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Is Bitcoin the 'Paris Hilton' of the Currency World? Or Are the Early Investors onto Something That Will Make Them Rich?

By

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Is Bitcoin the 'Paris Hilton' of the Currency World? Or Are the Early Investors onto Something That Will Make Them Rich?

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Abstract.
The bitcoin phenomenon, and the technological innovation that made it possible, is interesting - but for investors large and small, the more pertinent question is whether they should buy the digital currency or avoid it. We analyze a bitcoin investment from the standpoint of an investor with a diversified portfolio using both in-sample and out-of-sample settings. Within the in-sample setting, bitcoin does not yield added value to investors with utility function consistent with the mean-variance setting. On the other hand, they do offer diversification benefits to investors with negative exponential and power utility functions. However, these benefits are not preserved in the out-of-sample framework. In most cases, the optimal portfolios that include only the traditional asset classes appear to have superior performance.

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"…In a matter of months you won't be hearing about it (Bitcoin). It will go the same way of Paris Hilton. People will move on to the next thing." — *- Peter Leeds, Penny Stock newsletter*

"…November Trade recommendation: Buy Bitcoin" — *- Raoul Pal, Global Macro Investors*

I. Introduction

Technological development and increased use of the Internet have led to the proliferation of virtual communities. Some of these communities have created and circulated their own currency for exchanging goods and services. Bitcoin is the most popular among these virtual or digital currencies and has been in the news recently because of wild fluctuations in its value and significant venture capital investment in entities associated with it.[1] Bitcoin is relevant in several areas of the financial system and is therefore of interest to central banks, consumers and investors.[2]

The bitcoin phenomenon, and the technological innovation that made it possible, is interesting—but for investors, the more pertinent question is whether they should buy the digital currency or avoid it. In other words, should investors treat bitcoin as an asset class?[3] This is more so given the recent U.S. Internal Revenue Service decision to treat bitcoin and

[1] While bitcoin is the most widely known and used digital currency today, there are several dozen more digital currencies available, such as, Ripple, Litecoin and Peeercoin. Most of them are based on the same technology that Bitcoin pioneered.

[2] Cameron and Tyler Winklevoss, of Facebook fame, have filed with the Securities and Exchange Commission to launch an exchange-traded fund, called the Winklevoss Bitcoin Trust, that holds bitcoins. SecondMarket, a platform for investing in private assets, who has already launched a private fund called the Bitcoin Investment Trust to cater to wealthy investors, are now planning to open up a private bitcoin investment fund to ordinary investors. Other firms are also trying to ramp up efforts to build a more robust investment infrastructure for bitcoin and other digital currencies. Some merchants say they like bitcoin because they don't have to pay hefty credit-card transaction fees. Investors worried about inflation like the fact that the supply of bitcoins can't be manipulated by a central authority.

[3] Investors are backing bitcoin-related startups. And some firms are already trying to make it easier for investors to get involved. The German finance ministry has recognized it as a "unit of account" and senior U.S. government officials told a U.S. Senate Committee on November 18th, 2013 that virtual currencies had legitimate uses. But there have also been many cases of bitcoin theft. Exchanges that convert bitcoin to other currencies have collapsed or closed. Silk Road, an online forum where illicit goods and services are traded for bitcoin, was shut down by America's Federal Bureau of Investigation in October 2013 but has since reopened.

other virtual currencies like property, not currency, giving a potential boost to investors but imposing extensive record-keeping rules – and significant taxes – on its use.[4]

Brière *et al* (2013) calculated that between July 2010 and July 2013, bitcoin had an annualized return of more than 370% with 175% volatility. They found that its returns had a weak but significant correlation with gold and inflation-linked bonds, supporting the notion that some investors see bitcoin as an inflation fighter. They concluded that a small allocation to bitcoin—perhaps 3% of a well-diversified portfolio—could improve one's risk-return trade-off. A number of papers have studied the technical and computer-related aspects of bitcoin.[5] However, there are hardly any academic research which investigates the investment opportunities that bitcoin presents. This paper tries to fill this important gap in the literature and is one of the first studies in this area.

This paper improves upon Brière *et al* (2013) in a number of ways. First, it extends the sample period to include the months of August through January 2014 which showed the highest level of volatility in bitcoin price since its inception in 2009.[6] Second, Brière *et al* (2013) uses a mean-variance Spanning test to see whether adding bitcoin to a predetermined asset universe improves the risk-return profile of investors' portfolios. However, Daskalaki and Skiadopoulos (2011) have shown that this approach is subject to some shortcomings. This paper follows Daskalaki and Skiadopoulos (2011) and takes a more general approach by examining whether bitcoins should be included in an investors' portfolio in both in-sample and out-of-sample settings.

[4] On March 25, 2014 the IRS announced that it would treat bitcoin held by investors much like stocks or other intangible property. If bitcoin is held for investment, any gains would be treated as capital gains, meaning they would be subject to lower tax rates. The top long-term capital gains tax rate is 20%, while the top ordinary income-tax rate is 39.6%, although add-on taxes often make both rates somewhat higher. But as capital investments, loss deductions from bitcoin often would be limited, whereas currency losses can be easier to deduct up front.

[5] See, among others, Reid and Harrigan (2012) and Ron and Shamir (2013) and the references therein.

[6] The bitcoin price has fluctuated wildly. It rose from $13 in the beginning of 2013 to over $1200 per coin in December 2013, and has since fallen to about $550 as of end-February, 2014.

The rest of the paper is organized as follows. Section 2 describes the bitcoin network. Section 3 outlines the data set and the methodology. Section 4 sets the asset allocation framework and then compares both the in-sample and out-of-sample performance of optimal portfolios that contain bitcoins with those that do not contain bitcoins. The last section summarizes the results.

II. Bitcoin Network[7]

Bitcoin is the world's first completely decentralized peer-to-peer digital currency. A pseudonymous software developer published the Bitcoin Protocol (Nakamoto, 2009), which outlined the theory of a decentralized currency. In January 2009 open-source Bitcoin software was released and mining of the first bitcoins began. Bitcoin rocketed to prominence in 2013, when its value soared more than 10-fold in a two-month period, from $22 in February to a $266 in April. The price of a Bitcoin again rose to a record $1203 in December, 2013 before dropping to $550 in February 2014.[8] The nearly five-fold increase in the price since early November was fueled by rising expectations that the virtual currency will continue to grow as an alternative to traditional methods of payment.[9] At its peak, based on more than 11.8 million bitcoins issued, this digital currency held a market value of more than $6 billion.

Since its creation, Bitcoin has evolved from a mathematical proof of concept to a rapidly expanding economic network. It is used in business transactions worldwide and businesses

[7] For a more detailed discussion, see Chowdhury and Mendelson (2014) and Grossman *et al* (2014).

[8] While the virtual-currency craze has sent Bitcoin prices to record highs, the development of new trading exchanges has made it confusing for enthusiasts to identify the current price. One Bitcoin exchange, Tokyo-based Mt. Gox, on January 28, 2014 showed a price of $954, up from less than $14 at the beginning of the year. But another, Slovenia-based Bitstamp, showed a price of $808. The CoinDesk index of two exchanges—Bitstamp and Bulgaria-based BTC-e—was in between, at roughly $850. The differences among the exchanges reflect the scattered nature of bitcoin trading, say people who track the activity. Erratic volumes and technical issues are creating big price swings, and sometimes wildly uneven prices across trading marts.

[9] The prices are as of the writing of this draft on the Tokyo-based Mt. Gox exchange and on the Slovenia-based Bitstamp Exchange. Bitcoin has been in the news a great deal in recent months. In February 2014, after weeks marked by technological breakdowns, regulatory issues, and general questions over its viability, bitcoin was in turmoil. The Mt. Gox exchange filed for bankruptcy protection.

big and small have shown interest in integrating the Bitcoin platform into their operations and providing new services within the Bitcoin economy.[10] The momentum behind Bitcoin is growing as amateur investors, venture capitalists, and technology enthusiasts worldwide pump money into businesses that are trying to figure out how to use Bitcoin to buy and sell goods and services (Needleman and Ante, 2013). A growing number of merchants accept Bitcoin, because the associated transaction costs are generally lower than for credit or debit cards.

Instead of being produced on a printing press or by a central authority, bitcoins are generated by solving complicated algorithmic searches with powerful computers, a process known as mining.[11] Most Bitcoin users do not mine, they purchase or trade for bitcoin. Mining doesn't affect the average Bitcoin user much, but it is still an important part of the Bitcoin ecosystem.

All newly mined bitcoins, along with every transactions, are publicly recorded. This record is known as the blockchain. The blockchain records transaction details, it does not record any personal identifying information about senders or recipients. The blockchain is critical to maintaining the transparency of the Bitcoin system, and make counterfeiting or double spending impossible.

Miners that solve the proof of work formula are rewarded with newly minted bitcoins. Currently, miners are rewarded 25 bitcoins for each confirmed block of trasactions, which occurs on average every ten minutes. About every four years, the reward is halved,

[10] Commercial space venture Virgin Galactic which announced on November 22, 2013 that it would start accepting Bitcoins to reserve seat on future space trips – is just the latest of many businesses that have recently embraced the decentralized virtual payment system. Bitcoin is also accepted in diverse places like a Subway sandwich shop in Allentown, PA; Cheapair.com travel agency; Baidu (operator of a website in China); The University of Nicosia in Cyprus, etc. In January 2014, the Sacramento Kings NBA team became the first major professional sports franchise to accept bitcoins. At today's prices, courtside seats for the March 1, 2014 match-up against the Minnesota Timberwolves would have cost 0.28 bitcoins. In dollars, those seats were listed for as much as $257.

[11] Mining is the calculation of a hash of a block header, which includes, among other things, a reference to the previous block, a hash of a set of transactions and a nonce (a 32-bit/4-byte field whose value is set so that the hash of the block will contain a run of zeros). If the hash value is found to be less than the current target (which is inversely proportional to the difficulty), a new block is formed and the miner gets 50 newly generated Bitcoins. If the hash is not less than the current target, a new nonce is tried, and a new hash is calculated. This is done millions of times per second by each miner.

resulting in a finite money supply of 21 million bitcoins which is expected to be completely mined by 2140. At that point, it is expected that miners will be rewarded with transactions fees from individual payments rather than newly minted bitcoins to incentivize them to continue mining. There are currently about 12.5 million bitcoins in circulation.

III. Methodology

In recent years, a large number of studies have used the Markowitz (1952) mean-variance (MV) static asset allocation setting to study if the addition of a particular asset to a given asset menu improves investment opportunities (see DeRoon and Nijman (2001) for a detail survey). If the mean-variance frontier of the benchmark assets and the frontier of the benchmark plus the new assets have exactly one point in common, this is known as *intersection*. This means that there is one mean-variance utility function for which there is no benefit from adding the new assets. If the mean-variance frontier of the benchmark assets plus the new assets coincides with the frontier of the benchmark assets only, there is *spanning.* In this case, no mean-variance investor can benefit from adding the new assets to his optimal portfolio of the benchmark assets only (DeSantis, 1995).

However, Daskalaki and Skiadopoulos (2011) have shown that this approach is subject to several shortcomings. First, the Markowitz setting may not reflect accurately the gains from investing in a new asset since it is based on two assumptions, i.e. that either the distribution of the asset returns is normal or investor's preferences are described by a quadratic utility function. Neither of these two conditions is expected to hold. In particular, there is ample empirical evidence that asset returns are not distributed normally, especially for relatively short horizons (see e.g., Peiro, 1999, for stock indices, and Gorton and Rouwenhorst, 2006, Kat and Oomen, 2007, for commodity futures). In the case where the non-normality of returns is not taken into account in the optimal portfolio formation process, there is a utility loss (Jondeau and Rockinger, 2006). This is because a risk averse investor has a preference for positive skewness and dislikes high kurtosis and therefore these moments should be taken into account in the portfolio choice process. Furthermore, a quadratic utility function exhibits negative marginal utility after a certain finite wealth level and increasing absolute risk aversion with respect to wealth; both these features are not consistent with rational behavior.

Second, many papers cited in DeRoon and Nijman's suvey article asses the diversification benefits of investing in new assets by inspecting visually the relative position of efficient frontiers. But the comparison of the relative position of efficient frontiers should be set within a statistical framework. Third, most previous studies have investigated the benefits of investing in new assets within an in-sample setting. Daskalaki and Skiadopoulos (2011) argues that the portfolio choice should be examined in an out-of-sample setting given that the investor decides on the portfolio weights and the portfolio returns to be realized over the investment horizon is uncertain.

Therefore, this paper follows Daskalaki and Skiadopoulos (2011) and takes a more general approach to examining whether bitcoins should be included in an investors' portfolio. In particular, we consider an investor who allocates funds between equities, bonds, currencies, commodities, real estate and bitcoins in a standard static asset allocation context and make the following changes.

First, we perform our estimation within an in-sample setting so that comparison could be made with Brière *et al* (2013). The novelty though is that we employ rigorous tests instead of eyeballing the relative position of efficient frontiers based on the traditional and the traditional augmented with bitcoin asset universes, respectively. To this end, the regression-based spanning techniques are applied to test for spanning when investor preferences are described by utility functions that are consistent with the MV setting, as well as, a more general non-MV one (see e.g., Huberman and Kandel, 1987, DeRoon and Nijman, 2001, for MV spanning, and DeRoon *et al.,* 1996, for generalized non-MV spanning tests).

Second, in line with DeMiguel et al. (2009) and Kostakis et al. (2010), we also employ an *out-of-sample* setting. In particular, static one-period optimal portfolios are formed, their corresponding realized returns are calculated and their performance is evaluated under a number of performance measures. Third, optimal portfolios are constructed by taking into account the higher order moments of the returns distributions of the involved assets. To this end, direct utility maximization is performed (e.g., Cremers et al., 2005, Adler and Kritzman, 2007, Sharpe, 2007). The advantage of this approach compared to the MV optimization applied by previous studies is that the optimal portfolios can be derived by maximizing the expected utility of the investor for any assumed type of returns distribution and preferences.

IV. Data and Estimation Results

In this Section we perform statistical tests to analyze the investment potential of bitcoins. We consider the case of a U.S. investor holding a diversified portfolio which includes both traditional assets (stocks, bonds, hard currencies) and alternative investments (e.g., commodities, real estate). We examine the behavior of this investor when bitcoin is added to the portfolio.

The dataset consists of weekly closing prices of a number of indexes and bitcoin. We employ the S&P 500 total return index and the Barclays US Aggregate Bond Index to proxy the equity and bond market, respectively. To get exposure to alternative investments, such as, commodity asset class, we use individual future contracts on crude oil (NYMEX) and gold (COMEX). Crude oil is the world's most actively traded commodity. Futures contracts on light sweet crude oil (WTI) are traded on NYMEX. They are the world's largest volume futures contract on a physical commodity. Each futures contract has a 1,000 barrels contract size and its price is quoted in U.S. dollars per barrel. Gold has been a traditional investment vehicle since it serves as a hedge against inflation and a safe haven in periods of market crises (see e.g., Baur and McDermott, 2010). Each gold futures contract (traded on COMEX) has a contract size of 100 troy ounces and its price is quoted in U.S. dollars and cents per troy ounce.

Two currencies – euro and British pound – are also used. The sample period runs from August 1 2010 to January 24, 2014 for a total of 182 weeks. The data on bitcoin is taken from *Bitcoincharts* website while data on all the other variables are taken from *Bloomberg* and *Datastream*.

First, we investigate the economic benefits from investing in bitcoin by employing rigorous tests within an in-sample setting. To this end, the regression-based spanning techniques are applied to test for spanning when investor preferences are described by utility functions that are consistent with the MV setting, as well as, a more general non-MV one (see e.g., Huberman and Kandel, 1987, DeRoon and Nijman, 2001, for MV spanning, and DeRoon *et al*., 1996, for generalized non-MV spanning tests).

Next, we investigate whether the in-sample diversification benefits provided by bitcoin is preserved in an out-of-sample setting, too. For this, we calculate optimal portfolios and evaluate their relative performance separately for an asset universe that includes "traditional" asset classes (stock, bond, currencies, commodities, real estate) and an "augmented" one that also includes bitcoin.

The descriptive statistics are presented in Table 1. The average return for bitcoin is very high at 476% annually while the volatility is 258%. Both the average return and volatility is much higher than those reported in Briere *et al* (2013). Adding data from four additional months to their sample period significantly changed the results. This reflects the sharp increase in both price and degree of fluctuation during the last two months of our sample period. The kurtosis value of 16.10 for the bitcoin reflects the extreme risk involved in holding this asset. The skewness of 2.30 is higher than the 2.02 reported in Brière *et al* (2013) paper. The high skewness value can only be reached by sophisticated strategies such as volatility investments meant to hedge financial portfolios against crises (Brière *et al* 2010). This may indicate that bitcoin could act as a partial hedge against crises. The high Sharp ratio of 2.94 indicates that bitcoin is particularly attractive compared to other asset classes.

Table 2 reports the results of testing the spanning hypothesis when bitcoin is included in a traditional asset universe, consisting of equity, bonds, currencies, commodities and real estate. The Table shows the Wald test statistics and the respective *p*-values for testing the null hypothesis that there is spanning. The test is conducted for testing only MV spanning, MV and non-MV spanning jointly (MV & exponential, MV & power), as well as non-MV spanning (exponential, power). Risk aversion coefficients for a range of values are used (ARA, RRA=2,4,6,8,10) to conduct the non-MV spanning tests. The null hypothesis of MV spanning cannot be rejected at a 5% significance level. Therefore, the results suggest that under a MV setting, the performance of traditional portfolios, consisting of stocks, bonds, currencies, commodities and real estate, cannot be significantly improved by investing in bitcoin. On the other hand, in the non-MV case we can see that the spanning hypothesis is rejected for the bitcoin. Results hold regardless of whether testing is carried out for joint MV and non-MV spanning or for only non-MV spanning. The non-MV results support Brière *et al* (2013) suggestion that adding bitcoin would improve an investor's portfolio allocation.

Next, we test if the results are robust for out-of-sample performance of the traditional and augmented with bitcoin portfolios formed by direct maximization of expected utility. Tables 3, 4, and 5 show the results for the cases where the preferences of the investor are described by an exponential utility, power utility, and DA value function, respectively. Results are reported for the four performance measures and various levels of (absolute/relative) risk and disappointment aversion (DA), as well as two different sample sizes of the estimation window. To assess the statistical significance of the superiority in Sharpe Ratios, the *p*-values of Memmel's (2003) test are reported within parentheses. The null hypothesis is that the Sharpe ratios obtained from the traditional investment opportunity set and the investment opportunity set that also includes bitcoin are equal. We can see that the optimal portfolios formed based on the traditional investment opportunity set (column heading 'Trad') yield greater Sharpe ratios than the corresponding portfolio strategies based on the expanded investment opportunity set (column heading 'Exp'). These results are contrary to those reported in Brière *et al* (2013). Interestingly, we can see that for any given level of risk aversion, the Sharpe ratios decrease as the size of the rolling window increases. This indicates that the recent information should be weighted more heavily than past information (see also Kostakis *et al.,* 2010, for a similar finding).

The opportunity cost measure is negative in all cases. The negative sign indicates that the investor is willing to pay a premium in order to replace the optimal strategy that includes investment in bitcoin with the optimal one that invests only in the traditional assets. This implies that the investor is better off when the traditional investment opportunity set is considered. These results are in accordance with the ones obtained under the Sharpe ratios despite the fact the distribution of the optimal portfolio returns deviates from normality. Interestingly, in most of the cases, the opportunity cost decreases (in absolute terms) as the risk aversion increases. This implies that the investor becomes indifferent in utility terms between including or nor including bitcoin in her asset portfolio as she becomes more risk averse.

Furthermore, the portfolios that include only the traditional asset classes induce less portfolio turnover compared with the ones that also include bitcoin. Interestingly, we can see that in most cases the difference in the portfolio turnovers of the two strategies decreases as the risk aversion increases. This suggests that as the investor becomes more risk averse, she decreases her rebalancing activity since she is willing less to undertake an active bet. Finally, we can see that the return-loss measure that takes into account transaction costs is negative. The negative sign simply confirms the out-of-sample superiority of the portfolios that include only the traditional asset class, even after deducting the incurred transaction costs. In addition, we can see that the return-loss measure decreases (in absolute terms) as the risk aversion increases, just as was the case with the opportunity cost. These findings hold regardless of the assumed utility/value function, degree of the investor's relative/absolute risk aversion, degree of DA, and the employed size of the estimation window.

V. Concluding Remarks

Bitcoin has received significant attention because of its dramatic price appreciation, use in illicit transactions, and recent tax guidance on digital currencies issued by the IRS. The future of bitcoin is difficult to project. Its' utility may remain limited to those users that desire a degree of anonymity, or it could grow to become a globally accepted alternative to conventional money or an investment product.

With a view to assess bitcoin's role in the financial markets and its ultimate trajectory, this paper represents one of the first attempt to see whether an investor is better off by including bitcoin in a portfolio that consists of traditional asset classes, namely stocks, bonds, currencies, commodities and real estate. To this end, a more general approach than the mean-variance (MV) in-sample setting followed by the previous literature has been taken. In particular, we have followed Daskalaki and Skiadopoulos (2011) and extended the previous literature in two aspects. First, we have employed rigorous spanning tests within an in-sample setting that are consistent with MV as well as non-MV preferences. Second, the diversification benefits of bitcoin has been studied within an out-of-sample static non-MV framework. Optimal portfolios are formed under the traditional and augmented with bitcoin asset universes, separately, by taking into account the higher order moments of returns distribution. Next, their performance is evaluated. Various utility/value functions and degrees of risk aversion that describe the preferences of the individual investor have also been employed. Furthermore, a number of performance measures are used to compare the performance of the optimal portfolio based on traditional and augmented with bitcoin opportunity sets, respectively.

We find that within the in-sample setting, bitcoin does not yield added value to investors with utility function consistent with the MV setting. On the other hand, they do offer diversification benefits to investors with negative exponential and power utility functions.
However, these benefits are not preserved in the out-of-sample framework where the optimal portfolios that include only the traditional asset classes have superior performance. The findings are robust regardless of the performance measure, specification of utility function and the investment vehicle employed.

Given that the main question in this paper is better addressed in an out-of-sample setting, the results challenge the growing belief that adding bitcoin to an investor's traditional portfolio would lead to a better performance.

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Table 1

Descriptive Statistics, Weekly Return: August 1, 2010 – January 24, 2014

Statistics	Bitcoin	Euro	Pound	Stocks	Bonds	Gold	Oil	Real Estate
Mean	10.04	0.02%	0.08%	0.11%	0.06%	0.11%	0.18%	0.20%
Annual Mean	476.32	0.44%	0.83%	15.22%	1.99%	4.33%	10.36%	10.37%
Median	3.64%	0.08%	0.07%	0.66%	0.08%	0.20%	0.45%	0.30%
Maximum	1060	4.43%	3.60%	9.30%	2.82%	8.00%	14.08%	6.30%
Minimum	-20.88%	-2.95%	-1.60%	-10.15%	-3.80%	-8.16%	-15.1%	-8.90%
St. Deviation	124.14%	1.60%	1.24%	2.80%	1.15%	2.18%	3.20%	1.80%
Volatility	258.15%	10.20%	8.76%	20.24%	8.26%	22.10%	31.28%	16.36%
Skewness	2.30	0.30	0.22	-0.58	-0.18	-0.18	-0.28	-1.04
Kurtosis	16.10	3.90	3.14	4.80	2.92	4.12	5.18	5.36
Sharp Ratio	2.94	0.08	0.16	1.14	0.36	0.30	0.42	0.55
Observations	182	182	182	182	182	182	182	182

Table 2

Testing for Spanning using the Test Asset Bitcoin

	Wald Test Statistics (p-value)
Mean-Variance (MV)	0.42 (0.382)
MV and exponential	18.66 (0.001)
Exponential	12.10 (0.024)
MV and Power	82.54 (0.000)
Power	62.40 (0.011)

…………………………

Entries report the Wald test statistics and the respective *p*-values for the null hypothesis that a set of benchmark assets consisting of stocks, bonds, currencies, commodities and real estate spans a given test asset (bitcoin). The first row reports results for the null hypothesis that there is mean-variance spanning. The next row reports results for the null hypothesis that there is both mean-variance and exponential utility spanning with risk aversion coefficient ranging from 2 to 8. The third row reports results for the null hypothesis that there is spanning only for investors with exponential utility function. The forth row reports results for the null hypothesis that there is both mean-variance and power utility spanning with risk aversion coefficient ranging from 2 to 8. The last row presents the respective results when only power utility function is considered. All test statistics are based on a Newey-West covariance matrix with four lags.

Table 3

Direct Utility Maximization: Bitcoin and exponential utility function

	ARA=2		ARA = 4		ARA = 6		ARA = 8	
	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.
K = 26								
Sharpe ratio	0.40	0.47	0.44	0.51	0.50	0.72	0.52	0.75
(p-value)	(0.166)		(0.076)		(0.066)		(0.062)	
Opp. Cost	-3.64%		-3.18%		-2.44%		-2.06%	
Port. Turnover	60.72%	48.16%	54.12%	40.10%	46.22%	38.10%	33.18%	30.74%
Return-loss	-5.24%		-4.10		-3.66%		-3.10%	
K = 52								
Sharpe ratio	0.07	0.26	0.12	0.22	0.10	0.18	0.11	0.16
(p-value)	(0.220)		(0.177)		(0.161)		(0.154)	
Opp. Cost	-7.44%		-5.80%		-4.12%		-3.68%	
Port. Turnover	88.02%	60.18%	63.72%	54.10%	58.78%	40.98%	60.32%	34.10%
Return-loss	-4.89%		-4.28%		-4.10%		-2.42%	

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Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under an exponential utility. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes bitcoin. Results are reported for six month (k=26) and one year (k=52) rolling windows and different degrees of absolute risk aversion (ARA=2,4,6,8).

Table 4

Direct Utility Maximization: Bitcoin and power utility function

	RRA=2		RRA = 4		RRA = 6		RRA = 8	
	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.
K = 26								
Sharpe ratio	0.11	0.24	0.28	0.42	0.36	0.48	0.24	0.37
(p-value)	(0.282)		(0.310)		(0.266)		(0.322)	
Opp. Cost	-7.15%		-6.28%		-5.74%		-5.10%	
Port. Turnover	64.10%	48.93%	50.22%	40.92%	48.16%	34.11%	46.30%	30.32%
Return-loss	-5.11%		-4.42%		-2.64%		-2.12%	
K = 52								
Sharpe ratio	0.20	0.32	0.25	0.38	0.33	0.43	0.27	0.45
(p-value)	(0.202)		(0.198)		0.164)		0.241)	
Opp. Cost	-7.85%		-6.22%		-4.94%		-2.85%	
Port. Turnover	58.11%	50.24%	41.96%	26.06%	27.38%	20.14%	26.12%	18.35%
Return-loss	-8.15%		-6.35%		-4.90%		-2.40%	

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Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under an exponential utility. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes bitcoin. Results are reported for six month (k=26) and one year (k=52) rolling windows and different degrees of absolute risk aversion (RRA=2,4,6,8).

Table 5

Direct Utility Maximization: Bitcoin and disappointment aversion value function

	RRA=2		RRA = 4		RRA = 6		RRA = 8	
	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.	Exp.	Trad.
K = 26								
Sharpe ratio	0.39	0.48	0.40	0.50	0.36	0.42	0.37	0.44
(p-value)	(0.196)		(0.244)		(0.250)		(0.174)	
Opp. Cost	-6.74%		-5.40%		-3.25%		-1.76%	
Port. Turnover	60.32%	48.18%	50.33%	40.15%	46.34%	38.94%	38.35%	30.12%
Return-loss	-6.12%		-5.22%		-3.26%		-3.10%	
K = 52								
Sharpe ratio	0.48	0.56	0.44	0.52	0.38	0.41	0.42	0.49
(p-value)	(0.282)		(0.312)		(0.260)		(0.198)	
Opp. Cost	-4.42%		-3.23%		-3.12%		-2.75%	
Port. Turnover	81.13%	50.29%	76.22%	45.31%	60.56%	43.33%	40.19%	38.33%
Return-loss	-4.16%		-3.28%		-2.17%		-2.00%	

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Entries report the performance measures (annualized Sharpe Ratio, annualized Opportunity Cost, Portfolio Turnover, annualized Return-Loss) for the case where the expected utility is maximized under an exponential utility. The *p*-values of Memmel's (2003) test are also reported within parentheses; the null hypothesis is that the SR obtained from the traditional investment opportunity set is equal to that derived from the expanded set that includes bitcoin. Results are reported for six month (k=26) and one year (k=52) rolling windows and different degrees of absolute risk aversion (RRA=2,4,6,8).