Published in *J. Low Temp. Phys.* **148**, 251-255 (2007).

Nonlinear Second Sound Waves and Acoustic Turbulence in Superfluid ^{4}He

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The preliminary results of an investigation of nonlinear second sound waves in a high quality resonator filled with superfluid ^{4}He are presented and discussed. It is found that, for a sufficiently strong periodic driving force, a cascade of second sound waves is formed at harmonics of the driving frequency over the extremely wide range 1–100kHz. It can be described by a power law $A_\omega = \mathrm{const} \times \omega^{-m}$, where the scaling index $m \approx 1$. These observations can be attributed to the formation of a Kolmogorov-like turbulent cascade in the system of second sound waves, accompanied by a directed energy flux through the frequency scales. It manifests itself as a limitation of the amplitude of the standing waves, a distortion in the shape of the initially harmonic waves, and a reduction of the effective quality factor Q of the resonator.

PACS: 67.40.Pm, 67.60.Fp

1. INTRODUCTION

The stationary distribution of interacting waves in an incompressible liquid was first described theoretically more than sixty years ago[1]. During the last few decades nonlinear wave dynamics and wave turbulence have became topics of intensive theoretical investigation under a variety of different conditions[2–5]. Theoretical models have been developed to describe diverse manifestations of wave turbulence in physics, including waves on the surfaces of conventional and quantum liquids[2,6,7], nonlinear waves in superfluid helium[8], phonon turbulence in perfect crystals[9], turbulence in plasmas[10], and turbulence in helium gas flow at high Reynolds numbers[11].

In this paper we present and discuss some preliminary results of our ongoing investigations of acoustic turbulence in a system of nonlinear second sound waves in a high-quality resonator filled with superfluid ^{4}He. Important features of this system are its linear dispersion law for the wave spectrum $\omega_k = u_{20}\, k$, the large nonlinear coefficient[12] of the second sound velocity $u_2(\delta T) = u_{20} + \alpha_2 \times \delta T$ as a function of its temperature excursions δT, and the low dissipation in the second sound waves. The nonlinearity coefficient α_2 of roton second sound changes from $-\infty$ at T_λ to a large positive value ($\alpha_2 \approx 2$ K^{-1} at $T = 1.5$ K) within the experimentally convenient temperature range $1.5 < T < 2.17$ K. Furthermore, α_2 passes through zero at $T_\alpha = 1.88$ K, allowing us to identify unambiguously the effects of the nonlinearity. Thus, at temperatures $T \neq T_\alpha$, a propagating second sound pulse with $\delta T \sim 1$ mK transforms to a triangle with a shock (temperature discontinuity) formed at the front of the pulse if $T < T_\alpha$, or at the trailing edge of the pulse if $T > T_\alpha$ at a distance ~ 1 cm from the second sound generator[13] e.g. a thin-film heater).

For second sound waves in a high-Q resonator the amplitude of a standing wave should be Q times higher than the amplitude of the initial wave generated by a heater, due to resonant amplification. Except at *very* small driving amplitudes, therefore, we may expect a large Q to be associated with the distortion of initially harmonic waves, and in the appearance of multiple harmonics due to nonlinearity. At sufficiently high α_2, this will occur even for relatively small generator powers. We show below that this leads to the formation of a Kolmogorov-like cascade of second sound standing waves over the frequency range 1–100 kHz.

2. EXPERIMENTS

The resonator[8,14] was formed from a cylindrical quartz tube length $L = 7$ cm, diameter $D = 1.5$ cm, capped by a pair of parallel flat glass plates that carried the bolometer and heater. The sensitivity of the Sn-Cu bolometer[15] lay in the range 1.2–2.6 V/K for temperatures $T = 1.79 - 2.08$ K, and could be shifted to its optimal sensitivity by an external magnetic field. The bolometer signal was amplified, digitized by an analog-to-digital converter, and subsequently analyzed. The second sound waves were generated by a heater connected to a sinewave generator. The frequency f_d of the temperature waves from the heater (at twice the generator frequency f_g) could be varied within the range 50 Hz–50 kHz. The ac heat flux radiated by the heater lay in the range $1 < W < 50$ mW/cm^2.

In a linear approximation the resonant frequency for longitudinal modes in a resonator can be written as $f(p) = 1/2 \times c_{20} \times p/L$, where $p \geq 1$ is the

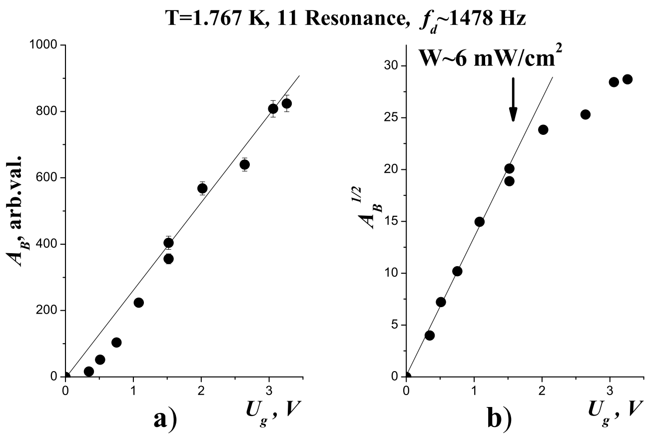

Fig. 1. The second sound amplitude (temperature variation A_B, in arbitrary units) in the resonator at the 11th resonance, $f_d = 1478$ Hz, as a function of applied voltage from the sinewave generator. Note that parts a) and b) have different ordinate scales. The temperature was $T = 1.767$ K, i.e. the nonlinear coefficient was positive. The highest amplitudes δT correspond to ~ 5 mK. The arrow indicates the critical heat flux density W_c for generation of a cascade of high frequency harmonics.

number of a resonant mode. The Q-factor of the resonator for resonant modes $p \geq 9$ lies in the range $1000 < Q < 7000$. At sufficiently small W the amplitude $A_B \propto \delta T$ of the standing wave should be given by

$$A_B \propto W \propto U_g^2$$

where U_g is the voltage of the sinusoidal wave applied to the heater.

This relationship was indeed observed for small enough applied signals ($W < 5$ mW/cm^2), as shown in Fig. 1b. Increase of the excitation above ~ 5 mW/cm^2 led to deviations from the line $\sqrt{A_B} \propto U_g$, and the results are then better described by $A_B \propto U_g$ (Fig. 1a). Attenuation of second sound waves by quantized vortexes created by the second sound heat flux can be shown to be small under the conditions of our measurements (heat flux density $W < 50$ mW/cm^2). Thus nonlinearity and viscous attenuation of sound play central roles in the dynamics of waves in the high-Q resonator. The leak of energy away from the driving frequency is accompanied by visible deformation of the initially sinusoidal temperature wave, due to the nonlinearity. It is this steepening of the harmonic wave that accounts for the multiple harmonics formed in its spectrum (Fig. 2) and for the transformation of energy from f_d to higher f. For larger $W > W_c$, the observed strong

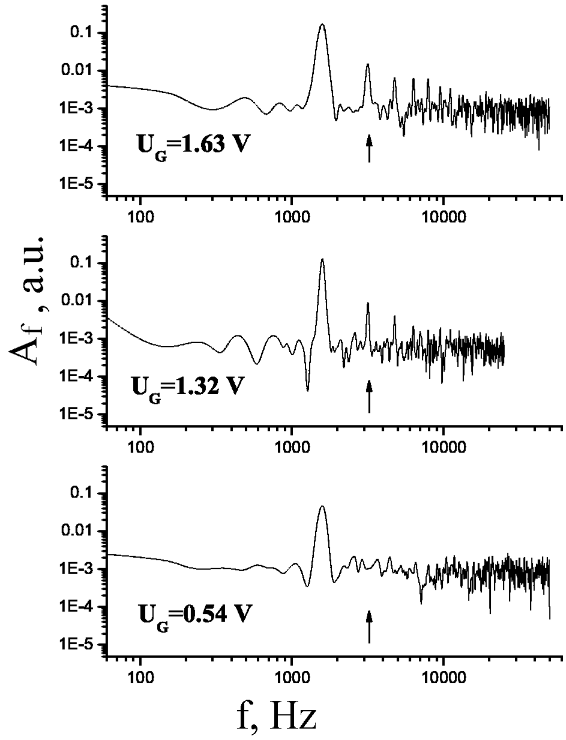

Fig. 2. Formation of high frequency harmonics with increasing the ac voltage U_G applied to the heater for $T = 1.767$ K, driving at the 11th resonant mode. The arrows indicate the second harmonic of the driving frequency (i.e. the 22nd resonant mode).

deviation of the amplitude from a linear dependence on the drive can thus be connected to the cascade generation of high frequency harmonics. The voltage $U_g = 1.63$ V corresponds to $W = W_c$ in Fig. 1.

Fig. 3 shows a typical spectrum of second sound standing waves observed at $W \approx 12$ mW/cm^2 at $T = 2.08$ K. The full line corresponds to a power-law dependence of the peak height $A_f \propto f^{-m}$, where $m = 1.6$. It follows from theoretical analysis[4] and numerical estimation[16] that the exponent m for developed acoustic turbulence (with a Kolmogorov-like cascade) should be close to unity. Our experimental data for the fully developed cascade, obtained at different temperatures and driving frequencies, yields a power-law dependence with $m = 1.5 \pm 0.3$.

In measurements at higher resonant modes we were able to observe the effect of the high frequency edge of Kolmogorov's spectrum as an abrupt

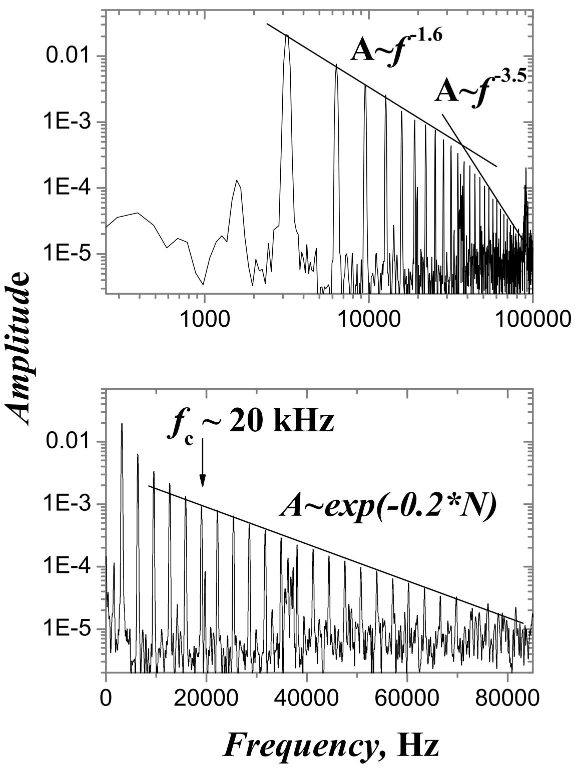

Fig. 3. Typical power spectra of second sound standing waves in the resonator for a heat flux density of 12 mW/cm^2 , $T = 2.08$ K, α_2 is negative, 31st resonance, $f_d = 3166$ Hz

change of slope on double-log scales at frequencies above $f_b \sim 30\ kHz$. Plotted on a log-linear scale (lower part of Fig. 3) the onset of dissipative processes evidently occurs near 20 kHz and can be described in terms of damping within the system of harmonics: $A \sim \exp(-0.2\, n)$, for $n \geq 6$, where n is number of the harmonic. The energy dissipation may be connected with the finite viscosity of helium at wavelengths shorter than $l \sim c_{20}/f_b \sim 4 \times 10^{-4}$ m, i.e. a few hundred microns.

3. CONCLUSIONS

By investigation of large-amplitude second sound in a high-Q resonator we have shown that the standing wave system exhibits a nonequlibrium stationary energy distribution characterised by linear dispersion. We infer that, for $T \neq T_\alpha$, fully-developed acoustic turbulence (a Kolmogorov-like cascade) can be formed, with energy flowing continuously from the lower (driving) frequency domain to the high frequency (damping) domain.

ACKNOWLEDGEMENTS

The investigations were supported by the Russian Foundation for Basic Research, projects Nos. 05-02-17849 and 06-02-17253, by the Presidium of the Russian Academy of Sciences within the programs "Quantum Macrophysics" and "Mathematical Methods in Nonlinear Dynamics", and by the Engineering and Physical Sciences Research Council (UK).

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