Blending as a Multi-Horizon Time Series Forecasting Tool

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BLENDING AS A MULTI-HORIZON TIME SERIES FORECASTING TOOL

by

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ABSTRACT

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Every day, millions of cubic feet of natural gas is transported through interstate pipelines and consumed by customers all over the United States of America. Gas distributors, responsible for sending natural gas to individual customers, are eager for an estimate of how much natural gas will be used in the near future. GasHour^{TM} software, a reliable forecasting tool from the Marquette University GasDay^{TM} lab, has been providing highly accurate hourly forecasts over the past few years. Our goal is to improve current GasHour forecasts, and my thesis presents an approach to achieve that using a blending technique.

This thesis includes detailed explanations of the multi-horizon forecasting technique employed by GasHour models. Several graphs are displayed to reveal the structure of hourly forecasts from GasHour. We present SMHF (Smoothing Multi-horizon Forecasts), a step-by-step method showing how a polynomial smoothing technique is applied to current GasHour predications. A slightly different approach of smoothing has also been introduced.

We compare RMSEs of both GasHour forecasts and smoothed ones. Different comparisons resulting from different situations have been demonstrated as well. Several conclusions have been reached. Based on the results, blending techniques can improve current GasHour forecasts. We look forward to applying this blending technique to other fields of forecasting.

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"No man is an island,
Entire of itself,
Every man is a piece of the continent,
A part of the main."

John Donne

I wish to express my sincere gratitude to all the people who have helped me in the past few years. Your encouragement is the source of my strength, and my work shall be the fruits shared by all of you.

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"We rise by lifting others."

Robert Ingersoll

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CHAPTER 1

Introduction to Natural Gas Consumption Forecasting

This chapter presents an overview of the natural gas industry and the motivation for the study undertaken in this thesis.

1.1 Natural Gas and the Industry

Natural gas is a vital component of the American supply of energy. According to the Energy Information Administration (EIA) [2; 53; 67], natural gas accounts for 24 percent of total energy consumed in the United States. Gas utilities across the country provide service to over 65 million residential customers and 5 million commercial enterprises [2]. Due to its growing popularity, natural gas consumption is experiencing the fastest growth among all fossil fuels [61], and it is expected to maintain that trend for the years to come [2; 3; 4].

Most of the natural gas demand comes from industrial use (32%), electric generation (24%), and residential use (22%) [12; 53]. Applications of natural gas in industry include providing base ingredients for certain products, metal preheating, food processing, drying, dehumidification, and so on [53]. Electricity generators contribute most to the current growth in natural gas demand [2; 4]. Estimates from the EIA [53] show that between 2009 and 2015, over 20 percent of new electricity capacity will come from natural gas-fired power plants. Based on the analysis from the Department of Energy (DOE) [53] in 2011, natural gas is the lowest cost conventional energy source for residential customers. Heating (space heating, water heating, and clothes drying) and cooking are the best known uses for natural gas around the homes [5; 53]. The American Gas Association (AGA) indicates that 62 million homes in the U.S., are heated using natural gas [2; 53].

The process of getting natural gas from wellheads to end customers is a complicated one, including exploration, extraction, production, transportation, storage, and distribution [52]. In the final distribution part, a nationwide delivery system, including 2.1 million miles of local utility distribution pipes and 0.3 million miles of transmission lines, transports natural gas to households and businesses all over the country [2]. Most of the consumers receive their natural gas from their local distribution companies (LDCs), utilities engaged in natural gas retail sales and delivery of gas to the end customers within a specific geographic area [2; 6].

Natural gas, as one of the cleanest, safest, and most useful forms of energy, will play an increasingly important role in meeting the United States' energy demand in the future [3; 53]. To use it efficiently and wisely, accurate natural gas demand forecasts attract more and more attention from pipeline and distribution companies.

1.2 Short-term Natural Gas Forecast

Natural gas demand forecasts can be classified as short-, medium- or long-term forecasts, based on the time horizons of these forecasts. Diebold (2008) defines forecast horizon as "the number of periods between today and the date of the forecast we make," which shares similarities with concepts built by Bes and Sethi (1988). More generally, a forecast time horizon can be the time (or time interval) between the time (or time interval) **at which** the forecasts are made and the time (or time interval) **for which** they are made. The short-term demand forecasts often refer to forecasts made 1-7 days ahead [37]. For gas suppliers and distributors, forecasts are needed both hourly and daily [43]. Usually, short-term demand for residential and commercial customers is influenced by many factors, including temperature, wind speed, day-of-the-week, and so on [14; 43]. Among them, temperature is the most significant factor, since natural gas is mostly used for space heating [5; 14]. Also, buildings using gas for heating tend to lose more heat on a windy day than a calm day, making wind speed a vital component affecting total natural gas consumption [14; 43]. Some commercial and industrial customers may shut down over the weekends, which explains why day-of-the-week should be considered as well [43; 68]. Besides the factors mentioned above, future gas consumption also is related to historical gas flow data and actual temperature information [14; 57]. The historical flow data is also called measured flow data, which was recorded and reported through LDCs. These data are used to represent actual flow data, which is unfeasible to access due to the structure of pipelines.

In the past decades, many approaches have been investigated to improve energy consumption forecasting. These methods include artificial neural networks, nonlinear regression, expert systems, stochastic models, and support vector machines [14; 50; 57]. Based on past experience, forecasting accuracy can be increased by proper understanding of the underlying principles of energy consumption. These principles sometimes are called “domain knowledge” [13; 22; 57; 68]. For instance, knowing the categories of the customer base and the characteristics of these categories can provide insightful assistance in identifying and analyzing the data patterns of gas usage for such customers [68].

1.3 GasDay Lab in Marquette University

GasDay™ project was established by Dr. Ronald Brown in 1993. This lab has an educational mission and also fosters research opportunities for undergraduate and graduate students by licensing and supporting software that serves local distribution companies (LDCs). GasDay has been developing and improving natural gas demand forecasting technology for more than 20 years, with products covering hourly, daily, monthly, and yearly gas demand forecasts. Currently, GasDay is providing service to 30 utilities in 24 states, accounting for more than one fifth of industrial, residential, and commercial natural gas demand in the United States [31].

The work presented in the thesis has mainly focused on hourly gas demand forecasts, although the methods developed can be applied to other forecasting fields.

1.4 Statement of the Problem

Natural gas price was capped and controlled before the early 1980s [1]. Starting in the year 1978 [51], a series of new acts intended to deregulate natural gas market were passed on the federal level. With the elimination of artificial ceiling prices on natural gas, demands from emerging customers have been greatly satisfied [12]. Now, the wholesale price of natural gas is market-driven, which sometimes contributes to high volatility in prices due to large swings in supply and demand [1; 12]. In some cases, the cost of natural gas purchased on the spot market can even be 10 times that purchased under contract [29]. This economic reality pushes LDCs to make wise decisions on how much gas they should buy from natural gas producers or traders, which is mainly based on the estimation of how much gas they need to deliver to their customers. If an LDC overestimates consumption, then they must either store the extra gas, for which most LDCs have no capacity, or pay a penalty for leaving the gas in the pipelines. If an LDC underestimates the usage, they may have to purchase gas from spot market to maintain service to the customers [29; 68].

GasHour, as an hourly gas consumption forecasting tool from the GasDay lab, has been used by several LDCs in the past decade. It produces forecasts up to 106 hours ahead, using multi-horizon forecasting technology. For each forecasted hour, an independent model with possibly different inputs is applied to obtain a forecast [63]. With several years' modification and enhancement, GasHour has become increasingly accurate in hourly gas demand forecasting, yet there is still much room for improvement.

Numerous papers and articles published in the past half century have shown that by combining multiple individual forecasts, forecasting accuracy can be improved significantly [19; 34]. Most research focuses on the aggregation of different sources of information, such as different forecasting methods or data. For example, some experts [74] use several mathematical models to make forecasts for the same time horizon and average them using different weights. The final forecast for that horizon is a combination of all forecasts from those models [7; 19; 27; 72]. The idea of combining methods, along with the multi-horizon forecasting technology GasHour is using, suggests a potential direction to improve the accuracy of GasHour forecasts. More specifically, the gas consumption forecast for one individual hour can be improved by combining forecasts generated for nearby hours, using approaches such as polynomial smoothing or filtering. Forecasting skill will be evaluated using RMSE and MAPE, measurements of forecast errors [26].

1.5 Outline and Organization

The first chapter presents a concise description of the problem and potential solution. The whole thesis contains four chapters.

In Chapter 2, we will review some literature regarding basics about forecasting, multi-horizon technology, and combining methods. Chapter 3 will present the approach, which is polynomial smoothing. The results of measurement and analysis, including RMSE, along with conclusions and future work, will be introduced in Chapter 4.

CHAPTER 2

Review of Forecasting Literature

In this chapter, we will introduce background information about technologies related to the field of forecasting.

2.1 Forecasting

Forecasting can be viewed as the process of stating what a future event looks like when the outcome of that event has not been observed yet. For instance, predicting the prices of the stock market at some specific future date is one kind of forecast [8; 70]. Weather forecasting, as another common example, dates back thousands of years when early civilizations used astronomical and meteorological events to monitor seasonal changes [28].

Modern forecasters use several forecasting methods which can be classified as either subjective or objective. Judgemental (Subjective) methods are used widely for important forecasts. Statistical (Objective) methods include extrapolation (such as moving averages, linear regression against time, or exponential smoothing) and econometric methods (typically using regression techniques to estimate the effects of causal variables) [8; 69].

Judgemental forecasting, in which instinct plays an important role in predicting what may happen [24], suffers from divided opinions over the past decade. Some authorities suggested that "judgement cannot be trusted" [69]. Others provided evidence that "judgement was strongly preferred as the most important method of practical sales forecasting" [69]. Usually, judgemental forecasts can be made with or without domain knowledge. However, evidence indicates that the quality of the forecasts improves with increasing accuracy of the provided information [58].

Statistical forecasting, often compared with judgemental forecasting, focuses on predicting the future based on the past information by developing a forecast through identifying inner patterns of the data. Since it uses mathematical formulas to identify the inner patterns and tests the results by mathematical standards, the forecast mentioned above is referred to as a statistical forecast [62].

Both statistical and judgemental forecasting have weaknesses and strengths. Considerable findings show that the accuracy of statistical forecasting can be improved by making judgemental adjustments based on domain knowledge [8]. In the field of energy consumption forecasting, domain knowledge is especially helpful. It would be easier for forecasters to select and apply forecasting methods with knowledge about customer behavior, industry operation processes, and data characteristics [22].

Econometric forecasting is also an important part of the field of forecasting.

In economics, forecasting methods include "guessing, expert judgement, extrapolation, leading indicators, surveys, time-series models, and econometric systems" [21]. Econometric methods rely on statistical procedures to estimate relationships for models specified on the basis of theory, prior studies, and domain knowledge. Given good prior knowledge about relationships and good data, econometric methods provide an ideal way to incorporate expert judgemental and quantitative information [8].

Armstrong [8] lists some principles which should be followed in the process of forecasting, including:

1. Use quantitative methods rather than judgemental methods, if enough data exists. If no data exist, use judgmental methods. When enough data exist, quantitative methods are expected to be more accurate.
2. Use simple methods unless substantial evidence exists that complexity helps. One of the most enduring and useful conclusions from research on forecasting is that simple methods are generally as accurate as complex methods.
3. Match the forecasting methods to the situation. All the guidelines and principles are based on the situation.

Forecasting plays an important role in many aspects of people's lives. For individuals, prediction of future activities is the key to personal success. For organizations, an accurate forecast of the future trends in the financial market means great increases in revenue. For government agencies, forecast results of both domestic and international affairs may cause enormous impact on current policies, which may influence many people [8]. After the oil crisis in the 1970s, climbing energy cost contributed to attracting more attention to the forecasting of future consumption [23]. When it comes to the natural gas industry, prediction made for future natural gas markets significantly impact related government policies, economic activities, and people's everyday life. China, as a rising economic power which consumes large amounts of energy resources, is facing grave environmental issues. One of the solutions, replacing traditional power plants with gas-fired generators, is hindered by uncertain opinions on future gas demand and prices [30]. It would cast enormous influence on the current process if accurate forecasts of future gas markets can be obtained.

Knowledge regarding forecasting has increased a lot. Many papers are published every year in different industries [8]. These studies help us broaden our horizons in applying different methods to various areas. The quality of forecasting also has improved over time. For example, errors in weather forecasts and political poll result forecasts have decreased for the past decades [8].

There are several performance criteria which can be used to measure accuracy of forecasts. Assume the forecast for time horizon h is represented as $\hat{s}_h$, and the measured value is s_h. The error is

$$e_h = \hat{s}_h - s_h \ . \tag{2.1}$$

Notice that in Equation 2.1 the errors are calculated by subtracting measured flow values from forecasted values, which may be the opposite of other people' work. The reason GasDay lab adopts this definition is that when the errors are positive numbers, it indicates overforecasting, while negative numbers correspond to underforecasting. Mean absolute percentage error (MAPE) and root mean square error (RMSE) are used to assess accuracy of the forecasts. Suppose there is a series of N forecasts. Then MAPE can be represented as [26; 55]

$$MAPE = \left(\sum_{h=1}^{N} \left| \frac{e_h}{s_h} \right| \right) * \frac{1}{N} * 100\% \ . \tag{2.2}$$

Then RMSE can be represented as

$$RMSE = \sqrt{\frac{\sum_{h=1}^{N} e_h^2}{N}} \ . \tag{2.3}$$

In the following section, we introduce linear regression, an important technique for building models.

2.2 Linear Regression

"Forecasting is inextricably linked to the building of statistical models." [26] Before forecasting a variable of interest, one must build a model for it and estimate parameters of that model using observed historical data. The model built presents the dynamic patterns in the data and the characteristics of the links between present and past [26]. As a basic approach for forecasting, we use linear regression to build models.

Linear regression is a widely used and powerful approach in modeling the relationship between a dependent variable Y and one or more independent variables X [73]. If there is only one independent variable X, then a simple linear regression process takes the form

$$Y = \beta_0 + \beta_1 X + \epsilon \ , \tag{2.4}$$

where β_0 is an offset, β_1 is the coefficient for X, and ϵ is an error term. The error term often is assumed to be independent and identically distributed following a normal distribution with mean 0 and variance σ^2. In our work, we make no such assumptions about the error.

At the same time, this process can be modeled as

$$\widehat{Y} = \beta_0 + \beta_1 X \ , \tag{2.5}$$

where β_0 is an offset, and β_1 is the coefficient for X.

When there are more than one independent variable X, multiple regression has the form

$$Y = \beta_0 + \beta_1 X_1 + ... + \beta_n X_n + \epsilon \ , \tag{2.6}$$

where β_0 is an offset, β_i is the coefficient for X_i $(i = 1, \ldots, n)$, and ϵ is an error term. Then the dependent variable $\widehat{Y}$ is modeled as

$$\widehat{Y} = \beta_0 + \beta_1 X_1 + ... + \beta_n X_n \ . \tag{2.7}$$

As a basic approach for forecasting, we use linear regression to build models used in this work.

Since the method used here combines forecasting at multiple time horizons, an introduction to multiple time horizon forecasting will be given in the next section.

2.3 Multi-horizon Forecasting

Literally, multi-horizon (period) forecasting means forecasts generated for multiple time horizons ahead [15; 33; 56]. Time horizon indicates a specific period of time, which can be a year, a season, a month, a week, a day, or even an hour. While one-period-ahead forecasting is usually the focus, multi-horizon forecasts attract much less attention [33; 41].

Under most circumstances, there are two methods to construct multi-horizon forecasts. One way is to estimate a horizon-specific model and apply it repeatedly over future time horizons. More specifically, one model is used iteratively at different time horizons in the future. If the model is autoregressive, the previously forecasted value is used as an input. This "plug-in" method, adopted by most time series forecasters, is often called "iterated multi-step" (IMS) forecasting [11; 17; 20; 33; 38; 39; 41; 49].

Suppose there are five time horizons from h to $h+4$. If we choose to use a simple linear regression model (one constant and one input) to demonstrate IMS forecasts, it will take the form as follows.

LR models from h to $h+4$:

$$\widehat{Y}_h = \beta_0 + \beta_1 X_h, \tag{2.8a}$$

$$\widehat{Y}_{h+1} = \beta_0 + \beta_1 X_{h+1}, \tag{2.8b}$$

$$\widehat{Y}_{h+2} = \beta_0 + \beta_1 X_{h+2}, \tag{2.8c}$$

$$\widehat{Y}_{h+3} = \beta_0 + \beta_1 X_{h+3}, \tag{2.8d}$$

$$\widehat{Y}_{h+4} = \beta_0 + \beta_1 X_{h+4}. \tag{2.8e}$$

Although IMS predication has been widely accepted and applied in different fields of forecasting, research results suggest that a "direct multi-step" (DMS) method can contribute to better and more accurate forecasts. Unlike IMS models, a new set of parameters, often with different input factors, is estimated for each time horizon in DMS forecasting. Evidence and proofs have suggested that in most situations, especially with high-order autoregressions and long forecasting horizons, multi-horizon forecasts generated from separated models for each forecast horizon achieve lower RMSE and MAPE than those generated using a single iterated model [11; 16; 17; 18; 33; 36; 41; 49; 59; 66].

Assume there is a one-step-ahead, order n autoregressive model,

$$\widehat{Y}_h = \beta_0 + \beta_1 Y_{h-1} + ... + \beta_n Y_{h-n} \ . \tag{2.9}$$

When this model is used to make a multi-horizon forecast as an IMS model, it takes the form

$$\widehat{Y}_h = \beta_0 + \beta_1 Y_{h-1} + ... + \beta_n Y_{h-n} \ , \tag{2.10}$$

$$\widehat{Y}_{h+1} = \beta_0 + \beta_1 \widehat{Y}_h + ... + \beta_n Y_{h-n+1} \ , \tag{2.11}$$

$$\vdots$$

$$\widehat{Y}_{h+n} = \beta_0 + \beta_1 \widehat{Y}_{h+n-1} + ... + \beta_n \widehat{Y}_h \ . \tag{2.12}$$

Note that to forecast $\widehat{Y}_{h+1}$ requires Y_h, which is unknown, so we take the forecasted value $\widehat{Y}_h$ instead. It is clear that every time a new forecast is obtained using previous forecasts. This affects accuracy since forecasting errors get accumulated in the process and may contribute to larger errors for newly-generated forecasts.

Ensemble forecasting, as an increasingly used method of weather forecasting, allows for uncertainties concerning initial conditions and the the role of chance [32]. This thesis draws on the insights of ensemble forecasting and applies them to multi-horizon forecasting.

2.4 Ensemble Forecasting

Ensemble techniques have become established recently as methods for generating probabilistic weather forecasts [54; 77]. The idea for ensemble forecasting is that repeated forecasts are made from the same initial time, with the initial conditions varied by an error whose magnitude reflects the degree of uncertainty of the observations [32]. Also, it implies different models for weather evolution. It has been discovered that ensemble forecasting not only can enable better weather forecast, but also can provide access to forecast future uncertainty [77]. Ensemble techniques combine multiple forecasts (same model with different initial conditions or different models) for the same time horizon, for example, temperature at 8:00 A.M. tomorrow. In contrast, our multi-horizon technique is to combine forecasts at 6:00 A.M., 7:00 A.M., 8:00 A.M., 9:00 A.M., and 10:00 A.M. to get an improved forecast for the temperature at 8:00 A.M.

Figure 2.1 presents a rough picture of the example above. The red stars are original temperature forecasts. To get a better forecast for 8:00 A.M., surrounding points are considered to generate a new forecast at 8:00 A.M. (blue cross).

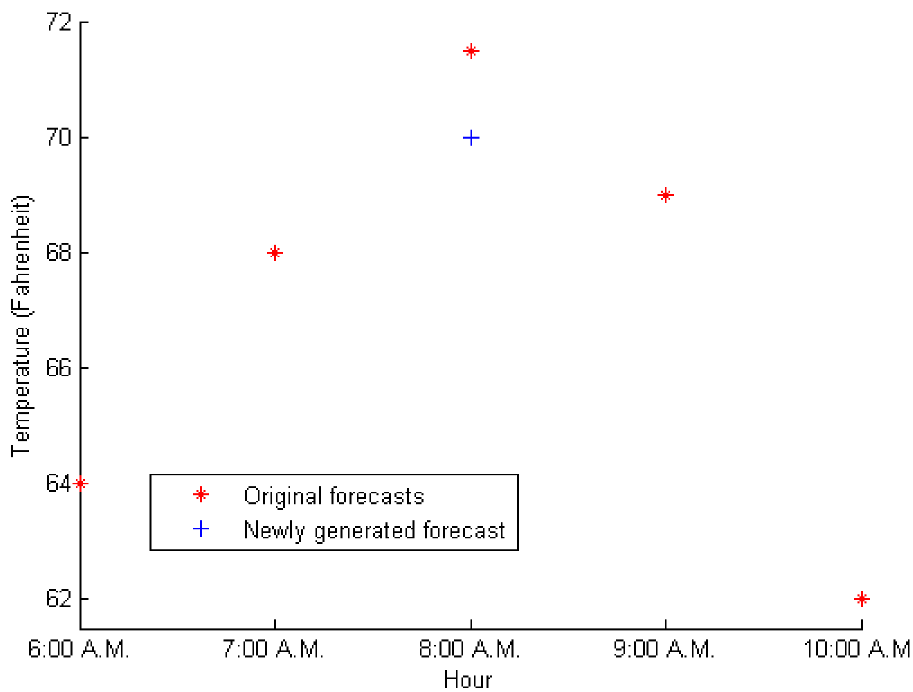

Figure 2.1: Example of multi-horizon ensemble forecasting

Ensemble forecast techniques are used widely in weather forecasting systems. They are recognized as an important component for real time prediction [75]. The primary implementation for ensemble forecasts under current circumstances is combining multiple models from different sources.

2.5 GasHour Software

Since the day GasDay$^{\text{TM}}$ lab was founded, Dr. Brown and his students have committed to develop more accurate and reliable tools generating gas consumption forecasts for hourly, daily, monthly, and yearly flow. GasDay's flagship product, GasDay$^{\text{TM}}$ software, generates highly accurate natural gas demand forecasts over a rolling eight-day period, allowing Local Distribution Companies (LDCs) sufficient time to plan gas supply for the week. Reliable estimates of natural gas consumption are critical for avoiding excess supply or shortages of natural gas, making GasDay a valuable product.

GasDay's hourly gas flow forecasting tool, GasHour$^{\text{TM}}$, generates hourly forecasts for time horizons 0 – 105 hours. These forecasts currently use no ensembling and rely on multi-horizon forecasting for accuracy. A set of 106 linear regression models, one for each time horizon, includes the only models currently used by GasHour. GasHour forecasts are accurate, although there is still room for improvement.

Since GasHour is using multi-horizon forecasting, it is important to express it appropriately. For multi-horizon models, there are three dimensions that need to be addressed: different time horizons, different inputs, and different parameters. Accordingly, we develop proper notation for each.

Currently, h used in GasHour models stands for a one-hour interval of time for which time horizon 0 is forecast. We label the model used for time horizon 0 as model 0. Based on different situations in which GasHour is fired, three cases need to be discussed. First comes the normal case. Assume now is 1:30 P.M., and there is no measured flow data starting at 1:00 P.M. After firing GasHour, the first forecast we get is for the time interval between 1:00 P.M. to 2:00 P.M., which is time horizon 0 in this case. Most LDCs use a bottom-of-the-hour designation, which means that they refer to this forecast as the 2:00 P.M. forecast. Measured flow data from 1:00 P.M. to 2:00 P.M. won't be available until 2:00 P.M. In another scenario, several hours' measured flow data are missing before GasHour is fired. For example, assume it is 1:30 P.M., and we do not have measured gas flow starting from 10:00 A.M. on the same day. When we fire GasHour, the first forecast we get is for the time interval between 10:00 A.M. and 11:00 A.M., which is time horizon 0. The third scenario is backtesting, where we fire GasHour for a time period for which we have measured gas flow. For example, assume it is 1:30 P.M., August 1st, 2013. We can adjust the time to be one year ago, which is 1:30 P.M., August 1st, 2012, and fire GasHour as if the time were 1:30 P.M., August 1st, 2012. In this case, the first forecast we get will be for the time horizon between 1:00 P.M. and 2:00 P.M., August 1st, 2012, and this time interval is called time horizon 0.

When GasHour is generating forecasts for the next 106 hours, it is using a unique and independent model for each hour, with possibly different inputs. Researchers from GasDay have created and mentioned different notations in their papers, yet their notations lack consistency and need to be unified.

To inform our development of appropriate notation for various inputs, Table 2.1 gathers what **others** have used.

Table 2.1: Previous GasDay and GasHour notations

	Taware 1998 [64]	Kennedy 2006 [42]	Tenneti 2009 [65]	Kiware 2010 [44]
Actual gas flow	S_k	s_k	S_k	s_k
Predicted gas flow	$\hat{S}_k$	$\widehat{s}_k$	N/A	$\widehat{s}_k$
Heating degree hour	d_h	N/A	N/A	N/A
Heating degree day	D_k	N/A	$\mathrm{HDD}_{\mathrm{k}}$	N/A
Day of the week (sine)	sdow	N/A	$\mathrm{DOW}_k^{s,1}$	N/A
Heating degree day reference 65	HDD_{65}	HDD_{65}	N/A	HDD65
Heating degree day reference 65 adjusted for wind	N/A	HDDW_{65}	$\mathrm{HDDw65}_{\mathrm{k}}$	HDDW65

There is a clear lack of consistency. Incorporating best practices in the GasDay lab, small changes will be made to these notations, as shown in Table 2.2, for some of the most important notations used in the forecasting models.

Table 2.2: Notation used in this thesis

Explanation	Symbols of components
Actual gas flow	s_h
Predicted gas flow	$\hat{s}_h$
Heating degree hour, ref 65, adjusted for wind	HDHW65_h
Heating degree hour, ref 55, adjusted for wind	HDHW55_h
Hour of the day indicator (sine)	$\sin\left(\frac{\text{HOD}_h{*2\pi}}{24}\right)$
Hour of the day indicator (cosine)	$\cos\left(\frac{\text{HOD}_h{*2\pi}}{24}\right)$
Actual gas flow one hour ago	s_{h-1}
Actual gas flow twenty-four hours ago	s_{h-24}
⋮	⋮

With modified inputs, the next step is to build a direct and simple notation for GasHour models. Following Lim [47] and taking forecast horizon hour 6 as an example, the notation can be represented as

$$\begin{aligned}\hat{s}_{h+6} = \beta_0 + \beta_1 \text{HDDW65}_{h+6} + \beta_2 \text{HDHW55}_{h+6} + \beta_3 \sin\left(\frac{\text{HOD}_{h+6} * 2\pi}{24}\right) \\ + \beta_4 s_{h-24+6} + \beta_5 s_{h-48+6} + \ldots + \beta_n \text{OtherInput} \ . \qquad (2.13)\end{aligned}$$

With this model at hand, it is possible for us to compare models at other hours, although they may use different inputs. For example, in Lim's thesis [47], some of the inputs for hour 6 are shown in Table 2.3. Other models at different hours may use different inputs. Accordingly, they have different models and different coefficients. Table 2.3 uses Lim's notation, "h" means hour 0 (current hour), "so" stands for sendout flow, "hod" and "how" are the hour of the day indicator and the hour of the week indicator, respectively.

Table 2.3: Some of the inputs for hour 6, using Lim's notation [47]

	Hour 6 input factors		Hour 6 input factors		Hour 6 input factors
1	offset	11	so(h-165)	21	so(h-19)
2	temp(h-163)	12	so(h-162)	22	so(h-18)
3	temp(h-139)	13	so(h-161)	23	so(h-5)
4	temp(h-92)	14	so(h-159)	24	so(h-1)
5	temp(h-20)	15	so(h-144)	25	(1 * hod)
6	temp(h-9)	16	so(h-138)	26	(2 * hod)
7	temp(h+2)	17	so(h-90)	27	(1 * how)
8	temp(h+4)	18	so(h-75)	28	(2 * how)
9	temp(h+5)	19	so(h-66)		
10	temp(h+6)	20	so(h-25)		

The models from the current GasHour are the basic models to be used in this work, so it is necessary to understand them. The way to combine forecasts from existing GasHour models using ensemble-like techniques across multiple time horizons is referred to in the literature as "blending," which we discuss in the next section.

2.6 Blending

One of the major findings of forecasting research over the last quarter century has been that greater predictive accuracy often can be achieved by combining forecasts from different methods or sources. Combination can be a process as straightforward as taking a simple average of the different forecasts, in which case the constituent forecasts are all weighted equally. Other, more sophisticated techniques are available, such as trying to estimate the individual weights that should be attached to the individual forecasts, so that those that are likely to be the most accurate receive a greater weight in the averaging process [19; 34].

If we have access to forecasts from different sources or methods, combing these forecasts is likely to be an effective way of improving accuracy [34]. Two reasons contribute to that phenomenon. One is that different models use different sets of information, and each model is likely to represent an incomplete view of the process that is driving the variable of interest. Therefore, combined forecasts are able to draw on a wider set of information. The other is that some of the constituent forecasting methods may be biased, in the sense that they consistently forecast too high or too low [34]. When several methods are combined, it is likely that biases in different directions counteract each other, thereby improving accuracy [8].

Combining forecasts is especially useful when forecasters are uncertain about the situations, uncertain about which method is most accurate, and when they wish to avoid large errors [8]. Armstrong [8] suggests the following steps for combining forecasts:

1. Use different data or different methods: Using several sources of data can add useful information and may also adjust for bias.
2. Use at least five forecasts when possible: When inexpensive, it is sensible to combine forecasts from at least five methods. As might be expected, adding more methods leads to diminishing rates of improvement.
3. Use formal procedures to combine forecasts: Combining should be done mechanically, and the procedure should be fully described.
4. Absent strong evidence to support unequal weighting of forecasts, use equal weights.
5. Use trimmed means.
6. Use the track record to vary the weights if evidence is strong.
7. Use domain knowledge to vary the weights on methods.

"Combining forecasts improves accuracy to the extent that the component forecasts contain useful and independent information" [8]. For this reason, it is ideal if the errors of the forecasts combined are uncorrelated with one another. Yet, forecasts are "almost always positively correlated and often highly so" [8].

It is unclear when the term "blending" first appeared in the forecasting literature. However, several papers from meteorology have already used it [10; 46]. In most of these papers, "blending" is referred to as the combination of several forecasts at the same time horizon, which is called ensemble forecasting by most authors.

For the purpose of this thesis, blending is a process by which multiple models are combined over multiple time horizons to produce one forecast for one time period, as illustrated in Figure 2.2. Here, forecast horizon H can be regarded as $h + k$, where k is a time period between when the forecasts are made and the time (time interval) these forecasts are made for. This means that H is k periods ahead of current time. Also, as the primary method in this thesis, blending is the key in showing that through combining temporally neighboring forecasts, the accuracy of a forecast for a certain time horizon may improve.

More generally, we also use the term blending for any ensemble-like technique combining multiple forecasts at each of several neighboring time horizons.

The idea of blending presents us with a general picture of future solutions. In

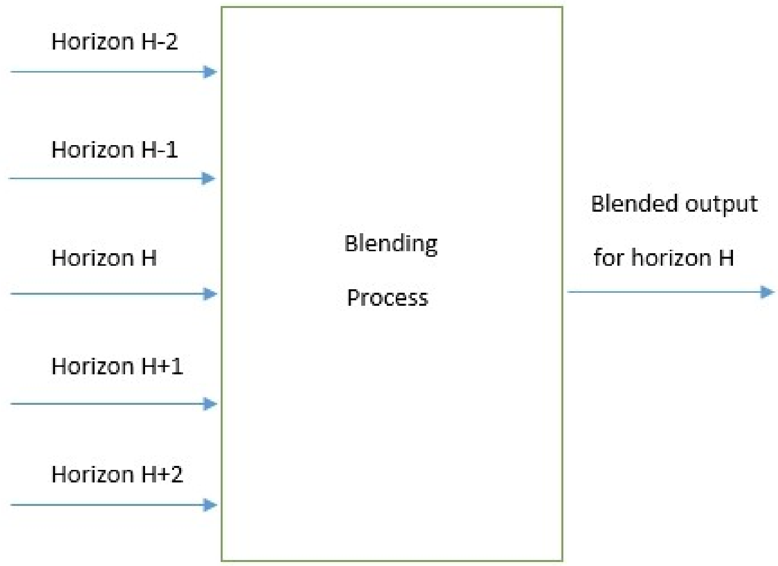

Figure 2.2: Blended forecast for time horizon H

the following sections, several methods useful for implementing a blending process are introduced.

2.7 Filtering

In signal processing, a filter is a device or a process that removes from a signal some unwanted component or feature [60]. Filtering is a class of signal processing algorithms. The defining feature of filters is the complete or partial suppression of some aspect of the signal. Often, this means removing some frequencies and not others to suppress interfering signals and reduce background noise [60].

For electrical data that are continuous in time (analog data), low-pass RC filters are used to clean the data [40]. Since the data here (hourly flow) is discrete, a digital filter should be applied.

The basic filter for the smoothing process is assumed to be a linear weighting of the input data values. The weighting factors are the coefficients of the filter. Thus the smoothed output data y_n are calculated from the original input data x_n by the weighting

$$y_n = \sum_{k=-N_p}^{N_p} b_k x_{n-k} \ , \tag{2.14}$$

where b_k = filter weighting coefficients, $2N_p$ = span of the filter in samples, and $2N_p$ + 1 = total number of terms in the filter. The output, y_n, is the convolution of the input, x_n, with the filter coefficient sequence, b_k.

Polynomial smoothing, which is a particular class of digital filters, will be introduced in the next section.

2.8 Polynomial Smoothing

Experimental data generally contain random noise [76]. A least squares technique is often used to fit a function to the data. The purpose of fitting a function rather than directly using raw data points is to decrease the error which results from the noise in the data points. The smoothing function, i.e., the function used for the least squares fit, should be chosen to provide a good fit to any real perturbations that exist in the data. For example, if it is known that a physical law demands the data to follow an exponential path, then nothing but an exponential function should be used [48].

If the data is sampled over a large period of time, and the trend of the data cannot be explained by a simple physical law, then the smoothing function is fit over a smaller segment of the data, and the newly evaluated midpoint of the segment should be picked out as the desired result. By sliding the smoothing interval (i.e., adding new points to the end of the interval while discarding points from the beginning of the interval), newly evaluated midpoints are obtained throughout the data, as Figure 2.3 shows. In this case, the choice of both a smoothing interval and a smoothing function must be considered [48]. In Figure 2.3, the smoothing interval contains five points. A parabola is fit through five original data points (stars in the Figure 2.3), and the newly generated point in the middle of the sliding window is acquired (cross points in Figure 2.3).

In most applications, a polynomial is chosen as the smoothing function [48]. Least-square polynomial smoothing, known as the Savitzky-Golay convolution method, commonly is used to eliminate random noise in data [76]. If the smoothing interval is fixed, a stepwise regression analysis can be used to determine the proper degree polynomial. In the more general case, both the smoothing interval and the polynomial degree are allowed to vary, and the optimum combination is to be

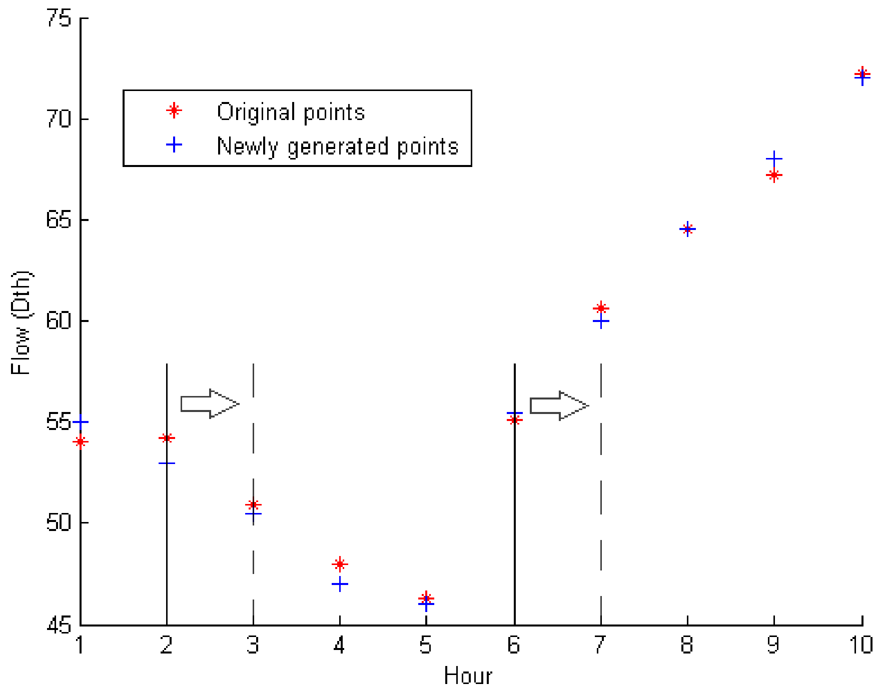

Figure 2.3: Sliding the smoothing interval

determined [48]. In the proposed method, low degree (degrees from 2 to 5) polynomials will be used with the number of points ranging from 4 to 9.

In the next chapter, detailed explanations and demonstrations of polynomial smoothing technology will be given.

CHAPTER 3

Smoothing Multi-horizon Forecasts (SMHF)

To obtain an accurate forecast, good models need to be constructed. To create these models, clean and abundant data are necessary for training these models. Much work has been done in improving the quality of data and building new models to increase the accuracy of forecasting. However, in the approach presented below, smoothing multi-horizon forecasts (SMHF), is the primary way reported here to improve forecast results. We emphasize finding interconnections between current forecasts, as suggested by Armstrong's advice for ensemble forecasting [8], and using polynomial smoothing or filters to combine these forecasts (blending).

In the first few sections, we introduce background information about smoothing techniques. Then, step by step, we unveil the components of the technology used in this thesis.

3.1 Smoothing in Forecasting

There always exists some random variation in collected data [76]. These irregular components can be neutralized or cancelled to some extent using a technique called "smoothing" [9].

The smoothing technique, which is different from traditional model-based methods, has been applied in the field of forecasting for a long time. It works well, especially when the number of samples is limited or the forecasting task is enormous [25]. People also use this technique when the time series is stable, without significant trend, seasonal, or cyclical effects [45].

In practice, moving average smoothing and exponential smoothing methods are mostly used. The first method is suitable for stationary time series data by discovering its underlying mean. Exponential smoothing generates forecasts using an exponentially weighted moving average of all previously observed values [25; 35].

In this thesis, the basic approach is not using smoothing as a forecasting technique, but as an approach to improve the quality of existing forecasts obtained through other methods. This process is achieved using least-square smoothing of the forecasts at multiple time horizons.

3.2 Least-square Smoothing

As a basic approach in estimating parameters in regression analysis, the method of least squares plays an important role in curve fitting. The best “fit” of the data can be found by minimizing the sum of squared deviations between actual and smoothed values. If the data to be fit has a linear trend, a least-square line should be employed. For complicated shapes involving high-degree polynomials, a least squares fit also can be used for approximation [71].

Suppose we use a polynomial of degree n to approximate a set of data $(t_1, y_1), (t_2, y_2), ..., (t_m, y_m)$, taking the form

$$f(t) = \alpha_0 + \alpha_1(t - t_j) + \alpha_2(t - t_j)^2 + ... + \alpha_n(t - t_j)^n \ , \tag{3.1}$$

where t_j is one of the points from the dataset. It is assumed that $n + 1 \leq m$. To find the best fitting polynomial, we minimize the squared errors between actual and smoothed values

$$\sum_{i=1}^{m}[y_i - f(t_i)]^2 = \sum_{i=1}^{m}[y_i - (\alpha_0 + \alpha_1(t_i - t_j) + \alpha_2(t_i - t_j)^2 + ... + \alpha_n(t_i - t_j)^n)]^2 \ , \tag{3.2}$$

to yield coefficients α of the best fitting polynomial. With the coefficients of the polynomial, we can evaluate at the desired point(s), which will be the central point in our work.

The purpose of using SMHF is to improve the quality of hourly gas consumption forecasts generated by GasHour, which is explained and demonstrated in detail in the following section.

3.3 GasHour^{TM} Data

Every hour when GasHour is fired, it generates a series of forecasts for the next 106 hours, including the hour when these forecasts are made. We can retrieve actual hourly flow data and hourly forecasts made by GasHour for the past year. These hourly forecasts are stored in an 8761*106 table, with rows of 106 forecasts made in each of the past 8761 hours. For better visual illustration, a 24-hour period time of GasHour forecasts obtained from a real operating area is displayed in Figure 3.1, except that the flow values have been scaled to hide the identity of the customer.

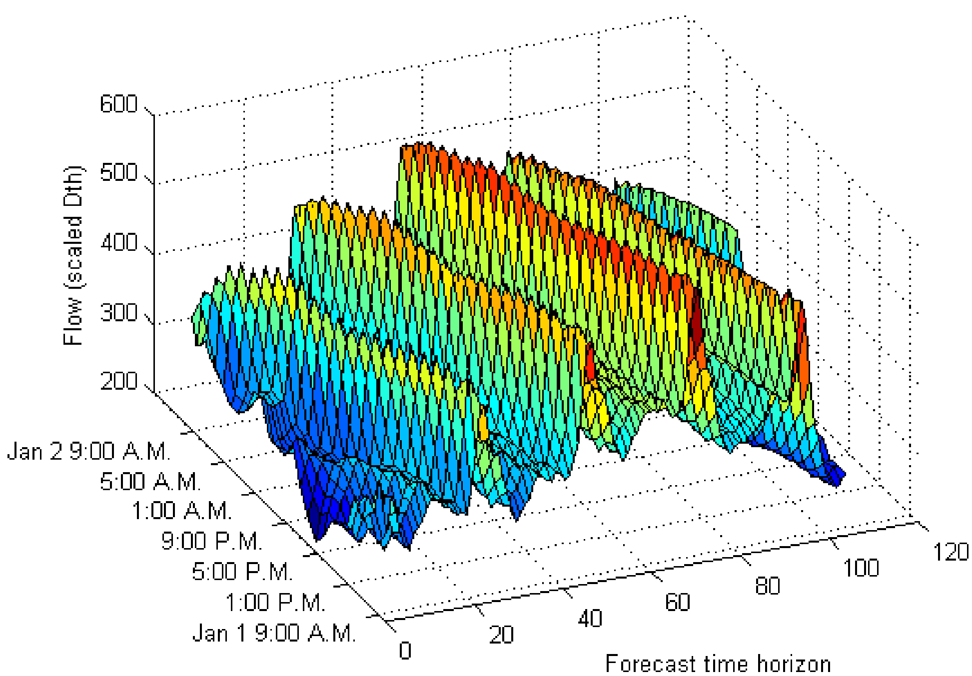

Figure 3.1: GasHour forecasts made during a 24-hour period on January 1, 2012.

The forecast time horizon axis stands for the 106 hourly forecasts made each hour when GasHour was fired. The time stamp axis corresponds to each hour of the day at which GasHour was fired, and the vertical axis shows the predicted scaled flow. Consider the hourly forecasts made at 11 A.M., as Figures 3.2 and 3.3 show.

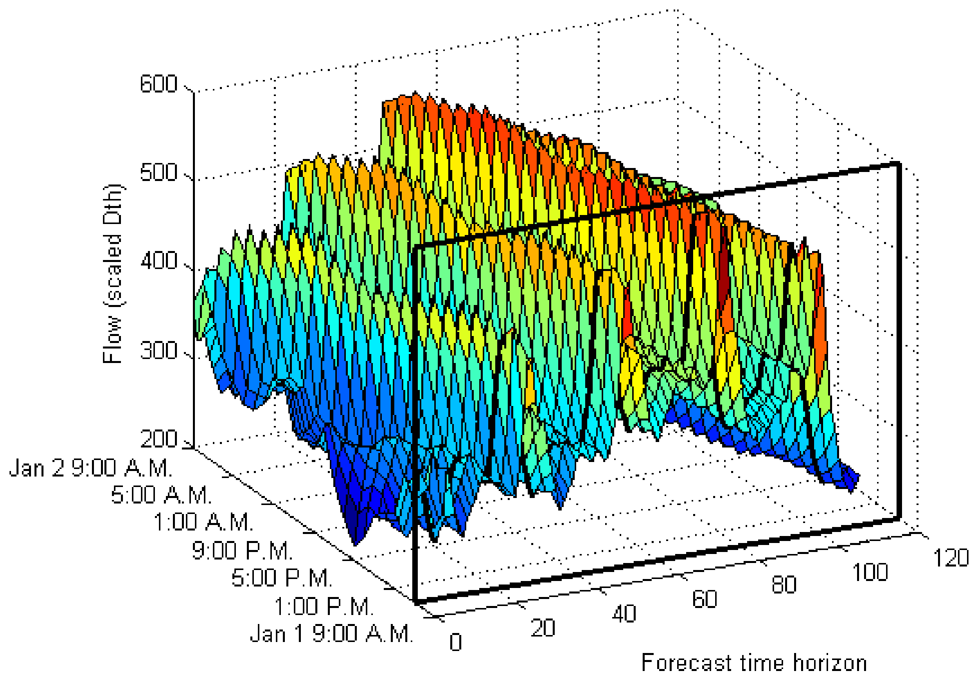

Figure 3.2: Cross-section showing relative position of GasHour forecasts made at 11 A.M. on January 1, 2012.

When run at 11 A.M., GasHour generated hourly gas consumption forecasts for the next 106 hours (Figure 3.3). The forecast made at time horizon 0 corresponds to the forecast made at 11 A.M.

Next, we view Figure 3.1 from another perspective. The cross-section in Figure 3.4 contains forecasts for the time horizons 6 hours from the time they were

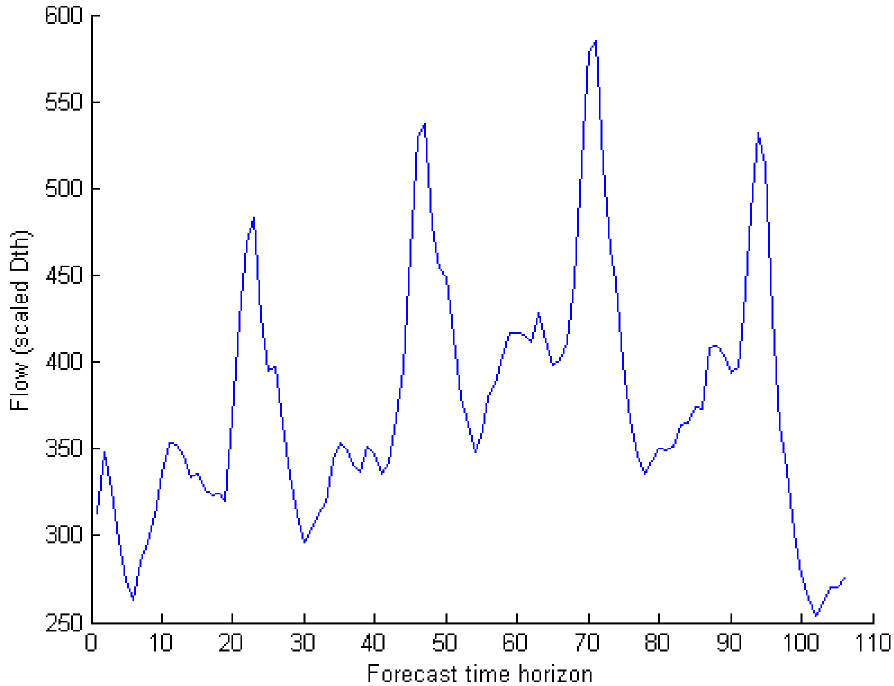

Figure 3.3: Cross-section of the surface shown in Figure 3.2 for forecasts made at 11 A.M.

made. To be more specific, the 25 forecasts in Figure 3.5, made at each hour from 9 A.M., January 1, 2012, to 9 A.M. on January 2, are actually for 3 P.M., January 1, 2012, to 3 P.M. on January 2.

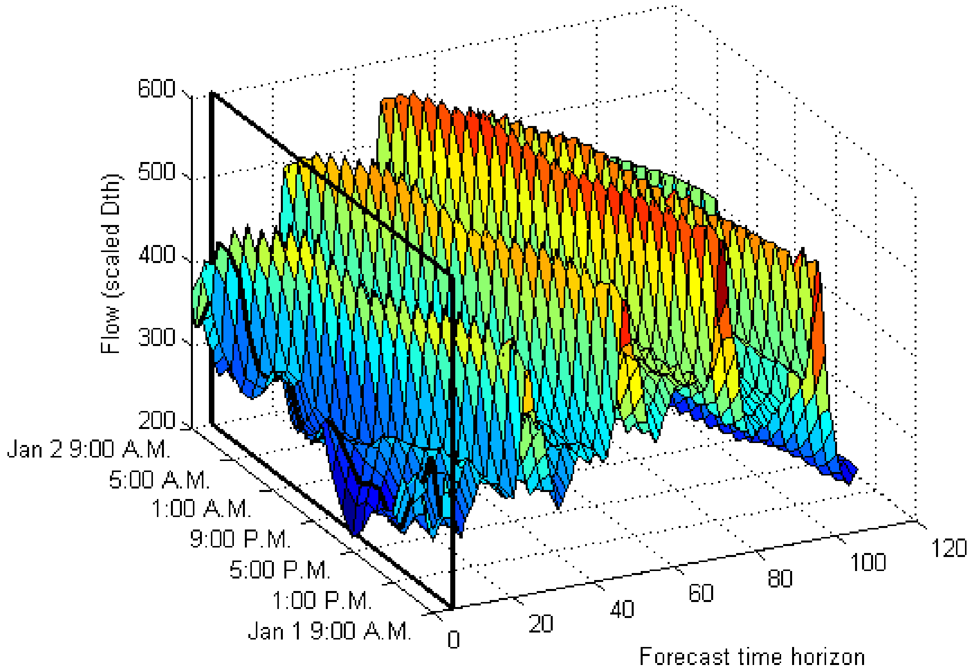

Figure 3.4: GasHour forecasts made for hour 6 starting 9 A.M. January 1, 2012, to 9 A.M. January 2, 2012.

Assume we want to make an hourly forecast for the time interval 24 hours from now, and GasHour is fired for every hour between now and then. Then a set of 24 hourly forecasts made for that time interval are obtained. The forecast horizon axis in Figure 3.6 represents the number of time horizons between the forecasted hour and the hour when the forecast is made. Hence, forecast horizon 0 corresponds to the forecast made during the designated hour, and 23 means the forecast made was 23 hours before the designated hour. It seems that GasHour is doing a better job when the number of intervening hours is declining, in other words, the forecasts with shorter time horizons. Generally speaking, the forecasts made at the designated hours (Flow value at forecast horizon 0 in Figure 3.6) are more accurate

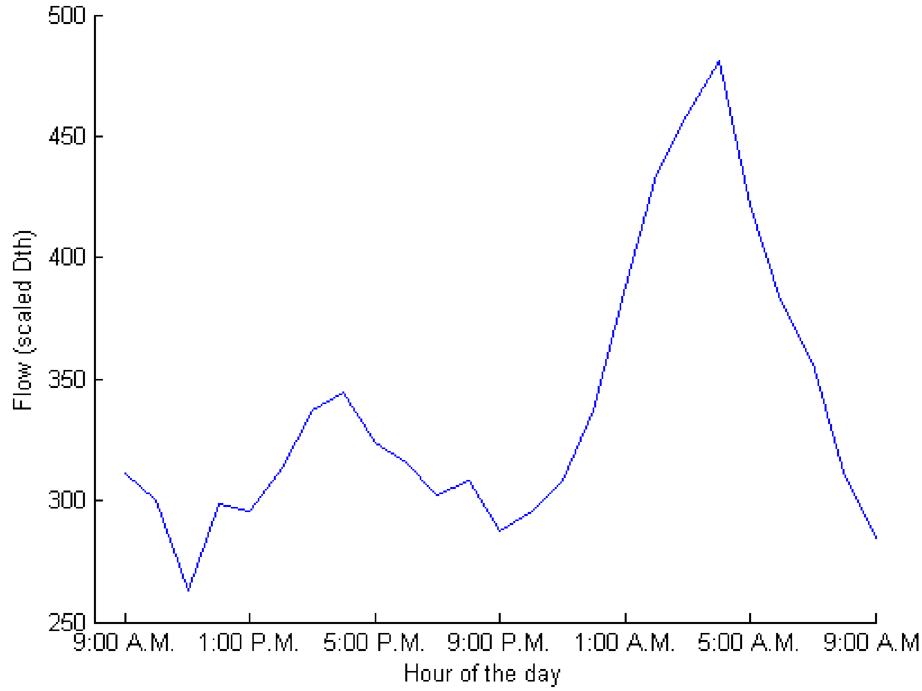

Figure 3.5: GasHour forecasts made for hour 6 starting 9 A.M. January 1, 2012, to 9 A.M. January 2, 2012.

than forecasts made at forecast horizon H (H is bigger than 0, for instance, forecasts made at forecast horizon 20). The red line stands for actual flow number for that particular hour, remaining constant, while forecasts vary.

Imagine viewing Figure 3.1 from above, as Figure 3.7 shows. The lines show the cross-sections discussed in the previous paragraphs.

The preceding discussion presents a picture of GasHour data. Next comes an introduction to blending techniques.

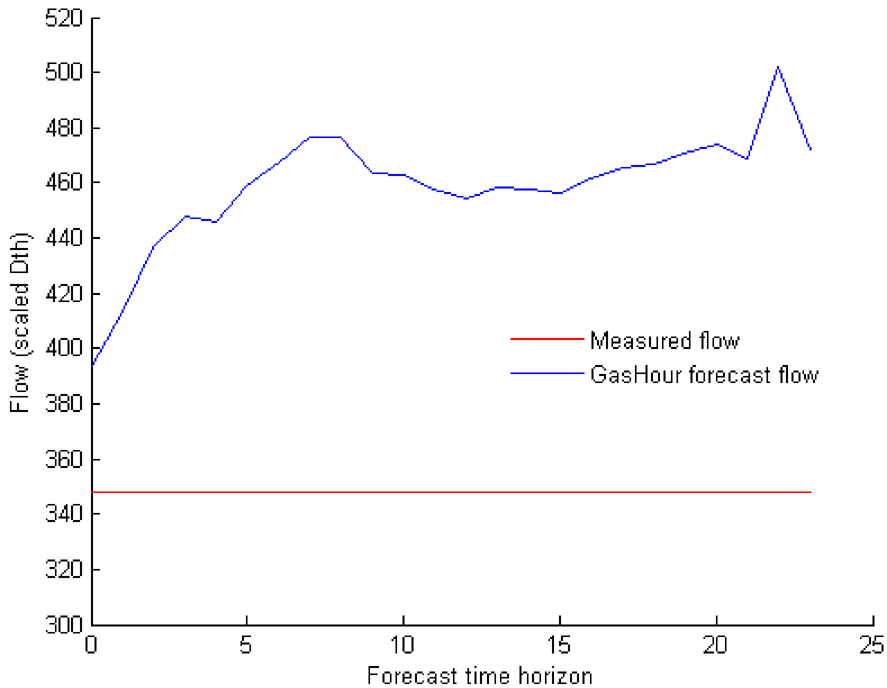

Figure 3.6: 24 GasHour forecasts made at different time horizons for one particular period.

3.4 Five Point Polynomial Fitting

Parabola fitting has been explored and practiced. To improve accuracy of forecasts at specific time horizons, we use parabolas to smooth them, using information from neighboring points.

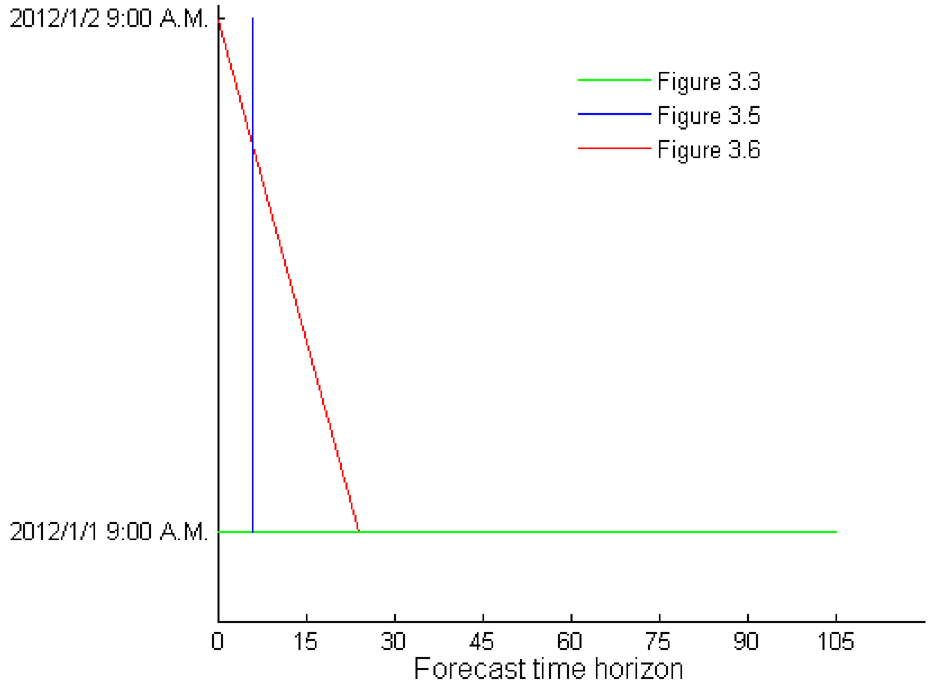

Figure 3.7: GasHour forecasts made for a 24-hour period.

Suppose an improved gas consumption forecast for time horizon H is desired. By running GasHour, a forecast for that time horizon has been generated. Neighboring data points including forecasts for hour $H+1$, $H-1$, $H+2$, and $H-2$ are also accessible. We fit a parabola through the five forecasts (time horizons H, $H+1$, $H-1$, $H+2$, and $H-2$) and obtain a smoothed forecast at time horizon H.

In Figure 3.8, red stars stand for the original forecasts. The blue cross is the smoothed forecast for time horizon H after fitting the original five forecasts.

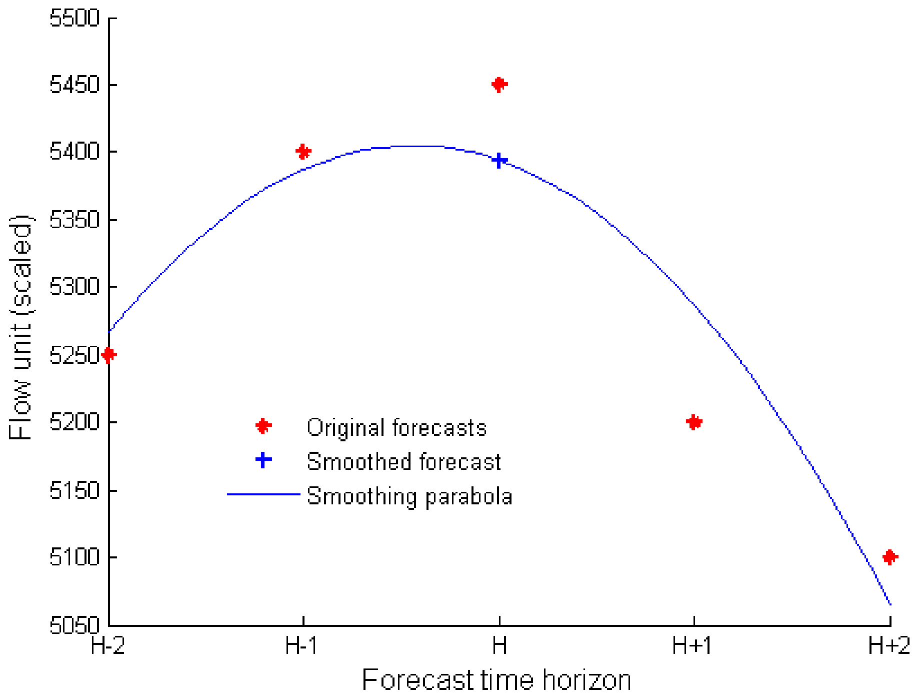

Figure 3.8: Five point parabola fitting

The above process has been realized through MATLAB programming. Algorithm 1 presents a basic idea of how it works.

Algorithm 1: Five point least-square smoothing

Data: input

Five forecasts $\widehat{Y}_{H-2}$, $\widehat{Y}_{H-1}$, $\widehat{Y}_{H}$, $\widehat{Y}_{H+1}$, and $\widehat{Y}_{H+2}$ for time horizons $H-2$, $H-1$, H, $H+1$, and $H+2$

Smoothing these five forecasts using a degree-2 polynomial:

$$f(t) = \alpha_0 + \alpha_1(t - t_i) + \alpha_2(t - t_i)^2 \ , \tag{3.3}$$

Here, $t_i = H$. Then, we evaluate $f(t)$ at the midpoint

$$f(t = H) = \alpha_0 \ . \tag{3.4}$$

Result: output

Improved forecast $\widehat{Y}^*_H$ for time horizon H

3.5 Data Formats and Requirements

The polynomial fitting process described in Section 3.4 is a simple example of smoothing five points with a quadratic polynomial. To apply the smoothing technique on real forecast data (such as the forecasts from GasHour), certain requirements must be met. First, the number of forecasts to be smoothed (for instance, 106 points generated by GasHour every time it is fired) should be larger than number of points to which the parabola is fitted (for instance, 5 points in the previous section). Let N be the total number of original data points needed to be smoothed and n be the number of points the parabola can smooth at a time, then $n \leq N$. Secondly, it is expedient, but not essential, for the points being fit to be equally spaced. The hourly forecasts from GasHour fit these criteria.

3.6 Polynomial Fitting

Generally, polynomial fitting can be applied under different circumstances, based on the choices of how many points to smooth and what degree of polynomial to deploy. Let p be the number of points to be smoothed each time and d be degree of polynomial, which should be at least 1 and no bigger than $p - 1$. Additionally, due to the characteristics of real GasHour data, p is usually an odd number and at least 5.

For the sake of better explanation, the above process can be expressed as the pseudo code in Algorithm 2.

Algorithm 2: General least-square smoothing

Data: input

p forecasts made for p time horizons, including $H-(p-1)/2$,

$H-(p-3)/2$, ... H, ... $H+(p-3)/2$, $H+(p-1)/2$.

Smoothing these p forecasts using a degree-d polynomial:

$$f(t) = \alpha_0 + \alpha_1(t-t_i) + \alpha_2(t-t_i)^2 + ... + \alpha_n(t-t_i)^d \ . \tag{3.5}$$

Here, $t_i = H$. Then, we evaluate $f(t)$ at the midpoint

$$f(t = H) = \alpha_0 \ . \tag{3.6}$$

Result: output

Improved forecast $\widehat{Y}_H^*$ for time horizon H

The polynomial fitting process in Algorithm 2 generates a smoothed value (improved forecast $\widehat{Y}_H^*$) at the midpoint H of the segment (all p points). When we slide a smoothing interval across the entire data set, the remaining points can be smoothed as well. For instance, with N total points to be smoothed, we can use a p-point smoothing interval across these points, which means adding new points to the end of the interval while discarding points from the beginning of it. Every time, a different set of p points is smoothed to yield a new evaluated value, replacing each of the original data points.

Take the following as an example. In Figure 3.9, the smoothing interval contains five points. Every time, a new parabola is fit through five original forecast points (red stars in Figure 3.9), and the newly generated point (blue cross and circle in Figure 3.9) in the middle of the sliding window are obtained.

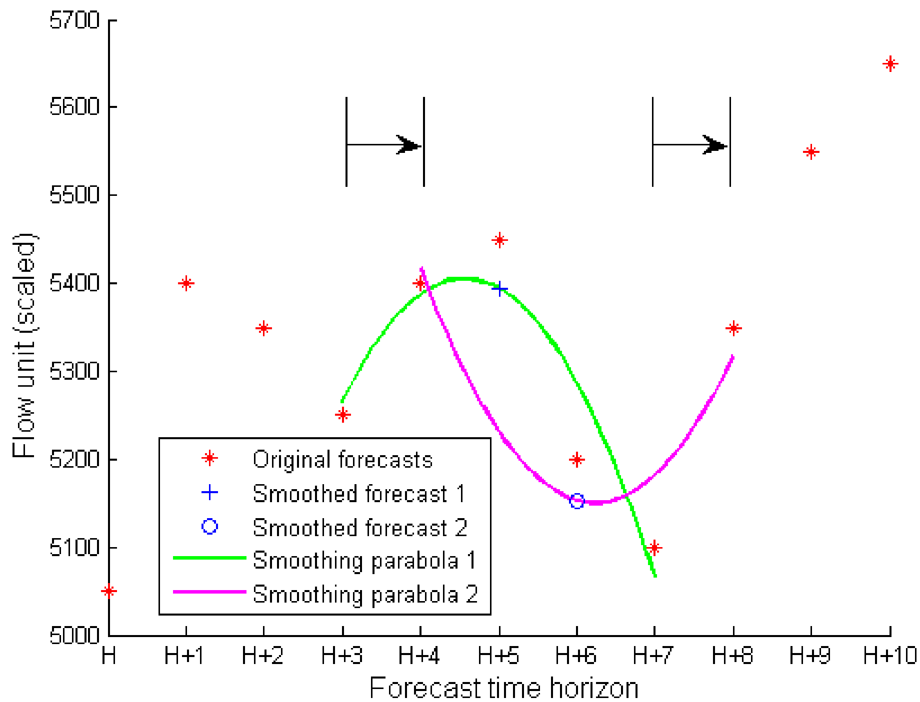

Figure 3.9: Sliding windows

As is shown in Figure 3.9, it is necessary to use additional points to smooth the first two time horizons in the forecast period (time horizon 0 and 1), so we use actual values Y_{-1} and Y_{-2} for “forecasts” $\widehat{Y}_{-1}$ and $\widehat{Y}_{-2}$. Similarly, when the sliding window moves over the last few points inside the original dataset, it is impossible to fit parabolas through them due to the lack of points. We either keep the original forecast points (which is actually adopted in our codes) or replace them using smoothed values from the last parabola fitted.

Figure 3.10 presents an example when 106 original GasHour forecasts are smoothed using quadratic polynomials. Since 5 points are chosen, we use two actual flow values at $t = -2$ and $t = -1$ for smoothing at $t = 0$ and $t = 1$, the first two forecasts in the dataset. For the last two forecasts ($t = 104$ and $t = 105$, respectively), we use original GasHour forecasts.

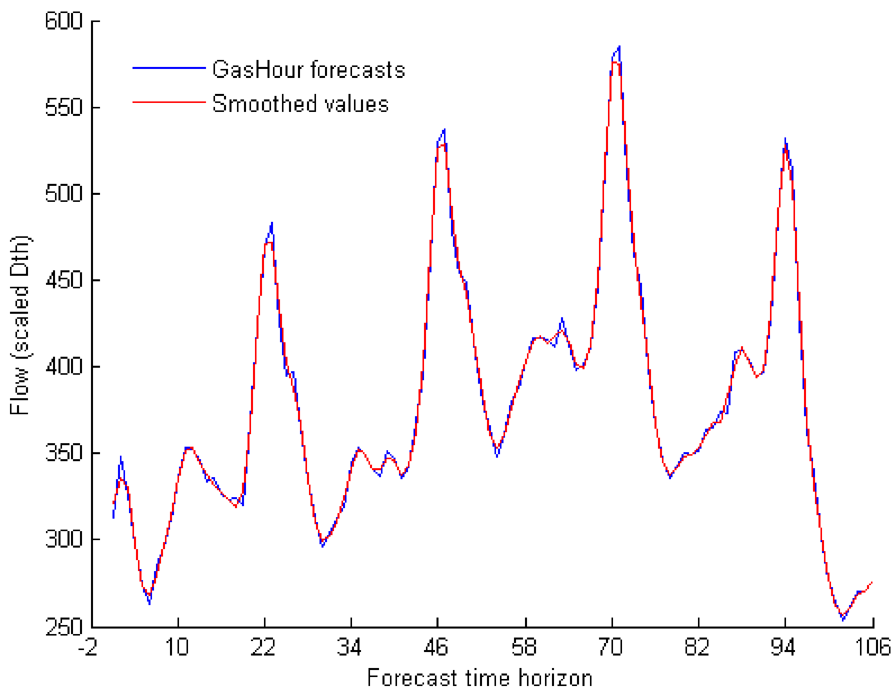

Figure 3.10: 106 points smoothing

This whole chapter presents the smoothing method I applied using MATLAB coding. Based on a certain number of points, least square is deployed to calculate the coefficients of each polynomial, which help generate smoothed values for each group of points. However, another strategy implies a different way of generating those smoothed forecasts through least square smoothing. This approach suggests that coefficients in Equation 2.14 can be precalculated. Imagine a parabolic approximation is applied to 5 points

$$y_n = b_0 + b_1 x + b_2 x^2 \ , \tag{3.7}$$

where $x = -2, -1, 0, 1, 2$, while b_0, b_1, and b_2 are coefficients. Through a similar process demonstrated in Section 3.2, the coefficients can be displayed as

$$b_0 = \frac{1}{35}(-3x[-2] + 12x[-1] + 17x[0] + 12x[1] - 3x[2]) \ , \tag{3.8}$$

$$b_1 = \frac{1}{10}(-2x[-2] - x[-1] + x[1] + 2x[2]) \ , \tag{3.9}$$

$$b_2 = \frac{1}{14}(2x[-2] - x[-1] - 2x[0] - x[1] + 2x[2]) \ . \tag{3.10}$$

When x is 0, the desired mid-point equals the value of b_0. Thus for a parabola through five points, when $N_p = 2$, the coefficients of Equation 2.14 are $b_{-2} = -\frac{3}{35}$, $b_{-1} = \frac{12}{35}$, $b_0 = \frac{17}{35}$, $b_1 = \frac{12}{35}$, and $b_2 = -\frac{3}{35}$. Through this method, it won't be necessary for us to apply least squares every time a group of new points is smoothed, thus saving some time for faster processing.

Additionally, when 7 points are smoothed each time (thus $x = -3$, -2, -1, 0, 1, 2, 3), the value of b_0 is

$$b_0 = \frac{1}{21}(-2x[-3] + 3x[-2] + 6x[-1] + 7x[0] + 6x[1] + 3x[2] - 2x[3])\ . \tag{3.11}$$

For smoothing 9 points (thus $x = -4, -3, -2, -1, 0, 1, 2, 3, 4$), the value of b_0 is

$$\begin{aligned} b_0 = \frac{1}{231}(&-21x[-4] + 14x[-3] + 39x[-2] + 54x[-1] \\ &+ 59x[0] + 54x[1] + 39x[2] + 14x[3] - 21x[4])\ . \end{aligned} \tag{3.12}$$

In the next Chapter, detailed graphs and results are given to show whether this blending methods works in improving GasHour forecasts.

CHAPTER 4

Improvement in Forecasting Accuracy with Blending

This chapter provides graphs and statistics to show the improvement of accuracy in GasHour forecasts after using the blending technology described in Chapter 3. RMSE results from different degrees of polynomials are displayed as well.

4.1 Measuring GasHour Forecasts

Usually, the accuracy of GasHour forecasts can be measured in the form of root mean square error (RMSE) or mean absolute percentage error (MAPE). We illustrate using the data from one GasHour operating area, which comes in the format described in Section 3.3. RMSEs for all 106 time horizons are calculated as suggested in Figure 4.1, using the matrices containing the measured hourly gas flow values and GasHour forecasts, from January 1, 2012, to December 31, 2012.

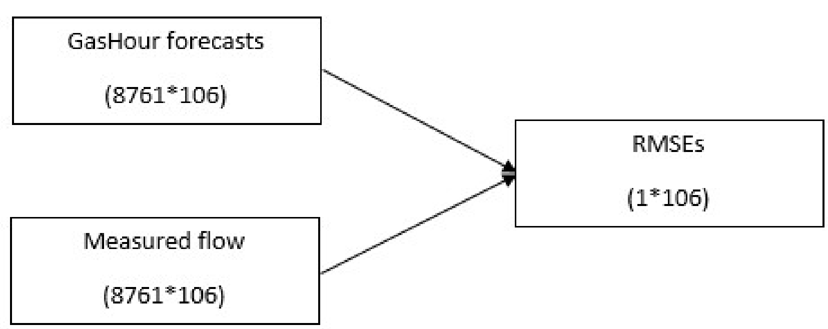

Figure 4.1: Measuring GasHour forecasts using RMSE

Since we are in the process of backtesting, as mentioned in Section 2.5, all the GasHour forecasts were generated using observed weather information. This is justified since we are trying to reduce modeling error, and error due to weather forecasting error is assumed to be uncorrelated with modeling error.

Due to the number of hours within a year, it is impractical to show all the smoothed forecasts each time GasHour is fired. Figures 4.2 and 4.3 show two examples from two individual hours inside that time period, each displaying 106 smoothed forecasts, along with their measured flow values and residues (smoothed values subtract measured flow values). These two examples are chosen based on the sum of the errors between smoothed forecasts and measured values for all 106 time horizons. Figure 4.2 presents the hour with the least sum of errors, which is the best performing set of 106 forecasts, while Figure 4.3 shows the hour with the worst performing set of 106 forecasts.

Figure 4.2 indicates that original GasHour forecasts are doing a good job in predicting hourly gas consumption, with a few exceptions. Those bigger-than-usual differences may occur under various circumstances, such as a drastic change of temperature or noisy data. On the other hand, Figure 4.3 exhibits a totally different situation, where the original GasHour predications fail to forecast almost all of the time horizons. Multiple factors may contribute to this phenomenon, including but not limited to, extremely corrupted data and disruptive events, such as a power

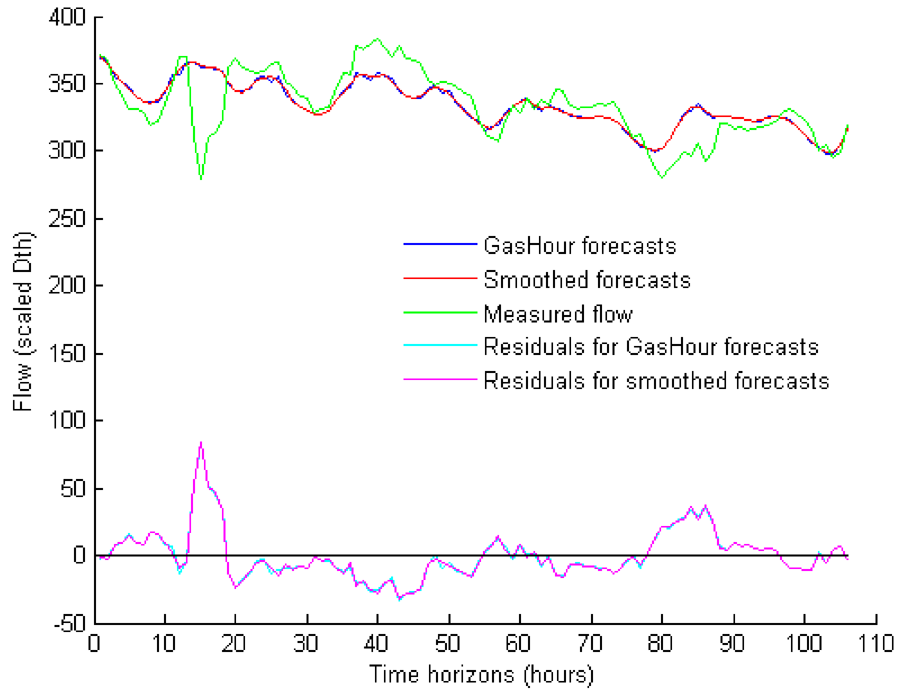

Figure 4.2: Smoothed forecasts, measured flow values, and residues for one particular hour

outage. The smoothed forecasts resemble the same trends as the original ones. After all, those smoothed forecasting values are obtained based on existing GasHour forecasts. However, the differences between those two stand out at some time horizons, such as around hour 24, an observation to which we will return later.

Figure 4.4 shows the RMSEs, summed over 8761 hours in the test year, for each of the 106 time horizons for that operating area. This process can be expressed as

$$e(H)_h = \widehat{Y}(H)_h - Y(H)_h \ , \tag{4.1}$$

where H ([0 105]) corresponds to the time horizon, and h ([1 8761]) stands for the

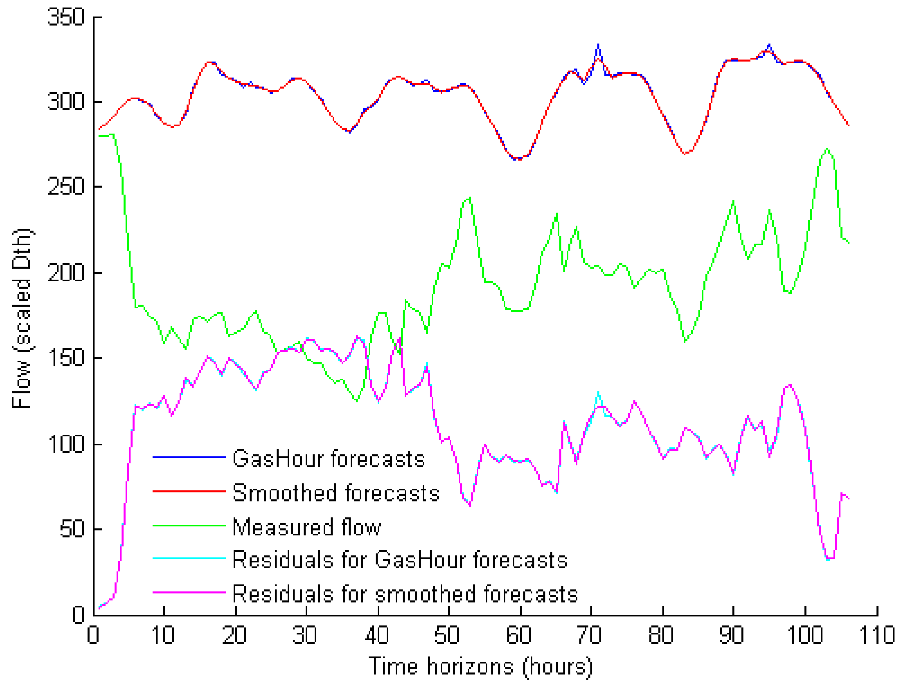

Figure 4.3: Smoothed forecasts, measured flow values, and residues for another particular hour

hour inside the test year. Then

$$RMSE_H = \sqrt{\frac{\sum_{h=1}^{8761} e(H)_h^2}{8761}}\ , \tag{4.2}$$

which will generate the RMSE for each time horizon (H).

It is clear that as the forecasted time horizon increases, the RMSE becomes larger, meaning that the accuracy of the forecasts are declining. At some point, the RMSE appears to approach a relatively stable state.

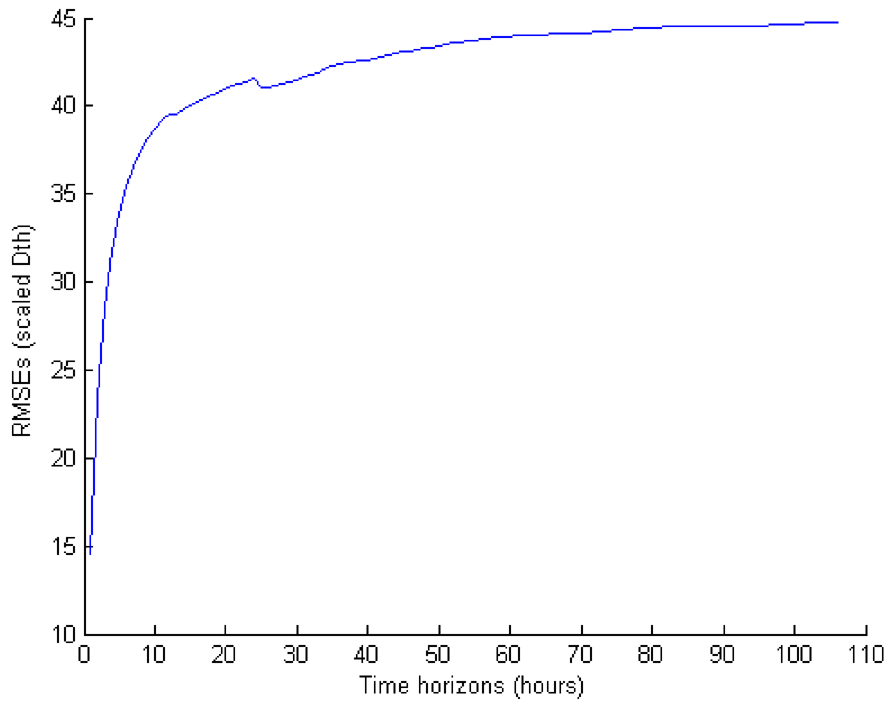

Figure 4.4: RMSEs for 106 time horizons

4.2 Measuring Smoothed GasHour Forecasts

Similar processes are applied when calculating RMSEs for smoothed GasHour forecasts. Suppose we choose to use a quadratic polynomial to smooth 5 points in the neighborhood of each time horizon. Then Algorithm 3 shows how to calculate the RMSEs for each time horizon (from time horizon 0 to time horizon 105) for the hourly forecasts, from January 1, 2012, to December 31, 2012.

Algorithm 3: RMSEs for smoothed GasHour forecasts

Data: 106 hourly forecasts made in each of 8761 hours.

```
for hour 3:8761
    for horizon 0:105
```

$\widehat{Y}_H^*$ from Algorithm 1

```
for horizon 0:105
```

RMSE for horizon across 8759 hours

Result: 106 RMSEs for time horizon 0 to time horizon 105

Similar to Figure 4.1, Figure 4.5 presents the flow chart of Algorithm 3. Notice that after smoothing 8761 hours of GasHour forecasts, there only exist 8759 rows of smoothed forecasts. Because for the first two hours inside the dataset, we have to use actual values instead of forecasts, as explained in Chapter 3, and they are not readily available to us in this dataset.

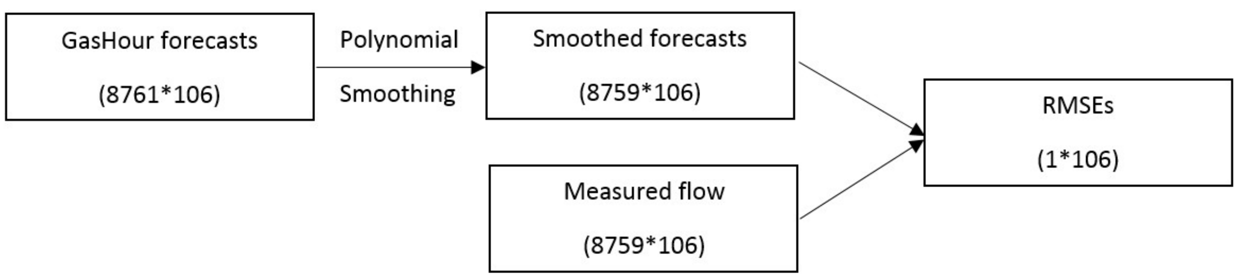

Figure 4.5: Measuring smoothed GasHour forecasts using RMSE

The red curve in Figure 4.6 corresponds to the RMSEs calculated for smoothed forecasts, while the blue curve stands for original ones.

Figure 4.7 is presented to illustrate the differences between them. The difference at each time horizon is calculated as

$$\text{DiffRMSE} = \text{RMSE}_{Original} - \text{RMSE}_{Smoothed} \ . \tag{4.3}$$

$\text{RMSE}_{Original}$ stands for RMSE calculated for original forecasts, and $\text{RMSE}_{Smoothed}$ means RMSE calculated for smoothed forecasts.

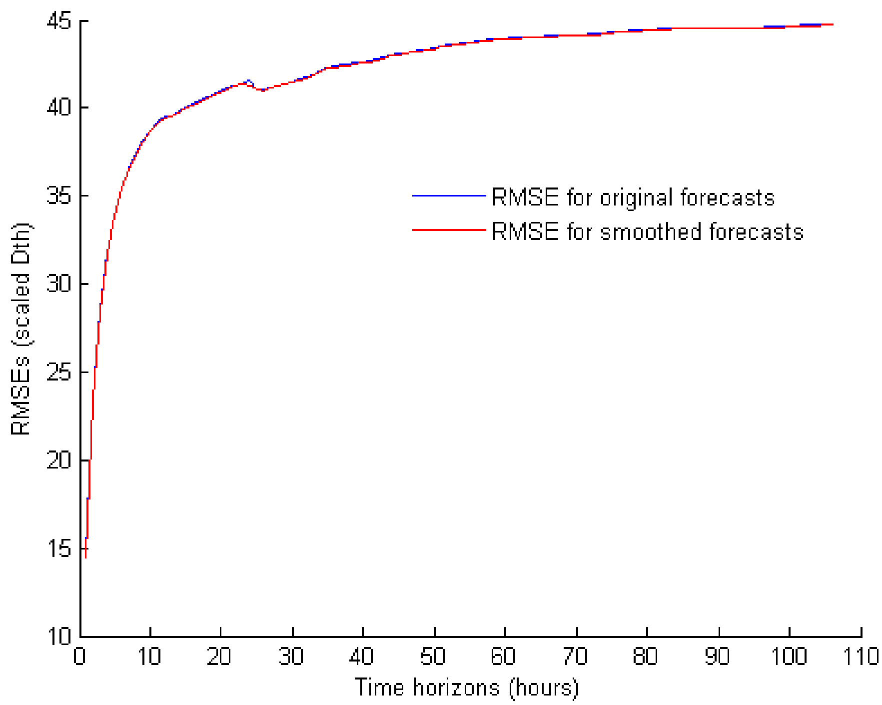

Figure 4.6: RMSEs for both original and smoothed forecasts

As concluded before, the quality of hourly flow forecasts deteriorate quickly

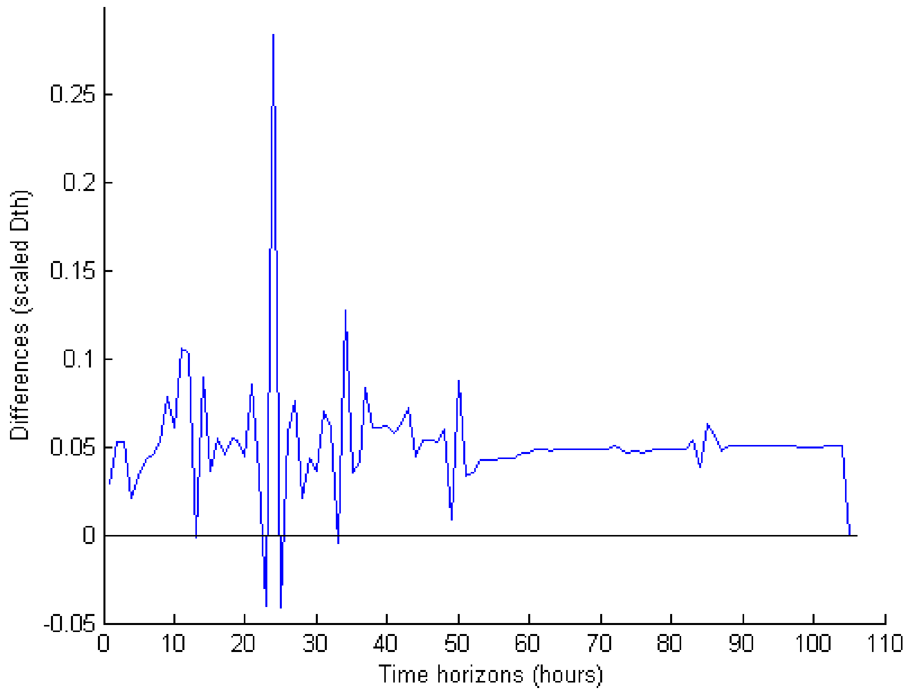

Figure 4.7: Differences between RMSEs for original and smoothed forecasts

with the increasing of time horizons, which can be seen from Figure 4.6. Also, the relative stability in RMSEs after a certain time horizons implies that if needed, the GasHour forecasting span can be extended to longer time horizons without losing too much credibility for the accuracy for those forecasts. Both Figure 4.6 and Figure 4.7 indicate improvement in accuracy, by showing that RMSEs for smoothed forecasts are slightly smaller than those for original forecasts for most of the time horizons. Those highly overlapped curves also show that smoothed forecasts are very similar to the original forecasts. The notch around hour 24 appears to reveal where polynomial smoothing might demonstrate its superiority. According to the t statistical test, the differences between two RMSEs are not statistically significant.

However, economically, these differences are meaningful to GasDay lab. Similar conclusions can be drawn from Figures 4.8 and 4.9, which are depicted using data from another operating area for the same time period.

After a certain time period (around time horizon 50), the differences shown in Figures 4.7 and 4.9 begin to stay in a relatively constant state, implying a mild, yet consistent, improvement. This suggests that our smoothing technique offers a consistent, modest improvement for forecasts at longer time horizons.

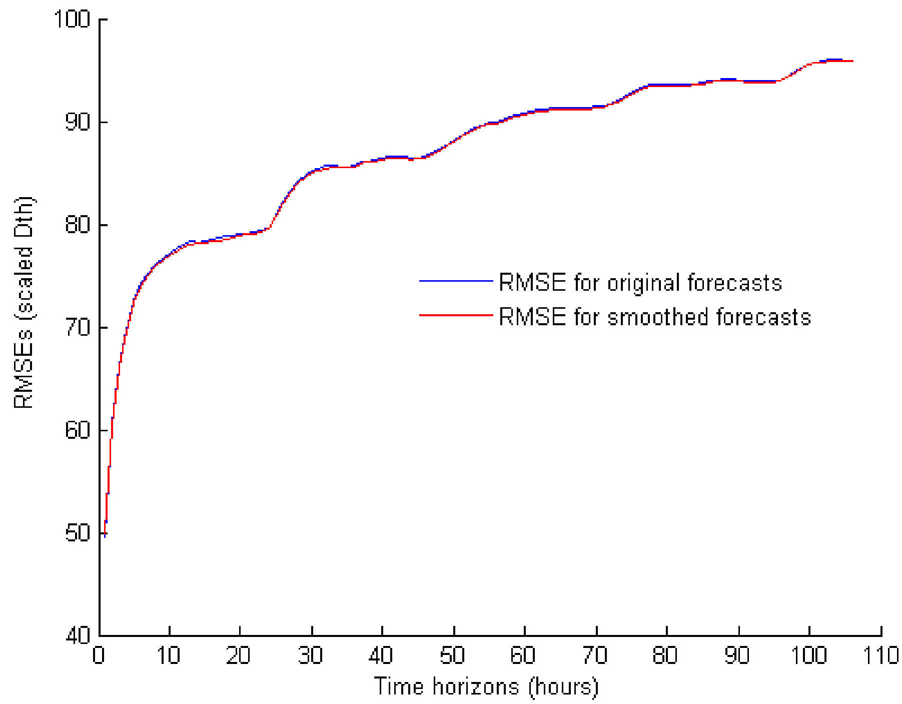

Figure 4.8: RMSEs for both original and smoothed forecasts

Usually, for the first few (for instance, 2 in Figure 4.9) time horizons at which the smoothed forecasts were generated by measured flow values and original forecasts, the RMSEs are larger than the original forecast RMSEs. It is not quite

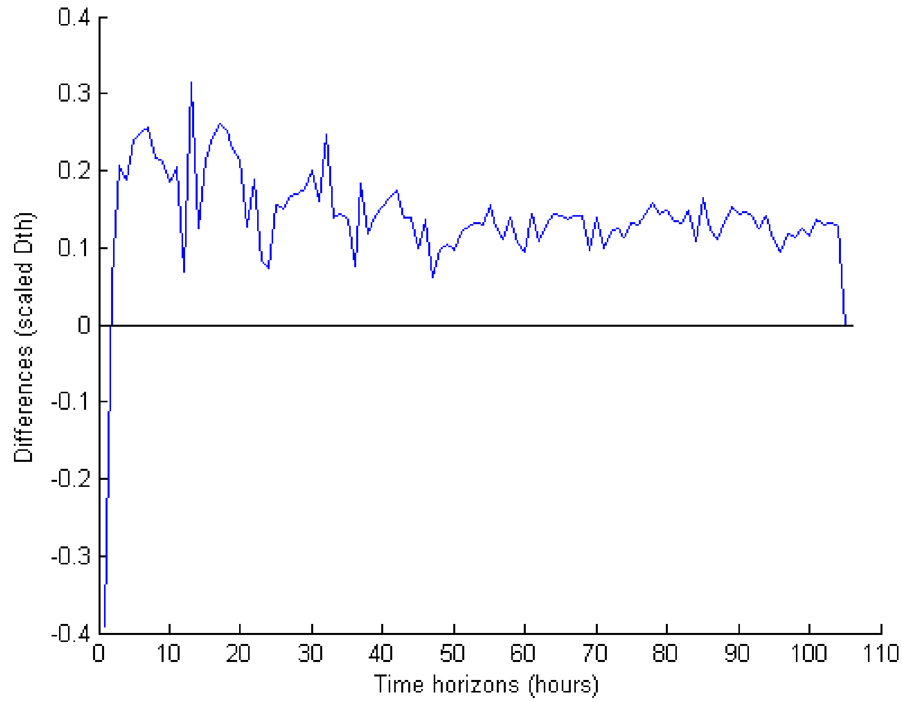

Figure 4.9: Differences between RMSEs for original and smoothed forecasts

clear what leads to this phenomenon. However, a reasonable speculation is that two different types of data (measured flow data and forecasts made by GasHour) are smoothed at the same time. To be more specific, measured flow data, which is recorded by gas providers on a certain time basis is often much more volatile, shares nothing like forecasts generated by GasHour, which are calculated mainly through linear regression models and are less volatile. The different characteristics between these two data steams might contribute to the relatively poor performance of the smoothing technique on the first few time horizons. Taking this uncertainty into account, it is recommended that our smoothing technique should not be applied for the first few points. After all, hourly forecasts from GasHour are usually more accurate when the time horizons are small.

For the last few (for instance, 2 in Figure 4.9) time horizons at which the original GasHour forecasts are kept instead of getting replaced by smoothed values, as discussed in Chapter 3, the RMSEs are the same for the original and the smoothed forecasts, explaining why the differences become 0 in Figure 4.9 for time horizons 104 and 105.

4.3 Measuring Smoothed GasHour Forecasts under More General Circumstances

The results shown in Section 4.2 suggest that smoothing using a quadratic polynomial in a five-point window can increase accuracy for current GasHour forecasts. For the purpose of fully experimenting with polynomial smoothing on GasHour data, the RMSEs when different degrees of polynomials, and different fitting windows, also should be presented.

Suppose a degree two polynomial is used to smooth over different numbers of points, e.g., seven or nine points, for the same operating area and same period of time as in Figure 4.1.

Figure 4.10 displays the differences of RMSEs between the original and the smoothed hourly forecasts based on Equation 4.3, using a quadratic polynomial to smooth 7 points in each fitting window. As explained previously, the first few (3) hours' smoothed forecasts perform badly and should be ignored. In this case, the original forecasts should be used. The peaks of differences around time horizon 24 echoes the previous conclusion that polynomial smoothing performs best for that period. The constant state of the differences starting from about time horizon 50 leads to future application with GasHour forecasting span.

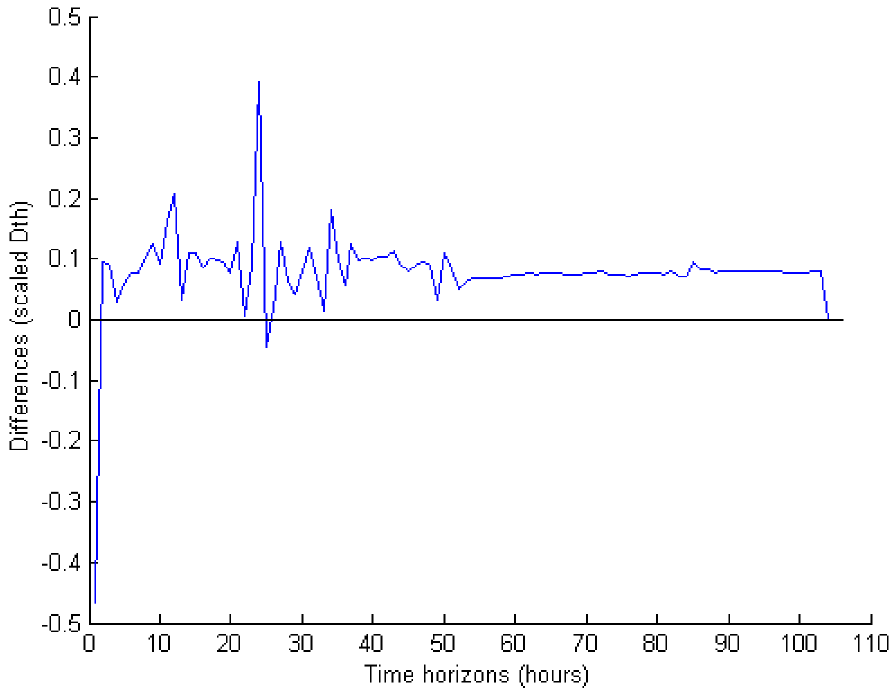

Figure 4.10: Differences between RMSEs for the original and the smoothed forecasts when using a quadratic polynomial with 7 points in each fitting window

Figure 4.11 displays the differences of RMSEs between original and smoothed hourly forecasts based on Equation 4.3, using a quadratic polynomial to smooth 9 points each time. As explained previously, the first few (4) hours' smoothed forecasts perform badly and should be ignored. In this case, the original forecasts should be used.

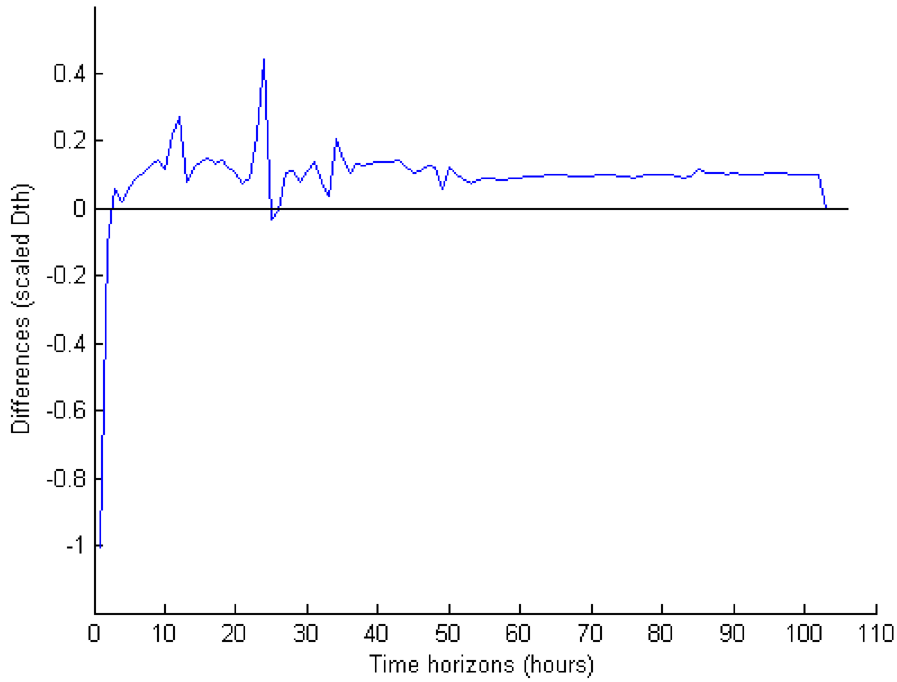

Figure 4.11: Differences between RMSEs for original and smoothed forecasts when using quadratic polynomial with 9 points in each fitting window

Compared to Figures 4.7 and 4.10, Figure 4.11 demonstrates that a larger number of points smoothed at once increases the forecasting accuracy to a certain extent for most of the time horizons, which is shown in Figures 4.12. Similar to previous observations, the outstanding differences around time horizon 24 add weight to the suggestion that smoothing should be applied for that time period.

Figures 4.10 and 4.11 show the differences in RMSEs when smoothing over

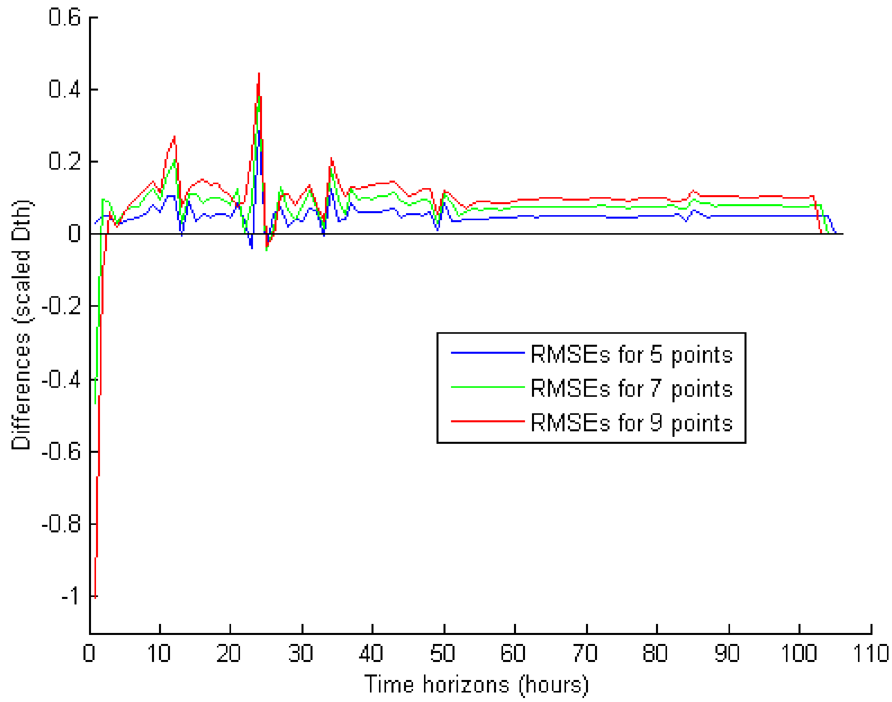

Figure 4.12: Differences between RMSEs for original and smoothed forecasts when using quadratic polynomial with 5, 7, and 9 points in each fitting window

different numbers of points. Next, suppose 7 points are smoothed each time, using different degree polynomials (3 and 4).

Figure 4.13 presents the differences of RMSEs between the original and the smoothed hourly forecasts, using a degree three polynomial over 7 points each time. As explained previously, the first few (3) hours' smoothed forecasts perform badly and should be ignored. In this case, the original forecasts should be used. The distinguished differences at hour 24, along with the stable stretch starting from time horizon 50, all correspond to previous results and conclusions.

Figure 4.14 presents the differences of RMSEs between original and

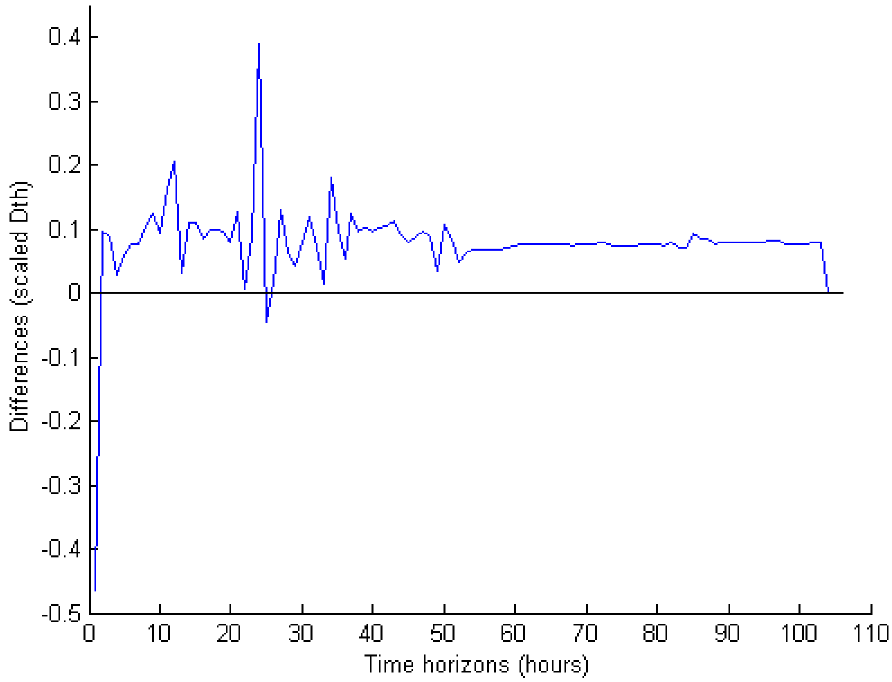

Figure 4.13: Differences between RMSEs for the original and the smoothed forecasts when using a degree three polynomial with 7 points in each fitting window

smoothed hourly forecasts, using a degree four polynomial over 7 points each time. As explained previously, the first few (3) hours' smoothed forecasts perform badly and should be ignored. In this case, the original forecasts should be used.

When comparing Figures 4.10, 4.13, and 4.14, it is evident that different degrees of polynomials do not essentially exert significant influences over the results, as shown in Figures 4.15. Since the results from degree two and three are the same, these two lines overlap each other, thus leaving two curves in total in the picture.

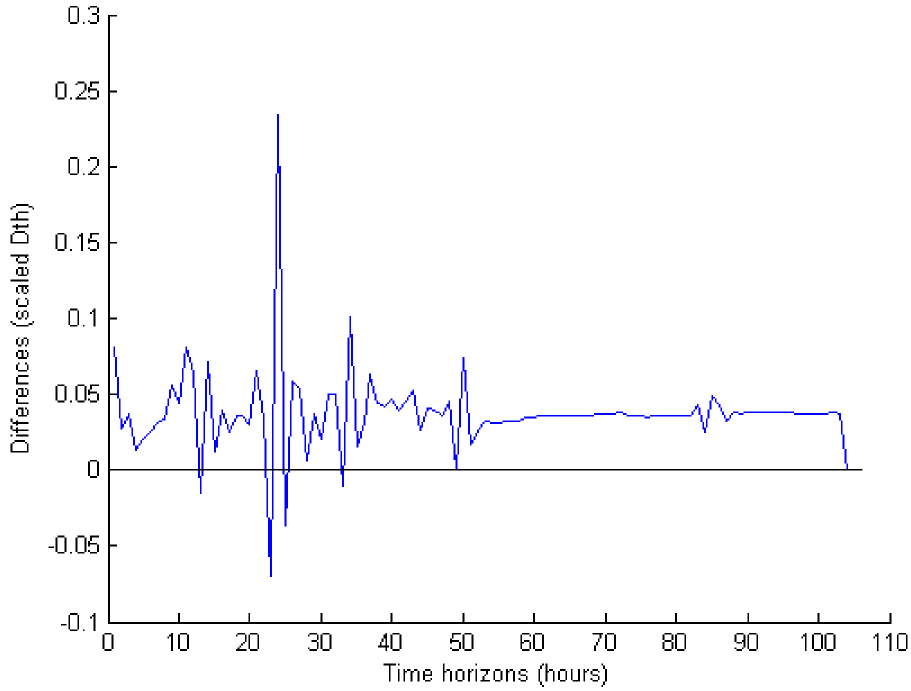

Figure 4.14: Differences between RMSEs for original and smoothed forecasts when using a degree four polynomial with 7 points in each fitting window

4.4 Conclusions

Graphs showing RMSEs from Section 4.2 and 4.3 have confirmed our expectation that polynomial smoothing can improve the accuracy of current GasHour forecasts. However, results in these pictures also indicate that improvement is relatively small. If we use 5-, 7-, or 9-point sliding windows for fitting, then the original GasHour forecasts, rather than the smoothed forecasts, should be used for the first 2, 3, or 4 time horizons, respectively. This decision is made due to the fact that smoothing different types of data at the same time may

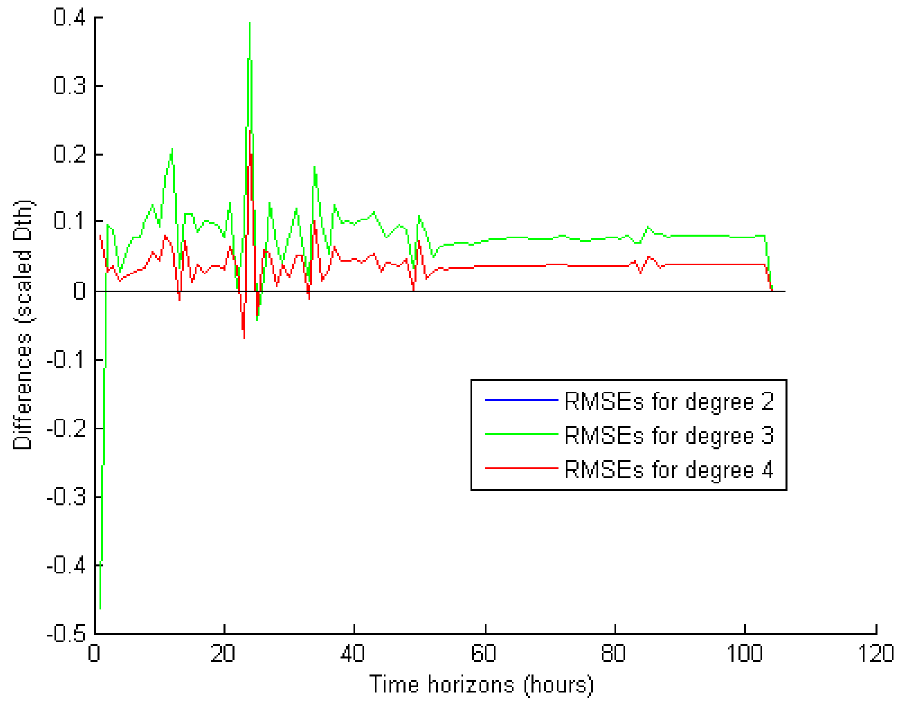

Figure 4.15: Differences between RMSEs for original and smoothed forecasts when using degree two, three, and four polynomials with 7 points in each fitting window

contaminate or add to the uncertainty of the smoothed results, as discussed in Section 4.2.

From Figures 4.10, 4.11, 4.13, and 4.14 shown in Section 4.3, different numbers of points in our sliding fitting window and different degrees of polynomial used for smoothing do not offer significantly different results. Overall, using a parabola smoothing 5 points is the best choice for now, considering factors such as performance results and convenience. Generally, we should use simple methods unless more complex ones prove better.

Based on all the figures showing differences between original and smoothed forecasts' RMSEs, our blending technique performs best around time horizon 24. The reason contributing to this phenomenon is related to the inputs GasHour forecasting models use. Recall in Chapter 2 that GasHour uses different independent variables in its models for each time horizon. Among the most important independent variables is an autoregressive term representing the most recently available measured flow value at the same time of the day. For example, if now is 1:15 P.M., forecast made for time horizon 0 uses the measured flow from 1:00 P.M. yesterday (24 hours earlier). Time horizon 23 for noon tomorrow uses the measured flow from noon today (24 hours earlier). However, time horizon 24 for 1:00 P.M. tomorrow cannot use the measured flow from 24 hours earlier (1:00 P.M. today) because that flow is not yet available. Instead, time horizon 24 for 1:00 P.M. tomorrow must use the measured flow from 1:00 P.M. yesterday (48 hours earlier). This jump backwards in the best available autoregressive measured flow leads to relatively larger errors in forecasts near 24 and 48 time horizons, which blending helps to reduce. Figure 4.16 demonstrates this phenomenon.

Although GasHour generates 106 hourly forecasts each time it is fired, not all of them are equally important to forecasters. The longer the time horizon, the less important the hourly forecast. Hence, the most important use of our blending technique is for the time horizons near 24 and 48.

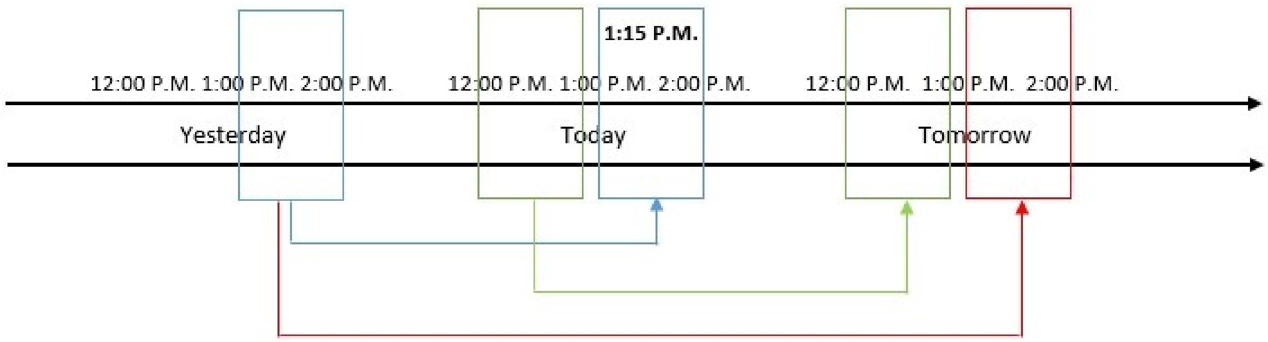

Figure 4.16: Measured flow value available after 24 hours

4.5 Future work

Polynomial smoothing has shown its promising role in increasing the accuracy of hourly flow forecasts. Our literature review in Chapter 2 has shown that ensemble forecasting generally produces quality forecasts. In this case, it is reasonable to extend the application of this blending technique into other GasDay forecasting tools, including GasDay, GasMonth, and GasYear, which correspond to daily, monthly, and yearly gas consumption predictions, respectively.

Our blending technique neutralizes noise in forecasts already generated by other forecasting tools. As stated before, forecasts from GasHour (or other GasDay products) are mostly generated through multi-horizon linear regression models, whose performance relies on the quality of their inputs, including weather information and measured history flow values. This triggers an interesting and potential point for us to explore the possibility of applying blending techniques over these model **inputs**. For instance, measured hourly flow values are always noisy, mainly due to the nature of the natural gas transportation structure. Smoothing those noisy data beforehand probably will help improve forecasting accuracy. The same idea also can be used for other inputs inside various models employed by the GasDay family of forecasting tools.

Generally speaking, our blending technique is a broader concept, including the idea of polynomial smoothing. There are many different ways that can help combine forecasting results, such as introducing a new set of models and weighting the forecasts according to different circumstances. Also, it is plausible to use a new filtering technique to deal with noise within either the forecasts or inputs.

Last but not least, we tend not to apply ensemble-like strategies, including polynomial smoothing in this thesis, under conditions when extreme events have occurred. Local maximal and minimal flow values appear when extreme events happen, such as a sudden drop in temperature. Flow forecasts made for these extreme situations are of vital importance to forecasters, which could help gas suppliers avoid great losses. However, smoothing techniques introduced here, along with other filtering methods, may moderate these extremes. Most of the time, we should weight original forecasts more heavily to maintain accuracy.

BIBLIOGRAPHY

[1] M. A. Albert, J. W. Mckinney, and R. B. Palmer, "Supply-demand forecast for gas utilities 2011-2020," Public Service Commission of West Virginia, Charleston, WV, Tech. Rep., 2011.

[2] American Gas Association, "About natural gas." [Online]. Available: http://www.aga.org/Kc/aboutnaturalgas/Pages/default.aspx

[3] American Gas Foundation, "Natural gas outlook to 2020," American Gas Foundation, Washington D.C., Tech. Rep., Feb. 2005.

[4] ——, "Direct use of natural gas," American Gas Foundation, Washington D.C., Tech. Rep., Apr. 2008.

[5] ——, "Natural gas end use: A vision for today and the future," American Gas Foundation, Washington D.C., Tech. Rep., Sep. 2009.

[6] ——, "Gas distribution infrastructure: Pipeline replacement and upgrades," American Gas Foundation, Washington D.C., Tech. Rep., Jul. 2012.

[7] J. S. Armstrong, "Combining forecasts," in *Principles of Forecasting: A Handbook for Researchers and Practitioners.* Boston, MA: Kluwer Academic, 2001, pp. 417–439.

[8] ——, *Principles of Forecasting: A Handbook for Researchers and Practitioners.* Boston, MA: Kluwer Academic, 2001.

[9] H. Arsham, "Smoothing techniques." [Online]. Available: http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/ForecaSmo.htm

[10] A. Atencia, T. Rigo, A. Sairouni, J. Bech, E. Vilaclara, J. Cunillera, M. C. Llasat, and L. Garrote, "Improving QPF by blending techniques at the meteorological service of Catalonia," *Natural Hazards and Earth System Sciences*, pp. 1443–1455, 2010.

[11] R. Bhansali, "Autoregressive model selection for multistep prediction," *Journal of Statistical Planning and Inference*, vol. 78, no. 12, pp. 295 – 305, 1999. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378375898002201

[12] Bipartisan Policy Center and the American Clean Skies Foundation, "Task force on ensuring stable natural gas markets," Bipartisan Policy Center and the American Clean Skies Foundation, Washington D.C., Tech. Rep., Mar. 2011.

[13] R. H. Brown, Y. Li, B. Pang, S. R. Vitullo, and G. F. Corliss, "Detrending daily natural gas demand data using domain knowledge," in *Proceedings of the 30th International Symposium on Forecasting.* San Diego, CA: International Institute of Forecasters, 2010.

[14] R. H. Brown and I. Matin, “Development of artificial neural network models to predict daily gas consumption,” in *Industrial Electronics, Control, and Instrumentation, 1995., Proceedings of the 1995 IEEE IECON 21st International Conference on*, vol. 2, 1995, pp. 1389–1394 vol.2.

[15] C. Capistrn, C. Constandse, and M. Ramos-Francia, “Multi-horizon inflation forecasts using disaggregated data,” *Economic Modelling*, vol. 27, no. 3, pp. 666 – 677, 2010. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0264999310000076

[16] G. Chevillon, “Multi-step forecasting in emerging economies: An investigation of the South African GDP,” *International Journal of Forecasting*, vol. 25, no. 3, pp. 602 – 628, 2009, special Section: Time Series Monitoring. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0169207008001507

[17] G. Chevillon and D. F. Hendry, “Non-parametric direct multi-step estimation for forecasting economic processes,” *International Journal of Forecasting*, vol. 21, no. 2, pp. 201 – 218, 2005. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S016920700400069X

[18] T. Clark and M. McCraken, “Evaluating direct multi-step forecasts,” Research Division, Federal Reserve Bank of Kansas City, Tech. Rep., Apr. 2011.

[19] R. T. Clemen, “Combining forecasts: A review and annotated bibliography,” *International Journal of Forecasting*, vol. 5, no. 4, pp. 559–583, 1989.

[20] M. P. Clements and D. F. Hendry, “Multi-step estimation for forecasting,” *Oxford Bulletin of Economics and Statistics*, vol. 58, no. 4, 2005.

[21] ——, “An overview of economic forecasting,” in *A Companion to Economic Forecasting*. Blackwell Publishing Ltd, 2007, pp. 1–18. [Online]. Available: http://dx.doi.org/10.1002/9780470996430.ch1

[22] G. F. Corliss, T. Sakauchi, S. R. Vitullo, and R. H. Brown, “Exploiting domain knowledge to forecast heating oil consumption,” in *Proceedings of the Laurier Centennial Conference: Applied Mathematics, Modeling, and Computational Science Conference*. Waterloo, Ontario, Canada: American Institute of Physics, July 2011, pp. 305–308.

[23] K. A. Cullen, “Forecasting electricity demand using regression and monte carlo simulation under conditions of insufficient data,” Master’s thesis, West Virginia University, College of Agriculture, Forestry, and Consumer Sciences, Morgantown, West Virginia, 1999.

[24] Customer Management, “Judgemental Forecasting,” http://www.customermanagementiq.com/glossary/judgmental-forecasting/ (Accessed 1 Sep. 2012).

[25] F. X. Diebold, “Unit roots, stochastic trends, ARIMA forecasting models, and smoothing,” in *Elements of Forecasting*. Mason, OH: South-Western, 2006, pp. 312–318.

[26] ——, *Elements of Forecasting*. Mason, OH: South-Western, 2008.

[27] F. X. Diebold and J. A. Lopez, "Forecast evaluation and combination," National Bureau of Economic Research, Working Paper 192, March 1996. [Online]. Available: http://www.nber.org/papers/t0192

[28] Earth Observatory, "Weather forecasting through the ages," http://earthobservatory.nasa.gov/Features/WxForecasting/wx2.php (Accessed 22 Oct. 2012).

[29] Energy and Environmental Analysis Inc., "The impacts of recent hurricanes on US gas markets for the upcoming winter," Energy and Environmental Analysis Inc., Tech. Rep., 2005.

[30] J. Feng, Y. Li, and J. Gong, "Cleaner air or cheaper energy: Gas-fired plants, bright future with many difficulties," *Southern Weekly*, p. C16, Aug 2013.

[31] GasDay^{TM} Lab, "GasHour." [Online]. Available: http://www.gasday.com/web/products&services_gashour.html

[32] B. Geerts and E. Linacre, "Chaos and ensemble forecasting," 12 1998, http://http://www-das.uwyo.edu/~geerts/cwx/notes/chap13/ensemble.html (Accessed 1 Sep. 2012).

[33] E. Ghyseis, R. I. Valkanov, and A. R. Serrano, "Multi-period forecasts of volatility: Direct, iterated, and mixed-data approaches." EFA 2009 Bergen Meetings Paper, Feb. 2009, http://www.unc.edu/~eghysels/papers/Var_9.pdf (Accessed 1 Aug. 2012).

[34] P. Goodwin, "New evidence on the value of combining forecasts," *Foresight: The International Journal of Applied Forecasting*, pp. 33–35, Dec. 2009.

[35] J. G. D. Gooijer and R. J. Hyndman, "25 years of time series forecasting," *International Journal of Forecasting*, vol. 22, pp. 443–473, 2006.

[36] B. E. Hansen, "Multi-step forecast model selection," Apr. 2010, unpublished paper.

[37] B. F. Hobbs, U. Helman, S. Jitprapaikulsarn, S. Konda, and D. Maratukulam, "Artificial neural networks for short-term energy forecasting: Accuracy and economic value," *Neurocomputing*, vol. 23, pp. 71–84, Jul. 1998.

[38] C. M. Hurvich and C.-L. Tsai, "Selection of a multistep linear predicator for short time series," *Statistica Sinica*, vol. 7, pp. 395–406, 1997.

[39] C.-K. Ing, "Multistep prediction in autoregressive processes," *Econometric Theory*, vol. 19, no. 2, pp. pp. 254–279, 2003. [Online]. Available: http://www.jstor.org/stable/3533353

[40] J. F. Kaiser and W. A. Reed, "Data smoothing using low-pass digital filters," *Review of Scientific Instruments*, vol. 48, no. 11, pp. 1447–1457, Nov. 1977.

[41] I.-B. Kang, "Multi-period forecasting using different models for different horizons:

an application to U.S. economic time series data," *International Journal of Forecasting*, vol. 19, no. 3, pp. 387 – 400, 2003. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0169207002000109

[42] R. Kennedy, "Detecting outliers and meter anomolies in natural gas customer flow data," Master's thesis, Marquette University, Department of Electrical and Computer Engineering, Milwaukee, WI, December 2006.

[43] A. Khotanzad, H. Elragal, and T. Lu, "Combination of artificial neural-network forecasters for prediction of natural gas consumption," *IEEE Transaction on Neural Networks*, vol. 11, no. 2, pp. 464–473, Mar. 2000.

[44] S. Kiware, "Detection of outliers in time series data," Master's thesis, Marquette University, Department of Mathematics, Statistics and Computer Science, Milwaukee, WI, March 2010.

[45] C. Lightner, "Forecasting techniques," Fayetteville State University, Tech. Rep., 2011, class PPT.

[46] S. Liguori and M. Rico-Ramirez, "Short-term hybrid forecasts blending NWP rainfall forecasts and radar nowcasts for probabilistic flow predicitions," http://hepex.nmpi.net/files/download/workshops/post-processing2/18%20HEPEX2011_Liguori.pdf (Accessed 17 Dec. 2012).

[47] H. L. E. Lim, "Computational intelligence models for short term natural gas demand forecasting," Master's thesis, Marquette University, Department of Electrical and Computer Engineering, Milwaukee, WI, August 2002.

[48] J. K. Luers and R. H. Wenning, "Polynomial smoothing: Linear vs cubic," *Technometrics*, vol. 13, no. 3, pp. 589–600, Aug. 1971.

[49] T. McElroyl, "When are direct multi-step and iterative forecasts identical," Center for Statistical Research and Methodology, U.S. Census Bureau, Washington,D.C., Tech. Rep.

[50] D. Mulholland, "State energy forecasting: An overview of methods," Jun. 2008, http://www.epa.gov/statelocalclimate/documents/pdf/presentations_stateenergyforecasting.pdf (Accessed 12 May. 2013).

[51] NaturalGas.org, "The history of regulation," http://www.naturalgas.org/regulation/history.asp (Accessed 6 May. 2013).

[52] ——, "Natural gas- from wellhead to burner tip," http://www.naturalgas.org/naturalgas/naturalgas.asp (Accessed 6 May. 2013).

[53] ——, "Uses of natural gas," http://www.naturalgas.org/overview/uses.asp (Accessed 6 May. 2013).

[54] D. Orrell, "Notes and correspondence ensemble forecasting in a system with model error," *Journal of the Atmospheric Sciences*, pp. 559–583, 1989.

[55] B. Pang, "The impact of additional weather inputs on gas load forecasting,"

Master's thesis, Marquette University, Department of Electrical and Computer Engineering, Milwaukee, WI, August 2012.

[56] A. J. Patton and A. Timmermann, "New tests of forecast optimality across multiple horizons," Mar. 2010.

[57] P. Potonik and E. Govekar, "Practical results of forecasting for the natural gas market," in *Natural Gas*. Sciyo, 2010, http://www.intechopen.com/books/natural-gas/practical-results-of-forecasting-for-the-natural-gas-market (Accessed 1 Aug. 2012).

[58] W. Remus, M. O'Connor, and K. Griggs, "Does reliable information improve the accurcy of judgemental forecasts?" *International Journal of Forecasting*, vol. 11, pp. 285–293, Jun. 1995.

[59] F. Schorfheide, "VAR forecasting under misspecification," *Journal of Econometrics*, vol. 128, no. 1, pp. 99 – 136, 2005. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0304407604001514

[60] B. A. Shenoi, *Introduction to Digital Signal Processing and Filter Design*. Hoboken, New Jersey: John Wiley & Sons, Inc., 2006.

[61] A. Sieminski, "International energy outlook 2013," Center for Strategic and International Studies, Tech. Rep., Jul. 2013, http://www.eia.gov/pressroom/presentations/sieminski_07252013.pdf (Accessed 22 Aug. 2013).

[62] Statistical Forecasting, "Statistical forecasting," 2006, http://www.statisticalforecasting.com/ (Accessed 22 Oct. 2012).

[63] C. Stochl, "Blending as a general purpose time series forecasting tool," GasDay Lab, Marquette University, Tech. Rep., 2011, unpublished paper.

[64] A. Taware, "Forecasting and identification methods applied to gas load estimation problems," Master's thesis, Marquette University, Department of Electrical and Computer Engineering, Milwaukee, WI, December 1998.

[65] S. Tenneti, "Identification of nontemperature-sensitive natural gas customers and forecasting their demand," Master's thesis, Marquette University, Department of Electrical and Computer Engineering, Milwaukee, WI, May 2009.

[66] G. C. Tiao and D. Xu, "Robustness of maximum likelihood estimates for multi-step predictions: The exponential smoothing case," *Biometrika*, vol. 80, no. 3, pp. pp. 623–641, 1993. [Online]. Available: http://www.jstor.org/stable/2337183

[67] U.S. Energy Information Administration, "FAQ about natural gas," http://www.eia.gov/naturalgas/ (Accessed 12 May. 2013).

[68] S. R. Vitullo, R. H. Brown, G. F. Corliss, and B. M. Marx, "Mathematical models for natural gas forecasting," *Canadian Applied Mathematics Quarterly*, vol. 17, no. 7, pp. 807–827, Jan. 2009.

[69] R. Webby and M. O'Connor, "Judgemental and statistical time series forecasting: A review of the literature," *International Journal of Forecasting*, vol. 12, pp. 91–118, Mar. 1996.

[70] Wikipedia, "Forecasting," http://en.wikipedia.org/wiki/Forecasting (Accessed 5 Dec. 2012).

[71] ——, "Least squares," http://http://en.wikipedia.org/wiki/Least_squares (Accessed 1 Jun. 2013).

[72] R. L. Winkler and S. Makridakis, "The combination of forecasts," *Journal of the Royal Statistical Society*, vol. 146, pp. 150–157, 1983.

[73] X. Yan and X. Su, *Linear Regression Analysis: Theory and Computing.* Singapore: World Scientific Publishing Co. Pte. Ltd., 2009.

[74] V. Zarnowitz, *An Appraisal of Short-Term Economic Forecasts*, ser. NBER Books. National Bureau of Economic Research, Inc, July 1967, no. zarn67-1. [Online]. Available: http://ideas.repec.org/b/nbr/nberbk/zarn67-1.html

[75] Z. Zhang and T. N. Krishnamurtii, "Ensemble forecasting of hurricane tracks," *Bullentin of the American Meteorological Society*, vol. 78, pp. 2785–2795, 1997.

[76] Z. Zhang, Z. Piao, X. Zeng, Y. Jinfu, and C. Xinhai, "Least-square polynomial smoothing for inductively coupled plasma atomic emission spectra," *Spectroscopy Letters: An International Journal for Rapid Communication*, vol. 25, pp. 375–388, 1992.

[77] Y. Zhu, "Ensemble forecast: A new approach to uncertainty and predictability," *Adavances in Atomospheric Sciences*, vol. 22, pp. 781–788, 2005.