Meson-Baryon Coupling Constants in Two-Flavor Lattice QCD

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Abstract. We evaluate the pseudoscalar-meson coupling constants and the strangeness-conserving and the strangeness-changing axial charges of octet baryons in lattice QCD with two flavors of dynamical quarks. We find that the coupling constants and the axial charges have rather weak quark-mass dependence and the breaking in SU(3)-flavor symmetry is small at each quark-mass point we consider.

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INTRODUCTION

Meson-baryon coupling constants and baryon axial charges are significant parameters for low-energy effective description of baryon sector. While the coupling constants provide a measure of baryon-baryon interactions in terms of One Boson Exchange (OBE) models, and production of mesons off the baryons, the axial charges enter in the loop graphs of chiral perturbation theory. The nucleon axial charge can be precisely determined from nuclear β-decay (the modern value is $g_{A,NN} = 1.2694(28)$ [1]), however, we do not have enough information about hyperon axial charges from experiment.

In the SU(3)-flavor [SU(3)$_F$] symmetric limit, one can classify the meson coupling constants and the axial charges of baryons in terms of the constants of two types of couplings, F and D [2]. This systematic classification, which phenomenologically works rather well but is not known *a priori* to hold, is expected to govern all the couplings. However as we move from the symmetric case to the realistic one, the SU(3)$_F$ breaking occurs as a result of the s-quark mass. The broken symmetry no longer provides a pattern for the couplings, and therefore they should be individually calculated based on the underlying theory, QCD.

In this framework, we have evaluated the pseudoscalar-meson coupling constants and the strangeness-conserving and the strangeness-changing axial charges of octet baryons in lattice QCD with two flavors of dynamical quarks. The evaluation of the coupling constants and the axial charges allows us to check whether the SU(3)$_F$ relations are well respected in the degenerate quark-mass case and to what extent this symmetry is broken as we restore the physical masses of quarks. In our calculations, we assume exact flavor-SU(2) symmetry and take u and the d quarks degenerate.

THE FORMULATION AND THE LATTICE SIMULATIONS

We refer the reader to Ref. [3, 4] for the lattice formulation and the details of the calculations of pseudoscalar-meson–octet-baryon coupling constants. The pseudoscalar current matrix element of the baryon states is written as

$$\langle \mathscr{B}(\mathbf{p})|P(0)|\mathscr{B}'(\mathbf{p}')\rangle = g_P(q^2)\bar{u}(\mathbf{p})i\gamma_5 u(\mathbf{p}'), \tag{1}$$

where $g_P(q^2)$ is the pseudoscalar form factor, $q_\mu = p'_\mu - p_\mu$ is the transferred four-momentum and $P(x) = \bar{\psi}(x)i\gamma_5\frac{\tau_3}{2}\psi(x)$ is the pseudoscalar current. As for the axial charges, we consider the baryon matrix elements of the isovector axial-vector current $A_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$, which can be written in the form

$$\langle \mathscr{B}(p)|A_\mu|\mathscr{B}'(p')\rangle = C_{\mathscr{B}\mathscr{B}'}\bar{u}(p)\left[\gamma_\mu\gamma_5 G_{A,\mathscr{B}\mathscr{B}'}(q^2) + \gamma_5\frac{q_\mu}{m_{\mathscr{B}}+m_{\mathscr{B}'}}G_{P,\mathscr{B}\mathscr{B}'}(q^2)\right]u(p). \tag{2}$$

Here $G_{A,\mathscr{B}\mathscr{B}'}(p^2)$ and $G_{P,\mathscr{B}\mathscr{B}'}(p^2)$ are the baryon axial and induced pseudoscalar form factors, respectively. The baryon axial charges are defined as the axial form factors at zero-momentum transfer, *viz.* $g_{A,\mathscr{B}\mathscr{B}'} = G_{A,\mathscr{B}\mathscr{B}'}(0)$. We compute the matrix element in Eq. (2) using the ratio method. For example, we construct the following ratio for the axial charges:

$$\begin{aligned} R(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) = \;& \frac{\langle F^{\mathscr{B}\mathscr{A}_\mu\mathscr{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle}{\langle F^{\mathscr{B}'}(t_2;\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle F^{\mathscr{B}}(t_2-t_1;\mathbf{p};\Gamma_4)\rangle}{\langle F^{\mathscr{B}'}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle}\right. \\ & \left.\times\frac{\langle F^{\mathscr{B}'}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle F^{\mathscr{B}'}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle F^{\mathscr{B}}(t_1;\mathbf{p};\Gamma_4)\rangle\langle F^{\mathscr{B}}(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}, \end{aligned} \tag{3}$$

where the baryonic two- and three-point correlation functions are respectively defined as

$$\langle F^{\mathscr{B}}(t;\mathbf{p};\Gamma_4)\rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\Gamma_4^{\alpha\alpha'}\langle vac|T[\eta^\alpha_{\mathscr{B}}(x)\bar{\eta}^{\alpha'}_{\mathscr{B}'}(0)]|vac\rangle, \tag{4}$$

$$\begin{aligned} \langle F^{\mathscr{B}\mathscr{A}_\mu\mathscr{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle = -i\textstyle\sum_{\mathbf{x_2},\mathbf{x_1}} e^{-i\mathbf{p}\cdot\mathbf{x_2}}e^{i\mathbf{q}\cdot\mathbf{x_1}} \\ \times\Gamma^{\alpha\alpha'}\langle vac|T[\eta^\alpha_{\mathscr{B}}(x_2)A_\mu(x_1)\bar{\eta}^{\alpha'}_{\mathscr{B}'}(0)]|vac\rangle, \end{aligned} \tag{5}$$

with $\Gamma \equiv \gamma_3\gamma_5\Gamma_4$ and $\Gamma_4 \equiv (1+\gamma_4)/2$. The baryon interpolating fields are given as

$$\eta_N(x) = \varepsilon^{abc}[u^{Ta}(x)C\gamma_5 d^b(x)]u^c(x), \tag{6}$$

$$\eta_\Xi(x) = \varepsilon^{abc}[s^{Ta}(x)C\gamma_5 d^b(x)]s^c(x), \tag{7}$$

$$\eta_\Sigma(x) = \varepsilon^{abc}[s^{Ta}(x)C\gamma_5 u^b(x)]u^c(x), \tag{8}$$

$$\begin{aligned} \eta_\Lambda(x) = \frac{1}{\sqrt{6}}\varepsilon^{abc}\{[u^{Ta}(x)C\gamma_5 s^b(x)]d^c(x) - [d^{Ta}(x)C \\ \times\gamma_5 s^b(x)]u^c(x) + 2[u^{Ta}(x)C\gamma_5 d^b(x)]s^c(x)\}, \end{aligned} \tag{9}$$

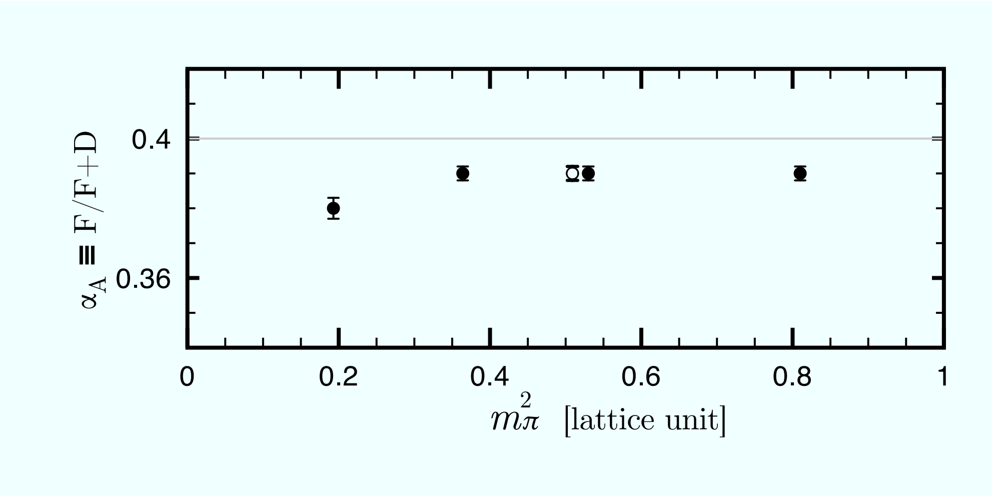

FIGURE 1. $\alpha_A = F/F+D$ ratio as a function of m_π^2. The empty circle denotes the SU(3)$_F$ limit.

where $C = \gamma_4\gamma_2$ and a, b, c are the color indices. t_1 is the time when the meson interacts with a quark and t_2 is the time when the final baryon state is annihilated. The ratio in Eq. (3) reduces to the desired form when $t_2 - t_1$ and $t_1 \gg a$, *viz.*

$$R(t_2,t_1;\mathbf{0},\mathbf{p};\Gamma;\mu) \xrightarrow[t_2-t_1\gg a]{t_1\gg a} \sqrt{\frac{E+m}{2m}}\, G_{A,\mathscr{B}\mathscr{B}'}(Q^2), \tag{10}$$

where m and E are the mass and the energy of the initial baryon and $Q^2 = -q^2$. We apply a procedure of seeking plateau regions as a function of t_1 in the ratio (10) and calculating the axial form factors $G_{A,\mathscr{B}\mathscr{B}'}(Q^2)$ at $Q^2 = 0$ in order to extract the axial charges $g_{A,\mathscr{B}\mathscr{B}'}$.

We employ a $16^3 \times 32$ lattice with two flavors of dynamical quarks and use the gauge configurations generated by the CP-PACS collaboration [5] with the renormalization group improved gauge action and the mean-field improved clover quark action. We use the gauge configurations at $\beta = 1.95$ with the clover coefficient $c_{SW} = 1.530$, which give a lattice spacing of $a = 0.1555(17)$ fm ($a^{-1} = 1.267$ GeV), which is determined from the ρ-meson mass. The simulations are carried out with four different hopping parameters for the sea and the u,d valence quarks, $\kappa_{sea}, \kappa_{val}^{u,d} = 0.1375$, 0.1390, 0.1400 and 0.1410, which correspond to quark masses of $\sim$ 150, 100, 65, and 35 MeV, and we use 490, 680, 680 and 490 such gauge configurations, respectively. The hopping parameter for the s valence quark is fixed to $\kappa_{val}^{s} = 0.1393$ so that the Kaon mass is reproduced [5], which corresponds to a quark mass of $\sim$ 90 MeV. We employ smeared source and smeared sink, which are separated by 8 lattice units in the temporal direction. Source and sink operators are smeared in a gauge-invariant manner with the root mean square radius of 0.6 fm. All the statistical errors are estimated via the jackknife analysis.

Our results are presented in Tables 1 and 2: We give the fitted values of $g_{\pi NN}$ and other meson-baryon coupling constants normalized with $g_{\pi NN}$ in Table 1. Here, $g_{M\mathscr{B}\mathscr{B}'}^R$ denotes $g_{M\mathscr{B}\mathscr{B}'}/g_{\pi NN}$, for various quark masses. In Table 2, we give the fitted values of the NN axial charge, $g_{A,NN}$, together with the fitted values of the strangeness-conserving

TABLE 1. The fitted values of the $\pi\Xi\Xi$, $K\Lambda\Xi$ and $K\Sigma\Xi$ coupling constants and the corresponding monopole masses normalized with $g_{\pi NN}$ and $\Lambda_{\pi NN}$, respectively. Here, we define $g^R_{M\mathscr{B}\mathscr{B}'} = g_{M\mathscr{B}\mathscr{B}'}/g_{\pi NN}$ and $\Lambda^R_{M\mathscr{B}\mathscr{B}'} = \Lambda_{M\mathscr{B}\mathscr{B}'}/\Lambda_{\pi NN}$.

$\kappa^{u,d}_{val}$	$g_{\pi NN}$	$g^R_{\pi\Xi\Xi}$	$g^R_{K\Lambda\Xi}$	$g^R_{K\Sigma\Xi}$
0.1375	13.953(412)	-0.227(18)	0.334(15)	-1.025(20)
0.1390	13.257(448)	-0.216(14)	0.348(16)	-1.037(18)
0.1393	13.236(478)	-0.217(14)	0.347(16)	-1.036(19)
0.1400	13.098(393)	-0.245(13)	0.313(14)	-0.998(10)
0.1410	12.834(1.092)	-0.273(26)	0.291(25)	-0.963(48)
$\kappa^{u,d}_{val}$	$g^R_{\pi\Sigma\Sigma}$	$g^R_{\pi\Lambda\Sigma}$	$g^R_{K\Lambda N}$	$g^R_{K\Sigma N}$
0.1375	0.759(11)	0.698(11)	-1.038(07)	0.231(14)
0.1390	0.785(12)	0.697(07)	-1.034(07)	0.209(12)
0.1393	0.789(13)	0.699(08)	-1.033(08)	0.209(13)
0.1400	0.781(13)	0.723(08)	-1.017(07)	0.242(15)
0.1410	0.781(38)	0.756(28)	-1.007(30)	0.260(30)

TABLE 2. The fitted value of the NN axial charge together with the fitted values of the strangeness-conserving $\Xi\Xi$, $\Sigma\Sigma$, $\Lambda\Sigma$ and strangeness-changing $\Lambda\Xi$, $\Sigma\Xi$, ΛN and ΣN axial charges normalized with $g_{A,NN}$. Here, we define $g^R_{A,\mathscr{B}\mathscr{B}'} = g_{A,\mathscr{B}\mathscr{B}'}/g_{A,NN}$. We also give the fitted value of $F/F+D$ at each quark mass.

$\kappa^{u,d}_{val}$	$g_{A,NN}$	$g^R_{A,\Xi\Xi}$	$g^R_{A,\Sigma\Sigma}$	$g^R_{A,\Lambda\Sigma}$	
0.1375	1.284(11)	0.218(05)	0.791(04)	1.223(05)	
0.1390	1.282(15)	0.220(04)	0.782(04)	1.221(04)	
0.1393	1.280(15)	0.221(04)	0.779(04)	1.221(04)	
0.1400	1.289(15)	0.221(04)	0.772(04)	1.218(04)	
0.1410	1.314(24)	0.228(06)	0.738(09)	1.221(12)	
$\kappa^{u,d}_{val}$	$g^R_{A,\Lambda\Xi}$	$g^R_{A,\Sigma\Xi}$	$g^R_{A,\Lambda N}$	$g^R_{A,\Sigma N}$	$F/F+D$
0.1375	0.564(09)	-0.994(03)	1.761(06)	0.212(04)	0.390(2)
0.1390	0.558(08)	-0.999(01)	1.776(04)	0.220(04)	0.390(2)
0.1393	0.559(09)	-1.000(01)	1.779(04)	0.221(04)	0.390(2)
0.1400	0.553(07)	-1.000(02)	1.790(05)	0.225(04)	0.390(2)
0.1410	0.511(14)	-0.977(11)	1.775(14)	0.258(08)	0.380(3)

and strangeness-changing axial charges normalized with $g_{A,NN}$ for various quark masses. We expect that the systematic errors cancel out to some degree in the ratios of the coupling constants. We also present the values of the ratios of the coupling constants $\alpha_A = F/F+D$ as obtained from a global fit. In the SU(3)$_F$ limit, where $\kappa^{u,d}_{val} \equiv \kappa^s_{val} = 0.1393$, we obtain $\alpha_A = F/F+D = 0.390(2)$. The value of α_A has a weak quark-mass dependence and as we approach the chiral point α_A tends to decrease. We illustrate this behavior in Fig. 1.

Our analysis reveals that the ratios of the coupling constants and the ratios of the axial charges show rather weak quark-mass dependence. We have allowed an SU(3)$_F$ breaking by varying the quark masses. Our results suggest that the $SU(3)_F$ for pseudoscalar-meson coupling constants and axial charges octet baryons are a good symmetry, which is broken by only a few percent. While we think that the present work reveals the $SU(3)_F$ pattern of couplings of octet baryons, there are a number of improvements to be considered in a future work. Our lattice is still coarse by modern standards and quark masses are too large to reach a definite conclusion about $SU(3)_F$ breaking. Simulations with more realistic setups with smaller lattice spacing and larger lattice size employing much lighter quarks and a dynamical *s*-quark are under way [6].

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