

# Collaborative coding and decoding techniques for multiple access channel

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Indexing terms: Coding, Decoding, Multiple access channel, Error control

**Abstract:** It is highly desirable to use simple and effective multiple access coding and decoding techniques which are capable of multiple access function and error control. The collaborative coding multiple access (CCMA) techniques potentially permit efficient simultaneous transmission by several users sharing a common channel, without subdivision in time, frequency or orthogonal codes. The authors investigate the performance of uniquely decodable CCMA schemes employing hard decision and maximum likelihood decoding techniques. A low complexity maximum likelihood decoding technique is presented. The reliability performance of various coding schemes employing these decoding techniques are carried out in the presence of AWGN conditions. The simulation results are presented in the form of symbol and codeword error rates as a function of signal to noise ratios. It is shown that uniquely decodable CCMA schemes permit the multiple access function to be combined with that of forward error correction.

## 1 Introduction

The multiple access channel (MAC) communication system is depicted in Fig. 1, in which there are  $T$  independent sources transmitting data to  $T$  separate destinations over a common discrete channel with one decoder serving  $T$  sinks. The inputs and their associated sources and encoders may be in different physical locations; for example, different rooms in a building or different mobiles in an area. The signals over the channel will interfere, superimpose or combine in some way. The single decoder at the receiver is required to unscramble and deliver the messages to their corresponding sinks, if possible, without errors. In the collaborative coding schemes, the  $T$  messages generated from the  $T$  sources are encoded independently such that they are interference free during simultaneous transmission over a common channel. Each user is provided with a code which enables the receiver to unscramble the individual information

streams, by detecting the resulting combined signal. The information theory of MAC communications was initiated in 1961 by Shannon in his fundamental paper [1],

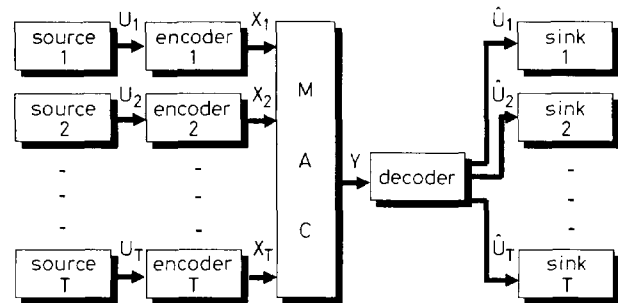


Fig. 1 Block diagram of  $T$ -user multiple access channel

and established in 1971 with a coding theorem developed in References 2 and 3. Surveys of the information theory approach are given in References 4–7 and various coding constructions for the  $T$ -user MAC are given in References 8–21.

In the following section of this paper,  $T$ -user CCMA techniques are described. Section 3 describes hard decision decoding of CCMA techniques. A low complexity maximum likelihood decoding technique is presented in Section 4 to utilise the error control capability. The generalised decoding procedure and algorithm are given. A particular two-user uniquely decodable scheme is taken as an example and analysed with this technique. The error probability analysis is given in Section 5. The reliability of various coding schemes employing hard decision and maximum likelihood decoding techniques are carried out by simulation in the presence of AWGN conditions. The simulation results and discussions are given in Section 6.

## 2 $T$ -user CCMA schemes

In Fig. 1, data from the  $i$ th source,  $U_i$ , where  $i = 1, 2, \dots, T$ , is encoded by the  $i$ th encoder according to a uniquely assigned block code  $C_i$  of length  $N$ . The resulting codeword vector  $X_i$  is then transmitted over the channel where it combines with the other  $(T - 1)$  codeword vectors to produce a composite codeword vector  $Y$  of length  $N$ . The transmitters are assumed to operate in perfect symbol and block synchronisation over a common discrete  $T$ -user MAC. At the receiving end the single decoder decodes  $Y$  into estimates of the original data streams  $\hat{U}_1, \hat{U}_2, \dots, \hat{U}_T$ .

The  $T$  codes  $C_1, C_2, \dots, C_T$  together are called a ' $T$ -user collaborative code', where each component is termed a 'constituent code'. If all the constituent codes are binary block codes then the codeword vector  $X_i$  is an

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Paper 10431 (E5, E7), first received 21st April and in revised form 29th November 1993

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$N$ -symbol binary vector. The transmission rate (in bits per channel use) of the  $i$ th constituent code  $C_i$  containing  $CW_i$  codewords each of length  $N$  and each codeword equally likely, is given by

$$R_i = \log_2(CW_i)/N \quad (1)$$

and the rate sum,  $R_{sum}$ , of all the users is given by

$$R_{sum} = \sum_{i=1}^T \log_2(CW_i)/N \quad (2)$$

Various block collaborative coding schemes have been constructed for different number of users ( $T \geq 2$ ) over the years for noiseless and noisy MACs [8–21]. These constructions have followed various approaches to achieve the bounds promised by the multiple access information theory. As an example, a simple coding scheme for a two-user uniquely decodable code with block length of  $N = 2$ , is given here [8–11]. The codewords for user 1 and 2 are  $C_1 = (00,11)$  and  $C_2 = (00,01,10)$ , respectively. This two-user code ( $C_1, C_2$ ) is uniquely decodable because all the received composite codewords are distinct as shown in Table 1. Therefore the decoder can unscramble the two messages without any ambiguity. Generally, in noiseless channel conditions, the decoder is capable of decoding every received composite codeword vector, without ambiguity, into  $T$  codewords that were transmitted by the  $T$  encoders. However, if the channel is noisy, the decoder chooses the codeword which is closest to the received, as measured by some metric distance.

**Table 1: Two-user uniquely decodable code**

|         | $C_1 + C_2$ | $(C_1)$ |       |
|---------|-------------|---------|-------|
|         |             | (0 0)   | (1 1) |
|         | (0 0)       | 0 0     | 1 1   |
| $(C_2)$ | (0 1)       | 0 1     | 1 2   |
|         | (1 0)       | 1 0     | 2 1   |

Uniquely decodable coding schemes can also have some error protection capability [8–21]. In particular, codes have been found for the two-user binary adder MAC with rates up to 1.292 bits per channel use which achieve the MAC function and offer some error protection capability. The decoding of  $T$ -user collaborative coding schemes is based here on two techniques, hard and soft decision decoding.

### 3 Hard decision (HD) decoding

In HD decoding of CCMA schemes the demodulator sets ( $T$ ) decision thresholds to detect the ( $T + 1$ ) possible signal levels transmitted by the  $T$ -user. Here, each received symbol is detected independently for  $N$  received symbols, hence is called symbol-by-symbol HD (SBS\_HD) decoding. However, this decoding technique cannot be used on its own to perform the full decoding process to deliver the individual users information to their intended destinations. This is due to the fact that sometimes in noisy conditions the SBS\_HD decoding results in a codeword which is not admissible. In this case the decoder will fail to deliver the individual users information. Therefore  $L$ -distance HD decoding is used in conjunction with SBS\_HD to complete the decoding process and resolve this ambiguity. This complete process is referred to as the HD\_CCMA decoding technique. The HD\_CCMA decoder calculates all the  $L$ -distances between the SBS\_HD codeword and all the possible

admissible codewords. Then, the codeword with the least  $L$ -distance is chosen. The  $L$ -distance between two  $N$ -symbol composite codewords  $Z$  and  $Z'$  ( $Z \neq Z'$ ) is defined as follows:

$$d_L(Z, Z') = \sum_{i=1}^N |z_i - z'_i| = \|Z - Z'\| \quad (3)$$

where the minus sign denotes real subtraction,  $|z_i - z'_i|$  denotes the absolute value of  $z_i - z'_i$  and the symbol  $\|Z - Z'\|$  means that the  $L$ -distance is a metric. This kind of decoding guarantees correct decoding in the noiseless uniquely decodable coding scheme. However, in the noisy case the number of errors which can be corrected under this decoding is  $t = \lfloor (d_{min} - 1)/2 \rfloor$ , where  $d_{min}$  is the minimum  $L$ -distance of a  $T$ -user collaborative code, which is the smallest value of  $d_L(Z, Z')$  over all  $Z \neq Z'$ , and  $\lfloor x \rfloor$  means an integer number less than or equal to  $x$ .

### 4 Maximum likelihood soft decision (MLSD) decoding

Consider a set of composite codewords each comprising  $N$  symbols and assume the received signal is  $W = (w_1, w_2, \dots, w_N)$ , where  $w_i$  is the magnitude of the element representing the  $i$ th symbol. In principle, the joint maximum likelihood (ML) decision carried out on the complete word is a very powerful detection technique. Therefore, if the actual signal magnitude of the  $N$ -symbol codeword is made available to the decoder, then an ML decoding for CCMA schemes can be performed. This is achieved by calculating the Euclidean distances between the received codeword and all the admissible codewords. The codeword with minimum Euclidean distance (MED) is chosen as the received codeword. Provided the codewords are all equally likely, this strategy is optimum in the sense that it minimises the probability of error in the decoder.

However, this technique is difficult to implement in practice, because this would require the storage of the precise amplitudes of all symbols as received. In addition, the decoding table becomes unmanageably large as the length of the code and the number of active users increases. Therefore, what is needed is a simple means of calculating the possible transmitted codewords from the received codeword with the least number of operations possible.

Here, a low complexity ML decoding technique is introduced. This technique has the reliability of ML decoding with less implementation complexity and reduces the number of computations required to decode a received codeword [22]. The decoding problem at the receiver can be defined as follows: 'given  $W = (w_1, w_2, \dots, w_N)$ , where  $w_i$  is a real valued scalar, it is necessary to decode the transmitted codeword in such a way that the total probability of codeword error is minimised'. For a given  $T$ -user CCMA code structure, two sets of 'admissible' and 'forbidden' codewords are defined. Assume  $A_i = (a_{i1}, a_{i2}, \dots, a_{iN})$  is the  $i$ th admissible codeword where  $i = 1, 2, \dots, N_a$ ;  $N_a$  is the number of admissible codewords given by

$$N_a = \prod_{l=1}^T CW_l \quad (4)$$

$CW_l$  is the number of codewords in the  $l$ th user, and  $a_{ij}$  is the  $j$ th symbol value of the  $i$ th admissible codeword where  $j = 1, 2, \dots, N$ . A set of forbidden codewords for certain error conditions is defined as  $F_i = (f_{i1}, f_{i2}, \dots, f_{iN})$ , where  $i = 1, 2, \dots, N_f$ ;  $N_f$  is the number

of forbidden codewords, and  $f_{ij}$  is the  $j$ th symbol value of the  $i$ th forbidden codeword where  $j = 1, 2, \dots, N$ .

If we assume the transmitted codeword is  $A_k = (a_{k1}, a_{k2}, \dots, a_{kN})$ , and the received codeword is  $W = (w_1, w_2, \dots, w_N)$ . Then, in order to construct the decision decoding table to decode the received codeword, the following procedure is required:

(i) Define the subset of admissible codewords nearest to each forbidden codeword for certain error conditions.

(ii) Calculate all the Euclidean distances between the received codeword  $W = (w_1, w_2, \dots, w_N)$  and all the codewords from the admissible codewords subset nearest to the  $i$ th forbidden codeword.

(iii) Choose the codeword with the MED. That is, if the generalised distance between  $W$  and the admissible codeword, say  $A_p \in (A_i)$  is minimum, then  $A_p = (a_{p1}, a_{p2}, \dots, a_{pN})$  is accepted as the transmitted codeword.

(iv) Comparison thresholds are then found for each forbidden codeword to form the decoding table for this decoding technique.

This decoding technique is also used in conjunction with SBS\_HD decoding. This arrangement allows one to correct some errors which cannot be corrected using SBS\_HD and HD\_CCMA decoding techniques. In addition this decoding scheme may perform both error detection and correction at the same time.

#### 4.1 Decoding algorithm

The generalised decoding algorithm steps for this MLSD\_CCMA decoding technique can be summarised as follows:

*Step 1:* Perform SBS\_HD decoding on the received codeword  $W = (w_1, w_2, \dots, w_N)$ .

*Step 2:* 'Error detection': perform error detection by checking the SBS\_HD decoded codeword, whether it is admissible or forbidden; if it is admissible go to step 4, else continue.

*Step 3:* 'Error correction': perform error correction by checking the decision thresholds (calculated for a given  $T$ -user coding scheme as shown in the procedure above in Section 4) stored in the decoding decision table for the current forbidden codeword.

*Step 4:* Individual users' information is then decoded by using the normal decoding procedure used in the noiseless case.

It can be seen from the above steps that it is not necessary to calculate the Euclidean distances between the received codeword and all the admissible codewords every time a codeword is received. It is only necessary to check certain conditions according to a decision decoding table. Therefore, in addition to performing MLSD decoding, the total number of operations are reduced compared to conventional techniques.

#### 4.2 Example: two-user coding scheme

As an example and for analysis of this MLSD\_CCMA decoding technique, the two-user code given in Table 1 is considered and referred to as code 1. That is, user 1 and user 2 codewords are  $C_1 = (00,11)$ ,  $C_2 = (00,01,10)$ , respectively. The set of admissible composite codewords is  $(00,10,01,11,12,21)$ . The single error conditions for this coding scheme are defined as follows: if the transmitted symbols from each user are  $(0,0)$  or  $(1,1)$  and the received composite symbol is 1; and if  $(0,1)$  or  $(1,0)$  are transmitted and either 0 or 2 is received, then a single error has occurred during transmission. Therefore, for this single error condition, the set of forbidden composite code-

words is  $(02,20,22)$  and the subset of admissible composite codewords nearest to each forbidden codeword are defined as shown in Table 2.

**Table 2: Forbidden and nearest admissible codewords for two-user code 1**

| Forbidden codewords | Nearest admissible codewords |
|---------------------|------------------------------|
| 02                  | → (12,01)                    |
| 20                  | → (21,10)                    |
| 22                  | → (21,12)                    |

The construction of the decoding decision table can be obtained by calculating the Euclidean distances between the received codeword  $W = (w_1, w_2)$  of a forbidden codeword and the nearest admissible codewords (defined in Table 2) as shown below:

(i) The Euclidean distances for the forbidden codeword 02:

$$d_1^2 = (w_1 - 1)^2 + (w_2 - 2)^2;$$

between  $W = (w_1, w_2)$  and 12,

$$d_2^2 = (w_1 - 0)^2 + (w_2 - 1)^2;$$

between  $W = (w_1, w_2)$  and 01,

$$d_1^2 - d_2^2 = -2w_1 - 2w_2 + 4;$$

then  $d_1^2 < d_2^2$  if  $w_1 + w_2 > 2$ ;

therefore  $d_1^2$  is the minimum distance.

(ii) The Euclidean distances for the forbidden codeword 20:

$$d_3^2 = (w_1 - 2)^2 + (w_2 - 1)^2;$$

between  $W = (w_1, w_2)$  and 21,

$$d_4^2 = (w_1 - 1)^2 + (w_2 - 0)^2;$$

between  $W = (w_1, w_2)$  and 10,

$$d_3^2 - d_4^2 = -2w_1 - 2w_2 + 4;$$

then  $d_4^2 > d_3^2$  if  $w_1 + w_2 > 2$ ;

therefore  $d_3^2$  is the minimum distance.

(iii) The Euclidean distances for the forbidden codeword 22:

$$d_5^2 = (w_1 - 2)^2 + (w_2 - 2)^2;$$

between  $W = (w_1, w_2)$  and 21,

$$d_6^2 = (w_1 - 1)^2 + (w_2 - 2)^2;$$

between  $W = (w_1, w_2)$  and 12,

$$d_5^2 - d_6^2 = -2w_1 + 2w_2;$$

then  $d_5^2 < d_6^2$  if  $w_1 > w_2$ ;

therefore  $d_5^2$  is the minimum distance

Therefore a decision table is constructed based on these calculations, to be used for the decoding purposes, as shown in Table 3.

**Table 3: Decoding decision table for two-user code 1**

| Forbidden codewords | Comparison thresholds | Decoding decision |
|---------------------|-----------------------|-------------------|
| 02                  | $w_1 + w_2 > 2$       | 12                |
|                     | $w_1 + w_2 < 2$       | 01                |
| 20                  | $w_1 + w_2 > 2$       | 21                |
|                     | $w_1 + w_2 < 2$       | 10                |
| 22                  | $w_1 > w_2$           | 21                |
|                     | $w_1 < w_2$           | 12                |

Suppose the transmitted codeword is (10) and the received soft information codeword is  $W = (1.6, 0.3)$ . Then, performing SBS\_HD decoding, the decoded codeword will be (20); this means there is a single error from the transmitted composite codeword. Employing the HD\_CCMA decoding technique will result in two possible codewords (10) and (21) which have the same minimum  $L$ -distance from the codeword (20). Therefore,

the HD\_CCMA decoder chooses either codeword with equal probability. If (10) is chosen then a single error has been corrected, however, if the codeword (21) is chosen then a double error has been introduced.

Now, employing the MLSD\_CCMA decoding technique, since an error is detected by the forbidden codeword (20) and  $(w_1 + w_2) < 2$ , then the output of the decoder is the codeword (10) as can be seen from Table 3. Thus, a single error has been corrected from the SBS\_HD decoder or a double error from the HD\_CCMA decoder (if the HD\_CCMA decoder had chosen the codeword (21)). Therefore, by employing MLSD\_CCMA decoding improvement in the decoding is possible, which allows some of the detected errors to be corrected. This decoding technique will give higher improvement gain if the code used has some error protection capability.

## 5 Error probability of MLSD\_CCMA decoding

The performance of the  $T$ -user MLSD\_CCMA decoding scheme is evaluated here in terms of the probability of composite codeword error. The reliability of individual user's sink information depends on how accurate the composite codeword is decoded. These analyses are carried out over an AWGN channel of zero mean and variance  $\sigma_N^2$ . If the total number of admissible composite codewords for a given  $T$ -user code is  $N_a$ , then the total probability of correct decision can be written as

$$P_{cc}(to) = \sum_{i=1}^{N_a} p(i)P_{cc}(i) \quad (5)$$

where  $p(i)$  is the probability of the  $i$ th admissible codeword,

$$\sum_{i=1}^{N_a} p(i) = 1 \quad (6)$$

and  $P_{cc}(i)$  is the probability of correct decision on the  $i$ th admissible codeword,

$$P_{cc}(i) = \iint \cdots \int_{\{G_i\}} f_i(w_1 w_2 \cdots w_N) dw_1 dw_2 \cdots dw_N \quad (7)$$

where  $\{G_i\}$  is the region of correct decision of the  $i$ th composite codeword, and  $f_i(w_1 w_2 \cdots w_N)$  is the joint PDF of the  $i$ th admissible codeword, which can be written for the AWGN as

$$f_i(w_1 w_2 \cdots w_N) = \left[ \frac{1}{\sqrt{(2\pi)\sigma_N}} \right]^N \exp \left\{ \sum_{j=1}^N \left[ \frac{-(w_j - a_{ij})^2}{2\sigma_N^2} \right] \right\} \quad (8)$$

where  $a_{ij}$  is the  $j$ th symbol in the  $i$ th admissible codeword, where  $j = 1, 2, \dots, N$  and  $i = 1, 2, \dots, N_a$ . Substituting back into eqn. 7 and 5, the total probability of correct decision is

$$P_{cc}(to) = \sum_{i=1}^{N_a} \left\langle p(i) \left[ \frac{1}{\sqrt{(2\pi)\sigma_N}} \right]^N \times \iint \cdots \int_{\{G_i\}} \exp \left\{ \sum_{j=1}^N \left[ \frac{-(w_j - a_{ij})^2}{2\sigma_N^2} \right] \right\} \times dw_1 dw_2 \cdots dw_N \right\rangle \quad (9)$$

and the total probability of error can be written as

$$P_{cc}(to) = 1 - \sum_{i=1}^{N_a} \left\langle p(i) \left[ \frac{1}{\sqrt{(2\pi)\sigma_N}} \right]^N \times \iint \cdots \int_{\{G_i\}} \exp \left\{ \sum_{j=1}^N \left[ \frac{-(w_j - a_{ij})^2}{2\sigma_N^2} \right] \right\} \times dw_1 dw_2 \cdots dw_N \right\rangle \quad (10)$$

The region of correct decision  $\{G_i\}$  represent the set of points for which the Euclidean distance to the  $i$ th admissible codeword is smaller than for all other admissible codewords. That is:

$$\{G_i\} = \{(w_1 w_2 \cdots w_N) : \sum_{j=1}^N (w_j - a_{ij})^2 < \sum_{j=1}^N (w_j - a_{kj})^2\} \quad (11)$$

where  $(w_1 w_2 \cdots w_N)$  is the set of points in  $\{G_i\}$ ; the colon sign ( $:$ ) means defined as;  $w_j$  are real-valued scalars which represent the coordinates for each point in the region  $\{G_i\}$ ;  $a_{ij}$  and  $a_{kj}$  are the  $j$ th symbols in the  $i$ th and  $k$ th admissible codewords, respectively, where  $i = 1, 2, \dots, N_a, j = 1, 2, \dots, N$ , and  $i \neq k$ .

Similarly, this procedure can also be used to calculate the composite symbol and codeword error rate for CCMA schemes employing hard decision decoding techniques.

## 6 Simulation results and discussions

The simulation is carried out to evaluate the reliability performance of CCMA schemes employing various coding and decoding schemes. Various two-user CCMA schemes are introduced first and used throughout the simulation analysis. These collaborative codes are chosen to be simple short codes with summary rate, in most cases, higher than one bit/channel use. In addition, they are chosen to have different error protection capability of the overall two-user code and its constituent codes. These codes are:

- (a) Code 1:  $C_1 = (00,11)$ ,  $C_2 = (00,01,10)$ ,  
 $CW_1 = 2$ ,  $N_1 = 2$ ,  $R_1 = 0.5$ ,  $d_{1min} = 2$ ,  
 $CW_2 = 3$ ,  $N_2 = 2$ ,  $R_2 = 0.792$ ,  $d_{2min} = 1$ ,  
 $R_{sum} = 1.292$ ,  $d_{min} = 1$ .
- (b) Code 2:  $C_1 = (000,111)$ ,  
 $C_2 = (000,001,010,011,100,101,110)$ ,  
 $CW_1 = 2$ ,  $N_1 = 3$ ,  $R_1 = 0.333$ ,  $d_{1min} = 3$ ,  
 $CW_2 = 7$ ,  $N_2 = 3$ ,  $R_2 = 0.935$ ,  $d_{2min} = 1$ ,  
 $R_{sum} = 1.269$ ,  $d_{min} = 1$ .
- (c) Code 3:  $C_1 = (0000,0011,1100,1111)$ ,  
 $C_2 = (0000,0001,0010,0100,0101,0110,1000,1001,1010)$ ,  
 $CW_1 = 4$ ,  $N_1 = 4$ ,  $R_1 = 0.5$ ,  $d_{1min} = 2$ ,  
 $CW_2 = 9$ ,  $N_2 = 4$ ,  $R_2 = 0.792$ ,  $d_{2min} = 1$ ,  
 $R_{sum} = 1.292$ ,  $d_{min} = 1$ .
- (d) Code 4:  $C_1 = (0000,0001,0011,1100,1110,1111)$ ,  
 $C_2 = (0000,0101,0110,1001,1010,1101)$ ,  
 $CW_1 = 6$ ,  $N_1 = 4$ ,  $R_1 = 0.646$ ,  $d_{1min} = 1$ ,  
 $CW_2 = 6$ ,  $N_2 = 4$ ,  $R_2 = 0.646$ ,  $d_{2min} = 1$ ,  
 $R_{sum} = 1.292$ ,  $d_{min} = 1$ .
- (e) Code 5:  $C_1 = (00,11)$ ,  $C_2 = (10,01)$ ,  
 $CW_1 = 2$ ,  $N_1 = 2$ ,  $R_1 = 0.5$ ,  $d_{1min} = 2$ ,  
 $CW_2 = 2$ ,  $N_2 = 2$ ,  $R_2 = 0.5$ ,  $d_{2min} = 2$ ,  
 $R_{sum} = 1.0$ ,  $d_{min} = 2$ .

It is assumed that the two-user CCMA communication system is in perfect synchronisation. Modulation and

demodulation are assumed to be available for these codes and considered to be part of the discrete channel. The channel is assumed to be AWGN of zero mean and variance  $\sigma_N^2$ . The ratio  $E/N_0$ , is defined here as the average signal energy per user to noise power spectral density given by  $\sigma_N^2 = N_0/2$ . The composite codeword error rate (CER) and the constituent users sink SER are calculated for each two-user collaborative code.

The composite CER versus  $E/N_0$ , employing HD\_CCMA decoding is shown in Fig. 2, for all five

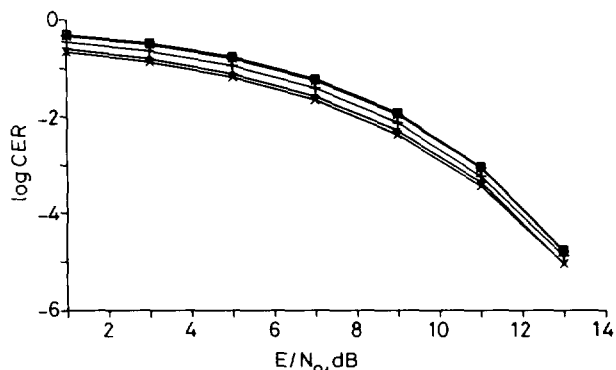


Fig. 2 HD\_CCMA decoder CER

- code 1
- + code 2
- \* code 3
- code 4
- × code 5

codes. It can be seen from this figure that the reliability of these codes are very similar, since their protect capability is the same under HD\_CCMA decoding. The small difference is due to the variation in the number of admissible and forbidden codewords from one code to another. The composite CER versus  $E/N_0$  employing the MLSD\_CCMA decoding is shown in Fig. 3 for all five

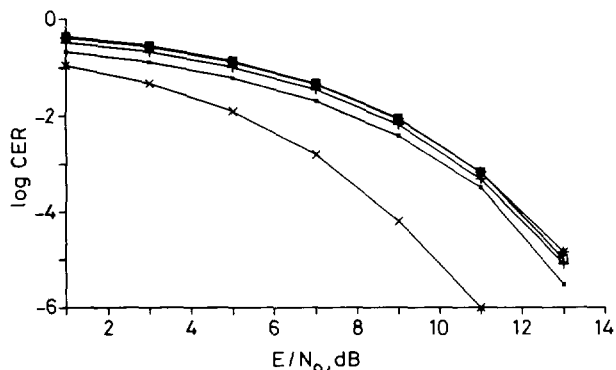


Fig. 3 MLSD\_CCMA decoder CER

- code 1
- + code 2
- \* code 3
- code 4
- × code 5

codes. It can be seen clearly that code 5 gives the best performance because its  $d_{min} = 2$ , which means that under this decoding a single error can be corrected.

For comparison purposes, and calculating the energy gain achieved by employing MLSD\_CCMA decoding, the CER for codes 1 and 5 is presented separately in Figs. 4 and 5, respectively. Also included with these Figures is the CER for SBS\_HD decoding. It can be seen from Fig. 4 that the MLSD\_CCMA decoding gives better performance with some detection gain (because  $d_{min} = 1$  for code 1). However, this gain is much higher when the code

employed has some error correction capability ( $d_{min} \geq 2$ ) as can be seen very clearly in Fig. 5. The gain achieved is more than 2.5 dB at an error probability of  $10^{-6}$ .

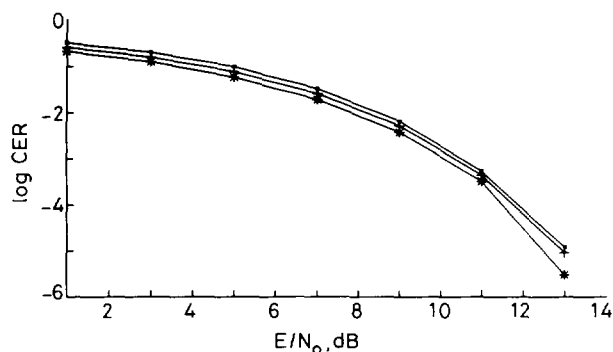


Fig. 4 CCMA decoding schemes CER (code 1)

- SBS\_HD
- + HD\_CCMA
- \* MLSD\_CCMA

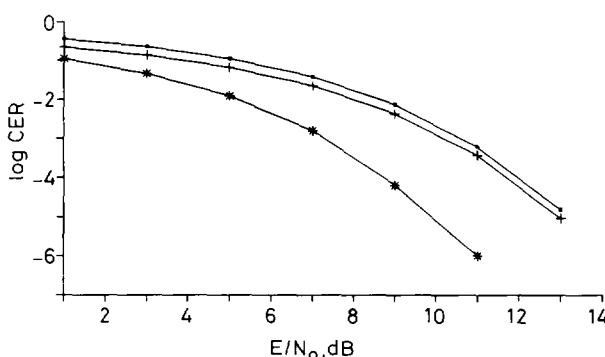


Fig. 5 CCMA decoding schemes CER (code 5)

- SBS\_HD
- + HD\_CCMA
- \* MLSD\_CCMA

The effect of employing these coding and decoding techniques is also investigated on the constituent codes and hence the user's sink data. The sink SER for each user is presented in Figs. 6–10 for all codes. It can be seen, for example in Fig. 6, that user 2 sink SER is very

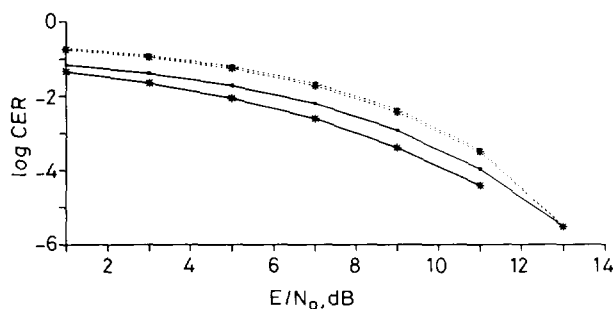


Fig. 6 CCMA decoding schemes users sink SER (code 1)

- User 1 HD\_CCMA
- \* User 1 MLSD\_CCMA
- User 2 HD\_CCMA
- \* User 2 MLSD\_CCMA

close for both MLSD\_CCMA and HD\_CCMA decoding techniques. However, user 1 reliability employing MLSD\_CCMA decoding is better than HD\_CCMA decoding because  $d_{1min} = 2$ . This gain is also shown in Figs. 7 and 8 for user 1 of code 2 and code 3, respectively. Since code 4 is a balanced code, the reliability of each user is very close as shown in Fig. 9. Code 5 is also a

balanced code with  $d_{1min} = d_{2min} = d_{min} = 2$ . Therefore, the sink SER is the same for each user as shown in Fig. 10. It can also be seen from Fig. 10 that a coding gain of

more than 2.5 dB at  $10^{-6}$  error probability is achievable employing MLSD\_CCMA over the HD\_CCMA decoding technique.

## 7 Conclusion

Collaborative coding and decoding techniques to utilise the MAC function and error control capability have been described. In particular, HD\_CCMA and MLSD\_CCMA decoding techniques were described in conjunction with SBS\_HD decoding. A low complexity MLSD decoding technique is introduced. Reliability performance evaluation has been carried out with various collaborative codes.

It has been shown that uniquely decodable CCMA schemes permit the multiple access function to be combined with that of forward error protection assuming symbol and block synchronisations are maintained. The MLSD\_CCMA decoding technique decreases the overall probability of error with some energy gain. The energy gain achieved is higher when the codes used have some error protection capability. A coding gain of more than 2.5 dB has been achieved employing MLSD\_CCMA over HD\_CCMA decoding technique.

Although the analyses in this paper have been carried out specifically for given two-user codes, generally the gain is achievable for any  $T$ -user coding scheme with error correction capability and a low complexity decoding technique which utilises this correction power. In addition, the synchronisation requirements should be reduced to a minimum, and, ideally, the system would be completely asynchronous.

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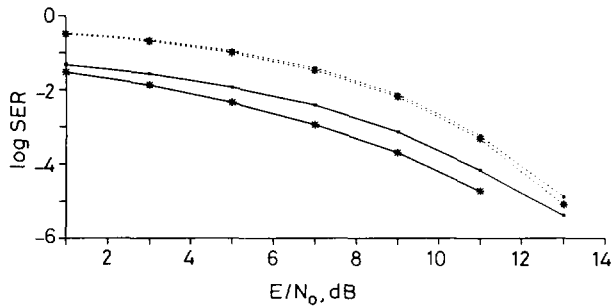


Fig. 7 CCMA decoding schemes users sink SER (code 2)

- User 1 HD\_CCMA
- \*-\* User 1 MLSD\_CCMA
- User 2 HD\_CCMA
- \*-\* User 2 MLSD\_CCMA

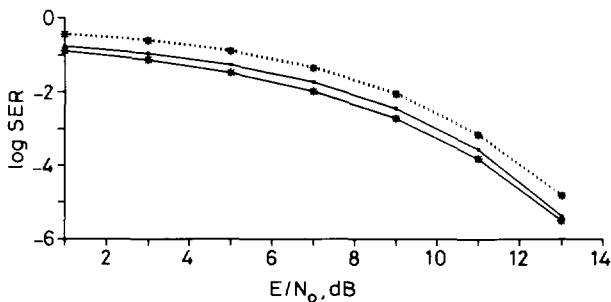


Fig. 8 CCMA decoding schemes users sink SER (code 3)

- User 1 HD\_CCMA
- \*-\* User 1 MLSD\_CCMA
- User 2 HD\_CCMA
- \*-\* User 2 MLSD\_CCMA

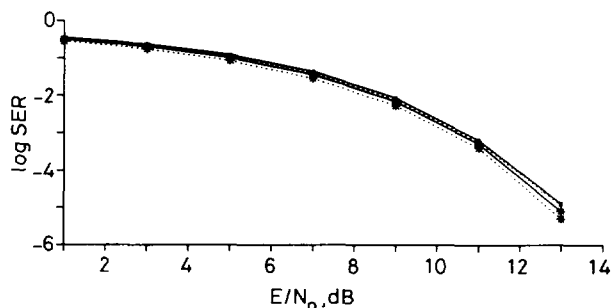


Fig. 9 CCMA decoding schemes users sink SER (code 4)

- User 1 HD\_CCMA
- \*-\* User 1 MLSD\_CCMA
- User 2 HD\_CCMA
- \*-\* User 2 MLSD\_CCMA

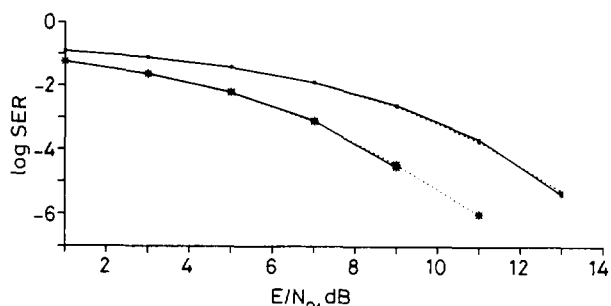


Fig. 10 CCMA decoding schemes users sink SER (code 5)

- User 1 HD\_CCMA
- \*-\* User 1 MLSD\_CCMA
- User 2 HD\_CCMA
- \*-\* User 2 MLSD\_CCMA

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