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A NOVEL DESIGN OPTIMIZATION OF A FAULT-TOLERANT AC PERMANENT MAGNET MACHINE-DRIVE SYSTEM

By Peng Zhang, B.S., M.S.

A Dissertation Submitted to the Faculty of the Graduate School, Marquette University, in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

> Milwaukee, Wisconsin December 2013

ABSTRACT

A NOVEL DESIGN OPTIMIZATION OF A FAULT-TOLERANT AC PERMANENT MAGNET MACHINE-DRIVE SYSTEM

Peng Zhang, B.S., M.S. Marquette University, 2013

In this dissertation, fault-tolerant capabilities of permanent magnet (PM) machines were investigated. The 12-slot 10-pole PM machines with V-type and spoketype PM layouts were selected as candidate topologies for fault-tolerant PM machine design optimization problems. The combination of 12-slot and 10-pole configuration for PM machines requires a fractional-slot concentrated winding (FSCW) layout, which can lead to especially significant PM losses in such machines. Thus, a hybrid method to compute the PM losses was investigated, which combines computationally efficient finite-element analysis (CE-FEA) with a new analytical formulation for PM eddy-current loss computation in sine-wave current regulated synchronous PM machines. These algorithms were applied to two FSCW PM machines with different circumferential and axial PM block segmentation arrangements. The accuracy of this method was validated by results from 2D and 3D time-stepping FEA.

The CE-FEA approach has the capabilities of calculating torque profiles, induced voltage waveforms, d and q-axes inductances, torque angle for maximum torque per ampere load condition, and stator core losses. The implementation techniques for such a method are presented. A combined design optimization method employing design of experiments (DOE) and differential evolution (DE) algorithms was developed. The DOE approaches were used to perform a sensitivity study from which significant independent design variables were selected for the DE design optimization procedure. Two optimization objectives are concurrently considered for minimizing material cost and power losses. The optimization results enabled the systematic comparison of four PM motor topologies: two different V-shape, flat bar-type and spoke-type, respectively. A study of the relative merits of each topology was determined.

An automated design optimization method using the CE-FEA and DE algorithms was utilized in the case study of a 12-slot 10-pole PM machine with V-type PM layout. An engineering decision process based on the Pareto-optimal front for two objectives, material cost and losses, is presented together with discussions on the tradeoffs between cost and performance. One optimal design was finally selected and prototyped. A set of experimental tests, including open circuit tests at various speeds and on-load tests under various load and speed conditions, were performed successfully, which validated the findings of this work.

ACKNOWLEDGEMENTS

Peng Zhang, B.S., M.S.

First of all, I would like to express my most sincere appreciation to Professor Nabeel A.O. Demerdash, my academic and dissertation advisor. I wish to thank him for giving me the opportunity to begin my Ph.D. studies at Marquette University and for his wise and trusted guidance in my research direction and writing techniques throughout the entire process. I would also like to express my sincere gratitude to my industrial co-advisor, Professor Dan M. Ionel, for his consistent contributions in brain-storming discussions and for his technical guidance in my industry related work during my Ph.D. studies. If it has not been for their continuous support and guidance, much of this work would not have been possible. Their suggestions, encouragement and trust helped me a lot in building my confidence for my future career.

I am very grateful to Professor Edwin E. Yaz. His guidance during our research group meetings have helped me to solve engineering mathematical problems and improve my communication skills as an engineer. I further would like to thank my committee members: Professor Ian Brown, Dr. Thomas W. Nehl, and Professor James E. Richie, who gave me advice and consistent support whenever I needed it.

I should also mention that my graduate studies at Marquette University were supported by Regal Beloit Corporation, the National Science Foundation NSF-GOALI Grant No: 1028348, and the software support from ANSYS Corporation. Furthermore, both Mr. Steven Stretz and Mr. Alan W. Yeadon from Regal Beloit Corporation have contributed with a lot of interesting discussions and valuable assistance in the design optimization and experimental testing of the electric machines subject of this research project. I am also very grateful to Dr. Marius Rosu and Mr. Mark Solveson from ANSYS Corporation for their consistent support through the ANSYS software packages made accessible to me in the Electric Machines and Drives Laboratory at Marquette University.

I would also like to thank my colleague Dr. Gennadi Y. Sizov, who has laid a solid foundation and made tremendous contributions to the automated design optimization tools for electric machines during his Ph.D. studies at Marquette University, which formed a solid basis for the start of my Ph.D. dissertation research.

Of course, I wish to thank my friends: Jiangbiao He, Alireza Fatemi, Andrew Strandt, Alia Strandt, Muyang Li, and Chad Somogyi. They are wonderful colleagues in the Electric Machines and Drives Lab at Marquette University.

Finally, I am grateful to my parents and husband for their patience and *love*. Without their consistent encouragement throughout my studies, this work would never have come into existence.

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CHAPTER 1 INTRODUCTION

In this chapter, the background of the topic in this dissertation is briefly introduced. Through a large amount of literature search, the recent trends in several topics related to the subject of this dissertation are reviewed. This includes different types of permanent magnet (PM) machines and their corresponding applications, as well as modeling and analysis approaches for electric machines and associated design optimization algorithms. Based on these previous investigations, the main objectives of this work are delineated below.

1.1 Background of the Problem

Over the last decade, brushless (BL) PM motor technology was established as the preferred choice for high efficiency applications [1], and because such motors have high torque and power to volume densities, as well as wide speed ranges. These qualities make PM machines more popular in applications of medical devices, hybrid electric vehicles (HEV) and other automotive applications, motion control and aerospace applications, and renewable wind energy systems [2]. Meanwhile, some typical drawbacks are associated with these popular PM machines. One of these drawbacks is the possible poor thermal dissipation of rotor losses which may cause overheating and consequently irreversible demagnetization of permanent magnets. Another typical and severe drawback is that when a PM machine runs under field weakening condition at high speed, and in the event of a partial failure in the control system of the drive, the high-speed rotating magnetic field produced by the PMs in the rotor will cause very large terminal voltages across the stator windings. This in turn, will cause catastrophic damage to the switches in any drive connected to such a machine. Some of the most common faults in such PM machines include failures of insulation in stator windings [3], and eccentricities resulting from the stator and rotor misalignment. Given these concerns, nowadays the need for the design of fault-tolerant PM machine-drive systems has become a high-priority topic to various investigators and users.

Another important endeavor in the design optimization of electric machines is the development of design optimization tools. For different topologies of electric machines, without an effective optimization procedure, conceivably an unreasonable comparison might be performed between the worst design for one topology and the best design for another topology. Depicted in Figures 1.1 and 1.2, are the performance points of three different machine topologies in the cost-losses plane, which are located in three different clusters bounded by the "Pareto-front" [4] in this cost-losses plane. This means that for one single topology, the performance varies a great deal with



Figure 1.1: Pareto-sets of three different electric machine topologies for two design objectives.



Figure 1.2: Pareto-sets of three different electric machine topologies for three design objectives.

the variation of geometric variables. In order to provide effective means for the systematic comparison between different machine topologies, a fast and automated design optimization method needs to be developed, investigated and implemented in this dissertation.

1.2 Review of the Literature

1.2.1 Fault-Tolerant Permanent Magnet Machines

The design of fault-tolerant PM machine drive systems can be classified into two main areas, the electric machine area and drive control area. Reference [5] gives a review of different topologies of fault-tolerant PM machines and drives. A fault-tolerant electric machine needs to sustain a comparable performance under faulty conditions to that performance when such a machine is healthy. Meanwhile, such a machine needs to fail safely without leading to a catastrophic damage to the associated system. These properties require that such fault-tolerant PM machines must have good electrical, magnetic, thermal and physical isolations [3], and good loss/thermal dissipation in such machine structures. Based on these requirements, various investigations on the design optimization of fault-tolerant machines have been performed and presented [6–23].

Compared with the integer-slot distributed windings (ISDWs), the factional-slot

concentrated windings (FSCWs) [6] have shorter end-winding connections, which means less copper losses and cost. Meanwhile, PM machines with FSCWs are also an excellent option for the design of modular electrical machines [7], which have the merits of physically, thermally and magnetically isolated windings. In reference [8], an approach was presented for the optimized combination of stator slot and rotor pole numbers to eliminate the magnetic coupling between phases for such fault-tolerant PM machines.

Multi-phase (greater than or equal to three-phase) stator windings, especially fivephase fault-tolerant PM machines have received substantial attention in the literature [9–11]. Reference [9] covers three fault types: the open circuit fault of a single phase, the open circuit fault of two nonadjacent phases, and the open circuit fault of two adjacent phases, for two motors with two different stator windings. The postfault current control strategies of five-phase PM machine was investigated in reference [10], which covers both the open circuit faults of one and two phases and the short circuit fault at the machine terminal of one phase. Another five-phase interior PM (IPM) machine with FSCWs was designed with low torque pulsation in reference [11].

In reference [13], the influence of parallel paths on PM machines' unbalanced magnetic pull with motor eccentricities was investigated. The authors concluded that unbalanced currents in the parallel paths of stator windings can reduce the unbalanced magnetic pull with the increase in the number of parallel paths.



Figure 1.3: Cross section of a DSPM machine.

In references [14] and [15], double-salient PM machines (DSPMs), Figure 1.3, were designed and analyzed, where the PMs were embedded in the stator, which leads to good thermal dissipation of losses in the PMs. In references [16] and [17], the design principles and analysis of flux-switching PM machines (FSPMs), Figure 1.4, were given, and the fault-tolerant capabilities of such machines were also described. The work in [18] compared the DSPM and FSPM classes of machines. For the FSPMs, the good fault-tolerant capability requires high manufacturing technology to assemble the combined structure of the modular stator cores and PMs. Meanwhile, some FSPMs with odd-number of rotor teeth will lead to significant magnetic asymmetry leading to radial forces which would cause various eccentricity faults.

In references [19–23], the spoke-type PM layout rotor, Figure 1.5, was adopted to



Figure 1.4: Cross section of a FSPM machine.



Figure 1.5: Cross section of a spoke-type PM machine.



Figure 1.6: Cross section of a novel spoke-type rotor in reference [23].

generate the concentrated effect of the air-gap flux, in which ferrite magnets were used to reduce the material cost. A 9-slot, 6- pole, spoke-type PM machine was designed as a brushless dc (BLDC) motor and compared with a prototype IPM machine in [22]. A 12-slot 10-pole spoke-type ferrite magnet machine with a novel rotor structure, Figure 1.6, was proposed and tested in [23], in which this motor was utilized in a traction application for low-speed Electric Vehicles (EV).

Based on the literature search on motor configurations provided above, in this dissertation the fault-tolerant topology study will focus on the stator geometry with FSCWs, proper selection of stator slots and rotor poles, and interior as well as spoke-type PM layouts in such rotors.

1.2.2 Modeling and Analysis of Electric Machines

There are two main branches in the modeling and analysis of electric machines including numerical and analytical methods. As an effective and powerful tool, the finite element analysis (FEA) and finite differences methods are the most frequently used numerical methods for analyzing magnetic field problems. An evaluation of these two numerical methods is provided in [24], from which FEA was found to be superior in improved accuracy, computer time and storage requirements, as well as programming flexibility (gridding) and implementation aspects. The FEA method possesses not only high accuracy but also general applicability for materials with non-linear magnetic characteristics and for magnetic circuits with complex geometric boundaries. This method has been used in the analysis of induction machines in [25, 26], PM machines in [27–29], and synchronous generators in [30]. In these references, two dimensional (2D) FEA methods were implemented. A more accurate magnetic field computation method is the three dimensional (3D) FEA method, which was investigated in [31–33]. Both 2D and 3D FEA methods require good meshing layouts in such machine models. More dense mesh elements lead to longer simulation times. Thus, these long simulation times associated with the FEA method, render such a method not practical for direct application in design optimization problems, when combined with population-based optimization techniques.

Analytical solutions can clearly express physical principles and be conveniently

used in machine design work. The most commonly used analytical solution is the magnetic equivalent circuit (MEC) modeling method, also known as permeance network models, as presented in [34–40]. In [34], the fundamentals of the MEC method and the corresponding application to the computation of induction machine dynamics have been presented. The simplifications associated with such a method were investigated and presented in [35]. In [36], the MEC method was utilized in the modeling and analysis of a field regulated reluctance machine. In [37], the MEC modeling method was used to simulate squirrel-cage rotor faults in a 5 hp induction machine. In the same reference, the accuracy of this modeling method was also validated by comparison to results obtained from a time-stepping FEA (TS-FEA) method. The MEC method was also implemented in the analysis of a line start PM machine in [38], a surface-mounted PM (SPM) machine in [39], and a synchronous machine in [40]. Although the MEC has fast computational speed and reasonable accuracy, the analytical expressions of such a method can be only obtained while the geometry of the field region is simple and the materials involved have linear (linearized) characteristics. For PM machines with complicated PM layouts, the accuracy of this MEC method might not be sufficiently satisfactory in the estimate of the performance characteristics of such machines.

In order to overcome the drawbacks of the analytical solution (MEC method), combined numerical-analytical methods were investigated and presented in [41–43]. Even though such combined modeling and analysis methods have led to improved accuracy, the corresponding computational speeds were not sufficiently fast for utilization in population-based optimization problems.

Based on this concern, efficient FE modeling methods have been reviewed by Sizov, et. al., in [44]. Efficient FE analysis techniques reviewed in this reference aim to minimize the computational effort required to obtain the maximum possible information about the performance of a device being modeled, through the least number of FE solutions. Recently, these authors have proposed a technique for Computationally Efficient-Finite Element Analysis (CE-FEA) in [45–47]. The method uses only a reduced set of magnetostatic field solutions in order to satisfactorily estimate sinewave current regulated BLPM motor performance. The accuracy of such a method has been validated by comparison to the TS-FEA method and experimental test results in [47]. Meanwhile, significant reduction of simulation times was also achieved, which led to corresponding applications of such a method to large design spaces for machine design optimization purposes [48, 49].

1.2.3 Design Optimization Methods

By and large, the design optimization of electric machines inherently has multiple design objectives that need to be achieved. Also in general, the objective functions are non-differentiable, non-continuous, and have multiple constraints. There are two main methods used in the design optimization of electric machines. One is the Design of Experiments (DOE) techniques combined with Response Surface Methodology (RSM) [50], and another is the population-based evolutionary algorithms [51]. The first method is more suitable for local design optimization problems with a limited number of geometric design variables. The latter one is more adaptable for global design optimization of electric machines with a significant number of geometric design variables.

In reference [52], direct and stochastic search algorithms for both single- and multi- objective design optimization problems were discussed. Meanwhile, in the same reference benchmark studies of comparing RSM and Differential Evolution (DE) were investigated. This paper shows that the DE approach is more effective than RSM when more parameters (design variables) need to be optimized.

The method of DOE was first developed and applied by Ronald A. Fisher in 1919 [50], which has the advantage of providing engineering insights for the interactions between design variables and design objectives. Furthermore, this method points out the sensitive ranges of all design variables for each design objective. The RSM approach [53], used as a data processing procedure in DOE methods, is widely used in many applications in the industrial world, particularly in situations where several input variables potentially influence some measured performance or quality characteristics of a product. This approach was first introduced by G. E. P. Box and K. B.

Wilson in 1951 [53].

One DOE method [54], the Full Factorial Design (FFD) technique, was employed to establish the design variable and objective space for applying RSM for the design optimization of slotless-type PM linear synchronous machines. In this reference, only three geometric variables were chosen to pursue the highest thrust and the lowest thrust ripples. The same approach was implemented for the design optimization of other types of PM machines, such as double-layer IPM motors in reference [55].

The Central Composite Design (CCD) technique is another DOE method, which can be implemented to estimate a second-degree polynomial model. This method was utilized in the design optimization of a PM reluctance motor (PRM) in [56], IPM machines with concentrated windings in [57], and SPM machines in [58]. In these referenced papers, only three to five independent design variables were considered for the DOE studies.

DOE combined with RSM approach was also useful in the design optimization of induction machines as presented in [59, 60]. In these references, the Box-Behnken experimental design was chosen over the common CCD method because of two factors. First, Box-Behnken requires fewer samples (costly simulations) versus the CCD method. Second, the sample points of the independent variables are not located at the extreme of their ranges. This can improve a model's robustness. Another application of the Stochastic Response Surface Methodology (SRSM) was presented in [61], in which the SRSM approach was implemented to analyze the manufacturing tolerance for a BLDC SPM machine.

Stochastic evolutionary optimization methods include Genetic Algorithms (GAs) [62], Evolutionary Programming (EP), Evolutionary Strategies (ES), Genetic Programming (GP), Differential Evolution (DE) [63], as well as the swarm intelligence algorithms [51]. The swarm intelligence algorithms also contain Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Bees algorithm, Bacterial Foraging Optimization (BFO), and so on. For electric machine design optimization, the DE, PSO and GA are the most popular choices, as described for example in [48, 49, 64– 72]. Comparisons between DE and other optimization algorithms have been reported in [63], and for electric machine optimization problems in a recent study [52]. The results show that although there is no guarantee that DE is the fastest method, it is nevertheless the one that typically yields the best results.

In [68], Barcaro *et. al.* utilized the GA method in the design optimization of a three-layer IPM machine for a high performance drive. A multi-objective is considered in the optimization process including the torque density and the sensorless detection capability. Meanwhile, the total losses in the motor and the minimum operating point in the PMs were set up as constraints. The GA method was also used in [64], in which Pellegrino and Cupertino focused on the rotor design of a three-layer IPM machine to achieve the optimized machine with maximum torque, minimum torque ripple, maximum flux weakening capability, and minimum rotor harmonic losses.

In reference [70], Arkadan *et. al.* implemented the PSO method in the design optimization of an axially laminated anisotropic (ALA) rotor synchronous reluctance machine (synRM). The objective of this optimization is to maximize the developed torque while minimizing torque ripples and the copper and core losses for traction applications.

The PSO method and GA method were compared by Duan *et. al.* in [73]. This reference used the design of a 15 kW SPM machine with an analytical model as a benchmark and compared the performance of PSO and GA in terms of their accuracy, the robustness to population size and algorithm coefficients. In the design optimization procedure, single weighted design optimization method was set up for the machine volume, weight, efficiency, weight of PM and the torque per Ampere as optimization variables. The results show that PSO has advantages over GA with regard to these optimization variables and is preferred over GA when computation time is a limiting factor.

The DE approach, which is another popular optimization method for machine design problems, has been utilized in several publications [65–67, 74]. In [66], Zarko *et. al.* implemented the FEA and DE method for minimizing the rotor inertia in the design optimization of a servo motor. In this reference, the cavity area in the rotor was maximized, and the PM dimension was also calculated to fit four different
International Electrotechnical Commission (IEC) frame sizes of servo motors while occupying the maximum space on the rotor circumference. In [67], two benchmark studies on the design optimization of PM machines were performed by Ouyang *et. al.*. The first case was regarding optimizing one single weighted objective function of maximum average torque, and minimum torque ripple, with constant air gap for IPM machines with a modular stator. In the second case, the rotor of an IPM machine with a conventional stator was design optimized to achieve the maximum average torque and maximum flux weakening capability (maximum normalized characteristic current). Here, the maximum normalized characteristic current is defined as follows:

$$i_{nc} = \frac{\lambda_{pm} - \lambda_d}{L_d} \tag{1.2.1}$$

where, λ_{pm} is the flux linkage of the PM, and λ_d is the flux linkage along the d-axis due to the armature current, while L_d is the d-axis inductance.

In reference [65], a cloud computing technique was implemented in the design optimization of PM machines utilizing FEA and DE algorithms. In this reference, only one objective, the torque density, was optimized for a 30 kW, 12-slot, 10-pole, SPM machine with a FSCW.

The DE algorithm was integrated into the Computationally Efficient FEA (CE-FEA) method by Sizov *et. al.* in [47, 48, 74]. The detailed explanation of the principle and implementation of the CE-FEA was presented in [46, 47]. In [48], the torque ripple was minimized and the "goodness", a measure of average torque production

with respect to total losses, was maximized for a 9-slot, 6-pole, IPM machine. In this reference, the decision-making by the assistance of a Pareto-set was described. The same procedure was implemented in multi-MW direct drive PM machines in [74], in which a reasonable systematic comparison between the fractional slot (FS) SPM, FS IPM, integer slot (IS) SPM and IS IPM were presented.

Several publications proposed combined design optimization methods using both of the statistical and population-based evolution algorithms [72, 75, 76]. In [75], the DOE method was combined with GA algorithm by Jolly *et. al.* to maximize the constant power speed range (CPSR) for a 36-slot, 4-pole IPM machine with several hundred watts. In this reference, the DOE and RSM methods were used to obtain the response surface (polynomial function) of the design objective, which was implemented in the GA algorithm instead of performing FEA for all designs. The same principle was implemented by Hasanien *et. al.* in the design optimization of PM-type Transverse Flux Linear Machines (TFLM) in [72]. In [76], Hasanien combined the DOE and PSO design optimization methods to reduce the machine's weight, maximize the thrust and minimize the detent forces of the TFLM. Here, the detent force is analogous to the cogging torque of a rotating PM machines.

Based on this literature search regarding the design optimization methods for electric machines, a new combined DOE and DE method is proposed to be utilized in conjunction with the CE-FEA method in the design optimization of fault-tolerant PM machines.

1.3 Statement of the Problem

For the investigation on topologies of fault-tolerant PM machines, the FSCWs, optimal combination of stator slots and rotor poles, as well as the interior and spoke-type PM layouts in rotors will in investigated in this dissertation. Based on these investigations, a 12-slot, 10-pole, IPM machine with a V-type PM layout will be optimally designed and built for the experimental calibration.

For PM machines with FSCWs, the eddy current losses in PMs can be especially significant because of a rich content of magnetic motive force (mmf) harmonics. In order to take account of these losses, a hybrid method must be investigated, which needs to combine the CE-FEA method with a new analytical formulation for the eddy current losses in PMs of such machines. The 3D end effects and the effect of PWM switching harmonics will be incorporated in such an analytical calculation. The accuracy of this method will be validated by comparison to the results obtained from 2D and 3D TS-FEA.

Before embarking on the design optimization process, the material properties of steel laminations, permanent magnets and copper will be described in this dissertation. Meanwhile, the FEA parametric models of different stator and rotor topologies will be included.

A combined design optimization method of DOE and DE will be developed and implemented in optimization problems of PM machines with fault-tolerant capabilities. In this combined design optimization method, the DOE approach will be used to perform the sensitivity studies of geometric variables for multiple design objectives. This procedure will be useful for designers to select geometric variables with significant effects on objectives, and also choose reasonable ranges for each geometric variable. In a following step, the DE algorithm will be utilized to perform the population-based optimization procedure. This developed design optimization procedure must be combined with the CE-FEA techniques to improve the computational speed.

By utilizing the combined design optimization method and CE-FEA approach, Pareto-fronts of PM machines with different geometric topologies can be obtained and compared. This will provide more systematic comparison between different types of PM machines. In essence, a fast and automated design optimization method will be developed in this dissertation research, for optimization of the design of fault-tolerant PM machines with FSCWs and interior as well as spoke-type PM layouts.

1.4 Dissertation Organization

In view of the problem background and the literature search, several fault-tolerant topologies for PM machines are discussed in Chapter 2, including the IPM machines and spoke-type ferrite magnet machines with fractional-slot concentrated windings. In Chapter 3, a new hybrid calculation method of the eddy-current losses in PMs is proposed for the CE-FEA method and the accuracy was validated by two case studies. In order to improve the stability of the design optimization procedure, the robust FEA parametric method for the geometry model is provided in Chapter 4. In Chapter 5, the combined design optimization method utilizing DOE and DE method is described and implemented for the design optimization of IPM machines with different rotor topologies. In Chapter 6, the implementation of the CE-FEA method and automated design optimization method are presented. In Chapter 7, this method is calibrated through a case study and the accuracy is validated by the experimental results. The conclusions, contributions and future works are discussed in Chapter 8.

CHAPTER 2

FAULT-TOLERANT PERMANENT MAGNET MACHINES

In this chapter, based on the fault-tolerant requirements for PM machines, stator winding layouts are investigated first. Then, a discussion is conducted for the selection of numbers of stator slots and rotor poles. At last, the various rotor PM layouts are compared. Finally, the V-type and the spoke-type PM layouts are adopted. This is in order to reduce the losses in the rotor portion of the magnetic circuits of this type of machines. This also leads to an increase in such rotors' thermal dissipation capability.

2.1 Introduction

Several important design principles for fault-tolerant PM machines were summarized by Mecrow *et. al.* in [3] and Mitcham *et. al.* in [8], from which two points are emphasized as follows:

• Stator windings of the PM machines are wound around alternate stator teeth with the concentrated winding layout, so that the coils of different phases are physically separated.



Figure 2.1: Different types of stator winding failures.

• The number of rotor poles must be close to the number of stator slots. Typically, the number of slots per pole should be in the range between 0.7-1.5. Meanwhile, proper selection of the number of stator slots and rotor poles can eliminate or reduce the magnetic coupling between different phases.

For electric machines, failures occur most often in stator windings, which constitute about 35%-37% of machine faults [77]. Different types of stator winding failures are shown in Figure 2.1, which include the turn-to-turn fault, coil-to-coil fault, open circuit fault, line-to-line fault, and line-to-ground fault. Among these faults, the first likely fault is the turn-to-turn short-circuit, which is usually due to insulation failures in several turns of a stator coil within one phase. When this type of fault happens, excessive heat in the shorted turns can be generated due to resulting large





circulating currents, which can develop rapidly into catastrophic failures. Therefore, properly choosing stator winding layouts can dramatically improve such electric machines' fault tolerant capabilities by providing strong magnetic, thermal and physical isolations.

2.2 Stator Windings With Fault-Tolerant Capabilities

The distribution of stator windings in an ac machine has a significant impact on the machine's performance characteristics. There are two main types of stator windings, distributed windings and concentrated windings, as shown in Figure 2.2.

A distributed winding generally results in a more sinusoidal MMF distribution, which makes it very popular in applications of PM brushless ac (BLAC) machines. However, because of the manufacturing limitation, the slot fill factor is generally low, which is around 35%-45% [6]. This means that over half of the slot is a combination of insulation and non-magnetic filler. This low slot fill factor has a significant effect on limiting the maximum torque and power densities that can be achieved with these types ac machines. Meanwhile, the long end-winding connection, which can be discerned in Figure 2.2 (a), causes large copper losses, which can lead to thermal issues in these ac machines.

Once again, for fault-tolerant electric machines, the stator windings are required to have good electrical, magnetic, and physical isolations between phases to prevent turnto-turn shorts from cascading into the occurrence of catastrophic winding failures. Based on these concerns, concentrated types of windings are more popular in the construction of fault-tolerant machines. The winding layout shown in Figure 2.2 (b), in which each coil surrounds only a single stator tooth, constitutes such a fault tolerant design. This type of stator winding has a reduced volume of copper used in the end-winding connections. This leads to some reduction in the stator copper losses. For this type of concentrated winding, the slot fill factor can be increased to 50% to 65%, if coupled with segmented stator structures [6], which can increase such machines' power density and torque density. This type of stator winding also eliminates the overlap between coils in the end winding region, which can reduce the chance for short circuit faults between different phases. Thus, PM machines with



Figure 2.3: Two different concentrated windings for 12-slot 10-pole PM machines.

FSCWs were investigated in this work. One of the key challenges of PM machines with FSCWs is the significantly increased rotor core loss, PM loss and sleeve loss in case of conductive magnet retainment sleeves. This is due to various mmf and airgap flux harmonic components inherent in such waveforms associated with these winding configurations. The calculation method for such PM losses caused in such PM machines with FSCWs will be discussed in Chapter 3.

There are two types of concentrated windings, which are alternate-teeth-wound and all-teeth-wound concentrated windings, as shown in parts (a) and (b) of Figures 2.3 and 2.4, respectively. Here, Figure 2.3 shows the winding layout for a 12-slot, 10-pole, PM machine, and Figure 2.4 is for a 12-slot, 8-pole, PM machine. These two slot and pole combinations are popular for low-rating three-phase PM machines with



Figure 2.4: Two different concentrated windings for 12-slot 8-pole PM machines.

fault-tolerant capabilities.

Compared with all-teeth-wound concentrated windings, the alternate-teeth-wound layout has more fault-tolerant capabilities because of the reduced winding contact in each stator slot between the various phases. Yet, this type of concentrated winding might lead to lower fundamental frequency winding factor and higher harmonic components in the back-emf waveforms for certain combinations of stator slots and rotor poles. For example, a typical back-emf waveform for a 12-slot, 10-pole, PM machine is shown in Figure 2.5 (a), and the corresponding harmonic breakdown/spectral analysis is given in Figure 2.5 (b). The harmonics with the order of multiple of three can be ignored because they are eliminated from the line-to-line voltage waveforms due to the Y-connection of the three-phase stator windings in such machines. For the 12-slot 10-pole combination, the alternate-teeth-wound stator winding provides more 5^{th} and 7^{th} order harmonics than the all-teeth-wound winding layout. However, this is not always the case. Different phenomenon was observed from the 12-slot, 8-pole, PM machines with alternate-teeth-wound and all-teeth-wound stator windings, for which the back-emf waveforms and harmonic analysis are shown in Figure 2.6 (a) and (b), respectively. Both of the alternate-teeth-wound and all-teeth-wound stator windings provide the same phase back-emf waveforms. Compared with the 12-slot 10-pole combination, the 12-slot 8-pole combination has higher 5^{th} and 7^{th} order harmonics and lower 11^{th} and 13^{th} order harmonics.

2.3 Different Combinations of Stator Slots and Rotor Poles

A more effective means of limiting inter-phase coupling is by the precise choice of the numbers of stator slots and rotor poles. Two popular slot and pole combinations for lower-rating three-phase PM machines with fault-tolerant capabilities are the 12slot 10-pole and 12-slot 8-pole topologies, which are shown in Figures 2.3 and 2.4, respectively.

In order to observe the magnetic coupling between the three phases, a finite element (FE) analysis was performed on a 12-slot, 10-pole, PM machine with two



Figure 2.5: Phase back-emfs and harmonics of 12-slot 10-pole PM machines with two different concentrated windings.



Figure 2.6: Phase back-emfs and harmonics of 12-slot 8-pole PM machines with two different concentrated windings.



Figure 2.7: Flux plot for 12-slot 10-pole PM machines with single phase excitation and un-magnetized magnets.

different stator winding layouts, for which only a single phase was excited and the PMs were not magnetized. The corresponding flux plots are shown in Figure 2.7. From this Figure, one can observe that the flux generated from phase-A only links (goes through) the teeth surrounded by the coils for phase-A. This means that there is no magnetic/flux coupling between the three phases for such 12-slot, 10-pole, PM machines with either one of the alternate-teeth-wound and all-teeth-wound stator windings.

The same type of FE analysis were repeated for a 12-slot, 8-pole, PM machine with alternate-teeth-wound and all-teeth-wound stator windings, and the corresponding flux plots are shown in Figure 2.8. One can observe that the flux established by the



Figure 2.8: Flux plot for 12-slot 8-pole PM machines with single phase excitation and un-magnetized magnets.

excitation of phase-A flows through all the stator teeth, which leads to a substantial magnetic coupling between the three phases of such a 12-slot, 8-pole, PM machine. Based on these observations and the back-emf comparison in the previous section, the 12-slot, 10-pole, PM machine with all-teeth-wound stator windings is selected to be the focus point of this work. Meanwhile, insulation material can be added between the two coil sides in one slot to reduce the physical contacts.

2.4 PM Layouts in the Rotor

For PM machines, there are five popular PM layouts in the rotors of such machines as shown in Figure 2.9, which includes SPM, IPM, PRM, permanent-magnet assisted



Figure 2.9: Rotor layouts for PM machines.



Figure 2.10: Torque and speed curves of PM machines. PRM: permanent-magnet reluctance machine; PMa-SynRM: permanent magnet assisted-synchronous reluctance machine.

synchronous reluctance machine (PMa-SynRM) and spoke-type PM machine. Their typical torque and speed capability profiles are shown in Figure 2.10, in which five types of PM machines are assumed to have the same type of stator winding layout. The operation speed range is significantly affected by the flux weakening capabilities of these designs, which are positively affected by the saliency ratio of inductances (L_q/L_d) of such PM machines. The saliency ratio of PM machines can be varied by changing the stator winding and PM layouts. For SPM machines, the saliency ration is a little larger than one, because of the almost equal reluctances along the d- and q-axes magnetic circuits. In the case of IPM and spoke-type layouts in the rotor, the saliency ratio is lower than two for machines with FSCWs, and is around the value of three for machines with distributed windings. For PRMs and PMa-SynRMs, the saliency ratio can achieve a value as high as five with effective optimization procedures,



Figure 2.11: Rotor layouts for IPM machines.

which leads to a larger speed range than the SPM, IPM and spoke-type PM machines.

Conventional SPM machines have advantages of simple control schemes and simple rotor structures, thus reducing the number of design variables [78–80]. However, this type of PM machine also has poor flux-weakening ability, which limits the speed range of such a machine as shown in Figure 2.10. Meanwhile, it is hard to hold such PMs in place when the machine runs at high speed without complex PM retaining structures. Besides these disadvantages, large air-gaps in SPM machines lead to very small synchronous inductances, which may not limit short-circuit currents in the event that winding shorts take place. Thus, such SPM machines are not suitable if fault-tolerant PM machine designs are required.

Two of the most popular IPM topologies are the flat bar-type and the V-type PM mounting configurations, which are shown in Figures 2.11 (a) and (b), respectively. In principle, in comparison with the flat bar-type PM layout, the V-type can have

higher saliency ratio and lower PM losses, making it more suitable for high speed and flux-weakening constant-power operation. The V-type IPM is in fact a common choice for hybrid and electric vehicle applications.

Spoke-type PM machines are known for their inherent flux concentration capability, as the flux per pole is contributed by two adjacent PMs, which are radially located and tangentially magnetized [19]. In principle, a spoke-type PM machine can achieve very high flux densities in the air-gap yielding increased specific power output. This capability allows the rare earth magnet material to be potentially replaced by ferrite magnets to achieve a competitive performance and reduce the material cost of this type of electric machines.

The PRMs, as presented in [81], and the PMa-SynRMs [82] have the most advantage of high saliency ratios, which leads to a wide speed range for these types of PM machines, as shown in Figure 2.10. This property makes them very popular in the applications of EVs, railway systems and elevator systems. However, the complicated rotor geometries bring out considerable difficulties for the design optimization of these two types of PM machines. Thus, they will not be investigated in this dissertation, and will be left for future work.

2.5 Summary

Summing up the investigations provided above, the 12-slot, 10-pole, PM machines with V-type and spoke-type PM layouts are selected as the candidate topologies for the fault-tolerant PM machine design to be investigated and optimized here in this work. These motors can be operated by sine-wave current regulated vector controlled power electronic drives, which are commonly referred to as PM synchronous or sinewave machines. Because of the importance of the losses in the PMs of such machines to the overall efficiency of PM motor-drive systems, the computation of such losses is the subject of the next chapter.

CHAPTER 3

COMPUTATION METHOD FOR PERMANENT MAGNET EDDY-CURRENT LOSSES

In this chapter, a hybrid method combines CE-FEA with a new analytical formulation for the computation of eddy-current losses in the PMs of sine-wave current regulated brushless synchronous machines. The CE-FEA only employs a reduced set of magnetostatic solutions yielding substantial reductions in the computational time as compared with conventional time-stepping FEA (TS-FEA). The 3D end effects and the effect of PWM switching harmonics were incorporated in the analytical calculations. The algorithms were applied to two fractional-slot concentrated-winding IPM machines with different circumferential and axial PM block segmentation arrangements. The method was validated versus 2D and 3D TS-FEA.

3.1 Introduction

The latest generations of BLPM sine-wave motors employ rare earth PMs, such as neodymium iron boron (NdFeB) magnets, which are electrically conductive and therefore prone to eddy-current losses. The satisfactory estimation of PM losses is very important not only for optimizing the design of high-efficiency motors, but also for the growing number of machines dedicated to fault-tolerant applications, in which local losses and heating are of particular concern. The PM losses can be especially significant in BLPM motors that have a rich content of mmf harmonics. This is the case for FSCW topologies, which in turn are recommended due to their potential benefits for lower material cost at specified performance and enhanced fault handling capability. Two such IPM machines serve as case studies in this chapter.

Calculation of rotor losses has been a common theme for different types of electric machines, e.g., [83–90]. In reference [83], Demerdash and Nehl presented the application of the concept of effective permeability using the FEA method to the calculation of eddy current problems in solid iron rotors of a turbogenerator. In reference [84], Krawczyk and Tegopoulos presented the methodology explanation and applications of the numerical analysis of eddy current problems.

It is well known that TS-FEA numerical solutions have, in principle, the advantage of high accuracy. However, this method's applicability, especially for optimization studies involving many candidate designs, is still limited due to the prohibitive requirements for computational resources. Thus, analytical techniques are often preferred for predicting the rotor PM losses at the design stage [85–88]. In reference [85], Deng has presented the analytical models for predicting the eddy-current losses in rotors due to the field variations caused by current commutation in the stator windings of such PM machines. In this reference, the stator current was assumed constant during periods between switching. The same method was improved by Deng and Nehl [86] to predict the effects of inverter high frequency PWM switching on eddy-current losses in a BLDC PM machine (an SPM machine). The accuracy of such an analytical approach presented in these two references was verified by the FEA computation. In reference [87], Atallah et. al. developed an analytical model to predict rotor-induced eddy currents in SPM machines, and to quantify the effectiveness of circumferentially segmenting the PMs in reducing the rotor losses. In reference [88], an improved analytical model was developed by Zhu *et. al.* to calculate the eddy-current losses in both the PMs and the retaining sleeve of an SPM machine in a traction application.

Such an analytical method is more suitable for SPM machines because of its corresponding simple rotor geometry. For interior and spoke-type PM machines, complicated rotor geometries bring about the challenges of the application of such an analytical method to the calculation of PM eddy current losses. The presence of 3D end effects further complicates the problems and hence hybrid analytical combined with FEA algorithms have been proposed by Yamazaki *et. al.* [91, 92]. The method introduced in this chapter is of the combined solution type, which is of particular interest as it leads to a satisfactory trade-off between accuracy and computational speed.

Recently, simplified and fast FEA techniques, such as the CE-FEA [45–47], have been coupled to large-scale design optimization procedures. The method uses only a reduced set of magnetostatic field solutions in order to satisfactorily estimate sinewave current regulated BLPM motor performance. The satisfactory accuracy of the this CE-FEA method, for calculating the torque profiles (including cogging torque), waveforms of induced voltages, and stator core losses, has been demonstrated in previous publications [47–49]. The work in this chapter brings further significant contributions that enable the calculation of PM eddy-current losses based on magnetic FEA solutions and on a theoretical development that includes the 3D end effects. The PWM switching losses in the PMs are also quantified, together with the effect of various PM block segmentation techniques, on two IPM example machines of the 12-slot 10-pole and 12-slot 8-pole type, respectively.

3.2 Electromagnetic Field Analysis Using CE-FEA

During steady-state operation, the rotor moves synchronously with the rotating airgap magnetic field. This rotation takes place in the presence of stator slots, discrete distribution of the windings, and time harmonics present in the phase currents due to PWM type power supplies. This causes a time-domain variation in the PM flux density under steady state conditions, that can be expressed as follows:

$$B(t) = B_0 + \sum_k B_k \cos(k\omega_1 t + \varphi_k) , \qquad (3.2.1)$$

where, B_k and φ_k , are the peak magnitude/amplitude and phase angle corresponding to the harmonic of order, k, and B_0 , is the dc component. Note that because the above Fourier series is expressed in term of the fundamental angular frequency of the ac stator current, ω_1 , the order of the rotor field harmonics, k, can be, in principle, a non-integer number as explained later.

The traditional approach for calculating the PM flux density waveform employs a time-consuming time-stepping transient FEA with a small time sampling/time step. The alternative approach proposed in this work builds upon the CE-FEA method, which was previously introduced with particular emphasis on the distribution of the magnetic field in the stators of brushless PM machines operated from sine-wave current regulated drives [45–47]. In that case, the CE-FEA can fully exploit both the electric and magnetic symmetries existent at the winding layout and slot pitch levels.

For the rotor field, the periodicity is identified at the pole pitch level, and under the eddy-current related assumptions specified in the next section, a relatively small number of magnetostatic solutions, together with a space-time transformation, are employed to "construct" (calculate) the PM flux density waveforms, the nature of the function of which can be expressed generically as follows:

$$B(r,\theta,t) = B\left(r,\theta + m\theta_p, t + \frac{m\theta_p}{\omega_1}\right) , \qquad (3.2.2)$$

where, r, is the radial position, and θ , is the electrical angular space position of a point within the rotor. Here, θ_p is the electrical pole-pitch, and m is an integer. By utilizing the CE-FEA approach, the computational effort is substantially reduced and the calculation speed is increased, as compared to obtaining equivalent results from the TS-FEA approach.

In principle, the application of the CE-FEA approach, with s magnetostatic solutions for a rotor field domain that includes n_p poles provides n solution points, where,

$$n = s \times n_p + 1 , \qquad (3.2.3)$$

That is, there are n points/samples on the rotor flux density waveform. The maximum harmonic order that can be used in this Fourier analysis is determined by the Nyquist criterion. In order to avoid any aliasing effects, this number should be higher than the order of any rotor harmonic that is expected to have a significant magnitude.

3.3 Eddy-Current Losses in Permanent Magnets

Rare earth PMs, such as those of the NdFeB type, are electrically conductive, and hence variations of the magnetic field with time produce eddy currents. In order to minimize these currents, a typical engineering approach is to "segment" the PMs, i.e. to employ multiple individual PM blocks both in the rotor axial direction as well as in the circumferential direction. The expectation is that the power losses in PMs will be minimized and that the eddy current effect will rather be resistance limited, such that these eddy currents will not change the original magnetic field distribution, which would be present in the machine should there be no eddy currents.

In order to reduce the eddy-current losses, it is also recommended to select the thickness of the PM blocks, h, along the magnetization direction to be smaller than the skin depth corresponding to the frequency of the highest order field harmonic that is expected to have a significant magnitude. This harmonic is typically generated by the PWM switching frequency, and further details regarding this topic are presented in reference [93].

The skin depth for a frequency, f, can be calculated as:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu_0 \mu_r}} \tag{3.3.1}$$

where, in the following case studies one assumes a typical constant value for the relative permeability, μ_r , of 1.05, and a PM resistivity, ρ , of 1.5×10^{-7} m/S, at an



Figure 3.1: Variation of skin depth with frequency for a typical NdFeB permanent magnet (PM).

operating temperature of 100°C, which yields the dependency plotted in Figure 3.1.

The method implemented in this chapter is based on the assumption, as it is generally the case in industrial practice, that the eddy current effect is resistance limited through the employment of adequate engineering design solutions, such as the aforementioned PM segmentation, which could be based on laborious computational methods [91, 92], or more often, based on practical experience. Other typical assumptions employed are that the PM material is isotropic and that there is no variation of the electromagnetic field in the axial z-direction (axial symmetry prevails).

The eddy current and flux density distributions in a PM are demonstrated in Figures 3.2 (a) and (b), respectively, and the corresponding eddy-current circulating loops are illustrated with dotted lines in Figure 3.3. In this figure, portions (a) and (b) represent the 3D and 2D view of a PM block, respectively. For the initial explanation,



Figure 3.2: Example electro-magnetic field distribution in a PM calculated by 3D FEA.



Figure 3.3: Schematic eddy-current loops in a rectangular PM block.

assuming that the magnetic field is uniformly distributed in space, a filamentary loop in the x - z plane, which is perpendicular to the PM direction of magnetization, extends along the y-direction to the full extent of the magnet thickness, h.

The variable for the axial direction, z, is not independent and can be expressed as a function of the PM width, w, height, h, and axial length, ℓ , as well as of the x-location, which is expressed as follows:

$$z(x) = \frac{\ell}{2} - \left(\frac{w}{2} - x\right)k_z = z_0 + k_z x , \qquad (3.3.2)$$

where,

$$k_z = \tan \alpha$$
, and $z_0 = \ell/2 - wk_z/2.$ (3.3.3)

Here, the magnetic flux through an eddy-current loop can be expressed as follows:

$$\phi(x,t) = B(t) \left[2x \cdot 2z(x) \right] = 4B(t) \left[z_0 x + k_z x^2 \right] . \tag{3.3.4}$$

Hence, the induced voltage in the eddy-current circulating loop is calculated from Faraday's law as shown in the following expression:

$$E(x,t) = -\frac{d\phi(x,t)}{dt} = -4\frac{dB(t)}{dt} \left[z_0 x + k_z x^2 \right] .$$
 (3.3.5)

The differential resistance of the eddy-current loop can be determined using the following expression:

$$dR(x) = \frac{4\left[k_e x + z(x)\right]\rho}{hdx} = \frac{4\rho}{h} \frac{\left(k_e + k_z\right)x + z_0}{dx} , \qquad (3.3.6)$$

where, k_e , is a coefficient with an original value equal to 1, which can be adjusted to correct for end effects. Here, $4[k_ex + z(x)]$ and [hdx] are the length and cross section area of each eddy-current circulating loop (the blue dash line in Figure 3.3). For example, if the PM is very long in comparison with the width, the angle α can be assumed to be zero, and in such a case, k_z as well as k_e , are equal to zero. Hence, the end effect contribution to the resistance is neglected and the resistance calculation is simplified to the following expression:

$$dR(x) = \frac{2\ell\rho}{hdx} . \tag{3.3.7}$$

Although this approach for modeling end effects is mostly based on geometry rather than physics, it is very useful as it enables, on one hand, the implementation of a means for calibrating, if necessary, the analytical results against other data provided by experiments or 3D-FEA. On the other hand, when the resistive end effects are neglected, the results can be compared against (quasi) 2D-FEA, which implicitly considers an ideal short circuit at the two axial ends of the eddy-current loop.

The power loss associated with one eddy current loop having the end effect resistance incorporated through (3.3.6) is equal to:

$$\frac{E^2(x,t)}{dR(x)} = \left(\frac{dB(t)}{dt}\right)^2 \frac{4\rho}{h} \frac{(z_0 x + k_z x^2)^2}{(k_e + k_z)x + z_0} dx .$$
(3.3.8)

Integrating over the entire PM block yields the total eddy-current loss, which is

deduced as follows:

$$P_{PM}(t) = \int_{0}^{w/2} \left(\frac{dB(t)}{dt}\right)^{2} \frac{4\rho}{h} \frac{(z_{0}x + k_{z}x^{2})^{2}}{(k_{e} + k_{z})x + z_{0}} dx$$

$$= \left(\frac{dB(t)}{dt}\right)^{2} \frac{4hk_{z}^{2}}{\rho(k_{e} + k_{z})} \left[\frac{w^{4}}{64} + \frac{(k_{z} + 2)z_{0}w^{3}}{24k_{z}(k_{e} + k_{z})} + \frac{z_{0}^{2}w^{2}}{8k_{z}^{2}(k_{e} + k_{z})^{2}} - \frac{z_{0}^{3}w}{2k_{z}^{2}(k_{e} + k_{z})^{3}} + \frac{z_{0}^{4}}{k_{z}^{2}(k_{e} + k_{z})^{4}} \ln \frac{w(k_{e} + k_{z}) + 2z_{0}}{2z_{0}}\right].$$
(3.3.9)

In the general case, the spatial distribution of the PM flux density is non-uniform. In principle, in order to increase the accuracy of loss calculation, the calculation of flux densities in one magnet block can be discretized in a computational grid with columns along the x-axis and rows along the y-axis. A 4 × 4 example grid is shown in Figure 3.4. The flux density within the grid is denoted by B_{ij} , where *i* and *j* are the indices for the row and column, respectively. The flux density in one PM block is still assumed to be independent of the location along the z-axis. That is, any B_{ij} has a single constant value derived from an average of a flux density distribution that varies with the *x* and *y* locations of each block.

In this case, the magnetic flux through a rectangular eddy current loop is provided by the following expression:

$$\phi_i(x,t) = \begin{cases} \left[B_{i2}(t) + B_{i3}(t)\right] 2xz(x) & \text{for } 0 \le |x| \le \frac{w}{4}, \\ \left[B_{i2}(t) + B_{i3}(t)\right] \frac{wz(x)}{2} + & (3.3.10) \\ \left[B_{i1}(t) + B_{i4}(t)\right] 2\left(x - \frac{w}{4}\right) z(x) & \text{for } \frac{w}{4} < |x| \le \frac{w}{2}. \end{cases}$$



Figure 3.4: Example computational grid used for calculating the magnetic flux in a PM.

The resistance of an eddy current loop considering the end effects is the same as that given by the expression (3.3.6), and the total eddy-current losses in the PM can be calculated as:

$$P_{PM}(t) = \sum_{i} \int_{0}^{w/2} \frac{h}{16\rho[(k_e + k_z)x + z_0]} \left(\frac{d\phi_i(x, t)}{dt}\right)^2 dx .$$
(3.3.11)

The resistive end effects can be ignored by setting $k_e = 0$ in equation (3.3.11).

3.4 Case Studies and Discussions

The methods previously presented were implemented using ANSYS electromagnetic FEA software [94]. The following results are provided for two case study IPM motor designs rated at 10 hp and 1800 r/min. The machines' stator magnetic circuit topologies are based on FSCW arrangements and a conventional internal flat bartype PM rotor topology. Results from the study of these two case study machines

Table 3.1: The number of PM blocks per pole in example segmentation schemes for a topology with one rotor slot per pole.

Segmentation scheme SEG1 SEG2 SEG3 SEG4	
Segmentation scheme SEG1 SEG2 SEG9 SEG4	-
Axial PM blocks1212	
Circumferential PM blocks 2 2 3 3	

are given next in sections 3.4.1 and 3.4.2. Such IPM designs are prone to relatively high PM losses due to the high harmonic content of the stator field and because of the proximity of the magnets to the air-gap. In order to minimize PM eddy-current losses, various segmentation arrangements with multiple PM blocks per rotor slot were considered, as specified in Table 3.1.

As a first step of the analysis, the FEA domain is modeled and the PMs are discretized for computational purposes in a uniform grid as shown in Figure 3.4. Secondly, the PM flux density waveforms are obtained using the CE-FEA approach and the results are analyzed both for harmonic content as well as for spatial variation. Finally, the PM eddy-current losses are calculated and compared with data obtained through 2D- and 3D-TS-FEA.

3.4.1 The First Case Study-an IPM Machine With 12 Slots and 10 Poles

For the 12-slot 10-pole IPM case study, the computational domain corresponding to the general electromagnetic periodicity comprises 5 poles as shown in Figure 3.5. For



Figure 3.5: Geometry of a 12-slot, 10-pole, IPM machine case study.

Table 3.2: Example harmonic spectrum of the flux density in the PMs of the 12-slot 10-pole IPM.

$f_1 = 150Hz$								
Frequency [Hz]	0	180	360	540	720	1080		
B[T]	0.870	0.045	0.094	0.003	0.013	0.003		

any point within a PM, a CE-FEA employing 7 magnetostatic solutions yields the discrete points shown on a flux density waveform in Figures 3.6 and 3.7. Using the CE-FEA techniques, the waveform corresponding to an entire time cycle is "constructed" (assembled) based on (3.2.2) and on the information provided by each individual pole, as illustrated in Figure 3.6 through the use of colored coded points and arrows.

In this case, there are 35 points on the resultant flux density waveform. That is, harmonics up to the 15-th order can be reasonably predicted (Table 3.2). The CE-FEA obtained waveform virtually overlaps the results obtained using the substantially more accurate and computationally more intensive conventional TS-FEA, see the


Figure 3.6: PM flux density waveform construction according to CE-FEA for the 12-slot 10-pole IPM motor case study.



Figure 3.7: PM flux density waveform at rated load operation calculated by CE-FEA and time-stepping (TS) 2D FEA.



Figure 3.8: Definition of one PM block's width, thickness and axial length.

black solid curve in Figure 3.7.

For reference purposes, the PM blocks employed in the SEG1 arrangement, see Table 3.1 and Figure 3.8, have a width of 18.44 mm, a thickness of 4.24 mm and an axial length of 83.15 mm. Accordingly, in the SEG2 arrangement, which uses two PM blocks per rotor length, the ratio of PM axial length per width is 2.257, and consequently the end effects are expected to be significant.

The spatial distribution of the flux density across the PM cross-section was obtained using a 4×4 grid as per Figure 3.4. In line with expectations for the example under consideration, the variation of both the flux density and its time derivative along the radial direction is slight. Meanwhile, more noticeable differences are observable along the circumferential direction as shown in Figures 3.9 and 3.10, respectively.

To evaluate the capabilities of the PM eddy current loss calculation method, even in its simpler formulation, only the average value of the flux density was considered for



Figure 3.9: Waveforms of flux densities at various points in a PM of the 12-slot 10-pole IPM with the SEG1 segmentation.



Figure 3.10: Waveforms of dB(t)/dt at various points in a PM of the 12-slot 10-pole IPM with the SEG1 segmentation.



Figure 3.11: Time variation of PM losses in the 12-slot 10-pole IPM with SEG1 (top graph) and SEG2 segmentations, respectively.



Figure 3.12: Time variation of PM losses in the 12-slot 10-pole IPM with SEG3 (top graph) and SEG4 segmentations, respectively.

Seg. type	2D-TS-FEA	3D-TS-FEA	CE-FEA without	CE-FEA with
			end effect	end effect
	[W]	[W]	[W]	[W]
SEG1	68.2	51.8	65.5	49.4
SEG2	68.2	44.6	65.5	45.0
SEG3	35.5	28.1	35.0	27.4
SEG4	35.5	26.2	35.0	26.2

Table 3.3: Average PM eddy-current losses for a 12-slot, 10-pole, IPM machine with different PM block segmentations.

each PM block, Figure 3.4, in conjunction with the loss calculation expression (3.3.9). The rated load results for the different PM segmentation arrangements of Table 3.1 are illustrated in Figures 3.11, 3.12 and summarized in Table 3.3. Reasonable engineering agreement is observed between the 3D-FEA and the new method at hand including the consideration of the end effects. Further validation is provided in Figures 3.11 and 3.12 as well as Table 3.3 through the correlation noted between the 2D-FEA results and the results of the new method at hand when the resistive end effects are neglected.

3.4.2 The Second Case Study-an IPM Machine With 12 Slots and 8 Poles

In the case of a 12-slot, 8-pole, IPM machine case study, the minimum domain required for FEA contains only two poles as shown in Figure 3.13. Similar to the



Figure 3.13: Geometry of the 12-slot 8-pole IPM motor case study.

previous case study, the CE-FEA model employed 7 magnetostatic solutions yielding in this case 15 points on a full cycle flux density waveform constructed under the steady-state space-time transformation and procedure schematically illustrated in Figures 3.14 and 3.15. Again, very good agreement was reached in comparison with the flux density waveform calculated from the conventional TS-FEA.

Using the resulting CE-FEA waveform data for this case study, the rotor field harmonics up to the 15-th order can be reasonably predicted as given in Table 3.4. It should be noted that, according to the obtained numerical results, the fundamental frequency of the rotor flux variation with time is different from the fundamental frequency, f_1 , of the stator mmf and air-gap revolving field. This can be observed both in the previous 12-slot 10-pole IPM case study, as well for the current 12-slot 8-pole IPM case study for which there are three electric cycles of the field inside the



Figure 3.14: PM flux density waveform construction according to CE-FEA for the 12-slot 8-pole IPM motor case study.



Figure 3.15: PM flux density waveform at rated load operation calculated by CE-FEA and time-stepping (TS) FEA.

Table 3.4: Example harmonic spectrum of the flux density in the PMs of the 12-slot 8-pole IPM.

$f_s = 120Hz$					
Frequency [Hz]	0	360	720	1080	1440
B [T]	0.916	0.049	0.011	0.005	0.002

Seg. type	2D-TS-FEA	3D-TS-FEA	CE-FEA without	CE-FEA with
			end effect	end effect
	[W]	[W]	[W]	[W]
SEG1	33.2	27.6	33.4	26.6
SEG2	33.2	22.3	33.4	23.7
SEG3	15.4	14.1	15.3	13.3
SEG4	15.4	11.6	15.3	12.6

 Table 3.5: Average PM eddy-current losses for the 12-slot 8-pole IPM.

PM for each one electric cycle of the stator fundamental field.

The variation of the flux density derivative with respect to time within a 4×4 grid, see Figure 3.4, is shown in Figure 3.16. In comparison with the 12-slot 10-pole IPM design, these flux density variations are lower, leading to reduced losses for comparable PM segmentation arrangements in the 12-slot 8-pole configuration as illustrated in the waveforms in Figures 3.17 and 3.18 as well as the average eddy-current loss results summarized in Table 3.5. For reference purposes, the SEG1 arrangement, Table 3.1, was utilized for PM segmentation, in the 12-slot 8-pole IPM case study with a width of 23 mm, a thickness of 4.24 mm and an axial length of 166.3 mm, see Figure 3.8.

It is interesting to note that, as indicated by the results plotted in Figures 3.17 and 3.18 and by the data given in Table 3.5, for this case study 12-slot 8-pole IPM



Figure 3.16: Waveforms of dB(t)/dt at various points in a PM of the 12-slot 10-pole IPM with the SEG1 segmentation.



Figure 3.17: Time variation of PM losses in the 12-slot, 8-pole, IPM machine with SEG1 (top graph) and SEG2 segmentations, respectively.



Figure 3.18: Time variation of PM losses in the 12-slot, 8-pole, IPM machine with SEG3 (top graph) and SEG4 segmentations, respectively.

machine, as well as for the previous 12-slot 10-pole case study, the most effective means for substantially reducing the PM eddy-current losses is the circumferential magnet segmentation approach. This observation might not be applicable to other types of PM machines, because PM losses would depend on the aspect ratio of the magnet width and axial length in relation to the pole pitch of each of the space harmonics that are causing the losses.

3.4.3 Discussion

PM eddy-current losses are very important as they can directly impact the heat generation, the rotor temperature and the motor efficiency. For example, in the worst case scenario, for the 10 hp IPM case studies, the PM losses can cause the motor efficiency to drop by 1 percentage point, a reduction of efficiency that can be very significant in many applications.

The developed computational method is sensitive to the effects of circumferential and axial magnet segmentations and is able to calculate the PM losses with satisfactory precision, as demonstrated in both IPM case studies.

The CE-FEA based calculation technique for PM eddy-current losses incorporates the end effects and the axial segmentation effects, which represents a major improvement over conventional 2D-FEA. At the same time, for the above two case studies, the results of the new method at hand are comparable with those results obtained

Table 3.6: Examples of computational time for different case studies.

IPM example	CE-FEA	2D-TS-FEA	3D-TS-FEA
12s 10p SEG1	$40 \sec$	$2 \min$	4 days
12s 8p SEG1	$30 \sec$	$1.5 \min$	2 days

from the 3D-FEA, while the computational resources are significantly reduced and the speed of computation is increased by several orders of magnitudes for such a PM loss calculation method. This major advantage is demonstrated in the computation times listed in Table 3.6, which corresponds to results obtained with comparable FE meshes and with 7 magnetostatic solutions for the CE-FEA approach, 42 time steps per electrical cycle for the 2D-TS-FEA, and with 42 time steps per electrical cycle for the 3D-TS-FEA. All the simulations were performed on an HP Z800 workstation with 12 cores (2 Xeon X5690 processors) and 32GB RAM memory with ANSYS Maxwell V14.0.

3.5 PWM Switching Losses in the PMs

The effect of the harmonics in the supplied current waveforms, including those associated with the PWM switching frequency, is not incorporated in the previously described CE-FEA technique. For this purpose, an extension of the method is investigated in this section. Explanations are provided for the generic case, in which the phase current waveform, i_a , contains, apart from the fundamental frequency component, one additional high-frequency component. Hence, the expression for the phase current waveform can be written as follows:

$$i_a(t) = I_{1pk} \cos\left(\omega_1 t + \varphi_1\right) + I_{Wpk} \cos\left(\omega_W t + \varphi_W\right), \qquad (3.5.1)$$

where, I_{1pk} and φ_1 , are the magnitude of the fundamental peak current, and its phase angle, respectively. Also here, I_{Wpk} , ω_W and φ_W are the magnitude of the high-frequency component's peak current, its angular frequency, and its phase angle, respectively.

The algorithm can of course be extended to include multiple time harmonics in the current waveform under the assumption that the contribution of each harmonic to the non-linear magnetic field is relatively low, such that superposition can be applied as a generally acceptable engineering approximation. Here, in the example current waveforms shown in Figure 3.19, the magnitude of the higher frequency PWM current component is set equal to 20% of the fundamental peak current. This PWM current component is modulated on top of the fundamental component to produce a typical current waveform for PM brushless motors supplied from power electronic inverters.

The variation of the flux densities in a PM block under the open-circuit condition is caused by the slotted stator structure under the influence of the traveling rotor magnetic field (tooth-slot induced pulsation). Further variation is exhibited under load conditions, and the difference between the two waveforms, which are calculated



Figure 3.19: Fundamental and high frequency PWM components of the phase current.

by the CE-FEA approach and shown in Figure 3.20, can be used to estimate the flux density in a PM block, B_a , due to the stator armature reaction caused by the fundamental current. Numerically, this PM flux density component, $B_a(t)$, can be expressed as a function of a permeance function of time, $\Lambda(t)$, and the stator mmf which can be expressed as a function of time, F(t), caused by the rotor rotating, thus yielding the following expression:

$$B_a(t) = \Lambda(t) \cdot F(t) . \tag{3.5.2}$$

Further simplification for calculating an equivalent permeance waveform with respect to time can be introduced by neglecting the high-order mmf space harmonics. In this case, only the stator fundamental mmf is present, which is a standing component in the rotor reference frame with a time-independent value proportional to the



Figure 3.20: PM flux densities at open circuit and on load with sine-wave rated current supply.

peak fundamental phase current.

The equivalent permeance function approach can be utilized also for the study of the high frequency fields in the PM. After superposition of the effects of this high frequency component and the effects of the low frequency CE-FEA data, this approach can provide satisfactory results. Such an approximated waveform is labeled as harmonic injection (HI) and is shown in Figure 3.21 together with the PM flux density waveform computed by the more laborious 2D-TS-FEA.

Using the previously described method, calculations of PM eddy-current losses were performed for the two IPM motor case studies in the SEG1 arrangement, see Table 3.1, operating under the rated load condition with a PWM switching frequency



Figure 3.21: PM flux density for the example PWM supply from Figure 3.19. The flux density in the top figure is obtained from a 2D-TS-FEA, and the flux density in the bottom one is calculated from the harmonic injection technique.

of 5 kHz and 8 kHz respectively and a PWM current ripple as illustrated in Figure 3.19. The results summarized in Figure 3.22 indicate satisfactory accuracy of the new method at hand, with reasonable engineering agreement between the CE-FEA and 2D-TS-FEA obtained results.

At the same time, the data is in line with expectations because the PM losses increase with the PWM switching frequency and they can be significant as compared to losses under pure sine-wave supply. This trend correlates with the reports of other authors, which are based on experimentation and other more laborious 3D-FEA based methods, e.g. [91].



Figure 3.22: PM eddy-current losses with PWM switching. Results are expressed in per unit. The 2D-TS-FEA with sinewave current supply was defined as the standard value for each motor.

3.6 Summary

The CE-FEA technique described in this chapter combines a relatively low number of magnetostatic field solutions coupled to space-time transformations, in conjunction with a new analytical formulation for calculating PM eddy-current losses. The results provided by two FSCW IPM machine case studies demonstrate satisfactory accuracy and significant decrease in the computational time as compared with the conventional approaches, which are based on the more time consuming TS-FEA method. Based on these advantages, the new method is considered to be particularly suitable for incorporation into large-scale design optimization tools in industrial environments.

Because this developed power loss calculation method incorporates the 3D end effects, it can be employed to study the impact on losses of PM block segmentation in the circumferential and axial directions, under the typical assumptions of resistance limited eddy currents. The sensitivity of the method to PWM switching harmonics was also successfully demonstrated on two IPM machine case studies.

Besides the calculation method for the PM eddy-current losses presented in this chapter, the CE-FEA method also has the capabilities of estimating torque profiles, induced voltage waveforms and stator core losses as described in [46, 47]. In order to improve the computational speed, the "distributed solve" function in ANSYS Maxwell software packages can be integrated into the implementation techniques for the CE-FEA approach. Such implementation techniques for the CE-FEA method in ANSYS Maxwell are the main subjects in the next chapter.

CHAPTER 4

IMPLEMENTATION OF COMPUTATIONALLY EFFICIENT FINITE-ELEMENT ANALYSIS IN ANSYS MAXWELL

In this chapter, a detailed procedure and principle of the implementation of the CE-FEA method with ANSYS Maxwell software packages is described. Before embarking on the CE-FEA principle, the basic conception of the phasor diagram for PM machines is presented in section 4.2. Then the CE-FEA calculation techniques for PM flux linkages and inductances are presented in section 4.3. These parameters are useful for the torque angle calculation for the maximum torque per ampere (MTPA) load condition, which is provided in section 4.4. In section 4.5, computation methods for the waveforms of flux densities in the stator teeth and yokes as well as the stator core losses are investigated. Then, the skew effects are taken into account in the CE-FEA method in section 4.6. At last, robust FEA parametric models for PM machines with different topologies are presented in section 4.7

4.1 Introduction

In the automated multi-objective design optimization procedure, shown in the flowchart of Figure 4.1 [74], there are several major modules, including preparation of parametric FEA models, CE-FEA implementation, a DE optimization algorithm, and decision-making from Pareto-sets. Here, the CE-FEA method is used to calculate the performances including the torque profile, emf/induced voltage waveforms and losses (stator iron, PM and copper). Meanwhile, material costs and masses, as well as resistances and inductances are also calculated for each design candidate. In the design optimization procedure, each design is assumed to operate under the maximum torque per ampere (MTPA) load condition. The distributed solve function package in ANSYS Maxwell software is utilized to improve the computational speed of this global design optimization. Previous publications provide detailed explanations of the CE-FEA method [46, 47, 74] and of DE algorithms [63]. This chapter mainly focuses on the implementation procedures of the CE-FEA techniques in the ANSYS Maxwell and MATLAB scripting functions.

There are numerous ways to determine the electromagnetic field distribution within an electric machine. For very simple geometries, for instance SPM machines, the magnetic field distribution can be found analytically [85–88]. However, in most cases, the field distribution can only be approximated. Magnetic field approximations



Figure 4.1: Flowchart of the automated design optimization utilizing the computationally efficient-FEA and differential evolution algorithm [73].

appear in two general forms. In the first, the direction of the magnetic field is assumed to be known everywhere within an electric machine. This leads to magnetic circuit analysis [34–40, 85–88], which is analogous to electric circuit analysis. In the other form, the electric machine is discretized geometrically using a meshing technique, and the magnetic field is numerically computed at discrete points in such an electric machine. From this information, the magnitude and direction of the magnetic field can be approximated throughout the whole electric machine. The FEA method is one of the numerical solutions, which is commonly utilized in the modeling and analysis of different types of electric machines [25–30].

Of these two magnetic field approximations, the FEA approach produces the most accurate results if the geometric discretization (meshing) is fine enough. The CE-FEA technique is an ultrafast FEA approach with significantly improved computational speed. This method still requires a detailed model of an electric machine, which includes the modeling of material properties as well as a transformable and robust parametric model.

4.2 Phasor Diagram

Before embarking on the CE-FEA method, some basic principles of modeling and analysis of PM machines are introduced here. First is the phasor diagram as shown in Figure 4.2. The dq-frame formulation in the phasor form can be expressed as



Figure 4.2: Phasor diagram of PM machines.

follows:

$$\bar{V} = \omega \bar{\lambda}_{pm} + R_s \bar{I} + j X_d \bar{I}_d + j X_q \bar{I}_q \tag{4.2.1}$$

where, \bar{V} and \bar{I} , are the terminal phase voltage and current phasors, respectively, and $\bar{\lambda}_{pm}$, is the PM flux linkage phasor, while R_s , is the phase resistance. Here, the subscripts d and q represent the d- and q-axes components, and X stands for reactances, $X = \omega L$, while L stands for inductances, and ω , is the electrical rotating speed (angular frequency) in *elec. rad./s.* This relationship is also shown in the dqphasor diagram of such PM machines in Figure 4.2. Here, the phase angle between the current phasor and the d-axis is defined as the torque angle, γ . The phase angle between the voltage phasor and current phasor is the power factor angle, φ .

4.3 PM Flux Linkages and Inductances

In the design optimization of PM machines, all the designs are assumed to be simulated under the MTPA load condition. Thus, in order to calculate the correct torque angle for this load condition, the PM flux linkages and dq-axes inductances are required. In this section, the methods to compute these three parameters are described.

When implementing the CE-FEA with ANSYS Maxwell software, there are two methods to calculate the d-axis and q-axis inductances. Both of the methods utilize Park's transformation

$$\underline{\mathbf{T}}_{s} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{4\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$
(4.3.1)

where, $\theta = \theta_0 + \omega t$, and θ_0 is the initial rotor position as shown in Figure 4.3.

From Park's transformation, the well-known dq-frame formulation of flux linkages is given in the following expression [95]:

$$\begin{cases} \lambda_d = \lambda_{pm} + L_d i_d \\ \lambda_q = L_q i_q \end{cases}$$
(4.3.2)

Method 1: The detailed procedure to utilize expression (4.3.2) and Park's transformation is described in the following steps:

1. With the simulation model running at $90^{\circ}e$ torque angle, one can obtain FEA solutions for a sufficient number of rotor positions. From these solutions, the



Figure 4.3: Phasor diagram of abc- and dq-reference frames.

three phase flux linkages can be exported. Under this load condition, the d-axis current is equal to zero. In this case, the flux linkage of permanent magnets can be calculated as follows:

$$\lambda_{pm} = \lambda_d = \frac{2}{3} \left[\cos(\theta) \lambda_a + \cos(\theta - 2\pi/3) \lambda_b + \cos(\theta - 4\pi/3) \lambda_c \right].$$
(4.3.3)

2. Simulating the FEA model under the load condition of a torque angle between $100^{\circ}e$ and $120^{\circ}e$, another set of three phase flux linkages, λ_{abc} , and currents, i_{abc} , can be obtained. After the application of the dq-transformation, the real time values of the dq-reference frame flux linkages, $\underline{\lambda}_{dq0}$, and currents, \underline{i}_{dq0} , can be expressed as follows:

$$\begin{cases}
\underline{\lambda}_{dq0} = \underline{T}_s \, \underline{\lambda}_{abc} \\
\underline{i}_{dq0} = \underline{T}_s \, \underline{i}_{abc}
\end{cases}$$
(4.3.4)

3. From the dq-frame formulation, the d-axis and q-axis inductances can hence be computed using the following expressions:

$$\begin{cases} L_d = \left(\lambda_d - \lambda_{pm}\right)/i_d \\ L_q = \lambda_q/i_q \end{cases}$$
(4.3.5)

Method 2: when implementing the CE-FEA approach with ANSYS Maxwell software packages, there is another method to calculate the dq-axes inductances. This method utilizes Park's transformation to calculate the d-axis and q-axis inductances directly from the three phase self and mutual inductance profiles. These profiles show how such self and mutual inductances vary with the rotor angular position, covering at least a complete ac cycle (2-pole pitches or more depending on the design of a machine). This requires one to activate/enable the inductance calculation function in the ANSYS Maxwell simulation software. One should notice that in the default state of the software this function is disabled. The detailed procedure to calculate the inductances is as follows:

- For a sufficient number of rotor positions (such as in the CE-FEA method) one obtains FEA solutions, with the simulation model running at 90° torque angle. The PM flux linkage can be calculated using the same procedure as in expression (4.3.3). Meanwhile, the three phase self and mutual inductance profiles can be obtained.
- 2. One conducts a Fourier analysis of these inductance profiles from which one

obtains expressions for self inductances, $L_{aa}(\theta)$, $L_{bb}(\theta)$ and $L_{cc}(\theta)$, and mutual inductances, $L_{ab}(\theta)$, $L_{bc}(\theta)$ and $L_{ca}(\theta)$ [96–100] as follows:

$$\begin{cases} L_{aa}(\theta) = L_{sa} + L_{sv}\cos(2\theta) \\ L_{bb}(\theta) = L_{sa} + L_{sv}\cos(2\theta - 4\pi/3) \\ L_{cc}(\theta) = L_{sa} + L_{sv}\cos(2\theta - 2\pi/3) \\ L_{ab}(\theta) = -L_{ma} + L_{mv}\cos(2\theta - 2\pi/3) \\ L_{bc}(\theta) = -L_{ma} + L_{mv}\cos(2\theta) \\ L_{ca}(\theta) = -L_{ma} + L_{mv}\cos(2\theta - 4\pi/3) \end{cases}$$
(4.3.6)

3. Through Park's transformation

$$\begin{bmatrix} L_d & 0 & 0\\ 0 & L_q & 0\\ 0 & 0 & L_0 \end{bmatrix} = \underline{T}_s \begin{bmatrix} L_{aa} & L_{ab} & L_{ac}\\ L_{ba} & L_{bb} & L_{bc}\\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \underline{T}_s^{-1}, \quad (4.3.7)$$

one can show that the d-axis and q-axis inductances, L_d and L_q , can be expressed as follows [96, 98]:

$$L_d = L_{sa} + L_{ma} + L_{mv} + \frac{1}{2}L_{sv}$$
(4.3.8)

$$L_q = L_{sa} + L_{ma} - L_{mv} - \frac{1}{2}L_{sv}$$
(4.3.9)

Case study: A 12-slot 10-pole IPM machine is chosen as a case study to compare the results obtained from the two above mentioned inductance calculation methods. The cross-section of this example machine is shown in Figure 4.4.

The first step in method 2 mentioned above is to determine the self and mutual inductance profiles versus rotor position utilizing the CE-FEA method. For this case



Figure 4.4: Cross section of a 12-slot 10-pole PM machine.

study, the constructed self- and mutual-inductances were compared to the inductance profiles from TS-FEA simulations, and the results are shown in Figures 4.5 and 4.6.

Based on Fourier analysis of the inductance profiles and the reconstruction of these inductance profiles utilizing expression (4.3.6), new inductance waveforms can be obtained, which only include the dc and second-order harmonic components. These waveforms were validated by the simulation results from the TS-FEA as shown in Figures 4.7 and 4.8. This slight difference in profiles is resulting from the inherently more rigorous nature of the TS-FEA computations in comparison to the cruder sampling rate of the CE-FEA approach. From these profiles, the average (dc) value and the peak value of the second-order harmonic of the self- and mutual-inductances, see equation (4.3.6), were provided as follows: $L_{sa} = 18.00 \, mH$, $L_{sv} = -2.63 \, mH$, $L_{ma} =$ $0.71 \, mH$, and $L_{mv} = -0.72 \, mH$. Utilizing expressions (4.3.8) and (4.3.9), the d- and



Figure 4.5: Self-inductance construction using the CE-FEA (validated by the TS-FEA).



Figure 4.6: Mutual-inductance construction using the CE-FEA (validated by the TS-FEA).



Figure 4.7: Self-inductance construction using Equ. (4.3.6) (validated by the TS-FEA).



Figure 4.8: Mutual-inductance construction using Equ. (4.3.6) (validated by the TS-FEA).

	Test	Method 1	Method 2
$L_d [\mathrm{mH}]$	10.59	11.8	11.12
$L_q \; [\mathrm{mH}]$	13.36	15.5	13.83
Saliency ratio, L_q/L_d	1.26	1.31	1.24

Table 4.1: Comparison of inductances, L_d and L_q . The test values of inductances were measured in the open circuit condition.

q-axes inductances were calculated, and the results are as follows: $L_d = 16.68 \ mH$, and $L_q = 20.74 \ mH$. Compared with the experimental results, these inductance values should be divided by 1.5. This ratio is from the inherent nature of the internal solver of the Maxwell software packages. The comparison between the two inductance calculation methods are provided in Table 4.1. In this table, the tested inductances were measured under the open circuit condition. From this table, one can observe that method 2 for inductance calculations provides higher accuracy than method 1.

4.4 Maximum Torque per Ampere

Here, the electromagnetic torque, T_e , developed by the PM machine can be expressed as follows:

$$T_e = \frac{3}{2} \frac{P}{2} \left(\lambda_d i_q - \lambda_q i_d \right) \tag{4.4.1}$$

where P is the number of poles. Substituting (4.3.2) into the above expression, the electromagnetic torque can be re-expressed as follows:

$$T_e = \frac{3}{2} \frac{P}{2} \left(\lambda_{pm} i_q + (L_d - L_q) i_d i_q \right).$$
(4.4.2)



Figure 4.9: Two components in the electromagnetic torque: magnetic (synchronous) and reluctance torques.

This torque expression identifies two torque components: (1) the magnetic (alignment/synchronous) torque component $\frac{3}{2}\frac{p}{2}\lambda_{pm}i_q$, and (2) the reluctance torque component $\frac{3}{2}\frac{P}{2}(L_d - L_q)i_di_q$.

Example: a 10 hp PM machine with parameters: P = 10, $I_{rms} = 12A$, $\lambda_{pm} = 0.342$, $L_d = 11mH$, and $L_q = 19mH$. The average electromagnetic torque varies with the torque angle, γ , as shown in Figure 4.9. In this figure the gross electromagnetic torque and its two components, the magnetic (alignment/synchronous) torque and the reluctance torque, mentioned above are shown separately.

In an SPM machine, the reluctance torque is very small or negligible due to the almost equal magnitudes of the d-axis inductance, L_d , and q-axis inductance, L_q . Thus the torque angle for the MTPA load condition is generally around 90° for SPM machines. However, for IPM machines, because of the existence of the reluctance component, the torque angle for the MTPA load condition is usually greater than 90°. One should notice that in an IPM machine, $L_q > L_d$, unlike a wound-field, salient-pole, synchronous machine, or an SPM machine.

Substituting for $i_d = I \cos(\gamma)$ and $i_q = I \sin(\gamma)$ into expression (4.4.2), the electromagnetic torque expression can be reformulated as follows:

$$Te = \frac{3}{2} \frac{P}{2} \left(\lambda_{pm} I \sin(\gamma) + (L_d - L_q) I^2 \sin(\gamma) \cos(\gamma) \right).$$
(4.4.3)

Equating the derivative of the electromagnetic torque expression to zero, equation (4.4.4), can yield the angle, γ , that gives the maximum torque, as given in expression (4.4.5) below.

$$\frac{dTe}{d\gamma} = \frac{3P}{4} \left[\lambda_{pm} I \cos(\gamma) + (L_d - L_q) I^2 (2\cos^2(\gamma) - 1) \right] = 0, \qquad (4.4.4)$$

$$\gamma = \arccos\left(\frac{-\lambda_{pm}I + \sqrt{\lambda_{pm}^2 I^2 + 8(L_d - L_q)^2 I^4}}{4(L_d - L_q)I^2}\right).$$
 (4.4.5)

This is the expression used to obtain the torque angle for the MTPA load condition.

4.5 Core Loss Calculation Method

4.5.1 Specific Core Loss Coefficients

Generally, the specific core loss data for steel laminations can be tested and obtained from different test equipment, which include an Epstein frame, a toroid tester, a single sheet tester, etc. [101]. These specific core loss test data can be used to estimate the material core loss coefficients. The modified Steinmetz formula for the specific core loss with the unit of Watts, W/lb or W/kg, is given as follows [102]:

$$w_{Fe} = k_h f B^{\alpha} + k_e f^2 B^2 + k_a f^{1.5} B^{1.5}, \qquad (4.5.1)$$

where, k_h , k_e and k_a , are the so-called hysteresis, eddy-current and excess loss coefficients, respectively. While, f, is the frequency in Hz of the sinusoidal field excitation, and B is the peak value of the field flux density in Tesla for the corresponding frequency.

In the ANSYS Maxwell software, the above formula is utilized to calculate core losses, for which the power exponent of the hysteresis losses, α , is equal to 2, and the excess loss is neglected. Based on the experimental results of specific core losses versus different frequencies, constant coefficients for the hysteresis and eddy-current core losses can be calculated, which are used to estimate the total core losses under different load conditions.

In the CE-FEA method, the excess loss is neglected, and the CAL2 model [103] can be used to estimate the core loss coefficients $k_h(f, B)$ and $k_e(f, B)$, which are used in the following specific core loss calculation model:

$$w_{Fe} = k_h(f, B)fB^2 + k_e(f, B)(fB)^2, \qquad (4.5.2)$$

where, the coefficients, k_h and k_e , are functions of the peak flux density, B, and the
frequency, f. Here, f stands for a range of frequencies.

Previously obtained results [104–106] demonstrated that, for certain frequency ranges, the k_h and k_e coefficients can be considered as functions of the flux density only. Thus, the third-order polynomials for these two coefficients with the lowest relative error values, as validated in [104–106], were utilized in the CE-FEA method, which are given as follows:

$$\begin{cases} k_h(B) = k_{h3}B^3 + k_{h2}B^2 + k_{h1}B + k_{h0} \\ k_e(B) = k_{e3}B^3 + k_{e2}B^2 + k_{e1}B + k_{e0} \end{cases}$$
(4.5.3)

Example: for the core material used in a 210-frame PM machine, the experimental results of specific core losses versus four different frequencies are available. The coefficients for the hysteresis and eddy-current core losses were estimated and shown in Figures 4.10 (a) and (b), respectively.

The specific core loss comparison between the experimental results and the CAL2 model interpolation and their relative error are provided in Figures 4.11 and 4.12, respectively. Here, $k_h(f, B)$ and $k_c(f, B)$ are calculated for a range of frequencies of 50Hz, 60Hz, 100Hz and 400Hz. From Figure 4.12, one can observe that the CAL2 model has a reasonable accuracy for estimating the specific core losses using the changeable hysteresis and eddy-current core loss coefficients.



Figure 4.10: Hysteresis and eddy-current core loss coefficients for a steel.



Figure 4.11: Specific core loss comparison between test and CAL2 model interpolation.



Figure 4.12: Relative error between W/lb losses calculated from test and CAL2 model interpolation.

4.5.2 Flux Densities in the Stator Core

The symmetry property of the magnetic circuits of electric machines results in the following relationships for the radial (r) and tangential (t) components, $B_{r,t}$, of stator core flux densities at different rotor positions [47]:

$$B_{r,t}\left(\left(t + \frac{k_s\theta_s}{\omega}\right), r, \theta\right) = B_{r,t}\left(t, r, \left(\theta + k_s\theta_s\right)\right)$$
(4.5.4)

where, t, is time, and k_s , is a positive integer, while θ_s , is the slot-pitch in electrical measure (electrical radians), as shown in Figure 4.13. At the middle of the back iron/yoke, for example, at point 1 in Figure 4.13, the flux lines are mostly along the tangential direction. Thus, the best way to calculate the flux density at that point is by utilizing the concept of "search coils" with a single turn around the yoke. The same approach is used to obtain the flux densities in the middle of a stator tooth, for instance, such as at point 2 in Figure 4.13.

Using several steps of FEA solutions, the flux density waveforms in the stator teeth and yoke can be reconstructed [47]. The Fourier series of the elemental flux densities can thus be created as follows:

$$B_{r,t}(\theta) = \sum_{n=1}^{n_{max}} B_n \cos(n\theta + \phi_n)$$
(4.5.5)

where, n, is the harmonic order, in which B_n and ϕ_n are the amplitude (peak) and the phase angle of the flux density for the n^{th} harmonic, respectively. Utilizing the polynomial functions in equations (4.5.3) and the flux density amplitude of each harmonic



Figure 4.13: Magnetic circuit symmetry and sampling points of flux densities in the stator core. (For the purpose of the core loss calculation, the junction area between the tooth and yoke is taken into account in the yoke area calculation.)

obtained above, the coefficients, k_h and k_c , can be obtained. These coefficients can be substituted into expression (4.5.2) with the corresponding frequency to estimate the specific core losses of stator teeth and yoke separately.

4.5.3 Total Core Losses in the Stator

Based on the specific core loss coefficients and constructed flux densities in the stator teeth and yoke, the total stator core losses can be calculated according to the following steps:

1. The specific hysteresis harmonic losses and eddy-current losses in the stator

teeth and yoke are calculated as follows:

$$w_h = \sum_{n=1}^{n_{max}} k_h(B_n)(nf_1)B_n^2, \ W/kg \ or \ W/lb$$
(4.5.6)

$$w_e = \sum_{n=1}^{n_{max}} k_c(B_n) (nf_1)^2 B_n^2, \ W/kg \ or \ W/lb$$
(4.5.7)

where, f_1 , is the fundamental frequency.

2. The total core losses in the stator can thus be calculated as follows:

$$P_{Fe_stator} = (w_{h_tooth} + w_{e_tooth})m_{tooth} + (w_{h_yoke} + w_{e_yoke})m_{yoke}, \qquad (4.5.8)$$

where, m_{tooth} and m_{yoke} , are the mass of the stator teeth and yoke, respectively.

4.6 Skew Effects

In this section, the method to take account of the skew effect into the performance calculation using a single CE-FEA evaluation is described. A non-skewed machine is simulated with a 2D-FEA solver assuming the sine-wave current supply. Based on the performance results of this non-skewed machine, the open circuit back-emf/induced voltage and the torque profile with skew can be calculated [107].

4.6.1 Flux Linkages and Induced Voltages

There are two methods to take account of the skew effect in the calculation of flux linkages and induced voltages.



Figure 4.14: Representation of the stator and/or rotor skew [108].

Method 1: in this method, the waveforms of flux linkages are phase shifted and averaged over a rotational angle equal to the skew angle, ρ , as shown in the Figure 4.14 [107]. This process can be expressed as follows:

$$\lambda_a = \frac{1}{\rho} \int_{-\frac{\rho}{2}}^{\frac{\rho}{2}} \lambda_a(\theta + \alpha) d\alpha.$$
(4.6.1)

Thus, the back-emf/induced voltage can be deduced from the derivative of the flux linkage, λ_a .

Method 2: in this method, the harmonic skew factor is applied to the individual harmonics of the flux linkage and induced voltage waveforms. The harmonic skew factor is provided in the following expression [108]:

$$k_{sn} = \frac{\sin\left(\frac{n\rho}{2}\right)}{\frac{n\rho}{2}} \tag{4.6.2}$$

Application of the harmonic skew factors to the Fourier series of the flux linkage and induced voltage waveforms that are readily available from a single non-skewed



Figure 4.15: Open circuit back-emf at 1800 r/min of the PM machine in Figure 4.4.

CE-FEA evaluation is depicted as follows [107]:

$$\lambda_a = \sum_{n=1}^{3s-1} k_{sn} \lambda_n \cos(n\theta + \phi_n), \qquad (4.6.3)$$

$$e_a = \omega \sum_{n=1}^{3s-1} nk_{sn} \lambda_n \sin(n\theta + \phi_n).$$
(4.6.4)

where, s, is the number of FEA solutions in the CE-FEA.

Example: A PM machine with 12-slot and 10-pole is shown in Figure 4.4. The open circuit back-emf waveforms at 1800 r/min are calibrated in Figure 4.15, which shows that method 2 with the harmonic skew factor provides a more accurate result in comparison with method 1. Thus, it is recommended that method 2 be applied to the calculation of flux linkages and induced voltages. Utilizing method 2 to calculate the



Figure 4.16: Induced voltages under a load condition.

induced voltage under a load condition, the obtained voltage waveform is compared with the time-stepping sliced transient FEA results, which are shown in Figure 4.16.

4.6.2 Torque Profiles

When implementing the CE-FEA method with ANSYS Maxwell software packages, there are three methods to evaluate the torque profiles including the skew effects.

Method 1: this method utilizes the field calculation function in the Maxwell software to obtain the energy profiles to calculate the torque profile, which is given in the following expression:

$$T_e = \frac{P}{2} \left(i_a \frac{d\lambda_a}{d\theta} + i_b \frac{d\lambda_b}{d\theta} + i_c \frac{d\lambda_c}{d\theta} \right) - \frac{dW}{d\theta}$$
(4.6.5)

where, λ_a , λ_b and λ_c , are the three phase flux linkages, which can be calculated by equation (4.6.3). Here, W, is the energy profile including the skew effect, which can be expressed in a periodic Fourier series as follows [107]:

$$W = \sum_{n=1}^{3s-1} k_{sn} W_n \cos(n\theta + \phi_n).$$
(4.6.6)

This method takes longer simulation time when implemented with ANSYS Maxwell software, thus it was not utilized in this work.

Method 2: in this method, the harmonic skew factor is applied to the torque profile obtained from a single CE-FEA evaluation. The procedure is as follows:

• A Fourier analysis is conducted on the torque profile obtained from a single CE-FEA evaluation, from which one obtains the harmonic expression for the torque as follows:

$$T_e = T_{avg} + \sum_{n=6,12} T_n \cos(n\omega t + \phi_n)$$
 (4.6.7)

• When considering the harmonic skew factor, the average component in the torque profile should be multiplied by the fundamental skew factor, which is deduced in the Appendix I. Thus, the torque profile with skew effect is expressed as follows:

$$T_e = k_{s1}T_{avg} + \sum_{n=6,12} 0.5(k_{s(n-1)} + k_{s(n+1)})T_n \cos\left(n\omega t + \phi_n\right).$$
(4.6.8)



Figure 4.17: Torque profiles with skew effects of a 12-slot 10-pole PM machine.

Method 3: alternatively, the torque including the skew effect also can be estimated by phase shifting, integrating, and averaging the torque profile obtained from a single CE-FEA evaluation [107], which can be expressed as follows:

$$T_{e} = \frac{1}{\rho} \int_{-\frac{\rho}{2}}^{\frac{\rho}{2}} T_{e}(\theta + \alpha) d\alpha.$$
 (4.6.9)

Example 1: A 12-slot 10-pole PM machine is shown in Figure 4.4. The calculated torque profiles with skew effects are compared with the result from the multi-sliced transient FEA simulation, which are shown in Figure 4.17. Their average torques and torque ripples are provided in Table 4.2. In this figure, the torque calculated from the multi-sliced transient FEA is the most accurate waveform for the real machine with skew. Here, the multi-sliced transient FEA means that several FEA evaluations were

Sliced 1 SFEA	1					
Calculation method	CE-FEA	CE-FEA	Multi-sliced			
	method 2	method 3	transient FEA			
Average torque [Nm]	41.4	41.9	41.4			
Error [%]	0.00	-1.21				
Torque ripple [%]	1.8	1.8	2.4			
Error [%]	25	25				

Table 4.2: Average torque and torque ripples of a 12-slot 10-pole PM machine. Errors are calculated by $\frac{\text{Sliced TSFEA}-\text{method 2 or 3}}{\text{Sliced TSFEA}}100\%$.

Figure 4.18: Cross-section of a 36-slot and 6-pole PM machine.

performed at the same torque angle with different skew angle shift, from which all the induced voltage and torque profiles were added up and then averaged to achieve the profiles with the skew effect.

Example 2: A 36-slot 6-pole PM machine is shown in Figure 4.18. The comparison between the torque profiles is provided in Figure 4.19, and the corresponding average torques and torque ripples are given in Table 4.3.



Figure 4.19: Torque profiles with skew effects of a 36-slot 6-pole PM machine.

Calculation method	CE-FEA	CE-FEA	Multi-sliced
	method 2	method 3	transient FEA
Average torque [Nm]	67.2	67.9	67.5
Error [%]	0.4	-0.6	
Torque ripple [%]	1.8	2.8	3.7
Error [%]	51.3	24.3	

Table 4.3: Average torque and torque ripples of a 36-slot 6-pole PM machine. Errors are calculated by $\frac{\text{Sliced TSFEA}-\text{method 2 or 3}}{\text{Sliced TSFEA}}100\%$.

The two examples presented above include two different types PM machines. One is a PM machine with FSCWs, and another is a PM machine with integer-slot distributed windings. From the results of both case studies, one can observe that the CE-FEA with method 2 can be used to calculate the average torque, and CE-FEA with method 3 can be utilized to estimate the torque ripple. The most accurate method is the CE-FEA with method 1, which has been validated in [107]. However, this method takes longer time because of the "field calculation function" in the AN-SYS Maxwell software package. Thus, when implementing the CE-FEA method with such a FEA software in a population-based design optimization problem, method 1, as shown in expression (4.6.5), is not recommended.

4.7 Parametric Modeling of Permanent Magnet Machines

A requisite step for the automated design optimization of PM machines is building a robust and flexible parametric model for each design optimization problem. In this section, the parametric modeling of PM machines using FEA software packages is described. In order to increase the robustness of the parametric model for the design optimization procedure, several geometric parameters are ratio parameterized, which are described separately for the stator slots and rotor poles. Meanwhile, the outer



Figure 4.20: Cross-section and geometric parameters of stator open slot.

boundary of the whole model was extended by 20% of the stator outer radius in the FEA model.

4.7.1 Stator Tooth and Slot Layouts

4.7.1.1 Open slot with wedges

For the stator with open slots and wedges shown in Figure 4.20, the descriptions of all the geometric parameters are given as follows:

- Input stator geometric parameters:
 - 1. N_s : number of stator slots
 - 2. R_{so} : stator outer radius

- 3. k_{si} : split ratio between the stator inner radius and outer radius, $k_{si} = R_{si}/R_{so}$
- 4. h_g : airgap height
- 5. k_{wt} : tooth width ratio, $k_{wt} = \alpha_{wt}/\alpha_s$
- 6. d_y : depth of yoke/back iron in the stator
- 7. d_w : depth of wedges
- 8. w_w : width of wedges
- 9. T_L : tooth tip length
- Auxiliary calculated geometric variables and expressions can be deduced based on the input geometric variables, which are listed as follows:
 - 1. α_s : slot pitch, mechanical degree, $\alpha_s = 360/N_s$
 - 2. R_{si} : stator inner radius, $R_{si} = k_{si}R_{so}$
 - 3. w_t : tooth width, $w_t = 2R_{si}\sin(k_{wt}\alpha_s/2)$
 - 4. R_{slot} : stator slot bottom radius, $R_{slot} = R_{so} d_y$
 - 5. $OO_s = w_t/2/\sin(\alpha_s/2)$

For the points defining the outlines of the stator slot and yoke shown in Figure 4.20, the corresponding x-y position/coordinate functions are expressed as follows:

$$\begin{cases} x_{p1} = R_{si} \\ y_{p1} = 0 \\ \\ x_{p2} = R_{so} - d_{y} \\ y_{p2} = 0 \\ \\ x_{p3} = R_{slot} \cos(Ang_{P3}), \text{ where } Ang_{P3} = \alpha_{s}/2 - \arcsin(w_{t}/2/R_{slot}) \\ y_{p3} = R_{slot} \sin(Ang_{P3}) \\ \\ x_{p4} = x_{p5} + x_{d}, \text{ where } x_{d} = \frac{y_{p5}(x_{p5} - OO_{S}) - y_{p17}(x_{p5} - OO_{S})}{y_{p17} + x_{p5} - OO_{S}} \\ y_{p4} = y_{p5} - x_{d} \\ \\ x_{p5} = x_{p6} + w_{w} \\ y_{p5} = y_{p6} \\ \\ x_{p6} = d_{w} + R_{si} - h_{g}/2 \\ y_{p6} = (x_{p6} - OO_{S}) \tan(\alpha_{s}/2) + T_{L} \\ \\ x_{p7} = x_{p6} \\ y_{p7} = (x_{p6} - OO_{S}) \tan(\alpha_{s}/2) \\ \\ x_{p8} = R_{si} \cos(\alpha_{s}/2 - \arcsin(w_{t}/2/R_{si})) \\ y_{p8} = R_{si} \sin(\alpha_{s}/2 - \arcsin(w_{t}/2/R_{si})) \\ \\ x_{p15} = x_{p5} + y_{p5} \\ y_{p15} = 0 \\ \\ \begin{cases} x_{p17} = x_{p5} \\ y_{p17} = (x_{p5} - OO_{S}) \tan(\alpha_{s}/2) \end{cases}$$



Figure 4.21: Cross-section and geometric parameters of stator 1.

$$\begin{cases} x_{p20} = x_{p4} + y_{p8} \sin(\alpha_s/2) \\ y_{p20} = y_{p4} - y_{p8} \cos(\alpha_s/2) \\ x_{p21} = x_{p22} + y_{p8} \sin(\alpha_s/2) \\ y_{p21} = y_{p22} - y_{p8} \cos(\alpha_s/2) \\ \end{cases}$$

$$\begin{cases} x_{p22} = x_{p2} - (x_{p2} - OO_S) \sin(\alpha_s/2) \sin(\alpha_s/2) \\ y_{p22} = (x_{p2} - OO_S) \sin(\alpha_s/2) \cos(\alpha_s/2) \end{cases}$$
(4.7.2)

4.7.1.2 Semi-closed slot type 1

For the semi-closed stator slot type 1 in Figure 4.21, several of the geometric parameters are the same as in the previous stator layout including: Ns, R_{so} , k_{si} , R_{si} , k_{wt} , d_y , α_s , and w_t . Extra geometric variables for the stator tooth tips, slot and coils are

- k_{so} : slot opening ratio, $k_{so} = \alpha_{so}/(\alpha_s \alpha_{wt})$, thus, the slot opening angle: $\alpha_{so} = k_{so}(\alpha_s - \alpha_{wt})$
- d_{t2} and d_{t3} : tooth tip depths as shown in Figure 4.21
- w_{so} : slot opening width, and, $w_{so} = 2R_{si}\sin(\alpha_{so}/2)$
- d_{ct} : insulation thickness between the tooth and coil
- d_{cc} : distance between two coils in one slot.

For the six points defining the outlines of the stator slot and yoke, the x-direction of the coordinate should align on the x-axis (1) in Figure 4.21. Thus, the x-y position functions for these points are expressed as follows:

$$\begin{cases} x_{p1} = R_{si} \\ y_{p1} = 0 \end{cases}$$

$$\begin{cases} x_{p2} = R_{si} \cos \left(\alpha_s / 2 - \alpha_{so} / 2 \right) \\ y_{p2} = R_{si} \sin \left(\alpha_s / 2 - \alpha_{so} / 2 \right) \\ y_{p3} = (R_{si} + d_{t2}) \cos \left(\alpha_s / 2 - \alpha_{so} / 2 \right) \\ y_{p3} = (R_{si} + d_{t2}) \sin \left(\alpha_s / 2 - \alpha_{so} / 2 \right) \\ \begin{cases} x_{p4} = x_{p3} + d_{t3} \\ y_{p4} = w_t / 2, \text{ where } w_t = 2x_4 \tan \left(\alpha_s k_{wt} / 2 \right) \end{cases}$$

$$(4.7.3)$$

$$\begin{cases} x_{p5} = R_{so} - d_y \\ y_{p5} = w_t/2 \\ x_{p6} = (R_{so} - d_y) \cos(\alpha_s/2) \\ y_{p6} = (R_{so} - d_y) \sin(\alpha_s/2) \end{cases}$$
(4.7.4)

Before introducing the functions for the four points defining the outline of the coil, several auxiliary functions are defined in the following expressions:

$$\begin{cases} R_{c1} = \sqrt{(x_{p4} + d_{ct})^2 + (y_{p4} + d_{ct})^2} \\ \alpha_{c1} = 2 \arctan\left(\frac{y_{p4} + d_{ct}}{x_{p4} + d_{ct}}\right) \\ R_{c2} = \sqrt{(x_{p5} + d_{ct})^2 + (y_{p5} + d_{ct})^2} \\ \alpha_{c2} = 2 \arctan\left(\frac{y_{p5} + d_{ct}}{x_{p5} + d_{ct}}\right) \end{cases}$$

$$(4.7.5)$$

For the four points defining the outline of the coil, the x-direction of the coordinate should align with the x-axis (2) in Figure 4.21. The x-y position functions of these four points are expressed as follows:

$$\begin{cases} x_{c1} = R_{c1} \cos(\alpha_s/2 - \alpha_{c1}/2) \\ y_{c1} = R_{c1} \sin(\alpha_s/2 - \alpha_{c1}/2) \\ x_{c2} = R_{c2} \cos(\alpha_s/2 - \alpha_{c2}/2) \\ y_{c2} = R_{c2} \sin(\alpha_s/2 - \alpha_{c2}/2) \\ x_{c3} = R_{so} - d_y - d_{ct} \\ y_{c3} = d_{cc}/2 \\ \end{cases}$$

$$\begin{cases} x_{c4} = (R_{si} + d_{t2} + d_{ct}) \cos(\alpha_{so}/2), \text{ where } \alpha_{so} = 2 \arcsin\left(\frac{w_{so}}{2R_{si}}\right) \\ y_{c4} = d_{cc}/2 \end{cases}$$

$$(4.7.6)$$



Figure 4.22: Cross-section and geometric parameters of stator 2.

4.7.1.3 Semi-closed stator slot type 2

Another type of semi-closed stator slot is shown in Figure 4.22. The only geometric difference from stator type 1 exists at the bottom of the slot, and the corresponding radius of this bottom arc is defined as R_{sb} . The distance between the stator inner radius and the center point of the bottom arc of the slot is d_s .

For the five points defining the outlines of the stator slot and yoke in Figure 4.22, the x-y position functions are expressed as follows:

$$\begin{cases} x_{p1} = \sqrt{R_{si}^2 - (w_{so}/2)^2} \\ y_{p1} = w_{so}/2 \\ \\ x_{p2} = x_{p1} + d_{t2} \\ y_{p2} = y_{p1} \\ \\ x_{p3} = x_{p2} + d_{t3} \\ y_{p3} = y_{p2} + d_{t3}/\tan(\beta) \\ \\ x_{p4} = R_{si} + d_s - R_{sb} \\ y_{p4} = R_{sb}, \text{ where, } R_{sb} = \frac{(R_{si} + d_s)\tan\left(\frac{\alpha_s}{2}\right) - \frac{w_t}{2\cos\left(\frac{\alpha_s}{2}\right)}}{1 + \tan\left(\frac{\alpha_s}{2}\right)} \end{cases}$$

$$(4.7.7)$$



Figure 4.23: Cross-section and geometric parameters of the V-type PM layout with four segments per pole.

$$\begin{cases} x_{p5} = R_{si} + d_s \\ y_{p5} = 0 \end{cases}$$
(4.7.8)

4.7.2 Rotor Pole Layouts

4.7.2.1 V-type PM layout with four segments per pole

For the V-type PM layout with four segments per pole as shown in Figure 4.23, the descriptions of all the geometric parameters are given as follows:

- Input rotor geometric variables:
 - 1. P: number of rotor poles

- 2. R_{ri} : rotor inner radius/shaft radius
- 3. h_{pm} : PM slot height along the magnetizing direction including the clearance, $h_{pm} = h_{pmi} + h_c$
- 4. w_{rad} : radial bridge width of the flux barrier on the top of PM
- 5. w_{Fe1} : bridge width between two flux barriers in the middle of one pole
- 6. w_{Fe2} : bridge width between two PM segmentations
- 7. h_c : PM clearance height along the magnetizing direction
- 8. d_q : depth of the flux barrier on the top of PM
- 9. τ_{pp} : pole arc, elec. deg.
- 10. k_{dpm} : PM depth ratio, $k_{dpm} = \frac{d_{pm}}{R_{ro} R_{ri}}$
- 11. k_{wpm} : PM width ratio, $k_{wpm} = \frac{w_{pm}}{w_{pm_max}}$, where, $w_{pm_max} = \sqrt{(y_{q3} - w_{Fe1}/2)^2 + (x_{q3} - (R_{ro} - d_{pm}))^2}$. Here, x_{q3} and y_{q3} are given in the following coordinate functions.
- 12. k_{wq} : ratio of the bridge width between two flux barriers of two adjacent poles, $k_{wq} = \frac{w_q}{w_{q_max}}$, where, $w_{q_max} = 2(\sqrt{x_{q30}^2 + y_{q30}^2})\sin(\alpha_p/2 - \arctan(y_{q30}/x_{q30}))$. Here, $x_{q30} = x_{q3} - h_{pm}\sin(\tau_P/2)$, and $y_{q30} = y_{q3} + h_{pm}\cos(\tau_P/2)$.
- Auxiliary calculated geometric variables and expressions can be deduced based on the input geometric variables, which are listed as follows:

- 1. α_p : pole pitch, mechanical degree, $\alpha_p = 360/P$
- 2. R_{ro} : rotor outer radius, $R_{ro} = R_{si} h_g$
- 3. w_{pm} : PM width of one segmentation, $w_{pm} = k_{wpm} w_{pm_max}$
- 4. w_q : bridge width between two flux barriers of two adjacent poles, $w_q = k_{wq} w_{q\text{-max}}$
- 5. h_{pmi} : PM height along the magnetizing direction, $h_{pmi} = h_{pm} h_c$

6.
$$\tau_p = 2 \arctan\left(\frac{y_{q3} - w_{Fe1}/2}{d_{pm} - (R_{ro} - x_{q3})}\right)$$

For the sixteen points defining the outlines of the geometry of the rotor crosssection shown in Figure 4.23, the x-y position/coordinate functions can be expressed as follows:

$$\begin{cases} x_{q1} = x_{q2} - h_{pm} \sin(\tau_p/2) \\ y_{q1} = w_{Fe1}/2 \\ \begin{cases} x_{q2} = R_{ro} - d_{pm} \\ y_{q2} = w_{Fe1}/2 \end{cases}$$

$$\begin{cases} x_{q3} = (R_{ro} - w_{rad}) \cos(\tau_{pp}/2) \\ y_{q3} = (R_{ro} - w_{rad}) \sin(\tau_{pp}/2) \end{cases}$$

$$\begin{cases} x_{q4} = (R_{ro} - w_{rad}) \cos\left(\alpha_p/2 - \arcsin\left(\frac{w_q}{2(R_{ro} - w_{rad})}\right)\right) \\ y_{q4} = (R_{ro} - w_{rad}) \sin\left(\alpha_p/2 - \arcsin\left(\frac{w_q}{2(R_{ro} - w_{rad})}\right)\right) \\ \end{cases}$$

$$\begin{cases} x_{q5} = x_{q4} - d_q \cos(\alpha_p/2) \\ y_{q5} = y_{q4} - d_q \sin(\alpha_p/2) \end{cases}$$

$$(4.7.9)$$

$$\begin{cases} x_{q6} = x_{q2} + (2w_{pm} + w_{Fe2})\cos(\tau_p/2) \\ y_{q6} = y_{q2} + (2w_{pm} + w_{Fe2})\sin(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q7} = x_{q6} - h_{pm}\sin(\tau_p/2) \\ y_{q7} = y_{q6} + h_{pm}\cos(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q8} = x_{q7} - w_{pm}\cos(\tau_p/2) \\ y_{q8} = y_{q7} - w_{pm}\sin(\tau_p/2) \\ y_{q9} = y_{q8} - h_{pm}\cos(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q10} = x_{q2} + w_{pm}\cos(\tau_p/2) \\ y_{q10} = y_{q2} + w_{pm}\sin(\tau_p/2) \\ y_{q10} = y_{q2} + w_{pm}\sin(\tau_p/2) \\ y_{q11} = x_{q10} - h_{pm}\sin(\tau_p/2) \\ y_{q12} = y_{q2} - h_{pm}\sin(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q13} = x_{PM1} = x_{q2} - h_{pm}\sin(\tau_p/2) \\ y_{q13} = y_{PM1} = y_{q2} + h_{pmi}\cos(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q14} = x_{PM4} = x_{q10} - h_{pmi}\sin(\tau_p/2) \\ y_{q13} = y_{PM1} = y_{q2} + h_{pmi}\cos(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q15} = x_{PM5} = x_{q9} - h_{pmi}\sin(\tau_p/2) \\ y_{q15} = y_{PM5} = y_{q9} + h_{pmi}\cos(\tau_p/2) \\ \end{cases}$$

$$\begin{cases} x_{q16} = x_{PM8} = x_{q6} - h_{pmi}\sin(\tau_p/2) \\ y_{q16} = y_{PM8} = y_{q6} + h_{pmi}\cos(\tau_p/2). \end{cases}$$



Figure 4.24: Cross-section and geometric parameters of the spoke-type PM layout.

4.7.2.2 Spoke-type PM layout

The input geometric parameters for the spoke-type PM layout shown in Figure 4.24 are listed as follows:

- 1. R_{sh} : rotor shaft radius
- 2. d_{br} : the depth of bridges on top of the magnets
- 3. k_{rc} : an auxiliary ratio of the magnet width, w_{pm} . It is defined as $k_{rc} = (R_{ro} R_{pm})/(R_{ro} R_{sh})$, where, R_{ro} , R_{pm} and R_{sh} are the rotor outer radius, magnet bottom radius and shaft radius, respectively, see Figure 4.24. Here, $R_{ro} = R_{si} h_g$.

- 4. k_{apm} : the magnet angle ratio, $k_{apm} = \alpha_{pm}/\alpha_{pm_max}$, where, $\alpha_{pm_max} = \alpha_p 2 \arcsin(1/2/R_{pm})$. Here, $R_{pm} = R_{ro} k_{rc}(R_{ro} R_{sh})$.
- 5. k_{wbr} : the magnet bridge width ratio, $k_{wbr} = w_{br}/h_{pm}$, where,

 $h_{pm} = 2R_{pm}\sin(\alpha_{pm}/2)$. Thus, the top bridge width of the PM, $w_{br} = k_{wbr}h_{pm}$.

For the six points defining the geometry of the rotor cross-section in Figure 4.24, the x-y position functions are expressed as follows:

$$\begin{cases} x_{q1} = x_{q4} - d_{br} - w_{pm} \\ y_{q1} = h_{pm}/2 \\ \begin{cases} x_{q2} = x_{q4} - d_{br} \\ y_{q2} = h_{pm}/2 \\ \end{cases}$$

$$\begin{cases} x_{q3} = x_{q4} - d_{br} \\ y_{q3} = w_{br}/2 \\ \end{cases}$$

$$\begin{cases} x_{q4} = \sqrt{R_{ro}^2 - (w_{br}/2)^2} \\ y_{q4} = w_{br}/2 \\ \end{cases}$$

$$\begin{cases} x_{q5} = x_{q2} \\ y_{q5} = y_{q2} + h_c \\ \end{cases}$$

$$\begin{cases} x_{q6} = \sqrt{R_{pm}^2 - y_{q6}^2} \\ y_{q6} = y_{q1} + h_c \end{cases}$$

$$(4.7.11)$$



Figure 4.25: Cross-section and geometry parameters of the morphing spoke-V-type PM layout.

4.7.2.3 Morphing spoke-V-Type PM layout

A morphing parametric model was developed for the spoke-V-type (SV) PM layout as shown in Figure 4.25. The input geometric parameters are described as follows:

- d_1 : distance between the PM outer flux barrier and the rotor outer circle
- d_2 : depth of the PM outer flux barrier
- w_q : width of the bridge between two adjacent PM segments along the q-axis
- β : PM tilt angle
- α_{fb} : flux barrier spanning angle

- d_3 : depth of the non-magnetic material for the PM inner flux barrier
- k_{wpm}: PM width ratio, w_{pm}/w_{pm_max}. Here, w_{pm_max} is the maximum possible width for PM segment limited by the other geometric variables as shown in Figure 4.25.

Based on these input variables, some calculated auxiliary geometric variables and expressions can be deduced, which are listed as follows:

- R_{r1} : radius of point 1, $R_{r1} = R_{ro} d_1$
- α_1 : spanning angle between point 1 and x-axis, $\alpha_1 = \arctan(y_{q1}/x_{q1})$
- h_{pm} : PM height along the magnetizing direction,

$$h_{pm} = \sqrt{(x_{q2} - x_{q1})^2 + (y_{q2} - y_{q1})^2} \cos\left(\beta + \arctan\left(\frac{x_{q1} - x_{q2}}{y_{q1} - y_{q2}}\right)\right)$$

For the eleven points defining the geometry of the rotor cross-section in Figure 4.25, the x - y position functions are expressed as follows:

$$\begin{cases} x_{q1} = \sqrt{R_{r1}^2 - (w_q/2)^2} \\ y_{q1} = w_q/2 \\ \begin{cases} x_{q2} = R_{r2} \cos(\alpha_{fb} + \alpha_1) \\ y_{q2} = R_{r2} \sin(\alpha_{fb} + \alpha_1) \\ \end{cases} \\ \begin{cases} x_{q3} = x_{q2} - d_2 \cos(\beta) \\ y_{q3} = y_{q2} + d_2 \sin(\beta) \\ \end{cases} \\ \begin{cases} x_{q4} = x_{q2} - h_{pm} \sin(\beta) \\ y_{q4} = y_{q2} - h_{pm} \cos(\beta) \end{cases} \end{cases}$$
(4.7.12)

$$\begin{cases} x_{q5} = x_{q4} - d_2 \cos(\beta) \\ y_{q5} = y_{q4} + d_2 \sin(\beta) \\ \\ \begin{cases} x_{q6} = x_{q3} - w_{pm} \cos(\beta) \\ y_{q6} = y_{q3} + w_{pm} \sin(\beta) \\ \\ y_{q6} = y_{q3} + w_{pm} \sin(\beta) \\ \\ y_{q7} = x_{q6} - h_{pm} \cos(\beta) \\ \\ \begin{cases} x_{q7} = x_{q6} - h_{pm} \cos(\beta) \\ \\ y_{q8} = y_{q6} - d_3 \cos(\beta) \\ \\ y_{q8} = y_{q6} + d_3 \sin(\beta) \\ \end{cases}$$
(4.7.13)
$$\begin{cases} x_{q9} = x_{q7} - d_3 \cos(\beta) \\ \\ y_{q9} = y_{q7} + d_3 \sin(\beta) \\ \\ \begin{cases} x_{q10} = \sqrt{x_{q6}^2 + y_{q6}^2} \cos(\alpha_p/2 - \arctan(y_{q6}/x_{q6})) \cos(\alpha_p/2) \\ \\ y_{q10} = \sqrt{x_{q6}^2 + y_{q6}^2} \cos(\alpha_p/2 - \arctan(y_{q9}/x_{q9})) \sin(\alpha_p/2) \\ \\ \end{cases}$$
(4.7.13)

When the q-axis bridge width, w_q , and the PM tilt angle, β , are not equal to zero, the SV-PM parametric model can be used as a V-type PM layout as shown in Figure 4.26 (a). At the extreme, for this SV-PM parametric model, w_q and β equal to zero, the layout morphs into a spoke-type PM configuration, as shown in Figure 4.26 (b). It should be noted that for such constructions the material for the bottom flux barrier has to be non-magnetic in order to prevent substantial magnetic leakage. When w_q and β are not equal to zero, the SV-PM parametric model can be used as a V-type PM layout as shown in Figure 4.26 (a).



Figure 4.26: Morphing cross-sections of the SV-PM layout parametric model.

4.7.2.4 Morphing flat-V-type PM layout

Another parametric model can morph between the flat bar-type and V-type PM layout, which is named FV-PM model as shown in Figure 4.27. This model has the same input geometric variables, except the PM width ratio, k_{wpm} , which is set up equaling to 1 for this FV-PM parametric model.

The x - y position expressions for points 1 through 3 and 6 through 9 are the same as in the previous morphing SV parametric model. The expressions for the other two points, 4 and 5, are given as follows:

$$\begin{cases} x_{q4} = x_{q5} + (y_{q5} - w_q/2) \tan(90^o - \beta) \\ y_{q4} = w_q/2 \\ \begin{cases} x_{q5} = x_{q3} - h_{pm} \cos(90^o - \beta) \\ y_{q5} = y_{q3} - h_{pm} \sin(90^o - \beta) \end{cases}$$

$$(4.7.14)$$

where, $h_{pm} = y_{q3} - w_q/2$.

For the FV-PM parametric model illustrated in Figure 4.27, and a PM tilt angle,



Figure 4.27: Cross-section and geometry parameters of the morphing flat-V-type PM layout.

 β , of 72°, the rotor geometry corresponds to a flat bar-type PM arrangement as shown in Figure 4.28 (a). For $\beta < 72^{\circ}$, the geometry morphs to a generic V-type PM layout as shown in Figure 4.28 (b).

4.8 Summary

This chapter focused on the implementation techniques for the CE-FEA method in the ANSYS Maxwell software packages. The calculation method for PM flux linkage and dq-axes inductances utilizing the CE-FEA method is presented first. These parameters can be utilized to calculate the torque angle for the MTPA load condition for each design in the automated design optimization procedure. The CE-FEA based



Figure 4.28: Morphing cross-sections of the FV-PM layout parametric model.

calculation procedure for the stator core losses was also presented in this chapter. Several methods for taking account of the skew effects into the calculation of the phase flux linkages, phase induced voltages and torque profiles were discussed. The accuracy of the CE-FEA method was validated by several case studies provided above and presented in a previous publication [47] by Sizov *et. al.*

This fast and accurate electromagnetic field analysis method, the CE-FEA technique, and the associated robust parametric modeling for PM machines with sine-wave current supplies were utilized in conjunction with design optimization techniques to achieve the desired highly efficient automated design optimization process subject of the next chapter. Therefore, in the next chapter, design optimization techniques will be investigated and implemented for PM machines with different geometric topologies.

CHAPTER 5

DESIGN OPTIMIZATION METHODS OF DESIGN OF EXPERIMENTS AND DIFFERENTIAL EVOLUTION

In this chapter, a combined design optimization method utilizing design of experiments (DOE) and differential evolution (DE) algorithms was investigated and implemented to provide practical insights in the multi-objective design optimization of PM machines with different rotor topologies. The basic principle of DOE and DE algorithms are introduced in sections 5.2 and 5.3, respectively. In section 5.4, this combined design optimization method was implemented on four PM machine case studies. All these PM machines have the combination of 12 slots and 10 poles with different rotor geometries, which include two different V-type, spoke-type, and flat bar-type PM layouts. Finally, a systematic comparison between these PM machines were performed.

5.1 Introduction

Based on the literature search in section 1.2.3 in Chapter 1, a combined DOE and DE design optimization method was developed and investigated in this Chapter. The detailed procedure for the new method is depicted in the flowchart of Figure 5.1. In this combined design optimization method, the Central Composite Design (CCD) approach combined with the Response Surface Method (RSM) was used to perform a sensitivity study. Based on the results, the design variables without significant effect on the objectives can be eliminated from the global DE optimization. The study is also useful for establishing the ranges for the selected design variables. The overall process contributes to the reduction of the simulation time and to the convergence of the DE algorithm. The DOE procedures are only desirable for a total number of design variables greater than five. A CE-FEA technique [46, 47] was employed to evaluate the electromagnetic performance of the candidate designs. This combined design optimization method was implemented on four PM machine case studies with different rotor topologies: namely the V-SV-shape, the spoke-type, the flat bar-type, and the V-FV-shape, which are shown in Figure 4.26 (a), 4.26 (b), 4.28 (a), and 4.28(b), respectively. The DE optimization results enable the systematic rationalized comparison between such four types of PM machines. A discussion on the relative merits of each topology is included.



Figure 5.1: Combined design optimization procedure. The performance estimation is based on the computationally efficient - finite element analysis (CE-FEA).
5.2 Design of Experiments and Response Surface Methodology

The DOE and RSM approaches are statistical and mathematical techniques useful for developing, improving and optimizing processes and products. In general, suppose that the electric machine designer is concerned with a motor/generator's performances, e.g. the efficiency, material cost, torque ripple, etc. These performances can be defined as objectives, y, which depend on the independent geometric variables (including the stator, rotor, airgap, and stack length). These geometric variables can be defined as the input variables $X = [x_1, x_2, \ldots, x_{D_v}]$, where D_v is the number of geometric design variables.

The relationship between the response/objective and design variables is given as follows:

$$y_n = f(x_1, x_2, \dots, x_{D_v}) + \varepsilon, \qquad (5.2.1)$$

where, $x_1, x_2, \ldots, x_{D_v}$, are usually called the natural variables, because they are expressed in the natural units of measurement, such as length unit, mm, and angle degree for electric machines. It is convenient to transform the natural variables to coded variables $C = [c_1, c_2, \ldots, c_{D_v}]$, which are defined as follows:

$$C = \frac{X - (X_{min} + X_{max})/2}{(X_{max} - X_{min})/2}.$$
(5.2.2)

These coded variables are defined to be dimensionless with zero mean and the same spread or standard deviation. In terms of the coded variables, the true response function can be reformulated as follows:

$$y = f(c_1, c_2, \dots, c_{D_v}), \tag{5.2.3}$$

In many cases, either a first-order or a second-order model for expression (5.2.3) can be used. The form of the first-order model only shows the main effects of input/design variables. If there is an interaction between these design variables, the second-order polynomial function is preferred, which is presented as follows:

$$y = \beta_0 + \sum_{i=1}^{D_v} \beta_i c_i + \sum_{i=1}^{D_v} \beta_{ii} c_i^2 + \sum_{i=1}^{D_v} \sum_{j=i+1}^{D_v} \beta_{ij} c_i c_j, \qquad (5.2.4)$$

where, β_0 , β_i , β_{ii} and β_{ij} , are the regression coefficients for the coded design variables c_i and c_j .

The second-order polynomial model is widely used in the RSM technique for several reasons [50], which are summarized as follows:

- The second-order model is very flexible, which can take on a wide variety of functional forms. Thus, it will often work well as an approximation to the true response surface.
- 2. It is easy to estimate the regression coefficients in the second-order model by using the method of least squares.

3. There is considerable practical experience indicating that second-order models work well in solving real response surface problems.

The second-order model, as given in expression (5.2.4), can be analyzed by multiple linear regression techniques.

For example, a second-order response surface model for two design variables is given as follows:

$$y = \beta_0 + \beta_1 c_1 + \beta_2 c_2 + \beta_{11} c_1^2 + \beta_{22} c_2^2 + \beta_{12} c_1 c_2 + \varepsilon, \qquad (5.2.5)$$

Let $\beta_3 = \beta_{11}$, $c_3 = c_1^2$, $\beta_4 = \beta_{22}$, $c_4 = c_2^2$, $\beta_5 = \beta_{12}$, and $c_5 = c_1c_2$, then the expression (5.2.5) can be rewritten as follows:

$$y = \beta_0 + \beta_1 c_1 + \beta_2 c_2 + \beta_3 c_3 + \beta_4 c_4 + \beta_5 c_5 + \varepsilon, \qquad (5.2.6)$$

This is a linear regression model. In general, if the regression coefficients of a model are linear, then this model is a linear regression model, regardless of the shape of the response surface that it generates.

5.2.1 Estimation of Regression Coefficients

In a multiple linear regression model, the method of least squares is typically used to estimate the regression coefficients. Suppose that the DOE method can generate N_r designs, there will be N_r responses for each design objective, which are y_1, y_2, \ldots , y_{N_r} . For each response, the second-order model can be expressed as follows:

$$y_i = \beta_0 + \sum_{j=1}^{D_v} \beta_j c_{ij} + \varepsilon_i, \ i = 1, 2, \dots, N_r$$
 (5.2.7)

This expression can be written in matrix notation as follows:

$$\underline{y} = \underline{C}\,\underline{\beta} + \underline{\varepsilon} \tag{5.2.8}$$

where,
$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_r} \end{bmatrix}$$
, $\underline{C} = \begin{bmatrix} 1 & c_{11} & c_{12} & \cdots & c_{1D_v} \\ 1 & c_{21} & c_{22} & \cdots & c_{2D_v} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & c_{N_r1} & c_{N_r2} & \cdots & c_{N_rD_v} \end{bmatrix}$, and $\underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{N_r} \end{bmatrix}$.

The least squares function is expressed as follows:

$$L = \sum_{i=1}^{N_r} \varepsilon_i^2 \tag{5.2.9}$$

$$= (\underline{y} - \underline{C}\underline{\beta})'(\underline{y} - \underline{C}\underline{\beta})$$
(5.2.10)

$$= \underline{y}'\underline{y} - 2\underline{\beta}'\underline{C}'\underline{y} + \underline{\beta}'\underline{C}'\underline{C}\underline{\beta}$$
(5.2.11)

The least squares estimators, \underline{b} , must satisfy the following expression:

$$\frac{dL}{d\underline{\beta}}\Big|_{\underline{b}} = -2\underline{C}'\underline{y} + 2\underline{C}'\underline{C}\underline{b} = 0$$
(5.2.12)

Thus, the least squares estimator of $\underline{\beta}$ is \underline{b} , which can be expressed as follows:

$$\underline{b} = (\underline{C'C})^{-1}\underline{C'y} \tag{5.2.13}$$

Here, the fitted regression model is

$$\underline{\hat{y}} = \underline{Cb}. \tag{5.2.14}$$



Figure 5.2: Parametric model of the 12-slot 10-pole spoke-type PM machine.

Thus the above is the computation method for the regression coefficients for the second-order polynomial function.

5.2.2 Central Composite Design

The model described by (5.2.4) contains $[1 + 2D_v + D_v(D_v + 1)/2]$ regression parameters. Therefore, the set of numerical experiments must comprise at least $[1 + 2D_v + D_v(D_v + 1)/2]$ distinct design samples/candidates. In addition, the design set must include at least three levels for each design variable to estimate the pure quadratic terms in equation (5.2.4). When the CCD method is implemented for generating a set of designs, the resulting are five levels, $[-\alpha, -1, 0, 1, \alpha]$ for the coded design variables.

This method will be explained through a case study for a 12-slot 10-pole spoketype PM machine with three design variables as shown in Figure 5.2. The three design variables are the PM width, w_{pm} , PM height along the magnetizing direction,

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Variables	Definition	Min	Max
w_{pm}	PM width [in]	0.50	1.0
h_{pm}	PM height [in]	0.20	0.30
w_T	stator tooth width [in]	0.25	0.435

Table 5.1: Definitions and ranges for three independent design variables of the spoketype PM machine depicted in Figure 5.2.

 h_{pm} , and stator tooth width, w_T , for which the corresponding value ranges are listed in Table 5.1. Two design objectives were investigated including the material cost and losses defined in expressions (5.2.15) and (5.2.16), respectively, which are listed as follows:

1. minimize the material cost with a weighted function,

$$y_1 = \min(c_{PM}m_{PM} + c_{Cu}m_{Cu} + c_{Fe}m_{Fe}), \qquad (5.2.15)$$

where, m_{PM} , m_{Cu} and m_{Fe} are the masses of PM, copper and laminated steel, respectively, and c_{PM} , c_{Cu} and c_{Fe} are their corresponding material cost's coefficients per unit of mass.

2. minimize the losses including the copper loss, P_{Cu} , stator core loss, P_{Fe} , and mechanical loss P_{me} :

$$y_2 = \min(P_{Cu} + P_{Fe} + P_{me}), \tag{5.2.16}$$

The CCD method generated 20 designs, which are given in Table 5.2. Utilizing the method of least squares described in the previous section, the regression coefficients,

Designs Cost Losses x_3 x_1 x_2 c_1 c_2 c_3 [\$][W][in] [in] [in]1 0 0 0 0.68 0.250.342510.25103.562 0.30.25-1 1 -1 11.64126.690.53 0.860.30.251 1 -1 10.01109.95 4 0.680.250.34250 0 0 10.25103.561.681893.99 50.680.33410.34250 0 9.67 60 0 0.68 0.250.34250 10.25103.567 0.2-1 1 0.50.435-1 12.61106.10 0.258 0.34250 0 0 10.25103.560.6890.16590.34250 -1.68180 11.76124.69 0.68 10 0 0 0 10.25103.560.68 0.250.342511 0.680.250.18690 0 -1.681812.63146.60121.68180 0.98270.250.34250 9.28 96.930 130.680.250.34250 0 10.25103.561 1 140.50.30.435-1 11.3892.10 1.68180 0 150.68 0.250.498110.80 82.41 1 1 1 80.36160.860.30.4359.16 171 -1 1 90.100.860.20.43510.33-1 180.50.20.25-1 -1 12.98149.62 190.37730.250.3425-1.68180 0 13.01127.15200.860.21 11.30132.070.25-1 -1

Table 5.2: Designs generated by the CCD method. x_1 and c_1 : w_{pm} ; x_2 and c_2 : h_{pm} ; x_3 and c_3 : w_T .



Figure 5.3: Variable effects and interaction on the cost objective.

 $\underline{\beta}$, can be calculated. For the material cost, the fitted second-order model based on the coded design variables is given as follows:

$$y_{cost} = 10.2486 - 1.0315c_1 - 0.6254c_2 - 0.4050c_3 + 0.3016c_1^2 + 0.1508c_2^2 + 0.5036c_3^2 + 0.0142c_1c_2 - 0.1475c_1c_3 + 0.0273c_2c_3.$$

The sensitivity study for the cost objective is shown in Figure 5.3, in which the per unit values of each regression coefficient are defined as follows: $\frac{\beta_1}{\beta_0}, \frac{\beta_2}{\beta_0}, \cdots, \frac{\beta_{23}}{\beta_0}$.

The same procedure was repeated for the loss objective, and the second-order model is given as follows:

$$y_{losses} = 103.614 - 8.264c_1 - 8.818c_2 - 18.864c_3 + 2.660c_1^2 + 1.706c_2^2 + 3.532c_3^2 + 0.633c_1c_2 + 0.820c_1c_3 + 2.666c_2c_3.$$

The corresponding sensitivity study for the loss objective is shown in Figure 5.4.



Figure 5.4: Variable effects and interaction on the loss objective.

From the examination of the results in Figures 5.3 and 5.4, it turns out that the PM width and PM height have more significant effects on the cost objective, and meanwhile the stator tooth width and PM height have more significant effects on the loss objective. For this multi-objective design optimization problem, all the three design variables should be included in the DE algorithm for design optimization to meet both the design objectives of minimum cost and minimum loss. There is no conflict between the loss and cost objectives in such a design optimization problem.

In this section, the principle of the CCD method and the corresponding calculation method for regression coefficients were explained through a PM machine case study with three design variables. This method can be extended to more complicated electric machine design problems, which will be included in section 5.4 of this chapter.

5.3 Differential Evolution

DE algorithms employ the following definitions:

- Population: $P_{Xg} = (X_{i,g})$, where the individual design index $i = 1, ..., N_p$, and the individual generation index $g = 1, ..., g_{max}$. A DE optimization contains a number of g_{max} generations, and each generation has N_p individual designs.
- Design constraints: $\Gamma_m(X) = f_{cm}(X) f_{crm} \leq 0$, where the constraint index m = 1, 2, ..., M, for which, $f_{cm}(X)$, is the mathematical function of the m_{th} physical variable, and M, is the total number of constraints, while, f_{crm} is the reference value of the m_{th} constraint.
- Design objectives: $\min(f_n(X))$, where the objective index n = 1, 2, ..., N, for which, $f_n(X)$, is the mathematical function of the n_{th} physical performance quantity, and N, is the total number of objectives.

The main DE process includes the procedures of initialization, mutation, crossover and selection, which are described in Figure 5.5, and discussed in this section, respectively.



Figure 5.5: Implementation procedure of the DE algorithm in the design optimization of electrical machines. The Lampinen's selection criteria is interpreted in the pseudo-code expression in (5.3.7).

5.3.1 Initialization

Before the initialization of the first generation, both upper and lower limits (bounds) for each design variable must be assigned. These limits are organized into two initialization vectors, X_L and X_U , for which subscripts, L and U, indicate the lower and upper bounds, respectively. Utilizing a random process, the initial values of the j^{th} design variable of the i^{th} vector in the first generation can be expressed as follows [63]:

$$x_{j,i,1} = rand_j(0,1) \cdot (x_{j,U} - x_{j,L}) + x_{j,L}$$
(5.3.1)

where, $0 \leq rand_j(0,1) < 1$, here j, indicates that a new random value is generated for each design variable.

After generating the first generation containing N_p designs/vectors, the design objectives and constraints can be evaluated through the utilization of the CE-FEA method.

5.3.2 Mutation

The differential mutation process adds a scaled difference between two randomly selected vectors to a third vector. The following expression shows how to combine three different, randomly chosen vectors to create a mutant vector, $V_{i,g}$ [63]:

$$V_{i,g} = X_{r1,g} + F \cdot (X_{r2,g} - X_{r3,g}).$$
(5.3.2)

where, F, is the scale factor, $F \in (0, 1+)$, which is a positive value without upper limit. However, its effective value is seldom greater than 1 [63]. This factor is used to control the rate at which the population evolves. In the above expression, g indicates the number of current generation. Here, the target index, i, specifies the vector index in the mutant generation, while, the subscripts r1, r2, and r3 are randomly selected vector indices per mutant, and $i \neq r1 \neq r2 \neq r3$. These mutually exclusive indices enable the DE algorithm to achieve both good convergence speed and probability of convergence with a relatively small population [63].

For the mutation scale factor, F, see [63], the recommended range for F is (0, 1), that is, 0 < F < 1. Beyond this range for F, when F > 1, the DE algorithm tends to be time-consuming and less reliable than if F < 1. In 2002, Zaharie proposed a method to calculate a critical value of F [63], which can be expressed as follows:

$$F_{crit} = \sqrt{\frac{(1 - C_r/2)}{N_p}}$$
(5.3.3)

where, F_{crit} , is the lower limit for F, and C_r , is the crossover probability, which is explained in the next "crossover" section. In reality, the larger value of F leads to a better diversity of populations and better convergence of the DE algorithm.

5.3.3 Crossover

The crossover procedure builds trial vectors, $U_{i,g}$, out of variable values that have been copied from two different vectors, $X_{i,g}$ and $V_{i,g}$, which can be formulated as follows [63]:

$$U_{i,g} = (u_{j,i,g}), \text{ where } u_{j,i,g} = \begin{cases} v_{j,i,g}, \text{ if } rand_j(0,1) \le C_r \\ x_{j,i,g}, \text{ otherwise} \end{cases}$$
(5.3.4)

where the crossover probability, $C_r \in [0, 1]$, that is $0 \leq C_r \leq 1$, is a user-defined value that controls the fraction of variables' values that are copied from the mutant process. In the above equation, if the random number of the j^{th} variable in the i^{th} vector, $rand_j(0, 1)$, is less than or equal to C_r , the trial variable of vector $U_{i,g}$ is equal to the mutant variable of vector $V_{i,g}$. Otherwise, the trial variable is copied from the variable in vector $X_{i,g}$. Through this process, a new trial generation can be produced. Consequently, utilizing the CE-FEA method, the design objectives and constraints can be calculated for the comparison and selection that take place in the next step.

In the mutant process of expression (5.3.2), the design variables' values generated in the mutant vector, $V_{i,g}$, can very easily violate such variables' limits, $x_{j,L}$, and $x_{j,U}$. Thus, at the end of the crossover procedure, the *resetting* process must be utilized to modify out-of-bounds variables so that the trial vectors satisfy all boundary constraints (upper and lower limits). There are two methods to perform the resetting procedure, which are described as follows [63]:

1. Random re-initialization:

$$u_{j,i,g} = x_{j,L} + rand_j(0,1) \cdot (x_{j,U} - x_{j,L}), \text{ if } (u_{j,i,g} < x_{j,L}) \text{ or } (u_{j,i,g} > x_{j,U})$$
 (5.3.5)

2. Bounce-back:

$$\begin{cases} u_{j,i,g} = x_{j,r1,g} + rand_j(0,1) \cdot (x_{j,L} - x_{j,r1,g}), \text{ if } (u_{j,i,g} < x_{j,L}) \\ u_{j,i,g} = x_{j,r1,g} + rand_j(0,1) \cdot (x_{j,U} - x_{j,r1,g}), \text{ if } (u_{j,i,g} > x_{j,U}) \end{cases}$$
(5.3.6)

In contrast to the random re-initialization process, the bounce-back strategy takes the progress toward the optimum objective into account by selecting a variable value that lies between its base value, $x_{j,r1,g}$, and the bound (lower or upper limit) being violated.

5.3.4 Selection

In the selection step, the trial vectors, $U_{i,g}$, are compared to the target vectors in the current generation, $X_{i,g}$, including the design constraints and objectives. Lampinen's selection criterion [63] is adopted here to perform this procedure, which is described as follows:

$$X_{i,g+1} = \begin{cases} U_{i,g}, \text{ if } \begin{cases} \Gamma_m(U_{i,g}) \le 0 \text{ and } \Gamma_m(X_{i,g}) \le 0, \\ f_n(U_{i,g}) \le f_n(X_{i,g}); \\ \text{or} \\ \begin{cases} \Gamma_m(U_{i,g}) \le 0, \\ \Gamma_m(X_{i,g}) > 0; \\ \text{or} \\ \begin{cases} \Gamma_m(U_{i,g}) > 0; \\ \text{max}(\Gamma_m(U_{i,g}), 0) \le \max(\Gamma_m(X_{i,g}), 0); \\ X_{i,g}, \text{ otherwise} \end{cases}$$
(5.3.7)

Once a new generation is obtained and analyzed by the CE-FEA approach, the process of mutation, recombination and selection is repeated until the stoping criteria are satisfied.

5.3.5 Stopping Criteria for DE Algorithms

Stopping criteria are needed to terminate the execution of the DE algorithm. Different stopping criteria were examined for unconstrained single-objective optimization in [109]. Ten stopping criteria were examined and classified into six classes: reference, exhaustion-based, improvement-based, movement-based, distribution-based and combined criteria in [110]. Recently, a criterion based on a combination of variations in the design space and the objectives has been proposed for electric machine problems [111]. The stopping criteria presented in the above references can work properly for design optimization problems with single-objective or a weighted function for multiobjective problems. For such a weighted function, there is no explicit rules/methods to define a correct weight for each objective. Meanwhile, in multi-objective design optimization problems, objectives often conflict, which also can complicate the definition of a weighted function for such a problem. Satisfying one objective may leave another unfulfilled. Accordingly, it is not always clear when to stop the search process for a better compromise. The typical stopping criterion for a multi-objective and multi-constraint design optimization problem is based on setting a maximum number of generations g_{max} [63], [110], which was used in this work.

5.3.6 Implementation of a DE Algorithm in MATLAB in Combination With CE-FEA

The programming of the DE algorithm in MATLAB software is explained in detail in [63]. In this section, the implementation technique of the DE algorithm in combination with the CE-FEA method in the design optimization of PM machines is the focus point.

5.3.6.1 General structure of a DE algorithm in MATLAB

The MATLAB script for the DE algorithm consists of the *.m files shown in Figure 5.6. A brief description of these files is provided as follows:



Figure 5.6: Files for the DE MATLAB code.

• **Rundeopt_dis.m**: This is the main script file for the definition of design specifications, including design variables and corresponding ranges, design constraints and objectives, as well as the population size, N_p , and number of generations, g_{max} . Meanwhile, the control variables of the DE algorithm, F and C_r , are also defined in this file. In this main file, three functions are invoked, which are "initialize_structure_12S10PV.m", "initialize_Sval.m" and "deopt_dis.m".

- initialize_structure_12S10PV.m: This script code is used to save all the variables and performance characteristics into a MATLAB structure named as "generation", which will be shown in the MATLAB workspace after completing the whole design optimization.
- initialize_Sval.m: This script code is used to save the design objectives and constraints temporarily during the design optimization procedure.
- deopt_dis.m: This file contains the main DE engine, including the initialization, mutation, crossover and selection procedures, in which three functions, "MTPA_12S10PV.m", "objfun_dis.m" and "left_win.m", are invoked.
- MTPA_12S10PV.m: This file contains the script code for invoking Maxwell models and calculating the torque angle for the MTPA load condition for each design, which was presented early in section 4.4. In this function, there are two subfunctions, "Fluxlinkage_pm.m" and "Inductance_100deg.m", which are used to calculate the PM flux linkage and dq-axes inductances, respectively. Based on these three calculated parameters, the torque angle for the MTPA load condition

can be computed, which is coded in the function "Inductance_100deg.m".

- objfun_dis.m: This file is used to evaluate the design constraint and objective functions, in which one subfunction file, "cefea_dis.m" is invoked to perform the CE-FEA process to calculate all the performance characteristics and parameters including the torque profiles, induced voltage waveforms, core losses, copper losses, PM losses, phase resistance, dq-axes inductances, masses, and material costs.
- **cefea_dis.m**: This function file is the main code for the implementation of the CE-FEA method, in which nine subfunctions are invoked, see Fig. 5.6, which are introduced as follows.
- **density_construction.m**: This code is used to construct the flux density waveforms in the middle of stator teeth and yoke. The corresponding principle was described in section 4.5 and reference [47].
- **flux_construction.m**: This code is used to construct three phase flux linkages in the CE-FEA method, which can be used to calculate three phase induced voltages.
- **flux_fourier.m**: This code is used to perform the FFT analysis on one phase flux linkage waveform.

- emf_fourier.m: This code is used to perform the FFT analysis on one phase voltage waveform.
- **density_fourier.m**: This code is used to perform the FFT analysis on flux density waveforms constructed above for the stator teeth and yoke.
- core_loss_C113.m: This code is used to save the core loss coefficients versus the variation of the flux density values, which is used in the core loss calculation.
- torque_FE_fourier.m: This code is used to perform the FFT analysis on the torque waveform/profile.
- emf_fourier_LL.m: This code is used to perform the FFT analysis on the line-to-line voltage.
- axial_scaling_loop.m: This function is used to scale the axial stack length to achieve the required shaft torque for all the designs.
- left_win.m: This is a function that defines the selection criterion in the DE algorithm described in expression (5.3.7).

The above is an explanation of the MATLAB script functions for the automated design optimization procedure utilizing the DE and CE-FEA methods, for the case study of a 12-slot 10-pole PM machine.

5.4 Case Studies and Systematic Comparison

In this section, the combined design optimization method presented above was implemented for four case studies including the V-type using SV-PM parametric model (V-SV), the spoke-type, the flat bar-type, and the V-type using FV-PM parametric model (V-FV) PM machines. Utilizing the Pareto-front from the DE design optimization results, the systematic rationalized comparison between these four topologies was performed.

The design specifications of these design optimization case studies are listed as follows:

- slot/pole combination: 12 slots and 10 poles
- four types of rotor topologies: V-SV-shape, spoke-type, flat bar-type, and V-FV-shape
- stator winding: concentrated winding, all-teeth-wound layout
- rated condition: 10 hp at 1800 r/min (the stack length will be scaled to achieve this rated condition.)
- stator outer diameter: 233.6 mm
- conductor current density: $4 A/mm^2$

- stator slot fill factor: 0.38
- stator winding temperature: $100^{\circ}C$
- PM operating at a temperature resulting in $\mu_R = 1.05$, and $B_r = 1.05 T$.

Two design objectives are described as follows:

- minimize the material cost with a weighted cost function given in expression (5.2.15)
- 2. minimize the losses including the copper loss, P_{Cu} , stator core loss, P_{Fe} , PM loss, P_{pm} , and mechanical loss P_{me} .

Meanwhile, three design constraints are set up as follows:

- 1. torque ripple under the rated load condition $\leq 15\%$, and
- 2. total harmonic distortion (THD) of the induced voltage at rated operation \leq 5%, and
- 3. minimum flux density in the PMs $\geq 0.3B_r$, where the remanent flux density, $B_r = 1.05T$.



Figure 5.7: Geometric variables for the SV-PM morphing model.

5.4.1 Case Study I: V-Type PM Machine Using an SV-PM Parametric Model (V-SV)

Using the previously described DOE and RSM techniques, a sensitivity study of the design variables was performed for the V-shape PM machine utilizing the SV-PM Parametric model shown in Figure 5.7, which is referred to as V-SV PM machine. Nine geometric independent variables were selected as specified in Table 5.3, and the ranges of such geometric variables were defined based on mechanical limitations and engineering experience. In this case, the un-coded design variable vector, X, in (5.2.4) is represented by $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, k_{wpm}, w_q]$. A total of 156 candidate designs were generated by the CCD method and analyzed by the CE-FEA approach.

The normalized regression coefficients in expression (5.2.4) were calculated and



Figure 5.8: Sensitivity study for the V-SV PM machine. Two geometric variables, d_{fb} and w_q , were eliminated from the DE optimization.

Table 5.3: Definitions and ranges for nine independent design variables of the V-type PM machine depicted in Figure 5.7. For the spoke-type the variables w_q and β are equal to zero, and d_{fb} was kept at its minimum value.

Variables	Definition	Min	Max
k_{si}	D_{si}/D_{so}	0.55	0.7
h_g	airgap height	$0.6\mathrm{mm}$	$1.2\mathrm{mm}$
w_T	tooth width	14.0mm	$30 \mathrm{mm}$
d_Y	yoke thickness	$12.0\mathrm{mm}$	$20 \mathrm{mm}$
$lpha_{fb}$	flux barrier angle	2.5^{o}	5.5^{o}
d_{fb}	PM top flux barrier depth	$0.5\mathrm{mm}$	$5\mathrm{mm}$
β	PM tilt angle	0^{o}	30^{o}
k_{wpm}	w_{pm}/w_{pm_max}	0.5	0.95
w_q	q-axis bridge width	$0.5\mathrm{mm}$	4mm

quantified as the sensitivity study results and given in Figure 5.8, where the firstorder regression coefficients are expressed in a per unit system defined as β_i/β_0 . From these results, one can observe that the objectives of material cost and losses are conflicting for all geometric independent variables with the exception of the the PM width ratio, k_{wpm} , and stator yoke thickness, d_Y . The flux barrier depth on top of the PM, d_{fb} , and q-axis bridge width, w_q , have no significant effects on any of the objectives. Furthermore, the increase of both of these two variables, leads to higher material cost, losses and torque ripple. The second-order regression coefficients in per unit for interaction effects between d_{fb} , w_q and the following variables: $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, k_{wpm}, w_q]$ are given in Table 5.4. From this table, it was found out that d_{fb} and w_q have no significant interaction effects with the other design variables on the two main design objectives, cost and losses. Consequently, d_{fb} and

· coll ajo,	and the l	$\cdots s_i, \cdots g, \infty$	$1, \infty I, \infty J$	$0, \approx j0, \approx j$	w pm, w q	1 101		1.1 11100011	
$d_{fb}*$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	β	k_{wpm}	w_q
Cost	0	0.001	0.002	0.001	0	0	0.003	-0.002	0
Loss	-0.003	0	-0.001	-0.003	0	0	0.003	-0.003	0
Ripple	0	-0.002	0	-0.006	0.002	0	0.004	-0.002	0
$w_q *$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	β	k_{wpm}	w_q
Cost	0	0	0	0	0	0	0.001	0	0
Loss	-0.002	0	-0.001	0	0	0	0.002	-0.002	0
Ripple	0.004	-0.005	-0.012	-0.004	-0.037	0	0.003	-0.006	-0.007

Table 5.4: Second-order regression coefficients in per unit for interaction effects between d_{fb} , w_q and $[k_{si}, h_q, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, k_{wpm}, w_q]$ for V-SV PM machines.

 w_q , should be fixed to their minimum values leading to simplifications in the DE optimization process, which was afterwards run with a total of seven design variables, $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, \beta, k_{wpm}].$

The DE algorithm employed 60 generations and 50 individual designs per generation. The scatter plot and the optimal Pareto-front for the material cost and losses of the resulting 3,000 designs are shown in Figure 5.9, where the torque ripple is presented using the color map. In this figure, the solid black line represents the Pareto-front for the optimal V-SV PM machines. As a general observation for all studied rotor configurations, it should be noted that, in line with expectations for a 12-slot 10-pole configuration, the torque ripple is typically low.



Figure 5.9: Scatter plot and Pareto-front for the V-SV PM machines.

5.4.2 Case Study II: Spoke-Type PM Machine Using an SV-PM Parametric Model

In this case study, the SV-PM morphing parametric model was studied in the geometrical configuration (spoke-type) depicted in Figure 4.26 (b). The geometric variables w_q , β , and d_{fb} , identified in Figure 5.7, were equal to 0 mm, 0°, and 0.5 mm, respectively. In the sensitivity study, there were six design variables, corresponding to the un-coded design variable vector $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, k_{wpm}]$, and having the ranges as given in Table 5.5.

Through the CCD method, a total of 53 designs were generated. The sensitivity study results for this spoke-type PM machine are shown in Figure 5.10. Based on

Table 5.5: Definitions and ranges for six independent design variables of the spoke-type PM machine depicted in Figure 5.7. For the spoke-type, the variables w_q and β are equal to zero, and d_{fb} was kept at its minimum value.

Variables	Definition	Min	Max
k_{si}	D_{si}/D_{so}	0.55	0.7
h_g	airgap height	$0.6\mathrm{mm}$	$1.2\mathrm{mm}$
w_T	tooth width	$14.0\mathrm{mm}$	$30 \mathrm{mm}$
d_Y	yoke thickness	$12.0\mathrm{mm}$	$20 \mathrm{mm}$
α_{fb}	flux barrier angle	2.5^{o}	5.5^{o}
k_{wpm}	w_{pm}/w_{pm_max}	0.5	0.95

examination of these results, one can observe that none of the six independent design variables can be eliminated from the DE design optimization process.

Accordingly, the DE optimization was performed with six independent design variables, which required 60 generations and 50 individual designs per generation. This resulted in a total 3,000 PM machine designs that were investigated, and the corresponding scatter plot and Pareto-front of this case study are shown in Figure 5.11.

5.4.3 Case Study III: Flat bar-Type PM Machine Using an FV-PM Parametric Model

In this case, a sensitivity study of the design variables was performed for the flat bar-type PM machine, for which the semi-closed stator slot (type 1 in section 4.7.1.2) and the morphing FV-PM model in section 4.7.2.4 were employed. Consequently, the



Figure 5.10: Sensitivity study for the spoke-type PM machine. No geometric variables could be eliminated from the DE optimization.



Figure 5.11: Scatter plot and Pareto-front for the spoke-type PM machines.

whole cross-section and geometric variables are shown in Figure 5.12.

The definitions for seven independent design variables are given in Figure 5.12 and Table 5.6. The un-coded variable vector was defined as $X = [k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, w_q]$. By utilizing the CCD method a total of 88 candidate designs were generated and analyzed by the CE-FEA approach. The normalized regression coefficients for this case study are shown in Figure 5.13.

For flat bar-type PM machines, the design objective of material cost conflicted with the loss objective. Two out of the seven independent design variables, the PM flux barrier depth, d_{fb} , and the q-axis bridge width between PM flux barriers, w_q , have a relatively insignificant and positive effect on the objectives of cost, losses and torque ripple. The second-order regression coefficients in per unit for interaction



Figure 5.12: Geometric variables for the FV-PM morphing model.

Variables Definition Min Max k_{si} D_{si}/D_{so} 0.550.7airgap height h_g 0.6mm 1.2mm tooth width 14.0mm 30.0mm w_T d_Y yoke thickness 12.0mm 20.0mm 5^{o} 2^{o} flux barrier angle α_{fb}

 $0.5 \mathrm{mm}$

 $0.0 \mathrm{mm}$

 $10 \mathrm{mm}$

4.0mm

PM flux barrier depth

q-axis bridge width

 d_{fb}

 w_q

 Table 5.6: Definitions and ranges for seven independent design variables of the flat

 bar-type PM machine depicted in Figure 5.12.



Figure 5.13: Sensitivity study for the flat bar-type PM machine. Two geometric variables, d_{fb} and w_q , were eliminated from the DE optimization.

- / - 1	L	- , - ,	J = / J = /	11	v	-	
$d_{fb}*$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	w_q
Cost	0	0.004	0.005	0.003	0	0	0.002
Loss	-0.007	0	0	-0.002	0	0	0.003
Ripple	-0.004	-0.002	-0.008	-0.002	0.003	-0.021	0.003
$w_q *$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	w_q
Cost	-0.001	0	0	0	-0.002	0.002	0
Loss	-0.006	-0.002	-0.003	-0.002	-0.001	0.003	0
Ripple	-0.002	0	-0.005	0	0.040	0.003	0.018

Table 5.7: Second-order regression coefficients in per unit for interaction effects between d_{fb} , w_q and $[k_{si}, h_q, w_T, d_Y, \alpha_{fb}, d_{fb}, w_q]$ for flat bar-type PM machines.

effects between d_{fb} , w_q and the following variables: $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, w_q]$ are given in Table 5.7. From this table, it was found out that d_{fb} and w_q have no significant interaction effects with the other design variables on the two main design objectives, cost and losses. Thus, these two geometric variables can be set to their minimum values, and only five geometric variables were left for the DE global design optimization, $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}]$.

Based on the DOE sensitivity, five independent variables $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}]$ were selected to be optimized using the DE algorithm. A total of 60 generations, each with 40 individuals, making a total population of 2,400 candidate designs were generated and studied by the CE-FEA approach. The scatter plot for the three design objectives, material cost, losses and torque ripple, is shown in Figure 5.14.



Figure 5.14: Scatter plot and Pareto-front for the flat bar-type PM machines.

5.4.4 Case Study IV: V-Type PM Machine Using an FV-PM Parametric Model (V-FV)

In this case study, the FV-PM parametric model, as shown in Figure 5.12, was utilized in the design optimization procedure, which is referred to here as the V-FV PM machine. Eight design variables were selected for this V-FV PM machine study. Thus, $X = [k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, w_q]$. The corresponding ranges are specified in Table 5.8. A total of 90 designs were generated by the CCD method.

The normalized regression coefficients for the material cost, losses, torque ripple are shown in Figure 5.15. The second-order regression coefficients in per unit for interaction effects between d_{fb} , w_q and the following variables: $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, w_q]$

Variables	Definition	Min	Max
k_{si}	D_{si}/D_{so}	0.55	0.7
h_g	airgap height	$0.6\mathrm{mm}$	$1.2\mathrm{mm}$
w_T	tooth width	14.0mm	$30.0 \mathrm{mm}$
d_Y	yoke thickness	$12.0\mathrm{mm}$	20.0mm
$lpha_{fb}$	flux barrier angle	2^{o}	5^{o}
d_{fb}	PM flux barrier depth	$0.5\mathrm{mm}$	$10 \mathrm{mm}$
β	PM tilt angle	5^{o}	70^{o}
w_q	q-axis bridge width	$0.0\mathrm{mm}$	4.0mm

Table 5.8: Definitions and ranges for eight independent design variables of the V-FV PM machine using the parametric model depicted in Figure 5.12.

are given in Table 5.9. Similar to the case study of the flat bar-type PM machines, the design objective of material cost conflicted with the loss objective. Two geometric variables, the PM flux barrier depth, d_{fb} , and the q-axis bridge width between PM flux barriers, w_q , were eliminated from the DE optimization, and both of them are kept at their minimum values. Thus, only six geometric variables were left for the DE global design optimization, $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, \beta]$. A total population of 3000 designs (60 generations and 50 individual designs per generation) were generated by the DE algorithm. The scatter plot of these designs is shown in Figure 5.16.



Figure 5.15: Sensitivity study for the V-FV PM machine. Two geometric variables, d_{fb} and w_q , were eliminated from the DE optimization.
<i>y</i> 1								
$d_{fb}*$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	β	w_q
Cost	0	0.002	0.005	0.002	0	0	0.004	0.002
Loss	-0.006	0	-0.003	-0.002	0	0.002	0.006	0.003
Ripple	0.005	-0.042	0	-0.021	-0.006	0	0	0
$w_q *$	k_{si}	h_g	w_T	d_Y	α_{fb}	d_{fb}	β	w_q
Cost	-0.002	0	0	0	-0.002	0.002	0.004	0
				•	0.005	0.001	0.00-	
Loss	-0.005	-0.001	-0.004	-0.002	-0.002	0.003	0.005	0
Loss Ripple	-0.005 0	-0.001 -0.004	-0.004 0.002	-0.002 -0.007	-0.002 0.047	0.003	$0.005 \\ 0.015$	0 0.025

Table 5.9: Second-order regression coefficients in per unit for interaction effects between d_{fb} , w_q and $[k_{si}, h_g, w_T, d_Y, \alpha_{fb}, d_{fb}, \beta, w_q]$ for V-FV PM machines.



Figure 5.16: Scatter plot and Pareto-front for the V-FV PM machines.



Figure 5.17: Optimal Pareto-fronts corresponding to Figures 5.9, 5.11, 5.14, and 5.16 on zoomed scales.

5.4.5 Systematic Comparisons Between the Four Case Studies

In order to provide a systematic comparison between the V-SV, spoke-type, flat bartype, and V-FV PM machines, the corresponding optimal Pareto-fronts are co-plotted and compared in Figure 5.17. For the given design problem formulation, including the variable ranges and the imposed constraints, the flat bar-type configuration is able to consistently deliver, somewhat surprisingly, both the lowest cost and the lowest losses.

For the V-type PM machines, different parametric models lead to different locations of the Pareto-fronts. This can be used to judge the goodness of the parametric FEA models. Regarding the V-SV optimal results, it should be kept in mind that the morphing geometry parametric model SV-PM inherently restricts some of the variable ranges. However, the V-FV design optimization provides the almost overlapping Pareto-front with the flat bar-type PM machine case study. The V-FV topology can provide designs with lower material cost and higher losses than the designs from the flat bar-type topology.

The spoke-type PM machines can match the results for much of the cost and loss objectives, but falls behind for very high-efficiency high-cost designs. This could be because the spoke lends itself to high magnetic loading designs, which in turn may call for silicon steel with lower specific losses than the one considered throughout this study. Also, the comparison from Figure 5.17 does not convey the potential advantages of the spoke in terms of increased protection against demagnetization during faults and the ability to employ lower energy magnets, such as sintered ferrite, as an alternative to higher cost neodymium iron boron, NdFeB [112].

From the comparison of the Pareto-fronts in Figure 5.17, one significant observation is that the flat bar-type, V-FV, and spoke-type PM machines can achieve the same objectives of material cost and losses, which is marked with a black circle in Figure 5.17. The corresponding cross-sections and flux plots of three optimal designs for these three topologies are shown in Figures 5.18, 5.19 and 5.20, which are referred to as F-PM, V-PM and S-PM in this work, respectively. Several important geometric



Figure 5.18: Cross-section and flux plot of the optimal design of the flat bar-type PM machine (F-PM).



Figure 5.19: Cross-section and flux plot of the optimal design of the V-FV PM machine (V-PM).

variables are given in Table 5.10.

One should notice that the losses presented in Figure 5.17 include the stator core loss, winding copper loss, PM eddy-current loss, and mechanical loss. Here, the mechanical loss is assumed to be constant for all the designs with 10 hp rating at 1800 r/min. The rotor core loss was not included in the design optimization. Thus, when comparing the optimal designs of these three topologies, the total losses are different as shown in Table 5.11. The S-PM machine has lowest rotor core loss, which leads



Figure 5.20: Cross-section and flux plot of the optimal design of the spoke-type PM machine (S-PM).

Geometric	Units	F-PM	V-PM	S-PM
variables				
Stator inner diameter	[mm]	134.02	136.88	133.01
Airgap height	[mm]	0.619	0.738	0.704
Tooth width	[mm]	21.58	20.27	16.52
Back iron depth	[mm]	13.14	13.30	14.38
PM width per pole	[mm]	30.36	31.02	29.38
PM height	[mm]	4.46	4.55	7.97
Stack length	[mm]	76.08	74.47	53.19

 Table 5.10: Geometric variables for the three candidate designs in Figures 5.18, 5.19

 and 5.20.

Performance	Units	F-PM	V-PM	S-PM
Torque angle	[deg.]	103.3	104.1	100.2
Electromagnetic torque	[Nm]	41.50	41.59	41.57
Electromagnetic power	[W]	7839.8	7836.0	7822.4
Copper loss	[W]	137.7	138.9	129.7
Input power	[W]	7977.5	7974.8	7952.1
Core loss	[W]	233.3	228.2	217.0
PM loss	[W]	22.5	18.8	17.3
Mechanical loss	[W]	91.0	91.0	91.0
Total losses without rotor core loss	[W]	442.1	441.3	442.8
Total losses	[W]	484.5	476.8	455.0
Output power	[W]	7493.0	7498.0	7497.1
Shaft torque	[Nm]	41.69	41.62	41.65
Efficiency	[%]	93.92	94.02	94.28
Material cost	[pu]	1.21	1.21	1.22
Total losses without rotor core loss	[pu]	0.86	0.86	0.86
Torque ripple	[%]	13.38	13.41	10.99
Mass distribution				
PM	[kg]	0.773	0.789	0.934
Copper	[kg]	3.405	3.434	3.207
Steel	[kg]	18.211	17.643	12.129
Total mass	[kg]	22.388	21.866	16.270

Table 5.11: Performances of the recommended designs in Figures 5.18, 5.19 and 5.20. Torque angle is defined as the phase shift between the d-axis and the current phasor.

to the most efficient machine. Meanwhile, this S-PM machine also has the shortest axial stack length, which leads to a higher torque density for this design compared with the F-PM and V-PM machines.

For the F-PM and V-PM machines, the same parametric FV-PM model was used with different geometric values. For the flat bar-type PM machine design, the PM tilt angle, β , is fixed to be 72°. For the optimal design V-PM, this angle is 69.79°,

Topology	DOE		L	ЭЕ	Total		
	D	Time	D	Time	D	Time	
V-SV	156	$55 \min$	3,000	17.5 h	3,156	$18.5 \mathrm{h}$	
Spoke	53	$15 \min$	3,000	$17.5~\mathrm{h}$	3,053	$17.8~\mathrm{h}$	
Flat	88	$31 \min$	2,400	14 h	2,488	$14.5~\mathrm{h}$	
V-FV	90	$32 \min$	3,000	$17.5~\mathrm{h}$	3,090	$18.0~\mathrm{h}$	

Table 5.12: Simulation time for the design optimization of the V-SV, spoke-type, flat bar-type, and V-FV PM machines. "D" stands for the number of candidate designs.

which leads to a slight-V topology, and hence the corresponding geometry shown in Figure 5.19 (a) is very similar to the optimal F-PM machine that is shown in Figure 5.18 (a).

These design optimization case studies were performed on an HP Z800 workstation with 12 cores (2 Xeon X5690 processors) and 32GB RAM memory. The "distributed solve" function in the ANSYS Maxwell software was utilized for parallel processing of the candidate designs [94]. The employed typical FEA models have 5,000-6,000 second-order triangular elements. A summary of simulation times is provided in Table 5.12 illustrating the fast computational speed of this combined design optimization method.

5.5 Summary

In this chapter, a combined design optimization method, utilizing the DOE and DE algorithms, was developed and implemented into four design optimization case studies

for PM machines with four different rotor topologies. These rotor topologies include the V-SV, spoke-type, flat bar-type, and V-FV PM layouts. Before embarking on these case studies, the principle of DOE and DE algorithms were first introduced. From the design optimization results for the four case studies, a systematic comparison between PM machines with four different rotor topologies was performed.

In order to verify the efficacy of the automated design optimization method in an industry environment, a 12-slot 10-pole PM machine with V-type PM layout in the rotor was designed, prototyped and experimentally calibrated as will be given in the next chapter. In this case study subject of the next chapter, only the DE algorithm was utilized in the automated design optimization procedure.

CHAPTER 6

CASE STUDY OF A 12-SLOT 10-POLE PERMANENT MAGNET MACHINE

In this chapter, the automated design optimization method was performed on a case study of a 12-slot 10-pole V-type IPM machine. A robust parametric CE-FEA model of such an IPM machine with concentrated windings, driven by a sine-wave current regulated power electronic drive, is laid out in section 6.2. A multi-objective and multi-constraint design optimization, including two objectives and three constraints, was executed on nine geometric design variables of such a PM machine in section 6.3. In section 6.4, an engineering decision process based on a Pareto-set of optimal designs and a tradeoff study leading to the selection of a recommended design are presented. Consequently, the optimal design was prototyped and tested successfully and the experimental calibration is given here in section 6.5.

6.1 Introduction

The latest developments in computer hardware and software technologies enabled substantial research work on automated design optimization of electric machines using the CE-FEA method and DE algorithms. The detailed procedures of such an automated design optimization method was presented in Figure 4.1 in Chapter 4. In such an automated design optimization procedure, there are several major modules, including preparation of parametric FEA models, CE-FEA implementation, a DE optimization algorithm, and design decision making from Pareto-sets.

Unlike the TS-FEA approach, the CE-FEA method only employs the minimum number of static field solutions such as in [46], [47]. Based on the pole-pitch and slot-pitch symmetrical and periodic property of the electromagnetic field in PM machines, the three phase flux linkages and flux density distributions in the stator core and PMs can be constructed using space-time transformation in [46], [47], [113]. As a consequence, the back-emf and induced voltage waveforms, ripple and average torque, as well as stator core losses can be calculated systematically using the CE-FEA technique [47], [74]. In PM machines with FSCWs, the PM eddy-current losses can be estimated using a hybrid method combining the CE-FEA method with a novel analytical formulation as outlined in Chapter 3. Furthermore, the minimum-effort calculation methods for the PM flux linkage, dq-axes inductances, torque angle for the MTPA load condition, together with further insights into the stator core losses, as well as the skew effects were described in the Chapter 4. The principle and implementation techniques of the DE algorithm have been presented in section 5.3 in Chapter 5.

New contributions described in the chapter include a robust parametric CE-FEA model of a 12- slot 10-pole concentrated winding IPM topology for a brushless (BL) machine driven by a sine-wave current regulated power electronic drive, and a systematic multi-objective design optimization case study. This includes an engineering decision process based on a Pareto-set of optimal designs, and a tradeoff study leading to the selection of a recommended design, which was prototyped and tested.

6.2 Parametric Modeling of a PM Machine

In this section, a 12-slot 10-pole IPM machine, with a V-type layout of permanent magnets in the rotor and a standard NEMA 210-frame, was parameterized and design optimized with the rated condition of 10 hp at 1800 r/min. The detailed parametric model is shown in Figure 6.1 with a zoom-in for the PM component and its parameters given in Figure 6.2.

In order to avoid the geometric conflicts in the automated design optimization procedure, design variables such as the stator inner diameter, D_{si} , tooth width, w_T ,



Figure 6.1: Parametric model of a 12-slot 10-pole BLPM machine.



Figure 6.2: Zoom in of the red rectangular in Figure 6.1.

Design variables	Definition	Min	Max
k_{si}	D_{si}/D_{so}	0.5	0.7
h_g	airgap height	$0.7\mathrm{mm}$	$1.3 \mathrm{mm}$
k_{wT}	α_T/α_s	0.35	0.55
d_Y	yoke thickness	$13.0\mathrm{mm}$	20.0mm
h_{pm}	PM height	$2.5\mathrm{mm}$	$5.0\mathrm{mm}$
k_{wpm}	$2w_{pm}/w_{pm_max}$	0.65	0.95
k_{dpm}	$2d_{pm}/(D_{ro}-D_{ri})$	0.15	0.65
w_q	q-axis bridge width	$0.5\mathrm{mm}$	4.0mm
$lpha_{pm}$	pole arc [elec. deg.]	95	130

Table 6.1: Definition and ranges of nine design variables depicted in Figures 6.1 and 6.2.

PM width, w_{pm} , and PM depth, d_{pm} , were defined using the ratio expressions of k_{si} , k_{wT} , k_{wpm} , and k_{dpm} , as also given in Table 6.1. Here, k_{si} is the split ratio between the stator inner diameter and outer diameter, and k_{wT} is the ratio between the tooth arc angle, α_T , and the slot pitch, $\alpha_s = 2\pi/N_s$, where, N_s is the number of stator slots. In the ratio expression of k_{wpm} , the maximum width of two magnets, w_{pm_max} , can be decided by the magnet depth, d_{pm} , and the pole arc, α_{pm} . In the design optimization, several geometric variables were fixed, such as the stator outer diameter, D_{so} , rotor inner diameter, D_{ri} , distances between PM segments, w_{Fe1} and w_{Fe2} , and the distance from the PM top flux barrier to the rotor outer diameter, w_{rad} . Based on these definitions and assumptions, the selected geometric variables for the DE design optimization are $[k_{si}, h_g, k_{wT}, d_Y, h_{pm}, k_{wpm}, k_{dpm}, w_q, \alpha_{pm}]$ with the corresponding variable ranges provided in Table 6.1.

In the manufacturing process, the slot of the magnet is always wider and thicker

than the actual PM physical cross-sectional dimensions, as shown by the clearances under the PMs in Figure 6.2. Here, the clearance under the PM, h_c , is aligned in series along the flux path in the magnetic circuit. This renders it having significant effects on the performance estimation in the FEA, which will lead to 2-3% difference in the open circuit back-emf estimation. Thus, when parameterizing the model, the clearance must be taken into account.

6.3 Design Optimization Using the DE Algorithm

In the automated design optimization, a DE algorithm was utilized to generate a set of candidate designs, which were analyzed with the CE-FEA method to estimate the torque and induced voltage waveforms, and the losses in the stator core and copper, and PMs [46], [47], [107], [113]. Meanwhile, material costs for the copper, steel lamination, and PM were also calculated. All the simulations were performed on an HP Z800 workstation with 12 cores (2 Xeon X5690 processors) and 32GB RAM memory. Parallel execution for CE-FEA was implemented in order to fully utilize the multiple CPUs and the distributed solve functions available within the ANSYS Maxwell software [94]. Overall, this resulted in a substantial increase of the computational speed as compared with the conventional TS-FEA method.

The DE algorithm aims to find a global minimum or maximum by iteratively improving a population of candidate designs until the stopping criterion is satisfied. The principles of DE optimization and its application to electrical machine problems were previously introduced in [63], [74], [112]. In the case of single-objective problems, the evolution and the "goodness" of the optimized design can be evaluated through simple comparison to other designs. In case of multi-objective problems with multiple constraints, where conflicts may exist between objectives, the stopping criteria and the decision-making based on a Pareto-front are more complicated [114], [111].

6.3.1 Problem Statement

A multi-objective optimization for this BLPM machine requires the DE algorithm to search for designs in order to:

- minimize losses: $P_{loss} = P_{Fe} + P_{Cu} + P_{pm} + P_{fw}$
- minimize the material cost: $Cost = c_{pm}m_{pm} + c_{Cu}m_{Cu} + c_{Fe}m_{Fe}$,

where, P_{Fe} , P_{Cu} , P_{pm} , and P_{fw} are the stator core losses, copper losses, magnet losses, and friction and windage losses, respectively, while m_{pm} , m_{Cu} , and m_{Fe} are the masses of the PM, copper and steel materials, respectively. Here, the specific material costs are denoted by c_{pm} , c_{Cu} and c_{Fe} .

Three design constraints are required and defined by the following expressions:

• the torque ripple under the rated load contion, $\frac{\max(T_e) - \min(T_e)}{average(T_e)} \leq 5\%$,

- the total harmonic distortion (THD) for the rated load induced voltage waveform $\leq 3\%$, and
- minimum flux density in the PMs, B_{min} , under rated load, should be equal to or greater than $0.3B_r$, where, for the PM material used here the retentivity, $B_r = 1.1T$.

In the design optimization procedure, the operating temperature in the windings and PMs for all the candidate designs was assumed to be $100^{\circ}C$. Meanwhile, all the candidate designs have the same slot fill factor and current density, which lead in each case to different ampere-turns due to the changed net slot areas. For each candidate design, the stack length was scaled to obtain a shaft torque of 42 Nm, which corresponds to 10.6 hp output power rating at 1800 r/min.

6.3.2 Design Optimization Results

Based on the previously introduced design specifications, the design optimization of this BLPM machine was performed utilizing the DE algorithm coupled with the electromagnetic CE-FEA. There were 70 individual designs per generation and 50 generations, which yielded a total of 3,500 design candidates. The results of the optimization study in the two-dimensional plane of material cost versus stator loss is shown in Figure 6.3.



Figure 6.3: Scattered plot for 3,500 candidate designs (50 DE generations, each with 70 individuals) analyzed with electromagnetic CE-FEA. Three recommended designs M-1, M-2, and M-3, are identified on the Pareto-front.

From the Pareto-optimal front, defined as the collection of results for which an improvement of one objective can only be achieved through the deterioration of another objective, three candidate designs were selected and labeled as M-1, M-2 and M-3. Design M-1 represents a high efficiency solution, and motor M-3 has lower cost, while machine M-2 is a compromise alternative. The cross sections of these three PM machines are provided in Figure 6.4, and the corresponding geometric variables are presented in Table 6.2, where design M-3 was defined as the reference/base for a per unit system.



Figure 6.4: Cross sections and flux plots of three recommended 12-slot 10-pole designs from the Pareto-front shown in Figure 6.3.

Geometric variables	M-1	M-2	M-3
Axial stack length	1.23	1	1
Stator inner diameter, D_{si}	1.06	0.99	1
Airgap height, h_g	1.82	1.12	1
Tooth width, w_T	1.12	1.05	1
Stator back iron thickness, d_Y	0.94	0.97	1
PM thickness, h_{pm}	0.90	0.88	1
PM width, w_{pm}	1.22	1.29	1
PM depth, d_{pm}	1.28	1.40	1
Q-axis bridge width, w_q	1.24	1.08	1
Pole arc, α_p	0.92	1.04	1

Table 6.2: Relative values for the geometric variables. Machine M-3 was selected as the reference for the other candidate designs.

6.4 Comparison Between Candidate Designs and Optimal Trade-off Studies

For the optimally designed M-1, M-2 and M-3 motors, the weights and material costs, and the performance characteristics at the rated power and rated speed of 1800 r/min are summarized in Tables 6.3 and 6.4, respectively. In industrial applications, such motors operate in a range of variable torque and speed, and in order to provide a more systematic comparison for the three candidate designs, the so-called efficiency maps have been calculated and are shown in Figures 6.5 (a), (b) and (c). On these efficiency maps, the black solid curve corresponds to a typical fan/pump load for the given 10 hp power rating.

Design M-3 was selected for prototyping and in serving as a performance reference,

	Weights [%]			Cost $[\%]$			
	M-1	M-2	M-3	M-1	M-2	M-3	
PM	3.5	3.5	3.1	70.2	70.2	66.6	
Steel	81.6	81.2	80.1	11.9	11.7	12.3	
Copper	14.9	15.3	16.8	17.9	18.1	21.1	

Table 6.3: Weight and cost distributions in percentage. For each design, the total weight and cost are set as the base value.

Table 6.4: Performance of the recommended motor designs from Figure 6.3.

			0	
Performance	units	M-1	M-2	M-3
Saliency ratio		1.17	1.24	1.29
Torque angle	[deg.]	96	97	99
Electromagnetic torque	[Nm]	42.31	42.83	42.87
Electromagnetic power	[W]	7975	8073	8081
Copper loss	[W]	145	124	133
Input power	[W]	8120	8197	8214
PM loss	[W]	18	20	16
Core loss	[W]	166	206	199
Mechanical loss	[W]	91	91	91
Total loss	[W]	420	440	439
Output power	[W]	7700	7757	7775
Shaft torque	[Nm]	41.69	41.62	41.65
Efficiency	[%]	94.83	94.63	94.65
Material cost	[pu]	1.00	0.84	0.78



Figure 6.5: Efficiency maps for three candidate optimum designs.

mainly due to the fact that it has the lowest cost, while still meeting the rated efficiency requirements, hence offering a good tradeoff between the two optimization objectives. The efficiency difference between M-3 and M-1 provided in Figure 6.6 (a), indicates, that for fan/pump applications motor M-1 can provide 0.3% to 0.8% higher efficiency than motor M-3. Nevertheless, design M-3 is superior for high torque low speed operation. The efficiency map difference from Figure 6.6 (b) shows that the M-3 motor has 0% to 0.5% higher efficiency than the M-2 motor.

6.5 Experimental Calibration

An IPM machine prototype based on the recommended M-3 design was built and tested on an active dyno set-up with a computer data acquisition system, as shown in Figures 6.7 and 6.8, respectively. The IPM prototype was energized from a commercially available Yaskawa A1000, sensorless controlled sine-wave drive.

6.5.1 Open Circuit Test

Prior to the load measurements, an open circuit test was performed under "cold" temperature conditions at a winding temperature of $35^{\circ}C$. The phase back-emf validation for open circuit operation at 1800 r/min provided in Figure 6.9 confirms the satisfactory accuracy of the CE-FEA method for such simulations.



Figure 6.6: Efficiency differences between three optimum candidate designs.



Figure 6.7: Test dyno for the 210-frame 10 hp BLPM machine.



Figure 6.8: Data acquisition system for the 210-frame 10hp BLPM machine.



Figure 6.9: Phase back-emf validation at 1800r/min.

6.5.2 On-Load Tests

A comprehensive on-load test for speeds from 600 r/min to 1800 r/min in increments of 300 r/min and for loads from 25% to 125% in increments of 25% of rated torque was performed. It should be noted that with the employed sensorless drive the user has limited control in accurately setting the torque angle, β , accordingly operation at exactly the predicted MTPA could not be ascertained. Instead, the rotor position was measured and this value together with the measured current value were employed in CE-FEA and TS-FEA calculations.

In line with expectations and with previous publications, e.g. [47], [74], the results for the two FEA techniques are in satisfactory agreement, while CE-FEA is one order of magnitude faster. Current and voltage waveforms measured at rated load operation (1800 r/min and 100% load) are shown in Figure 6.10. Another set of three phase current and voltage waveforms are shown in Figure 6.11, which were tested at 600 r/min under 25% load condition. A sample of computed and measured data is provided in Table 6.5.

6.5.3 Discussion for the Loss Separation

In Table 6.5, the calculated copper losses just include the loss component corresponding to the dc resistance part at the measured winding temperature of $35^{\circ}C$. The calculated core losses and PM losses were computed by the two-dimensional (2D) TS-FEA method. For the core loss calculation, the TS-FEA method utilized verified specific core loss coefficients k_h and k_e , which were validated based on a set of open-circuit loss separation tests for a 10 hp prototype PM machine. From such tests, the friction and windage losses were measured separately, for which the PMs were not inserted into the rotor laminations.

When the motor runs at the same speed at various load conditions, the flux density distributions in the stator core do not change significantly, which can be observed from the time-domain flux density waveforms in Figure 6.12 for four distinct locations in the stator core at five load conditions. In this figure, the variation of flux densities at the center points of two adjacent stator teeth and two locations in the yoke were shown, respectively. The locations of these sampling points are shown in Figure 6.13.



Figure 6.10: Three phase current and voltage waveforms at 1800 r/min under rated load condition.



Figure 6.11: Three phase current and voltage waveforms at 600 r/min under 25% load condition.

				С	alcula	ited		Tested	Loss
Speed	Load	Ι	P_{Cu}	P_{Fe}	P_{pm}	P_{fw}	Total	loss	Diff.
r/min	%	Arms	W	W	W	W	W	W	W
	25	3.3	11	97	3	91	200	219	19
	50	6.3	41	106	5	91	241	290	49
1800	75	9.4	90	112	9	91	297	383	86
	100	13.0	172	106	13	91	374	530	156
	125	17.6	319	102	20	91	518	807	289
	25	3.2	10	71	2	69	152	174	23
	50	6.2	39	75	3	69	187	225	39
1500	75	9.2	88	82	6	69	244	324	83
	100	12.4	159	84	9	69	321	443	129
	125	15.8	257	88	13	69	428	614	197
	25	3.2	11	48	1	49	109	138	29
	50	6.2	39	56	2	49	146	177	32
1200	75	9.2	87	63	4	49	203	260	61
	100	12.4	157	69	6	49	281	367	93
	125	15.7	253	70	8	49	382	517	146
	25	3.2	10	31	1	32	74	96	23
	50	6.2	40	33	1	32	106	144	40
900	75	9.2	87	36	2	32	157	205	51
	100	12.4	157	39	3	32	232	305	80
	125	15.7	254	41	5	32	332	434	112
	25	3.1	10	17	0	17	45	65	20
	50	6.1	39	19	1	17	75	98	25
600	75	9.1	85	20	1	17	124	155	35
	100	12.3	156	22	2	17	196	240	50
	125	15.7	252	23	2	17	294	351	67

Table 6.5: Losses for different load and speed conditions. Here, P_{Cu} is calculated based on the measured dc resistance, and P_{Fe} and P_{pm} were calculated using the TS-FEA, while P_{fw} is estimated based on a 10 hp prototype IPM machine.



Figure 6.12: Flux densities in the stator teeth and yoke (four points in Fig. 6.13) at 1800 r/min under different load conditions. (a) Point 1 for tooth. (b) Point 2 for tooth. (c) Point 3 for yoke. (d) Point 4 for yoke.

In the Steinmetz formula in expression (4.5.1), the specific core loss only depends on the flux densities and frequencies in the stator core . Thus, when this PM machine was operated at the same speed under various load conditions, the core losses did not vary significantly. This becomes evident upon examination of the results in Table 6.5.

The tested losses provided in Table 6.5 were equal to the difference between the



Figure 6.13: Flux distribution in the optimal design under the rated load condition.

measured input power and output power of the tested machine, while the output power of such a machine was calculated from the measured shaft torque. The differences between the calculated losses and tested losses are given in the last column in Table 6.5. It was found out that these loss differences, P_{dif} , have a linear relationship with the square of phase current, I^2 , plots of which at various speeds are shown in Figure 6.14. These linear relationships can be expressed as follows:

$$P_{dif} = a I^2 + b (6.5.1)$$

For each operation speed, such a linear expression was computed and provided in Figure 6.14. These differences between the calculated and test losses stem from the



Figure 6.14: Linear relationship between the loss difference and the rms square values of the phase current. P_{dif} is the loss difference between the calculated results and test results.

extra ac copper losses in the stator windings, which are caused by the skin and proximity effects resulting from the fringing flux around the stator slots, a depiction of which is shown in Figure 6.13 [115–117].

6.6 Summary

The method presented in this work for the large-scale design optimization of currentregulated synchronous PM machines based on the CE-FEA method was demonstrated on a concentrated winding 12-slot 10-pole IPM case study rated at 10 hp. Based on a robust parametric model, nine independent variables were selected for a DE optimization with the concurrent objectives of minimizing losses and material cost. A total of 3,500 candidate designs were analyzed and automatically compared yielding a Pareto-set of recommended designs. An engineering analysis and discussion of trade-offs between several candidate designs under variable speed operation were performed. This led to a reasonable selection of one design, which has been prototyped and successfully tested. The differences between the calculated losses and test losses were calibrated. This calibration points to excess ohmic losses in the stator coils attributable to fringing flux proximity effects and skin effects, in the stator conductors, which will be the subject of future investigations. Overall, the experimental tests confirmed the results of the design optimization study and soundness of the approach presented in this work.

In the next chapter, the conclusions, contributions of this work will be summarized. Based on the obtained research results, possible future work in continuation of this research will be presented.

CHAPTER 7

CONCLUSIONS, CONTRIBUTIONS AND FUTURE WORK

In this chapter, the conclusions and main contributions resulting from these research activities associated with this dissertation are summarized. This is followed by some recommendations regarding possible research directions for future work.

7.1 Summary and Conclusions

This dissertation focused on the study of a novel design optimization technique for fault-tolerant permanent magnet (PM) machine-drive systems. In Chapter 1, the problem background regarding this research topic was introduced. Through a relatively extensive literature search, the recent trends in several topics related to the subject of this dissertation were reviewed. This includes different types of PM machines and their corresponding applications, as well as modeling and analysis approaches for electric machines and associated design optimization algorithms. Based on this literature search, the main objectives of this work were delineated in Chapter 1.

In Chapter 2, several fault-tolerant topologies for PM machines were discussed. Based on the fault-tolerant requirements for PM machines, the 12-slot, 10-pole, PM machines with V-type and spoke-type PM layouts were selected as the candidate topologies for the fault-tolerant PM machine design to be investigated and optimized in this dissertation.

Accordingly, the combination of 12-slot and 10-pole configuration requires the stator windings for PM machines to be of the fractional-slot concentrated winding (FSCW) type. Thus, the PM losses can be especially significant because of the expected rich harmonic content in the armature mmf. Consequently, in Chapter 3, a hybrid method which combines the computationally efficient finite element analysis (CE-FEA) method with a new analytical formulation was developed to compute the eddy-current losses in the PMs of sine-wave current regulated brushless PM machines subject of this dissertation. The results provided by two FSCW interior permanent magnet (IPM) machine case studies demonstrated satisfactory accuracy of PM loss calculation and significant decrease in the associated computational time as compared with the well-known, though time-consuming, time-stepping finite element analysis (TS-FEA) method. Based on these advantages, the new PM loss calculation method is considered to be particularly suitable for incorporation into large-scale design optimization tools in industrial environments. Because this developed PM loss calculation method incorporates the 3D end effects, it can be employed to study the impact on losses of PM block segmentation in the circumferential and axial directions, under the typical assumptions of resistance limited eddy currents in the PMs. The sensitivity of the method to PWM switching harmonics was also successfully demonstrated on two IPM machine case studies.

In Chapter 4, a detailed procedure and principle of the implementation of the CE-FEA method within ANSYS-Maxwell software packages was described. First, the calculation method for PM flux linkage and dq-axes inductances was presented. Then, these parameters were utilized to calculate the torque angle for the maximum torque per ampere (MTPA) load condition for each design case in the automated design optimization procedure. The CE-FEA based calculation procedure for the stator core losses was also presented in this chapter. Several methods for taking account of the skew effects into the calculation of the phase flux linkages, phase induced voltages and torque profiles were discussed. The accuracy of the CE-FEA method was validated by several case studies provided in this chapter. For the automated design optimization of PM machines, a requisite step is building a robust and flexible parametric model for each design optimization problem. In this Chapter, the parametric modeling of PM machines using FEA software packages was described. In order to increase the robustness of the parametric model for the design optimization procedure, several geometric parameters were ratio parameterized to avoid geometry conflicts, which were described separately for the stator slots and rotor poles.

In Chapter 5, a combined design optimization method, utilizing the design of experiments (DOE) and differential evolution (DE) algorithms, was developed. According to this procedure, based on a DOE sensitivity study, design variables with
significant or with conflicting effects on the multiple optimization objectives, for example the total material cost, power losses and torque ripples, were selected to be independent parameters for a global DE optimization. This resulted in a reduction of the design space, which in turn led to fewer candidate designs to be considered per generation/population. Further advantages in terms of reducing the computational effort for the DE optimization were provided through narrower ranges for the variables, as per the DOE findings. This combined two-pronged design optimization method was implemented into design optimization case studies for PM machines with four different rotor topologies. These rotor topologies include the two different V-shape, spoke-type, and flat bar-type PM layouts. The optimal DE results for the 10 hp 1,800 r/min example rating represent the basis for a systematic comparison between these four IPM motor topologies. The data provides interesting insights into the relative merits of each configuration for the specified objectives and constraints, which were detailed in this chapter.

In Chapter 6, the automated design optimization method utilizing the CE-FEA techniques and a DE algorithm was implemented for a case study of a 12-slot 10-pole V-type PM machine which is in demand for several industrial drive applications. One optimal design was selected based on an engineering analysis and discussion of trade-offs under variable speed operation. The final selected design has been prototyped and successfully tested. The differences between the calculated losses and test losses

were calibrated and explained. The experimental tests confirmed the results of the design optimization study and approach presented in this dissertation.

7.2 Contributions

The main contributions resulting from this dissertation's work can best be summarized as follows:

- A comparison between PM machines with different rotor topologies and stator winding layouts was performed for fault-tolerant PM machine investigations. Finally, a combination of 12-slot and 10-pole configuration was selected to be investigated in this dissertation, which requires a FSCW layout in the stator. Meanwhile, based on the results of this work, the V-type and spoke-type PM layouts in the rotor were recommended for the design of such fault-tolerant PM machines.
- 2. A new analytical formulation for the calculation of the PM eddy-current losses was developed to be combined with the CE-FEA method. The results from two FSCW IPM machine case studies demonstrated satisfactory accuracy and significant decrease in the computational time as compared to the 2D and 3D TS-FEA method. This method can be utilized in large-scale design optimization problems for PM machines with FSCWs to calculate the PM eddy-current losses.

- 3. The automated design optimization method, including the novel idea of utilizing the CE-FEA and DE algorithms was implemented, using the MATLAB script function and ANSYS Maxwell software. The detailed implementation techniques for such a method were clearly described. The "distributed solve" function package in ANSYS-Maxwell software was utilized to take advantage of parallel processing, leading to improvements in the computational speed of this global design optimization method, by a factor of more than "2" for the case studies considered in this dissertation.
- 4. A new combined design optimization method utilizing DOE and DE algorithms was developed and coupled to the CE-FEA techniques. The central composite design (CCD) approach, which is the most popular DOE method, was utilized to perform design variables' sensitivity studies. The response surface methodology (RSM) approach was utilized to generate the response surface function (second-order polynomial function for the CCD method) for each design objective. This process contributed to the reduction of the simulation time to practical ranges between zero and seven hours per case study, and to the successful convergence of the DE algorithm for the whole design optimization procedure in all the case studies considered here in this dissertation.

In addition to the above mentioned main contributions, this dissertation also contributed to the following investigations: First, the PM machines with different stator and rotor topologies were ratio parameterized in this dissertation. This can increase the robustness of the parametric model for the design optimization procedure. A detailed explanation for these parametric models were given in this dissertation.

Second, the efficacy of the automated design optimization method was verified in an industrial environment through the application to a 10 hp, 12-slot, 10-pole, Vtype, PM machine case study. An optimal design was selected based on an engineering comparison and tradeoffs study of three candidate designs. The selected design was prototyped and successfully tested to validate the findings of this work.

7.3 Possible Future Work

Based on the research results and progress obtained from this dissertation, as well as earlier research work by others, possible research directions in continuation of this work should be considered. Some of this future work may include the following:

1. In the modeling analysis of PM machines with sine-wave current supply utilizing the CE-FEA method, the rotor core loss has not been investigated. Especially for PM machines with FSCWs, which have mmfs with rich harmonic content, that will cause higher rotor core losses. The symmetric and periodic properties of the magnetic field in the rotor need to be first investigated. Then, space-time transformation techniques can be accurately employed for the rotor magnetic fields' construction in the CE-FEA method, for purposes of rotor loss calculations.

- 2. A more accurate modeling method that will consider the PWM carrier frequency effects into the calculation of the PM eddy-current loss and core loss need to be investigated, especially for the integration of such loss computation into the CE-FEA based design optimization techniques. For the PM eddy-current losses, more sampling grids in the PMs can be implemented to observe the magnetic field distribution in such PMs. This will provide more accurate results for the computation of PM losses especially with PWM excitations.
- 3. The CE-FEA method can be implemented into different electric machine designs, for example the surface-mounted permanent magnet (SPM) machines, synchronous reluctance machines (Syn-RMs), PM assisted Syn-RMs, etc. Meanwhile, the combined design optimization method utilizing DOE and DE algorithms can be implemented for these design optimization problems. This will provide more systematic comparison between different electric machines, for example the comparison between PM machines and Syn-RM machines.
- 4. Different mutation and crossover strategies as well as stopping criteria for DE algorithms can be implemented into the automated design optimization tools

to study the convergence property of such design optimization problems.

- 5. Multi-physics modeling techniques including the embedded thermal and vibration analysis need to be investigated for incorporation to a systematic design optimization of electric machines.
- 6. Modeling and analysis methods for studying of skin and proximity effects on the stator winding ohmic losses for PM machines need to be investigated. This part of the stator winding ohmic losses is more significant for PM machines with FSCWs, because of the wider slot opening in comparison to PM machines with integer-slot distributed windings, which results in larger amounts of flux fringing into such stator slots of FSCW configurations.

BIBLOGRAPHY

- D. M. Ionel, "High-Efficiency Variable-Speed Electric Motor Drive Technologies for Energy Savings in the US Residential Sector," in 2010 12th International Conference on Optimization of Electrical and Electronic Equipment (OPTIM), May 2010, pp. 1403 –1414.
- [2] X. Yang, D. Patterson, and J. Hudgins, "Permanent Magnet Generator Design and Control for Large Wind Turbines," in 2012 IEEE Power Electronics and Machines in Wind Applications (PEMWA),, 2012, pp. 1–5.
- [3] B. Mecrow, A. Jack, J. Haylock, and J. Coles, "Fault-Tolerant Permanent Magnet Machine Drives," *IEE Proceedings-Electric Power Applications*, vol. 143, no. 6, pp. 437–442, 1996.
- [4] P. Y. Papalambros and D. J. Wilde, *Principles of Optimal Design: Modeling* and Computation. Cambridge University Press, 2000.
- [5] A. El-Refaie, "Fault-Tolerant Permanent Magnet Machines: a Review," IET Electric Power Applications, vol. 5, no. 1, pp. 59–74, 2011.
- [6] A. EL-Refaie, "Fractional-Slot Concentrated-Windings Synchronous Permanent Magnet Machines: Opportunities and Challenges," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 1, pp. 107–121, 2010.
- [7] W. Ouyang and T. Lipo, "Modular Permanent Magnet Machine With Fault Tolerant Capability," in Annual IEEE Applied Power Electronics Conference and Exposition (APEC), 2009, pp. 930–937.
- [8] A. Mitcham, G. Antonopoulos, and J. J. A. Cullen, "Favourable Slot and Pole Number Combinations for Fault-Tolerant PM Machines," *IEE Proceedings-Electric Power Applications*, vol. 151, no. 5, pp. 520–525, 2004.
- [9] N. Bianchi, S. Bolognani, and M. D. Pre, "Impact of Stator Winding of a Five-Phase Permanent-Magnet Motor on Postfault Operations," *IEEE Transactions* on Industrial Electronics, vol. 55, no. 5, pp. 1978–1987, 2008.

- [10] N. Bianchi, S. Bolognani, and M. Pre, "Strategies for the Fault-Tolerant Current Control of a Five-Phase Permanent-Magnet Motor," *IEEE Transactions on Industry Applications*, vol. 43, no. 4, pp. 960–970, 2007.
- [11] L. Parsa, H. Toliyat, and A. Goodarzi, "Five-Phase Interior Permanent-Magnet Motors With Low Torque Pulsation," *IEEE Transactions on Industry Applications*, vol. 43, no. 1, pp. 40–46, 2007.
- [12] J. He, G. Liang, and S. Wang, "The Design of a Switched Reluctance Motor Drive System," in *Informatics in Control, Automation and Robotics, 2009. CAR* '09. International Asia Conference on, 2009, pp. 445–449.
- [13] I. Brown, D. Ionel, and D. Dorrell, "Influence of Parallel Paths on Current-Regulated Sine-Wave Interior-Permanent-Magnet Machines With Rotor Eccentricity," *IEEE Transactions on Industry Applications*, vol. 48, no. 2, pp. 642– 652, 2012.
- [14] R. Deodhar, S. Andersson, I. Boldea, and T. J. E. Miller, "The Flux-Reversal Machine: a new Brushless Doubly-Salient Permanent-Magnet Machine," *IEEE Transactions on Industry Applications*, vol. 33, no. 4, pp. 925–934, 1997.
- [15] Y. Liao, F. Liang, and T. Lipo, "A Novel Permanent Magnet Motor With Doubly Salient Structure," *IEEE Transactions on Industry Applications*, vol. 31, no. 5, pp. 1069–1078, 1995.
- [16] Z. Zhu and J. Chen, "Advanced Flux-Switching Permanent Magnet Brushless Machines," *IEEE Transactions on Magnetics*, vol. 46, no. 6, pp. 1447–1453, 2010.
- [17] J. Chen and Z. Zhu, "Winding Configurations and Optimal Stator and Rotor Pole Combination of Flux-Switching PM Brushless AC Machines," *IEEE Transactions on Energy Conversion*, vol. 25, no. 2, pp. 293–302, 2010.
- [18] W. Hua, M. Cheng, H. Jia, and X. Fu, "Comparative Study of Flux-Switching and Doubly-Salient PM Machines Particularly on Torque Capability," in *IEEE Industry Applications Society Annual Meeting (IEEE-IAS)*, 2008, pp. 1–8.
- [19] D. M. Ionel, R. J. Heideman, and R. P. Bartos, "Spoke Permanent Magnet Rotors for Electrical Machines and Methods of Manufacturing Same," U.S. Patent 7 148 598, Oct. 23, 2003.

- [20] K.-C. Kim and J. Lee, "The Dynamic Analysis of a Spoke-Type Permanent Magnet Generator With Large Overhang," *IEEE Transactions on Magnetics*, vol. 41, no. 10, pp. 3805–3807, 2005.
- [21] D. Dorrell, M.-F. Hsieh, and A. Knight, "Alternative Rotor Designs for High Performance Brushless Permanent Magnet Machines for Hybrid Electric Vehicles," *IEEE Transactions on Magnetics*, vol. 48, no. 2, pp. 835–838, 2012.
- [22] H.-W. Kim, K.-T. Kim, B.-W. Kim, J. Hur, and Y.-S. Jo, "Design of new Spoke Type Brushless DC Motor for Neodymium Permanent Magnet Free," in *IEEE Vehicle Power and Propulsion Conference (VPPC)*, 2012, pp. 133–137.
- [23] S.-I. Kim, J. Cho, S. Park, T. Park, and S. Lim, "Characteristics Comparison of a Conventional and Modified Spoke-Type Ferrite Magnet Motor for Traction Drives of low-Speed Electric Vehicles," in *IEEE Energy Conversion Congress* and Exposition (ECCE), 2012, pp. 3048–3054.
- [24] N. Demerdash and T. Nehl, "An Evaluation of the Methods of Finite Elements and Finite Differences in the Solution of Nonlinear Electromagnetic Fields in Electrical Machines," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-98, no. 1, pp. 74–87, 1979.
- [25] N. Demerdash, J. Bangura, and A. Arkadan, "A Time-Stepping Coupled Finite Element-State Space Model for Induction Motor Drives. I. Model Formulation and Machine Parameter Computation," *IEEE Transactions on Energy Conver*sion, vol. 14, no. 4, pp. 1465–1471, 1999.
- [26] J. Bangura, F. N. Isaac, N. Demerdash, and A. Arkadan, "A Time-Stepping Coupled Finite Element-State Space Model for Induction Motor Drives. II. Machine Performance Computation and Verification," *IEEE Transactions on En*ergy Conversion, vol. 14, no. 4, pp. 1472–1478, 1999.
- [27] F. Fouad, T. Nehl, and N. Demerdash, "Magnetic Field Modeling of Permanent Magnet Type Electronically Operated Synchronous Machines Using Finite Elements," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 9, pp. 4125–4135, 1981.
- [28] F. Deng and N. A. Demerdash, "Comprehensive Salient-Pole Synchronous Machine Parametric Design Analysis Using Time-Step Finite Element-State Space Modeling Techniques," *IEEE Transactions on Energy Conversion*, vol. 13, no. 3,

pp. 221-229, 1998.

- [29] D. Ionel, M. Balchin, J. Eastham, and E. Demeter, "Finite Element Analysis of Brushless DC Motors for Flux Weakening Operation," *IEEE Transactions on Magnetics*, vol. 32, no. 5, pp. 5040–5042, 1996.
- [30] E. Deng and N. A. O. Demerdash, "A Coupled Finite-Element State-Space Approach for Synchronous Generators. I. Model Development," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 2, pp. 775–784, 1996.
- [31] N. Demerdash, T. Nehl, and F. Fouad, "Finite Element Formulation and Analysis of Three Dimensional Magnetic Field Problems," *IEEE Transactions on Magnetics*, vol. 16, no. 5, pp. 1092–1094, 1980.
- [32] R. Wang and N. Demerdash, "On the Effects of Grid ill-Conditioning in Three Dimensional Finite Element Vector Potential Magnetostatic Field Computations," *IEEE Transactions on Magnetics*, vol. 26, no. 5, pp. 2190–2192, 1990.
- [33] N. A. O. Demerdash and M. Alhamadi, "Three-Dimensional Finite Element Analysis of Permanent Magnet Brushless DC Motor Drives-Status of the State of the art," *IEEE Transactions on Industrial Electronics*, vol. 43, no. 2, pp. 268–275, 1996.
- [34] V. Ostovic, "Magnetic Equivalent Circuit Presentation of Electric Machines," Electric Machines and Power Systems, vol. 12, pp. 407–437, June 1987.
- [35] ——, "A Simplified Approach to Magnetic Equivalent-Circuit Modeling of Induction Machines," *IEEE Transactions on Industry Applications*, vol. 24, no. 2, pp. 308–316, March/April 1988.
- [36] J. Law, T. Busch, and T. Lipo, "Magnetic Circuit Modelling of the Field Regulated Reluctance Machine. Part I: Model Development," *IEEE Transactions* on Energy Conversion, vol. 11, no. 1, pp. 49–55, 1996.
- [37] G. Y. Sizov, Analysis, Modeling, and Diagnostics of Adjacent and Nonadjacent Broken Rotor Bars in Squirrel-Cage Induction Machines. Marquette University, 2007.
- [38] S.-K. Baik, Y.-K. Kwon, E.-Y. Lee, J. deuk Lee, J.-P. Hong, Y.-C. Kim, T.-S.

Moon, H.-J. Park, and W. sik Kwon, "Design and Analysis of 1 MW Synchronous Machine via 3D Magnetic Field Calculation," *IEEE Transactions on Applied Superconductivity*, vol. 17, no. 2, pp. 1549–1552, 2007.

- [39] M.-F. Hsieh and Y.-C. Hsu, "A Generalized Magnetic Circuit Modeling Approach for Design of Surface Permanent-Magnet Machines," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 2, pp. 779–792, 2012.
- [40] X. Lu, K. Iyer, K. Mukherjee, and N. Kar, "Development of a Novel Magnetic Circuit Model for Design of Premium Efficiency Three-Phase Line Start Permanent Magnet Machines With Improved Starting Performance," *IEEE Transactions on Magnetics*, vol. 49, no. 7, pp. 3965–3968, 2013.
- [41] A. A. Abdel-Razek, J. L. Coulomb, M. Feliachi, and J. Sabonnadiere, "The Calculation of Electromagnetic Torque in Saturated Electric Machines Within Combined Numerical and Analytical Solutions of the Field Equations," *IEEE Transactions on Magnetics*, vol. 17, no. 6, pp. 3250–3252, 1981.
- [42] Z. Liu, C. Bi, H. C. Tan, and T. S. Low, "A Combined Numerical and Analytical Approach for Magnetic Field Analysis of Permanent Magnet Machines," *IEEE Transactions on Magnetics*, vol. 31, no. 3, pp. 1372–1375, 1995.
- [43] Y.-J. Zhang, J.-Z. Jiang, and G.-Z. Tu, "Numerical-Analytical Method for Magnetic Field Computation in Rotational Electric Machines," *Journal of Shanghai* University (English Edition), vol. 7, no. 3, pp. 270–274, 2003.
- [44] G. Sizov, D. Ionel, and N. A. O. Demerdash, "A Review of Efficient FE Modeling Techniques Eith Applications to PM AC Machines," in 2011 IEEE Power and Energy Society General Meeting, 2011, pp. 1–6.
- [45] D. M. Ionel and M. Popescu, "Finite-Element Surrogate Model for Electric Machines with Revolving Field-Application to IPM Motors," *IEEE Transactions* on *Industry Applications*, vol. 46, no. 6, pp. 2424 –2433, Nov.-Dec. 2010.
- [46] D. M. Ionel and M. M. Popescu, "Ultrafast Finite-Element Analysis of Brushless PM Machines Based on SpaceTime Transformations," *IEEE Transactions on Industry Applications*, vol. 47, no. 2, pp. 744–753, March-April 2011.
- [47] G. Y. Sizov, D. M. Ionel, and N. A. O. Demerdash, "Modeling and Parametric

Design of Permanent-Magnet AC Machines Using Computationally Efficient-Finite Element Analysis," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 6, pp. 2403–2413, June 2012.

- [48] G. Sizov, D. Ionel, and N. Demerdash, "Multi-Objective Optimization of PM AC Machines Using Computationally Efficient-FEA and Differential Evolution," in *IEEE International Electric Machines Drives Conference 2011* (*IEMDC '11*), may 2011, pp. 1528-1533.
- [49] G. Y. Sizov, P. Zhang, D. M. Ionel, N. A. O. Demerdash, and M. Rosu, "Automated Bi-Objective Design Optimization of Multi-MW Direct-Drive PM Machines Using CE-FEA and Differential Evolution," in *IEEE Energy Conversion Congress and Exposition (ECCE)*, 2011, pp. 3672–3678.
- [50] R. O. Kuehl, Design of Experiments: Statistical Principle of Research Design and Anlysis, Second edition. Duxbury/Thomson Learning, 2000.
- [51] S. Das and P. Suganthan, "Differential Evolution: A Survey of the State-ofthe-Art," *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 1, pp. 4–31, 2011.
- [52] Y. Duan and D. M. Ionel, "A Review of Recent Developments in Electrical Machine Design Optimization Methods With a Permanent-Magnet Synchronous Motor Benchmark Study," *IEEE Transactions on Industry Applications*, vol. 49, no. 3, pp. 1268–1275, 2013.
- [53] R. H. Myers and D. C. Montgomery, Response Surface Methodology: Process and Product Optimization Using Designed Experiments. John Wiley & Sons, 2002.
- [54] S.-I. Kim, J.-P. Hong, Y.-K. Kim, H. Nam, and H.-I. Cho, "Optimal Design of Slotless-Type PMLSM Considering Multiple Responses by Response Surface Methodology," *IEEE Transactions on Magnetics*, vol. 42, no. 4, pp. 1219–1222, 2006.
- [55] L. Fang, J.-W. Jung, J.-P. Hong, and J.-H. Lee, "Study on High-Efficiency Performance in Interior Permanent-Magnet Synchronous Motor With Double-Layer PM Design," *IEEE Transactions on Magnetics*, vol. 44, no. 11, pp. 4393– 4396, 2008.

- [56] P. Zhang, S.-O. Kwon, L. Fang, and J. pyo Hong, "The Design and Analysis of a High Efficiency Permanent Magnet Reluctance Motor," in *International Conference on Electrical Machines and Systems (ICEMS)*, 2006, pp. 1–7.
- [57] J. woo Jung, P. Zhang, J.-P. Hong, and J. young Lee, "Design for Total Harmonic Distortion Reduction of Concentric Winding Type IPMSM for Integrated Starter and Generator," in *International Conference on Electrical Machines and* Systems (ICEMS), 2006, pp. 1–7.
- [58] Y.-K. Kim, S.-H. Rhyu, and I.-S. Jung, "Robust Design Based on the Response Surface Method for BLDC Motor," in *International Conference on Electrical Machines and Systems (ICEMS)*, 2010, pp. 1159–1162.
- [59] I. Brown and R. Lorenz, "Response Surface Methodologies for the Design of Induction Machine Self-Sensing Rotor Position Saliencies," in *International Conference on Electrical Machines and Systems (ICEMS)*, 2007, pp. 1354–1359.
- [60] I. P. Brown and R. D. Lorenz, "Induction Machine Design Methodology for Self-Sensing: Balancing Saliencies and Power Conversion Properties," *IEEE Transactions on Industry Applications*, vol. 47, no. 1, pp. 79–87, 2011.
- [61] Y.-K. Kim, J.-P. Hong, and J. Hur, "Torque Characteristic Analysis Considering the Manufacturing Tolerance for Electric Machine by Stochastic Response Surface Method," *IEEE Transactions on Industry Applications*, vol. 39, no. 3, pp. 713–719, 2003.
- [62] M. Mitchell, An Introduction to Genetic Algorithms, 3rd ed. MIT Press, 1998.
- [63] K. V. Price, R. M. Storn, and J. A. Lampinen, *Differential Evolution-A Practical Approach to Global Optimization*. Springer-Verlag Berlin Heidelberg, 2005.
- [64] G. Pellegrino and F. Cupertino, "FEA-based Multi-Objective Optimization of IPM Motor Design Including Rotor Losses," in *IEEE Energy Conversion Congress and Exposition (ECCE)*, Sept. 2010, pp. 3659–3666.
- [65] W. Jiang, T. M. Jahns, T. A. Lipo, W. Taylor, and Y. Suziki, "Machine Design Optimization Based on Finite Element Analysis in a High-Throughput Computing Environment," in *IEEE Energy Conversion Congress and Exposition(ECCE)*, Sept. 2012, pp. 869–876.

- [66] D. Zarko, D. Ban, and D. Goricki, "Improvement of a Servo Motor Design Including Optimization and Cost Analysis," in 12th International Power Electronics and Motion Control Conference (EPE-PEMC), 2006, pp. 302–307.
- [67] W. Ouyang, D. Zarko, and T. A. Lipo, "Permanent Magnet Machine Design Practice and Optimization," in 41st IEEE Industry Applications Society (IAS) Annual Meeting Conference, vol. 4, Oct. 2006, pp. 1905 –1911.
- [68] M. Barcaro, N. Bianchi, and F. Magnussen, "Permanent-Magnet Optimization in Permanent-Magnet-Assisted Synchronous Reluctance Motor for a Wide Constant-Power Speed Range," *IEEE Transaction on Industrial Electronics*, vol. 59, no. 6, pp. 2495–2502, June 2012.
- [69] F. Parasiliti, M. Villani, S. Lucidi, and F. Rinaldi, "Finite-Element-Based Multiobjective Design Optimization Procedure of Interior Permanent Magnet Synchronous Motors for Wide Constant-Power Region Operation," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 6, pp. 2503–2514, June 2012.
- [70] A. Arkadan, M. ElBsat, and M. Mneimneh, "Particle Swarm Design Optimization of ALA Rotor SynRM for Traction Applications," *IEEE Transaction on Magnetics*, vol. 45, no. 3, pp. 956–959, March 2009.
- [71] L. Jolly, M. Jabbar, and L. Qinghua, "Optimization of the Constant Power Speed Range of a Saturated Permanent-Magnet Synchronous Motor," *IEEE Transactions on Industry Applications*, vol. 42, no. 4, pp. 1024–1030, 2006.
- [72] H. Hasanien, A. Abd-Rabou, and S. Sakr, "Design Optimization of Transverse Flux Linear Motor for Weight Reduction and Performance Improvement Using Response Surface Methodology and Genetic Algorithms," *IEEE Transactions* on Energy Conversion, vol. 25, no. 3, pp. 598–605, 2010.
- [73] Y. Duan, R. Harley, and T. Habetler, "Comparison of Particle Swarm Optimization and Genetic Algorithm in the Design of Permanent Magnet Motors," in *IEEE 6th International Power Electronics and Motion Control Conference* (*IPEMC*), 2009, pp. 822–825.
- [74] G. Sizov, P. Zhang, D. Ionel, N. Demerdash, and M. Rosu, "Automated Multi-Objective Design Optimization of PM AC Machines Using Computationally Efficient- FEA and Differential Evolution," *IEEE Transactions on Industry Applications*, vol. 49, no. 5, pp. 2086–2096, 2013.

- [75] L. Jolly, M. Jabbar, and L. Qinghua, "Design Optimization of Permanent Magnet Motors Using Response Surface Methodology and Genetic Algorithms," *IEEE Transactions on Magnetics*, vol. 41, no. 10, pp. 3928–3930, 2005.
- [76] H. Hasanien, "Particle Swarm Design Optimization of Transverse Flux Linear Motor for Weight Reduction and Improvement of Thrust Force," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 9, pp. 4048–4056, 2011.
- [77] A. Bonnett and G. Soukup, "Cause and Analysis of Stator and Rotor Failures in Three-Phase Squirrel-Cage Induction Motors," *IEEE Transactions on Industry Applications*, vol. 28, no. 4, pp. 921–937, 1992.
- [78] M. Degner, R. Van Maaren, A. Fahim, D. Novotny, R. Lorenz, and C. Syverson, "A Rotor Lamination Design for Surface Permanent Magnet Retention at High Speeds," *IEEE Transactions on Industry Applications*, vol. 32, no. 2, pp. 380– 385, 1996.
- [79] N. Bianchi and S. Bolognani, "Design Techniques for Reducing the Cogging Torque in Surface-Mounted PM Motors," *IEEE Transactions on Industry Applications*, vol. 38, no. 5, pp. 1259–1265, 2002.
- [80] A. EL-Refaie and T. Jahns, "Scalability of Surface PM Machines With Concentrated Windings Designed to Achieve Wide Speed Ranges of Constant-Power Operation," *IEEE Transactions on Energy Conversion*, vol. 21, no. 2, pp. 362– 369, 2006.
- [81] K. Sakai, T. Hattori, N. Takahashi, M. Arata, and T. Tajima, "High Efficiency and High Performance Motor for Energy Saving in Systems," in *IEEE Power Engineering Society Winter Meeting*, vol. 3, 2001, pp. 1413–1418.
- [82] H. Murakami, Y. Honda, S. Morimoto, and Y. Takeda, "A Permanent Magnet-Assisted Synchronous Reluctance Motor," *Electrical Engineering in Japan*, vol. 142, no. 4, pp. 66–74, 2003.
- [83] N. A. O. Demerdash and T. Nehl, "Use of Numerical Analysis of Nonlinear Eddy Current Problems by Finite Elements in the Determination of Parameters of Electrical Machines Eith Solid Iron Rotors," *IEEE Transactions on Magnetics*, vol. 15, no. 6, pp. 1482 – 1484, Nov. 1979.

- [84] A. Krawczyk and J. Tegopoulous, Numerical Modelling of Eddy Currents. Oxford University Press, 1993.
- [85] F. Deng, "Commutation-Caused Eddy-Current Losses in Permanent-Magnet Brushless DC Motors," *IEEE Transactions on Magnetics*, vol. 33, no. 5, pp. 4310–4318, 1997.
- [86] F. Deng and T. Nehl, "Analytical Modeling of Eddy-Current Losses Caused by Pulse-Width-Modulation Switching in Permanent-Magnet Brushless Direct-Current Motors," *IEEE Transactions on Magnetics*, vol. 34, no. 5, pp. 3728 –3736, Sep. 1998.
- [87] K. Atallah, D. Howe, P. Mellor, and D. Stone, "Rotor Loss in Permanent-Magnet Brushless AC Machines," *IEEE Transactions on Industry Applications*, vol. 36, no. 6, pp. 1612–1618, 2000.
- [88] Z. Zhu, K. Ng, N. Schofield, and D. Howe, "Improved Analytical Modelling of Rotor Eddy Current Loss in Brushless Machines Equipped With Surface-Mounted Permanent Magnets," *IEE Proceedings-Electric Power Applications*, vol. 151, no. 6, pp. 641 – 650, Nov. 2004.
- [89] A. Knight and Y. Zhan, "Identification of Flux Density Harmoncis and Resulting Iron Losses in Induction Machines With Nonsinusoidal Supplies," *IEEE Transactions on Magnetics*, vol. 44, no. 6, pp. 1562 –1565, June 2008.
- [90] T. Miller, M. McGilp, and K. Klontz, "Approximate Methods for Calculating Rotor Losses in Permanent-Magnet Brushless Machines," in *IEEE International Electric Machines and Drives Conference 2009 (IEMDC '09)*, May 2009, pp. 1–8.
- [91] K. Yamazaki and A. Abe, "Loss Investigation of Interior Permanent-Magnet Motors Considering Carrier Harmoncis and Magnet Eddy Currents," *IEEE Transactions on Industry Applications*, vol. 45, no. 2, pp. 659–665, March-April 2009.
- [92] K. Yamazaki and S. Watari, "Loss Analysis of Permanent-Magnet Motor Considering Carrier Harmonics of PWM Inverter Using Combination of 2-D and 3-D Finite-Element Method," *IEEE Transactions on Magnetics*, vol. 41, no. 5, pp. 1980–1983, May 2005.

- [93] W.-Y. Huang, A. Bettayeb, R. Kaczmarek, and J.-C. Vannier, "Optimization of Magnet Segmentation for Reduction of Eddy-Current Losses in Permanent Magnet Synchronous Machine," *IEEE Transactions on Energy Conversion*, vol. 25, no. 2, pp. 381–387, June 2010.
- [94] ANSYS Maxwell. [Online]. Available: http://www.ansys.com/
- [95] A. E. Fitzgerald, J. Charles Kingsley, and S. D. Umans, *Electric Machinery*, Sixth Edition. McGraw-Hill Higher Education, 2003.
- [96] R. Wang and N. A. Demerdash, "Extra High Speed Modified Lundell Alternator Parameters and Open/Short-Circuit Characteristics From Global 3-D-FE Magnetic Field Solutions," *IEEE Transactions on Energy Conversion*, vol. 7, no. 2, pp. 330 –341, Jun 1992.
- [97] R. Wang, and N. A. Demerdash, "Computation of Load Performance and Other Parameters of Extra High Speed Modified Lundell Alternators From 3D-FE Magnetic Field Solutions," *IEEE Transactions on Energy Conversion*, vol. 7, no. 2, pp. 342 –352, Jun. 1992.
- [98] F. Deng and N. A. Demerdash, "CFE-SS Approach for Salient-Pole Synchronous Generators Under Unbalances," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no. 1, pp. 142–162, Jan. 1997.
- [99] S. Chaudhry, S. Ahmed-Zaid, and N. A. Demerdash, "Coupled Finite-Element State-Space Modeling of Turbogenerators in the abc Frame of Reference-the no-Load Case," *IEEE Transactions on Energy Conversion*, vol. 10, no. 1, pp. 56–62, Mar. 1995.
- [100] S. Chaudhry, S. AhmedZaid, and N. A. Demerdash, "Coupled Finite-Element State-Space Modeling of Turbogenerators in the abc Frame of Reference-the Short-Circuit and Load Cases Including Saturated Parameters," *IEEE Transactions on Energy Conversion*, vol. 10, no. 1, pp. 63–70, Mar. 1995.
- [101] X. Yang, D. Patterson, and J. Hudgins, "Core Loss Measurement in a Fabricated Stator of a Single-Sided Axial Flux Permanent Magnet Machine," in 2013 IEEE International Electric Machines Drives Conference (IEMDC), 2013, pp. 612– 617.

- [102] G. Bertotti, "General Properties of Power Losses in Soft Ferromagnetic Materials," *IEEE Transactions on Magnetics*, vol. 24, no. 1, pp. 621–630, Jan 1988.
- [103] D. Ionel, M. Popescu, M. McGilp, T. Miller, S. Dellinger, and R. Heideman, "Computation of Core Losses in Electrical Machines Using Improved Models for Laminated Steel," *IEEE Transactions on Industry Applications*, vol. 43, no. 6, pp. 1554–1564, 2007.
- [104] A. Boglietti, A. Cavagnino, D. Ionel, M. Popescu, D. Staton, and S. Vaschetto, "A General Model to Predict the Iron Losses in PWM Inverter-Fed Induction Motors," *IEEE Transactions on Industry Applications*, vol. 46, no. 5, pp. 1882– 1890, 2010.
- [105] D. Ionel, M. Popescu, C. Cossar, M. McGilp, A. Boglietti, and A. Cavagnino, "A General Model of the Laminated Steel Losses in Electric Motors with PWM Voltage Supply," in *IEEE Industry Applications Society Annual Meeting (IAS)*, 2008, pp. 1–7.
- [106] M. Popescu, D. Ionel, A. Boglietti, A. Cavagnino, C. Cossar, and M. McGilp, "A General Model for Estimating the Laminated Steel Losses Under PWM Voltage Supply," *IEEE Transactions on Industry Applications*, vol. 46, no. 4, pp. 1389–1396, 2010.
- [107] G. Sizov, P. Zhang, D. Ionel, N. Demerdash, I. Brown, A. Smith, and M. Solveson, "Modeling and Analysis of Effects of Skew on Torque Ripple and Stator Tooth Forces in Permanent Magnet AC Machines," in *IEEE Energy Conversion Congress and Exposition 2012 (ECCE)*, sept. 2012, pp. 3055 –3061.
- [108] V. H. Juha Pyrhonen, Tapani Jokinen, Design of Rotating Electrical Machines. John Wiley & Sons, Ltd., 2008.
- [109] K. Zielinski and R. Laur, "Stopping Criteria for a Constrained Single-Objective Particle Swarm Optimization Algorithm," *Informatica (03505596)*, vol. 31, pp. 51–59, March 2007.
- [110] K. Zielinski, P. Weitkemper, R. Laur, and R. L. K.-D. Kammeyer, K. Zielinski, "Examination of Stopping Criteria for Differential Evolution based on a Power Allocation Problem," in 10th International Conference on Optimization of Electrical and Electronic Equipment, Brasov, Romania, May 2006.

- [111] Y. Duan and D. Ionel, "Non-Linear Scaling Rules for Brushless PM Synchronous Machines Based on Optimal Design Studies for a Wide Range of Power Ratings," in *IEEE Energy Conversion Congress and Exposition 2012 (ECCE)*, 2012, pp. 2334–2341.
- [112] P. Zhang, G. Sizov, D. M. Ionel, and N. Demerdash, "Design Optimization of Spoke-Type Ferrite Magnet Machines by Combined Design of Experiments and Differential Evolution Algorithms," in *IEEE International Electric Machines* Drives Conference 2013 (IEMDC), 2013, pp. 958–964.
- [113] P. Zhang, G. Sizov, J. He, D. M. Ionel, and N. Demerdash, "Calculation of Magnet Losses in Concentrated-Winding Permanent Magnet Synchronous Machines Using a Computationally Efficient-Finite Element method," *IEEE Transactions* on Industry Applications, vol. 49, no. 6, pp. 2524–2532, 2013.
- [114] N. Vaks, S. Sudhoff, and S. Pekarek, "Investigation of Tradeoffs between Efficiency, Mass, Cost, Service Factor, and Power Factor in Induction Machines," in SAE Int. J. Aerosp. 3(1), 2010, pp. 159–167.
- [115] C. Sullivan, "Computationally Efficient Winding Loss Calculation With Multiple Windings, Arbitrary Waveforms, and two-Dimensional or Three-Dimensional Field Geometry," *IEEE Transactions on Power Electronics*, vol. 16, no. 1, pp. 142–150, 2001.
- [116] S. Iwasaki, R. Deodhar, Y. Liu, A. Pride, Z. Zhu, and J. Bremner, "Influence of PWM on the Proximity Loss in Permanent-Magnet Brushless AC Machines," *IEEE Transactions on Industry Applications*, vol. 45, no. 4, pp. 1359–1367, 2009.
- [117] M. Popescu and D. G. Dorrell, "Skin Effect and Proximity Losses in High Speed Brushless Permanent Magnet Motors," in *IEEE Energy Conversion Congress* and Exposition (ECCE), 2013, pp. 3521–3527.

APPENDIX I

Three phase induced voltages with skew effects:

$$\begin{cases} e_a = \sum_n k_{sn} E_n \cos(n\omega t + \phi_n) \\ e_b = \sum_n k_{sn} E_n \cos\left(n\left(\omega t - \frac{2\pi}{3}\right) + \phi_n\right) \\ e_c = \sum_n k_{sn} E_n \cos\left(n\left(\omega t - \frac{4\pi}{3}\right) + \phi_n\right) \end{cases}$$

where, n = 1, 3, 5, 7.

Three phase currents:

$$\begin{cases} i_a = \sum_n I_n \cos(n\omega t + \varphi_n) \\ i_b = \sum_n I_n \cos\left(n\left(\omega t - \frac{2\pi}{3}\right) + \varphi_n\right) \\ i_c = \sum_n I_n \cos\left(n\left(\omega t - \frac{4\pi}{3}\right) + \varphi_n\right) \end{cases}$$

The electromagnetic power:

$$P_e = e_a i_a + e_b i_b + e_c i_c.$$

Substituting $e_{a,b,c}$ and $i_{a,b,c}$ into the expression for P_e , the electromagnetic power can be deduced as follows:

$$P_e = P_{avg} + P_6 + P_{12}$$

where, P_{avg} and P_6 , are shown as follows:

$$P_{avg} = \frac{3}{2} \left[k_{s1} E_1 I_1 \cos(\phi_1 - \varphi_1) + k_{s3} E_3 I_3 \cos(\phi_3 - \varphi_3) + k_{s5} E_5 I_5 \cos(\phi_5 - \varphi_5) + k_{s7} E_7 I_7 \cos(\phi_7 - \varphi_7) \right]$$

$$P_6 = \frac{3}{2} \left[k_{s5} E_5 I_1 \cos(6\omega t + \phi_5 - \varphi_1) + k_{s7} E_7 I_1 \cos(6\omega t + \phi_7 - \varphi_1) \right]$$

APPENDIX II

ACO	Ant Colony Optimization
ALA	Axially Laminated Anisotropic
BFO	Bacterial Foraging Optimization
BL	Brushless
CCD	Central Composite Design
CE-FEA	Computationally Efficient-Finite Element Analysis
CPSR	Constant Power Speed Range
DE	Differential Evolution
DOE	Design of Experiments
DSPM	Double-Salient Permanent-Magnet Machine
EP	Evolutionary Programming
EV	Electric Vehicles
ES	Evolutionary Strategies
FEA	Finite Element Analysis
FFD	Full Factorial Design
FSCW	Fractional Slot Concentrated Winding
FSPM	Flux-Switching Permanent-Magnet Machine
GA	Genetic Algorithms
GP	Genetic Programming
HEV	Hybrid Electric Vehicle

HEV Hybrid Electric Vehicle IEC International Electrotechnical Commission IPM Interior Permanent Magnet ISDW Interger-Slot Distributed Winding MEC Magnetic Equivalent Circuit MTPA Maximum Torque per Ampere \mathbf{PM} Permanent Magnet PRM Permanent-Magnet Reluctance Machine PSO Particle Swarm Optimization RSM Response Surface Methodology SPM Surface-mounted Permanent Magnet SVSpoke-V-type SynRM Synchronous Reluctance Machine TFLM Transverse Flux Linear Machines TS-FEA Time Stepping-Finite Element Analysis