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Renegotiations and the Diffusion of a Technology with Positive Externalities

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Abstract

We consider the problem of technology transfer. We specifically treat a technology such that an adoption of the technology by a player increases the other players' payoffs but may decrease the adopter's own payoff. The technology is transferred through negotiations. The property of the technology gives players the incentive to deviate from the negotiation for free-riding on the other players' adoptions. We formulate this situation by the theory of social situations and investigate whether full diffusion is possible provided that full diffusion achieves the social optimum. We show that full diffusion is always achieved in the initial negotiation when renegotiations are allowed after implementing an agreement, whereas full diffusion may fail to be achieved if no such renegotiation is allowed.

Keywords: technology with positive externalities, full diffusion, renegotiation after implementing an agreement, theory of social situations, optimistic stable standard of behavior

JEL Classification: C71 · D62 · H87

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1 Introduction

We consider the problem of technology transfer through negotiations. A technology is a kind of information. A distinguishing feature of information as a commodity is its irreversibility. One player can transfer information while retaining the information he has because the player can replicate the information either without cost or with a negligible cost. Specifically, we treat a technology where adoption by one player benefits the others. Typical examples include water purification technology for an open-access water resource and technology for reducing pollution emission abatement costs. More generally, cost reduction technology for a public good with differentiated cost is also an example. We examine whether full diffusion is possible provided that full diffusion achieves the social optimum.

We will refer to such a technology as a technology with positive externalities (henceforth, TPE). Many studies have pointed out that the adoption of a TPE may not serve the adopting player's own interest. Buchholz and Konrad (1994) considered an international pollution emission reduction model where each player strategically chooses his emission reduction technology prior to the noncooperative or cooperative emission reduction decisions. They showed that there are incentives for the players to choose higher-cost technologies in both cases. Lee (2001) considered a two-country model of the pollution emission reduction technology transfer. He showed that the recipient's welfare may decrease even if a simultaneous income transfer is allowed. Cornes and Hartley (2007) showed that in a voluntary provision game of a public good with differentiated unit costs, a player's payoff at a Nash equilibrium may decrease when his own unit cost is reduced.

Furthermore, there may involve an additional cost for installing the TPE, even if the TPE adoption itself benefits the recipient. Some examples are given by Jaffe *et al.* (2005): the cost to learn the new technology, the cost to purchase new equipment, and the cost to adapt the new technology to their own circumstances. Strunland (1996) showed that the existence of such costs may prevent technology transfer in a two-country model even though the technology transfer itself is Pareto-improving.

The payoff structure of the TPE transfer is summarized as follows. First, an adoption of the TPE by one player increases the other players' payoffs. Second, an adoption of the TPE *may* decrease the adopting player's own payoff. Therefore, our model includes a typical social conflict along the lines of the prisoners' dilemma and the game of chicken. Further, we will allow side payments that will be described as the monetary transfer. Under such a payoff structure, we consider the following TPE transfer process. The TPE is originally held by only one player. Agents, including the original holder, form a coalition and negotiate cooperatively for the TPE and the monetary transfer. Despite the optimality of full diffusion, players may find it preferable to refuse the technology and monetary transfers and free-ride on the benefits generated by the other players' adoptions. Then, such a player will deviate from the negotiation, and full diffusion may not be sustained.

To formulate such a situation, we need to consider a hybrid model that incorporates both the (cooperative) coalitional negotiations and (noncooperative) deviations for free-riding. In the coalitional negotiations, players are allowed to deviate from the negotiation by starting another negotiation by a subcoalition. This kind of deviation is just like a standard deviation in coalitional games. Additionally, players are allowed to deviate from the negotiation for free-riding on the other players' adoptions. Several studies have investigated models incorporating both cooperative and noncooperative behavior. Carraro and Siniscalco (1993) considered the international environmental agreement model. An agreement is defined by a coalition, where participants can transfer money within it. They also allow the players to deviate noncooperatively from the coalition to free-ride the remaining players' efforts for the environment. Furusawa and Konishi (2011) defined the free-riding proof core for a public good economy. The free-riding proof core requires the robustness not only to the standard coalitional improvement, but also to noncooperative deviations from contributing for a public good for free-riding. Both models exogenously assume cooperation in the remaining coalition after a (noncooperative) free-riding deviation. However, remaining players also behave for their own interest. It should be considered what happens in the negotiation held by the remaining players. Furusawa and Konishi (2011) required that the coalitional improvement be achieved with an allocation that is credible in the sense of Ray (1989). We also impose this requirement.

To capture such a negotiation process, we formulate our model by the theory of social situations (Greenberg, 1990). We employ the optimistic stable standard of behavior (henceforth, OSSB) as the solution concept. Greenberg (1990) argued that the OSSB is an acceptable recommendation for rational players. In this sense, our main purpose is as follows: whether the full diffusion of the TPE is acceptable for the players at all.

We first consider a situation where the standard coalitional deviations and the free-riding deviations are allowed. In this model, the OSSB may not include the full diffusion. We next consider a situation where the players are additionally allowed to renegotiate after reaching an agreement and implementing the agreement. Then, the result is drastically changed, and the OSSB always includes the full diffusion of the TPE. Indeed, the OSSB recommends that the full diffusion be achieved in the initial negotiation.

Therefore, the possibility of renegotiations after implementing an agreement plays an important role. Similar concepts of renegotiation have been employed in the literature. Carraro and Siniscalco (1993) considered renegotiations among participating and nonparticipating players concerning the international environmental agreement, where participating players commit themselves not to deviate from the agreement afterward. Gomes (2005) employed a similar renegotiation concept in the game of coalitional multilateral contracts with externalities.[1] In his model, players form a coalition and write a contract within the coalition. After writing a contract in each coalition, some coalitions merge, and the merged coalitions can rewrite the contract if all the members agree. In these studies, the coalitions sequentially expand in general, and the grand coalition is eventually formed after a finite number of commitments. In contrast, in the persent paper, the full diffusion of the TPE is always acceptable for players in the initial negotiation.

Another related study is Muto and Nakayama (1994), who also employ the theory of social situations to investigate the trade of information where resales are freely allowed. They addressed an information with negative externalities such as the cost reduction technology in a Cournot market. They showed that if the initial trade is appropriate, then no further resale by new holders takes place and the initial holder can retain its benefit despite the fact that resales are possible. They called such a trade the resale-proof trade, which was originally defined by Nakayama *et al.* (1991).[2] Our problem is a kind of inverse problem of the resale-proofness in the following sense. The full diffusion may be intuitively prevented since recipients of the TPE may

[1]Other multilateral contracts models including renegotiations after a commitments has been considered by Seidmann and Winter (1998), Okada (2000), and Gomes and Jehiel (2005).

[2]See also Muto (1986, 1990) and Nakayama and Quintas (1991).

have incentives to deviate from the negotiation even though the full diffusion is efficient. Our analysis shows that the possibility of renegotiations after implementing agreements, which corresponds to the possibility of resales in Muto and Nakayama (1994), makes the full diffusion of the TPE acceptable.

The rest of this paper is organized as follows. In the next section, we briefly introduce the theory of social situations. In section 3, we define a model of TPE transfer where standard coalitional deviations and the free-riding deviation are allowed and show that full diffusion may not be achieved. In section 4, we define a model of TPE transfer that allows renegotiations after agreements are implemented. The main result of this paper is stated in this section. The proof of the main theorem is relegated to the Appendix. In the final section, we conclude with some remarks.

2 Theory of social situations

We briefly introduce the theory of social situations established by Greenberg (1990). The framework in the theory of social situations is a *situation* that is a tuple (γ, Γ), where γ is an *inducement correspondence* and Γ is a set of *positions*. A position is a triple $G = (N(G), X(G), (u_i(G))_{i \in N(G)})$, where $N(G)$ is the set of players at G, $X(G)$ is the feasible outcomes at G, and $u_i(G)$ is the payoff function of player $i \in N(G)$ at G.

The inducement correspondence is a mapping γ that assigns $\gamma(S|G, x) \subseteq \Gamma$ for any $G \in \Gamma$, $S \subseteq N(G)$, and $x \in X(G)$. An alternative position $G' \in \gamma(S|G, x)$ is a position that is inducible from G via S when x is taken as an outcome. A situation requires that Γ be closed under γ. For a situation (γ, Γ), a mapping σ that assigns $\sigma(G) \subseteq X(G)$ for any $G \in \Gamma$ is said to be a standard of behavior (henceforth, SB). The purpose of the theory of social situations is to investigate the stability of the SB.[3]

Definition 1 *An SB σ for a situation (γ, Γ) is said to be an optimistic stable standard of behavior (henceforth, OSSB) iff σ satisfies the following.*

The optimistic internal stability *For all $G \in \Gamma$, $x \in \sigma(G)$ implies that there do not exist $S \subseteq N(G)$, $H \in \gamma(S|G, x)$, and $y \in \sigma(H)$ such that $u_i(H)(y) > u_i(G)(x)$ for all $i \in S$.*

The optimistic external stability *For all $G \in \Gamma$, $x \in X(G) \setminus \sigma(G)$ implies that there exist $S \subseteq N(G)$, $H \in \Gamma(S|G, x)$, and $y \in \sigma(H)$ such that $u_i(H)(y) > u_i(G)(x)$ for all $i \in S$.*

Let σ be an SB in (γ, Γ) and $G \in \Gamma$. The optimistic dominion of G relative to σ via $T \subseteq N(G)$ is defined by

$$ODOM^T(\sigma, G) = \left\{ x \in X(G) \;\middle|\; \begin{array}{c} \exists H \in \gamma(T|G, x), \exists y \in \sigma(H), \\ u_i(H)(y) > u_i(G)(x), \forall i \in T \end{array} \right\}.$$

Then, the optimistic dominion of G relative to σ is defined by

$$ODOM(\sigma, G) = \bigcup_{T \subseteq N(G)} ODOM^T(\sigma, G).$$

Remark 1 We can restate the definitions of the optimistic stability in terms of the optimistic dominion of G as follows (see Greenberg, 1990). Let σ be an SB for a situation (γ, Γ). Then,

- σ satisfies the optimistic internal stability if and only if $\sigma(G) \subseteq X(G) \setminus ODOM(\sigma, G)$ for all $G \in \Gamma$;
- σ satisfies the optimistic external stability if and only if $\sigma(G) \supseteq X(G) \setminus ODOM(\sigma, G)$ for all $G \in \Gamma$;

[3]We omit the definition of the conservative stability of the SB, which is an another stability concept in the theory of social situations. See Greenberg (1990) for details.

- σ is an OSSB if and only if $\sigma(G) = X(G) \setminus ODOM(\sigma, G)$ for all $G \in \Gamma$.

The last restatement will play an important role in the proof of the main results.

In general, a situation may admit no OSSB or multiple OSSB. Greenberg (1990) showed that if a situation is *hierarchical*, the OSSB uniquely exists.

Definition 2 *A situation* (γ, Γ) *is said to be hierarchical iff*

(a) *there exists a finite partition* $\{\Gamma_1, ..., \Gamma_k\}$ *of* Γ *such that for all* $h = 1, ..., k$ *and any* $G \in \Gamma_h$, $\left(\gamma, \{G\} \cup \left(\bigcup_{h+1 \leq h \leq k} \Gamma_h\right)\right)$ *is a situation, and*

(b) *for any* $G \in \Gamma$, *there exists at most one* $S \subseteq N(G)$ *such that* $G \in \gamma(S|G, x)$ *for some* $x \in X(G)$.

3 The TPE transfer situation

Let $N = \{1, ..., n\}$ $(n \geq 2)$ be the set of players. A nonempty subset of N is called a coalition. Let $\mathcal{N} = \{T \subseteq N | T \neq \emptyset\}$ be the set of coalitions. For each coalition S, let $\mathcal{N}(S) = \{T \in \mathcal{N} | T \cap S \neq \emptyset\}$ be the set of coalitions each of which has a nonempty intersection with S. For each coalition $S \in \mathcal{N}$, let $|S|$ denote the cardinality of S.

We consider a situation where a technology is transferred among the players. We say this technology is a technology with positive externalities (TPE) in the sense that an additional TPE transfer to a player benefits the other players. This technology is initially held by one player, say, player 1. A transfer of the TPE may involve monetary transfers. These can include the expense of the TPE (*i.e.*, from the recipient to the donor) and the subsidy for adopting the TPE (*i.e.*, from the donor to the recipient). We say an

n-dimensional vector x is a transfer vector if $\sum_{i\in N} x_i = 0$. Let $\mathbb{R}^n_0$ denote the set of transfer vectors. For each $x \in \mathbb{R}^n_0$ and $i \in N$, $x_i > 0$ means that player i receives an amount of money x_i, and $x_i < 0$ means that player i pays an amount of money $-x_i$.

The state, which describes (i) the holder(s) of the TPE and (ii) the monetary transfer, is defined by a coalition $S \in \mathcal{N}(\{1\})$ and a monetary transfer vector $z \in \mathbb{R}^n_0$. Therefore, the set of states is defined by

$$\Theta = \mathcal{N}(\{1\}) \times \mathbb{R}^n_0.$$

A special state is the *initial* state, where the TPE has not been transferred yet and no monetary transfer has taken place. The initial state is defined by $\bar{\theta} = (\{1\}; 0, ..., 0)$. For simplicity, we identify the phrase "adopting the TPE" with the phrase "the TPE is transferred". Therefore, given a state (S, z), S is the set of players who have adopted the TPE as well as the set of players who have transferred the TPE.

For each $i \in N$ and $S \in \mathcal{N}(\{1\})$, $f_i(S)$ defines the payoff of i when the TPE is diffused among the members in S. Assume that f_i measures the payoff in terms of money for each $i \in N$. We will be able to represent the total payoff in a quasi-linear form: for each $i \in N$, the payoff function $u_i : \Theta \to \mathbb{R}$ is defined as $u_i(S, z) = f_i(S) + z_i$ for any $(S, z) \in \Theta$. We assume the following conditions on f_i for each $i \in N$.

Assumption 1 **(a)** *For each $i \in N$, $f_i(S) < f_i(T)$ for all $S, T \in \mathcal{N}(\{1\})$ with $S \subsetneq T$ and either $i \in S$ or $i \notin T$.*

(b) *$\sum_{i\in N} f_i(N) > \sum_{i\in N} f_i(S)$ for all $S \in \mathcal{N}(\{1\}) \setminus \{N\}$.*

Assumption 1(a) represents the positive externalities of the technology. Assumption 1(b) requires the full diffusion of the TPE be socially optimum.

Assumption 1 does not exclude the possibility that the adoption of the TPE may decrease the adopter's own payoff; *e.g.*, for each $S \in \mathcal{N}(\{1\})$ and $i \notin S$, it may be $f_i(S) > f_i(S \cup \{i\})$. This may be caused by the effect of the technological change through the interactions among players (Buchholz and Konrad, 1994; Lee, 2001; Cornes and Hartley, 2007) or some cost of implementing the TPE (Strunland, 1996; Jaffe *et al.*, 2005).

A state (S, z) is said to be

- individually rational iff $f_i(S) + z_i \geq f_i(\{1\})$ for all $i \in N$;
- (weakly) Pareto efficient iff there exists no $(S', z') \in \Theta$ such that $f_i(S') + z'_i > f_i(S) + z_i$ for all $i \in N$;
- a full diffusion state iff $S = N$.

Note that a state is a full diffusion state if and only if it is Pareto efficient by Assumption 1(b).

Now, we define the TPE transfer situation.

Definition 3 *Let* $\Gamma = \{G(T) \,|T \in \mathcal{N}(\{1\})\}$ *be the set of positions, where for all* $T \in \mathcal{N}(\{1\})$,

$$\begin{aligned} &N(G(T)) = T, \\ &X(G(T)) = \{(R, x) \in \Theta | 1 \in R \subseteq T, x_i = 0, \forall i \in N \setminus T\}, \\ &u_i(G(T)) = u_i \textit{ for all } i \in N, \end{aligned}$$

and let γ *be the inducement correspondence such that*

$$\gamma(Q|G(T), (R, x)) = \begin{cases} \{G(Q)\} & \textit{if } Q \in \mathcal{N}(\{1\}), Q \subseteq T; \\ \{G(T \setminus Q)\} & \textit{if } Q \subseteq T \setminus \{1\}; \\ \emptyset & \textit{otherwise} \end{cases}$$

for each $T \in \mathcal{N}(\{1\})$ *and* $(R, x) \in X(G(T))$. *Then the situation* (γ, Γ) *is said to be a TPE transfer (henceforce, TPET) situation.*

Note that $(R, x) \in X(G(T))$ implies that $\sum_{i \in T} x_i = 0$ for all $T \in \mathcal{N}(\{1\})$.

The TPET situation describes the following process. Let $T \in \mathcal{N}(\{1\})$. At position $G(T)$, the players in T negotiate the TPE transfer and the monetary transfer within T. The states available in this negotiation are denoted by $X(G(T))$. Note that $X(G(T))$ includes the states where some players in T are excluded from the TPE transfer and the states where some players are involved only in the monetary transfer. It might seem strange, but we allow such kinds of transfers by following the spirit of the theory of social situations expressed by Greenberg (1990, p.10) "An outcome is a feasible alternative; it need not be a 'predicted,' 'reasonable,' or 'rational' alternative." Of course, there is a possibility of no transfer, which is indicated by the initial state $\bar{\theta}$.

Once a state (R, z) is proposed at $G(T)$, the inducement correspondence γ allows the following two kinds of deviations. First, a coalition Q with $1 \in Q \subseteq T$ is allowed to deviate and renegotiate by itself. This deviation would be successful if Q can agree on a more preferable state. This deviation examines whether (R, z) is a robust agreement within T against the standard notion of deviations in the coalitional game. Second, a coalition $Q \subseteq T \setminus \{1\}$ is allowed to deviate from (R, z) by refusing any technology and monetary transfer and to leave the negotiation. This deviation would be successful if Q finds it preferable to leave the negotiation and enjoy the benefit from the resulting state in the negotiation by $T \setminus Q$. Therefore, this deviation examines whether players keep participating in the negotiation.

We show that the TPET situation has the following property.

Proposition 1 *The TPET situation admits the unique OSSB.*

Proof. Let (γ, Γ) be a TPET situation. It suffices to show that (γ, Γ) is hierarchical as mentioned in Section 2. For each $k = 1, ..., n$, define

$$\Gamma_k = \{G(T) \in \Gamma \mid |T| = n - k + 1\}.$$

Obviously, $(\Gamma_1, ..., \Gamma_n)$ is a partition of Γ. We show that this partition satisfies Definition 2(a).

Fix an arbitrary $G \in \Gamma$. Let $k = 1, ..., n$ such that $G \in \Gamma_k$. Then, there exists some $T \in \mathcal{N}(\{1\})$ such that $G = G(T)$. Note that $k = n - |T| + 1$. Fix an arbitrary $\bar{G} \in \{G(T)\} \cup \bigcup_{k \leq h \leq n} \Gamma_h$. Then, there exists some $R \in \mathcal{N}(\{1\})$ such that $\bar{G} = G(R)$ and either $R = T$ or $|R| < |T|$. For any $(R', x) \in X(G(R))$, $\gamma(Q|G(R), (R', x)) \neq \emptyset$ implies $Q \subseteq R \setminus \{1\}$ or $Q \in \mathcal{N}(\{1\})$ with $Q \subseteq R$.

First, assume $Q \subseteq R \setminus \{1\}$. Then, $\gamma(Q|G(R), (R', x)) = \{G(R \setminus Q)\}$ for any $(R', x) \in X(G(R))$. By $|R \setminus Q| < |R|$, $G(R \setminus Q) \in \bigcup_{k \leq h \leq n+1} \Gamma_h$. Next, assume $Q \in \mathcal{N}(\{1\})$ with $Q \subseteq R$. Then, $\gamma(Q|G(R), (R', x)) = \{G(Q)\}$ for any $(R', x) \in X(G(R))$. By $Q = R$ or $|Q| < |R|$, either $G(Q) = G(R)$ or $G(Q) \in \bigcup_{k \leq h \leq n+1} \Gamma_h$. Hence $\left(\gamma, \{G\} \cup \bigcup_{k+1 \leq h \leq n+1} \Gamma_h\right)$ is a situation.

Definition 2(b) immediately follows from the definition of γ, since for all $S \in \mathcal{N}(\{1\})$ and any $(S', x) \in X(G(S))$, $G(S) \in \gamma(Q|G(S), (S', x))$ implies $Q = S$. Hence (γ, Γ) is hierarchical and admits the unique OSSB. ■

Despite the fact that Proposition 1 assures the existence and the uniqueness of the OSSB, it is difficult to characterize the OSSB for the TPET situation completely. Indeed, it is more difficult than for the more complex model that appears in the later section. We restrict our attention to showing that full diffusion may *fail* to be achieved in the OSSB of the TPET situation. To this end, we consider the following simple situation.

Example 1 This example is the lake of Shapley and Shubik (1969) with a reinterpretation and a restricted assumption. There are $n(\geq 3)$ factories operating with a lake (an open-access water resource). Each firm discharges water after operation. The water discharge of each factory pollutes the lake except for that of factory 1, which has an environmentally sound technology.

We treat the environmentally sound technology as the TPE. If a factory adopts the TPE, then the factory no longer pollutes the lake, but it must bear $B > 0$ of the installation cost. When k factories have not adopted the technology, each firm must purify the water with cost $kD > 0$ before operation. Assume that $2D < B < 3D$. Once the environmentally sound technology is diffused among the members in $S \in \mathcal{N}(\{1\})$, the payoff of each factory is

$$f_i(S) = \begin{cases} -(n - |S|)D & \text{if } i \in \{1\} \cup (N \setminus S), \\ -(n - |S|)D - B & \text{if } i \in S. \end{cases} \tag{1}$$

It can be easily confirmed that this example meets Assumption 1. Note that f_i further satisfies $f_i(S) > f_i(S \cup \{i\})$ for all $i \in N$ and all $S \in \mathcal{N}(\{1\})$ with $i \notin S$. □

In Example 1, the following proposition follows.[4]

Proposition 2 *Let σ be the OSSB for a TPET situation (γ, Γ) defined by Example 1. Let $T \in \mathcal{N}(\{1\})$. Then, $\sigma(G(N)) \neq \emptyset$ if and only if n is odd. Formally,*

(a) *If $|T| \leq 2$, then $\sigma(G(T)) = \{\bar{\theta}\}$.*

(b) *If $|T| > 2$ and $|T|$ is odd, then there exists some $(T, x) \in X(G(T))$ such that $(T, x) \in \sigma(G(T))$.*

(c) *If $|T| > 2$ and $|T|$ is even, then $\sigma(G(T)) = \emptyset$.*

Preceding the proof of Proposition 2, we prove a lemma.

[4]Such a "rotational" outcome frequently appears in the social dilemma situation formulated by the theory of social situations. See for example, Masuzawa (2005) and Nakanishi (2009).

Lemma 1 *Let* (γ, Γ) *be a TPET situation defined by Example 1. Let* $S \in \mathcal{N}(\{1\})$ *with* $|S| > 1$.

(a) *There exists no* $(S, x) \in X(G(S))$ *satisfying both* $f_1(S) + x_1 \geq f_1(\{1\})$ *and* $f_i(S) + x_i \geq f_i(S \setminus \{i\})$ *for all* $i \in S \setminus \{1\}$.

(b) *Let* $R \in \mathcal{N}(\{1\})$ *with* $R \subsetneq S$. *Then,* $\sum_{i \in S} f_i(S) > \sum_{i \in S} f_i(R)$ *if and only if* $|S| \geq 3$.

Proof. Let (γ, Γ) be a TPET situation defined by Example 1. Let $S \in \mathcal{N}(\{1\})$ with $|S| > 1$. For notational simplicity, let $s = |S|$.

(a) Suppose that there exists some $(S, x) \in X(G(S))$ such that $f_1(S) + x_1 \geq f_1(\{1\})$ and $f_i(S) + x_i \geq f_i(S \setminus \{i\})$ for all $i \in S \setminus \{1\}$. Aggregating these inequalities over S,

$$\sum_{i \in S} f_i(S) = \sum_{i \in S} (f_i(S) + x_i) \geq f_1(\{1\}) + \sum_{i \in S \setminus \{1\}} f_i(S \setminus \{i\}) \tag{2}$$

by $\sum_{i \in S} x_i = 0$. On the other hand, by (1)

$$\sum_{i \in S} f_i(S) = -s(n-s)D - (s-1)B,$$

$$\begin{aligned} f_1(\{1\}) + \sum_{i \in S \setminus \{1\}} f_i(S \setminus \{i\}) &= -(n-1)D + (s-1)(-(n-s+1)D) \\ &= -s(n-s)D - 2(s-1)D. \end{aligned}$$

This contradicts (2) by $2D < B$.

(b) Fix an arbitrary $R \in \mathcal{N}(\{1\})$ with $R \subsetneq S$. Denote $r = |R|$. By (1),

$$\begin{aligned} \sum_{i \in S} f_i(S) - \sum_{i \in S} f_i(R) &= [-s(n-s)D - (s-1)B] \\ &\quad - [-s(n-r)D - (r-1)B] \\ &= (s-r)sD - (s-r)B \\ &= (s-r)(sD - B) \end{aligned}$$

By $s > r$ and $2D < B < 3D$, $(s-r)(sD-B) > 0$ if and only if $s \geq 3$. ■

Proof of Proposition 2. Let σ be the OSSB of a TPET situation (γ, Γ) defined by Example 1. Fix an arbitrary $T \in \mathcal{N}(\{1\})$. The proof proceeds by a mathematical induction on $|T|$. We first show (a). Then, we show (b) and (c) assuming the proof is done for all $T' \subsetneq T$.

(a) It can be easily confirmed that $\sigma(G(\{1\})) = \{\bar{\theta}\}$ since $X(G(\{1\})) = \{\bar{\theta}\}$, $\gamma(\{1\}|G(\{1\}), \bar{\theta}) = \{G(\{1\})\}$, and $\gamma(Q|G(\{1\}), \bar{\theta}) = \emptyset$ for all $Q \neq \{1\}$. Let $T = \{1, i\}$, where $i \in N \setminus \{1\}$. Fix an arbitrary $(R, x) \in X(G(T))$ with $(R, x) \neq \bar{\theta}$. Then, either $f_1(R) + x_1 < f_1(\{1\})$ or $f_i(R) + x_i < f_i(\{1\})$ by Lemma 1(a). Thus, $(R, x) \in ODOM(\sigma, G(T))$ since $\sigma(G(\{1\})) = \{\bar{\theta}\}$ and $G(\{1\}) \in \gamma(\{1\}|G(T), (R, x)) \cap \gamma(\{i\}|G(T), (R, x))$. Hence $\{\bar{\theta}\} \supseteq X(G(T)) \setminus ODOM(\sigma, G(T)) = \sigma(G(T))$. Then, it follows that $\bar{\theta} \notin ODOM(\sigma, G(T))$ from $\sigma(G(\{1\})) = \{\bar{\theta}\}$ and $\{\bar{\theta}\} \supseteq \sigma(G(T))$. Hence $\sigma(G(T)) = \{\bar{\theta}\}$.

(b) Assume that $|T| > 2$ and $|T|$ is odd. Denote $t = |T|$. Assume that the proof is done for all $T' \subsetneq T$. Define $x_1 = (t-1)(2D - B)$ and $x_i = B - 2D$ for each $i \in T \setminus \{1\}$. Then,

$$\begin{aligned} f_i(T) + x_i &= -(n-t)D - B + (B - 2D) \\ &= -(n-(t-2))D \end{aligned} \tag{3}$$

for all $i \in T \setminus \{1\}$. For each $R \in \mathcal{N}(\{1\})$ with $R \subseteq T$ and $r = |R|$,

$$\begin{aligned} &\sum_{i \in R}(f_i(T) + x_i) - \sum_{i \in R} f_i(R) \\ &\quad = [r(-(n-t)D) - (r-1)B + (t-r)(2D-B)] \\ &\qquad\quad - [r(-(n-r))D - (r-1)B] \\ &\quad = r(t-r)D + (t-r)(2D-B) \\ &\quad = (t-r)((r+2)D - B) \\ &\quad \geq 0 \end{aligned} \tag{4}$$

by $B < 3D$.

We claim that $(T,x) \notin ODOM(\sigma, G(T))$. First, we show that $(T,x) \notin ODOM^T(\sigma, G(T))$. If there exists some $(R,y) \in X(G(T))$ such that $f_i(R) + y_i > f_i(T) + x_i$ for all $i \in T$, then $\sum_{i\in T} f_i(R) > \sum_{i\in T} f_i(T)$ by $\sum_{i\in T} x_i = \sum_{i\in T} y_i = 0$. This contradicts Lemma 1(b) by $|T| \geq 3$. Thus, $(T,x) \notin ODOM^T(\sigma, G(T))$.

Next, fix an arbitrary $Q \in \mathcal{N}(\{1\})$ with $Q \subsetneq T$. We begin with the case where $|Q| = 1$, 2 or $|T| - 1$. Then, $\sigma(G(Q)) = \emptyset$ or $\sigma(G(Q)) = \{\bar{\theta}\}$ by the induction hypothesis. Thus, $(T,x) \notin ODOM^Q(\sigma, G(T))$ since $\{G(Q)\} = \gamma(Q|G(T),(T,x))$ and $f_1(T) + x_1 \geq f_1(\{1\})$. Then, consider the case where $3 \leq |Q| < |T| - 1$. Suppose that there exists some $(Q', y') \in X(G(Q))$ such that $f_i(Q') + y'_i > f_i(T) + x_i$ for all $i \in Q$. Then,

$$\sum_{i\in Q} f_i(Q) \geq \sum_{i\in Q} f_i(Q') > \sum_{i\in Q}(f_i(T) + x_i)$$

by $|Q| \geq 3$, Lemma 1(b), and $\sum_{i\in Q} y'_i = 0$. This contradicts (4). Therefore, $(T,x) \notin ODOM^Q(\sigma, G(T))$.

Finally, fix an arbitrary $P \subseteq T \setminus \{1\}$. First, assume that $|P| = 1$. Then, $\sigma(G(T \setminus P)) = \emptyset$ or $\sigma(G(T \setminus P)) = \{\bar{\theta}\}$ by the induction hypothesis, where the latter holds only if $|T| = 3$. Thus, $(T,x) \notin ODOM^P(\sigma, G(T))$ since $\{G(T \setminus P)\} = \gamma(P|G(T),(T,x))$ and

$$f_i(T) + x_i = -(n - |T| + 2)D \geq -(n-1)D = f_i(\{1\})$$

for all $i \in T \setminus \{1\}$ by $|T| \geq 3$ and (3). Next, assume that $|P| > 1$. For any $(P', z') \in X(G(T\setminus P))$, $|P'| \leq |T\setminus P| \leq |T| - 2$. For any $(P', z') \in X(G(T\setminus P))$ and all $i \in P$,

$$f_i(P') \leq f_i(T \setminus P) \leq -(n - (|T| - 2))D = f_i(T) + x_i$$

by the choice of (T,x) and (3). Thus, $(T,x) \notin ODOM^P(\sigma, G(T))$.

Hence $(T, x) \in X(G(T)) \setminus ODOM(\sigma, G(T)) = \sigma(G(T))$.

(c) Assume that $|T| > 2$ and $|T|$ is even. Assume that the proof is done for all $T' \subsetneq T$. Fix an arbitrary $(R, x) \in X(G(T))$. Suppose that $f_1(R) + x_1 \geq f_1(\{1\})$ and $f_i(R) + x_i \geq f_i(T \setminus \{i\})$ for all $i \in T \setminus \{1\}$. Aggregating these inequalities over T,

$$\sum_{i \in T} f_i(T) \geq \sum_{i \in T} f_i(R) \geq f_1(\{1\}) + \sum_{i \in T \setminus \{1\}} f_i(T \setminus \{i\})$$

by $\sum_{i \in T} x_i = 0$ and Lemma 1(b). Then, there exists some $(\bar{x}_i)_{i \in T}$ such that $\sum_{i \in T} \bar{x}_i = 0$, $f_1(T) + \bar{x}_1 \geq f_1(\{1\})$, and $f_i(T) + \bar{x}_i \geq f_i(T \setminus \{i\})$ for all $i \in T \setminus \{1\}$. This contradicts Lemma 1(a). Hence either $f_1(R) + x_1 < f_1(\{1\})$ or there exists some $k \in T \setminus \{1\}$ with $f_k(R) + x_k < f_k(T \setminus \{k\})$.

If the former holds, then $(R, x) \in ODOM^{\{1\}}(\sigma, G(T))$ by $G(\{1\}) \in \gamma(\{1\}|G(T), (R, x))$ and $\sigma(G(\{1\})) = \{\bar{\theta}\}$. If the latter holds, then $(R, x) \in ODOM^{\{k\}}(\sigma, G(T))$ since $G(T \setminus \{k\}) \in \gamma(\{k\}|G(T), (R, x))$ and $(T \setminus \{k\}, \bar{y}) \in \sigma(G(T \setminus \{k\}))$ for some $\bar{y} \in \mathbb{R}^n_0$ by the induction hypothesis. Hence $(R, x) \in ODOM(\sigma, G(T))$. ■

4 Renegotiations after implementing agreements

In this section, we define the TPET-RIA situation, which is similar to the TPET situation but allows renegotiations after implementing agreements. In the TPET situation, coalitions may deviate from a negotiation before reaching an agreement at the negotiation. The TPET-RIA situation additionally allows coalitions to start new negotiations after an agreement is reached in the current negotiation and is implemented. We introduce the formal definition of the TPET-RIA situation.

Definition 4 *Let* $\bar{\Gamma} = \left\{ G^{(S,z)}(T) \,|(S,z) \in \Theta, T \in \mathcal{N}(S) \right\}$ *be the set of positions, where for any* $\theta = (S,z) \in \Theta$ *and* $T \in \mathcal{N}(S)$*,*

$$\begin{aligned} &N(G^\theta(T)) = N, \\ &X(G^\theta(T)) = \left\{ (R,x) \in \Theta \,\middle|\, \begin{array}{l} S \subseteq R \subseteq S \cup T, \\ x_i = z_i, \forall i \in N \setminus T, \end{array} \right\}, \\ &u_i(G^\theta(T)) = u_i \text{ for all } i \in N. \end{aligned}$$

Let $\bar{\gamma}$ *be the inducement correspondence that is defined by the following two steps. First, for each* $Q \in \mathcal{N}$*,* $\theta = (S,z) \in \Theta$*,* $T \in \mathcal{N}(S)$*, and* $(R,x) \in X(G^\theta(T))$*, define*

$$\gamma'(Q|G^\theta(T),(R,x)) = \begin{cases} \{G^\theta(Q)\} & \text{if } S \cap Q \neq \emptyset, Q \subseteq T; \\ \{G^\theta(T \setminus Q)\} & \text{if } Q \subseteq T \setminus S; \\ \emptyset & \text{otherwise.} \end{cases}$$

Next, for each $Q \in \mathcal{N}$*,* $\theta = (S,z) \in \Theta$*,* $T \in \mathcal{N}(S)$*, and* $(R,x) \in X(G^\theta(T))$*, define*

$$\begin{aligned} &\bar{\gamma}(Q|G^\theta(T),(R,x)) \\ &= \begin{cases} \gamma'(Q|G^\theta(T),(R,x)) \cup \{G^{(R,x)}(Q)\} & \text{if } Q \supseteq T, (R,x) \neq \theta; \\ \gamma'(Q|G^\theta(T),(R,x)) & \text{otherwise,} \end{cases} \end{aligned}$$

Then, a situation $(\bar{\gamma}, \bar{\Gamma})$ *is said to be a TPE transfer with renegotiations after implementing an agreement (henceforth, TPET-RIA) situation.*

Note that for any $G^{(S,z)}(T) \in \bar{\Gamma}$, $(R,x) \in X(G^\theta(T))$ implies $\sum_{i \in T} z_i = \sum_{i \in T} x_i$.

The TPET-RIA situation describes the following process. Consider that a state $\theta = (S,z) \in \Theta$ is actually achieved, and $T \in \mathcal{N}$ with $S \cap T \neq \emptyset$ is participating in a negotiation denoted by a position $G^\theta(T)$. Note that S, the current TPE holders, will keep holding the TPE by its irreversibility, even if one is not participating in a current negotiation. Note also that at least one current TPE holder must be included in the negotiation for the TPE transfer. In the same manner as the TPET situation, the states available in this negotiation are defined by $X(G^{\theta}(T))$. In this sense, $X(G^{\theta}(T))$ is a natural extension of $X(G(T))$, which appeared in Section 3. The situation starts with $G^{\bar{\theta}}(N)$, the initial state and a negotiation among all players.

Consider that a state (R, z) is once proposed at $G^{\theta}(T)$. Two kinds of deviations defined in the TPET situation are naturally extended to the TPET-RIA situation defined in γ'. These kinds of deviations are done *before* reaching or implementing an agreement. In addition to these, one more kind of deviation is allowed. The players in T, who are the participants in the current negotiation, agree on (R, z) and start an another negotiation after implementing (R, z). This new kind of deviation is conducted by a superset Q of T. Namely, the players in T must be included in Q since they are required to agree and actually achieve (R, z); additionally, the players in T are allowed to invite the players outside T, and the invited players participate the new negotiation if they accept the invitation. We call this process a *renegotiation after implementing an agreement*.

Two remarks on the renegotiation after implementing an agreement follow. First, when a negotiation concludes without any TPE and monetary transfer, *i.e.* θ appears as a result at $G^{\theta}(T)$, we regard it as an agreement to stop any further TPE and monetary transfer. Therefore, we do not consider the renegotiation after implementing θ. Second, unlike the TPET situation, the players not participating in a negotiation, $N(G^{\theta}(T)) \setminus T = N \setminus T$, are involved in the position $G^{\theta}(T)$. The players in $N \setminus T$ are regarded as just waiting for a possible invitation for renegotiation after implementing an agreement, and they do not voluntarily return to the negotiation without this invitation.

Note that we allow negotiations held by a coalition without the original holder. In this sense, we are assuming that it is allowed to transfer the TPE without the permission of the original holder. This may happen by the incomplete intellectual property right (henceforth, IPR) protection for the TPE. For example, the original holder (player 1) renounces the intellectual property right, or some illegal imitations are overlooked.

Unfortunately, in contrast to the TPET situation, the TPET-RIA situation is never hierarchical.

Proposition 3 *The TPET-RIA situation* $(\bar{\gamma}, \bar{\Gamma})$ *is not hierarchical.*

Proof. Let $(\bar{\gamma}, \bar{\Gamma})$ be a TPET-RIA situation. Fix an arbitrary $(S, x) \in \Theta$. Fix an arbitrary $T \in \mathcal{N}(S)$ with $|T| \geq 2$. By $|T| \geq 2$, we can choose some $(S, y) \in X(G^{(S,x)}(T))$ with $x \neq y$. Note that $x_i = y_i$ for all $i \in N \setminus T$. Thus, $G^{(S,y)}(T) \in \bar{\gamma}(T|G^{(S,x)}(T), (S, y))$. By $x_i = y_i$ for all $i \in N \setminus T$, $(S, x) \in X(G^{(S,y)}(T))$. By $x \neq y$, $G^{(S,x)}(T) \in \bar{\gamma}(T|G^{(S,y)}(T), (S, x))$.

Now, fix an arbitrary partition $P(\bar{\Gamma}) = \left(\bar{\Gamma}_k\right)_{k=1}^m$. Let k and k' be integers such that $G^{(S,x)}(T) \in \bar{\Gamma}_k$ and $G^{(S,y)}(T) \in \bar{\Gamma}_{k'}$, respectively. If $k \geq k'$, then $P(\bar{\Gamma})$ does not satisfy Definition 2(a) by $G^{(S,y)}(T) \in \bar{\gamma}(T|G^{(S,x)}(T), (S, x))$. On the other hand, if $k < k'$, then $P(\bar{\Gamma})$ does not satisfy Definition 2(a) by $G^{(S,x)}(T) \in \bar{\gamma}(T|G^{(S,y)}(T), (S, y))$. ■

We can, however, prove the existence and uniqueness of the OSSB for a TPET-RIA situation. The main result of this paper is stated by Theorem 1.

Theorem 1 *There exists the unique OSSB* σ *that assigns the set of individually rational full diffusion states at the initial negotiation* $G^{\bar{\theta}}(N)$*, i.e.*

$$\sigma(G^{\bar{\theta}}(N)) = \left\{ (N, x) \in X(G^{\theta}(T)) \mid f_i(N) + x_i \geq f_i(\{1\}), \forall i \in N \right\}. \quad (5)$$

Formally, for any $\theta = (S, z) \in \Theta$ *and* $T \in \mathcal{N}$ *with* $S \cap T \neq \emptyset$*,*

(a) *if* $S = N$*, then* $\sigma(G^\theta(T)) = \{(N, z)\}$*;*

(b) *if* $S \neq N$ *and* $S \cup T \neq N$*, then* $\sigma(G^\theta(T)) = \{(S, z)\}$*;*

(c) *if* $S \neq N$*,* $S \cup T = N$*, and* $\sum_{i \in T} f_i(N) \leq \sum_{i \in T} f_i(S)$*, then*

$$\sigma(G^\theta(T)) = \{(N, x) \in X(G^\theta(T)) |\ f_i(N) + x_i = f_i(S) + z_i, \forall i \in T\} \\ \cup \{(S, z)\};$$

(d) *if* $S \neq N$*,* $S \cup T = N$*,* $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(S)$*, and* $|S \cap T| = 1$*, then*

$$\sigma(G^\theta(T)) = \left\{(N, x) \in X(G^\theta(T)) \mid f_i(N) + x_i \geq f_i(S) + z_i, \forall i \in T\right\};$$

(e) *if* $S \neq N$*,* $S \cup T = N$*,* $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(S)$*, and* $|S \cap T| > 1$*, then*

$$\sigma(G^\theta(T)) = \left\{(N, x) \in X(G^\theta(T)) \;\middle|\; \begin{array}{l} x_i \leq z_i, \forall i \in S \cap T; \\ f_i(N) + x_i \geq f_i(S) + z_i, \forall i \in T \end{array} \right\}.$$

The proof of Theorem 1 is relegated to the Appendix.

Remark 2 The SB σ defined by Theorem 1(a)-(e) is nonempty for any position. The nonemptiness of $\sigma(G^\theta(T))$ is obvious for (a)-(c), follows from Assumption 1(b) for (d), and follows from Lemma 2 in the Appendix for (e).

Remark 3 In this section, we remarked that we are assuming the incomplete protection of the IPR. If the IPR is completely protected, the TPE cannot be transferred without permission from the original holder (player 1); that is, any negotiating coalition must include player 1. We can obtain a similar result with Theorem 1 in this alternative model. Indeed, the most important result of this paper remains unchanged; that is, the OSSB assigns the individually rational full diffusion states in the initial negotiation. The proof of Theorem 1 applies to this case with slight modifications. In this sense, whether the IPR is completely protected or not is not a large matter here.

5 Concluding remarks

We consider TPE transfer problems by employing the theory of social situations. In the TPET situation, it is shown that full diffusion may not be acceptable for the players in the OSSB. On the other hand, full diffusion is always acceptable for the players in the OSSB for the TPET-RIA situation. Moreover, the OSSB recommends achieving the individually rational full diffusion state in the initial negotiation. Two remarks are in order.

First, the OSSB for the TPET-RIA situation says nothing about how to share the profit from the TPE diffusion since the OSSB assigns all the Pareto efficient and individually rational states at the initial negotiation. A further analysis of the profit-sharing problem may be necessary.

Second, there are some directions in which the model of this paper can be extended. For example, we may consider the case where there are multiple TPEs held by some players. Moreover, the players may have options to innovate the TPEs. In this case, the innovation cost is also taken into account for considering the efficiency. It should be examined whether efficient diffusion can be achieved in these cases. However, these extensions make the model complex, so we remain these problems for future research.

Appendix: proof of Theorem 1

This appendix is devoted to the proof of Theorem 1.

Define a partition $(\bar{\Gamma}_k)_{k=1}^{n^2}$ of $\bar{\Gamma}$ such that

$$\bar{\Gamma}_k = \{G^{(S,z)}(T)|\ n|S| - |T| + 1 = k\} \text{ for each } k = 1, ..., n^2.$$

Note that this partition does not satisfy Definition 2(a) as shown in Proposition 3, but it will be useful in the following proofs. For each position

$G^{(S,z)}(T)$, define

$$\bar{\Gamma}(G^{(S,z)}(T)) = \bigcup_{n|S|-|T|+1<k\leq n^2} \bar{\Gamma}_k.$$

Throughout the following proofs, we employ these notations.

Preceding to the proof of Theorem 1, we state and prove three lemmas. Lemmas 2 and 3 prove properties that frequently appear in the subsequent lemmas.

Lemma 2 *Let $\theta = (S, z) \in \Theta$ with $S \neq N$. Let $T \in \mathcal{N}(S)$ with $S \cup T = N$ and $\sum_{i\in T} f_i(N) > \sum_{i\in T} f_i(S)$. Then, there exists some $(N, y) \in X(G^\theta(T))$ such that $f_i(N) + y_i > f_i(S) + z_i$ for all $i \in T$ and $y_i \leq z_i$ for all $i \in S \cap T$.*

Proof. Fix an arbitrary $\theta = (S, z) \in \Theta$ with $S \neq N$. Let $T \in \mathcal{N}(S)$ with $S \cup T = N$ and $\sum_{i\in T} f_i(N) > \sum_{i\in T} f_i(S)$. Define $\delta_i = f_i(N) - f_i(S)$ for each $i \in T$. Define

$$\varepsilon = \min\left(\{\delta_i | i \in S \cap T\} \cup \left\{\frac{\sum_{i\in T} \delta_i}{|T|}\right\}\right),$$
$$\varepsilon' = \frac{\sum_{i\in T} \delta_i - |S \cap T|\varepsilon}{|T \setminus S|}.$$

Note that $\varepsilon > 0$ since $\delta_i > 0$ for all $i \in S \cap T$ by Assumption 1(a), and $\sum_{i\in T} \delta_i = \sum_{i\in T} f_i(N) - \sum_{i\in T} f_i(S) > 0$. Note also that $\varepsilon' > 0$ by

$$\sum_{i\in T} \delta_i - |S \cap T|\varepsilon \geq \sum_{i\in T} \delta_i \left(1 - \frac{|S \cap T|}{|T|}\right) > 0,$$

where the last strict inequality follows from $T \setminus S \neq \emptyset$, which is due to $S \cup T = N$ and $S \neq N$. Define

$$y_i = \begin{cases} z_i - \delta_i + \varepsilon & \text{if } i \in S \cap T \\ z_i - \delta_i + \varepsilon' & \text{if } i \in T \setminus S \\ z_i & \text{if } i \in N \setminus T. \end{cases} \tag{6}$$

Then, $(N, y) \in X(G^\theta(T))$ by (6) and

$$\sum_{i\in T} y_i = \sum_{i\in T} z_i - \sum_{i\in T} \delta_i + |S\cap T|\varepsilon + \left(\sum_{i\in T} \delta_i - |S\cap T|\varepsilon\right) = \sum_{i\in T} z_i.$$

By $\varepsilon > 0$ and $\varepsilon' > 0$, $f_i(N) + y_i > f_i(S) + z_i$ for all $i \in T$. Further, $y_i \leq z_i$ for all $i \in S \cap T$ by $\varepsilon \leq \delta_i$ for all $i \in S \cap T$. ■

Lemma 3 *Let σ be an SB for a TPET-RIA situation $(\bar{\gamma}, \bar{\Gamma})$. Let $\theta = (S, z) \in \Theta$ with $S \neq N$. Let $T \in \mathcal{N}(S)$ with $S \cup T = N$. Assume that*

$$\sigma(G) \text{ satisfies Theorem 1 for any } G \in \bar{\Gamma}(G^\theta(T)). \tag{7}$$

(a) *For any $(R, x) \in X(G^\theta(T))$ with $f_i(R) + x_i \geq f_i(S) + z_i$ for all $i \in T$, $(R, x) \notin ODOM^Q(\sigma, G^\theta(T))$ for all $Q \subseteq T \setminus S$ and all $Q \subseteq T$ with $S \cup Q \neq N$.*

(b) *For any $(R, x) \in X(G^\theta(T))$ with $f_i(R) + x_i < f_i(S) + z_i$ for some $i \in T$, $(N, x) \in ODOM(\sigma, G^\theta(T))$.*

(c) *For any $(R, x) \in X(G^\theta(T))$ with $R \neq S, N$, $(R, x) \in ODOM(\sigma, G^\theta(T))$.*

(d) *For any $(N, x) \in X(G^\theta(T))$ with $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$, $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ for all $Q \supseteq T$ if either (i) $\sigma(G^\theta(T))$ satisfies Theorem 1 or (ii) σ is an OSSB.*

Proof. Let σ be an SB for a TPET-RIA situation $(\bar{\gamma}, \bar{\Gamma})$. Fix an arbitrary $\theta = (S, z) \in \Theta$ with $S \neq N$ and an arbitrary $T \in \mathcal{N}(S)$ with $S \cup T = N$. Assume (7).

(a) Fix an arbitrary $(R, x) \in X(G^\theta(T))$ with $f_i(R) + x_i \geq f_i(S) + z_i$ for all $i \in N$. First, fix an arbitrary $Q \subseteq T \setminus S$. Then, $\{G^\theta(T \setminus Q)\} = \bar{\gamma}(Q|G^\theta(T), (R, x))$. We have $G^\theta(T \setminus Q) \in \bar{\Gamma}(G^\theta(T))$ by $|T \setminus Q| < |T|$. Thus,

$\sigma(G^\theta(T \setminus Q)) = \{(S, z)\}$ by (7). Therefore, $(R, x) \notin ODOM^Q(\sigma, G^\theta(T))$ by $f_i(R) + x_i \geq f_i(S) + z_i$ for all $i \in T$.

Next, fix an arbitrary $Q' \subseteq T$ with $Q' \cap S \neq \emptyset$ and $Q' \cup S \neq N$. Note that $Q' \subsetneq T$ by $S \cup T = N$. Then, $G^\theta(Q') \in \bar{\Gamma}(G^\theta(T))$ by $|Q'| < |T|$, and $\sigma(G^\theta(Q')) = \{(S, z)\}$ by (7). Therefore, $(R, x) \notin ODOM^{Q'}(\sigma, G^\theta(T))$ by $f_i(R)+x_i \geq f_i(S)+z_i$ for all $i \in Q' \subseteq T$ and $\{G^\theta(Q')\} = \bar{\gamma}(Q'|G^\theta(T), (R, x))$.

(b) Let $(R, x) \in X(G^\theta(T))$. Assume that there exists some $k \in T$ such that $f_k(R) + x_k < f_k(S) + z_k$. First, assume $k \in S \cap T$. In this case, $\{G^\theta(\{k\})\} = \bar{\gamma}(\{k\}|G^\theta(T), (N, x))$. By $|T| > 1$, $G^\theta(\{k\}) \in \bar{\Gamma}(G^\theta(T))$. Thus, $\sigma(G^\theta(\{k\})) = \{(S, z)\}$ by $k \in S$ and (7). Hence $(R, x) \in ODOM^{\{k\}}(\sigma, G^\theta(T))$.

Next, assume $k \in T\setminus S$. In this case, $\{G^\theta(T\setminus\{k\})\} = \bar{\gamma}(\{k\}|G^\theta(T), (R, x))$. By $|T \setminus \{k\}| < |T|$, $G^\theta(T \setminus \{k\}) \in \bar{\Gamma}(G^\theta(T))$. Thus, $\sigma(G^\theta(T \setminus \{k\})) = \{(S, z)\}$ by $S \cup (T \setminus \{k\}) \neq N$ and (7). Hence $(R, x) \in ODOM^{\{k\}}(\sigma, G^\theta(T))$.

(c) Fix an arbitrary $(R, y) \in X(G^\theta(T))$ with $R \neq N$ and $R \neq S$. Then, $G^{(R,y)}(N) \in \bar{\Gamma}(G^\theta(T))$ since $n|R| - n + 1 = (|R| - 1)n + 1 > n|S| - |T| + 1$ by $|R| > |S|$ and $|T| > 0$. Thus, $\sigma(G^{(R,y)}(N))$ satisfies Theorem 1. Then, we can find some $(N, y') \in \sigma(G^{(R,y)}(N))$ such that $f_i(N) + y'_i > f_i(R) + y_i$ for all $i \in N$ by Lemma 2, $R \neq N$, and Assumption 1(b). Hence $(R, y) \in ODOM^N(\sigma, G^\theta(T))$ since $G^{(R,y)}(N) \in \bar{\gamma}(N|G^\theta(T), (R, y))$ by $R \neq S$.

(d) Fix an arbitrary $(N, x) \in X(G^\theta(T))$ with $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$. Assume that either (i) or (ii) holds. For all $Q \supseteq T$, $G \in \bar{\gamma}(Q|G^\theta(T), (N, x))$ implies either $G = G^{(N,x)}(Q)$ or $G = G^\theta(T)$, where the latter takes place only if $Q = T$. If $G = G^{(N,x)}(Q)$, then for any $(N, y) \in X(G^{(N,x)}(Q))$, $f_j(N) + y_j \leq f_j(N) + x_j$ for some $j \in Q$. Thus, assume $G = G^\theta(T)$.

Suppose that there exists some $(R, y) \in \sigma(G^\theta(T))$ such that $f_i(R) + y_i > f_i(N) + x_i$ for all $i \in T$. Then, either $R = N$ or $R = S$ by (i) itself if (i) holds, and by (c) of this lemma if (ii) holds. By $(R, y), (N, x) \in X(G^\theta(T))$, $\sum_{i \in T} x_i = \sum_{i \in T} y_i = \sum_{i \in T} z_i$. Thus, $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(N)$ if $R = N$ since $f_i(R) + y_i > f_i(N) + x_i$ for all $i \in T$, and $\sum_{i \in T} f_i(S) > \sum_{i \in T} f_i(S)$ if $R = S$ since $f_i(R) + y_i > f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$. A contradiction occurs in each case. Hence $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ for all $Q \supseteq T$. ■

Lemma 4, which follows, will be useful in the proofs of Theorem 1(c)-(e).

Lemma 4 *Let σ be an SB for a TPET-RIA situation $(\bar{\gamma}, \bar{\Gamma})$. Let $\theta = (S, z) \in \Theta$ with $S \neq N$, and $T \in \mathcal{N}(S)$ with $S \cup T = N$. Assume that*

$$\sigma(G) \text{ satisfies Theorem 1 for any } G \in \bar{\Gamma}(G^\theta(T)). \tag{8}$$

Further assume that either (i) $\sigma(G^\theta(T))$ satisfies Theorem 1 or (ii) σ is an OSSB.

(a)

$$\begin{aligned} &\left\{ (N, x) \in X(G^\theta(T)) \middle| \begin{array}{l} x_i \leq z_i, \forall i \in S \cap T; \\ f_i(N) + x_i \geq f_i(S) + z_i, \forall i \in T \end{array} \right\} \\ &\subseteq X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) \\ &\subseteq \{(N, x) \in X(G^\theta(T)) | f_i(N) + x_i \geq f_i(S) + z_i, \forall i \in T\} \cup \{(S, z)\}. \end{aligned}$$

(b) *If $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(S)$, then $(S, z) \in ODOM(\sigma, G^\theta(T))$.*

Proof. Let σ be an SB for a TPET-RIA situation $(\bar{\gamma}, \bar{\Gamma})$. Fix an arbitrary $\theta = (S, z) \in \Theta$ with $S \neq N$, and an arbitrary $T \in \mathcal{N}(S)$ with $S \cup T = N$. Assume (8) and that either (i) or (ii) holds.

(a) The latter inclusion immediately follows from Lemma 3(b) and (c). Therefore, we turn to the proof of the former inclusion.

Fix an arbitrary $(N, x) \in X(G^\theta(T))$ such that $x_i \leq z_i$ for all $i \in T \cap S$ and $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$. Let $Q \in \mathcal{N}$. If $\bar{\gamma}(Q|G^\theta(T), (N, x)) \neq \emptyset$, then either $Q \subseteq T \setminus S$, $Q \supseteq T$, or $Q \subsetneq T$ with $Q \in \mathcal{N}(S)$. If $Q \subseteq T \setminus S$, then $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(a). If $Q \supseteq T$, then $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(d).

Assume, therefore, that $Q \subsetneq T$ with $Q \in \mathcal{N}(S)$. In this case, $\{G^\theta(Q)\} = \bar{\gamma}(Q|G^\theta(T), (N, x))$. If $S \cup Q \neq N$, then $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(a). Therefore, assume that $S \cup Q = N$. Suppose that there exists some $(R, y) \in \sigma(G^\theta(Q))$ such that $f_i(R)+y_i > f_i(N)+x_i$ for all $i \in Q$. By $|Q| < |T|$, $G^\theta(Q) \in \bar{\Gamma}(G^\theta(T))$. Thus, either $R = N$ or $R = S$ by (8). If $R = S$, then $f_i(S) + y_i > f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in Q$. Aggregating these inequalities over Q, $\sum_{i\in Q} f_i(S) > \sum_{i\in Q} f_i(S)$ by $\sum_{i\in Q} y_i = \sum_{i\in Q} z_i$. This is a contradiction. Assume $R = N$. Then, $y_i > x_i$ for all $i \in Q$ by $f_i(N) + y_i > f_i(N) + x_i$ for all $i \in Q$. Note that $S \cup Q = N$ and $Q \subsetneq T$ imply $T \setminus Q \subseteq S \cap T$. Then, by $x_i \leq z_i$ for all $i \in T \setminus Q \subseteq S \cap T$ and $\sum_{i\in T} x_i = \sum_{i\in T} z_i$,

$$\sum_{i\in Q} y_i > \sum_{i\in Q} x_i \geq \sum_{i\in Q} x_i + \sum_{i\in T\setminus Q} (x_i - z_i) = \sum_{i\in T} x_i - \sum_{i\in T\setminus Q} z_i = \sum_{i\in Q} z_i.$$

This contradicts $(N, y) \in X(G^\theta(Q))$. Thus, $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$. Hence $(N, x) \in X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T))$.

(b) Assume that $\sum_{i\in T} f_i(N) > \sum_{i\in T} f_i(S)$. We have

$$\sigma(G^\theta(T)) \supseteq \left\{ (N, x) \in X(G^\theta(T)) \,\middle|\, \begin{array}{l} x_i \leq z_i, \forall i \in S \cap T, \\ f_i(N) + z_i \geq f_i(S) + z_i, \forall i \in T \end{array} \right\}$$

by (i) itself if (i) holds, and by Lemma 4(a) if (ii) holds. Then, there exists some $(N, z') \in \sigma(G^\theta(T))$ with $f_i(N) + z'_i > f_i(S) + z_i$ for all $i \in T$ by Lemma 2 and $\sum_{i\in T} f_i(N) > \sum_{i\in T} f_i(S)$. Hence $(S, z) \in ODOM^T(\sigma, G^\theta(T))$. ■

Now, we turn to the proof of Theorem 1. It suffices to show that (a)-(e) are true since (5) immediately follows from (d).

Proof of Theorem 1. Let σ be an SB for a TPET-RIA situation $(\bar{\gamma}, \bar{\Gamma})$. Assume that either (i) σ satisfies Theorem 1 or (ii) σ is an OSSB. We prove that for any $G \in \bar{\Gamma}$, $X(G) \setminus ODOM(\sigma, G)$ coincides with the corresponding one of the right-hand side of (a)-(e) of Theorem 1 provided that either (i) or (ii) is satisfied. If (i) holds, then it is proved that σ satisfying Theorem 1 is an OSSB, and if (ii) holds, then its uniqueness is proved. The proof proceeds by a reverse mathematical induction on Γ_k, *i.e.* we prove Theorem 1 for any $G \in \Gamma_k$, assuming that the proof is done for any $G' \in \bar{\Gamma}(G)$.

We start with the induction base. Fix an arbitrary $G \in \bar{\Gamma}_{n^2}$. Then, there exist some $\eta = (N, z) \in \Theta$ and $i \in N$ such that $G = G^{(N,z)}(\{i\})$. It is straightforward that $(N, z) \notin ODOM(\sigma, G^\eta(\{i\}))$ by $X(G^\eta(\{i\})) = \{(N, z)\}$, $\bar{\gamma}(\{i\}|G^\eta(\{i\}), (N, z)) = \{G^\eta(\{i\})\}$, and $\bar{\gamma}(Q|G^\eta(\{i\}), (N, z)) = \emptyset$ for all $Q \neq \{i\}$. Therefore, $X(G^{(N,z)}(\{i\})) \setminus ODOM(\sigma, G^\theta(\{i\})) = \{(N, z)\}$.

Fix an arbitrary $\theta = (S, z) \in \Theta$ and an arbitrary $T \in \mathcal{N}(S)$. Assume that the proof is done for any $G \in \bar{\Gamma}(G^\theta(T))$. Note that $\sigma(G)$ satisfies Theorem 1 for any $G \in \bar{\Gamma}(G^\theta(T))$ by (i) itself if (i) holds and by the induction hypothesis if (ii) holds.

(a) Assume that $S = N$. If $|T| = 1$, then the proof is done as the induction base. Therefore, assume that $|T| > 1$. Fix an arbitrary $(N, x) \in X(G^\theta(T))$ such that $x \neq z$. Then, there exists some $k \in T$ such that $x_k < z_k$. Note that $\sigma(G^\theta(\{k\})) = \{(N, z)\}$ by the induction hypothesis. Thus, we obtain $(N, x) \in ODOM^{\{k\}}(\sigma, G^\theta(T))$ by $G^\theta(\{k\}) \in \bar{\gamma}(\{k\}|G^\theta(T), (N, x))$ and $(N, z) \in \sigma(G^\theta(\{k\}))$. Hence

$$X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) \subseteq \{(N, z)\}. \tag{9}$$

We turn to proving $(N, z) \notin ODOM(\sigma, G^\theta(T))$. Let $Q \in \mathcal{N}$. If $G \in \bar{\gamma}(Q|G^\theta(T), (N, z))$, then $G = G^\theta(Q)$ and $Q \subseteq T$. If $Q \subsetneq T$, then $G^\theta(Q) \in \bar{\Gamma}(G^\theta(T))$. Thus, $(N, z) \notin ODOM^Q(\sigma, G^\theta(T))$ since $\sigma(G^\theta(Q)) = \{(N, z)\}$ by the induction hypothesis. If $Q = T$, then $(N, z) \notin ODOM^T(\sigma, G^\theta(T))$ since $\sigma(G^\theta(T)) \subseteq \{(N, z)\}$ by (i) itself if (i) holds and by (9) if (ii) holds. Therefore, $(N, z) \notin ODOM(\sigma, G^\theta(T))$. Hence $X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) = \{(N, z)\}$.

(b) Assume that $S \neq N$ and $S \cup T \neq N$. First, fix an arbitrary $(R, x) \in X(G^\theta(T))$ such that $R \neq S$. Since $n|R| - n + 1 = n(|R| - 1) + 1 > n|S| - |T| + 1$ by $|R| > |S|$, $\sigma(G^{(R,x)}(N))$ satisfies Theorem 1. Then, we can find some $(N, x') \in \sigma(G^{(R,x)}(N))$ such that $f_i(N) + x'_i > f_i(R) + x_i$ for all $i \in N$ by Lemma 2 and Assumption 1(b). Thus, $(R, x) \in ODOM^N(\sigma, G^\theta(T))$ since $G^{(R,x)}(N) \in \bar{\gamma}(N|G^\theta(T), (R, x))$.

Next, fix an arbitrary $(S, x) \in X(G^\theta(T))$ with $x \neq z$. Note that $|T| > 1$ if such (S, x) exists. Since $n|S| - |Q| + 1 > n|S| - |T| + 1$ for any $Q \subsetneq T$, $\sigma(G^\theta(Q)) = \{(S, z)\}$ for all $Q \subsetneq T$ by (i) itself if (i) holds and by the induction hypothesis if (ii) holds. By $x \neq z$ and $\sum_{i \in T} x_i = \sum_{i \in T} z_i$, there exists some $k \in T$ such that $x_k < z_k$. Assume $k \in S \cap T$. Then, $\sigma(G^\theta(\{k\})) = \{(S, z)\}$ by $\{k\} \subsetneq T$, (i) itself if (i) holds, and the induction hypothesis if (ii) holds. Thus, $(S, x) \in ODOM^{\{k\}}(\sigma, G^\theta(T))$ since $G^\theta(\{k\}) \in \bar{\gamma}(\{k\}|G^\theta(T), (S, x))$. Assume $k \in T \setminus S$. Then, $\sigma(G^\theta(T \setminus \{k\})) = \{(S, z)\}$ by $T \setminus \{k\} \subsetneq T$, (i) itself if (i) holds, and the induction hypothesis if (ii) holds. Thus, $(S, x) \in ODOM^{\{k\}}(\sigma, G^\theta(T))$ since $G^\theta(T \setminus \{k\}) \in \bar{\gamma}(\{k\}|G^\theta(T), (S, x))$. Hence

$$X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) \subseteq \{(S, z)\}. \tag{10}$$

We turn to showing $(S, z) \notin ODOM(\sigma, G^\theta(T))$. By the construction of $\bar{\gamma}$, $G \in \bar{\gamma}(Q|G^\theta(T), (S, z))$ implies either $G = G^\theta(T \setminus Q)$ or $G = G^\theta(Q)$. First, assume that $G = G^\theta(T \setminus Q)$. Then, $Q \subseteq T \setminus S$. Thus, $G^\theta(T \setminus Q) \in \bar{\Gamma}(G^\theta(T))$ by $|T \setminus Q| < |T|$, and $\sigma(G^\theta(T \setminus Q)) = \{(S, z)\}$ by (i) itself if (i) holds and by the induction hypothesis if (ii) holds. Therefore, $(S, z) \notin ODOM^Q(\sigma, G^\theta(T))$. Next, assume that $G = G^\theta(Q)$. Then, $Q \subseteq T$. If

$Q \neq T$, then $G^\theta(Q) \in \bar{\Gamma}(G^\theta(T))$ by $|Q| < |T|$. Thus, $\sigma(G^\theta(Q)) = \{(S, z)\}$ by the induction hypothesis, and $(S, z) \notin ODOM^Q(\sigma, G^\theta(T))$. Let $Q = T$. We have $\sigma(G^\theta(T)) \subseteq \{(S, z)\}$ by (i) itself if (i) holds, and by (10) if (ii) holds. Thus, $(S, z) \notin ODOM^T(\sigma, G^\theta(T))$. Hence $X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) = \{(S, z)\}$.

(c) Assume that $S \neq N$, $S \cup T = N$, and $\sum_{i \in T} f_i(N) \leq \sum_{i \in T} f_i(S)$. Fix an arbitrary $(N, x) \in X(G^\theta(T))$ such that $f_i(N) + x_i = f_i(S) + z_i$ for all $i \in T$, which exists only if $\sum_{i \in T} f_i(N) = \sum_{i \in T} f_i(S)$. By Assumption 1(a), $x_i < z_i$ for all $i \in S \cap T$. Then, $(N, x) \notin ODOM(\sigma, G^\theta(T))$ by Lemma 4(a). Therefore, it suffices to show that $(S, z) \notin ODOM(\sigma, G^\theta(T))$ by Lemma 4(a).

Let $Q \in \mathcal{N}$. If $\bar{\gamma}(Q|G^\theta(T), (S, z)) \neq \emptyset$, then either $Q \subseteq N \setminus S$ or $Q \subseteq T$ with $Q \in \mathcal{N}(S)$ by the definition of $\bar{\gamma}$. If $Q \subseteq N \setminus S$, then $(S, z) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(a). Assume that $Q \subseteq T$. Then, $\{G^\theta(Q)\} = \bar{\gamma}(Q|G^\theta(T), (S, z))$. We distinguish two cases. First, consider the case with $Q \cup S \neq N$. Then, $(S, z) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(a). Next, consider the case where $Q \cup S = N$. In this case, $T \setminus Q \subseteq S$ by $Q \subseteq T$. By Assumption 1(a), $f_i(S) < f_i(N)$ for all $i \in T \setminus Q \subseteq S$. Thus, $\sum_{i \in Q} f_i(N) \leq \sum_{i \in Q} f_i(S)$ by $\sum_{i \in T} f_i(N) \leq \sum_{i \in T} f_i(S)$. Then, $(R, y) \in \sigma(G^\theta(Q))$ implies either $(R, y) = (S, z)$ or $R = N$ with $f_i(N) + y_i = f_i(S) + z_i$ for all $i \in T$ by (i) itself if (i) holds and by the second inclusion of Lemma 4(a) and $\sum_{i \in Q} f_i(N) \leq \sum_{i \in Q} f_i(S)$ if (ii) holds. Therefore, $(S, z) \notin ODOM^Q(\sigma, G^\theta(T))$. Thus, $(S, z) \notin ODOM(\sigma, G^\theta(T))$. Hence

$$
\begin{aligned}
&X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) \\
&\quad = \{(N, x) \in X(G^\theta(T)) | f_i(N) + x_i = f_i(S) + z_i, \forall i \in T\} \cup \{(S, z)\}.
\end{aligned}
$$

(d) Assume that $S \neq N$, $S \cup T = N$, $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(S)$, and

$|S \cap T| = 1$. Fix an arbitrary $(N, x) \in X(G^\theta(T))$ such that $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$ and $x_k > z_k$ where $\{k\} = S \cap T$. It suffices to show $(N, x) \notin ODOM(\sigma, G^\theta(T))$ by Lemma 4(a) and (b).

Let $Q \in \mathcal{N}$. If $\bar{\gamma}(Q|G^\theta(T), (N, x)) \neq \emptyset$, then either $Q \subseteq T \setminus S$, $Q \supseteq T$, or $Q \subsetneq T$ with $Q \in \mathcal{N}(S)$. If $Q \subseteq T \setminus S$, then $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ follows from Lemma 3(a). If $Q \supseteq T$, then $(N, x) \notin ODOM(\sigma, G^\theta(T))$ follows from Lemma 3(d). Therefore, assume $Q \subsetneq T$ with $Q \in \mathcal{N}(S)$. In this case, $\{G^\theta(Q)\} = \bar{\gamma}(Q|G^\theta(T), (N, x))$. By $|Q| < |T|$, $G^\theta(Q) \in \bar{\Gamma}(G^\theta(T))$. By $|S \cap T| = 1$ and $Q \cap S \neq \emptyset$, $Q \cup S \neq N$. Thus, $\sigma(G^\theta(Q)) = \{(S, z)\}$ by (i) itself if (i) holds and by the induction hypothesis if (ii) holds. Therefore, $(N, x) \notin ODOM^Q(\sigma, G^\theta(T))$ since $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$. Thus, $(N, x) \notin ODOM(\sigma, G^\theta(T))$. Hence

$$X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) = \left\{(N, x) \in X(G^\theta(T)) \,\middle|\, f_i(N) + z_i \geq f_i(S) + z_i, \forall i \in T \right\}.$$

(e) Assume that $S \neq N$, $S \cup T = N$, $\sum_{i \in T} f_i(N) > \sum_{i \in T} f_i(S)$, and $|S \cap T| > 1$. Fix an arbitrary $(N, x) \in X(G^\theta(T))$ such that $f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in T$ and there exists some $k \in S \cap T$ with $x_k > z_k$. It suffices to show $(N, x) \in ODOM(\sigma, G^\theta(T))$ by Lemma 4(a) and (b). We distinguish the proof two cases.

Case 1. $\sum_{i \in (S \cap T) \setminus \{k\}} x_i \geq \sum_{i \in (S \cap T) \setminus \{k\}} z_i$.

Let $h \in \arg\min\{x_j - z_j | \; j \in (S \cap T) \setminus \{k\}\}$. If $x_h \geq z_h$, then $x_j \geq z_j$ for all $j \in (S \cap T) \setminus \{h\}$, and $\sum_{i \in (S \cap T) \setminus \{h\}} x_i > \sum_{i \in (S \cap T) \setminus \{h\}} z_i$ by $x_k > z_k$. If $x_h < z_h$, then $(S \cap T) \setminus \{k, h\} \neq \emptyset$ and $\sum_{i \in (S \cap T) \setminus \{k,h\}} x_i > \sum_{i \in (S \cap T) \setminus \{k,h\}} z_i$. Thus, $\sum_{i \in (S \cap T) \setminus \{h\}} x_i > \sum_{i \in (S \cap T) \setminus \{h\}} z_i$ by $x_k > z_k$.

Define $\varepsilon = \sum_{i \in (S \cap T) \setminus \{h\}} (x_i - z_i) > 0$ and

$$y_i = \begin{cases} x_i + \frac{\varepsilon}{|T \setminus S| + 1} & \text{if } i \in \{h\} \cup T \setminus S, \\ z_i & \text{otherwise.} \end{cases} \tag{11}$$

By (11) and $\sum_{i\in T} x_i = \sum_{i\in T} z_i$,

$$\begin{aligned}\sum_{i\in\{h\}\cup(T\setminus S)} y_i &= \sum_{i\in\{h\}\cup(T\setminus S)} x_i + \varepsilon \\ &= \sum_{i\in\{h\}\cup(T\setminus S)} x_i + \sum_{i\in(S\cap T)\setminus\{h\}} x_i - \sum_{i\in(S\cap T)\setminus\{h\}} z_i \\ &= \sum_{i\in T} x_i - \sum_{i\in(S\cap T)\setminus\{h\}} z_i \\ &= \sum_{i\in\{h\}\cup(T\setminus S)} z_i.\end{aligned} \tag{12}$$

Thus, $(N,y)\in X(G^\theta(\{h\}\cup(T\setminus S)))$ by (11) and (12). By (11) and the choice of (N,x), $f_i(N)+y_i > f_i(N)+x_i \geq f_i(S)+z_i$ for all $i\in\{h\}\cup(T\setminus S)$. Denote $R=\{h\}\cup(T\setminus S)$. Then, $G^\theta(R)\in\bar{\Gamma}(G^\theta(T))$ by $|R|=|T\setminus S|+1<|T\setminus S|+|S\cap T|=|T|$. By (12),

$$\sum_{i\in R}(f_i(N)-f_i(S)) = \sum_{i\in R}(f_i(N)+y_i) - \sum_{i\in R}(f_i(S)+z_i) > 0.$$

Therefore, $(N,y)\in\sigma(G^\theta(R))$ by $f_i(N)+y_i > f_i(S)+z_i$ for all $i\in R$, $|R\cap S|=|\{h\}|=1$, (i) itself if (i) holds, and the induction hypothesis if (ii) holds. By $R\subseteq T$ and $R\cap S=\{h\}\neq\emptyset$, $G^\theta(R)\in\bar{\gamma}(R|G^\theta(T),(N,x))$. Hence $(N,x)\in ODOM^R(\sigma,G^\theta(T))$.

Case 2. $\sum_{i\in(S\cap T)\setminus\{k\}} x_i < \sum_{i\in(S\cap T)\setminus\{k\}} z_i$.

Let $P=\{i\in S\cap T | x_i < z_i\}$. Note that $P\neq\emptyset$ and $k\notin P$. Define $\varepsilon=\sum_{i\in(S\cap T)\setminus P}(x_i-z_i)$ and $\delta=\min\left(\{z_i-x_i | i\in P\}\cup\left\{\frac{\varepsilon}{|T\setminus S|+|P|}\right\}\right)$. It is easy to see that $\varepsilon>0$ as well as $\delta>0$ by $x_k>z_k$ and the definition of P. Define $\delta'=\frac{\varepsilon-|P|\delta}{|T\setminus S|}$. We have $\delta'>0$ by $P\neq\emptyset$ and

$$\varepsilon-|P|\delta \geq \varepsilon - \frac{|P|\varepsilon}{|T\setminus S|+|P|} = \frac{|T\setminus S|}{|T\setminus S|+|P|}\varepsilon > 0.$$

Further, define

$$y_i = \begin{cases} x_i + \delta & \text{if } i \in P \\ x_i + \delta' & \text{if } i \in T \setminus S \\ z_i & \text{otherwise.} \end{cases} \tag{13}$$

By (13) and $\sum_{i \in T} x_i = \sum_{i \in T} z_i$,

$$\begin{aligned} \sum_{i \in P \cup (T \setminus S)} y_i &= \sum_{i \in P} y_i + \sum_{i \in T \setminus S} y_i \\ &= \left(\sum_{i \in P} x_i + |P|\delta \right) + \left(\sum_{i \in T \setminus S} x_i + (\varepsilon - |P|\delta) \right) \\ &= \sum_{i \in P \cup (T \setminus S)} x_i + \sum_{i \in (S \cap T) \setminus P} x_i - \sum_{i \in (S \cap T) \setminus P} z_i \\ &= \sum_{i \in T} x_i - \sum_{i \in (S \cap T) \setminus P} z_i \\ &= \sum_{i \in P \cup (T \setminus S)} z_i. \end{aligned} \tag{14}$$

Then, $(N, y) \in X(G^\theta(P \cup (T \setminus S)))$ by (13) and (14). We also have $f_i(N) + y_i > f_i(N) + x_i \geq f_i(S) + z_i$ for all $i \in P \cup (T \setminus S)$, and $y_i \leq z_i$ for all $i \in P$ by $\delta \leq z_i - x_i$ for all $i \in P$. Denote $R' = P \cup (T \setminus S)$. Then, $G^\theta(R') \in \bar{\Gamma}(G^\theta(T))$ by $|R'| = |P| + |T \setminus S| \leq |(S \cap T) \setminus \{k\}| + |T \setminus S| < |T|$. Moreover, $\sum_{i \in R'}(f_i(N) - f_i(S)) > 0$ by (14) and

$$\sum_{i \in R'} (f_i(S) + z_i) \leq \sum_{i \in R'} (f_i(N) + x_i) < \sum_{i \in R'} (f_i(N) + y_i).$$

Thus, $(N, y) \in \sigma(G^\theta(R'))$ by (i) itself if (i) holds and by the induction hypothesis if (ii) holds. We have $G^\theta(R') \in \bar{\gamma}(R' | G^\theta(T), (N, x))$ by $R' \subseteq T$ and $R' \cap S = P \neq \emptyset$. Hence $(N, x) \in ODOM^{R'}(\sigma, G^\theta(T))$.

By Cases 1 and 2, $(N, x) \in ODOM(\sigma, G^\theta(T))$.

Next, we show that $(S, z) \in ODOM(\sigma, G^\theta(T))$. Note that

$$\sigma(G^\theta(T)) \supseteq \left\{ (N, x) \in X(G^\theta(T)) \;\middle|\; \begin{array}{l} x_i \leq z_i, \forall i \in S \cap T, \\ f_i(N) + z_i \geq f_i(S) + z_i, \forall i \in T \end{array} \right\}$$

by (i) itself if (i) holds and by Lemma 4(a) if (ii) holds. Then, we can find some $(N, z') \in \sigma(G^\theta(T))$ such that $f_i(N) + z'_i > f_i(S) + z_i$ for all $i \in T$ by Lemma 2 and $\sum_{i\in T} f_i(N) > \sum_{i\in T} f_i(S)$. Thus, $(S, z) \in ODOM(\sigma, G^\theta(T))$. Hence

$$\begin{aligned} &X(G^\theta(T)) \setminus ODOM(\sigma, G^\theta(T)) \\ &\quad = \left\{ (N, x) \in X(G^\theta(T)) \;\middle|\; \begin{array}{l} x_i \le z_i, \forall i \in S \cap T, \\ f_i(N) + z_i \ge f_i(S) + z_i, \forall i \in T \end{array} \right\}. \end{aligned}$$

■

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