant K_3 such that

$$d(\partial D(u(t)), \partial D(\phi(x; \lambda))) \leq K_3 e^{-K_2((1-q)/2)t}. \tag{iii}$$

Let $u(x, t)$ be a solution of the problem (1.1)-(1.3) and $\phi(x)$ a solution of the problem (1.5)-(1.6) for $\lambda = 1$. Then $\phi(x)$ is a lower solution of the problem

(1.1)-(1.3) and $\overline{u}(x,t) \equiv 1$ is an upper solution of the problem (1.1)-(1.3). Hence we have :

Theorem 1.4.(Cf. [1], [10].) Let $u(x,t)$ be a solution of the problem (1.1)-(1.3) and $\phi(x)$ a solution of the problem (1.5)-(1.6) for $\lambda = 1$. Then the following convergences hold:

$$\text{(1.15)} \qquad \textit{if} \quad \phi(x) \leq u_0(x) \leq 1, \ \textit{then} \ u(x,t) \to \phi(x) \ \textit{in a.e.} \ \Omega \quad \textit{as} \quad t \to \infty,$$

and

$$\text{(1.16)} \qquad \textit{if} \ u_0(x) \leq \phi(x), \ \textit{then} \ u(x,t) \to \phi(x) \ \textit{in a.e.} \ \Omega \quad \textit{as} \quad t \to \infty.$$

There is no result on the decay estimate of solution $u(x,t)$ of the problem (1.1)-(1.3). In this note we shall study the decay estimate on

$$\|\nabla(u^m(x,t) - \phi^m(x;\lambda))\|_{L^2(\Omega)} \ \textit{in} \ t$$

for the solution $u(x,t)$ of the problem (1.1)-(1.3).

2 Some results on the dead core

We assume that $0 < \lambda \leq 1, 0 < p < m$ and $1 < m$.
Let $u(x,t)$ be a solution of the following special problem :

$$\text{(2.1)} \qquad u_t - \Delta_x(u^m) = -\, u^p \quad \text{in} \quad Q = \Omega \times R^+,$$

$$\text{(2.2)} \qquad u(x,t) = 1 \quad \text{on} \quad \Gamma = \partial\Omega \times R^+,$$

$$\text{(2.3)} \qquad u(x,0) = 1 \quad \text{in} \quad \Omega.$$

For the dead core of a solution $u(x,t)$ of the problem (2.1)-(2.3) we set

$$D(u(t)) = \{x \in \Omega : u(x,t) = 0\} \quad (0 < t < \infty).$$

Let $\phi(x;\lambda)$ be a solution of the following problem (2.4):

$$\text{(2.4)} \qquad -\Delta_x(\phi^m) = -\lambda\phi^p \quad \text{in} \quad \Omega, \quad \phi(x) = 1 \quad \text{on} \quad \partial\Omega.$$

Here $\lambda(\in (0,1])$ is a constant. We denote a dead core of $\phi(x:\lambda)$ by $D(\phi(x;\lambda))$.

Lemma 2.1 If $D(\phi(x;1))$ is not empty, then there exists a positive numbers $\lambda^*(< 1)$ such that if $0 < \lambda < \lambda^*$, then $D(\phi(x;\lambda))$ is empty, and if $\lambda \geq \lambda^*$ then there is a dead core $D(\phi(x;\lambda))$ such that if $\lambda^* \leq \lambda < \mu \leq 1$, then

$$D(\phi(x;\lambda)) \subset intD(\phi(x;\mu)).$$

Remark. If Ω is large enough, then $D(\phi(x;1))$ is not empty (See [12]).

Let $z(t;\lambda)$ be a solution of the following problem :

$$z_t = -\lambda z^p \quad (0 < t), \quad z(0) = 1. \tag{2.5}$$

Then the solution of the problem (2.5) is given by

$$if \quad p \neq 1, \ then \quad z(t;\lambda) = \{1 + \lambda(p-1)t\}_+^{-1/(p-1)},$$

and

$$if \quad p = 1, \ then \quad z(t;\lambda) = e^{-\lambda t}.$$

Here $\{f\}_+ = \mathrm{Max}(f, 0)$.

By the Comparison Principle we have

Lemma 2.2. Let $\lambda(\in (0,1))$ be a constant. For the solution $u(x,t)$ of the problem (2.1)-(2.3) we have

$$\mathrm{Max}\{z^m(t;1), \phi^m(x;1)\} \leq u^m(x,t) \tag{2.6}$$

$$\leq \phi^m(x;\lambda) + z^m(t;1-\lambda) \quad in \quad \Omega \times (0,T)$$

where $T = \infty$ if $1 \leq p$ and $T = 1/((1-\lambda)(1-p))$ if $0 < p < 1$.
Proof. For any positive $\lambda(< 1)$ we set

$$\Phi(x;\lambda) = \phi(x;\lambda)^m, Z(t;\lambda) = z(t;\lambda)^m$$

and

$$U(x,t) = \Phi(x;\lambda) + Z(t;1-\lambda).$$

Then we have

$$L(U) \equiv (U^{1/m})_t - \Delta U = (\Phi + Z)^{(1-m)/m} z^{m-1} z_t - \Delta\Phi$$

$$\geq -(1-\lambda)Z^{(1-m)/m} Z^{(m-1)/m} Z^{p/m} - \lambda\Phi^{p/m}$$

$$= -(1-\lambda)Z^{p/m} - \lambda\Phi^{p/m} \geq -(Z+\Phi)^{p/m} = -U^{p/m}.$$

Thus the function $U(x,t)$ satisfies the following

$$(U^{1/m})_t - \Delta U \geq -U^{p/m} \quad in \quad \Omega \times (0,T),$$

$$U(\partial\Omega, t) \geq 1 \quad (0 < t < T),$$

$$U(x,0) \geq 1 \quad (x \in \Omega).$$

Here

$$T = \begin{cases} \infty & if\ 1 \leq p \\ 1/((\lambda - 1)(1-p)) & if\ 0 < p < 1. \end{cases}$$

Therefore we obtain that

$$\mathrm{Max}\{\Phi(x;1), Z(t;1)\} \leq u^m(x,t) \leq \Phi(x;\lambda) + Z(t;1-\lambda) \quad in \quad \Omega \times (0,T) \tag{2.7}$$

where $T = \infty$ if $1 \leq p$ and $T = 1/((1-\lambda)(1-p))$ if $0 < p < 1$.

From (2.7) we can conclude that
Theorem 2.1. Suppose that $0 < \lambda < 1$ and $D(\phi(x;1)) \neq \emptyset$. Let $u(x,t)$ be a solution of the problem (2.1)-(2.3). We have :
(i) If $1 \leq p < m$, then $0 < u(x,t)$ in $\Omega \times R^+$.
(ii) If $0 < p < 1 < m$, then $0 < u(x,t)$ in $(\Omega - D(\phi(x;1))) \times R^+$.
(iii) If $1 < p < m$, then $0 < u(x,t) \leq (1 + (1-\lambda)(p-1)t)^{-1/(p-1)}$ in $D(\phi(x;\lambda)) \times R^+$.
(iv) If $1 = p$, then $u(x,t) \leq e^{-(1-\lambda)t/m}$ in $D(\phi(x;\lambda)) \times R^+$.
(v) If $0 < p < 1 < m$, then $u(x,t) = 0$ in $D(\phi(x;\lambda)) \times [1/(1-\lambda)(1-p), \infty)$.

From Theorem 2.1 (v) it follows that
Corollary 2.1. Suppose that $0 < p < 1$ and $\lambda < 1$ and $D(\phi(x;1))$ is not empty. There exists a time t^* such that if $0 < t < t^*$, then $D(u(t)) = \emptyset$ and $t \geq t^*$ then $D(u(t)) \neq \emptyset$. The sequence of the set $D(u(t)) = \{x \in \Omega : u(x,t) = 0\}$ is strictly increasing in t and converges to the set $D(u(\infty)) = D(\phi(x;1))$.

3 Decay Estimates

In this section we shall give some decay estimates on the solution $u(x,t)$ of the problem (1.1)-(1.3).
Theorem 3.1.(Cf. [2].) If we impose a natural condition on the initial value $u_0(x)$ such that

$$\Delta(u_0(x))^m - \lambda(u_0(x))^p \leq a < 0 \quad (x \in \Omega), \tag{3.1}$$

then the solution $u(x,t)$ of the problem (1.1)-(1.3) is nonincreasing in t , that is, $u(x,t+\tau) \leq u(x,t)$ for any $\tau (> 0)$ at every $x \in \Omega$.

Let $\phi(x;\lambda)$ be a solution of the problem (1.5)-(1.6). If $u_0(x) \geq \phi(x;\lambda)$, then $\phi(x;\lambda)$ is a lower solution of (1.1)-(1.3) ,that is, $u(x,t) \geq \phi(x;\lambda)$ in $\Omega \times [0,\infty)$. From the result of [9] we know that
Theorem 3.2.(Cf. [9].) Let $u(x,t)$ be a solution of the problem (1.1)-(1.3) under the condition $u_0(x) \geq \phi(x;\lambda)$. Then it follows that

$$\|u^m(x,t) - \phi^m(x;\lambda)\|_{L^2(\Omega)} \to 0 \ as \ t \to \infty.$$

We set $\delta(x,t) \equiv u(x,t) - \phi(x;\lambda)$.
Theroem 3.3.(Cf. [2].) Let $u(x,t)$ be a solution of the problem (1.1)-(1.3) . Suppose that $u_0(x) \geq \phi(x;\lambda)$. Then we have
(a) If $p \geq m (> 1)$, then

$$0 \leq \delta(x,t) \leq z(t),$$

where $z(t)$ is the solution of

$$z_t = -\lambda z^p \ (t > 0); z(0) = 1.$$

(b) If $p < m$, then

$$0 \leq \delta(x,t) \leq \theta(t),$$

where $\theta(t)$ is the solution of

$$\theta_t = -\gamma(\theta)^m, \theta(0) = 1, \gamma = \lambda\frac{p}{m}.$$

By Theorem 3.3 we define the function $\Lambda(t)$ by

$$(3.2)\quad \Lambda(t) = \begin{cases} (1+\lambda(p-1)t)^{-\frac{m}{p-1}}, & (p \geq m > 1) \ \ or \ \ (p > 1, m = 1) \\ e^{-\lambda t}, & p = m = 1 \\ (1+\theta(m-1)t)^{-\frac{m}{m-1}}, & m > 1, 0 < p < m \\ e^{-\theta t}, & 0 < p < 1 = m \end{cases}$$

where

$$\theta = \frac{p\lambda}{m}2^{\frac{p}{m}-1}.$$

Let $\phi(x;\lambda)$ be a solution of the problem (1.5)-(1.6). If $u_0(x) \leq \phi(x;\lambda)$, then $\phi(x;\lambda)$ is an upper solution of (1.1)-(1.3) ,that is, $u(x,t) \leq \phi(x;\lambda)$. From the result of [9] we know that

Theorem 3.4.(Cf. [9].) Let $u(x,t)$ be a solution of the problem (1.1)-(1.3) under the condition $u_0(x) \leq \phi(x;\lambda)$. Then it follows that

$$\|u^m(x,t) - \phi^m(x;\lambda)\|_{L^2(\Omega)} \to 0 \ as \ t \to \infty.$$

In the below we shall consider the large time behaviour of

$$\int_t^{t+1}\int_\Omega |\nabla(u^m(x,t) - \phi^m(x;\lambda))|^2 dxds$$

as $t \to \infty$.

Theorem 3.5. Let $u(x,t)$ be a solution of the problem (1.1)-(1.3) and $\phi(x;\lambda)$ a statinary solution of (1.5)-(1.6) . Assume that $u_0(x)$ satisfies (3.1) and $u_0(x) \geq \phi(x;\lambda)$ in Ω. Then for any $t > 0$ we have

$$\int_t^{t+1} ds \int_\Omega |\ \nabla(u^m(x,s;\lambda) - \phi^m(x;\lambda))\ |^2 \ dx$$

$$\leq \sqrt{m\ \Lambda(t)\ mes(\Omega)}\left\{\sqrt{\beta} + \sqrt{m\ \Lambda(t)\ mes(\Omega)}\right\}.$$

Here β is a constant which is determined in (3.8).

Proof. By Theorem 3.1 $u(x,t)$ is nonincreasing in t at every point $x \in \Omega$.

Set $\eta(x,t) \equiv u^m(x,t) - \phi^m(x;\lambda)$.
The functions $u(x,t)$ and $\phi(x) \equiv \phi(x;\lambda)$ satisfies :

$$u_t - \Delta(u^m - \phi^m) = -\lambda(u^p - \phi^p) \quad in \quad \Omega \times R^+. \tag{3.3}$$

Multiply (3.3) by $\eta(x,t)$ and integrate in Ω , then

$$\int_\Omega u_t \eta dx + \int_\Omega |\nabla \eta|^2 dx + \lambda \int_\Omega (u^p - \phi^p)\eta dx = 0. \tag{3.4}$$

By Theorem 3.3, we know $u(x,t) - \phi(x;\lambda) \le \Lambda(t)$ $(t > 0)$ and then we have

$$0 < \int_\Omega \eta(x,t)dx = \int_\Omega (u^m(x,t) - \phi^m(x;\lambda))dx \le m\ \Lambda(t)\ mes(\Omega). \tag{3.5}$$

We now return to the equation (3.3). Multiply (3.2) by η_t and integrate it over Ω, then by using $\eta_t(x,t) = (u^m)_t(x,t)$, we have

$$\int_\Omega u_t \eta_t dx + \frac{1}{2}\frac{d}{dt}\int_\Omega |\nabla \eta|^2 dx + \int_\Omega (u^p - \phi^p)\eta_t dx = 0, \tag{3.6}$$

$$\int_\Omega (u^p - \phi^p)\eta_t dx \ge \int_\Omega u^p (u^m)_t dx$$

and so we have

$$\int_\Omega u_t \eta_t dx + \frac{1}{2}\frac{d}{dt}\int_\Omega |\nabla \eta|^2 dx + \int_\Omega u^p (u^m)_t dx \le 0. \tag{3.7}$$

Integrating (3.7) in t on $[0,t]$, then for any $t > 0$

$$\int_0^t (\int_\Omega u_t (u^m)_t dx) ds + \frac{1}{2}\int_\Omega |\nabla \eta(x,t)|^2 dx + \frac{m}{(p+m)}\int_\Omega u(x,t)^{p+m} dx$$

$$\le \frac{1}{2}\int_\Omega |\nabla \eta(x,0)|^2 dx + \frac{m}{(p+m)}\int_\Omega u_0(x)^{p+m} dx \equiv \beta. \tag{3.8}$$

Integrating (3.7) in t on $[t, t+1]$, then we hvae

$$\int_t^{t+1} (\int_\Omega u_t (u^m)_t dx) ds + \frac{1}{2}\int_\Omega |\nabla \eta(x,t+1)|^2 dx + \frac{m}{(p+m)}\int_\Omega u(x,t+1)^{p+m} dx$$

$$= \frac{1}{2}\int_\Omega |\nabla \eta(x,t)|^2 dx + \frac{m}{(p+m)}\int_\Omega u(x,t)^{p+m} dx \le \beta,$$

and then it follows that for any $t > 0$

$$\int_t^{t+1}\int_\Omega u_t (u^m)_t dx ds \le \beta. \tag{3.9}$$

From (3.4) we have

$$\text{(3.10)} \qquad \int_t^{t+1} \int_\Omega |\nabla\eta|^2\, dx = -\int_t^{t+1} \int_\Omega u_t \eta dx - \lambda \int_t^{t+1} \int_\Omega (u^p - \phi^p)\eta dx.$$

By Theorem 3.3, for any $t(>0)$ we have

$$\text{(3.11)} \qquad \left| \int_t^{t+1} \int_\Omega (u^p - \phi^p)\eta dx \right| \leq \int_\Omega \eta(x,t) dx$$

$$\leq m\ \Lambda(t)\ mes(\Omega).$$

Writing

$$-\eta u_t = -(\delta^{\frac{1}{2}} u^{\frac{1}{2}})(u_t u^{\frac{m-1}{2}})\delta^{\frac{1}{2}} u^{-\frac{m}{2}} \leq -(\delta^{\frac{1}{2}} u^{\frac{1}{2}})(u_t u^{\frac{m-1}{2}}),$$

and using the Schwarz's ineaqlity, we obtain

$$\text{(3.12)} \qquad \left| -\int_t^{t+1} \int_\Omega u_t \eta dx \right|$$

$$\leq \left[\sqrt{\int_t^{t+1} ds \int_\Omega \eta u dx} \right] \left[\sqrt{\int_t^{t+1} ds \int_\Omega u_t^2 u^{m-1} dx} \right] \leq \sqrt{m\ \Lambda(t)\ mes(\Omega)\beta}.$$

Hence we have

$$\int_t^{t+1} ds \int_\Omega |\nabla\eta|^2\, dx \leq \left[\sqrt{\int_t^{t+1} ds \int_\Omega \eta u dx} \right] \left[\sqrt{\int_t^{t+1} ds \int_\Omega u_t^2 u^{m-1} dx} \right]$$

$$\leq \sqrt{m\ \Lambda(t)\ mes(\Omega)} \left\{ \sqrt{\beta} + \sqrt{m\ \Lambda(t)\ mes(\Omega)} \right\}.$$

Therefore we may conclude that

$$\int_t^{t+1} ds \int_\Omega |\nabla\eta|^2\, dx \leq \sqrt{m\ \Lambda(t)\ mes(\Omega)} \left\{ \sqrt{\beta} + \sqrt{m\ \Lambda(t)\ mes(\Omega)} \right\}.$$

Acknowlgement. This research was partially supported by the Research Grant of Toyama First Bank , 1997.

References

[1] D.G.Aronson, M.Crandall and L.A.Peletier, Stabilization of solutions of a degenerate diffusion problem, Nonlinear Anal.6(1982),p.1001-1022.

[2] C.Bandle, T.Nanbu and I.Stakgold, Porous medium equation with absorption, SIAM Jour. Math. Anal. Vol.29(1998) , p.1268-1278.

[3] C.Bandle, M.A.Pozio and A.Tesei, The asymptotic behavior of the solutions of degenerate parabolic equations, Trans.Amer.Math.Soc. Vol.303(1987) , p.487-501.

[4] C.Bandle and I.Stakgold, The formation of the dead core in parabolic reaction-diffusion problems, Trans.Amer.Math.Soc.286(1984),p.275-293.

[5] M.Bertsch, A class of degenerate diffusion equations with singular nonlinear term, Nonlinear Anal., Vol.7(1983),p.117-127.

[6] M.Bertsch, T.Nanbu and L.A.Peletier, Decay of solutions of a degenerate nonlinear diffusion equation, Nonlinear Anal. 6(1982), p.378-412.

[7] J.I.Diaz, Nonlinear Partial Differential Equations and Free Boundaries, Pitman, London, 1985

[8] R.Kershner, Degenerate parobolic equations with general non-linearities, Nonlinear Ana.4(1980), p.1043-1062.

[9] M.Langlais and D.Phillips, Stabilization of solutions of nonlinear and degenerate evolution equations, Nonlinear Anal., Vol.9(1985), p.321-333.

[10] P.De Mottoni, A.Schiaffino and A.Tesei, Attractivity properties of nonnegative solutions for a class of nonlinear degenerate parabolic problems, Ann. Mat. Pura Appl.136(1984), p.35-48.

[11] R.Ricci, Large time behavior of the solution of the heat equation with nonlinear strong absorption, Jour.Diff.Eq.79(1989),p.1-13.

[12] I.Stakgold, Partial extinction in reaction-diffusion, 224, Univ. Di Bari, 1987.