

What stochastic resonance can and cannot do

Stochastic resonance (SR)^{1–3} is often defined as a noise-induced rise (and then fall, for higher noise intensities) of the signal-to-noise-ratio (SNR) of a weak narrow-band signal in a nonlinear system. Various applications of this phenomenon are being explored. In particular, the possibility that SR might play a role in enabling biological cells to respond to extremely weak 50-60 Hz electromagnetic fields, far below the thermal noise level, has recently been discussed^{4,5}. Given the interest in SR, we suggest that its place within the broader physics context should be specified more clearly: in particular, it should be stated what SR can, and cannot, be expected to do.

The very idea of SR may seem counterintuitive, at first glance. However, soon after SR was discovered for bistable systems, it was realized that it amounted to a fairly straightforward extension of earlier work by Debye⁶ on reorienting polar molecules, and that the occurrence of SR in the general case could be treated⁷ using a traditional technique of statistical physics⁸: linear response theory (LRT). It is well-known in the LRT context that the response of a system to signals in certain frequency ranges can be strongly increased by noise, e.g. just by raising the temperature. Examples range from currents in electron tubes to optical absorption near absorption edges in semiconductors. The thresholdless model considered by Bezrukov and Vodyanoy⁹, which has been the subject of the recent discussions^{4,5}, displays noise-induced increases of both the signal and the SNR. Their formula for the SNR represents a special case of the general LRT expression given earlier⁷ and subsequently applied to many different systems^{7,10–12}, including some¹¹ without response thresholds.

An important consequence^{10,13} of LRT, apposite to the recent correspondence^{4,5}, is that, for a system driven by a signal and Gaussian noise, the SNR at the output, R_{out} , does not exceed that at the input, R_{in} . For a linear system $R_{\text{out}} = R_{\text{in}}$, and the SNR decreases with the increasing noise intensity. For a nonlinear system the ratio $R_{\text{out}}/R_{\text{in}}$ may be small, and then the provision of additional noise can sometimes help to increase the SNR at the output, back towards its value at the input. It is this latter effect which constitutes SR. Quite regardless of parameter choice^{4,5}, therefore, the SNR of the 50-60 Hz signal inside the biological cell (output signal) cannot be expected to exceed that of the external signal coming from the environment (input signal).

SR can ameliorate quite dramatically the SNR degradation of a noisy signal caused by its transduction through a nonlinear element. It does not, however, provide a mechanism whereby the SNR of the input signal can meaningfully be enhanced.

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