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Multi-objective Optimisation in Air-Conditioning Systems: Comfort/Discomfort Definition by IF Sets*

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Abstract: The problem of multi-objective optimisation of air-conditioning (AC) systems is treated in the paper in the framework of intuitionistic fuzzy (IF) set theory. The nature of the problem is multi-objective one with requirements for minimal costs (generally, life cycle costs; more specifically, energy costs) and maximal occupants' *comfort* (minimal *discomfort*). Moreover, its definition by conventional means is bounded to a number of restrictions and assumptions, which are often far from the real-life situations. Attempts have been made to formulate and solve this problem by means of the fuzzy optimisation [4]. The present paper makes further step by exploring the innovative concept of IF sets [6] into definition of the trickiest issue: *comfort* and *discomfort* definition. The new approach allows to formulate more precisely the problem which compromises energy saving and thermal comfort satisfaction under given constraints. The resulting IF optimisation problem could be solved numerically or, under some assumptions, analytically [1]-[2]. An example illustrates the viability of the proposed approach. A solution which significantly (with 38%) improves comfort is found which is more energetically expensive with only 0.6%. This illustrates the possibility to use the approach for trade-off analysis in multi-objective optimisation of AC systems.

Keywords: *IF sets, IF optimisation, multi-objective optimisation, air-conditioning systems, comfort/discomfort*

1. Introduction

In the last few decades there are an increasing interest in the optimum design and control of AC systems [4], [9]-[19]. Besides of economical efficiency, it has ecological aspects lowering the "green house" effect, fuel consumption and, hence, reducing emission of pollutants while keeping occupants' comfort [10], [22]. The aim is to design and control AC system such that to minimise the life cycle costs, including capital, operational costs (mainly energy consumption), maintenance and commissioning costs etc. while minimising the feeling of *discomfort* of occupants and keeping as maximum as possible productive environmental conditions. Solution of this problem is usually based on random search or gradient-based techniques [9]-[10]. In last few years successful application of GA to this problem has been reported [5], [9].

The specific of the AC systems, by differ from most other technical systems, is the fact that human and its *comfort* are in the centre of the problem and, as a result, control and optimisation objectives as well as some constraints are *highly subjective, ill defined* and

* So far as the laws of mathematics refer to reality, they are not certain. And so far they are certain, they do not refer to reality. **Albert Einstein**

different factors like clothing, metabolism, air movements, occupants' activity, solar radiation etc. have been seriously studied [15]-[20]. Human *comfort* is a complex non-linear, time-varying function of temperature, humidity, airflow rate, clothing level, activity, lighting, shading, noise etc. It is important to mention that the *comfort* requirements are *specific* for each individual or group of individuals and are *not constant.* The fact that natural temperature could be different in different locations and could vary with cultural and other differences, that occupants could have different feeling of comfort has been reported [22]-[23].

Current practice is to design and operate AC systems under assumptions of occupants' *comfort* requirements based on statistical design criteria contained within standards like ISO7730 [21], known as PPD and PMV[1]. In reality, the thermal sensations and needs of occupants vary from simple design standards [23]. Occupants' *preferences* are *subjective, individual*, their presence in the occupied zone and *level of activity* is time varying [15]-[19].

Another important issue in the AC systems control is their energy efficiency. The latest data from the International Energy Agency (http://www.iea.org) shows that significant part of the primary energy is used in buildings and AC system account for the major part of it. This problem is increasingly important nowadays, when ecological and economical concurrent requirements (the ozone hole, gas emissions and surging petrol prices, cost effectiveness as a condition for higher competitiveness etc.) are pressing and have to be balanced with the *comfort* needs. Additional potential for higher energy efficiency exist in finding and keeping more economical set-points or regimes, based on more adequate representation of humans' *feelings* of *comfort* and *discomfort*, of keeping a really productive environment [15]. It could lead to a twofold improvement:

- ✓ more customised, specific user-oriented AC system;
- ✓ higher energy efficiency.

A general formulation of the optimisation problem of an AC system in the framework of IF sets theory is proposed in the present paper. IF sets have been introduced recently [6] as an extension and generalisation of the fuzzy set concept. They consider not only the degree of membership to a given set, but also the degree of non-membership such that the sum of both values is less than one [7]. Due to the symmetrical treatment of objectives and constraints this multi-objective problem is transformed to a non-fuzzy (crisp, conventional) one [1-2], [24]-[26] that in some cases could be, solved analytically [25]. The building thermal model and plant models are not considered in this presentation in order to simplify the problem.

An example of a multi-objective IF optimisation problem illustrates the viability of the proposed approach. A solution, which significantly improves *comfort* and reduces *discomfort* feeling, is found. This illustrates the possibility to use IF optimisation for trade-off analysis in multi-objective optimisation of AC systems.

2. Some definitions over IF sets

IF set – basic definition [6]:

Definition 1

The IF set **A** is a mapping $R \rightarrow [0;1]x[0;1]$, which is defined by a pair $<\mu_A(x); \nu_A(x)>$ [6]

where $\mu_A(x)$ denotes the degree of membership of x to the set **A**;

$\nu_A(x)$ is the degree of non-membership (rejection) of x to **A**;

$\mu_A(x) + \nu_A(x) < 1$.

[1] from Predicted Percentage of Dissatisfied people and Predicted Mean Vote respectively

It is interesting to note, that **A** become a *conventional* fuzzy set, when $\mu_B(x)+\nu_B(x)=1$.

2.1 *Crispification*: de-fuzzification over IF sets [3]

Crispification operators have been introduced by the author in [3] as a mapping $[0;1]x[0;1] \rightarrow R$ in analogy to de-fuzzification operation over *conventional* fuzzy sets. The result of this operation is a crisp value which is representative for the given IF set as a whole. This operation is necessary in controllers to derive the final control action (fuzzy or IF set point can not be given to the servo in a control system) which will be realised. It is necessary also in decision making and expert systems for elicitation of information. A brief summary of definitions of four crispification operators and their main properties are presented bellow.

Definition 2a [3]

The centre-of-gravity (CoG) crispification operation over IF sets is defined as follows

$$x^0_{IF\_CoG} = \frac{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))x_i}{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))}; \qquad N = card(x); \quad for\ \mu(x_i) > \nu(x_i) \qquad (1)$$

This operator's definition is based on the difference between the degree of membership and non-membership (rejection), which have to be positive.

Definition 2b [3]

The mean-of-maxima (MoM) crispification operator is defined in analogy to the MoM de-fuzzification operator over $(\mu(x_i)-\nu(x_i))$.

$$x^0_{IF\_MoM} = \frac{\sum_{j=1}^{L} x_j^1}{2L}; \qquad x^1=\{x \mid \mu(x^1)-\nu(x^1) = \max_{i=1}^{N} (\mu(x_i)-\nu(x_i))\} \qquad (2)$$

Definition 2c [3]

Mean-of Equilibria (MoE) is introduced as a new operator in [3], which differ from de-fuzzification of *conventional* fuzzy sets). It averages all points with maximal $\mu(x_i)$ (the most acceptable points) and these with minimal $\nu(x_i)$ (the less non-acceptable):

$$x^0_{MoE} = \frac{\sum_{j=1}^{M} x_j^m}{2M} + \frac{\sum_{j=1}^{K} x_j^k}{2K} \qquad (3)$$

$$x^m=\{x \mid \mu(x^m) = {}_N max,^{i=1}\ \mu(x_i)\}; \qquad x^k=\{x \mid \nu(x^k) = {}_N min,^{i=1}\ \nu(x_i)\}$$

Definition 2d

Finally, an analogue of BADD operator is introduced for crispification. It is over the difference $\mu(x_i)-\nu(x_i)$:

$$x^0_{IF\_BADD} = \frac{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))^\alpha x_i}{\sum_{i=1}^{N} (\mu(x_i) - \nu(x_i))^\alpha} \qquad for\ \mu(x_i) > \nu(x_i)$$

(5)

Basic features of these operators are similar with these of their analogies:

The CoG crispification operator gives all possible solutions in which the degree of acceptance is higher than the degree of non-acceptance $(\mu(x_i)>\nu(x_i))$. However, it averages *good* and *poor* solutions (although it gives them different weights). The MoM and MoE crispification operations give information about the best solution(s) (MoE gives it in the sense of higher degree of acceptance and lower degree of non-acceptance). However they ignore information about the other possible solutions. The BADD crispification operator has the analogous property as BADD de-fuzzification operator:

- it implies CoG cispification operator for α=1 (it implies CoG, if ν=0 also);
- it approaches MoM crispification operator for $\alpha \rightarrow \propto$ and approaches MoM crispification when ν=0 also;
- it implies middle average (MA) when α=0:

$$IF\_BADD(\alpha=1;\nu>0) = IF\_CoG$$
$$IF\_BADD(\alpha=1;\nu=0) = IF\_CoG$$
$$IF\_BADD(\alpha \rightarrow \propto;\nu>0) = IF\_MoM$$
$$IF\_BADD(\alpha \rightarrow \propto;\nu=0) = IF\_MoM$$
$$IF\_BADD(\alpha = 0) = IF\_MA$$

α/ν	0	Positive
0	*IF_CoG*	*IF_CoG*
1	*IF_MoM*	*IF_MoM*
$\propto$	*IF_MA*	

Table 1

Crispification definitions [3] in combination with the conjunction and disjunction operations over IF sets [7] allow to develop new engineering applications of IF sets such as IF optimisation, IF decision support system, IF controllers, IF expert systems etc.

2.2. IF optimisation [1]-[2]

The concept of IF optimisation problem is introduced and investigated in [1]-[2] by the author. Ideas about IF sets and optimisation and decision making have been also considered in [7]-[8]. The original definition of [1]-[2] and the approach proposed there for solving the IF optimisation problem will be briefly presented here, as it will be used in AC system optimisation problem later. The benefit of the IF optimisation problems is twofold: they give richest apparatus for formulation of optimisation problems and, on the other hand, the solution of IF optimisation problem can satisfy the objective(s) with higher degree than the analogous fuzzy optimisation problem and the crisp one.

Quite often, it is difficult to describe the constraints of an optimisation problem by crisp relation's (equalities and/or non-equalities) [35]. Practically, a small violation of a given constraint is admissible and it can lead to a more efficient solution of the real problem [35]. Objective formulation represents, in fact, a subjective estimation of a possible effect of a given value of the objective function. Fuzzy optimisation formulations [29,32,34] are more flexible and allow finding solutions, which are more adequate to the real problem. One of the poorly studied problems in this field is definition of degrees of membership and rejection (non-membership) [30,36]. The principles of fuzzy optimisation problems are critically studied in [24]-[26]. However, mainly the transformations and the solution procedures [23], [25], [31], [33] have been investigated. Applying IF sets concept it is possible to reformulate optimisation problem using degrees of rejection of constraints and values of the objective, which are non-admissible.

In general, an optimisation problem includes objective(s) and constraints. In fuzzy optimisation problems (fuzzy mathematical programming [35], fuzzy optimal control [28], linear programming with fuzzy parameters [32] etc.) the objective(s) and/or constraints or parameters and relations are represented by fuzzy sets. These fuzzy sets explain the degree of satisfaction of the respective condition and are expressed by their membership functions [36].

Let us consider an optimisation problem:

$$f_i(x) \dashrightarrow min \qquad i=1,\dots,p \qquad (6)$$

subject to $g_j(x) \le 0$ $j=1,...,q$

where x denotes unknowns

$f_i(x)$ denotes objective functions

$g_j(x)$ denotes constraints (non-equalities)

p denotes the number of objectives

q denotes the number of constraints

The solution of this crisp optimisation problem satisfies all constraints exactly. In the analogous fuzzy optimisation problem (7) the degree of satisfaction of objective(s) AND of constraints is maximised (8).

$$f_i(x) \longrightarrow \widetilde{\min} \qquad i=1,...,p \tag{7}$$

subject to $g_j(x) \mathrel{\tilde{\le}} 0$ $j=1,...,q$

where $\widetilde{\min}$ denotes fuzzy minimisation

$\tilde{\le}$ denotes fuzzy inequality

It is transformed via Bellman-Zadeh's approach [27] to the following crisp (non-fuzzy) optimisation problem:

To maximise the degree of membership (acceptance) of the objective(s) AND constraints to the respective fuzzy sets:

$$max\ \mu_i(x) \qquad x \in R^n \qquad i = 1,...,p+q \tag{8}$$

subject to $0 \le \mu_i(x) \le 1$

where $\mu_i(x)$denotes degree of acceptance of x to the respective fuzzy sets

In the case when the degree of rejection (non-membership) is defined simultaneously with the degree of acceptance (membership) and when both these degrees are not complementary, IF sets can be used as a more general and reach tool for describing this uncertainty [6]. It is possible to represent deeply existing nuances in problem formulation defining objective(s) and constraints (or part of them) by IF sets, i.e. by pairs of membership ($\mu_i(x)$) and rejection ($\nu_i(x)$) functions. An IF optimisation problem is formulated as follows [3]:

To maximise the degree of acceptance of IF objective(s) AND constraints AND to minimise the degree of rejection of IF objective(s) AND constraints:

$$max,_x \{\mu_i(x)\} \qquad x \in R^n \qquad i = 1,...,p+q \tag{9}$$

$$min,_x \{\nu_i(x)\} \qquad i = 1,...,p+q$$

subject to $\nu_i(x) >,\_\ 0$ $i = 1,...,p+q$

$\mu_i(x) >,\_\ \nu_i(x)$ $i = 1,...,p+q$

$\mu_i(x) + \nu_i(x) <,\_\ 1$ $i = 1,...,p+q$

where $\mu_i(x)$ denotes degree of membership of x to the i-th IF set

$\nu_i(x)$ denotes degree of rejection of x from the i-th IF set

It is possible that a part of constraints and objective(s) are IF and other are fuzzy or crisp.

IF optimisation problem such as fuzzy optimisation problems can be represented as a two-stage process which includes aggregation of constraints and objective(s) and de-fuzzification (maximisation of aggregated function) [24, 26]. Usually applied Bellman-Zadeh's approach [27] for fuzzy optimisation problem solving realises min-aggregator. Conjunction of IF sets is defined as [6]:

$$G \cup C = \{ <x,\ \mu_G(x) \cap \mu_C(x),\ \nu_G(x) \cup \nu_C(x)>,\ x \in R^n\} \tag{10}$$

where G denotes an IF objective (gain)

C denotes an IF constraint

This operator can be easily generalised and applied to IF optimisation problem:

$$D = \{<x,\ \mu_D(x),\ \nu_D(x)>,\ x \in R^n\} \tag{11}$$

$$\mu_D = \bigcap_{i=1}^{p+q} \mu_i$$

$$\nu_D = \bigcup_{i=1}^{p+q} \nu_i$$

where D denotes the IF set of the decision

Min-aggregator could be used for conjunction and max-operator for disjunction:

$$\mu_D = \min_{i=1}^{p+q} \mu_i \qquad x \in R^n \tag{12}$$

$$\mu_D \leq \mu_i$$

$$\nu_D = \max_{i=1}^{q} \nu_i \qquad x \in R^n$$

$$\nu_D \geq \nu_i$$

It can be transformed to the following system of equations:

$$\begin{aligned} &\alpha <,\_\ \mu_i(x) && i=1,\ldots,p+q \\ &\beta >,\_\ \nu_i(x) && i=1,\ldots,p+q \\ &\alpha >,\_\ \beta \\ &\beta >,\_\ 0 \\ &\alpha >,\_\ 0 \\ &\alpha + \beta <,\_\ 1 \end{aligned} \tag{13}$$

where α denotes the minimal acceptable degree of objective(s) and constraints

β denotes the maximal degree of rejection of objective(s) and constraints

Finally IF optimisation problem can be transformed [1]-[2] to the following crisp (non-fuzzy) optimisation problem which can be easily solved numerically by simplex, gradient-based or genetic algorithms [5] or, in some cases [25] analytically:

$$(\alpha-\beta) \rightarrow \max \tag{14}$$

subject to

$$\begin{aligned} &\alpha <,\_\ \mu_i(x) && i=1,\ldots,p+q \\ &\beta >,\_\ \nu_i(x) && i=1,\ldots,p+q \\ &\alpha >,\_\ \beta \\ &\beta >,\_\ 0 \\ &\alpha + \beta <,\_\ 1 \end{aligned}$$

In general, the solution of IF optimisation problem is different than the solution of the analogous fuzzy problem and the degrees of satisfaction of a given objective or constraint in an IF optimisation problem can be higher or lower. It depends on the formulation of the respective functions of acceptance and rejection. This property of the IF optimisation problem is analogous to the property of fuzzy optimisation problems when in some cases (it depends on the membership function) the solution can satisfy the objective function *better*, but the price for this is the *worse* satisfaction of some constraints. A practical example of multi-objective optimisation of AC systems is considered in the next section.

3. Optimisation of AC systems as an IF optimisation problem

3.1 AC optimisation - an multi-objective problem

In general, the problem of AC system optimisation is formulated as a multi-objective optimisation, which aims to minimise costs (life-cycle costs, maintenance, commissioning, capital and running - mainly energy costs) and to maximise the *comfort* (minimise *discomfort*) subject to number of constraints [4]. Building and thermal plant models are considered as constraints as well as different physical and technical limitations over different components of the AC system (fans, coils, heaters, air-handling units etc.):

$$\begin{aligned} &J_1 = Costs\ (AC\ system\ variables{:}\ T,\ RH,\ m,\ ...) \rightarrow min \\ &J_2 = Comfort\ (AC\ system\ variables{:}\ T,\ RH,\ m,\ ...) \rightarrow max \\ &J_3 = Discomfort\ (AC\ system\ variables{:}\ T,\ RH,\ m,\ ...) \rightarrow min \end{aligned} \quad (15)$$

Subject to $Constraints\ (AC\ system\ variables{:}\ T,\ RH,\ m,\ ...) = 0$

where RH denotes relative humidity, %

m denotes mass flow rate, kg/s

Practically, the optimisation problem is solved as an uni-objective one where *comfort* requirements are considered as a constraint [4]-[5], [9]-[10].

$$J = Costs\ (T,\ RH,\ m,\ ...) \rightarrow min \quad (16)$$

Subject to $|\ PPD\ (T,\ RH,\ m,\ ...)\ | \leq 10\%$

$Constraints\ (T,\ RH,\ m,\ ...) = 0$

This simplification makes the problem a rough approximation of the real intention of the designer. It is so, if conventional technique is applied, because it simplifies the solution of the problem. Fuzzy sets and IF sets, specifically, offer a tool for coming to grip with this problem.

Discomfort, dissatisfaction as well as *comfort*, which is normally a more restrictive category, could be formulated with an appropriate, subjectively specified membership function (Fig.2).

The problem of AC system optimisation became fuzzy multi-objective one:

$$\begin{aligned} &Costs\ (T,\ RH,\ m,\ ...) \rightarrow \widetilde{\min} && : \mu_{costs} \\ &Discomfort\ (T,\ RH,\ m,\ ...) \rightarrow \widetilde{\min} && : \nu_{discomfort} \\ &Comfort\ (T,\ RH,\ m,\ ...) \rightarrow \widetilde{\max} && : \mu_{comfort} \end{aligned} \quad (17)$$

Subject to $Constraints\ (T,\ RH,\ m,\ ...) \cong 0 \quad : \mu_{constraints}$

where ~ denotes fuzzy

It should be noted that constraints and costs could be fuzzy, IF or crisp. We consider fuzzy description for them without loss of generality for the sake of simplification of the expressions. Building model (which could be of the fuzzy rule-base type) as well as models of components of the thermal system are not considered due to simplicity.

3.2 Partial Load Ratio definition by IF sets

Formulations of constraints with IF sets allow a higher flexibility to describe better the really existing uncertainty. For example, fans and coils are used with a part of their design load range: the loads lower than given thresholds are not so effective. Parameter Partial Load Ratio (PLR) represents the load used as a ratio to the designed load. Practically, loads lower than given threshold are not practically used (dashed line on the Fig.1). Very often, $PLR^{critical}$ is 20% of the design load (L^{design}). Fuzzy constraints over the fan's load (bold line μ_L on Fig.1) give additional flexibility in description. According to this fuzzy constraint, PLR in a range of [0.15; 0.25] are acceptable, but with degree less than 1. Introduction of a non-membership (rejection) function, however, (bold line ν_L on Fig.1) enhances even further this flexibility: combination of μ_L and ν_L defines an IF set over PLR (Fig.1). Its meaning is that:

- Loads up to 15% of the L^{design} are *definitely not acceptable*;
- Loads higher than 25% of the L^{design} are *definitely acceptable*;
- Loads in the range between 15% and 20% of the L^{design} are *partially acceptable* with the rate of rejection (non-acceptance) decreasing steeply until 20% of the L^{design};
- Loads in the range between 20% and 25% of the L^{design} are not rejected, but still are *not fully acceptable* (*acceptable with some reservations*) as $\mu_L<1$.

The last zone makes the main distinction with the conventional fuzzy sets and as it is seen it enriches the options for description of uncertainties, which exists in practice and the real life*.

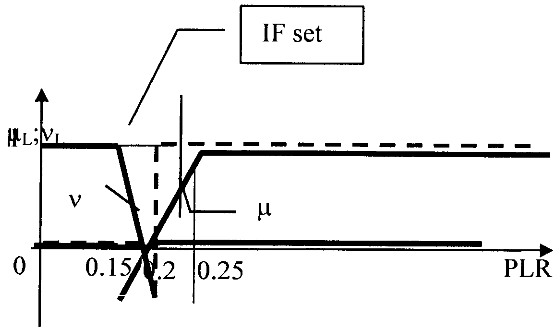

Fig.1 Partial load ratio constraint

3.2 *Comfort/discomfort* defintion by IF sets

Definition of *comfort/discomfort* requirements (normally treated by statistical categories of PPD and PMV) could be described more realistically using IF sets. A typical case of divergence in *comfort* and *discomfort* (which is not simply a complementary to the *comfort*) is given on Fig.2. Similar representations could be made for different physical variables describing human *comfort* and *discomfort* (T, relative humidity, lighting, shading, noise, air quality, solar radiation, air movement etc.). The meaning of this graphic is:

- for the values of the physical variable (T in this case) less than A and more than D the conditions are *definitely uncomfortable*;
- values between B and C determine the *acceptable comfort zone*;
- For the zones AB and CD the *discomfort feeling* is *dominating* the *feeling of comfort.*

This representation is more natural, because people tent to be more pretentious, demanding and easier say 'it is uncomfortable', even if, certain physical variable under consideration lay inside the pre-scribed by ISO standard band. One possible reason is that some other parameter of the internal climate is out of the range of the comfort zone. Another is the individual pessimism, conservatism and demands.

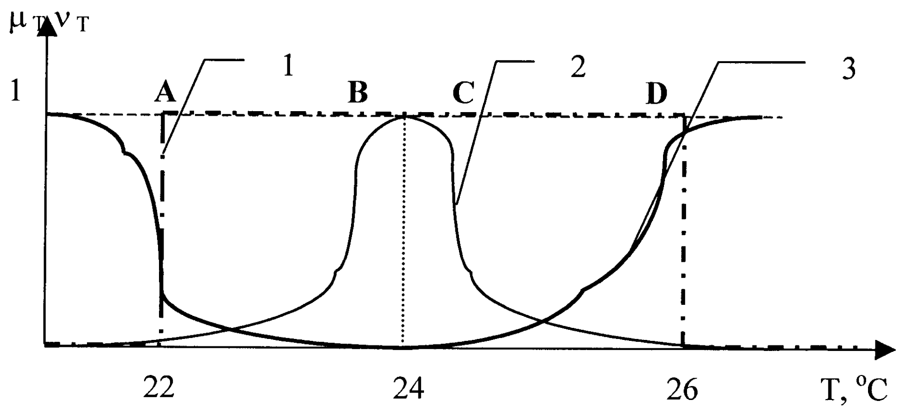

Fig.2 IF description of comfort/*discomfort*

(1 - conventional case; **2 - *comfort* description;** 3 - discomfort description)

From the Fig. 2 one could also see that for this particular example of thermal comfort, *slightly* higher temperatures, which are still in the standard *comfort* band (25-26 °C) are with higher degree of rejection ν_T (*dissatisfaction, discomfort*) than *slightly* lower temperatures (22-23 °C). That means that the non-membership (*discomfort*) function could be non-symmetrical. The practical meaning is that this person(s) tend to *prefer slightly lower* temperature than normal to *slightly higher*. Such degree of flexibility and freedom is difficult, if not possible, to have even using description with *conventional* fuzzy sets. In practice, when statistical measures are used instead of fuzzy and IF sets, such nuances are simply ignored. Our point is that using more complex and realistic description possess potential for much more customised and effective systems to be build in engineering practice increasing machine intelligence quotient.

4. Solving Multi-objective IF optimisation problem of AC system control

Solution of the optimisation problem (17) could be found numerically [1-2]. For some special cases it could be found analytically [25]. We consider a simple example of an AC system optimisation in order to illustrate the proposed approach.

Suppose that cumulative costs (capital and running) depends on PPD as follows (Fig.3) [4]:

$$Costs = 9.9 + \frac{1}{|PPD|} \tag{18}$$

and suppose for simplicity that constraints are satisfied for all variables considered, the following optimal solution is achieved:

$$Costs^0 = 10\ k£ \qquad PPD^0 = \pm 10\% \tag{19}$$

Practically, the dependence of Costs on PPD is more complex and includes building and plant models often it is dynamic problem [4]-[5], [9]-[10]. Variables are usually control parameters of the plant (supply air temperature, mass flow rate etc.). However, the nature of the problem is the same.

If apply conventional optimisation problem (15) comfort characteristics will be in the required range, but on one of its marginal values (+ or - 10% dissatisfied).

Suppose that *comfort* and *discomfort are* described in a similar way as shown on Fig.2 (with the only difference that instead of T and marginal values of 22 and 26 °C the dependence is in respect to PPD with ±10% dissatisfaction as boundary conditions).

$$\mu_{costs/comfort} = \begin{cases} 1-(0.1\,\mathrm{PPD})^2; & |\,\mathrm{PPD}\,| < 10\% \\ 0..................; & |\,\mathrm{PPD}\,| \geq 10\% \end{cases} \tag{20}$$

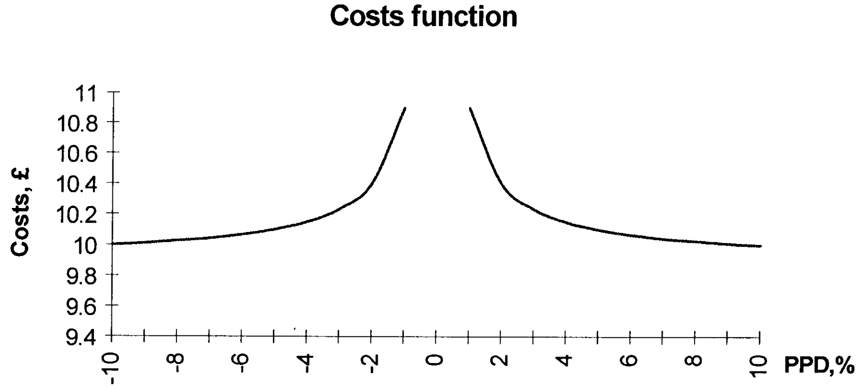

Fig.3 Costs as a function of PPD

Suppose that costs are given with the following membership function [4]:

$$\mu_{costs} = \begin{cases} 1 \ldots\ldots\ldots\ldots\ldots\ldots; & \text{Costs} < 10\text{ k£} \\ 0.1/(9.9 - \text{Costs}); & \text{Costs} \geq 10\text{ k£} \end{cases} \tag{21}$$

$$Costs = 9.9 + \frac{1}{|PPD|}$$

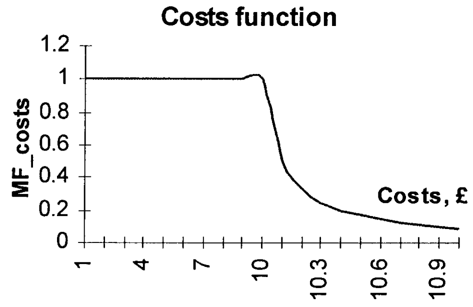

Fig.4 Costs function (*conventional* fuzzy set)

Then the following IF multi-objective optimisation problem arise [4]:

$$\mu_{result} \rightarrow max \tag{22}$$

$$\nu_{discomfort} \rightarrow min$$

subject to $\mu_{result} = \mu_{costs}\ AND\ \mu_{comfort}\ AND\ \mu_{constraints}$

Its solution could be found analytically for some cases [25] from:

$$\mu_{costs}(PPD) = min(\mu_{comfort}(PPD),\ \nu_{discomfort}) \tag{23}$$

$$PPD^2 + 10\ |PPD|\ -100 = 0 \tag{24}$$

Note that objective function is a *conventional* fuzzy set, *comfort/discomfort* are IF sets and other constraints could be either fuzzy, IF or crisp sets.

It is supposed for simplicity that constraints are satisfied for all values of considered variables. The fuzzy optimal solution is:

$$Costs^{opt} = £\ 10\ 060\ (-0.6\%) \tag{25}$$

$PPD^{opt} = \pm 6.2\%$ *(+38%* comparing to the standard *10% dissatisfaction*)

It is seen that significant improvement with more than 1/3 of the *comfort* requirements could be achieved with marginally higher (with only 0.6% or £60) costs. For the more complex (and realistic) IF optimisation problems the solution could be found numerically using gradient-based or genetic algorithms [5].

5. Conclusions

Optimisation of air-conditioning systems is treated in the paper in the framework of IF optimisation. This new approach allows to formulate more precisely the problem, which compromises energy saving and thermal *comfort/discomfort* satisfaction under given constraints. This fuzzy optimisation problem is solved analytically under some assumptions.

IF optimisation of AC systems allow us to formulate problem more flexibly, user-friendly and to analyse more precisely and deeply the whole range of possible solutions. It helps us to avoid necessity of scaling and normalisation applied in other approaches (when penalty functions are used) as well as infeasible solutions (infeasible solutions has degree of rejection 1 and are simply not considered in the final result). The problem is formulated in more natural IF multi-objective fashion and is solved efficiently. This is possible analytically in some cases and either numerically in general.

A practical engineering example illustrates the viability of the proposed approach. More customised solution (significantly improving *comfort* - with more than 38%) is found for the marginally higher energy expense of £60 or 0.6% more. In general, it is possible to find less energetically expensive solutions, which in the same time are more customised. This, however, depends on the specific preferences of occupants and could not be guaranteed for all practical cases.

6. Acknowledgement

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