Method of Tolerance Allocation to Maintain Rotary Balance of Multi-Component Bodies

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by

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ABSTRACT

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Vibration of rotating machinery caused by mass imbalance is the most frequent source of unwanted disturbing forces and also the most preventable. In the case of a CT scanner, unwanted vibration in the equipment causes artifacts to the X-ray image, and therefore all measures are taken to eliminate imbalance. The CT scanner is a multi-component rotating body, therefore making it a challenge to account for many discrete components, each with unique variation. This research developed the equations for static and dynamic balance including considerations for inertia. The variation of the components was studied using two models: a sensitivity analysis and a statistical approach. A method was developed to allocate tolerances for mass and center of gravity to the discrete components in order to produce a system capable of being balanced yet manufacturable.

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DEDICATION

To my parents for their faith, love, and support. To my sister, who is my rock, for all the ways you've shaped me; and with forever love to my fiancé, Adam Thiel.

All the glory to my Lord and Savior Jesus Christ.

For it is by grace you have been saved, through faith – and this is not from yourselves, it is the gift of God.

Ephesians 2:8

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CHAPTER 1: INTRODUCTION

1.1 Rotor Balance Need

Machines utilizing rotating shafts are prevalent in nearly every industry, from automotive to aerospace to medical devices. With the advancement of industry, such devices have developed to reach impressive speeds and carry high load; however, with these advancements comes increasing need to control unwanted consequences such as vibration.

Unwanted vibration in running machinery is a concern for many reasons, the most obvious of which are noise, wear, and fatigue. The forces transmitted from vibrations pass through the bearing to the foundation structure; they may cause damage to the machine and its surroundings [1]. Furthermore, this disturbing force reduces the machine's life [2]. As equipment operates at higher speeds, these vibrations also usually become higher, as they are a function of the operating speed of the machinery.

The most common source of vibration in rotating equipment is imbalance [2]. "Imbalance" or "unbalance" is the state of being out of equilibrium or out of proportion [3]. In the case of rotating machinery, such as machining tools and turbine engines, imbalance refers to unequally distributed mass or inertia about the axis of rotation.

An example of rotating machinery in the medical industry is the Computed Tomograpy (CT) Scanner used for diagnostic imaging. This device is also troubled by unwanted vibration and great effort is taken to ensure the vibration does not cause undesirable effects. The main piece of equipment, called the gantry, consists of a stationary frame that houses a rotating hollow cylinder which carries an X-ray tube, detector module, heat exchangers, and other imaging electronics. The rotating cylinder is mounted to the stationary frame via a large diameter bearing and spun at speeds high enough to produce over 165 kilojoules of energy. The distribution of mass of the rotating components must be designed to prevent imbalanced loading, and therefore minimize vibration.

In addition to wear, noise, and fatigue, significant vibrations of a CT scanner impact the quality of the X-ray image. The vibrations in the structure cause a misalignment between the X-ray tube and the detector, resulting in a blurry image. This result could cause a number of undesired outcomes, including additional radiation dose to the patient required from a rescan. Therefore, it is imperative to control the imbalance of the gantry to maintain a reasonable amount of vibration and guarantee good image quality.

1.2 Detection, Prevention, and Correction of Imbalance

Detection of imbalance is relatively straightforward and is available in the literature. Imbalance can be determined by analyzing the vibration spectrum, specifically the amplitude and phase of the once per revolution component [2]. From this data, the mass and location of the heavy spot(s) can be calculated and corrected by adding or removing mass at the appropriate location(s). Although a seemingly simple process, the correction is greatly complicated with complex rotors such as those that cannot be modified at any desired location. Additionally, the process is complicated by the difficulty to measure the vibration once the device is installed in its operating environment.

To ease the difficulties of correcting for imbalance once the assembly is built, the imbalance can be predicted, modeled, and improved before the components are manufactured and assembled. Proper up-front design will lead to prevention of imbalance and limit the need for correction after manufacturing.

1.3 Goal of this Research

The study of rotor dynamics and shaft vibration is well published. However, no literature was found identifying the factors that contribute most to imbalance. Nor, was data found discussing the impact of part or assembly variation impact to imbalance.

This research focused on the theory of rotor balance and examined the various factors that contribute to imbalance. Focus was given to the analysis of a multiple body system and the contribution each component had on the collective system. Specially, the contributors to imbalance such as mass and mass distribution were studied to discover the impact to imbalance and the impact the variation of each of these factors to the variation of the system imbalance. A method to allocate tolerances on each of the mass properties was developed to effectively control the variation of the assembly.

1.4 Overview of this Thesis

In Chapter 2, the definitions of the various types of imbalance and how to correct for them are discussed. Additionally, the causes of imbalance are further explored. The uniqueness of the application to a CT scanner is investigated, and various alternative balancing methods briefly described.

In Chapter 3, the equations for a balance model by Newtonian mechanics are theoretically developed. The equations capture the transfer functions used to relate the system geometry and operating speeds to the forces causing undesirable vibration due to imbalance. These drive the design capability and provide a guide to modify the inputs for a desired output given the transfer functions.

Chapter 4 explored techniques at analyzing the force and moment equations developed in Chapter 3. This modeling allowed for a means to evaluate the success of the design given the variation of the individual components making up the system. Furthermore, it provided a guide to design acceptable limits of part variation that will lead to tolerance allocation.

Two modeling approaches were uses to evaluate the system performance given the transfer functions. These two models reflect both the nominal solution and the contribution of variation of each component to the total solution. A sensitivity analysis was conducted using calculated sensitivities to compare the contributions of each component and its variation on the total solution. A second modeling method, a statistical approach via a Monte Carlo simulation, was also built, which used a random number generator to calculate many trials of the imbalance solution.

Chapter 5 discussed the application of the sensitivity and simulated models to allocate component tolerances. The expected part variation was estimated based on factors such as manufacturing processes, service replacement cycles, and cost. The estimated variation was analyzed using the two balance models, and adjustments were made as necessary. The estimated variation was iterated until the system functioned in its acceptable bounds and the component variations were achievable. The expected variations were allocated as official tolerances and converted to more measureable quantities, where necessary.

In conclusion, chapter 6 reviewed the information published in this research and identifies the areas of new contribution to the field of rotor dynamics. Further research topics are also suggested.

Chapter 2: Background and Literature Review

The term mass imbalance has different meanings for different analyses, for example, a simple shaft operating at slow speeds may only consider imbalance in a single plane; however, a satellite requires a thorough investigation of inertia. Most equipment lies somewhere between these two extremes, as the case of the CT scanner.

2.1 Definitions – Static and Dynamic Balance

There exist two classes of imbalance: static and dynamic. Static imbalance is a planar imbalance resulting in vertical motion of the rotor from heavy spots on the rotating mass. Dynamic imbalance is an imbalance over the depth of the rotor and causes the rotor to wobble when rotated. Both types of imbalance affect the rotor of a CT scanner and need to be considered in order to effectively control the motion of the rotating mass.

Static imbalance exists when the principal axis of inertia is displaced parallel to the shaft's axis of rotation [4] and the center of mass does not lie along the axis of rotation. This type of imbalance can be corrected with a single mass either added or removed from the rotor perpendicular from the rotor's axis of rotation at its center of mass. Figure 2.1 demonstrates static imbalance.

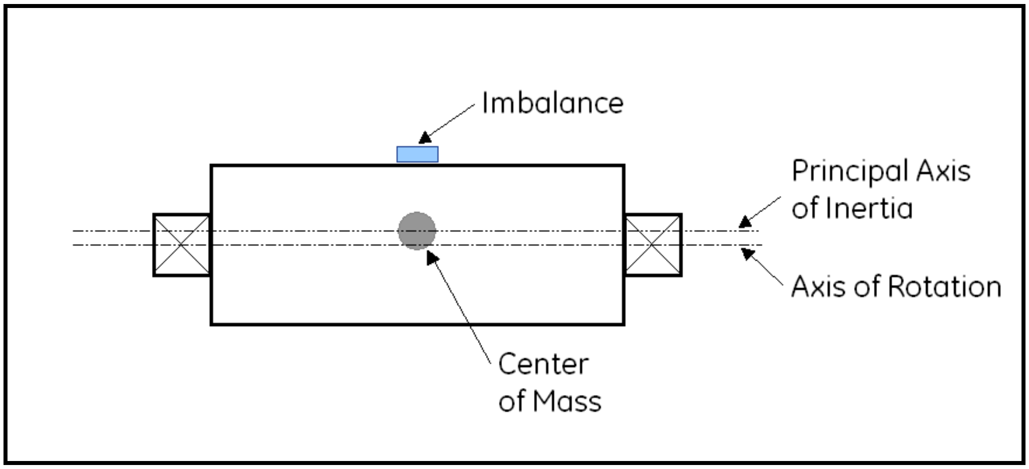

Figure 2.1: Static Imbalance

In the case of dynamic imbalance, the principal axis of inertia is displaced from the axis of rotation in a non-parallel orientation, as shown in figure 2.2, and the center of mass may or may not lie along the axis of rotation. If dynamic imbalance is distributed along the length of the rotor, as it is in many cases, then a static balancing procedure cannot be used to determine the correction masses [1]. Most often, two masses are required to correct a dynamic imbalance.

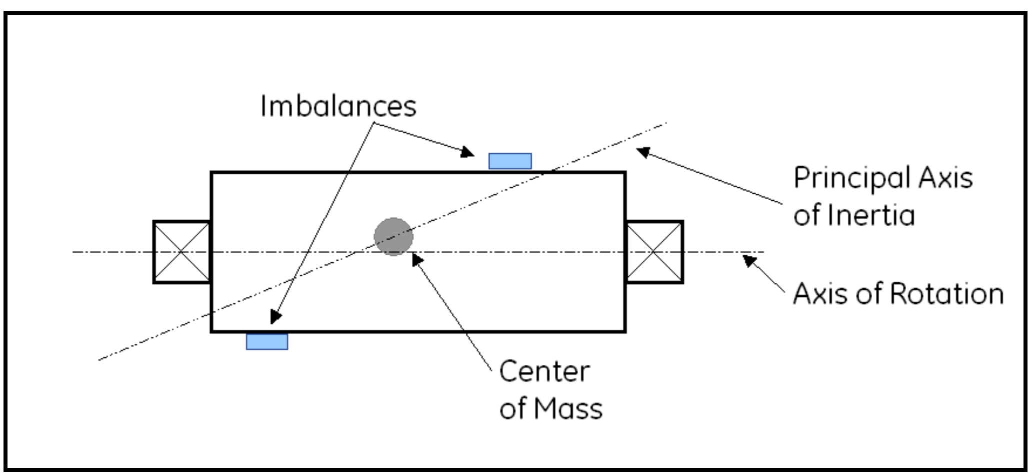

Figure 2.2: Dynamic Imbalance

Additional definitions used in this document are defined below.

Component: Part of the rotating mass assembly

Rotor: Object used to mount the imaging components and other electronics

Rotating Mass: The sum of all things rotating including the rotor and the components mounted to it, bearing rotor, etc.

Variable weights: Collection of weights used to alter the center of gravity (CG) and the mass distribution of the rotating mass *for each unique unit* to achieve the desired balanced state; used to account for the variation of the components

Permanent weights: Collection of weights used to alter the mass and CG of the rotating mass *for each product platform* to achieve the desired balanced state; used to bring the nominal system CG to the desired location

2.2 Causes of Imbalance

Imbalance is caused from a number of sources. In the case of a CT scanner, mass is distributed on the rotor by a number of discrete components placed at various radial and axial distances, and each component has a unique mass distribution itself. The assembly of these components to the rotor has inherent variation due to the manufacturing and assembly processes.

The variation of mass and mass distribution of each component is attributed to the component makeup, manufacturing process, and subassembly processes. Sources include precision of the machining process for some parts and the consistency of the material, often times by the casting process. In general, the material density and forming processes vary. Presence of liquid in the component and how the liquid is contained, such as a bladder, and whether the liquid is allowed to move or expand can also contribute to the variation of the component's mass properties. These variations of each component cause variation to the total system's mass distribution and therefore its imbalance.

2.3 Uniqueness of Multi-body Cantilevered System

When referring to the literature, the vast majority available on rotating systems consist of a single shaft with a single or number of disks mounted along the axis of rotation. Most systems consist of two bearings with the disks or flanges mounted between them. In the case of most CT scanners; however, many differences from the common rotary shaft exist.

The rotor geometry and bearing placement of a CT scanner is unique compared to common rotary shafts. The rotating mass of a CT scanner is a hollow cylinder instead of a solid shaft, which forces the CG of the components to lie at various radial distances instead of along the axis of rotation. The bearing placement is unique in that the interface between the stationary and rotating components is generally a single bearing, therefore making the load cantilevered instead of being supported on both ends.

The shear nature of the number of components presents a challenge to the imbalance model. Each component has a unique contribution to the balance solution based on its mass and location, and each component has a unique amount of variation associated with it. In order to maintain an acceptable level of model fidelity, the majority of the individual components must be considered in the analysis before a system model is meaningful.

Lastly, the CT scanner has additional challenges due to the limitations of component placement on the rotor. The geometry of certain components and their position on the rotor cannot be altered due to the system function, and correction masses cannot be added or removed at any desired location due to occupancy by other components. Many tradeoffs need to be considered before components are moved or changed.

The analysis of a CT scanner is quite different from the vast offering of published works on rotary dynamics. Although the physics behind such an analysis are the same, the application to this device is very different. Primarily, a multi-body analysis for a CT scanner must consider each component with unique variation of mass and CG. The common methods used for rotary balance theory and the practice of tolerance allocation is unique for their application to a CT scanner.

2.4 Previous CT Balance Work

Existing mathematical models to calculate CT imbalance have been developed and used in industry. These models calculate the nominal imbalance of the rotating assembly given the mass and location of the components on the assembly. They assume each component is a point mass and are used to design the counterweights required to bring the system within an acceptable imbalance limit. They are also useful to understand the manufacturing trend data by using them backward, in essence, and calculating the imbalance of the system given the counterweights used to balance it.

Techniques to measure the imbalance of a CT rotor have also been developed. One solution utilizes two strain sensors attached to the stationary structure to measure the vibration spectrum in the form of material strain. A 2-plane method (required for rotors of appreciable width) is used to collect data from 3 trials, an initial run and two with a known imbalance. A software algorithm collects the vibration data from the sensors for each of the 3 trials and calculates the initial imbalance.

Lastly, techniques to correct for imbalance have been implemented in industry. Most methods consist of masses mounted to the rotor at various axial and radial distances so the net reaction corrects for the imbalance of the system.

2.5 Alternative Vibration Correction Solutions

Alternative solutions to offset the effects of imbalance exist in other industries that could be utilized on a CT system. Some of these methods correct for the imbalance by altering the system's mass distribution in such a way the imbalance is counteracted. This approach can be implemented in such a way the technique is considered active, or online balancing. Alternative methods (such as fluid or magnetic bearings) don't correct the imbalance but rather counter the moment load caused by the imbalance.

Active techniques including movable masses, such as that discussed by Green, Friswell, Champneys, and Lieven [5], deploy two or more masses that are free to travel around a race, filled with a viscous fluid, at a fixed distance from the shaft center. The balls position themselves so they counteract any residual unbalance; however, the effectiveness of the device is limited to the amount of imbalance it corrects. Such a technique is useful when the imbalance of the rotor changes in time, such as in the case of machine tools; however, it is considered unnecessary when the imbalance is steady in time.

Another approach to compensating for imbalanced loading is implementing a fluid film or magnetic bearing. The fluid film bearing does not necessarily correct for imbalance but rather provide a means of damping to possibly attenuate the vibration of the rotor. A magnetic bearing is similar in that its implementation will not improve imbalance but uses an alternative means to correct for the moment at the bearings. It utilizes magnetic forces in the bearing to adjust for the misalignment that occurs when a moment is produced [6, 7]. Both bearing types will compensate for the loading caused by the imbalanced rotating mass; however, they do not solve the problem at the root and have a high cost associated with them.

The alternative solutions are meaningful in some design spaces but not in the application of a CT scanner because they fail to correct for imbalance, a necessary element in reducing vibration of the focal spot in a reliable manner over the life of the product. The only means to achieving a safe, reliable, smooth operation is to design and manufacture a truly balanced system. Although the nature of the multi-body system presents challenges to imbalance, the mass distribution can be designed and corrected with careful analysis to produce a reliably balanced machine.

Chapter 3: Developing the Transfer Function and Variables

3.1 Objective

As discussed in chapter 2, rotary imbalance is based on the location of the center of gravity and orientation of the principal axes. If the system's center of gravity is not properly located, static imbalance will result; if the principal axis of rotation is not properly aligned, dynamic imbalance will result. In order to design a system with minimal imbalance, transfer functions were developed to calculate the imbalance from the factors that cause it.

The equations were developed in a general form to relate each of the individual component's mass properties to the total system imbalance. They were created for both static and dynamic imbalance using a Cartesian coordinate system. Considerations were taken to establish guidelines for when a component is considered a simple particle and when its inertial properties were included. Transformations from local to global coordinate systems were also developed for transferring inertial properties. The imbalance relationships provided a means to objectively evaluate the system and created a foundation for further analysis, such as the impact of part variation.

3.2 Theory

The imbalance equations were developed using a Newtonian approach with a rigid body assumption. In such a case, the rotational forces and moments produced by the components were summed about a selected point. Any resulting forces and moments were considered the imbalance. The rigid body assumption was helpful to prevent the consideration of elastic effects and was safely made because the deformation of the components was very small compared to the overall dimensions of the rotating mass [8].

The origin of the global coordinate system was chosen for geometric convenience yet satisfied the rules of simplifying the angular momentum equation. The origin needed to be either the center of mass, must not accelerate, or accelerates directly toward or away from the center of mass [9]. The first criterion, the center of mass, is a challenging location for this analysis, as the location of the CG changes depending on the specific scanner; the third option is not applicable as it can only be used for a balanced body. It was assumed that any point along the axis of rotation does not accelerate (or at least at an insignificant amount for this analysis) and therefore, any location along the axis of rotation was assumed to be acceptable. A convenient location along that axis was selected as the origin to the global coordinate system (Figure 3.1).

The global coordinate system was selected to be a moving Cartesian coordinate system fixed to the rotor by choice so that inertia of the reference frame was constant. It is noted in capital letters (X, Y, Z) with its Y-axis oriented toward the X-ray tube and rotates about the Z-axis.

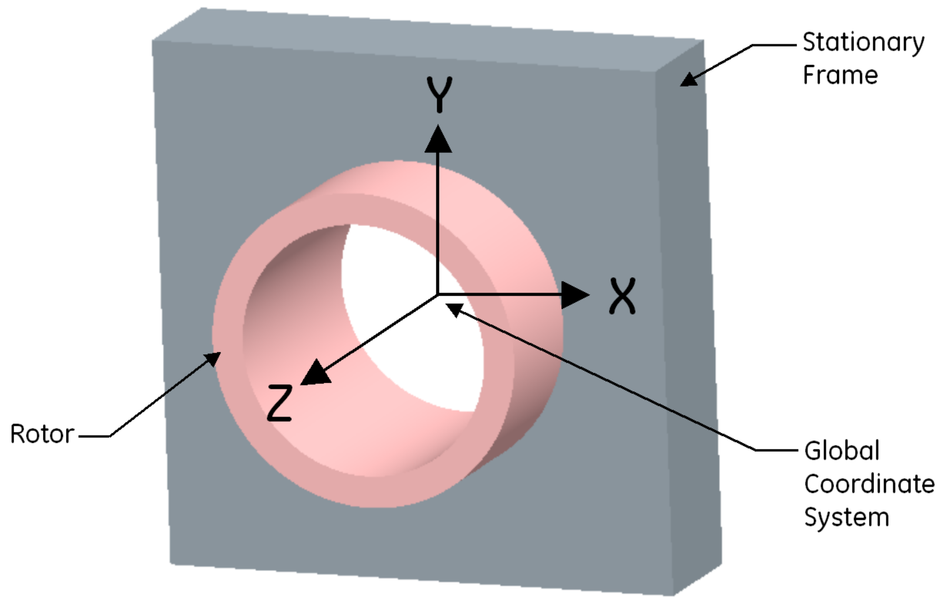

Figure 3.1: System with Global Coordinate Frame

The major components that are part of the rotating mass, including the rotor and components mounted to it, were included in the force and moment equations derived from Newton's laws. The forces and moments created from each component's mass, inertia, and location was considered. Each component and its coordinate system was located at a position in the global reference frame noted with lower case letters and numerical subscript, such as (x_1, y_1, z_1), as shown in figure 3.2.

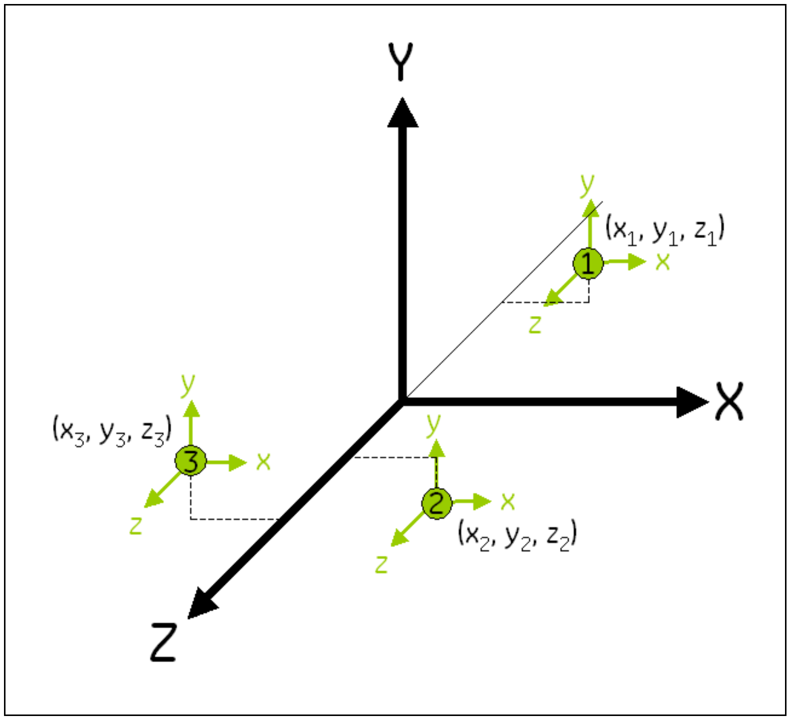

Figure 3.2: Global Coordinate System with Various Masses

In a statically and dynamically balanced state and in the absence of gravity, the reaction forces are zero whether or not the system is rotating [4]. Using Newton's second law, the sum of the forces is equal to the mass of a particle times the acceleration of the particle as provided in equation 3.1.

$$\sum \vec{F} = m\vec{a} \tag{3.1}$$

In the case of multiple masses, Newton's law was written as:

$$\sum \vec{F}_i = \left(\sum m_i \vec{a}_i\right) \tag{3.2}$$

Due to the unique situation of the CT scanner, this equation was simplified. The acceleration in the radial direction ($\hat{e}_r$) for a rotating body with constant angular velocity along the Z-axis, ω_Z, and constant radius, r, was written [9]:

$$\vec{a} = -r\omega_Z^2\hat{e}_r \tag{3.3}$$

This angular acceleration expression was substituted back in equation 3.2 and the angular velocity term moved to the left side of the equation. Expressing in Cartesian (X, Y, Z) components:

$$\frac{\sum \vec{F}}{\omega_Z^2} = \sum m(X\hat{i} + Y\hat{j} + Z\hat{k}) \tag{3.4}$$

Static balance results when equation 3.4 is equal to zero. Expressing in each coordinate, the equations for static balance are:

$$\sum m_i x_i = 0 \tag{3.5}$$

$$\sum m_i y_i = 0 \tag{3.6}$$

$$\sum m_i z_i = 0 \tag{3.7}$$

When the CG is located along the axis of rotation, as stated in Chapter 2, static balance results. This condition is desired, as the reaction forces are what cause vibration. Note equation 3.7 is formulated; however, it is not needed in order to calculate planar (static) balance, as the location of the CG along the rotation axis is not necessary.

The moment equations for rotary motion follow a similar analysis. Using the body-fixed global (X, Y, Z) coordinate system, the sum of the moments was [9]:

$$\sum \vec{M} = [I]\vec{\alpha} + \vec{\omega} \times [I]\vec{\omega} \tag{3.8}$$

Where $\vec{\omega}$ is the angular velocity, $\vec{\alpha}$ is the angular acceleration, and $[I]$ is the inertia matrix for the entire system. The inertia matrix is defined as shown in equation 3.9.

$$[I] = \begin{bmatrix} I_{XX} & -I_{XY} & -I_{XZ} \\ -I_{XY} & I_{YY} & -I_{YZ} \\ -I_{XZ} & -I_{YZ} & I_{ZZ} \end{bmatrix} \tag{3.9}$$

A special case arises when the rotation is about a fixed axis, as in the case of a CT scanner. In normal operating condition the angular acceleration is zero, and the angular velocity is only in one direction, here, the Z-axis:

$$\vec{\omega} = \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega_Z \end{bmatrix} \tag{3.10}$$

In this case, the moment equation of 3.8 was simplified to:

$$\sum \vec{M} = \begin{bmatrix} I_{YZ}\omega_Z{}^2 \\ -I_{XZ}\omega_Z{}^2 \\ 0 \end{bmatrix} \tag{3.11}$$

The moment vector was broken down into its respective components:

$$\sum M_X = \sum I_{XZ}\omega_Z^2 = 0 \tag{3.12}$$

$$\sum M_Y = \sum I_{YZ}\omega_Z^2 = 0 \tag{3.13}$$

Similar to the sum of force equations, the angular velocity can be moved to the left side of the equation and drops out when set to zero. As stated in Chapter 2, dynamic balance requires the principal axis to be aligned with rotation axis. Therefore, dynamic balance is achieved when the following two equations are met [4]:

$$\sum I_{XZ} = 0 \tag{3.14}$$

$$\sum I_{YZ} = 0 \tag{3.15}$$

3.3 Transforming Local Coordinates to Global Coordinates

Often, it was convenient to express the location of the CG of a component in a coordinate system local to the component. Many of the components on the system are assemblies themselves, consisting of electronics, cooling devices, and other components. In order to find the CG of a subassembly, a local coordinate system was used. A local coordinate system was also necessary when measuring a component on load plates to find the CG of the subassembly.

In order to use the local CG information for the system-level analysis, it had to be converted to global coordinates. To do so, rotation transformations and/or translation transformations were used to express them in the global coordinate system. Rotation transformations were required when the local coordinate system was not oriented in the same direction as the global coordinate system. Translations were needed when the origin of the local coordinate system did not coincide with the global coordinate system.

A rotation transformation was performed by use of a rotation matrix, [R]. This matrix was defined dependent about what axis the rotation was to be performed. The following matrices were used for rotations about the subscript noted, where the angle θ is defined as positive about by the right hand rule [9]:

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{bmatrix} \quad (3.18)$$

$$[R_y] = \begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \quad (3.19)$$

$$[R_z] = \begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.20)$$

In the following example, a rotation transformation was used to orient a component's local coordinate system (x, y, z) in the same direction as the global coordinate system (X, Y, Z) about the Z-axis, as shown in figure 3.3. Equation 3.21 shows the applied rotation.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R]\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3.21}$$

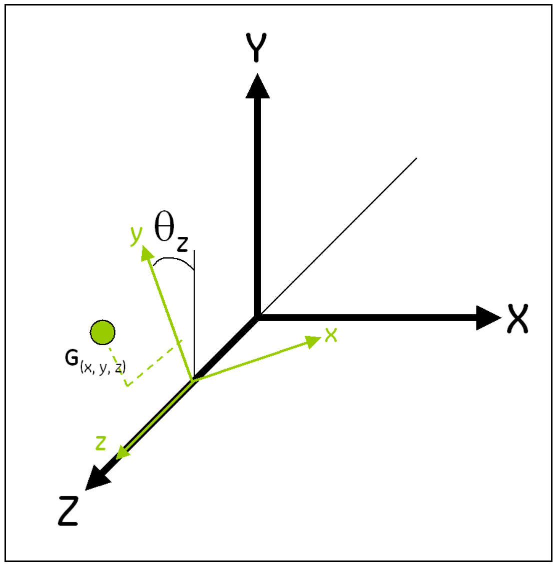

Figure 3.3: Rotation Transformation

Additional tools were occasionally applied to manipulate the coordinate systems, such as using a body-fixed process to make multiple rotations in one calculation.

Once the inertia of a component was expressed in a coordinate system that is aligned with the global coordinate system, a translation was made to express the local coordinate system coincident with the global. Using the parallel axis theorem, defined as matrix [P] in equation 3.22, the center of mass of the component was translated to the desired location. The terms along the diagonal of this matrix correspond with the moments of inertia and the off-diagonal terms with the products of inertia.

$$[P]=\begin{bmatrix} m(y^2+z^2) & m(xy) & m(xz) \\ m(xy) & m(x^2+z^2) & m(yz) \\ m(xz) & m(yz) & m(x^2+y^2) \end{bmatrix} \tag{3.22}$$

The translation was made by simply adding the parallel axis matrix to the local, oriented inertia matrix. This final step was important to express the components of a system in one global coordinate system.

$$\left[I_{(X,Y,Z)}\right]=\left[I_{(x,y,z)}\right]+[P]$$

$$=\begin{bmatrix} I_{xx}+m(y^2+z^2) & -I_{xy}+m(xy) & -I_{xz}+m(xz) \\ -I_{xy}+m(xy) & I_{yy}+m(x^2+z^2) & -I_{yz}+m(yz) \\ -I_{xz}+m(xz) & -I_{yz}+m(yz) & I_{zz}+m(x^2+y^2) \end{bmatrix} \tag{3.23}$$

Note that once this inertia matrix was substituted back into the moment equation (eq. 3.13) most terms of [P] drop out, but the products of inertia about the XZ and YZ planes remain.

$$\sum M=\begin{bmatrix} I_{YZ}\omega_z^{\,2} \\ -I_{XZ}\omega_z^{\,2} \\ 0 \end{bmatrix}=\begin{bmatrix} (-I_{yz}+m(yz)) \\ -(-I_{xz}+m(xz)) \\ 0 \end{bmatrix}\cdot\omega_z^2 \tag{3.24}$$

After breaking down the moment vector in terms of X and Y and removing the angular velocity term, the equations for dynamic imbalance were:

$$\sum\left[I_{yz}+m(yz)\right]=0 \tag{3.25}$$

$$\sum\left[I_{xz}+m(xz)\right]=0 \tag{3.26}$$

3.4 Particle Mass or Inertial Body

It was desired to use the inertia term (non-parallel axis term) in the dynamic imbalance equations for every component; however, this presented a challenge, as the component's inertia was difficult to measure. It was also often times challenging to estimate. It was desired to identify a means to determine when the component inertia was large enough to consider in the imbalance equations and when it could be considered negligible.

In some instances, the inertia of a component expressed in the global coordinate system may be heavily influenced by either its unique inertia [$I_{(x,y,z)}$] or by the parallel axis contribution [P]. From equation 3.24, the total inertia is the sum of these two terms. When a component was assumed a particle, or point mass, the component was assumed to be dimensionless, which forces the [$I_{(x,y,z)}$] term to zero, leaving only the parallel axis matrix [P] terms. This assumption is valid if the physical dimensions of the body are much smaller than the path followed by the body [8].

In most cases, this assumption can be made safely, as the component's products of inertia are very small compared to the parallel axis terms. When the inertia terms can safely be estimated (either by hand calculations of simple shapes or by CAD data), they should be considered in the dynamic balance equation. One way to evaluate the inertia term contribution is shown in equation 3.27, which was evaluated based on a ratio of the standalone component's inertia [$I_{(x,y,z)}$] to the inertia of the component once mounted to the rotor [$I_{(X,Y,Z)}$]. If the ratio was above, for example, 1%, the component's products of inertia should be considered in the analysis. Specifically, the Contribution, C, of the component's inertia (due to its symmetry) compared to the total inertia was calculated:

$$C = \frac{\left[I_{(x,y,z)}\right]}{\left[I_{(X,Y,Z)}\right]} \cdot 100 = \frac{\left[I_{(x,y,z)}\right]}{\left[I_{(x,y,z)}\right] + [P]} \cdot 100 \qquad (3.27)$$

As mentioned, the inertia term was often times difficult to estimate because the component was a subassembly composed of electronics, liquid, and other difficult to measure materials. Consideration was made for the orientation of the component to the rotation axis. For example, if the main pieces of the component were assumed to be a simple shape (rectangle, circle) and were mounted parallel to the rotation axis, the products of inertia were assumed zero.

Alternatively, if a component's CG was along the rotation axis, as the case of the bearing rotor, the rotor, or a collection of large bolts, the parallel axis contribution of the inertia was zero. In this case, the contribution of the component's inertia was significant (compared to the parallel axis contribution) and was therefore included in the dynamic balance equations.

The component's inertial term [$I_{(X,Y,Z)}$] was included on a case-by-case basis. A practical justification was made for those components whose inertia was not significant compared to the parallel axis contribution. This approach induces some amount of error; however, it was taken knowing it was in the bounds of the error of the analysis. This verification was confirmed by reviewing the vibration data of actual systems. The first order of the vibration, known to be caused by imbalance [2], was very small compared to the specification limit. Furthermore, the known first critical frequencies (calculated from the natural frequencies of the structure) are much higher (10 times or more) than the operating speed. For this reason, the assumptions made for the inertia terms were shown to be a good practical judgment for this application.

In summary, the equations for a statically and dynamically balanced system were developed using Newton's Laws of motion and are shown in eq. 3.28-31. Planar motion of a rigid body was assumed and the inertia was considered for dynamic balance. From here, they were further analyzed to investigate which terms were most sensitive to affecting the system's total imbalance due to variation.

$$\begin{aligned} \sum m_i x_i &= 0 \\ \sum m_i y_i &= 0 \end{aligned} \qquad (3.28\text{-}29)$$

$$\begin{aligned} \sum m_i x_i z_i + \sum I_{XZ} &= 0 \\ \sum m_i y_i z_i + \sum I_{YZ} &= 0 \end{aligned} \qquad (3.30\text{-}31)$$

3.5 Definition of Contributing Factors

From the equations defined in section 3.4, the factors contributing to imbalance were shown as the mass, inertia, and CG for any component on the rotating mass.

The nominal value for each of these parameters was first set based system function requirements. For example, the X-ray tube was always placed directly across from the detector. A heat exchanger was always placed directly adjacent to the component it is cooling. The exact location and mass of these components was altered when necessary; however, their nominal values were driven by the total system design including required geometry, minimum cable and hose lengths, and other factors that took priority to mass distribution for balance conditions.

Although most components could not be relocated, the location of the center of mass and the products of inertia of the system could still be altered by the addition of permanent weights. They were added to the rotating mass to bring the nominal location of the CG as close to a balanced state as possible. The permanent weights were designed by adjusting their masses and location to force the static and dynamic imbalance equations (Equations 3.28-31) to zero.

Once the nominal location of the CG was as close to a balanced solution as possible, the variation was addressed. The total system imbalance variation was a function of the variation of each component, as shown in the following chapter.

Once the desired outcome was understood and the contributing factors known, the analysis was conducted to adjust the inputs of mass, CG, inertia, and variation to achieve the desired system imbalance result. These system requirements flowed down to the subcomponents and allowed for appropriate tolerance allocation. The process of tolerance allocation and the methods for gantry balance are not unique by themselves; however, the application of allocating tolerances for gantry balance was not previously researched or published and provided a significant contribution to CT mechanical design.

Chapter 4: Multi-Dimensional Variation Analysis

Analysis of the static and dynamic balance equations provided understanding of the system's nominal imbalance given each of the components inputs (mass, CG, inertia). Furthermore, these equations were used to conduct a variation analysis, which provided insight to the impact of variation on the success of the system's balance. The goal of the variation analysis was to create a means to evaluate the contribution of each component's inputs to the success of balance at the system level. Conducting such an analysis allowed for a justified approach to allocate tolerances to the components.

Two methods were used to analyze the system variation. The first was a sensitivity analysis to determine how the components contributed to the output compared to one another. Second, a statistical approach was used to randomly sample each component's variation and analyze the result of the system imbalance. Both methods were required for a complete analysis.

4.1 Sensitivity Analysis

In many mechanical assemblies, a variation analysis or tolerance stack was conducted on a one-dimensional assembly, such as positional fits of assembled components where the dimensions are laid out along a single coordinate axis. In the case of an imbalance study, the variation analysis was multi-dimensional because the center of gravity was expressed in a Cartesian (X, Y, Z) coordinate system and the mass was also considered. All of these variations affect the proper function of the system and required simultaneous analysis, which was done using a sensitivity analysis.

Sensitivity is defined as an indicator of the effect of a variable on the model's output [10]. In multi-dimensional variation analysis, there exist several variables that affect the outcome. The sensitivities were found by taking the partial derivative of each equation from equation 3.28-31, expressed as θ, with respect to each variable, X_i, [10], as shown in equation 4.1.

$$S_i = \frac{\partial \Theta}{\partial X_i} \tag{4.1}$$

The partial derivatives were evaluated at the nominal value of each of the terms, where the nominal value is defined by equal bilateral tolerances. Equal bilateral tolerances refer to a tolerance of plus or minus a uniform amount. If unilateral tolerances or unequal tolerance are desired[1], they are derived from the equal bilateral tolerance. From equation 4.1, components with small nominal values likely have a small sensitivity and therefore a small contribution to the variation of the outcome.

---

[1]Unilateral tolerances refer to a range that is not uniform on both sides of the nominal value, such as plus zero and minus some value.

Once the partial derivatives for each of the imbalance equations were obtained and calculated for each component, the variation for each equation was evaluated. The variation, V_i, was defined as the product of the sensitivity, S_i, and the equal bilateral tolerance, t_i, as shown in equation 4.2.

$$V_i = S_i \cdot t_i \tag{4.2}$$

The variation of static and dynamic imbalance was evaluated using the sensitivity for each factor and its variation, and shown below in equation 4.3. As explained above, the partial derivative of each term of the imbalance equations was taken with respect to each dimension (m, x, y, z) then multiplied by is variation. The notation ΔX, ΔY, etc., refers to the variation of the imbalance equation.

$$\begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta I_{XZ,i} \\ \Delta I_{YZ,i} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial m} & \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} & \frac{\partial X}{\partial z} \\ \frac{\partial Y}{\partial m} & \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} & \frac{\partial Y}{\partial z} \\ \frac{\partial I_{XZ}}{\partial m} & \frac{\partial I_{XZ}}{\partial x} & \frac{\partial I_{XZ}}{\partial y} & \frac{\partial I_{XZ}}{\partial z} \\ \frac{\partial I_{YZ}}{\partial m} & \frac{\partial I_{YZ}}{\partial x} & \frac{\partial I_{YZ}}{\partial y} & \frac{\partial I_{YZ}}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} \Delta m_i \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} \tag{4.3}$$

$$= \begin{bmatrix} x_i & m_i & 0 & 0 \\ y_i & 0 & m_i & 0 \\ x_i z_i & m_i z_i & 0 & m_i x_i \\ y_i z_i & 0 & m_i z_i & m_i y_i \end{bmatrix} \cdot \begin{bmatrix} \Delta m_i \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix}$$

Note that in the case of dynamic imbalance, only the parallel axis contribution was considered and the local inertia terms omitted. This omission is a limitation of the sensitivity analysis, as the partial derivative of the local inertia term would be very complex, and perhaps not possible. The total system imbalance variation was the sum of each component:

$$\Delta X = \sum \Delta X_i \tag{4.4}$$

$$\Delta Y = \sum \Delta Y_i \tag{4.5}$$

$$\Delta I_{XZ} = \sum \Delta I_{XZ,i} \tag{4.6}$$

$$\Delta I_{YZ} = \sum \Delta I_{YZ,i} \tag{4.7}$$

It would be logical to attempt to estimate the desired component variation given the partial derivative and the contribution to system imbalance. Moving the partial derivative matrix (i.e., the Jacobian) to the other side of the equation by inverting it could achieve this goal; however, this step was not possible, as the partial derivative matrix was singular, and therefore could not be inverted. Furthermore, taking the pseudo-inverse of the matrix is not feasible due to the mixed units on the components [11]. Thus, other methods must be found.

Instead of rearranging the equation and solving for the variation of each component, an estimated value was assumed and the imbalance variation was evaluated. In this guess-and-check approach, the total system variation was evaluated after each change to the input variables.

By analyzing each imbalance variation relationship, it was clear which factors impacted the outcome for a given component. For example, the variation for static imbalance in X (in eq. 4.3) was defined by the sum of the nominal Y-term times the variation in mass and the nominal mass times the variation in Y. For this reason, a component such as the X-ray tube, which was a heavy component located at a large radial distance in Y, had a large contribution to the variation of the ΔY term.

Furthermore, the ratio of each component's contribution to imbalance variation was compared to one another, mainly to identify the highest contributors. Careful control of these components resulted in the largest impact for control of total imbalance variation. A stacked bar graph was created to show the contribution of each component to the variation. It was helpful to show the absolute value of each contribution to more easily compare one bar to another with all bars on the same side of the axis.

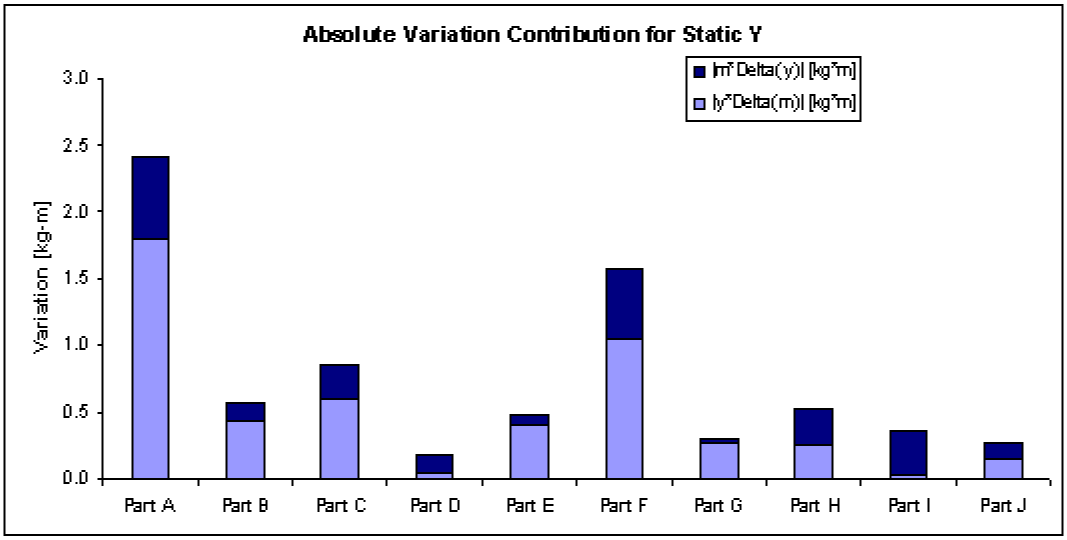

Figure 4.1: Contribution Plot

This analysis provides detailed insight into the contributions of each component comparatively, but failed to show the success of the system-level balance design. In order to show the result at the system level, a statistical approach was used.

4.2 Statistical Simulation

Statistical simulations, often used to interpret trends from historic data, was also useful to estimate future data based on the variation. A statistical model helped predict the most likely outcome and the most likely variation for that outcome. From these models, an estimate of system performance was made during the early stages of development [10].

The likelihood of an acceptably balanced system and the variation of the outcome were predicted using a Monte Carlo model. The model used the equations developed in Chapter 3 and varied the mass and location of the center of gravity within an estimated variation. The model was established by assigning a uniform distribution to each of the variables of mass and CG, and then randomly generating values from within the assigned range. The model then calculated the total imbalance given the selected value and repeated for a set number of trials. The total system static and dynamic imbalance was computed for all of the trials. These trials were tabulated and the resulting data set analyzed to study the variation of the system imbalance given the estimated variation of the components.

Once the variation of the system imbalance was estimated by the statistical model, a pass or fail criteria was established. The capability envelope of the counter balance weights established the limit for this particular design. As mentioned in section 3.5, the counter balance weights allowed for adjustment of the location of the system's center of gravity and the alignment of the principal axis of rotation. They provided a means to compensate for the variation of the piece parts and allowed a means to widen the specification limits for the component's mass and CG tolerances.

The capability envelope was constructed by performing a Monte Carlo simulation on the combinations of counter weights. Each of the masses and locations of the weights were randomly selected by the model and the imbalance was summed to make the total capability envelope. The imbalance data was extracted from the capability model and plotted.

Finally, the variation of the component variation model was compared to the capability envelope. If the variation of the system imbalance fell within the bounds of the capability, the variation of the piece-parts was considered acceptable. If not, changes needed to be considered, either to the nominal factors, the estimated variation of the piece-parts, or the system capability to correct of imbalance must be increased. The variation was shown graphically compared to the capability envelope by overlaying the imbalance variation on the capability envelope, as shown in figure 4.2 (note: the variation data was inverted before comparing to the capability data). The comparison of the two plots is objective relies on the visual data for acceptance.

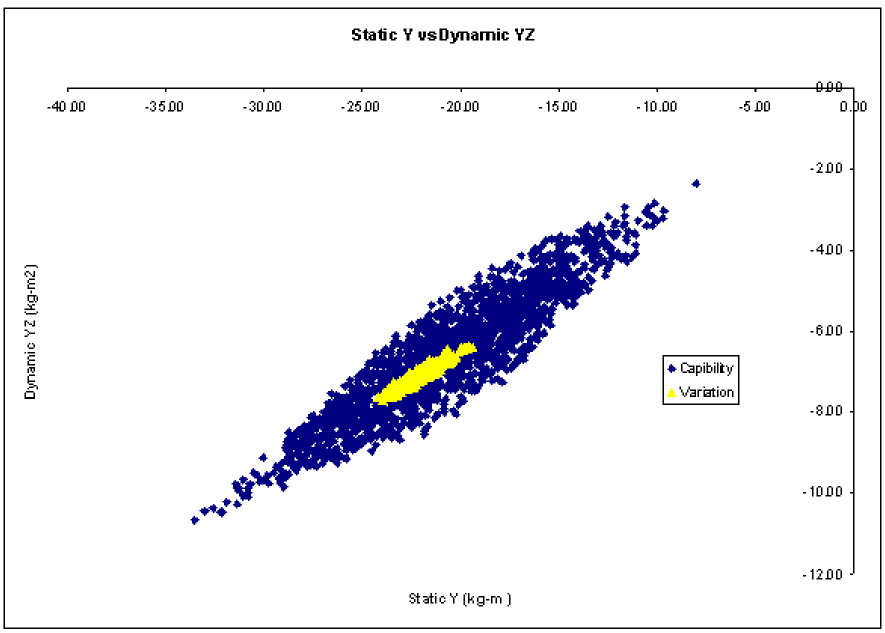

Figure 4.2: Variation and Capability Plot

The four measures of imbalance defined in Chapter 3 were named: Static X ($\Sigma m_i x_i$), Static Y ($\Sigma m_i y_i$), Dynamic XZ ($\Sigma Ixz+m_i x_i z_i$), and Dynamic YZ ($\Sigma Iyz+m_i y_i z_i$). These four measures made six unique combinations if plotted on a 2-axis graph. It was discovered that the Static X verses Dynamic XZ and Static Y verses Dynamic YZ were the two most reveling plots because when the variation data was centered over the capability data in these two plots, the remaining four graphs were also centered. Therefore, these two plots are the most frequent plots analyzed.

4.3 Model Comparison

The sensitivity analysis and the statistical model each had advantages and disadvantages. The sensitivity analysis provided insight to the contributions of each component because a numerical value could be calculated for each component's contribution to imbalance. It objectively identified the components that produced a large contribution to the system's imbalance variation from those with a less significant contribution. It showed the inner workings of the imbalance model and made clear how each component contributes to the outcome. For example, a heavy component with tight tolerances on mass and CG was often more of a contributor to system variation than a light component with wider variation of mass and CG. These differences were observed clearly with the contribution plots.

On the downside, the sensitivity analysis did not include component inertia, but only the parallel axis contribution for dynamic imbalance. It also did not show whether the system could be balanced, but rather a comparison of the contributions of each of the components.

The statistical model, on the other hand, showed the big picture. It clearly showed the success on the system-level by providing a means to visually compare the variation to the capability. It provided a means to establish a pass or fail criteria of all of the components variation collectively. It also showed whether the design was centered in the capability or skewed in one or more directions. Furthermore, it was possible to include the component's local inertia term.

Although effective at providing a visual of success at the system level, it was difficult to quantify a pass or fail result. Furthermore, it was difficult to measure the impact of the component 's variation without conducting many tedious trials and comparing the size of the point cloud.

As shown, models show unique perspectives of the system's balance variation, but only collectively do they tell the complete story.

Chapter 5: Proposed Method of Tolerance Allocation

It was understood from the analysis shown in Chapter 4 how a component's nominal mass, CG, and the variation of these factors contributed to the variation of the system imbalance. The theory was straightforward; however, the process of allocating tolerances for these specifications was not so clear.

The end goal of this research was a method to allocate tolerances on mass and CG for the individual components such that when the system is assembled its imbalance is acceptable for the system function every time. The tolerances needed to be allocated in a way that was achievable for the manufacturing process of each component without adding excessive cost. With so many variables and tradeoffs to evaluate, the allocation process frequently required a business case to select which components were allowed more variation than others. Furthermore, the models developed to evaluate tolerance allocation needed to be maintainable over time so component tolerances could be re-evaluated during their lifecycle if necessary.

5.1 Considerations for Tolerance Allocation

A truly engineered solution required consideration of all of the impacts of a tolerance allocation besides the pure mathematical result on the imbalance solution. Some of these factors included cost, service schedule, and ongoing part changes. The tradeoffs between these factors influenced the ability to appropriately select how to divide the system's variation among the components.

From a theoretical tolerance analysis perspective, success at the assembly level could easily be achieved with tight tolerances on each component; however, tighter tolerances induce a higher cost due to increased manufacturing precision. Most components in the system were sub-assemblies themselves, and were composed of unique components, built by unique assembly processes and by unique manufacturers. Any deviation from the nominal will cause a quantifiable cost, or loss, as quantified by Japanese engineer Dr. Genichi Taguchi [10].

Several factors were considered in order to establish the acceptable limits to the variation of the system. The first consideration was impact of variation during the initial product assembly. The variation of the components must collectively result in a balance state within limits that do not result in noticeable vibration to the gantry during rotation. Tests were conducted to apply a known imbalance (combinations of both static and dynamic) and observed the resulting vibration of the gantry. Additionally, studies were conducted to impose a limit on the maximum vibration that would cause an X-Ray image artifact. The variation of the system imbalance was established such that it fell within this acceptable vibration limit.

The second factor was the consideration of the variation of the components over the life of the product. The components that make up the initial product build were expected to be replaced over the life of the product. The replacement components must be similar enough to the original component such that when replaced the new component does not cause a noticeable impact to the system imbalance. Additional time spent balancing the CT system after a component replacement cost the service company and clinical site time and money while the system was not being used.

Lastly, the likelihood of a component to change over its life due to product updates was considered. Such changes are necessary to maintain a product platform by incorporating new features, improve safety and manufacturability, and decrease cost. These essential updates often require a change to the rotating mass components and occasionally impacts a component's contribution to imbalance or its contribution to variation. Therefore, it was beneficial to allow as much flexibility as possible to the component's tolerance allocation to accommodate such changes.

5.2 Procedure for Tolerance Allocation

The procedure for evaluating variation and eventually allocating tolerances was an iterative approach. First, the acceptance criterion of the system was established and the component's mass and CG data gathered. The system's nominal imbalance was calculated and if necessary, was altered by placing permanent masses in appropriate locations so it was centered about the capability. The variation for each of the components was estimated and the system imbalance was evaluated. The resulting system imbalance variation was compared to the acceptance criteria. Then, the sensitivities were calculated and used as tools to compare and adjust the component's variation as needed. Once the variation of the system imbalance was acceptable to the design capability, the tolerances were allocated. Refer to Figure 5.1 for a process map of the allocation process.

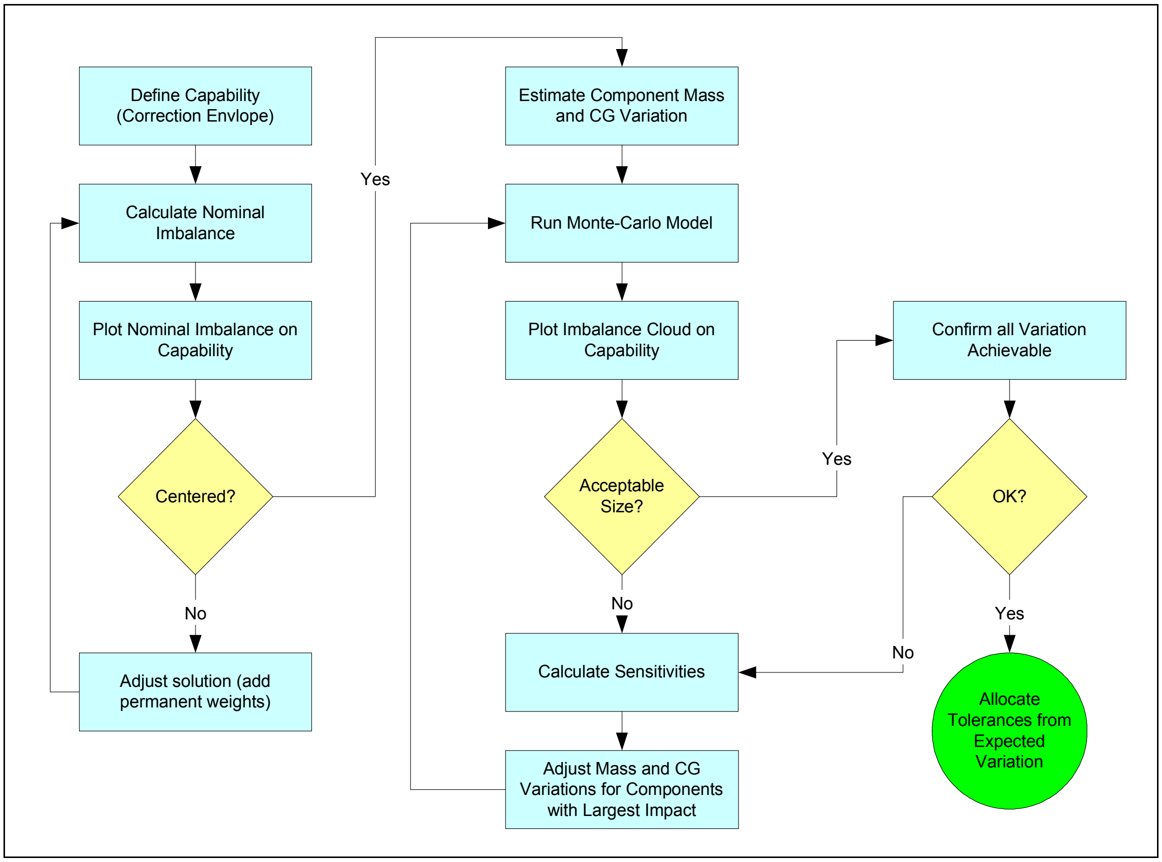

Figure 5.1: Tolerance Allocation Process Map

The first step in evaluating the variation was to understand the range of acceptance. For this design, it was decided the acceptable limitation of imbalance due to variation was defined by the ability of the system to correct for imbalance by use of variable weights. Consideration for this decision included the impact of imbalance to vibration and the effectivity of system to correct for imbalance.

The static and dynamic imbalance created by each combination of variable weights was tabulated by a Monte Carlo simulation and plotted in 2 two-axes graphs: Static X verses Dynamic XZ and Static Y vs Dynamic YZ. The area represented by this envelope was the capability for the system to correct for imbalance and was therefore the target envelope for the allowable variation of the system imbalance (not including design margin).

Next, the expected nominal static and dynamic imbalance of the system was evaluated. The mass and CG was gathered for each component. In most cases, the CG was given in a local coordinate system that needed to be expressed in a common global coordinate system. Once the data for all components was gathered and expressed in the global coordinates, the imbalance was calculated for each component and summed together (equations 3.28-31). This resulting summation of components is the total system's nominal imbalance. The nominal solution is important to understand where the variation will be centered in relation to the correction envelope.

Once the nominal imbalance was calculated, it was inverted, and then plotted over the imbalance correction envelope created by the variable weight combinations, thus creating the Variation and Capability Plot like that shown in figure 4.2. The two plots should overlap; ideally, the nominal imbalance should be centered on the correction envelope. A centered design will allow for the most variation possible and will allow design margin for component changes that may cause a shift to the nominal solution.

If the expected nominal imbalance of the system did not lie on the correction envelope or was not centered, the design was altered to center it. The nominal imbalance of the system was altered by adding or removing mass, usually in the form of permanent weights at a location on the rotating mass that was available for such a change yet favorable to the imbalance solution.

Once the nominal solution was centered about the correction envelope, the variation was evaluated. To estimate the variation of the system, the Monte Carlo model from Chapter 4 was used. The values for the variation of each component were estimated based on historical data, component manufacturing process, complexity of the subassembly, and experience of the engineer.

The components were first assigned an expected variation based on historical data, when available. Most of the major components had similar components on existing systems, so the previous specifications and manufacturing data was reviewed as a guideline. Second, was the consideration for how frequent the previously produced components experience design changes. The components that change less frequently were expected to have smaller variation, and thus a tighter variation was estimated. Next, the complexity of the component was considered: those with many parts were assigned a larger expected variation than those components made of few components, especially those cast and machined. Lastly, was simply the experience of the engineer who made a reasonable estimate of the variation, especially in cases where previous component data was not available.

Once an estimate for the variation of the mass and CG for each component was established, the Monte-Carlo simulation was run and the resulting imbalance data tabulated and plotted on the balance capability envelope. The size of the point cloud of the variation imbalance data indicated the amount of expected imbalance variation of the system.

The size of the imbalance point cloud should be small enough so the correction envelope can be seen around the circumference of the variation to allow for adequate design margin. The plots created from the Monte Carlo model provide a system-level visual of design acceptance; however, it is subjective and does not reveal anything about the components, only the system as a whole.

The sensitivity analysis was used to compare the contribution of each component in relation to each other. The contributing factors of mass and CG were altered one component at a time, as needed, starting with the largest contributors first. The changes were made while considering the limitations of the manufacturing capability of the component. After each component was evaluated, the Monte Carlo model was updated with the new allocations and rerun. This iterative approach was continued until a desired system imbalance variation was achieved. It provided a tool to show the stakeholders the contribution of each component and helped provide decision guidance for the adjustment of acceptable variation.

Once the size of the imbalance cloud was an acceptable, it was further evaluated to ensure all allocated tolerances were appropriate. Although the parts with the highest contribution were the easiest to adjust for a successful system result, all allocations must be realistic to manufacture. It was occasionally necessary to adjust multiple moderately contributing components to achieve the desired system imbalance instead of adjusting the highest contributors. This was done to balance the cost and manufacturability of each component. Once all of the variations were considered acceptable, the tolerances of mass and CG were allocated to the components.

5.3 Tolerance Allocation Process Example

Below is an example of how the process was used. First, the mass and CG of the parts were gathered and the nominal imbalance calculated (table 5.1).

Component:	Mass - NOM	CGx	CGy	CGz	Qty	X_i	Y_i	$I_{XZ,i}$	$I_{YZ,i}$
	[kg]	[mm]	[mm]	[mm]	on/off	[kg-m]	[kg-m]	[kg-m2]	[kg-m2]
Part A	120	-20.00	600.00	300	1	-2.40	72.00	-0.72	21.60
Part B	35	145.00	425.00	275	1	5.08	14.88	1.40	4.09
Part C	85	275.00	300.00	150	1	23.38	25.50	3.51	3.83
Part D	60	300.00	-35.00	250	1	18.00	-2.10	4.50	-0.53
Part E	40	215.00	-400.00	60	1	8.60	-16.00	0.52	-0.96
Part F	105	5.00	-525.00	350	1	0.53	-55.13	0.18	-19.29
Part G	20	-250.00	-525.00	250	1	-5.00	-10.50	-1.22	-1.20
Part H	55	-325.00	-250.00	200	1	-17.88	-13.75	-3.58	-2.57
Part I	70	-375.00	15.00	125	1	-26.25	1.05	-3.28	0.13
Part J	20	-250.00	300.00	325	1	-5.00	6.00	-1.63	1.95
						ΣX_i	ΣY_i	$\Sigma I_{XZ,i}$	$\Sigma I_{YZ,i}$
						-0.95	**21.95**	**-0.32**	**7.05**
						0.95	-21.95	0.32	-7.05

Table 5.1: Imbalance Model

The correction envelope was defined based on the capability of the variable weight stacks and the nominal imbalance of the system was plotted on the same set of graphs (figures 5.2-3).

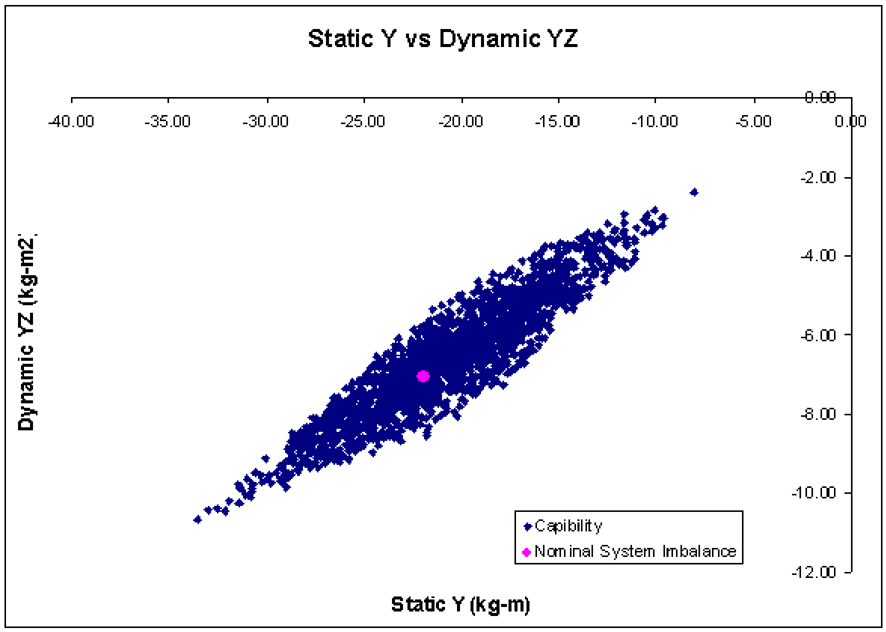

Figure 5.2: Capability Plot with Nominal Imbalance (Static Y vs. Dynamic YZ)

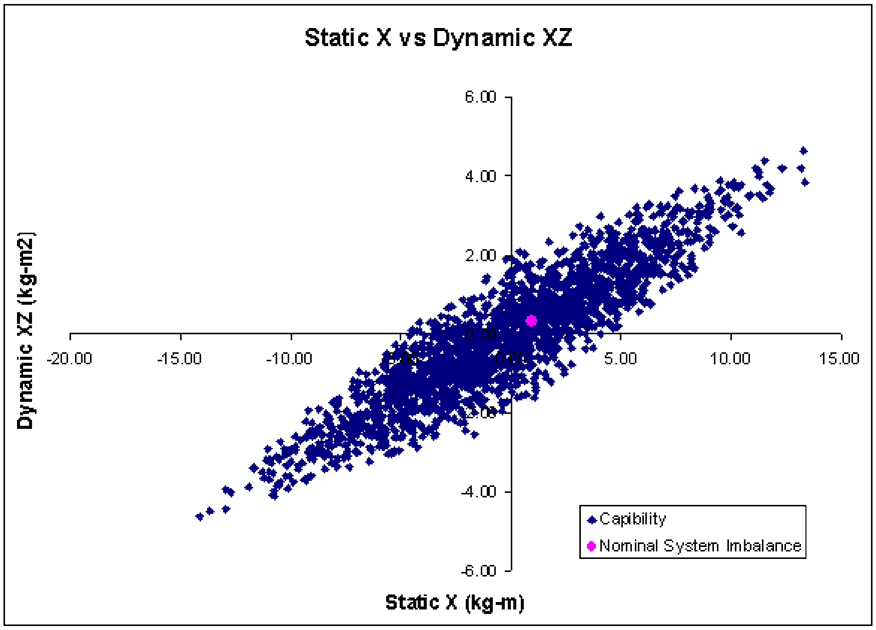

Figure 5.3: Capability Plot with Nominal Imbalance (Static X vs. Dynamic XZ)

The initial component placement yielded a solution centered in the bounds of the correction envelope. If it had not been centered, either the mass or locations of the components would have needed adjustment or a permanent weight would have been required to bring the nominal solution in the center the capability envelope.

Next, the component's variation was estimated and put in the Monte-Carlo model (table 5.2). The simulation was run and the data extracted, inverted, and plotted over the capability envelope (figures 5.4-5).

Component:	Mass - NOM [kg]	Δm [kg]	CGx [mm]	Δx [mm]	CGy [mm]	Δy [mm]	CGz [mm]	Δz [mm]	Qty on/off	X_i [kg-m]	Y_i [kg-m]	$I_{XZ,i}$ [kg-m2]	$I_{YZ,i}$ [kg-m2]
Part A	120	5.0	-20.00	5.00	600.00	5.00	300	5	1	-2.40	72.00	-0.72	21.60
Part B	35	1.0	145.00	4.00	425.00	4.00	275	4	1	5.08	14.88	1.40	4.09
Part C	85	2.0	275.00	3.00	300.00	3.00	150	3	1	23.38	25.50	3.51	3.83
Part D	60	1.5	300.00	2.00	-35.00	2.00	250	2	1	18.00	-2.10	4.50	-0.53
Part E	40	1.0	215.00	2.00	-400.00	2.00	60	2	1	8.60	-16.00	0.52	-0.96
Part F	105	2.0	5.00	5.00	-525.00	5.00	350	5	1	0.53	-55.13	0.18	-19.29
Part G	20	0.5	-250.00	2.00	-525.00	2.00	250	2	1	-5.00	-10.50	-1.22	-1.20
Part H	55	1.0	-325.00	7.00	-250.00	7.00	200	7	1	-17.88	-13.75	-3.58	-2.57
Part I	70	1.5	-375.00	5.00	15.00	5.00	125	5	1	-26.25	1.05	-3.28	0.13
Part J	20	0.5	-250.00	6.00	300.00	6.00	325	6	1	-5.00	6.00	-1.63	1.95
										ΣX_i	ΣY_i	$\Sigma I_{XZ,i}$	$\Sigma I_{YZ,i}$
										-0.95	**21.95**	**-0.32**	**7.05**
									Inverted:	0.95	-21.95	0.32	-7.05

Table 5.2: Imbalance Model with Variation Estimates

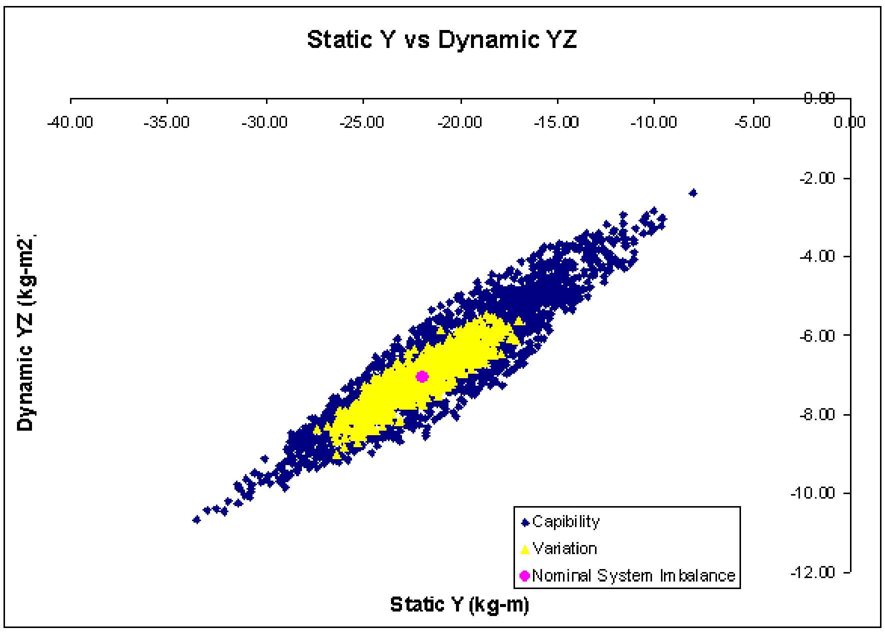

Figure 5.4: Capability and Variation Plot (Static Y vs. Dynamic YZ)

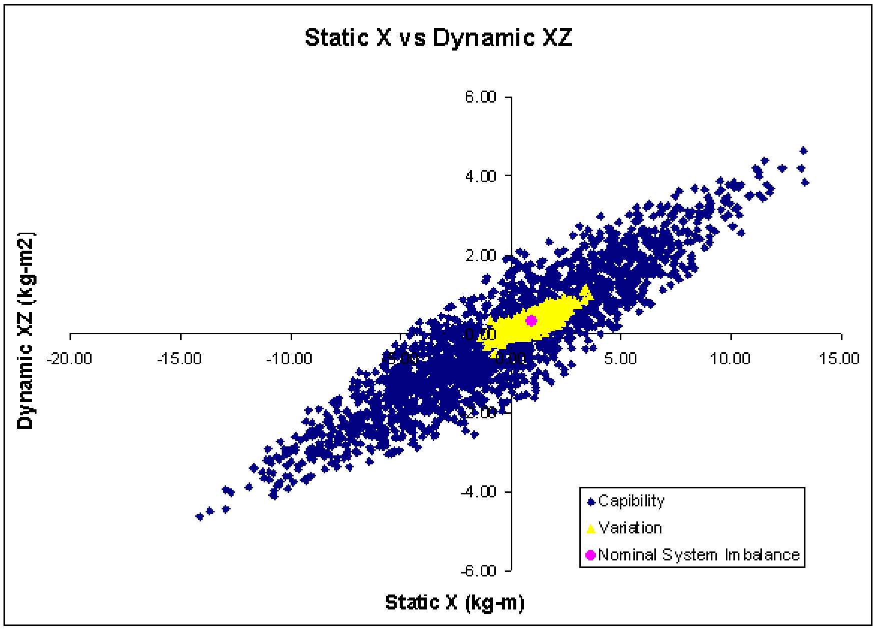

Figure 5.5: Capability and Variation Plot (Static X vs. Dynamic XZ)

Although the data was contained inside the capability envelope, the variation was far too large to have adequate control the design as it did not have acceptable margin between the variation and the capability. The contributions of each component were analyzed using the contribution plots derived from the sensitivities (figures 5.6-9). The plots were used as a guide to understand which component had the largest contribution to the variation, and therefore had the most impact to reducing the size of the variation. These components were adjusted first, along with any other components that had an estimated variation larger than necessary to produce the component.

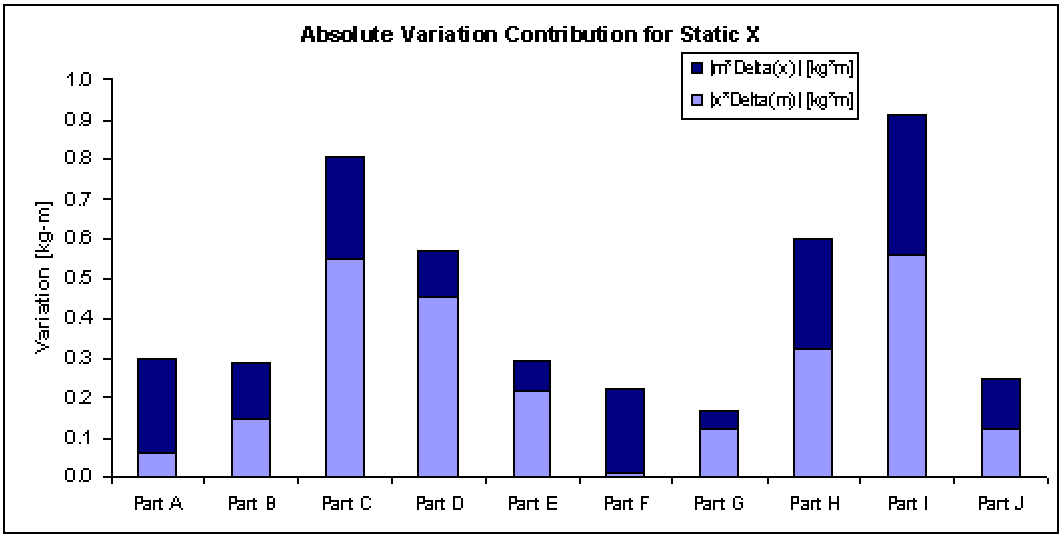

Figure 5.6: Contribution Plot (Static X)

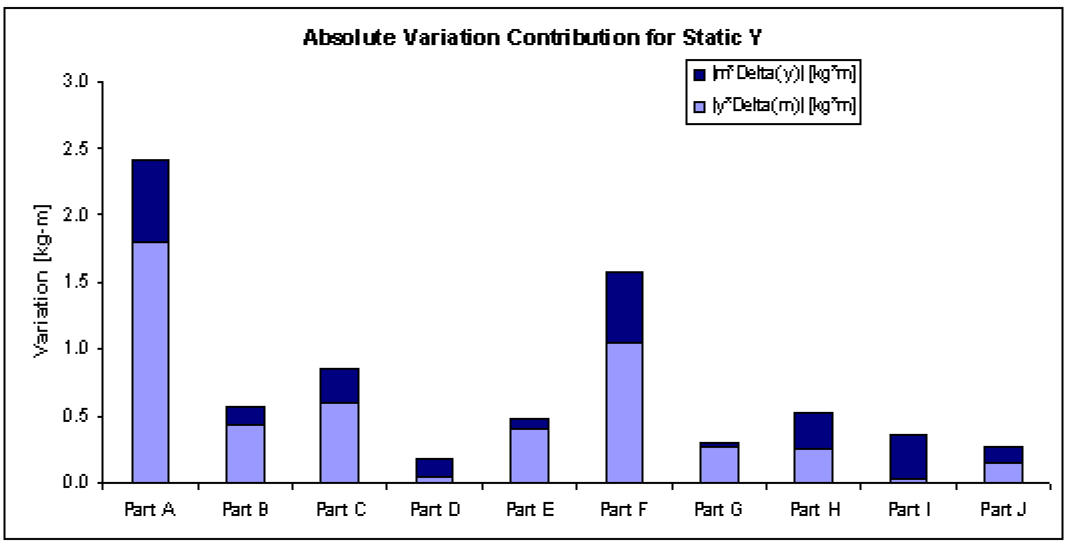

Figure 5.7: Contribution Plot (Static Y)

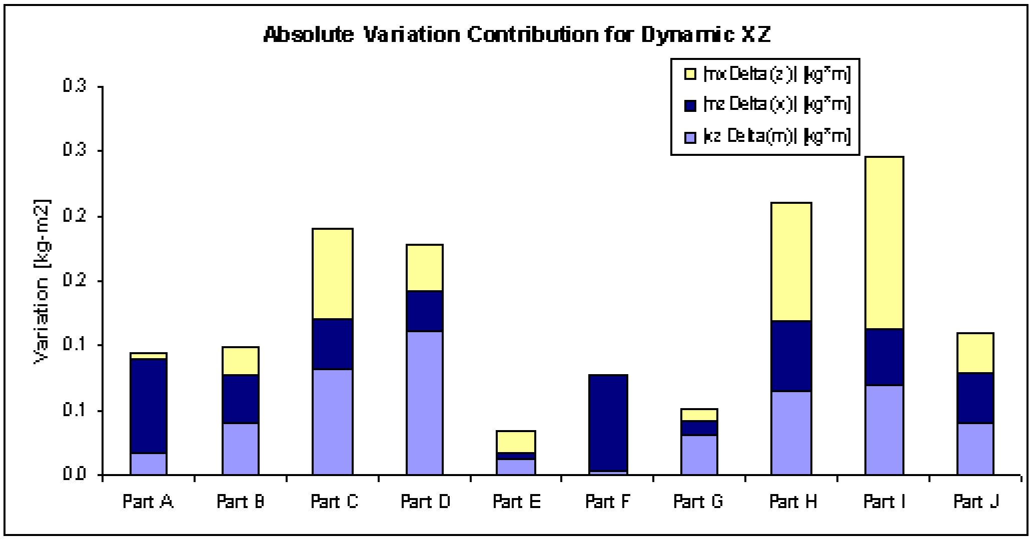

Figure 5.8: Contribution Plot (Dynamic XZ)

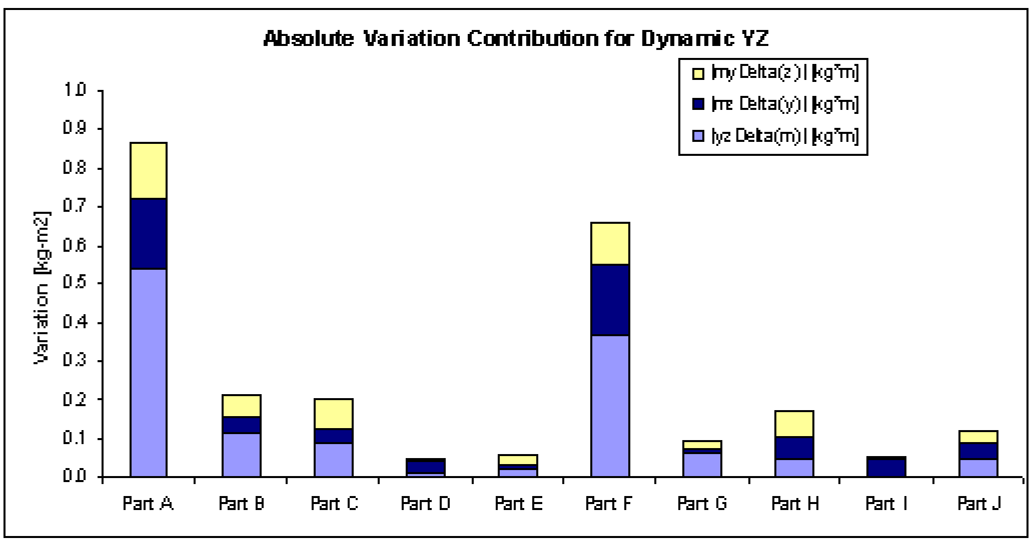

Figure 5.9: Contribution Plot (Dynamic YZ)

It was observed from the contribution plots that the distribution of variation in X for all of the components was well distributed; however, the variation in the Y direction was primarily consumed by two components: Parts A and F. The variation of the mass and CG of these two components was reduced until the contributions to imbalance were more equally distributed. The Monte Carlo simulation was rerun and the overall system variation was observed to be an acceptable amount (figure 5.10-11).

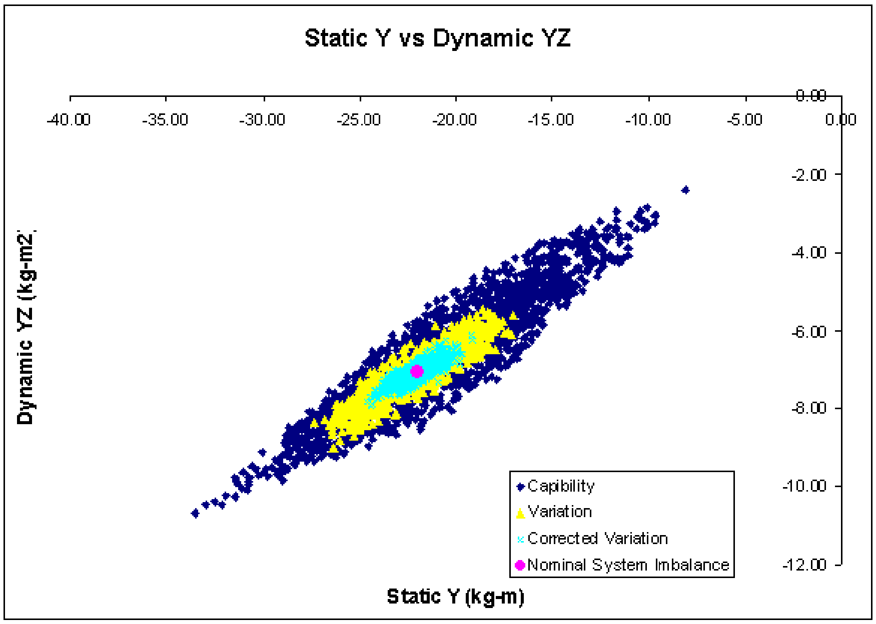

Figure 5.10: Corrected Capability and Variation Plot (Static Y vs. Dynamic YZ)

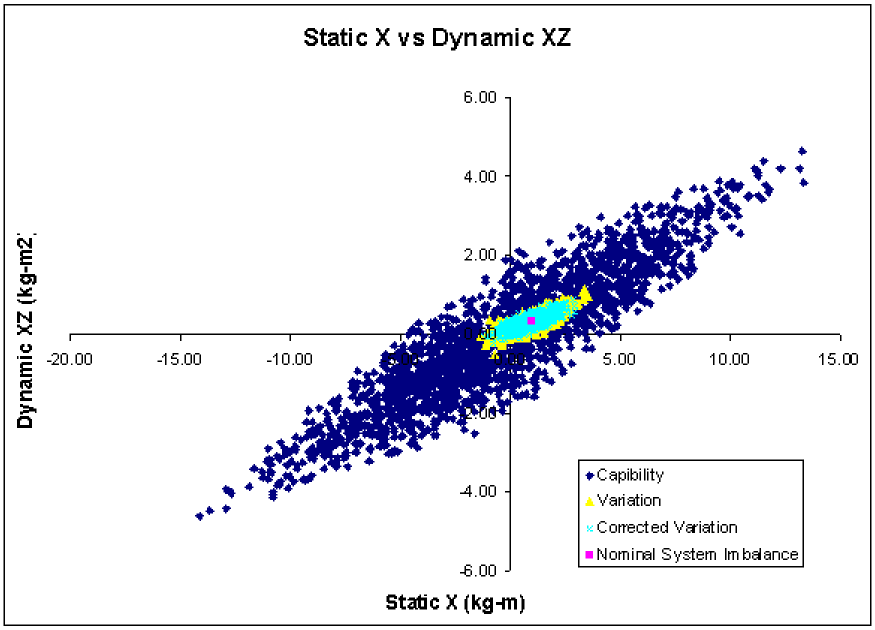

Figure 5.11: Corrected Capability and Variation Plot (Static X vs. Dynamic XZ)

It was shown by the variation and capability plots (figures 5.10-11) the variation was acceptable compared to the capability for correction. The adjustments made to the variation of the components were sufficient to control the variation of imbalance at the system level. If the changes had not made a significant enough impact, the contribution plots would have been used as guidance to further reduce the variation of components.

Finally, after the variation and capability plots showed a successful design, each component's variation was reviewed to ensure the assigned values were achievable based on the component composition and manufacturing process. If all estimated variations were achievable, the estimated variations were allocated as the required tolerances. This model was established in a way that it can be updated over the life of the product to maintain the system variation within acceptable bounds.

Chapter 6: Conclusions

The vibration of rotating machines has been studied thoroughly from causes to prevention to diagnostics. It is well documented that the main cause of vibration in rotating machinery is mass imbalance. Equipment has been developed to detect for imbalance and suggest a means to correct for it.

For the CT scanner, the affects of mass imbalance reach beyond noise, wear, and fatigue, as it also impacts the ability for the equipment to produce a high-quality image. Without a clear image, the device is not competitive in the market and can even negatively affect patient diagnosis. For this reason, mass imbalance must be carefully studied and controlled to eliminate the opportunity of poor performance.

6.1 Summary of Contributions

This research presented the formulation for calculating static and dynamic imbalance. Considerations were taken to include component inertia where necessary and otherwise justified the point-mass assumption. Translation and rotation transformations were introduced to utilize local coordinate systems.

The balance equations were further analyzed by conducting a variation analysis to understand the impact of discrete component's variation to the total system variation. A sensitivity model was developed to calculate the Jacobian matrix for each of the balance equations. The calculated sensitivities for each component provided a means to compare each of the component's contribution to imbalance. Additionally, a statistical simulation was created to predict the system imbalance given the variation of the components. It was compared to the system's allowable variation for pass or fail success.

Both models were used to evaluate the criteria for success of the design, including system capability, component manufacturability, service schedule, and cost. They revealed information both in the contributions of the discrete components and the variation at the system-level. They identified the mass properties and variations that contribute most to imbalance variation and provided a means to develop a method to allocate tolerances of mass and center of gravity to the discrete components that composed the rotating mass assembly.

6.2 Prospect of Future Work

Some of the possibilities for future development work in the field of balance to multi-body systems include the following suggestions:

- System for measuring the products of inertia of components in a time and cost effective way. Current methods include simplified approaches to measuring the moments of inertia (not helpful in this type of analysis) or complex, expensive machines to measure the products of inertia. If a means were available to measure the local products of inertia, then building the case for including the inertia or using the particle mass assumption would be clearer. Also, if a means existed to measure the products of inertia, one could implement a process control around this specification. Without an ability to take a measurement, it is not possible to verify the components are manufactured within a range; therefore, it is not possible to implement a specification limit.

- Device for online correction of significant imbalance, such as a balance ring. There are potential benefits of a method to adjust the CG and principal axis of inertia of the gantry without taking measurements, calculating the correction mass(s) and location(s), and implementing the solution. Such a device could increase the allowable system imbalance variation that would increase the allowable variation of the components, and would also reduce the time and cost required to assemble and service the system.
- Method of calculating the local component inertia in polar or cylindrical coordinates. There are potential advantages to conducting this type of analysis in cylindrical coordinates, but the current limitation is the inability to express local inertia of the components in anything other than the classically developed Cartesian coordinates. Due to the rotational mechanics of this analysis, a polar coordinate system is not only more intuitive, but may ease the ability to express the results of the sensitivity analysis and statistical simulations.

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