Parallelised Gaussian Mixture Filtering for Vehicular Traffic Flow Estimation

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Abstract: Large traffic network systems require handling huge amounts of data, often distributed over a large geographical region in space and time. Centralised processing is not then the right choice in such cases. In this paper we develop a parallelised Gaussian Mixture Model filter (GMMF) for traffic networks aimed to: 1) work with high amounts of data and heterogenous data (from different sensor modalities), 2) provide robustness in the presence of sparse and missing sensor data, 3) able to incorporate different models in different traffic segments and represent various traffic regimes, 4) able to cope with multimodalities (e.g., due to multimodal measurement likelihood or multimodal state probability density functions). The efficiency of the parallelised GMMF is investigated over traffic flows based on macroscopic modelling and compared with a centralised GMMF.

The proposed GMM approach is general, it is applicable to systems where the overall state vector can be partitioned into state components (subsets), corresponding to certain geographical regions, such that most of the interactions take place within the subsets. The performance of the paralellised and centralised GMMFs is investigated and evaluated in terms of accuracy and complexity.

Keywords: Parallelised Gaussian Mixture filters, vehicular traffic state estimation, multimodal probability density function.

1 Motivation

The vehicular traffic dynamics inherently exhibits multiple regimes and this complex and nonlinear behaviour requires methods able to represent well multimodal probability density functions. In such situations the Gaussian Muxture Model (GMM) approach [AS72] is the preferred choice. The GMM filter (GMMF) is appealing and powerful for solving nonlinear estimation problems and these are the cases when [ALS07]: 1) the system and/ or measurement noises can be represented using Gaussian mixtures, 2) the measurement likelihood function is multimodal and is hence well represented by a Gaussian mixture, 3) the system dynamics can be multimodal and the GMMF is a good choice for capturing this multimodality in the probability density function of the system state.

This paper presents a parallelised Gaussian Mixture filter (GMMF) for vehicular traffic flow estimation. Other works on Gaussian Mixture filtering have been proposed to solve different problems, e.g., on positioning and navigation in [AL08a, ALCL$^+$08, AL08b, ALS07]. However, to our knowledge the application of the GMM approach to vehicular traffic problems is limited. A work related with the GMMF is the centralised mixture Kalman filter (a sequential Monte Carlo approach combined with Kalman filtering) [SMH03] developed based on first order traffic models (without speed estimates). Switching state space piecewise affine models are used to model the multimodalities due to congested and free-flow traffic modes.

Our observations of the conditional likelihood of particle filters for vehicular traffic [MBH07] is that it can be multimodal (as it can be seen on Fig. 1) and this is one of our motivations for applying the GMMF to vehicular traffic problems. Another reason for multimodality is in the traffic behaviour itself: in presence of different regimes (e.g., free flow, congested, synchronised).

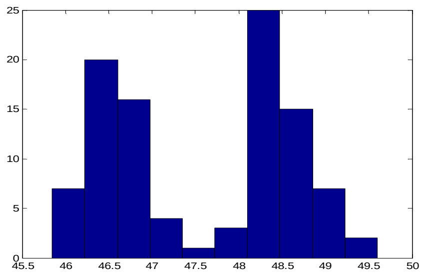

Figure 1: Multimodality observed in the likelihood for the measured real traffic data (number of vehicles) within the particle filter developed in [MBH07].

One of the challenges in the Gaussian mixture filtering is the choice of the number of Gaussian components and the adaptation of this number. Different methods are available in the literature, e.g., the Figueiredo-Jain algorithm [M. 02] and other components reduction methods [ALS07] such as merging and forgetting.

The remaining part of this paper is organised in the following way. Section 2 presents the theoretical background for the Bayesian estimation, Section 3 introduces the centralised Gaussian Mixture Model filtering approach. Section 4 presents the parallelised GMMF for vehicular traffic. Section 5 presents results from the parallelised GMMF. Finally, conclusions are given in Section 6.

2 Bayesian Estimation

Consider the discrete-time nonlinear non-Gaussian system model

$$x_k = f(x_{k-1}) + v_{k-1}, \tag{1}$$
$$z_k = h(x_k) + n_k, \tag{2}$$

where the system state x_k has to be estimated in time $k = 1, 2, \ldots$, z_k is the measurement vector, v_k is the state noise, n_k is the measurement noise. The noises v_k and n_k are assumed to be mutually independent and independent of the initial state x_0.

Since the system and the measurements are stochastic, the exact state cannot be inferred from the measurements, only the probability density function (pdf) of the state $p(x_k|z_{1:k})$ can be determined given all past and current measurements $z_{1:k} \triangleq \{z_1, \ldots, z_k\}$ from sample step 1 to k. So, the goal of the state estimation problem is to determine the conditional (posterior) pdf $p(x_k|z_{1:k})$.

The posterior can be determined recursively according to the *prediction* and *measurement update* steps.

Prediction (prior):

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1}p(x_{k-1})|z_{1:k-1})dx_{k-1} \tag{3}$$

Update (posterior):

$$\begin{aligned} p(x_k|z_{1:k}) &= \frac{p(z_{1:k}|x_k)p(x_k|z_{1:k})}{p(z_k|z_{1:k-1})} \\ &= \frac{p(z_{1:k}|x_k)p(x_k|z_{1:k})}{\int p(z_k|x_k)p(x_k|z_{1:k-1})dx_k} \end{aligned} \tag{4}$$

The transition pdf in (3) is

$$p(x_k|x_{k-1}) = p_{v_{k-1}}(x_k - f(x_{k-1})) \tag{5}$$

and the likelihood

$$p(z_k|x_k) = p_{n_k}(z_k - h(x_k)) \tag{6}$$

In the next section we introduce the general formulation of Gaussian mixture model filtering and we present two approaches for parallelisation. Although the parallelisation is explained for traffic networks, the same approach can be followed for other processes where the overall state can be partitioned into subsets of states where the interaction between the states takes mainly place *within* one subset.

3 A Gaussian Mixture Model Filter (GMMF) for Traffic Estimation

In this paper we consider the problem of traffic flow estimation by a mixture of Gaussian components. In the GMMF [AS72] both the prior state probability density function (pdf) $p(x_k|z_{1:k-1})$ and the posterior pdf $p(x_k|z_{1:k})$ can be Gaussian mixtures

$$p(x) = \sum_{i=1}^{m_x} \alpha_i \mathcal{N}_\Sigma^x(x, \mu_i, \Sigma_i), \tag{7}$$

where $\mathcal{N}^x(x, \mu_i, \Sigma_i)$ is the Gaussian pdf with vector mean μ_i and covariance matrix Σ_i; α_i are weights, such that $\sum_{i=1}^{m_x} \alpha_i = 1$; m_x is the number of Gaussian mixture components.

The likelihood can also be represented as a Gaussian mixture [ALS07]

$$p(z|x) = \sum_{j=1}^{m_z} \beta_j \mathcal{N}_R^n(z, \mu_j^n, R_j), \tag{8}$$

where the mean vector μ_j^n and the covariance matrix R_j for each mixture component are used to form the mixture and represent the measurement pdf; m is the number of Gaussian components, such that $\sum_{i=1}^{m_z} \beta_j = 1$.

The mean vectors and covariance matrices of the Gaussian mixture (7), resp. of (8) can be calculated by two main approaches [PKIK06]: maximum likelihood estimation (Expectation Maximisation) or Bayesian estimation.

Within recursive Bayesian estimation, in accordance with (4), (7) and (8) the posterior state pdf can be calculated with the Gaussian mixture [ALS07]

$$\begin{aligned} p(x|z) &= \frac{\sum_{j=1}^{m_z} \beta_j \mathcal{N}_R^n(z, \mu_j^n, R_j) \sum_{i=1}^{m_x} \alpha_i \mathcal{N}_\Sigma^x(x, \mu_i, \Sigma_i)}{\int \sum_{j=1}^{m_z} \beta_j \mathcal{N}_R^n(z, \mu_j^n, R_j) \sum_{i=1}^{m_x} \alpha_i \mathcal{N}_\Sigma^x(x, \mu_i, \Sigma_i) dx} \\ &= \frac{\sum_{j=1}^{m_z} \sum_{i=1}^{m_x} \alpha_i \beta_j \mathcal{N}_R^n(z, \mu_j^n, R_j) \mathcal{N}_\Sigma^x(x, \mu_i, \Sigma_i)}{\sum_{j=1}^{m_z} \sum_{i=1}^{m_x} \int \alpha_i \beta_j \mathcal{N}^n(z, \mu_j^n, R_j) \mathcal{N}^x(x, \mu_i, \Sigma_i) dx} \\ &= \frac{\sum_{j=1}^{m_z} \sum_{i=1}^{m_x} \alpha_i \beta_j \mathcal{N}_{P_{i,j}}^n(z, \mu_j^n, P_{i,j}) \mathcal{N}_{\hat{P}i,j}^x(x, \hat{x}_{i,j}, \hat{P}_{i,j})}{\sum_{j=1}^{m_z} \sum_{i=1}^{m_x} \int \alpha_i \beta_j \mathcal{N}^n(z, \mu_j^n, P_{i,j})}, \end{aligned} \tag{9}$$

where

$$P_{i,j} = H_j \Sigma_i H_j' + R_j \tag{10}$$

$$\hat{x}_{i,j} = \mu_i + K_{i,j}(z - H_j \mu_i), \tag{11}$$

$$\hat{P}_{i,j} = (\Sigma_i^{-1} + H_j' R^{-1} H_j)^{-1} \quad (12)$$

$$= (I - K_{i,j} H_j)\Sigma_i \quad (13)$$

$$K_{i,j} = \Sigma_i H_j' P_{i,j}^{-1} \quad (14)$$

where I is the identity matrix and $'$ denotes matrix transpose operation.

The mean vector of the mixture (7), can be calculated as [AL08b]

$$\mu = \sum_{i=1}^{m_x} \alpha_i \mu_i \quad (15)$$

and the covariance matrix is in the form

$$\Sigma = \sum_{i=1}^{m_x} \alpha_i (\Sigma_i + (\mu_i - \mu)(\mu_i - \mu)'). \quad (16)$$

The second approach for estimating the GMM parameters is the Expectation Maximisation (EM). Given the data z, the EM algorithm calculates iteratively the maximum likelihood parameter estimates. During the expectation step (E-step), the log-likelihood function is formed

$$Q(\theta, \theta') = E[ln\mathcal{L}(\theta|z, \theta')] \quad (17)$$

where θ' is the previous estimate for the GMM parameters, θ is the new estimate, $E(.)$ is the mathematical expectation operation. The maximisation (M-step) is to maximise $Q(\theta, \theta')$ with respect to θ and set:

$$\theta' \leftarrow \arg\max_\theta \mathrm{Q}(\theta, \theta') \quad (18)$$

The steps are repeated until convergence is reached based on a suitably selected threshold. In our implementation the GMMF is used to propagate the posterior state pdf $p(x)$, after performing the prediction step using METANET traffic model [PB89, KPD$^+$02]. The multimidality in the traffic behaviour stems from high speed/ low density and low speed/ high density. In our implementation the update of the parameters $\theta = \{\alpha_1, \mu_1, \Sigma_1, \ldots, \alpha_{m_x}, \mu_{m_x}, \Sigma_{m_x}\}$ of the GMMF is performed using the Expectation Maximisation (EM) algorithm [PKIK06].

4 A Parallelised Gaussian Mixture Model Filter

When centralised filtering methods are applied to large traffic networks, the computational complexity may become too high for running in real-time on a single Central Processing Unit (CPU). One way to tackle this problem is the parallelisation of the filters. A schematic representation of a parallelised scheme is shown on Figure 2. The boundary state estimates from the previous region are initial states estimates (inflow) for the next

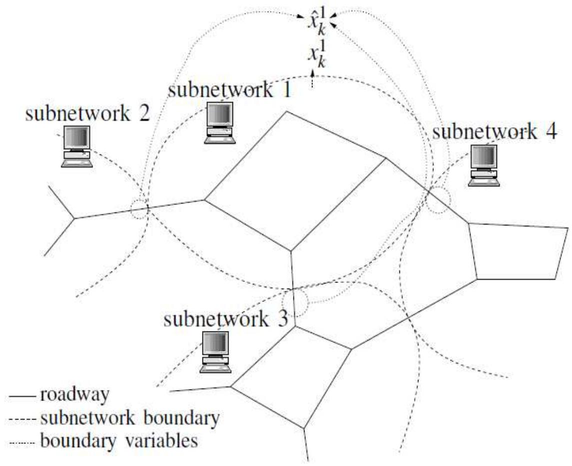

Figure 2: An example of partitioning a traffic network into subnetworks for parallelised simulation [HMB07].

geographic region and this enables speeding up considerably the computational process. Estimates of the number of vehicles crossing the boundaries between two regions and their respective speeds are transmitted.

The traffic network is subdivided into several subnetworks (corresponding to geographical regions), where each PU is responsible for one subnetwork and the *relevant* variables of the neighboring segments are communicated (as illustrated in Fig. 2). The state of the traffic network and the measurements can be correspondingly partitioned into S subvectors $x_k^s, s = 1, ..., S$ with $x_k = [(x_k^1)^T, (x_k^2)^T, \dots, (x_k^S)^T]^T$, and $z_k = [(z_k^1)^T, \dots, (z_k^S)^T]^T$. The system (1)-(2) can now be described by

$$x_k^s = f_k^s(x_{k-1}^s, \hat{x}_{k-1}^s, v_{k-1}^s), \tag{19}$$
$$z_k^s = h_k^s(x_k^s, n_k^s), \tag{20}$$

where $s = 1, ..., S$, and the vector $\hat{x}_{k-1}^s$ collects all neighboring state variables that act as an input to subnetwork s.

Instead of sending particles and/ or their weights to the CPU or the statistics of the particles as it is in the implemented parallelised particle filters in [HMB07], in the parallelised GMMF only the estimates of the boundary states and their covariances are transferred via the boundaries. This provides decomposition by region and also by mode. The state estimates are formed based on the average sum (15).

The different parallelised GMM filters can operate in general with a different number of

Gaussian components, and only whose with the most significant weights can contribute to the formation of the estimates for the particular region.

5 Results

The experiment is performed with simulated data over the same scenario that was presented in [HMB07]. Two filters are implemented: a centralised GMMF and a parallelised GMMF, both coping with the multimodality present in the likelihood function.

5.1 Lay-out

The network considered in [HMB07] consists of a 2-lane motorway link of 10 segments of 1 km each. In the parallelised approach this link is divided into two sublinks ("subnetworks") consisting of respectively the first and the last five segments.

5.2 Scenario

Two different scenarios are used to evaluate the filter performance: one with downstream propagating waves (in free-flow) and another with an upstream propagating shock wave as shown in Fig. 3. These scenarios are defined by selecting the upstream and downstream boundary conditions. The motivation to select these two scenarios is to have both conditions where information propagates forward and where information propagates backward over the sublink boundaries. The state and measurement noises are taken to be Gaussian (although any other distribution could be taken) with state noise variances, var$(\xi^v_{m,i}(k)) = 0.5\,$(km/h)^2, var$(\xi^\rho_{m,i}(k)) = 0.5\,$(veh/km/lane)^2, and measurement noise variances var$(n^v_{m,i}(k)) = 4\,$(km/h)^2, and var$(n^q_{m,i}(k)) = 22500\ (\mathrm{veh/h})^2$.

Results for segments 1-5 and 5-10 are shown respectively on Figures 4 and are obtained with two Gaussian mixture components, with the centralised GMMF (the measurements from segments 1 and 10 are processed in one centre), based on 10 Monte Carlo experiments. The Root-Mean Squared Errors are shown in Figure 6. The same results are shown for the parallelised GMM (two GMM algorithms run in parallel) on Figures 5 and 7.

The computational time of the centralised GMMF, is approximately 1.8 times more than the computational time of the parallelised GMMF. In terms of accuracy the performance of the centralised and parallelised GMMF seem to be comparable.

Current research is focused on studying the influence of the number of Gaussian mixture elements on the estimation process and on comparison of the GMM algorithms with other estimators.

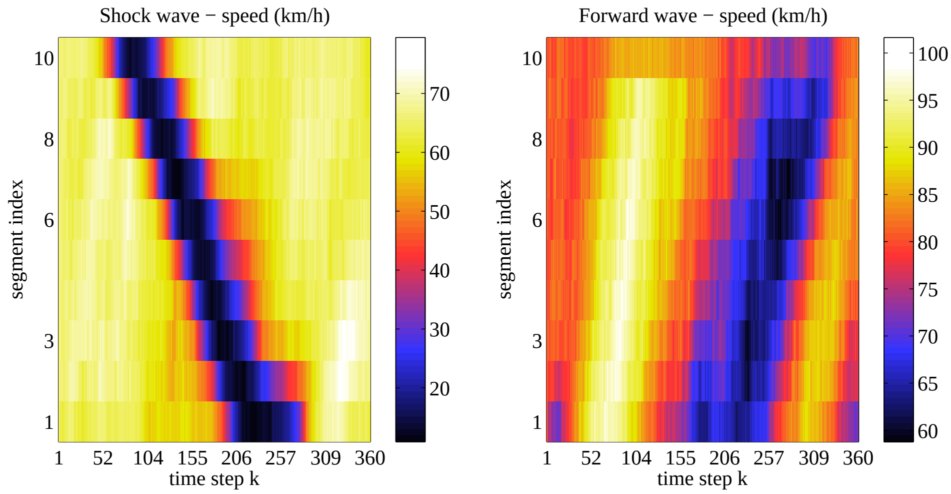

Figure 3: The shock wave (left) and the forward wave (right) scenario, used for the evaluation of the filters. The travel direction is from segment 1 to 10. The colours indicate the speed. Please note the difference in colour bar scales: the shock wave scenario includes a wider range of speed since it also contains congested traffic.

6 Conclusions

This paper presents a parallelised Gaussian Mixture Model filter for large traffic networks. The approach is able to cope with multimodalities both in the state probability density function and in the likelihood function. Compared with the parallelised particle filters developed in [HMB07], the parallelised GMMF requires less communications and less computational costs. It is flexible and can easily be adapted to the real-time demands of large traffic networks.

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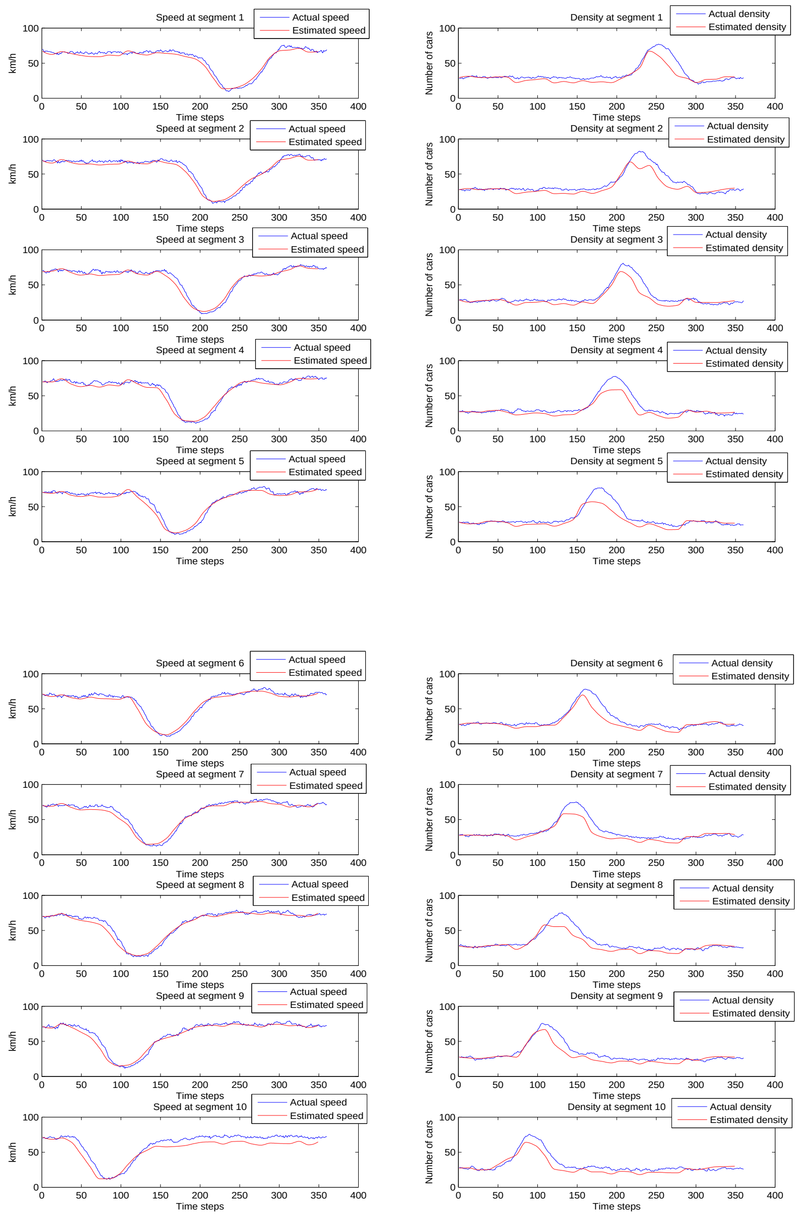

Figure 4: Results obtained with the centralised GMMF (actual vs. estimated states) for segments 1 to 10 with the shock wave traffic scenario shown on Figure 3 (left).

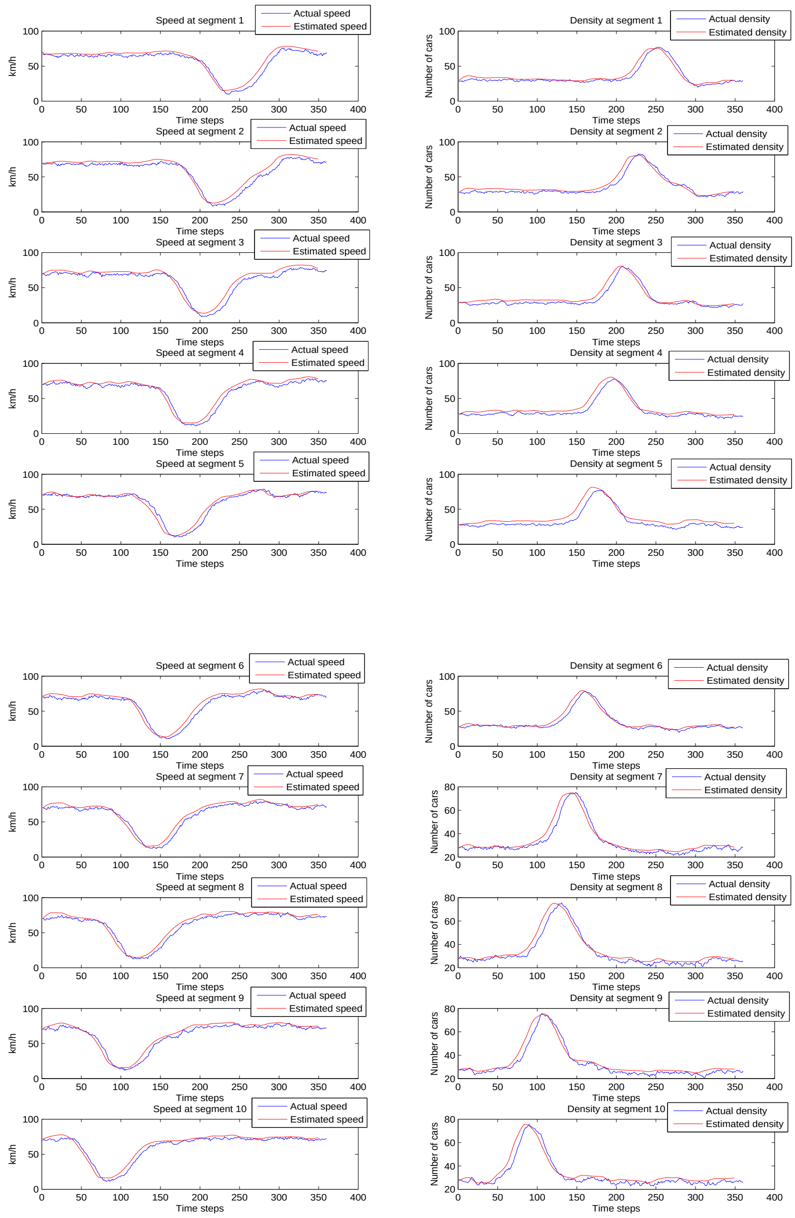

Figure 5: Results obtained with the parallelised GMMF (actual vs. estimated states) for segments 1 to 10 with the shock wave traffic scenario shown on Figure 3 (left), from 10 runs.

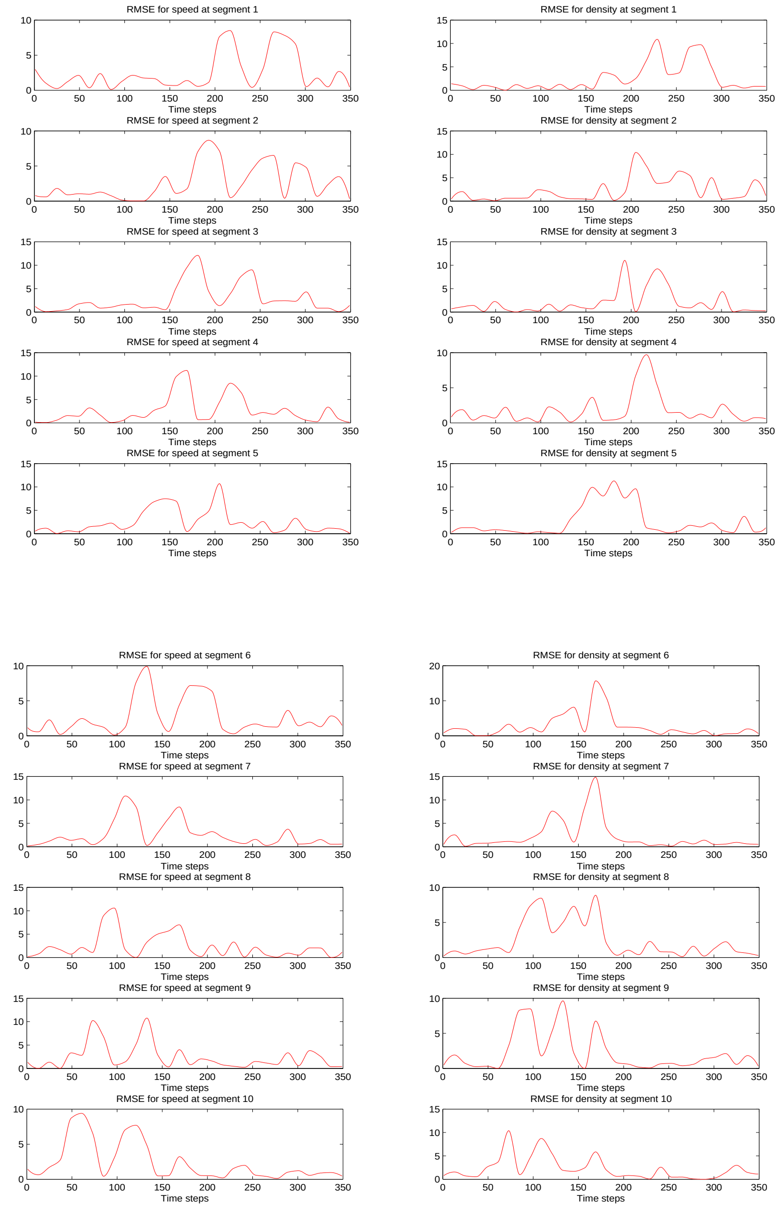

Figure 6: Root Mean Squared Errors obtained with the centralised GMMF (from 10 runs).

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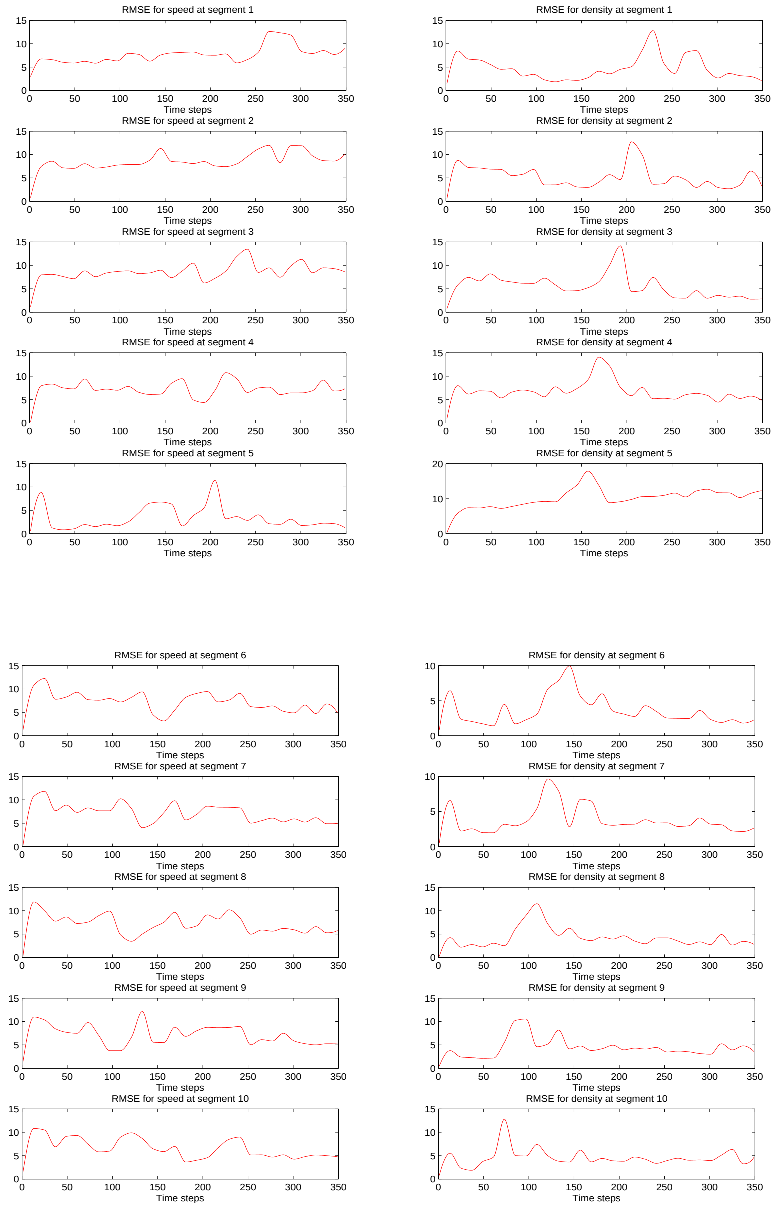

Figure 7: Root Mean Squared Errors obtained with the parallelised GMMF (from 10 runs).

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