Computational Comparison of Five Maximal Covering Models for Locating Ambulances

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Abstract

We categorize existing maximum coverage optimization models for locating ambulances based on whether uncertainty about (1) ambulance availability and (2) response times is incorporated. We use data from Edmonton, Alberta, Canada to test five different models, using the approximate hypercube model to compare solution quality between models. We find that the basic maximum covering model which ignores these two sources of uncertainty generates solutions that perform far worse than those generated by more sophisticated models. The model that incorporates both sources of uncertainty generates a configuration that covers up to 26% more than the demand covered by the basic model with the same number of ambulances.

1. Introduction

Emergency Medical Services (EMS) must balance cost and quality of service when planning their operations. Quality of service has multiple attributes, including response times, the type of care that EMS staff are trained to provide, and the equipment to which they have access. We will focus on response time—the time from contacting EMS until the patient is reached. Response time performance is typically measured as a cumulative fraction (referred to as the *fractile method of reporting*), for example the fraction of life-threatening calls reached in 8:59 minutes or less. Such performance measures are recommended by industry experts (Fitch, 2005) and in standards prepared by the National Fire Protection Association (2004, section 5.3.3.4.3). Regulations based on the EMS Act of 1973 in the US specified that 95% of calls should be reached in 10 minutes (Revelle, et al., 1977). A 2004 survey of the 200 largest cities in the US indicated that over three quarters of EMS agencies that provide transport to hospital use a target of 8:59 minutes or less and report the fraction of calls reached within this time standard (as opposed to, say, the average response time). The single most common standard for urban areas, at least in North America, appears to be to reach 90% of life-threatening calls in 8:59 minutes or less (Fitch, 2005).

Given the nature of such performance standards, it is not surprising that location theorists have formulated optimization problems to locate a fixed number of ambulance stations and to allocate a fixed number of ambulances to stations so as to optimize either (1) the average response time or (2) the demand that can be reached within some time standard. These two performance measures were discussed in an early survey paper by Chaiken and Larson (1972) and both measures have an associated stream of research in the operations research and location theory literature.

Jarvis (1975) developed the best known approach to minimizing measure (1), the average response time—a locate-allocate heuristic that use the approximate hypercube model (Larson, 1975) to evaluate solutions. We focus primarily on measure (2) because it corresponds to the measurement standard that is predominant in current practice. Research that focuses on measure (2) uses the concept of *coverage*, where a demand location is assumed to be covered by an ambulance station if the distance (or travel time) between the two is less than or equal to some threshold.

The first paper on locating EMS units optimally introduced the set covering location model (Toregas et al., 1971). This is a binary programming model which finds the minimum number of EMS units required to cover all demand locations in the service area. Unfortunately the optimal solution to this model requires an excessive number of EMS units since it requires complete coverage and disregards the cost of the system. Church and ReVelle (1974) proposed a more practical alternative: the maximal covering location problem (MCLP). The maximal covering location model fixes the number of EMS stations, and seeks to maximize the coverage of demand points in the service area. The binary integer program can be solved relatively easily using commercial software as its LP relaxation usually produces all-integer solutions. It has been used in practice for locating ambulance stations (Eaton et al., 1985), and it may be the most influential of all ambulance station location models.

Despite its appeal due to simplicity and solvability, MCLP is imperfect. First, it assumes that response times are known and deterministic. In reality, response times are highly uncertain because of factors such as variable pre-travel delays, traffic congestion, weather, local events, and hour of day. Second, MCLP assumes that the nearest EMS unit to a demand location is always available. A unit that is responding to a call is unavailable to respond to a subsequent call while it travels to the service site, provides aid on-site, transports the patient to the hospital, and completes paper work and maintenance (cleaning and stocking of materials) on the unit. Even though an EMS system is designed for low utilization, in most EMS systems the utilization rates are at least 25% and assuming near-zero utilization or a large number of units at each station is not realistic. To see how these two imperfections result in miscalculations, we present two examples.

Example 1: Consider an ambulance station with one vehicle and two demand locations A and B, with a response time threshold of 8 minutes. Suppose that the response time from the station to A is N(7.5, 2.5) and the response time to B is N(8.5, 2.5), where N(μ, σ) denotes a normally distributed random variable with mean μ and standard deviation σ. MCLP, using average travel times, considers A to be covered and B not to be covered. However, A is covered with a probability of 0.58 and B is covered with a probability of 0.42, if there is a unit available at the station. Assuming Pr{ambulance busy} = 0.3, we compute the probability of coverage for A and B as 0.405 and 0.295, a far cry from 1 and 0 as estimated by MCLP. Such miscalculations can result in poor location selections, as illustrated in our second example.

Example 2: Consider the one-dimensional problem with 4 demand locations in Figure 1.

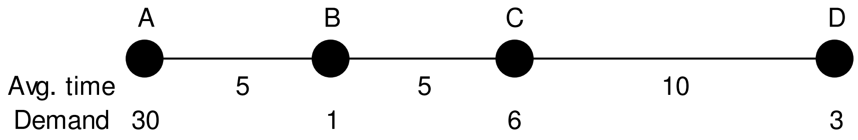


Figure 1: Example to demonstrate the differences between the models studied in this paper numerically. The average travel times are given in minutes.

Throughout this example we assume a response target of 8 minutes, additive response times (for example, the average response time from A to C is 5 + 5 = 10 minutes), and we consider locating two ambulances, limiting the candidate locations to demand nodes. If we locate the two ambulances at nodes B and D and ignore uncertainty, we cover all demand points. Hence, one optimal solution to MCLP is (B, D) with 100% coverage.

Now suppose we incorporate ambulance availability, and assume that Pr{ambulance busy} = 0.3 for each ambulance at each location, independent of the status of the other ambulance. The expected coverage provided by (B, D), the optimal solution to MCLP, is (1 − 0.3) × 40 = 28. However, we get an expected coverage of 33.7 by locating both ambulances at B ((1 − 0.3) × 37

+ 0.3 × (1 − 0.3) × 37), and this is the maximal expected coverage. This is because the majority of the demand is at nodes A, B, and C, and the secondary coverage of these nodes provides a higher incremental expected value than the primary coverage of node D.

Suppose we do not incorporate ambulance availability, but we model response time uncertainty. Assume that all response times follow a lognormal probability distribution where the travel time given in Figure 1 is equal to the mean, and the standard deviation is equal to half of the mean. We use a lognormal instead of a normal distribution to avoid negative response times—see further discussion in Section 4. In this case, the model protects the highest demand points by locating the two ambulances at A and C. Finally, suppose we combine both sources of uncertainty. The optimal solution is (A, B). Table 1 provides a summary of the results along with the assessment of each solution under each objective.

Table 1: A summary of the optimal solutions of the four models discussed and the evaluations of the optimal solutions under each objective.

			Objective function			
Model	**Optimal ambulance locations**		**(1)**	**(2)**	**(3)**	**(4)**
(1) MCLP	B	D	100.0%	70.0%	79.8%	57.0%
(2) Ambulances busy	B	B	92.5%	84.2%	72.8%	66.2%
(3) Response times uncertain	A	C	92.5%	65.3%	93.7%	70.6%
(4) Both	A	B	92.5%	81.0%	89.6%	76.1%

Even in this simplistic example we get four different optimal solutions. Furthermore, the optimal solution of one model does not necessarily perform well under the others. MCLP spreads the resources for an illusion of complete coverage. Yet the optimal solution to MCLP is the worst of the four solutions when measured using the objective of the most realistic model (4) because MCLP ignores both sources of uncertainty. The MCLP solution is 19 percentage points below the best possible, which is a major degradation of performance for EMS system design. The other two models that take into account only one source of variability perform better than MCLP, but they both fall far short of the optimum.

We note that the MCLP was originally proposed as a tool for determining the "location of ambulance dispatch points" (Revelle et al., 1977) as opposed to the allocation of ambulances to such dispatch points. Our focus is on the allocation of ambulances to dispatch points or stations. The MCLP can be used for that purpose, but it is subject to the limitations that we have just illustrated. We will use the term "station" throughout the paper instead of "dispatch point," with the understanding that in some cities, a "station" could simply be a convenient street corner location or a parking lot.

In this paper, we will compare the four models that were illustrated in Example 2 (including two variations of the last model). In the next section, we survey literature that is directly related to our study. We refer the reader to Swersey (1994), Marianov and Revelle (1995), Brotcorne et al. (2003) for general overviews of the literature on ambulance location.

2. Related Literature

We are not the first to realize the limitations of MCLP that were illustrated in Section 1. Other researchers have developed models that take into account the two sources of uncertainty. Daskin (1983) proposed the maximal expected covering location problem (MEXCLP—an extension of MCLP) to account for the probability that an EMS unit is busy. Daskin assumed the probability p that a unit is busy is the same for every unit. Assuming that the probabilities of individual units being busy are independent, if demand node i with call rate h_i is covered by m units, the expected coverage for node i is $h_i \times (1 - p^m)$. MEXCLP maximizes expected coverage over all demand nodes and finds the optimal location for a given number of units. Unlike MCLP, MEXCLP can locate multiple units at the same station, limited by the capacity of the station. Given that ambulances are typically busy at least 30% of the time, MEXCLP is considerably more realistic than MCLP. Saydam and McKnew (1985) studied the same problem and offered a separable programming formulation that they found could solve larger instances to optimality than Daskin's formulation.

Daskin (1987) also worked on the second shortcoming of MCLP—deterministic response times. In essence, MCLP is a "black-and-white" representation of reality, where all demand points within some threshold distance are considered covered and all other points are not covered. The extension described in Daskin (1987) incorporates probabilistic coverage by explicitly modeling response time uncertainty. Problem data includes the probability of responding from a station to a demand point within a given threshold time.

While the two imperfections of MCLP were treated relatively early, their combined treatment took longer to arrive. Goldberg and Paz (1991) were the first, to our knowledge, to formulate a mathematical program that addressed both sources of uncertainty. They allowed ambulance busy probabilities to vary between stations and used pairwise exchange heuristics to optimize expected coverage, as evaluated by the approximate hypercube model. Ingolfsson et al. (2006) made the same assumptions but used a different solution heuristic—one that iterates between solving a nonlinear integer program and the approximate hypercube model. Table 2 summarizes the maximal covering models that we have discussed.

The MCLP (top left quadrant) is a linear integer program and is the simplest to solve. Moving to the right or down from the top left quadrant incorporates more reality into the model at the expense of solving a more complex optimization model.

There is another stream of related research on optimization models that attempts to maximize demand that is "covered with α-reliability," for example, Revelle and Hogan (1989). Borras and Pastor (2002) compare four such *maximum availability models*. Their work is similar to ours in that they use the approximate hypercube model to evaluate solutions to idealized optimization models. In a follow-on paper, Borras and Pastor (2003) present a maximum availability model that is solved by iterating between solving an optimization problem and evaluation with the approximate hypercube model—an approach that is similar to the one we use to solve maximum expected coverage models. As discussed in Erkut et al. (2006), the objective function of maximum availability models does not correspond directly to the performance measures used in

EMS systems. It is not clear how to choose the reliability level α in a way that is consistent with common EMS performance targets. Most models in this category consider response times to be deterministic. Marianov and Revelle (1996) is an exception, but their model contains an additional parameter β and it is not obvious how to choose a value for this parameter. For all of these reasons, maximum availability models are difficult to apply in practice. Therefore, we have not included them in our comparison.

Table 2: Classification of maximal covering location models with respect to inclusion/exclusion of vehicle availability and response time uncertainty. (MCLP = Maximum Coverage Location Problem, MEXCLP = Maximum Expected Coverage Location Problem, PR = Probabilistic Response Times, SSBP = Station-Specific Busy Probabilities.)

	Units always available	Uncertain unit availability
Deterministic response times	MCLP (Church and Revelle, 1974)	MEXCLP (Daskin, 1983; Saydam and McKnew, 1985)
Probabilitistic response times	MCLP + PR (Daskin, 1987)	Constant busy probability: MEXCLP + PR (Ingolfsson et al., 2003)
		Station-specific busy probabilities: MEXCLP + PR + SSBP (Goldberg and Paz, 1991; Ingolfsson et al., 2006)

To compare the quality of solutions generated by the different covering models, we will use the approximate hypercube model first introduced by Larson (1975) and later extended by Jarvis (1985). This descriptive model uses more realistic assumptions about the behavior of the system than any of the covering models. In particular, demand from different demand nodes is assumed to follow independent Poisson processes, each call is responded to by the closest available ambulance (a *fixed dispatch policy*), and the average time that an ambulance is busy responding to a call (the "service time") depends on both the call location and the station location. Although the service time distribution is assumed to be negative exponential, as Jarvis (1985) argues, the model is relatively insensitive to the shape of the service time distribution beyond its mean. We use a version of the approximate hypercube that allows multiple vehicles per station (Budge et al., 2005).

In the remainder of the paper, we analyze the five different types of covering models listed in Table 2; in particular, we explore whether or not the incremental improvement in solution quality justifies the added model complexity. We compare the performance of the five model types using data from the EMS system of Edmonton, Canada.

We present the formulations for the five model types in Section 3. Section 4 describes how the busy and coverage probabilities for the EMS units can be computed. Experimental results are provided in Section 5, and concluding remarks are given in Section 6.

3. Models

3.1 The Maximal Covering Location Problem

Define m: the number of demand nodes,
n: the number of candidate locations,
q: the maximum number of stations,
d_i: the average service demand per time unit generated by node i,
t_c: the coverage time standard,
t_{ji}: the travel time from candidate location j to demand node i,
t_d: the pre-travel delay,

$$x_j = \begin{cases} 1, & \text{if candidate location } j \text{ is selected} \\ 0, & \text{otherwise} \end{cases},$$

$$y_i = \begin{cases} 1, & \text{if demand node } i \text{ is covered} \\ 0, & \text{otherwise} \end{cases}, \text{ and}$$

$$a_{ij} = \begin{cases} 1, & \text{if if demand node } i \text{ is covered by candidate location } j, \text{i.e.} \, t_{ji} + t_d \le t_c \\ 0, & \text{otherwise} \end{cases}.$$

The formulation for the MCLP follows:

MCLP:

$$\max \quad \sum_{i=1}^{m} d_i y_i \tag{1}$$

$$\text{s.t.} \ \sum_{j=1}^{n} a_{ij} x_j \ge y_i \text{ , } i = 1, \ldots, m \tag{2}$$

$$\sum_{j=1}^{n} x_j \le q \tag{3}$$

$$x_j \in \{0,1\}, j = 1, \ldots, n \tag{4}$$

$$y_i \in \{0,1\}, i = 1, \ldots, m \tag{5}$$

The objective function (1) maximizes total demand covered. Constraints (2) state that demand node i can only be covered if at least one candidate location that covers i is selected. Constraint (3) limits the number of facilities to q. In this model, each station houses at most one ambulance, as mentioned in Section 1. Indeed, when there is no uncertainty regarding ambulance availability or response times, no benefit can be derived from collocating ambulances.

3.2 The Maximum Expected Covering Location Problem

Daskin (1983) formulated the maximum expected covering location problem (MEXCLP) as an integer program, as follows.

Let q denote the maximum number of EMS units,
p denote the average fraction of time an EMS unit is busy,
c_j be the maximum number of EMS units that can be stationed at candidate location j,
z_j be the number of EMS units allocated to station j, and

$$\hat{y}_{ik} = \begin{cases} 1, & \text{if demand node } i \text{ is covered by at least } k \text{ units} \\ 0, & \text{otherwise} \end{cases}.$$

MEXCLP:

$$\max \quad \sum_{i=1}^{m} d_i \sum_{k=1}^{q} (1-p) p^{k-1} \hat{y}_{ik} \tag{6}$$

$$\text{s.t.} \ \sum_{k=1}^{q} \hat{y}_{ik} \le \sum_{j=1}^{n} a_{ij} z_j \ , \ i = 1,\ldots, m \tag{7}$$

$$\sum_{j=1}^{n} z_j \le q \tag{8}$$

$$\hat{y}_{ik} \in \{0,1\}, \ i = 1,\ldots,m, \ k = 1,\ldots, q \tag{9}$$

$$z_j \in \{0,1,\ldots,c_j\}, \ j = 1,\ldots, n \tag{10}$$

The inner summation of the objective function (6) calculates the probability that there will be an EMS unit available to service demand node *i*. Therefore, the objective function maximizes the expected coverage of demand nodes. Constraints (7) state that the actual total number of EMS units covering node *i* (LHS of constraint) cannot exceed the total number of EMS units that can cover node *i* (RHS). Constraint (8) enforces at most *q* EMS units to be allocated to all open candidate locations.

3.3 The Maximal Covering Location Problem with Probabilistic Response

Let $y_{ij} = \begin{cases} 1, & \text{if demand node } i \text{ is covered by candidate location } j \\ 0, & \text{otherwise} \end{cases}$, and
P_{ij} = the probability that station j covers demand node i.

Daskin (1987) provides the following formulation for the Maximal Covering Location Problem with Probabilistic Response Times:

MCLP+PR:

$$\max \quad \sum_{i=1}^{m} d_i \sum_{j=1}^{n} P_{ij} y_{ij} \tag{11}$$

$$\text{s.t.} \ \sum_{i=1}^{m} y_{ij} \leq m x_j \, , j = 1, \ldots, n \tag{12}$$

$$\sum_{j=1}^{n} y_{ij} = 1, \, i = 1, \ldots, m \tag{13}$$

$$\sum_{j=1}^{n} x_j \leq q \tag{14}$$

$$x_j \in \{0,1\}, j = 1, \ldots, n \tag{15}$$

$$y_{ij} \in \{0,1\}, \, i = 1, \ldots, m, \; j = 1, \ldots, n \tag{16}$$

Objective function (11) maximizes the total expected demand covered, taking into account the coverage probabilities. Constraints (12) and (13) ensure that a demand node is assigned to only one open EMS facility. Constraint (14) requires that at most q candidate locations be chosen. As in MCLP, each candidate location houses at most one vehicle.

3.4 The Maximum Expected Covering Location Problem with Probabilistic Response Time

Let $i(j)$ denote the j^{th} preferred station for demand node i and
z_j = the number of EMS units allocated to station j.

Ingolfsson et al. (2003) formulated the Maximal Expected Covering Location Problem with Probabilistic Response Times as follows:

MEXCLP+PR:

$$\max \quad \sum_{i=1}^{m} d_i \sum_{j=1}^{n} P_{i,i(j)} \, p^{\sum_{u=1}^{j-1} z_{i(u)}} \left(1 - p^{z_{i(j)}}\right) \tag{17}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} z_j \leq q \tag{18}$$

$$z_j \in \{0,1,\ldots,c_j\}, j = 1,\ldots,n, \tag{19}$$

All variables and parameters in the model are as previously described. Objective function (17) maximizes the total expected demand covered accounting for the coverage probabilities P_{ij} and the busy fraction p. Constraint (18) ensures that at most q EMS units are assigned to open candidate locations, with at most c_j units in location j.

While the model above is quite accurate, it is still hindered by the fact that the busy probability of ambulances located in different stations may differ considerably due to demand densities in the vicinity of the stations. Ingolfsson et al. (2006) overcomes this shortcoming. Let p_j denote the average fraction of time an EMS unit at station j is busy. The formulation for the Maximal Expected Covering Location Problem with Probabilistic Response Time and Station Specific Busy Probabilities is as follows:

MEXCLP+PR+SSBP:

$$\max \quad \sum_{i=1}^{m} d_i \sum_{j=1}^{n} P_{i,i(j)} \prod_{u=1}^{j-1} p_{i(u)}^{z_{i(u)}} \left(1 - p_{i(j)}^{z_{i(j)}}\right) \tag{20}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} z_j \le q \tag{21}$$

$$z_j \in \{0, 1, \ldots, c_j\}, \; j = 1, \ldots, n \tag{22}$$

Note that this model is the same as the model for MEXCLP+PR except for the busy probability values.

3.5 Model size

Table 3 provides the number of constraints and variables for each of the five models. Although adding more complexity to the basic MCLP model makes it more realistic, the resulting models are either non-linear or contain more constraints and variables.

Table 3: Model dimensions.

Model	Objective	Constraints	Variables
MCLP	Linear	m + 1 linear	m + n binary
MEXCLP	Linear	m + 1 linear	nm binary, n bounded integer
MCLP+PR	Linear	n + m + 1 linear	$n(m + 1)$ binary
MEXCLP+PR	Nonlinear	1 linear	n bounded integer
MEXCLP+PR+SSBP	Nonlinear	1 linear	n bounded integer

4. Determining Model Parameters

The optimization models presented in the previous section that involve uncertainty require input values for the coverage probability parameter P_{ij}, the system-wide busy fraction parameter p, and the station specific busy fraction parameter p_j. In this section, we discuss how to compute these values. Note that the coverage probabilities P_{ij} are true inputs, whereas the busy fractions are outputs that depend on the allocation of ambulances to stations. We overcome this difficulty by iterating between solving an optimization model and estimating busy fractions.

4.1 Coverage Probabilities

Both MCLP+PR and MEXCLP+PR require the coverage probabilities P_{ij} as input. Although there is no explicit station preference in MCLP+PR, we assume that the station preference for demand node *i* is based on the travel time between the demand node and the stations, with the most preferred station being the closest one.

We denote the response time for a station-node pair as R_{ij}, with mean μ_{ij}, standard deviation σ_{ij}, and coefficient of variation $c_{ij} = \sigma_{ij} / \mu_{ij}$. The mean and standard deviation of the response time depend on the distribution of both the travel time and the pre-travel delay. We assume that the response times are lognormally distributed. See Ingolfsson et al. (2006) for further discussion and empirical evidence that support the lognormality assumption. Other non-negative distributions, such as a log-logistic distribution or a gamma distribution could be used instead. The main consideration in choosing a distribution is to accurately model the tail probability $\Pr\{R_{ij} > t_c\}$.

If R_{ij} is lognormally distributed, then $\ln(R_{ij})$ is normally distributed, with the following mean and variance:

$$\mathrm{E}[\ln(R_{ij})] = \ln(\mu_{ij}) - 0.5\ln\left(1 + c_{ij}^2\right), \text{ and} \tag{23}$$

$$\mathrm{var}[\ln(R_{ij})] = \ln\left(1 + c_{ij}^2\right), \tag{24}$$

Therefore, we have

$$P_{ij} \quad = \quad \Pr\{R_{ij} \le t_c\} = \Pr\left\{\ln(R_{ij}) \le \ln(t_c)\right\} = \Phi\left(\frac{\ln(t_c) - \ln(\mu_{ij}) + 0.5\ln(1 + c_{ij}^2)}{\sqrt{\ln(1 + c_{ij}^2)}}\right) \tag{25}$$

where Φ is the cumulative standard Normal distribution function and t_c is the coverage time threshold.

4.2 Calculating the System-Wide Busy Fraction Parameter, p

For the models MEXCLP and MEXCLP+PR, we require the parameter *p*, which is the average fraction of time that an EMS unit is busy. The average busy fraction *p* can be estimated as

$$p = \lambda\,(1 - \mathrm{B}(\lambda\tau(z), q))\,\tau(z) / q \tag{26}$$

where $\lambda = \sum_{i=1}^{m} d_i$ is the total arrival rate of calls to the system, $\tau(z)$ is the average time that a vehicle is tied up with a call as a function of the vehicle allocation vector *z*, *q* is the total number of vehicles, and $B(r,s) = r^s / s! \Big/ \sum_{i=0}^{s} r^i / i!$ is the Erlang loss function, which measures the fraction of lost calls in an M/G/*s*/*s* queueing system. The total arrival rate is fixed, and the total number of vehicles is fixed as well since the constraint $\sum_{j=1}^{m} z_j \le q$ will be tight. However, the "average service time" τ will depend on how the q vehicles are distributed across the city because this will influence the average response time to a call. The average service time $\tau(z)$ consists of the average response time, the average time spent at the call location, and the average time to travel to and remain at a hospital:

$$\tau(z) = \mathrm{E}[T_{\text{response}}] + \mathrm{E}[T_{\text{on scene}}] + \mathrm{E}[T_{\text{hospital}}] \tag{27}$$

This assumes that the ambulance is available to take calls when it is on its way from a hospital back to a station. Only the first component ($\mathrm{E}[T_{\text{response}}]$) is assumed to depend on how vehicles are allocated to stations. This component can be calculated as

$$\mathrm{E}[T_{\text{response}}] = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{d_i}{\lambda} f_{i(j)}(z)\mathrm{E}[R_{i,i(j)}] \tag{28}$$

where $f_{i(j)}(z)$ is the probability that the j^{th} preferred station is the one that responds to a call from demand node i. To calculate $f_{i(j)}(z)$, let $z_{i(j)}$ be the number of vehicles at the j^{th} preferred station for demand node i, and we thus have

$$f_{i(j)}(z) = p^{\sum_{u=1}^{j-1} z_{i(u)}}(1 - p^{z_{i(j)}})\,. \tag{29}$$

The algorithm for iterating on the busy fraction p is as follows:

Step 0: Initialize p to p_{in}, where p_{in} can be determined by assuming that all calls are responded to by the most preferred station, i.e., setting $f_{i(1)}(z) = 1$ for all i (and $f_{i(j)}(z) = 0$ for all $j \ge 2$) and then using (26), (27), and (28). Set $cnt = 1$ and choose a smoothing parameter $\gamma \in (0,1)$.

Step 1: Solve the optimization model. Denote the vector of z_i variables in the solution by z^*_{cnt}. If a convergence criterion is satisfied, stop.

Step 2: Estimate p_{out} using the solution z^*_{cnt} and equations (26) to (29). Set $p_{\text{in}} = \gamma p_{\text{out}} + (1 - \gamma)p_{\text{in}}$ and $cnt = cnt + 1$, and return to Step 1.

There are two possible ways that the algorithm may converge: first, if both the solution and the busy fraction have converged, i.e., $z^*_{cnt-1} = z^*_{cnt}$ and $|p_{\text{in}} - p_{\text{out}}| < \varepsilon$ (we used $\varepsilon = 10^{-6}$). Second, if the busy fraction has converged (i.e., $|p_{\text{in}} - p_{\text{out}}| < \varepsilon$) and the solution has converged to a repeating cycle of solutions. In the experiments reported in the next section, the length of the cycle was at most two.

4.3 Calculating the Station Specific Busy Fraction Parameter, p_j

In model MEXCLP+PR+SSBP, we use the approximate hypercube model to estimate station specific busy fractions. Budge et al. (2005) describe the version of the approximate hypercube model that we use and Ingolfsson et al. (2006) discuss the iteration between solving the mathematical program and estimating the busy fractions.

We made one modification to the iteration procedure described in Ingolfsson et al. (2006): if a station was allocated no ambulances, then we set the busy probability for that station equal to the average busy fraction for the other stations. In Ingolfsson et al. (2006), the busy probability for such stations was set to 100%, which meant that the optimal solution to the next mathematical program to be solved would allocate no ambulances to such stations. We found that this modification led to higher quality solutions.

5. Computational Results

Our computational experiments were carried out on a data set provided by the EMS department of the City of Edmonton. The data set contains expected travel times from 16 ambulance stations to 180 demand points, and the fraction of demand generated at each of the demand points. We limit our analysis to the 16 current ambulance stations, and focus on the allocation of ambulances to those stations. We scaled total demand to keep the ratio between the "offered load" (total demand per time unit multiplied by the average time per call) and the number of EMS units equal to 0.3.

We solved the linear models using CPLEX 9.1 and the nonlinear models using the student version of GAMS 22.0. Solving the linear models took no more than a CPU second. The computation times and the number of iterations between solving an optimization problem and estimating busy probabilities for the nonlinear models are shown in Table 4. The computation times include the time to run the approximate hypercube model, which was less than one second in all cases. The total CPU time to solve an instance varied between 4 and 659 seconds and was highest for intermediate values of q. The number of iterations varied from 2 to 10 and was consistently higher for the MEXCLP + PR model. The CPU time per iteration, for a given q, was similar for the two nonlinear models.

For the system-wide busy fraction heuristic, we chose the smoothing parameter as $\gamma = 0.8$. Notably, different values of γ did not affect the final solution. Our choices for parameters were guided by analysis of real data. We assumed that the total average time spent at the call location, and the average time to travel to and remain at a hospital, $E[T_{\text{on scene}}] + E[T_{\text{hospital}}]$, is 2691 seconds (about 45 minutes). When calculating the coverage probability parameter P_{ij}, we assumed $c_{ij} = 0.3$. The initial system-wide busy probability p was set to 0.3. In MCLP, we set a_{ij} to 1 if the expected response time was less than the threshold time t_c.

Table 4: Total CPU time (including evaluation with the approximate hypercube model) in seconds and number of iterations for the MEXCLP+PR and MEXLPC+PR+SSBP models.

	Total CPU time in seconds		Number of iterations	
q	MEXCLP+PR	MEXCLP+PR+SSBP	MEXCLP+PR	MEXCLP+PR+SSBP
1	12	4	4	2
2	52	26	4	2
3	133	49	5	2
4	347	126	6	3
5	294	126	6	2
6	373	98	7	2
7	477	138	7	2
8	659	163	8	2
9	555	207	8	3
10	545	214	8	3
11	563	219	8	4
12	625	237	8	3
13	445	189	8	3
14	248	110	8	3
15	183	53	8	2
16	241	214	8	5
17	165	89	8	3
18	182	79	8	3
19	181	71	8	3
20	152	71	8	4
21	127	43	8	2
22	95	57	8	4
23	99	28	8	2
24	59	36	8	4
25	39	17	8	2

We evaluated the outcome of each model by computing the dispatch probabilities $f_{i(j)}(z)$ for the ambulance configuration at hand using the approximate hypercube model, and using them, together with the coverage probabilities P_{ij}, to compute the expected coverage. The results are summarized in Table 5 and Figure 2. In the last row of Table 5, we show the average percent deviation from the best solution value. Note that the maximum expected coverage that can be achieved for the example problem on hand is 94.8%. One would need to add new station locations to achieve greater expected coverage.

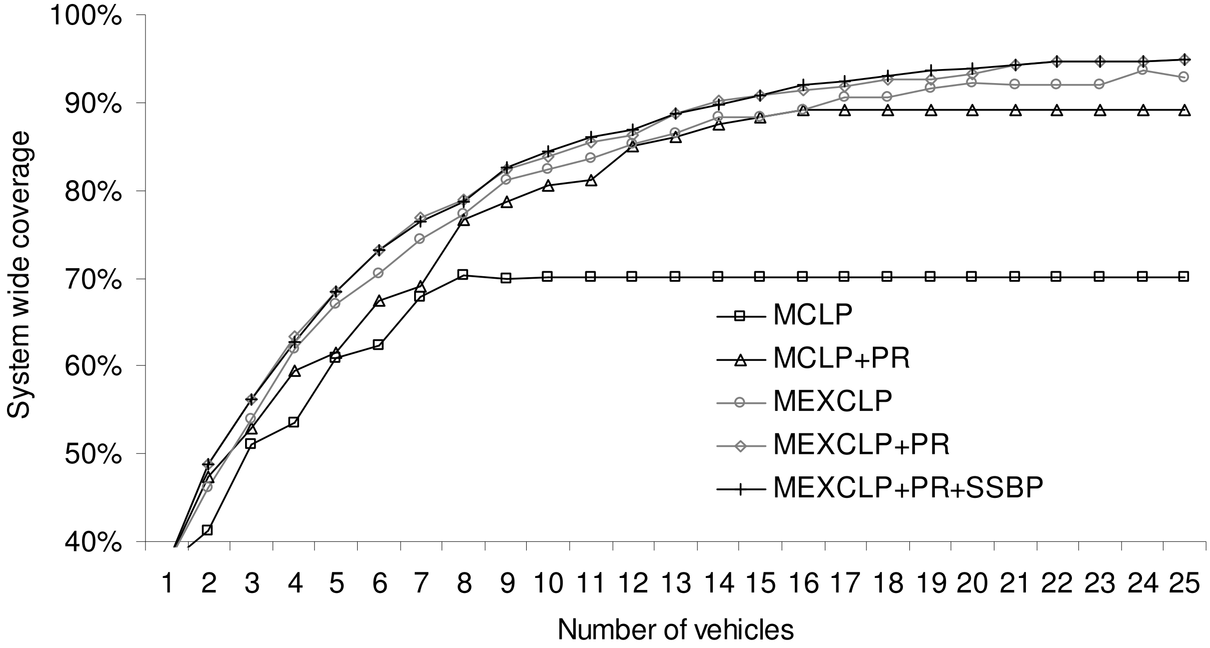


Figure 2: Expected coverage for the different models, evaluated using the approximate hypercube model.

The performance of MCLP stalls at q = 10, at which point the coverage reaches its peak and the model locates no more stations. Interestingly, the MCLP solution for q = 8 is slightly better than the one for $q \geq 10$. MCLP and MCLP+PR share the same limitation of being able to locate no more than a single ambulance at every location, and hence MCLP+PR shows no improvement after q = 16. In many EMS systems, the number of ambulances is larger than the number of stations, because of fixed and operating costs of stations. MCLP and MCLP+PR are inadequate for real world ambulance allocation in such EMS systems.

The worst performance is that of MCLP, with an average deviation of 19.1% from the best found solution. This is not surprising, considering that MCLP ignores both sources of uncertainty. MCLP+PR performs much better with an average deviation of 4.9%, and MEXCLP performs better still, with an average deviation of 2.5%. It seems that the incorporation of ambulance availability improves the solution quality more than the incorporation of response time uncertainty, if one were to incorporate only one source of uncertainty.

The models that account for both sources of uncertainty (MEXCLP+PR and MEXCLP+PR+SSBP) find the best solution for all values of q. These two models find the same solution in 11 of the 25 instances. MEXCLP+PR outperforms MEXCLP+PR+SSBP in 5 instances and the opposite is true in 9 instances. However, the instances in which MEXCLP+PR+SSBP yields a better solution are those with $q \geq 16$, which are more realistic. Considering that approximating the station specific busy probabilities takes a negligible amount of time, we conclude that MEXCLP+PR+SSBP outperforms the other four models.

Table 5: Expected coverage for the different models, evaluated using the approximate hypercube model. Coverage value for best known solution is shown in boldface font, for each value of q.

	Percent coverage of high priority demand				
				MEXCLP	MEXCLP+
q	MCLP	MCLP+PR	MEXCLP	+PR	PR+SSBP
1	**37.3%**	**37.3%**	**37.3%**	**37.3%**	**37.3%**
2	41.3%	47.3%	46.2%	**48.9%**	**48.9%**
3	51.0%	52.8%	53.9%	**56.2%**	**56.2%**
4	53.6%	59.4%	61.8%	**63.3%**	62.6%
5	60.8%	61.4%	66.9%	**68.4%**	**68.4%**
6	62.3%	67.4%	70.5%	**73.1%**	**73.1%**
7	67.8%	69.1%	74.4%	**76.8%**	76.4%
8	70.3%	76.6%	77.2%	**78.9%**	78.6%
9	69.9%	78.8%	81.2%	82.4%	**82.5%**
10	70.2%	80.6%	82.4%	83.9%	**84.5%**
11	70.2%	81.2%	83.5%	85.5%	**86.0%**
12	70.2%	85.0%	85.2%	86.3%	**87.0%**
13	70.2%	86.1%	86.5%	**88.8%**	**88.8%**
14	70.2%	87.6%	88.4%	**90.2%**	89.8%
15	70.2%	88.4%	88.4%	**90.8%**	90.8%
16	70.2%	89.2%	89.2%	91.4%	**91.9%**
17	70.2%	89.2%	90.5%	91.8%	**92.3%**
18	70.2%	89.2%	90.5%	92.5%	**93.1%**
19	70.2%	89.2%	91.6%	92.7%	**93.6%**
20	70.2%	89.2%	92.2%	93.2%	**93.9%**
21	70.2%	89.2%	92.1%	**94.4%**	**94.4%**
22	70.2%	89.2%	92.0%	**94.8%**	**94.8%**
23	70.2%	89.2%	92.0%	**94.7%**	**94.7%**
24	70.2%	89.2%	93.7%	**94.7%**	**94.7%**
25	70.2%	89.2%	92.7%	**94.8%**	**94.8%**
	Percent deviation from best known solution				
Maximum	26.0%	10.2%	5.6%	1.5%	1.0%
Average	19.1%	4.9%	2.5%	0.3%	0.1%

The approximate hypercube model allows us to estimate various other performance measures, including the fraction of calls that are "lost" (shown in Table 6) and the average response time to calls that are not lost (shown in Table 7). In practice, lost calls are typically responded to by a backup system, such as supervisor vehicles. The models we compare do not account explicitly for lost calls. However, all of them attempt to maximize the fraction of calls that are covered. Calls that are not covered either take longer than t_c time units to respond to or they are lost.

Therefore, one would expect that the models will tend to minimize the fraction of lost calls. Table 6 shows that, as expected, the loss probability decreases with *q*, for all models. The loss probability drops below 1% for all models for $q > 9$. MEXCLP+PR and MEXCLP+PR+SSBP consistently provide the lowest loss probability.

As Table 7 shows, the average response almost always decreases with *q*. Comparing the models using average response time as the yardstick, we see that MCLP typically results in the lowest average response time for $q \leq 10$ but is dominated by the other models thereafter. MCLP+PR performs best for $11 \leq q \leq 16$. MEXCLP, MEXCLP+PR, and MEXCLP+PR+SSBP are best for $q > 16$. We have not compared coverage models to models that attempt to minimize average response time.

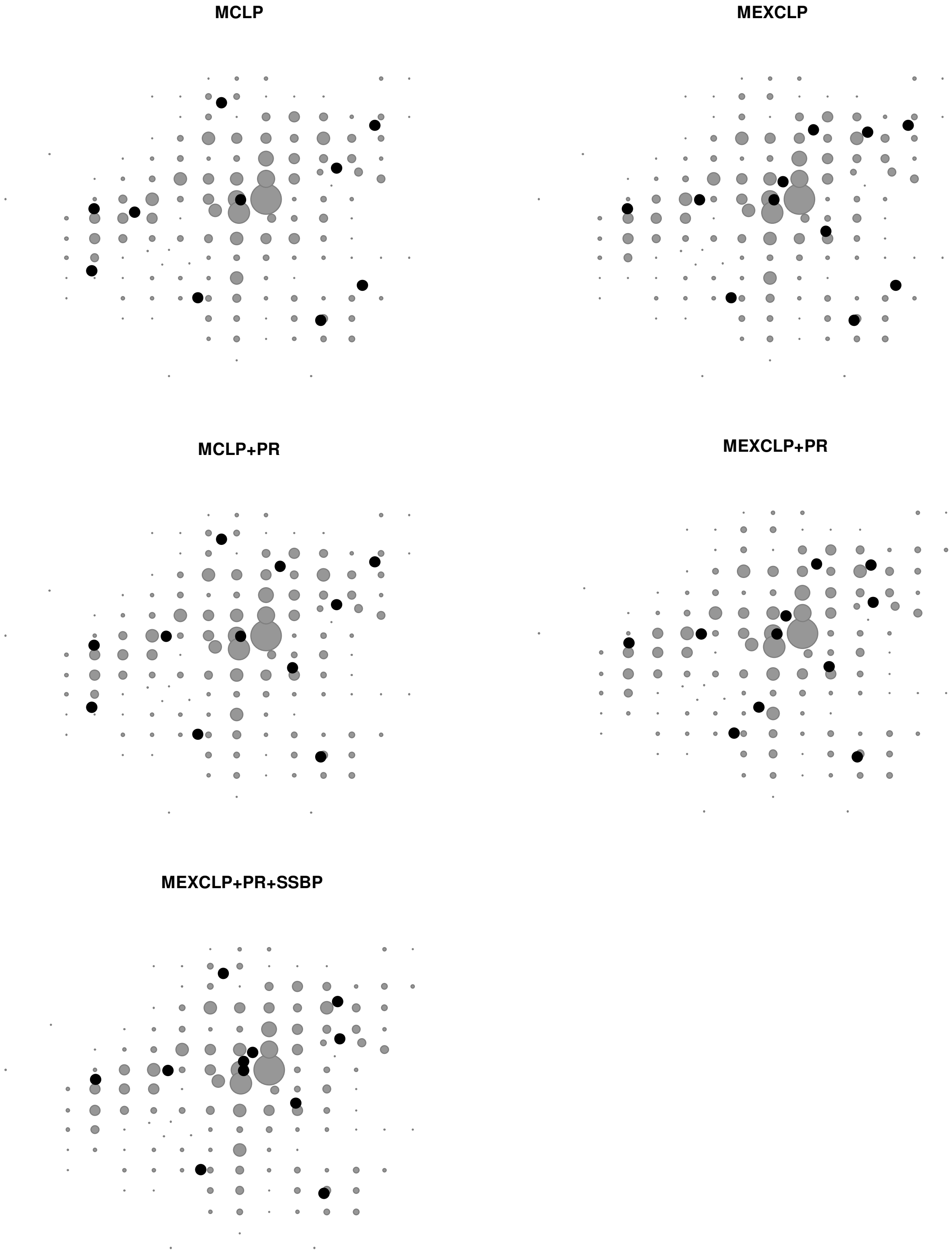
Table 6: Loss probabilities, evaluated using the approximate hypercube model. The lowest loss probability is shown in bold font, for each value of *q*.

	Loss probability				
				MEXCLP	MEXCLP +
q	MCLP	MCLP + PR	MEXCLP	+ PR	PR + SSBP
1	**0.28**	**0.28**	**0.28**	**0.28**	**0.28**
2	0.15	0.15	0.15	**0.14**	**0.14**
3	0.088	0.088	0.086	**0.084**	**0.084**
4	0.056	0.055	0.052	**0.051**	**0.051**
5	0.036	0.036	0.034	**0.033**	**0.033**
6	0.024	0.023	**0.021**	**0.021**	**0.021**
7	0.016	0.015	0.014	**0.013**	0.014
8	0.011	0.0094	0.0095	**0.0088**	0.0091
9	0.0075	0.0063	0.0058	0.0058	**0.0057**
10	0.0051	0.0042	0.0040	0.0038	**0.0037**
11	0.0051	0.0029	0.0027	**0.0025**	**0.0025**
12	0.0051	0.0018	0.0018	**0.0017**	**0.0017**
13	0.0051	0.0013	0.0012	**0.0011**	**0.0011**
14	0.0051	0.0008	0.0008	**0.0007**	**0.0007**
15	0.0051	0.0006	0.0006	**0.0005**	**0.0005**
16	0.0051	0.0004	0.0004	**0.0003**	**0.0003**
17	N/A	N/A	**0.0002**	**0.0002**	**0.0002**
18	N/A	N/A	0.0002	0.0002	**0.0001**
19	N/A	N/A	**0.0001**	**0.0001**	**0.0001**
20	N/A	N/A	**0.0001**	**0.0001**	**0.0001**
21	N/A	N/A	0.0001	**0.0000**	**0.0000**
22	N/A	N/A	**0.0000**	**0.0000**	**0.0000**
23	N/A	N/A	**0.0000**	**0.0000**	**0.0000**
24	N/A	N/A	**0.0000**	**0.0000**	**0.0000**
25	N/A	N/A	**0.0000**	**0.0000**	**0.0000**

Table 7: Average response time for the different models, evaluated using the approximate hypercube model. The lowest average response time is shown in boldface font, for each value of *q*.

	Average response time in minutes				
				MEXCLP	MEXCLP +
q	MCLP	MCLP + PR	MEXCLP	+ PR	PR + SSBP
1	**9.0**	**9.0**	**9.0**	**9.0**	**9.0**
2	10.4	10.4	**10.2**	10.5	10.5
3	**9.6**	9.9	9.8	10.2	10.2
4	9.5	**9.4**	9.5	9.5	**9.4**
5	**9.0**	**9.0**	**9.0**	9.1	9.1
6	**8.5**	**8.5**	8.8	8.7	8.7
7	**8.1**	8.2	8.3	8.5	8.4
8	**7.8**	7.9	**7.8**	8.1	8.0
9	**7.4**	7.6	7.6	7.6	7.6
10	**7.2**	7.3	7.3	7.4	7.3
11	7.2	**6.9**	7.1	7.2	7.2
12	7.2	**6.7**	6.8	6.8	7.0
13	7.2	**6.4**	6.7	6.7	6.7
14	7.2	**6.2**	6.4	6.5	6.5
15	7.2	**6.1**	**6.1**	6.3	6.3
16	7.2	**6.0**	**6.0**	6.2	6.3
17	7.2	6.0	**5.9**	**5.9**	6.0
18	7.2	6.0	5.9	**5.8**	5.9
19	7.2	6.0	**5.8**	**5.8**	**5.8**
20	7.2	6.0	**5.8**	**5.8**	**5.8**
21	7.2	6.0	5.8	**5.7**	**5.7**
22	7.2	6.0	**5.7**	**5.7**	**5.7**
23	7.2	6.0	**5.7**	**5.7**	**5.7**
24	7.2	6.0	**5.7**	**5.7**	**5.7**
25	7.2	6.0	**5.7**	**5.7**	**5.7**

We analyzed the nature of the optimal solutions for various values of *q*, to obtain greater insight into how the more sophisticated models achieve their superior performance. Figures 3 and 4 illustrate the ambulance allocation prescribed by the five models for $q = 11$ and 19, respectively. In all cases, the magnitude of demand is shown using grey circles with areas proportional to demand and the locations of ambulances are shown using black circles of constant size.



MCLP

MEXCLP

MCLP+PR

MEXCLP+PR

MEXCLP+PR+SSBP

Figure 3: Ambulance allocation for q = 11.

With 11 ambulances to be allocated to 16 stations (Figure 3), MCLP uses only 10 stations, because this achieves the maximum possible coverage, under the assumptions of that model. The MCLP+PR is qualitatively similar in that it spreads the ambulances throughout the city relatively evenly. The difference is that this model uses all 11 available ambulances. The MEXCLP and MEXCLP+PR solutions locate two ambulances in close proximity to each other near the city center, where most of the demand is concentrated. MEXCLP+PR+SSBP concentrates the ambulances even more strongly near the city centre, with two ambulances at one central station and one at the other.

With 19 available ambulances (Figure 4), the MCLP solution remains the same—using only 10 ambulances. MCLP+PR uses all 16 stations, but realizes no benefit from "doubling up" at any of the stations, because it ignores ambulance availability. MEXCLP doubles up at three stations, but none of those stations are in the city center. MEXCLP+PR also doubles up at three stations, one of them in the city centre. MEXCLP+PR+SSBP allocates three ambulances to the most central station and two ambulances to another station close to the city center. Discussions with EMS practitioners in Edmonton suggest that they would always double up at the most centrally located stations before doubling up at outlying stations. Thus, the solutions to MEXCLP+PR and MEXCLP+PR+SSBP are likely to be more acceptable to them than the solution to MEXCLP.

Next, we designated three of the 16 stations (shown with black circles in Figure 5) as central and the other thirteen as outlying (grey circles), and we computed the fraction of the ambulance fleet that was allocated to central stations for each model. The results are shown in Figure 6. MCLP settles down at around 10%, MCLP+PR and MEXCLP at around 20%, and MEXCLP+PR and MEXLCP+PR+SSBP at around 30%. The last figure corresponds roughly to the fraction of the total demand that occurs near the city center. We see from this that the improved expected coverage of the MEXCLP+PR and MEXCLP+PR+SSBP models is achieved through concentration of resources in areas where demand is highest so as to serve this demand with high probability.

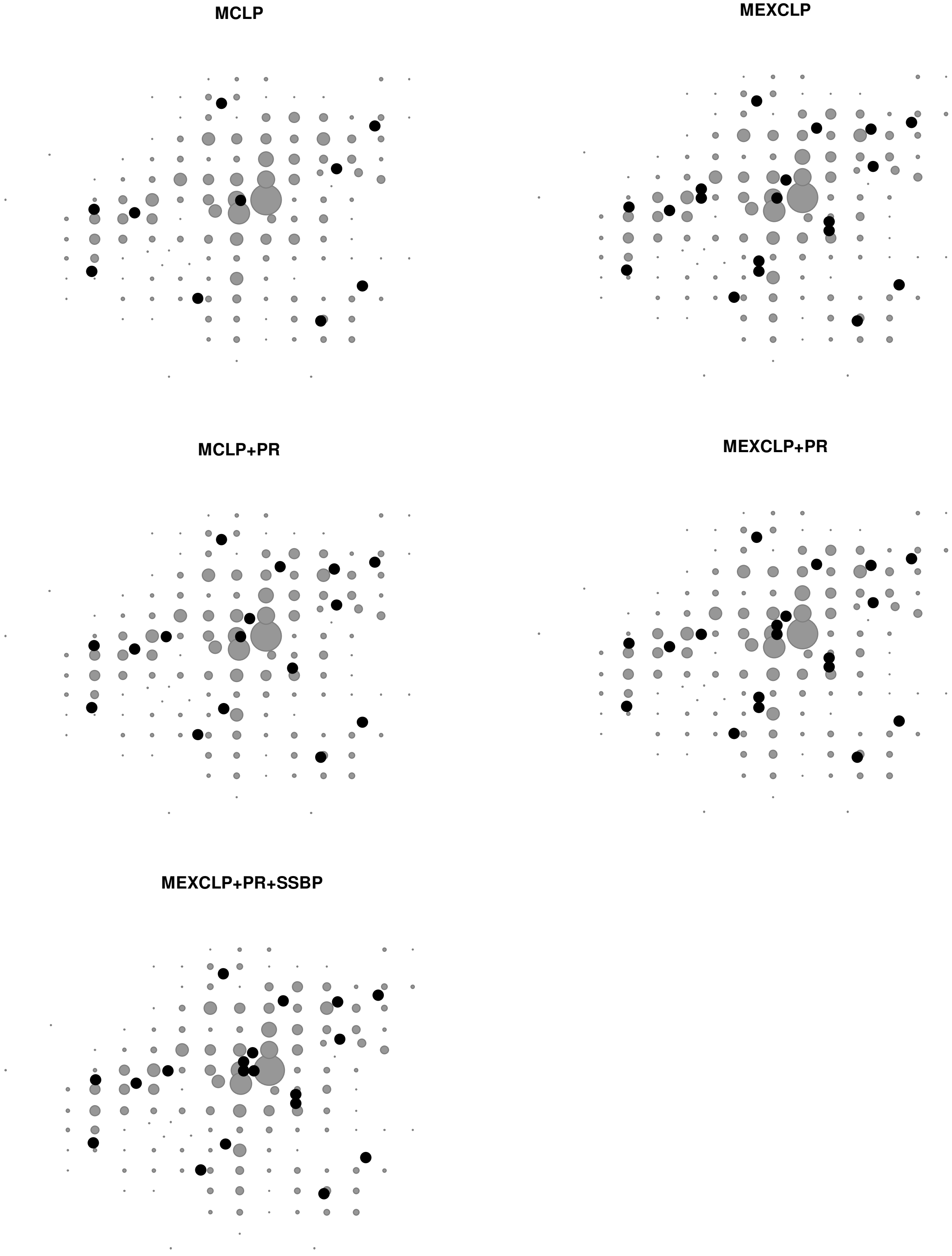


Figure 4: Ambulance allocation for *q* = 19.

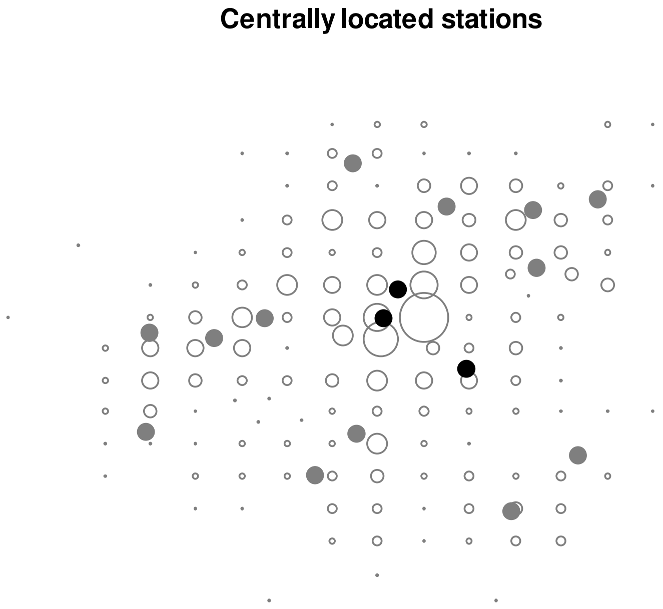


Figure 5: Central stations (black circles) and outlying stations (grey circles). The areas of the white circles are proportional to demand.

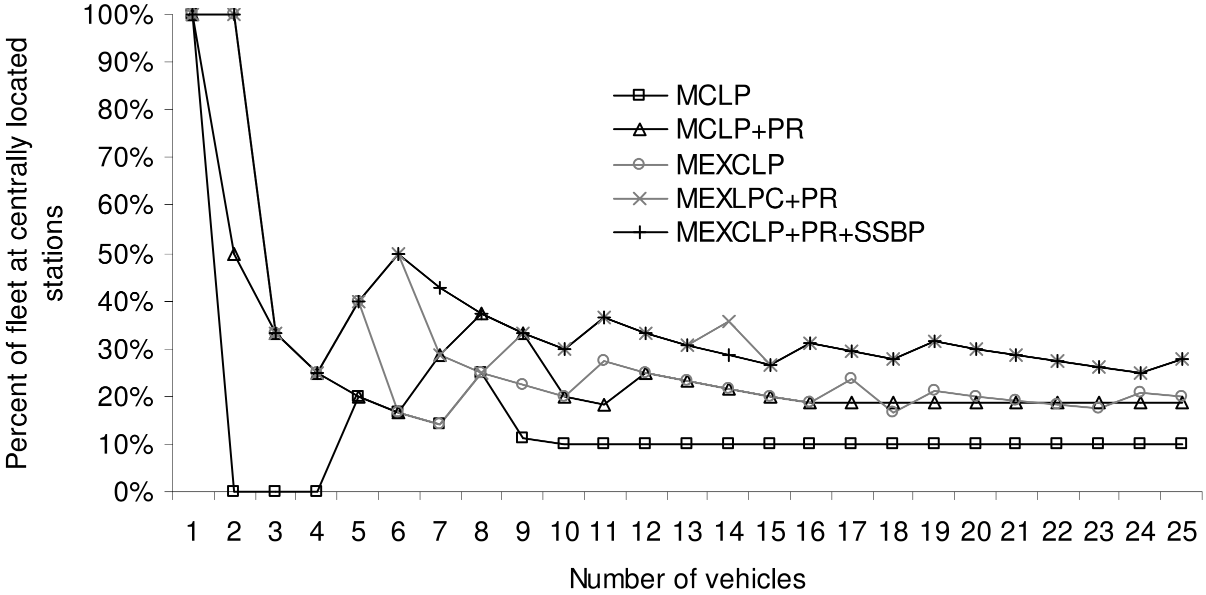


Figure 6: Fraction of ambulances allocated to central stations.

6. Concluding Remarks

In this study, we categorize and compare existing maximum coverage ambulance location models. Our categorization is based on the factors of uncertainty incorporated into the model. The comparison uses real world data from Edmonton, Canada. We observe that the models that incorporate uncertain factors yield considerably better performance. Given that the more realistic models are sufficiently tractable to be solved in at most a few minutes for the realistic problem instances that we used, the added model complexity appears to be warranted.

None of the models we compare is guaranteed to globally optimize expected coverage, as evaluated with the approximate hypercube model. It might be possible to achieve greater expected coverage by use of a heuristic approach, such as the Goldberg and Paz (1991) pairwise exchange heuristic. The performance of such heuristics might be improved by using a starting solution generated by, say, the MEXCLP+PR+SSPB model.

The Edmonton data we used aggregated demand to 180 points, based on a division of the city into "quadrants" that Edmonton EMS uses to collect data. A finer aggregation with more demand points would increase the effort required to solve the optimization models. However, we expect that all of the models would remain easily solvable for the Edmonton data even if the number of demand aggregation points were to increase considerably. However, as the number of station locations increases, the models become considerably more difficult to solve, particularly MEXCLP+PR and MEXCLP+PR+SSBP using standard nonlinear programming solvers. Specialized methods for solving MEXCLP+PR and MEXCLP+PR+SSBP may therefore be required. See Francis et al (2006) for a recent survey on aggregation errors in location models and Current and Schilling (1990) and Daskin et al (1989) for analysis of aggregation errors in deterministic maximal covering models.

One issue that we believe could be illuminated by additional research is how aggregation interacts with level of model detail. Specifically, it might be that the models which classify each demand point as either covered or not, are more susceptible to aggregation errors than the more sophisticated models which assign a coverage value that can be anywhere between 0 and 1 to each demand point. Thus, one could improve the analysis either through finer aggregation or through incorporating additional model features. More research is needed to understand the pros and cons of these two approaches.

System-wide coverage is a widely used performance measure in EMS systems. However, one could argue that the real performance measure ought to be the number of lives saved by the system. In a related paper (Erdogan et al., 2006), we demonstrate that models that incorporate uncertainty about ambulance availability and response times not only result in better coverage estimates, but also cause coverage to be a better proxy for lives saved.

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