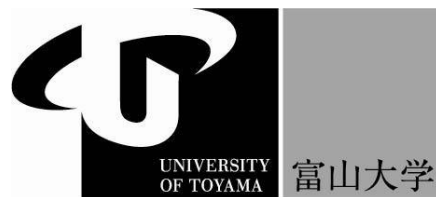


Working Paper No. 297

**Harrodian Dynamics and the Hoover Curve
: Japanese Case**

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28 December 2015



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This Draft: 28 December 2015

First Draft: 17 January 2011

Abstract

Hoover (2008) developed an empirical test with respect to an implication from Harrod (1939)'s dynamic theory. More specifically, the gross domestic product (GDP) gap should be inversely related to the difference between the natural and proper warranted rate of growth. I call this hypothesis the Hoover curve. Hoover derived a downward-sloping regression line of the U.S. economy during the period 1930–2005. In this paper I discuss the Harroddian dynamics of economic growth and show a Hoover curve with a structural change in the Japanese economy during the period 1957–2009.

Key words: Harroddian instability, warranted rate of growth, natural rate of growth, GDP gap, structural change

JEL classifications: E12, E20, E32

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1. Introduction

Roy Harrod is a well-known pioneer of the modern theory of economic growth. However, his theory has been declined by the neoclassical school since Robert Solow(1956). Solow argued that Harrod's knife-edge of equilibrium growth disappears once a fixed-proportions production function is changed to a smooth neoclassical production function.

Hoover (2008) discussed why Harrod's growth theory has been discarded. Hoover examined the Harrod(1939) article and pointed out that the Harrodian instability does not depend upon a fixed-proportions production function and that unlike Harrod, Solow assume that *ex ante* savings and investment are always equal to *ex post* savings and investment.¹ Moreover he argued that Harrod conjectured that the relationship of the proper warranted and natural rate of growth determines the *likelihood* of the economy operating below full employment for any length of time.² Furthermore, he provides an empirical test regarding an implication from Harrod's dynamic theory. More specifically, the gross domestic product (GDP) gap should be inversely related to the difference between the natural and proper warranted rate of growth. I call this hypothesis the Hoover curve. He obtained a downward-sloping regression line in the case of the U.S. economy during the period of 1930–2005.

In Section 2 of this paper I discuss the Harrodian dynamics of economic growth with respect to the economic growth rate, G , and the warranted rate, G_w , i.e. the

¹ See Bioanosky and Hoover (2009, pp. 4–11), Hagemann (2009), and Halsmayer and Hoover (2015) for recent discussion of Harrod–Domar and Solow.

² Hoover (2008, pp. 19–20).

instability principle for trade-cycle analysis, and provide a formal formulation of the Harrodian instability. In Section 3 I argue that a problem of a discrepancy between the natural rate, G_n , and the proper warranted rate, G_{pw} , is concerned with a trend or structural analysis of the dynamic economy. It illustrates the dynamics of the economy in the two cases of difference of the natural and proper warranted rate of growth. In Section 4 the Hoover curve is discussed in relation to the U.S. economy. In Section 5 I show a Hoover curve with a structural change in the case of the Japanese economy during the period 1957–2009. In Section 6 I briefly discuss the trends and changes within the Japanese economy since 1957 as an analysis by the Hoover curve. Finally, in Section 6 I provide conclusions.

2. Harrodian dynamics 1: G and G_W

Harrod (1939) stated that a dynamic theory is ‘thinking in terms of trends of increase’ (p. 15), put the rate of growth of income or output as G and defined ‘dynamic as referring to propositions in which a rate of growth appears as an unknown variable’ (p. 17).

Harrod defined the warranted rate of growth, G_W , as follows: ‘The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or to state the matter others, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth’(p. 16). He defined the following equation as the fundamental equation:

$$G_W = s / C , \quad (1)$$

where s is the savings rate and C is the value of the capital required for the production of a unit increment of output. G_W is ‘the value of which is determined by certain “fundamental conditions” namely, the propensity to save and the state of technology, etc.’ (p. 17). ‘This [s] may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc.’ (p. 16). ‘It [C] may be expected to vary as income grows and in a different phase of trade cycle; it may be somewhat dependent on the rate of interest’ (p. 17).

He called the following equation a truism:

$$G = s / C_p . \quad (2)$$

‘It is a truism, depending on the proposition that actual saving in a period ... is equal to the additional to the capital stock. Total saving is equal to sx_0 . The addition to the capital stock is equal to $C_p(x_1 - x_0)$. This follows from the definition of C_p G is the rate of increase in total output which actually occurs; C_p is the increment of the stock of capital divided by the increment of total output which actually occurs’ (p. 18).

He proceeded with the instability principle as follows: ‘Now suppose that there is a departure from the warranted rate of growth. Suppose excessive output, so that G exceeds G_W . The consequence will be that C_p , the actual increase of capital goods per unit increment of output, falls below C , that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. G , instead of returning to G_W , will move farther from it in an upward direction, and farther. ... Similarly, if G falls below G_W , there will be a redundancy of

capital goods, and a depression influence will be exerted; this will cause a further divergence and a still stronger depression influence; and so on. ... A departure from equilibrium, instead of being self-righting, will be self-aggravating. G_W presents a moving equilibrium, but a highly unstable. Of interest this for trade-cycle analysis!' (p. 22)

The Harrod's dynamics of the growth rate can be briefly described as the following differential equation:

$$dG / dt = \alpha(G - G_W), \quad (3)$$

where α is a positive constant and t is time. If G_W is constant, then the stationary point ($G=G_W$) is unstable. Equation (3) is unsuitable for the long-term but is suitable for trade-cycle analysis. s and C are variables that depend upon the phase of trade cycles and the interest rate. Therefore, the value for G_W changes over a trade cycle. Equation (3) can describe a turning point when G is equal to G_W .³ Harrod did not provide a complete dynamic system for the economy, he just proposed an idea of dynamic thinking.

Okishio (1964) provided a formal proof of the instability of Harrod-Domar's model with attention to the capital utilization rate and the investment function, both, in the cases of the fixed coefficient and the flexible production function. His system in the case of a fixed coefficient production is as follows:

$$sY = I, \quad (4)$$

$$dK / dt = I, \quad (5)$$

³ Harrod (1973, Chapter 3) discussed in detail the lower and upper turning points.

$$dg / dt = \alpha(\delta - 1), \quad (6)$$

$$\delta \equiv Y / \sigma K, \quad (7) \text{ and}$$

$$g \equiv I / K, \quad (8)$$

where \equiv : definition, Y : output, I : investment demand, s : savings rate (constant, $0 < s < 1$), δ : rate of capital utilization, σ : normal output-capital (constant), K : capital stock, g : growth rate of capital, α : positive constant. Equation (4) is an equilibrium condition of the goods market. Output is determined by a principle of effective demand. Equation (5) means that capital accumulation is equal to investment demand. Equation (6) is an investment function of the Harrod-Okishio type. The utilization rate indicates the current level of capital shortage or excess capacity. If the utilization rate is greater than 1, i.e. capital shortage, then capitalists increase the capital growth rate more than the previous year's level and *vice versa*. Equation (7) is a definition of the utilization rate of capital equipment. Equation (8) is a definition of g .

The system of Equation (4) – (8) is reduced to the following equation.

$$dg / dt = \alpha(g / s\sigma - 1). \quad (9)$$

The stationary point ($g^* = s\sigma$) is clearly unstable and has a 'knife-edge' property.⁴

Equation (9) can be translated in a Keynesian style. Let

$$y = Y / K,$$

then, from Equation (4) and (8) we get:

$$y = \frac{Y}{I} \frac{I}{K} = \frac{1}{s} g.$$

⁴ Yoshida (1999) considered the instability of the Harrodian model with a flexible production function.

The time-derivative of the above equation and Equation (9) yields the following equation:

$$dy/dt = (\alpha/s\sigma)(y - \sigma). \quad (10)$$

If the actual growth rate exceeds the warranted rate, then $y > \sigma$ and y increases, leading to even more excess demand relative to capital. The economy moves into an explosive excess demand trajectory in which, despite rising investment, a growing shortage of capital stock relative to demand persists. If the actual growth rate is less than the warranted rate, then $y < \sigma$ and y falls, leading to an even larger excess capacity. The growth rate declines and, despite the slower growth (or absolute decline) of investment, a growing surplus of capital stock relative to demand exists.

3. Harrodian dynamics 2: G_N and G_{PW}

Harrod (1939) proposed the concept of the natural rate of growth: ‘This[the natural rate of growth] is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense’ (p. 30). He introduced the concept of the proper warranted rate of growth: ‘Indeed, there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity. Consideration may be given to that warranted rate which would obtain in conditions of full employment; this may be regarded as the warranted rate “proper” to the economy’ (p. 30).⁵

⁵ The ‘proper’ warranted rate of growth term disappeared in Harrod (1948, 1973).

Then, he put forward his long-run or trend analysis: 'If the proper warranted rate is above this[the natural rate], there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment. The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profits. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profits booms' (p. 30).

Now, let us assume that we are proceeding on the full employment growth path and that, at a time, the natural rate of growth is less than the proper warranted rate of growth, perhaps, because of an increase in the savings rate. As such, a decrease in consumption demand occurs and the actual rate of growth will decrease because the actual rate (= the natural rate) is less than the warranted rate. Under - employment now exists for several periods. The growth rate decreases as long as it is smaller than the warranted rate.

Harrod's reasoning of long-run analysis is based on his short-run instability principle. Even in the long-run, he does not postulate the full employment of labour and

Harrod (1973, p. 36) made a distinction between a 'normal' and 'special' warranted rate. The normal rate is the initial warranted rate as pertaining to a steady advance and the special rate changes under the influence of a boom or slump. Harrod (1973, Chapter 7) discussed some problems and conflicts that arose from the discrepancy of the normal warranted and natural rate of growth.

capital unlike Solow and the neoclassical growth theory.⁶ A problem of the discrepancy of the natural and proper warranted rate of growth concerns a long-run trend or the structure of an unemployment economy. In contrast, the neoclassical school presumes a full employment economy and considers the stability of the steady growth of the economy.

This discrepancy problem can be formally analyzed as follows:

$$N = nY, \quad (11)$$

$$dL/dt = \lambda L, \quad (12) \text{ and}$$

$$e \equiv N/L, \quad (13)$$

where N : employment, n : labour coefficient (constant), L : labour supply, λ : growth rate of labour force(constant), e : employment rate. Equation (11) means that employment is determined by output. Equation (12) means that labour supply increases at the constant growth rate. Equation (13) is a definition of the employment rate. We assume $e < 1$ and analyze the underemployment economy. Let

$$k = K/L. \quad (14)$$

Then Equation (13) yields, from Equations (4), (7), (8) and (11), we get

$$e = nkg/s. \quad (15)$$

The time-differentiation of Equation (14) yields

⁶ Nikaido (1975, 1980) discussed the instability of the equilibrium growth path with the Harrodian investment function under a neoclassical production function. Fanti and Manfredi (2009), Sportelli (2000) and Yoshida (2007) are recent elaborations of the Harrodian dynamic models. Hein *et al.* (2011) provides a clear explanation of Harrodian instability, comparing it to Keynesian instability. Moudud (2009) discussed the role of the fiscal and taxation policy of warranted growth. See Dalgaard and Erickson (2006) and Shaikh (2009) on policy issues.

$$d \log k / dt = g - \lambda . \quad (16)$$

We shall focus on the movement of e and g . From Equation (9), the time-differentiation of Equation (15), and Equation (16), we get the following equation:

$$d \log e / dt = (g - \lambda) + \alpha \left(\frac{1}{s\sigma} - \frac{1}{g} \right) \equiv E(g) . \quad (17)$$

$E(g)$ has the following properties:

$$E(0) = -\infty, E(s\sigma) = s\sigma - \lambda, E(\lambda) = \alpha \left(\frac{1}{s\sigma} - \frac{1}{\lambda} \right),$$

$$dE(g)/dg > 0 \text{ for } g > 0 .$$

We denote g^e as a value of g such that $E(g^e) = 0$.

Then

$$\lambda < g^e < s\sigma \text{ if } \lambda < s\sigma ,$$

$$\lambda > g^e > s\sigma \text{ if } \lambda > s\sigma .$$

< Figure 1 >

< Figure 2 >

The system of Equations (9) and (17) describes the movement of g and e . Figures 1 and 2 illustrate the phase diagrams. Figure 1 is the case where the proper warranted rate, $s\sigma$, is greater than the natural rate of growth, λ . In Figure 1, the capital growth rate and the employment rate are declining even if the employment rate moves upwards for a while, after the capital growth rate is below the proper warranted rate. This indicates that a chronic tendency to depression exists. Figure 2 is the case where the proper warranted rate is less than the natural rate. In Figure2, the capital growth rate

and the employment rate are moving upwards even if the employment rate falls for a while, after the capital growth rate is above the warranted rate. This implies that a tendency of a profit boom occurs.

4. Hoover Curve of the U.S.

Hoover (2008) took Harrod's analysis as an empirically testable proposition: 'Harrod's conjecture says that an economy in which the proper warranted rate of growth (G_{PW}) stands above the natural rate of growth will display "a chronic tendency to depression" and an economy in which the proper warranted rate stands below the natural rate may be frequently driven towards full employment'(p.21). 'One implication of Harrod's hypothesis is that the output gap should be inversely related to the difference between the natural and the warranted rate ($G_N - G_{PW}$)' (p.22).

Hoover constructed a potential output series (Y_t^{pot}) for the U. S. for the years 1929–2005. The time series is generated from a Cobb-Douglas production function with the available labour force and capital stock. The level of the total factor productivity is estimated to grow smoothly along the upper bounds of the actual total-factor productivity. From the time series for the potential output and the capital stock, estimates of capital-output ratio (C) and the net savings rate (s) can be constructed, eventually yielding an estimate of the proper warranted rate of growth rate for each period: $G_{PW,t} = s_t / C_t$. The natural rate (G_N) is just the rate of growth of potential output for each period. The output gap is simply the percentage by which actual output falls short of the potential output each period: $Gap_t = (Y_t^{pot} - Y_t) / Y_t^{pot}$.

By a regression analysis, Hoover obtained a downward-sloping regression line:

$$Gap = -5.9(G_N - G_{PW}) + 16.1 \quad (18)$$

$$R^2 = 0.40$$

His Figure 5⁷ illustrates that ‘this is indeed the case: when the U.S. economy has a proper warranted rate of growth that was low relative to the natural rate, it has operated near to full potential’ (p. 22).

5. Hoover curve of Japan

I apply Hoover’s method to the Japanese economy. First I construct a potential output series for Japan for the years of 1956–2009 (The data construction details are found in Appendix 2) and then estimate the potential GDP. The GDP *Gap* and G_N are estimated for the period 1957–2009. G_{PW} is estimated as a trend of the capital growth rate. By a regression analysis, the following equation is estimated:

$$Gap = -2.257(G_N - G_{PW}) - 8.579 \quad (19)$$

(-3.794) (3.910)

$$R^2 = 0.220 \quad DW = 0.095$$

The determinant co-efficient is rather small. A scattered diagram with connected lines and years is shown in Figure 3. Looking at the time configuration of *Gap* and the difference of G_N and G_{PW} , it is easily observed that there are two Hoover curves. Hence, some structural change must have occurred since the 1970s. It is well known that in the early 1970s, the rapid growth period ended: therefore, 1970 was a turning point.⁸

⁷ Figure5 in Hoover (2008) is reproduced in Appendix 1. Prof. Hoover added the years and lines on it and provided the Figure to me.

⁸ See Nakamura (1995, Chapter 6) and Yoshikawa (1995, pp. 25–26 and p. 38) on the

< Figure 3 >

I add a dummy variable during 1972–2009 and estimate the regression again as follows:

$$Gap = -5.172(G_W - G_{PW}) - 18.89Dummy + 12.946 \quad (20)$$

$$\begin{matrix} (-10.458) & (-8.955) & (8.884) \end{matrix}$$

$$AR^2 = 0.688 \quad DW = 0.637$$

The determinant co-efficient is improved and the estimated values are all sufficiently significant (at the 1% level). The value of the slope is similar to the case of the U.S.A. The two estimated lines are illustrated in Figure 4, implying that the Japanese economy structurally changed in the early 1970s. If the G_{PW} falls, then the estimation line changes. The estimation line moves to the left by 3.65%,⁹ implying that G_{PW} falls from 7.22% to 3.57%.

< Figure 4 >

The shift of the estimated line would reflect the decline in the proper warranted rate of growth, $G_{PW}=s/C$, with a smaller net savings rate, s , and/or larger capital-output ratio, C , in the long run. In an open economy, from saving and investment balances, it follows that:

$$s = s_p(1 - d - t) - (g - t) - (x - m) \quad (21)$$

where s_p : private net savings rate of NDP, d : depreciation rate of GDP, g : government expenditure rate of GDP, t : tax rate, x : export rate, m : import rate. A decrease in s is

end of rapid growth. See Boyer and Yamada (2000) and Okishio (1992, Chapter 2) on the Japanese economy from 1950s –1990s.

⁹ The Hoover curve $Gap = -a(G_N - G_{PW}) + b$ is shifted down if G_{PW} is smaller. Let the new Hoover curve be $Gap = -a(G_N - (G_{PW} - \beta)) + b = -a(G_N - G_{PW}) - a\beta + b$. Then from Equation (20), the value of β is calculated as $\beta = 18.89/5.172 = 3.6523$.

caused by (1) a decline in s_p , (2) an increase in d , (3) an increase in t , (4) an increase in the full employment budget deficits rate, $g - t$, and (5) an increase in the foreign trade surplus rate, $x - m$. These changes in the Japanese economy have already been pointed out by many authors.¹⁰

6. Trends and changes of the Japanese economy

Figures 5 and 6 illustrate that GDP Gap and the gap between G_N and G_{PW} , respectively, in a time series form. By observing the ups and downs in these figures, the following periods are easily distinguished and corresponded to the economic topics in the Japanese economy:

< Figure 5 >

< Figure 6 >

□ 1957–1970 rapid economic growth:

A declining trend in the GDP gap and a zero gap in 1970. An increasing trend in G_N . The difference between G_N and G_{PW} disappeared in 1970.

□ 1971–1973 full employment and yen-evaluation:

Almost zero GDP gap (i.e. full employment). G_N exceeded G_{PW} .

□ 1974–1985 oil shocks and stable growth:

After sudden increase in the GDP gap, the gap was relatively constant. After the sudden fall of G_N below G_{PW} , there was a stable difference because of the falling G_N and the decline in the G_{PW} .

¹⁰ See Uni (2000) for a trend of C , and Ihori (2007, p. 109) and Hiromatsu *et al.* (1998, p. 110) for full employment budget deficits.

- 1986–1990 bubble economy:

A diminishing GDP gap. An increasing trend in G_N .

- 1991–2001 lost 10years:

An increasing GDP gap and a peak in 2002. An enlarged difference between G_N and G_{PW} owing to a decline in G_N .

- 2002–2007 Koizumi structural reform:

A diminishing but high level GDP gap. A small recovery of the difference between G_N and G_{PW} .

- 2008–2009 Lehman shock and after:

A sudden increase in the GDP gap owing to the Lehman shock in September 2008.

In Figure 6, the proper warranted rate of growth, G_{PW} , is a constant. The ups and downs in the graph reflect the movement of the natural rate, G_N . Broadly speaking, there is an increasing trend of the G_N until the early 1970s, a low level of G_N during the early 1980s and a further decreasing trend of G_N after the early 1990s. Supposing that a downward shift of G_{PW} occurred in the late 1970s and early 1980s, I conclude that the bubble economy during late 1980s appeared in the condition where G_N was greater than G_{PW} .¹¹

7. Conclusions

¹¹ Considering a shift down of G_{PW} by 3.65% after 1972, the horizontal line moves down by the same degree as Figure 6.

The Harrodian instability principle, which comes from a discrepancy between the actual and warranted rate of growth, is a short run tool used for trade-cycle analysis. A discrepancy problem between the natural and proper warranted rate concerns the long-run or trend analysis and is related to the structure of the economy. A Hoover curve is a testable proposition of the long run or trend of the economy.

In this paper, I showed a Hoover curve with a structural change in the case of the Japanese economy during the time period 1957–2009. This implied an increasing trend in the natural rate of growth, G_N , until the early 1970s, a low level of G_N during the early 1980s, a decreasing trend of G_N after the early 1990s, and a shift down in the proper warranted rate of growth, G_{PW} , after the late 1970s. I also briefly discussed some trends and changes in the Japanese economy using a Hoover curve since the 1950s. I concluded that the bubble economy during the late 1980s appeared in the condition where G_N was greater than G_{PW} .

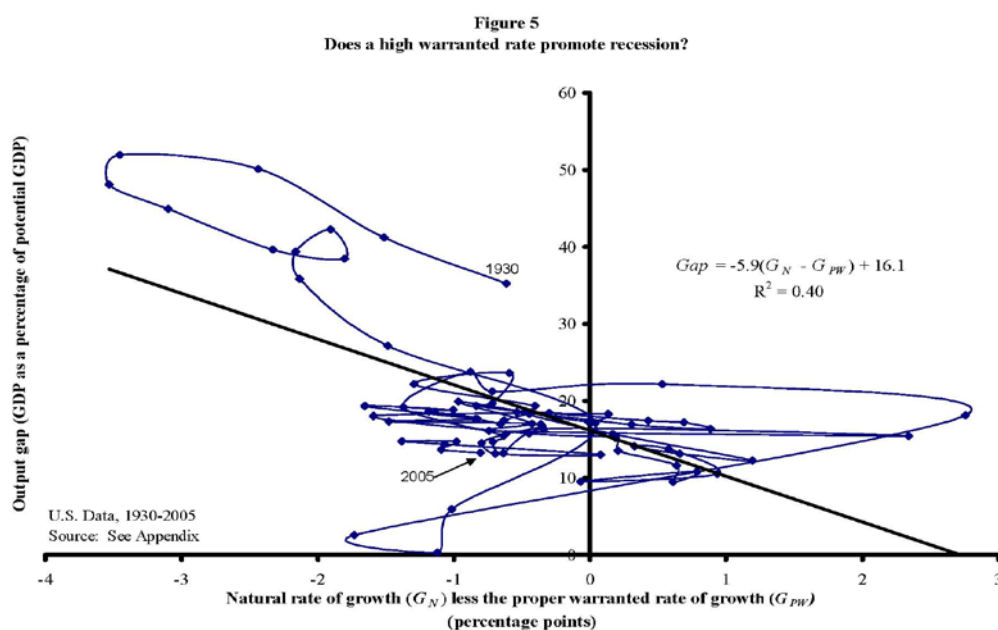
Undoubtedly further investigations should analyze the Japanese economy in more detail. It is clear that the Harrodian dynamic theory has an empirical evidence and is effective to analyze the economic growth and trade cycles.

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Appendix 1: Hoover Curve of the U.S.



Source: Hoover, Kevin D. 'Was Harrod Right?,' *Working paper*, Department of Economics, Duke University, 25 May 2008.

Appendix 2: Japanese Data Construction

1) Data Source

ESRI(economic and Social Research Institute) Website

(<http://www.esri.cao.go.jp/jp/sna/menu.html>), accessed on 6 May 2011.

Gross domestic product (NY)

Real gross domestic product (Y), fixed-based approach on 2000

Compensation of Employees

Private capital stock (K)

National Account for 2009(benchmark year=2000, 93SNA):

real GDP, nominal GDP and Compensation of employees for 1980-2009

National Account for 2000

real GDP, nominal GDP and Compensation of employees for 1955-1998

a series of real GDP(benchmark year=2000) is made from connecting two series at the 1980.

Ministry of Internal Affairs and Communications Website
 (<http://www.stat.go.jp/data/roudou/index.htm>), accessed on 6 May 2011.
Labour Force (LF): Source: Labour Force Survey

2) Construction of the Output Gap.

$$\text{Labor share in output}(\alpha): \alpha_t = \frac{\text{compensation of employees}}{NY_t - \text{personal firm income}}.$$

Personal firm income includes personal income in agriculture and fisheries.

$$\alpha = \text{the mean of } \alpha_t = 0.5746.$$

$$\text{Total-factor productivity (A): } A_t = \frac{Y_t}{LF_t^\alpha K_{t-1}^{(1-\alpha)}},$$

where K_{t-1} means the value of capital at the end of the year $t-1$ (=the beginning of the year t). At the above definition, the use of the labor force (LF) rather than employment has the effect of incorporating the “inefficiency” of the unemployment into A .

Full employment total-factor productivity (A^{full}): an exponential trend is fitted to A by ordinal least squares:

$$\ln \hat{A}_t = 0.0061 \text{time} - 9.9538. \quad \ln A_t^{full} = \ln \hat{A}_t + 0.1829.$$

The additional constant has the effect of shifting the whole path of \hat{A}_t so that it forms the outer envelope of the A_t .

$$\text{Potential GDP } (Y_t^{pot}): Y_t^{pot} = A_t^{full} LF_t^\alpha K_{t-1}^{(1-\alpha)}.$$

$$\text{GDPGap } (Gap_t): Gap_t = \frac{Y_t^{pot} - Y_t}{Y_t^{pot}}.$$

3) Construction of the Natural Rate of growth.

$$\text{Natural Rate of Growth } (G_N): G_N = \frac{Y_t^{pot} - Y_{t-1}^{pot}}{Y_{t-1}^{pot}}$$

4) Construction of the Proper Warranted Rate of Growth.

Trend capital (\hat{K}): An exponential trend is fitted to K by ordinary least square:

$$\ln \hat{K}_t = 0.0671 \text{time} - 125.44$$

$$\text{Capital-output Ratio (C): } C_t = \frac{\hat{K}_{t-1}}{Y_t^{pot}}.$$

$$\text{Net Savings Rate (s): } s_t = \frac{\hat{K}_t - \hat{K}_{t-1}}{Y_t^{pot}}.$$

$$\text{Proper Warranted Rate of Growth (G}_{PW_t}): G_{PW_t} = \frac{s_t}{C_t} = \frac{\hat{K}_t - \hat{K}_{t-1}}{\hat{K}_{t-1}} = 7.216\%.$$

Figures

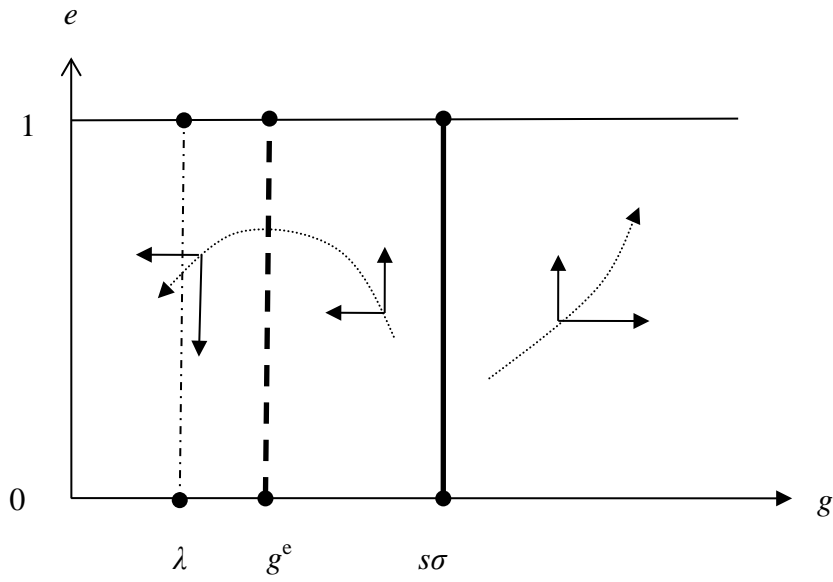


Figure 1. g - e phase diagram in Case1 (proper warranted rate > natural rate: $s\sigma > \lambda$)

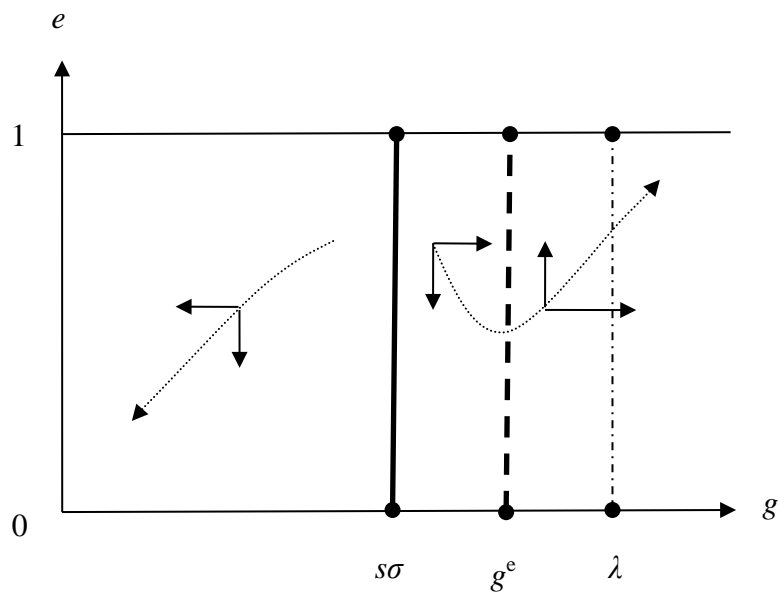


Figure 2. g - e phase diagram in Case 2 (proper warranted rate < natural rate: $s\sigma < \lambda$)

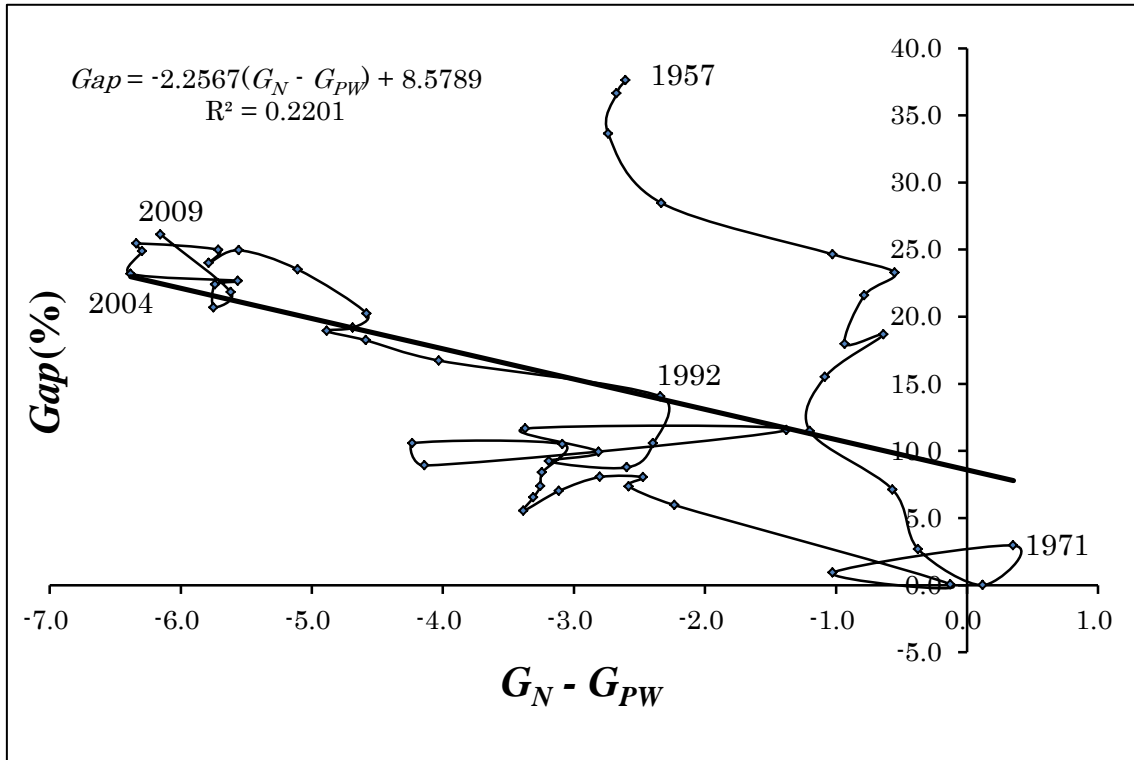


Figure 3. GDP and Growth rate gap, Japan, 1957-2009

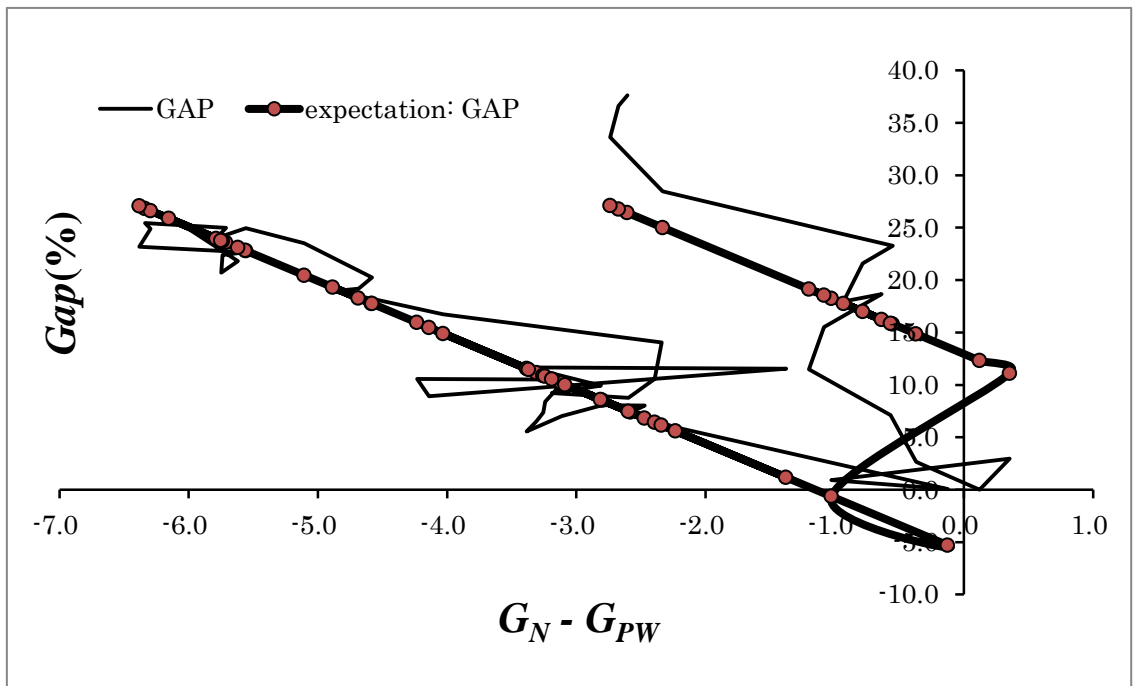


Figure 4. Shift of Hoover curve, Japan

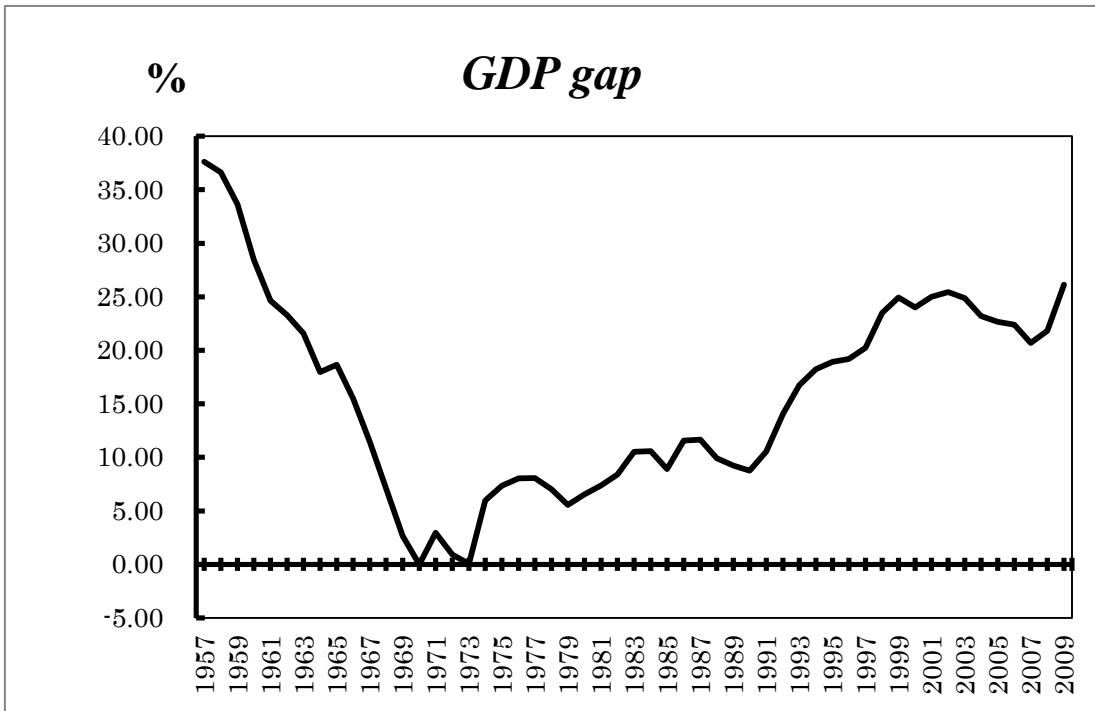


Figure 5. GDP gap, 1957-2009

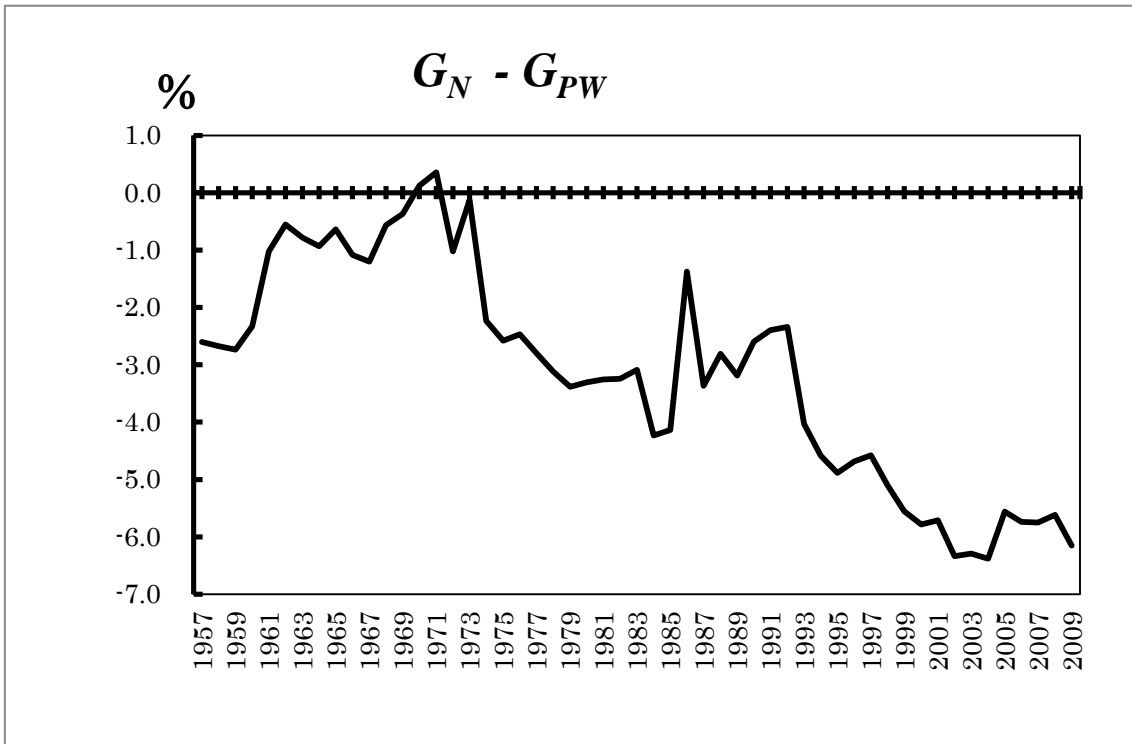


Figure 6. Growth rate gap, 1957-2009