JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 104, NO. A3, PAGES 4559–4565, MARCH 1, 1999

Frequency-doubled density perturbations driven by ULF pulsations

Ian R. Mann
Department of Physics, University of York, York, England

Gareth Chisham [1]
Astronomy Unit, Queen Mary and Westfield College, London

James A. Wanliss
Department of Physics, University of Alberta, Edmonton, Alberta, Canada

Abstract. We present Active Magnetospheric Particle Tracer Explorers Ion Release Module (AMPTE IRM) observations of a wave packet of compressional Pc5 ULF oscillations between 0915 and 1030 UT on October 28, 1984. The waves are observed in the local morning near the equatorial plane, have a dominant period of ~ 380 s, and most probably have a fundamental field-aligned harmonic structure. The waves have previously been interpreted as a magnetospheric waveguide mode propagating downtail [*Mann et al.*, 1998]. At the time of the maximum amplitude of the waves, AMPTE IRM observes oscillations in the background density at a frequency twice that of the coincident ULF pulsations. We develop a theory which explains the generation of frequency-doubled density fluctuations through large-amplitude radial displacements of curved flux tubes in a nonadiabatic background plasma pressure distribution. The theory predicts a phase locking between the peaks and troughs of the radial velocity field and the peaks in the frequency-doubled density oscillations which is observed by AMPTE IRM. The observations strongly support the hypothesis that the frequency doubling occurs as a result of finite amplitude effects.

1. Introduction

Ultralow frequency (ULF) pulsations can be driven by sources which are internal or external to the magnetosphere. Internal sources include drift-bounce resonance with energetic protons [e.g., *Southwood et al.*, 1969; *Southwood*, 1976], internal plasma instabilities such as the drift-mirror instability [*Hasegawa*, 1969], ballooning instabilities [*Chan et al.*, 1994], or mechanisms comprising combinations of these such as the drift Alfvén ballooning mode [*Chen and Hasegawa*, 1991]. These internally driven waves tend to have large azimuthal wavenumbers (m) and satisfy total pressure balance in the E-W direction so that [e.g., *Southwood*, 1976]

$$\delta P + \frac{B_0 b_{\parallel}}{\mu_0} = 0. \qquad (1)$$

[1]Now at British Antarctic Survey, Natural Environment Research Council, Cambridge, England

Paper number 1998JA900136.
0148-0227/99/1998JA900136$09.00

Frequency doubling in this class of internally driven high-m pulsations was originally considered by *Higuchi et al.* [1986] in an attempt to explain observations of the compressional magnetic component which oscillated at a frequency twice that of the transverse components [e.g., *Coleman* [1970]] (see also *Takahashi et al.* [1990]). The *Higuchi et al.* [1986] theory relies upon the meridional displacement of field lines in a background nonadiabatic plasma pressure distribution, requires the waves to be odd-mode field-aligned harmonics, and generates a frequency-doubled pressure perturbation. The pressure perturbation then drives a frequency-doubled compressional magnetic component via (1). As pointed out by *Southwood and Kivelson* [1997], high-m modes driven by the drift-bounce resonance mechanism are usually second-harmonic waves, and hence the *Higuchi et al.* [1986] theory cannot explain the high-m wave frequency doubling. *Southwood and Kivelson* [1997] go on to derive a new theory for this frequency doubling based on the orbits of the mirroring protons responsible for driving the waves.

ULF waves can also be driven by sources which are external to the magnetosphere. The solar wind acts as a direct source for these waves by impulsively driving global magnetospheric cavity/waveguide modes [e.g., *Kivelson and Southwood*, 1986; *Wright*, 1994] or by driving Kelvin-Helmholtz vortices on the magnetopause which inject fast mode energy deep into the magnetosphere [e.g., *Samson et al.*, 1971]. In contrast to the internally driven waves, the externally driven waves typically have low azimuthal wavenumbers and hence are not governed by (1).

In this paper we consider Active Magnetospheric Particle Tracer Explorers Ion Release Module (AMPTE IRM) observations of low-m compressional waves which have previously been interpreted as a tailwards propagating magnetospheric waveguide mode in a study using multiple satellites and ground-based magnetometer data [*Mann et al.*, 1998]. When the waves have maximum amplitude, AMPTE IRM observes simultaneous density perturbations at twice the frequency of the ULF waves. Since our observations are of large amplitude waves, which because of their long periods are most probably fundamental field-aligned harmonics, we can modify the theory of *Higuchi et al.* [1986] and apply it to our low-m wave observations. We interpret the frequency doubling as a consequence of the large-amplitude meridional displacement of curved magnetospheric flux tubes in nonadiabatic radial background plasma pressure distributions.

2. AMPTE IRM Observations

In this section we present observations from the magnetometer experiment [*Lühr et al.*, 1985] and plasma experiment [*Paschmann et al.*, 1985] on board AMPTE IRM. Figure 1 shows time series of the magnetic field components B_r, B_ϕ, and B_z, the proton velocities v_r, v_ϕ, and v_z, and the electron and proton number densities n_e and n_p observed between 0910 and 1030 UT on October 28, 1984 (the subscripts r, ϕ, and z represent cylindrical polar GSM coordinates). The oscillations in B_r and B_z appear to be in phase, especially where they have maximum amplitude around 0945 UT. Using the diagnostics of *Elphinstone et al.* [1995], this suggests that the waves are radially standing as would be expected for a waveguide mode between its reflection and turning points. The ULF waves are seen in the local morning around 0800 LT at $L \sim 8.5$, very close to the equatorial plane, and most probably represent the oscillations of a compressional magnetospheric waveguide mode (this event has been previously discussed in detail by *Mann et al.* [1998]). The wave packet displays very large amplitude oscillations in the velocity components ($v_r \sim \pm 50$ km s^{-1}; $v_\phi \sim \pm 70$ km s^{-1}), combined with small magnetic signatures (maximum amplitudes approximately a few nanoteslas). On the ground, conjugate magnetometer observations show amplitudes $\sim \pm 10$ nT [*Mann et al.*, 1998]. This suggests that waves represent an odd-mode field-aligned harmonic, and since they have a long dominant period $\sim$ 380 s, they are most probably the fundamental mode. In this paper we concentrate upon the perturbations in the

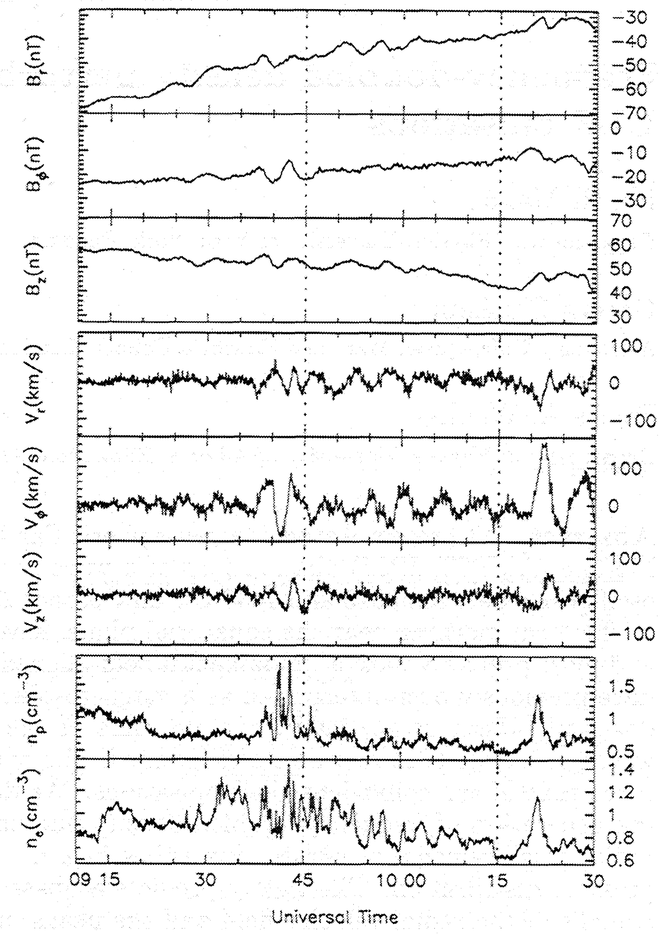

Figure 1. Time series for **B**, **v**, n_e, and n_p between 0910 and 1030 UT, 28 October, 1984.

background density observed during this wave packet; the maximum amplitude density fluctuations occurring at the time that the compressional ULF pulsation also has maximum amplitude.

In Figure 2 we show the time series of v_r, v_ϕ, and n_e between 0945 and 1015 UT (the interval contained within the dashed lines in Figure 1), when the waves have a particularly regular oscillatory structure. Figure 2a shows raw data, while Figure 2b shows low-pass filtered data where the high-frequency variations have been removed. A striking feature of this figure is that a frequency component exists in the density perturbations which is twice that of the velocity fields. Moreover, the peaks in the electron density perturbations n_e are phase locked to the peaks and troughs of v_r and v_ϕ. The phase locking is highlighted by the vertical lines drawn in Figure 2 which show the times of the maximum enhancements in n_e. Returning to Figure 1, we see that the density only seems to be strongly perturbed when the coexisting compressional ULF waves have a very large amplitude. This feature, along with the phase locking apparent in Figure 2, are important for our theoretical interpretation of this observation, and are points to which we return in section 3.

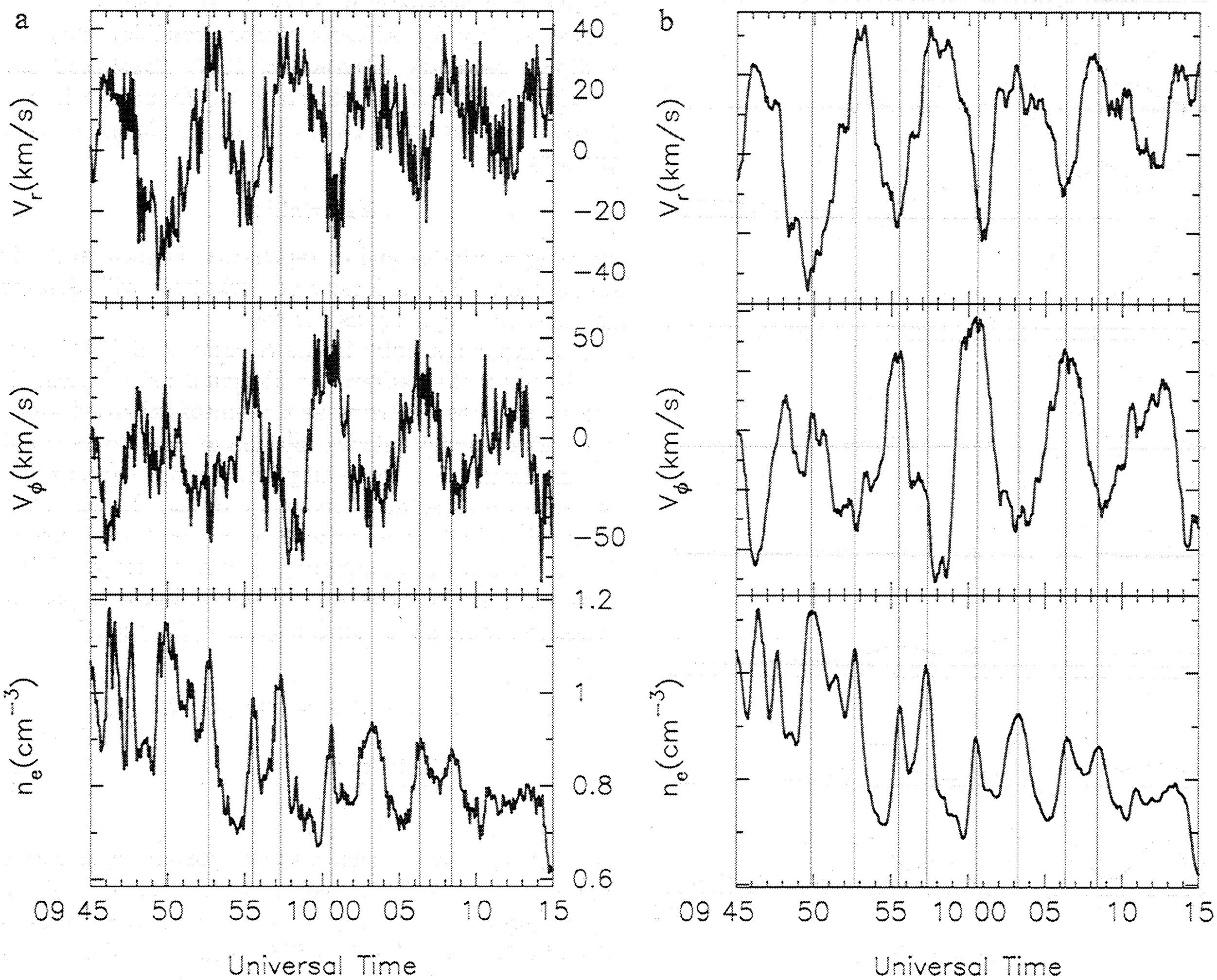

Figure 2. Variables v_r, v_ϕ, and n_e between 0945 and 1015 UT, October 28, 1984: (a) raw data and (b) low-pass filtered data.

In Figure 3 we show the power spectra for the interval between 0945 and 1015 UT. As expected, the spectra are dominated by the peak at $\sim$ 3 mHz, corresponding to the 380 s period waveguide mode (analyzing the spectra from a longer time series improves the resolution of the spectral peak putting it closer to 380 s, i.e., around 2.6 mHz). In the spectra of the densities n_e and n_p, however, there is an additional spectral peak at $\sim$ 6 mHz. This is the frequency-doubled density perturbation, driven by the fundamental mode ULF pulsation. There are also small spectral peaks in v_r and v_z (and possibly in B_r) at $\sim$ 6 mHz, similar to those seen in the density, but at a much lower power compared to that at the fundamental frequency. We consider a theoretical explanation for the observed frequency doubling in the following section.

3. Frequency Doubling Theory

In their analysis of high-m pulsations, *Higuchi et al.* [1986] presented a simple theory to explain the observations of frequency doubling in the compressional magnetic component (at 2ω) of waves with a transverse frequency ω. They argued that as a flux tube oscillates in meridian planes, pressure changes in the flux tube produce the frequency doubling as a nonlinear effect at 2ω.

Pressure changes in the flux tube can occur through several processes. First, the pressure can change via the advection of a background pressure gradient. This generates an advective pressure change which in linear theory is given by

$$\delta P^a = -\xi . \nabla P \qquad (2)$$

where ξ represents the flux tube displacement. Second, if the flux tube volume changes during a wave cycle, this will introduce plasma compression/expansion when the flux tube volume is decreased/increased. Changes in flux tube volume can be excited by the compressibility of the wave fields. Fast waves have $\nabla . \mathbf{v} \neq 0$ and will drive pressure changes which oscillate with the wave frequency. Flux tube volume changes can also occur as a result of the geometry of the oscillating flux tube. In a curved magnetic geometry, such as a dipole, Alfvénic meridional displacements will produce velocity

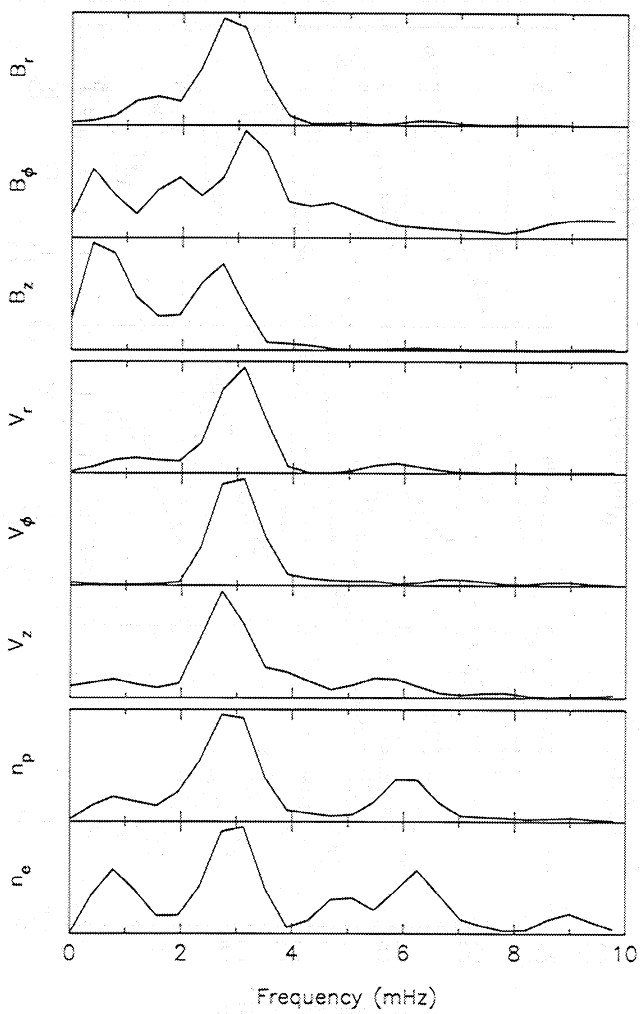

Figure 3. Power spectra for **B**, **v**, n_e, and n_p between 0945 and 1015 UT, October 28, 1984.

fields with $\nabla.\mathbf{v} \neq 0$ due to the background curvature of the field lines [e.g., *Southwood and Saunders*, 1985]. If the plasma behaves adiabatically, then the compressional pressure changes which result from changes in flux tube volume are governed by

$$\delta P^c = -\frac{\gamma P}{V}\delta V \tag{3}$$

where we have used the ideal gas law $PV^\gamma = \text{const}$, γ is the polytropic index, and V is the flux tube volume [e.g., *Southwood and Kivelson*, 1997]. The total pressure variation is given by

$$\delta P = \delta P^a + \delta P^c, \tag{4}$$

with both the terms on the right-hand side of the above equation linearly oscillating at the wave frequency ω.

In a magnetospheric plasma with an earthwards directed background plasma pressure gradient, the term δP^a and the part of δP^c which results from the dipole field line curvature oscillate in antiphase and tend to cancel each other out. If the background pressure profile results from adiabatic injection, then these two terms cancel exactly for Alfvénic (incompressible) waves in a dipole geometry [*Southwood*, 1977; *Southwood and Kivelson*, 1997]. If the plasma is locally adiabatic, the pressure changes will drive changes in plasma density given by

$$\delta P = c_s^2 \delta\rho \tag{5}$$

where $c_s^2 = \gamma P_0/\rho_0$ and c_s represents the local acoustic sound speed. These density perturbations will oscillate at the same frequency as the wave.

The important point in the *Higuchi et al.* [1986] theory is that if the background plasma density is nonadiabatic, perhaps occurring as a result of enhanced losses from adiabatically injected plasma as one moves closer to the Earth, then large-amplitude waves can excite a pressure response at a frequency of 2ω. In this case, we need to include additional terms in (2) to account for the nonlinear background pressure profile. If we consider a Taylor expansion of the background plasma pressure profile about an L shell $L = L_0$, then

$$\begin{aligned} P(L) &= P(L_0 + \Delta L) = P(L_0) + \Delta L \left.\frac{dP}{dL}\right|_{L=L_0} \\ &+ \frac{(\Delta L)^2}{2}\left.\frac{d^2P}{dL^2}\right|_{L=L_0} + \cdots \end{aligned} \tag{6}$$

Considering waves which displace plasma in meridian planes, the advective pressure change seen at position L_0 in response to a wave displacement $\xi = \xi_0 \cos\omega t$ is given by $\delta P^a = P(L - \xi) - P(L)$ (since the advection is moving plasma from the position $L_0 - \xi$ to L_0). Then replacing $\Delta L = -\xi$ in (6) to describe $P(L_0 - \xi)$ and including terms up to the quadratic in ξ we have $\delta P^a = P(L_0 - \xi) - P(L_0)$, therefore

$$\delta P^a = -\xi \left.\frac{dP}{dL}\right|_{L=L_0} + \frac{\xi^2}{2}\left.\frac{d^2P}{dL^2}\right|_{L=L_0} = \delta P_1^a + \delta P_2^a. \tag{7}$$

The component δP_2^a can be clearly seen to have a frequency-doubled component since

$$\begin{aligned} \delta P_2^a &= +\left.\frac{d^2P}{dL^2}\right|_{L=L_0} \frac{\xi_0^2}{2}\cos^2\omega t \\ &= \left.\frac{d^2P}{dL^2}\right|_{L=L_0} \frac{\xi_0^2}{4}(1 + \cos 2\omega t). \end{aligned} \tag{8}$$

This nonlinear frequency doubling effect will be maximized for waves with large meridional displacements (i.e., large ξ_0) and will occur at locations with background pressure profiles which are strongly nonlinear (i.e., where $|d^2P/dL^2|$ is large).

When *Higuchi et al.* [1986] applied this theory to their high-m wave observations, they argued that the pressure perturbations would drive parallel magnetic field components $b_{\|}$ through (1). They suggested that this

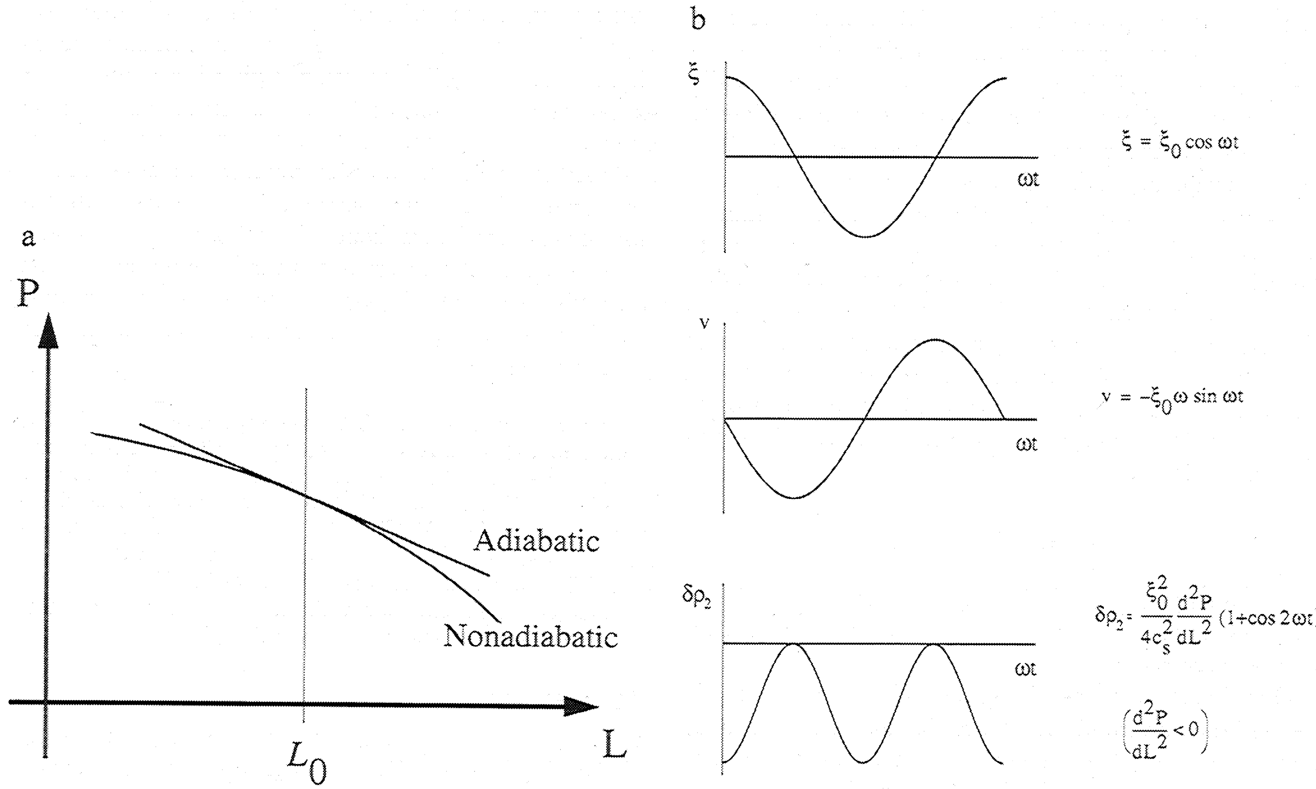

Figure 4. (a) Schematic diagram of adiabatic and nonadiabatic background plasma pressure distributions as a function of L (adapted from *Southwood and Kivelson* [1997]). (b) Schematic diagram of the pulsations radial displacement ξ, velocity v, and the resulting frequency-doubled density perturbation $\delta\rho_2$ as a function of time.

could explain the observation of a frequency-doubled component in the parallel magnetic component (at 2ω) compared to the transverse oscillation frequency at ω. As pointed out by *Southwood and Kivelson* [1997], for this theory to work the wave's displacement must be symmetric about the equatorial plane, and hence it relies on an odd-mode field-aligned harmonic. This is not generally applicable to high-m waves driven by drift-bounce resonance because they are usually second harmonics. However, the theory can be applied to our observations since they are likely to have a fundamental mode structure in the field-aligned direction.

Consequently, we suggest that the frequency-doubled density perturbations $\delta\rho_2 = \delta P_2/c_s^2$ (via (5)) observed by AMPTE IRM result from the large-amplitude meridional oscillations of waves at frequency ω in the presence of a strongly nonlinear background plasma pressure distribution. We cannot directly measure $|d^2P/dL^2|$; however, knowing that the waves have a maximum radial velocity of ~ 50 km s^{-1}, and assuming a wave period of 380 s, we obtain a displacement amplitude $\xi_0 = 0.47$ R_E. Hence the overall meridional flux tube displacement over one cycle is very large, ~ 1 R_E, and could allow the nonlinear advective pressure change δP_2^a to be important. The density perturbations observed with a frequency $\sim \omega$ in Figure 3 almost certainly result from the compression of the plasma by the fast mode during a wave cycle (i.e., δP_1^a and δP_1^c do not cancel exactly).

Ground-based European Incoherent Scatter magnetometer cross observations of waves with the same period and dominant amplitude between 0925 and 1000 UT suggest the waves have $|m| < 5$ [see *Mann et al.*, 1998]. Since the waves have low m, consistent with the interpretation of *Mann et al.* [1998] that the waves represent a compressional magnetospheric waveguide mode, they are not governed by (1). This explains why the frequency doubling is observed in the density perturbations but not in the compressional magnetic component B_z in Figure 3. An important test for this theory is the phase locking between the radial velocity v_r and the density perturbations $\delta\rho_2$. In Figure 2 we see that the peaks in the density perturbation occur at the peaks and troughs in v_r. The phase locking between these two wave trains is remarkable and very striking. If we again assume that the wave's normal displacement is given by $\xi = \xi_0 \cos\omega t$, then the radial velocity $v_r = d\xi/dt = -\xi_0\omega \sin\omega t$. Equation (8) shows that δP_2^a has an amplitude proportional to d^2P/dL^2. Assuming that the background plasma pressure profile is as illustrated in Figure 4a, where enhanced particle losses closer to the Earth might be responsible for the departure from an adiabatic distribution, then $d^2P/dL^2 < 0$.

In this case the peaks in the density profile should be mode locked to the peaks and troughs in v_r as is observed in the AMPTE IRM data. This is schematically illustrated in Figure 4b and provides strong evidence that this nonlinear mechanism is responsible for driving the frequency doubled density perturbations.

As we discussed earlier, Figure 3 suggests that it may be possible for a frequency-doubled component to be driven in the velocity and magnetic fields, in addition to the densities n_e and n_p. It is possible that the frequency-doubled density perturbation could couple to these fields and drive a frequency-doubled component. Certainly, a nonlinear pressure gradient may couple to both the velocity and magnetic fields through the equation of motion

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla P + \mathbf{j}_\wedge \mathbf{B}. \qquad (9)$$

Figure 3 suggests that this might be the case, with the coupling to the velocity field apparently stronger than to the magnetic field.

Our theory requires the waves to have an odd-mode field-aligned harmonic structure (in this case the fundamental) and to have large radial displacements, both of which are observed. Combining this with the fact that the frequency-doubled density perturbation is phase locked to the radial velocity field, the observations provide excellent evidence for the applicability of the theory.

4. Conclusions

We have presented AMPTE IRM observations of frequency-doubled plasma density perturbations which are driven by large-amplitude coincident ULF pulsations. The ULF waves are observed in the local morning, near the equatorial plane, and are believed to be low-m compressional magnetospheric waveguide modes which were probably driven by an impulsive stimulus from the solar wind [*Mann et al.*, 1998]. We have modified the high-m wave theory of *Higuchi et al.* [1986] and applied it to our low-m wave observations in order to explain the frequency doubling. The large-amplitude fundamental mode waves cause a radial displacement of flux tubes, which in the presence of a nonadiabatic background plasma pressure distribution can produce nonlinear pressure and hence density perturbations with a frequency twice that of the driving ULF wave. The theory we have developed predicts a phase locking between the radial velocity v_r and the frequency-doubled density perturbations $\delta\rho_2$. This phase locking is observed and provides very convincing evidence that the advection of a nonlinear background plasma pressure profile as described above is the mechanism responsible for creating the frequency doubling.

Moreover, neither the magnetic nor the velocity fields showed any significant frequency-doubled power. Hence it is unlikely that the frequency-doubling occurred as a result of the solar wind driving additional higher field aligned harmonics, or through the fundamental field-aligned mode coupling to other higher harmonics. Our explanation in terms of finite amplitude effects may explain why only the density, and not the magnetic and velocity fields, show a significant frequency-doubled component. To our knowledge, this is the first observation of this kind of frequency doubling phenomena and we believe that the observations can be explained by the finite displacement of magnetic flux tubes by compressional waves in regions of nonlinear local background plasma pressure.

Acknowledgments. G.C. was supported by a PPARC grant and I.R.M. was supported by a PPARC Fellowship. The authors thank Tony Allen for assistance with coordinate transforms. H. Lühr and G. Paschmann were the Lead Investigators for the AMPTE IRM magnetic field and plasma instruments, respectively, and the data were provided by the U.K. STP data handling facility at DRAL, U.K.

Janet G. Luhmann thanks Andrew Wright and another referee for their assistance in evaluating this paper.

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G. Chisham, British Antarctic Survey, Natural Environment Research Council, High Cross, Madingley Road, Cambridge, CB3 0ET, England (gchi@bas.ac.uk)

I. R. Mann, Department of Physics, University of York, Heslington, York, YO10 5DD, England (ian@aurora.york.ac.uk)

J. A. Wanliss, Department of Physics, University of Alberta, Edmonton, Alberta, Canada, T6G 2J1 (wanliss@space.ualberta.ca)

(Received September 24, 1998; revised November 19, 1998; accepted November 19, 1998.)