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Giant pulsations: A nonlinear phenomenon

O. A. Pokhotelov,[1] Y. G. Khabazin,[1] I. R. Mann,[2] D. K. Milling,[2]
P. K. Shukla,[3] and L. Stenflo[4]

Abstract. Previous treatments of the excitation of Alfvén waves by drift-bounce resonance have been dominated by the linear approximation. We present a nonlinear treatment of the excitation of giant pulsations (Pgs) by drift-bounce resonance and show that nonlinear behavior becomes important well within the typical lifetimes of Pg wave packets. In particular, we show that the nonlinear phase mixing of the resonant particles trapped in the wave fields is of great importance for Pgs, and hence these nonlinearities must be included in calculations of the growth rates of Pgs. We believe that Pg behavior can be described by monochromatic modes exhibiting strong nonlinear wave-particle interactions during injections of resonant particles into the magnetosphere and that the observed growth and damping of Pg wave packets may provide an indication of the temporal variations in the driving resonant particle source population.

1. Introduction

Giant pulsations (Pgs) were first observed by *Birkeland* [1901] at the dawn of the last century. Since that time they have attracted research interest because of their long lifetimes (wave trains can last for up to an hour or more), their beautiful sinusoidal appearance, and their often complex and intriguing behavior (see the review by *Brekke et al.* [1987]). The term "giant" originates from the fact that Pg amplitudes can typically be ~10 nT, occasionally even reaching ~50 nT at auroral latitudes on the ground [e.g., *Green*, 1979]. Pgs are most often observed during the night and early morning hours. They are almost exlusively auroral zone phenomena and are characterized by a latitudinal localization and moderately large azimuthal wavenumber, m ~20-40. Pgs are dominantly polarized in the D component on the ground (geomagnetic east-west), have periods between 60 and 200 s, and hence can be considered as a special class of Pc4 pulsations (see, e.g., *Chisham et al.* [1997], and references therein).

It is thought that Pgs may be driven in the Earth's magnetosphere through drift-bounce resonance with energetic ring current protons [*Southwood et al.*, 1969; *Southwood*, 1973, 1976], and many observational studies have supported this hypothesis [e.g., *Glassmeier*, 1980; *Poulter et al.*, 1983; *Chisham et al.*, 1992]. *Chisham et al.* [1992] showed that the azimuthal phase velocities of Pgs matched well with the azimuthal drift velocities of ~10-20 keV protons, which suggests that particles of this energy may be responsible for the excitation of Pgs by drift-bounce resonance. While satellites have also observed Pgs in situ in the magnetosphere [e.g., *Kokubun et al.*, 1989; *Takahashi et al.*, 1992] and evidence of resonant interactions between particles and Pgs has been seen [*Kokubun et al.*, 1989], convincing observational evidence for coincident unstable particle distributions has yet to be found. Poloidally polarized waves were seen in the afternoon sector by *Hughes et al.* [1978] in association with a bump-on-tail distribution around 10 keV, although these waves were different from Pgs in that they were observed to have m ~ 100 (much larger than Pgs), were situated in the afternoon sector, and had no ground-based magnetic signature because of their screening from ground-based magnetometers by the ionosphere [e.g., *Hughes and Southwood*, 1976].

One of the most striking features that distinguishes Pgs from other types of geomagnetic variations is that they are remarkably monochromatic. Their spectrum is so narrow that any theoretical model of Pgs should be constructed in the framework of a monochromatic approximation. A linear treatment of the drift-bounce resonance was considered by *Southwood* [1973] and was then substantially developed by a number of authors [*Mikhailovskii and Pokhotelov*, 1975; *Karpman et al.*, 1977]. However, the linear approximation is only valid if the amplitude of the pulsations is so small that the particle distribution function varies rather slowly under the action of the wave. A general approach to the study of nonlinear effects during drift-bounce interactions was developed by *Meyerson and Pokhotelov* [1978] in the framework of the random phase approximation. How-

[1] Institute of Physics of the Earth, Moscow, Russia.
[2] Department of Physics, University of York, York, England.
[3] Institut für Theoretische Physik IV, Ruhr-Universität Bochum, Bochum, Germany.
[4] Department of Plasma Physics, Umeå University, Umeå, Sweden.

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ever, such an approximation is only valid for broadband wave packets. Pgs represent an entirely different case which corresponds to the excitation of a single mode, usually thought to be the second field-aligned harmonic, the Pg spectrum being so narrow that the random phase approximation is not applicable.

In this paper we consider a theoretical treatment of the nonlinear excitation of Pgs by drift-bounce resonance. In order for linear theory to be applicable, the resonant particles must only interact once with the waves for a wave packet of typical duration. For typical Pg amplitudes one finds that the nonlinear timescale for particle trapping in the waves potential is quite fast, being of the order of a few wave cycles. Since Pgs can typically last for several hours, the particles will interact nonlinearly with the waves. Consequently, the nonlinear behavior of the particles in the wave fields must be considered and a nonlinear theory for Pg excitation developed. Since we would expect this particle trapping to saturate the drift-bounce resonance instability within the same timescale of a few periods, then the instability that is responsible for Pg excitation can only be fully described with a nonlinear approach.

Several questions remain concerning how Pgs may be excited. In particular, Pgs are believed to be excited by westward drifting ions through drift-bounce resonance, following their injection from the tail. While poloidal Pc4 pulsations with azimuthal wavenumbers $m \sim 100$ that are believed to be driven by drift-bounce resonance are often observed in the afternoon sector through which the ions drift first [e.g., *Hughes et al.*, 1978], Pgs with $m \sim$ 20-40 remain, in general, confined to the morningside. *Chisham* [1996] suggests that if Pgs are driven by protons with energies around 10-20 keV, then a westward drifting population generated by a tail injection will lie on drift trajectories that exit the dayside magnetopause before reaching the morning sector under typical geomagnetic conditions. Only under conditions of extreme quiet, when the dawn-dusk electric field is low, can the $\mathbf{E} \times \mathbf{B}$ drift effect be sufficiently reduced to allow this population to reach the morning sector and drive Pgs through drift-bounce resonance. Since the same particles will have traversed the afternoonside magnetosphere irrespective of geomagnetic activity, it is not clear why Pgs are not excited in the afternoon local time sector. Since we believe that nonlinear effects become important well within the lifetime of Pg wave packets, then if questions about Pg excitation are to be answered, it is important to fully understand their nonlinear growth. Moreover, since nonlinear saturation is expected to occur early in the Pg wave train, modulation in Pg wave packets may provide information about the temporal variations in the drifting resonant particle distributions which are believed to have excited the waves. Information about these temporal source variations might provide some additional new insight into these questions.

In this paper we first present the observational features of a typical Pg and then derive the theoretical details of a nonlinear drift-bounce resonance excitation mechanism. We then apply the theoretical results to derive the saturation and wave packet modulation that results from the nonlinear excitation of Pgs by a range of injected particle drivers. Finally, we discuss the implications of our work for Pgs in the magnetosphere and suggest that their dynamics are affected by these nonlinear wave particle interactions in a nontrivial way.

2. Observational Features of Pgs

Pgs appear in ground magnetometer data as large-amplitude, monochromatic waves of long duration, often displaying a distinctive packet structure with spatial and temporal variations. They appear in the morning sector within a narrow band stretching along the geomagnetic parallel near the equatorial boundary of the auroral zone [e.g., *Chisham and Orr*, 1994]. A unique feature of Pg time series is the exceptionally long duration of their wave trains. It is this feature that significantly distinguishes them from other Pc3-4 pulsations which are damped out on significantly shorter timescales. The ground magnetic field signatures of Pgs are also dominated by the D component, while other Pc4 pulsations tend to be dominated by the H component [e.g., *Chisham et al.*, 1997]. Accounting for the 90^o polarization rotation expected for Alfvén waves propagating through the ionosphere [e.g., *Hughes and Southwood*, 1976], Pgs are expected to be predominantly radially polarized in the magnetosphere, as is confirmed by satellite observations [e.g., *Kokubun et al.*, 1989]. According to *Chisham and Orr* [1991], Pgs are usually even mode (second field-aligned harmonic), although there is some evidence that they may also sometimes be represented by odd-mode waves [*Takahashi et al.*, 1992; *Green*, 1979, 1985; *Hillebrand et al.*, 1982].

A typical magnetogram of a Pg pulsation (D component, filtered with a high pass filter cutoff at 300 s) recorded on November 11, 1995, by the International Monitor for Auroral Geomagnetic Effects (IMAGE; 10 s resolution) and the U.K. Sub-Auroral Magnetometer Network (SAMNET; 1 s resolution) fluxgate magnetometers is shown in Figure 1. The station codes and coordinates are listed in Table 1. The Pg event starts at about 0015 UT and lasts until nearly 0300 UT. The power spectrum of the signal from SAMNET's Oulu station (geographic coordinates, $\varphi = 65.1°$N, $\lambda = 25.8°$E; geomagnetic coordinates, $\Phi = 61.5^o$, $\Lambda = 105.9^o$) is given in Figure 2. One can see that the main part of the signal represents a narrow frequency band with $\delta\omega/\omega \leq 0.1$. The central frequency is quasi-stable but undergoes a gradual decrease over the course of the event. Unlike pearl pulsations [*Bud'ko et al.*, 1972], the Pg's spectra do not exhibit frequency splitting, that is, they do not show the appearance of sidebands. This fact

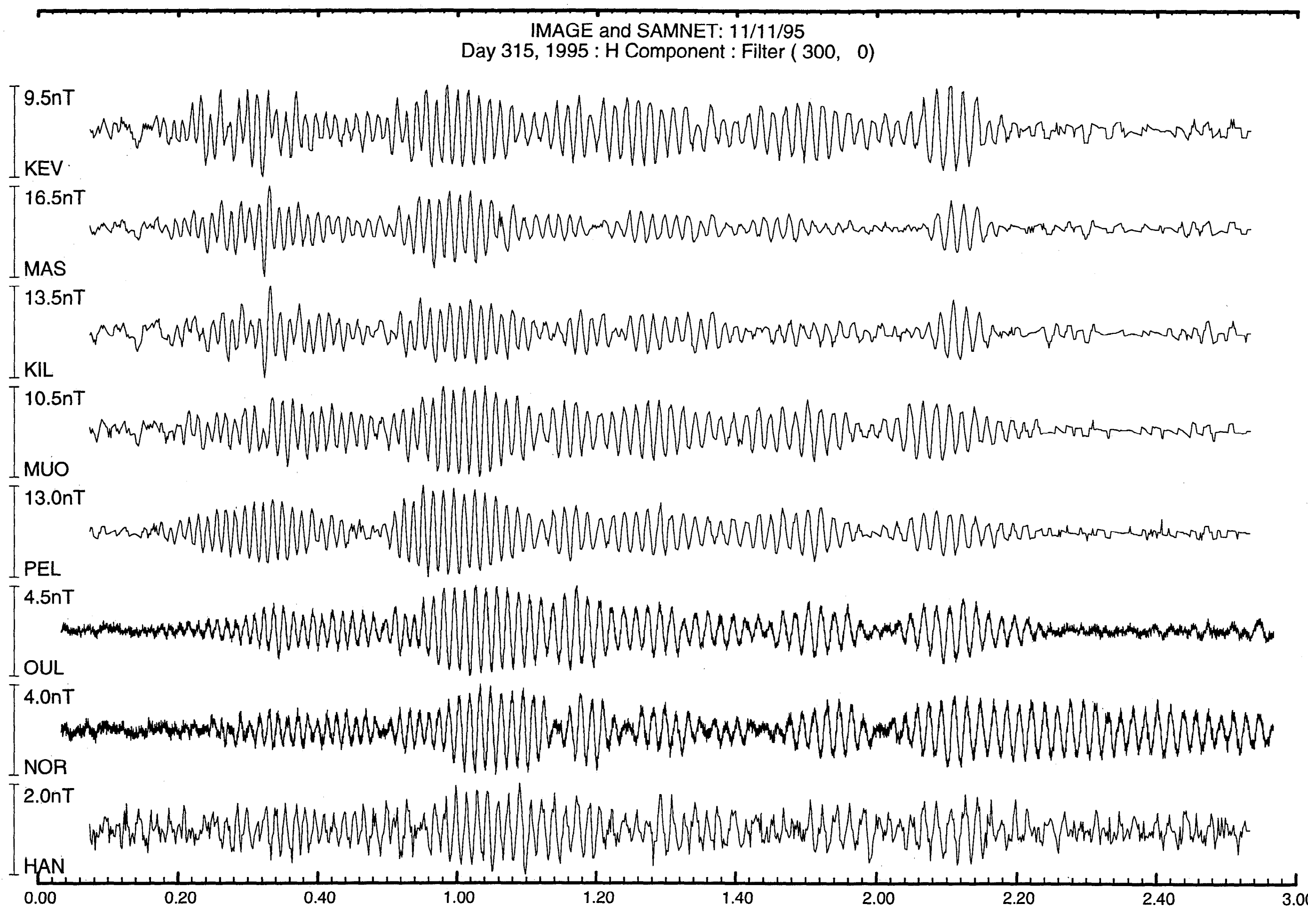

Figure 1. Magnetogram (D component, high-pass filtered at 300 s) of the giant pulsation (Pg) recorded on November 11, 1995, by the International Monitor for Auroral Geomagnetic Effects (IMAGE) and the U.K. Sub-Auroral Magnetometer Network (SAMNET) fluxgate magnetometers. See Table 1 for abbreviation definitions.

can be understood if we recall that the normal mode frequencies in the magnetosphere are quantized by bounce frequencies which significantly exceed the sideband frequency shift $\delta\omega_s \sim \omega(b/B_0)^{1/2} \ll \omega_b$. This fact prevents the system from being unstable against a sideband instability. The absence of sidebands in the Pg spectrum allows us to interpret the amplitude modulations as a variation in the monochromatic signal amplitude. We shall now proceed to develop the nonlinear theory for the evolution of Pgs and show how the wave growth depends on the form of the resonant particle injection.

3. Excitation and Nonlinear Evolution of Pgs

In this section we derive the details of the excitation of Pgs by drift-bounce resonant particles. Guided by the observed narrowband Pg frequency spectrum, we adopt a quasi-monochromatic approach. In section 3.1 we use a Hamiltonian approach to derive the details of charged particle motion in a background dipole magnetic field supporting poloidal Alfvén wave perturbations. Then in section 3.2 we derive nonlinear drift-bounce resonance growth rates by integrating over the particle distribution functions.

3.1. Hamiltonian Description of Particle Motion

The general form of the Hamiltonian [e.g., *Goldstein*, 1965; *Meirovich*, 1970] is

$$H = \frac{1}{2M}\left(\mathbf{P} - e\mathbf{A}\right)^2 , \qquad (1)$$

where $\mathbf{P}$ is the generalized momentum, e is the charge of the particle, M is a particle mass, and $\mathbf{A}$ is the vector potential of the magnetic field. However, the corresponding canonical equations can be solved in general form only for the simplest magnetic configurations, and most magnetospheres do not belong to this class of configurations. In the case of the Earth's magnetosphere, many approaches have been developed; below we shall use the so-called adiabatic theory [*Northrop*, 1963], where the Hamiltonian equations are obtained

Table 1. IMAGE and SAMNET Site Codes and Coordinates

			Geographic Coordinates		Geomagnetic Coordinates	
Site Name	Array	Code	Latitude, °N	Longitude, °E	Latitude, °N	Longitude, °E
Kevo	IMAGE	KEV	69.76	27.01	66.21	109.73
Masi	IMAGE	MAS	69.56	23.70	66.07	106.92
Kilpisjarvi	IMAGE	KIL	69.02	20.79	65.78	104.31
Muonio	IMAGE	MUO	68.02	23.53	64.62	105.70
Pello	IMAGE	PEL	66.90	24.08	63.46	105.38
Oulu	SAMNET	OUL	65.10	25.85	61.52	105.94
Nordli	SAMNET	NOR	64.37	13.36	61.43	95.49
Hankasalmi	IMAGE	HAN	62.30	26.65	58.62	104.99

IMAGE is the International Monitor for Auroral Geomagnetic Effects and SAMNET is the U.K. Sub-Auroral Magnetometer Network.

only for the averaged particle motion. The ambient geomagnetic field of the magnetosphere is assumed to be dipolar. The magnetic $\delta\mathbf{B}$ and electric $\delta\mathbf{E}$ fields of the Pg pulsations will be considered as perturbations. Thus we have

$$\delta H = -\, e\mathbf{v}\cdot\delta\mathbf{A}, \tag{2}$$

where $\mathbf{v}$ is the particle velocity and $\delta\mathbf{A}$ is the perturbation of the vector potential. In order to use properly the results of the adiabatic theory [*Northrop*, 1963], we should average (2).

According to *Northrop* [1963], for the description of particle motion in a magnetic field it is convenient to introduce coordinates (α, β, s) connected with the magnetic field lines. Here $\alpha(\mathbf{r})$ and $\beta(\mathbf{r})$ are two parameters denoting the field line on which they are constant and $s(\mathbf{r})$ is the distance along the field line measured from the equator. The parameters α and β can be chosen so that the vector potential $\mathbf{A}$ of the external geomagnetic field will be equal to $(\alpha\nabla)\beta$ and the magnetic field equal to $\nabla\alpha \times \nabla\beta$. For an axisymmetrical dipolar magnetic field it is convenient to identify α with the radial coordinate and to choose β to be equal to the azimuth ϕ. Then $\mathbf{A} = (\alpha\nabla)\phi$, where $\alpha = B_E R_E^2 L^{-1}$ and $LR_E = R/\sin^2\vartheta$. Here R and ϑ are the spherical polar radius and latitude, L is the McIlwain parameter, B_E is the equatorial magnetic field strength at the Earth surface, and R_E is the Earth radius. The vector potential $\mathbf{A}$ has only one nonzero component, namely, $A_\phi = B_E R_E^2 L^{-1}(\nabla\phi)_\phi$.

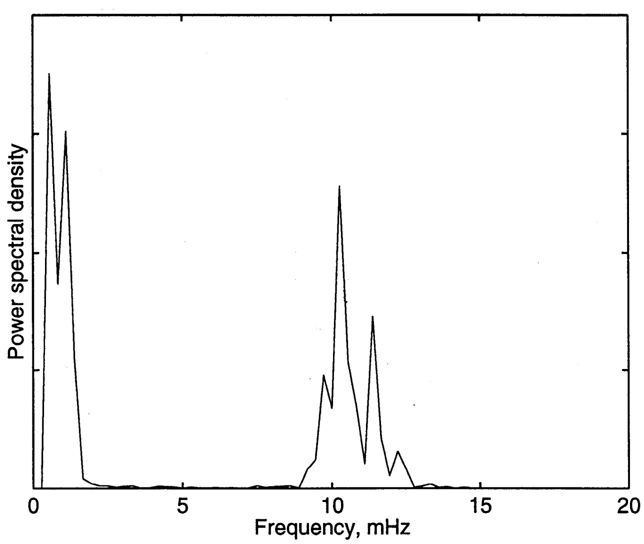

Figure 2. Power spectral estimates of the Pg observed at SAMNET's Oulu station on November 11, 1995, showing enhancements slightly above 10 mHz. No harmonics are observed. Close single peaks reflect the pulsation's modulation.

Since the frequencies of the Pg pulsations ($f \sim 10$ mHz) are considerably less than the particle gyrofrequency (~1.5 Hz for a proton in a 100 nT field), we make use of the drift approximation. According to *Northrop* [1963], the particle motion in the dipolar magnetic field can be divided into three parts. First, the particles are rotating around the field line with the gyrofrequency ω_c; in addition, the particles are oscillating along the field lines with the bounce frequency ω_b, and, finally, they are also drifting in the azimuthal direction around the Earth with the drift frequency ω_d. The three adiabatic invariants μ, J, and Φ correspond to these periodic particle motions in the three different directions. They are constant for the motion in the ambient geomagnetic field and are defined as

$$\mu = \frac{Mv_\perp^2}{2B}, \tag{3}$$

$$J = (2\pi)^{-1} M \oint v_\parallel ds, \tag{4}$$

$$\Phi = (2\pi)^{-1} \oint e\mathbf{A}\cdot d\mathbf{l}, \tag{5}$$

where $v_\perp$ and $v_\parallel$ are the velocity components perpendicular and parallel to the magnetic field, respectively. The first integral is taken over the bounce orbit. The second is taken over the drift trajectory around the dipole; owing to axial symmetry of the dipolar field, it is reduced to multiplication and yields $\Phi = eB_E R_E^2 L^{-1}$. The adiabatic invariants μ, J, and Φ in (3-5) are expressed in terms of spatial coordinates and momenta, where J and Φ differ from those of *Northrop* [1963] by factors introduced here for notational convenience. Actually, in our case, J measures the amplitude of the bounce oscillation and Φ shows the radial distance of the particle trajectory from the dipole.

Below, these invariants will be used as independent variables. In the case of Pg pulsations, μ will be constant even in the presence of the disturbances, and thus it will appear only as a parameter in the subsequent expressions. In order to use the Hamiltonian method in the calculations [*Nayfeh*, 1973], we shall introduce variables that are canonically conjugated to J and Φ. For J, such a variable is $\Psi = \omega_b \int ds/v_\parallel$, i.e., the bounce phase, and for Φ it is ϕ, i.e., the azimuth. The equations of particle motion expressed in terms of the adiabatic invariants have the Hamiltonian form [*Nayfeh*, 1973]

$$\frac{dJ}{dt} = -\frac{\partial H}{\partial \Psi}, \qquad \frac{d\Psi}{dt} = \frac{\partial H}{\partial J}, \tag{6}$$

$$\frac{d\Phi}{dt} = -\frac{\partial H}{\partial \phi}, \qquad \frac{d\phi}{dt} = \frac{\partial H}{\partial \Phi}. \tag{7}$$

The general explicit expression for the averaged Hamiltonian H is not known; however, for our purposes it is not needed. All that we need in the following are the bounce-averaged equations. For the unperturbed particle motion in an axisymmetrical dipolar magnetic field the Hamiltonian equations take the form [*Northrop*, 1963]

$$\dot{J} = 0, \qquad \dot{\Psi} = \frac{\partial H}{\partial J} = \omega_b, \tag{8}$$

$$\dot{\Phi} = 0, \qquad \left\langle \dot{\phi} \right\rangle = \frac{\partial H}{\partial \Phi} = \bar{\omega}_d, \tag{9}$$

where the dot stands for d/dt and the bar denotes the average over the bounce motion. Thus (8-9) state that an unperturbed energetic particle is oscillating along the field line with a frequency ω_b and is drifting in the azimuthal direction with average angular frequency $\bar{\omega}_d$ without displacement across the field line in the radial direction. As a consequence the azimuthal component of the electric field in the Alfvén wave perturbation is the dominant one (as the component along the main magnetic field is zero). The fields thus have the form

$$\delta E_\phi = E_\phi(\Phi, s)\cos(\omega t - m\phi), \tag{10}$$

$$\delta A_\phi = -\frac{1}{\omega} E_\phi \sin(\omega t - m\phi), \tag{11}$$

where m is an integer. The additional term in the Hamiltonian due to the Alfvén wave perturbation is

$$\delta H = -e\mathbf{v}\cdot\delta\mathbf{A} = \frac{e}{\omega} r\bar{\omega}_d E_\phi \sin(\omega t - m\phi). \tag{12}$$

As was mentioned above, the expression for the perturbed Hamiltonian must be averaged over the bounce phase. For this purpose we make use of the Fourier transform

$$er\frac{\bar{\omega}_d}{\omega} E_\phi = 2\left(\frac{\alpha_0}{2} + \sum_{N=1}^{\infty} \alpha_N \cos N\Psi\right), \tag{13}$$

where

$$\alpha_N = \frac{e}{2\pi\omega}\int_{-\pi}^{\pi} d\Psi r\bar{\omega}_d E_\phi \cos N\Psi,$$

assuming for simplicity that the electric field is an even function with respect to the equator.

The drift-bounce resonance between a particle and a wave occurs when

$$\omega = N\omega_b + m\bar{\omega}_d, \tag{14}$$

where N is an integer, and takes the values $0, \pm 1, \pm 2, \ldots$ Using the standard averaging method [*Nayfeh*, 1973], in the vicinity of the resonance we derive the reduced motion equations for the resonant particles, i.e., particles with $|\delta\omega| \ll \omega_b$ ($\delta\omega = N\omega_b + m\bar{\omega}_d - \omega$). Thus

$$\dot{\Phi} = m\alpha_N \cos(N\psi + m\varphi + \delta\omega t), \tag{15}$$

$$\dot{\varphi} = -\frac{\partial \alpha_N}{\partial \Phi}\sin(N\psi + m\varphi + \delta\omega t), \tag{16}$$

$$\dot{J} = N\alpha_N \cos(N\psi + m\varphi + \delta\omega t), \tag{17}$$

$$\dot{\psi} = -\frac{\partial \alpha_N}{\partial J}\sin(N\psi + m\varphi + \delta\omega t), \tag{18}$$

where φ and ψ denote the so-called slow variables: $\varphi = \phi - \bar{\omega}_d t$ and $\psi = \Psi - \omega_b t$.

To the same approximation we have

$$\delta H = -\alpha_N \sin(N\psi + m\varphi + \delta\omega t), \tag{19}$$

$$\frac{d\delta H}{dt} = \left(\frac{\partial \delta H}{\partial t}\right)_{\Psi,\,\phi} = \omega\alpha_N \cos(N\psi + m\varphi + \delta\omega t). \tag{20}$$

From equations (15)-(20) the next integrals of motion are

$$K = H + \delta H - \frac{\omega}{m}\Phi = \text{const}, \tag{21}$$

$$X = N\Phi - mJ = \text{const}. \tag{22}$$

The first integral K is a consequence of the time-azimuthal dependence of the wave $\omega t - m\phi$ and valid for all particles, while the second integral X refers only to the resonant particles. It means that the variations of Φ and J due to resonant interactions with the wave are proportional to each other.

Introducing a new variable, $\lambda = N\psi + m\varphi + \delta\omega t$, and its canonically conjugated momentum Y, given by $Y = (1/2)(\Phi/m + J/N)$, we can rewrite (15-18) in the Hamiltonian form

$$\dot{Y} = \alpha_N \cos\lambda = -\left(\frac{\partial K}{\partial \lambda}\right)_{X,\,Y}, \tag{23}$$

$$\dot{\lambda} = \delta\omega(\mu, J, \Phi) - \left(\frac{\partial \alpha_N}{\partial Y}\right)_X \sin\lambda = \left(\frac{\partial K}{\partial Y}\right)_{X,\,\lambda}, \tag{24}$$

where K is now expressed in terms of new variables Y and λ; that is,

$$K = H - \omega Y - \alpha_N \sin\lambda. \tag{25}$$

From the conservation of the new Hamiltonian K we can obtain the dependence $Y(t)$ in quadrature or

$$t = \int_{Y(0)}^{Y(t)} dY \left[\alpha_N^2(Y) - (K - H + \omega Y)^2\right]^{-1/2}. \tag{26}$$

As follows from (26), the particle motion is periodic with time and takes place in the region of variation of Y where the expression in the square root is positive. For a more complete investigation we expand the Hamiltonian near the mean value of $Y = Y_r$; that is,

$$K = H_r - \omega Y_r - \alpha_N(Y_r)\sin\lambda + \left(\frac{\partial H}{\partial Y_r}\right)_X (Y - Y_r) - \omega\,(Y - Y_r) + \frac{1}{2}\left(\frac{\partial^2 H}{\partial Y_r^2}\right)_X (Y - Y_r)^2. \tag{27}$$

Using the resonant condition (14), $(\partial H/\partial Y_r)_X = N\omega_b + m\bar{\omega}_d = \omega$, and omitting the constant terms, we obtain

$$K = \frac{1}{2}\Omega'\,(Y - Y_r)^2 + \alpha_N(Y_r)(1 - \sin\lambda), \tag{28}$$

where $\Omega = (\partial H/\partial Y_r)_X = N\omega_b + m\bar{\omega}_d$, and $\Omega' = (\partial\Omega/\partial Y_r)_X$. The resonant region width ΔY, as can be seen from (28), is

$$\Delta Y = \left[4\alpha_N\,(Y_r)\,/\Omega'\right]^{1/2}. \tag{29}$$

Introducing a new variable, $\xi = \pi/4 - \lambda/2$, we rewrite (23) and (24) as

$$\dot{Y} = \alpha_N \sin 2\xi, \qquad \dot{\xi} = -\frac{1}{2}\Omega'(Y - Y_r), \tag{30}$$

which can be reduced to the standard nonlinear pendulum equation

$$\tau_w^2\dot{\xi}^2 + \sin^2\xi = \kappa^{-2}, \tag{31}$$

where $\tau_w^{-2} = \alpha_N\Omega'$ is the inverse square of the characteristic nonlinear time, and $\kappa^2 = 2\alpha_N/K$.

The phase portrait in the (Y,ξ) plane of the Hamiltonian system (30) is shown in Figure 3. The separatrix separates two types of particles. Particles with $0 < \kappa^2 < 1$ are untrapped and those with $\kappa^2 > 1$ are trapped, similar to the case of Langmuir waves [*O'Neil*, 1965]. However, in our case the particles are trapped in phase space. Thus they can be considered as phase-trapped and phase-untrapped particles. The solution of (31) for $0 < \kappa^2 < 1$ can be expressed in terms of elliptical functions as

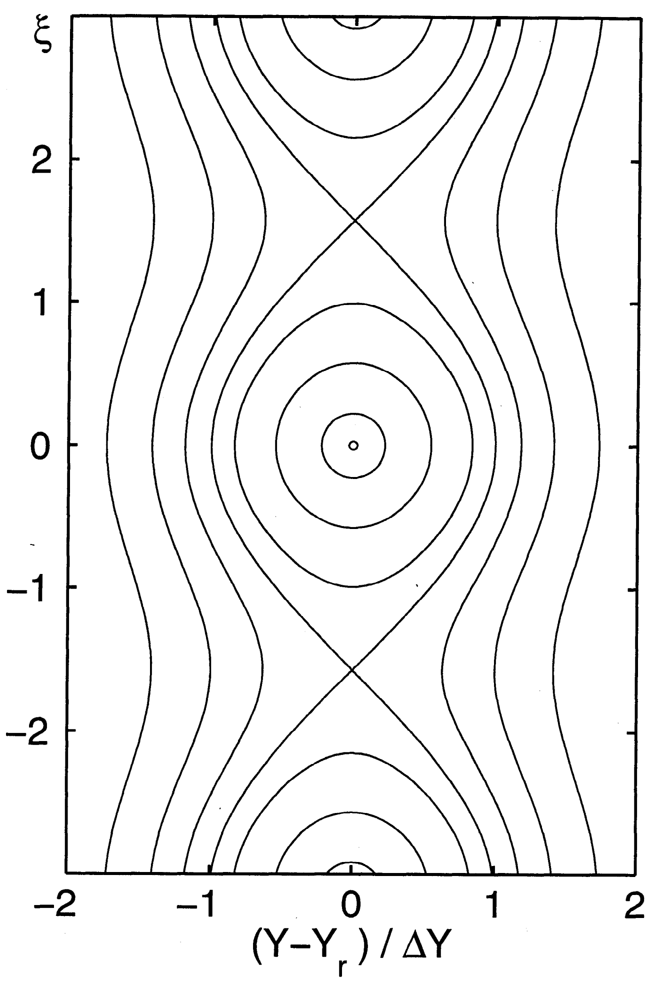

Figure 3. Trajectories of the imaging points in the phase (Y,ξ) plane.

$$F(\xi,\kappa) = F(\xi',\kappa) + (t - t')/\kappa\tau_w, \tag{32}$$

or

$$\xi = am[F(\xi',\kappa) + (1/\kappa\tau_w)(t - t'),\kappa], \tag{33}$$

$$\dot{\xi} = (1/\kappa\tau_w)dn[F(\xi',\kappa) + (1/\kappa\tau_w)(t - t'),\kappa], \tag{34}$$

where $F(\xi,\kappa) = \int_0^\xi d\varphi/(1 - \kappa^2\sin^2\varphi)^{1/2}$ is the elliptical integral of the first kind and $am(F,\kappa)$ and $dn(F,\kappa)$ are Jacobian amplitude and delta, respectively, of amplitude functions [*Abramowitz and Stegun*, 1964]. For particles with $\kappa^2 > 1$ it is convenient to transform to the parameter $1/\kappa$ according to the formula

$$dn\left[F(\xi',\kappa) + (1/\kappa\tau_w)(t - t'),\kappa\right] = cn\left[\kappa F(\xi',\kappa) + (1/\tau_w)(t - t'), 1/\kappa\right], \tag{35}$$

where $cn(F,\kappa)$ is the Jacobian cosine function of the amplitude. Expressions (33) and (34) determine the coordinate ξ and the velocity $\dot{\xi}$ of the particle at the moment t, given ξ' and $\dot{\xi}'$ at the moment t'. The corresponding expression for Y can be obtained with the help of (30). Returning to the initial variables Φ and J, we can express their dependence on time t through time t'.

3.2. Nonlinear Pg Growth Rates

Using the explicit expressions (33) and (34) for the particle trajectories, we can calculate the distribution function of the resonant particles starting from the Vlasov equation for the distribution function of the resonant particles

$$\frac{\partial f_{\rm res}}{\partial t} + \{H + \delta H,\ f_{\rm res}\} = Q(\mu, J, \Phi, t), \qquad (36)$$

where $Q(\mu, J, \Phi)$ is the source of the resonant particles, normalized by the condition

$$\int Q(\mu, J, \Phi, t)\, d\Gamma = \frac{dN}{dt}, \qquad (37)$$

where $d\Gamma = d\mu dJ d\Phi d\psi d\varphi = d\mu dX dY d\psi d\varphi$ is the cell of the phase space and dN/dt is the change in the total number of resonant particles in the volume corresponding to the entire region of the wave localization. Equation (36) can be solved by standard methods of integration along the trajectories. Supposing that the resonant particles are absent at $t = 0$, we can write for $t > 0$

$$f_{\rm res} = \int_0^t Q(\mu, J(t'), \Phi(t'), t')\, dt'. \qquad (38)$$

Since we are only interested in the particle motion in the narrow resonant region around $\Delta Y = Y - Y_r \sim \left[\alpha_N(Y_r)/\Omega'\right]^{1/2} \ll Y_r$, we expand the source function in (36) in powers of ΔY; that is,

$$Q(\mu, J, \Phi, t) \simeq Q(\mu, J_r, \Phi_r, t) + \left(\frac{\partial Q}{\partial Y}\right)_{\mu,\ X} (Y - Y_r). \qquad (39)$$

Thus the expression for the distribution function of the resonant particles (38) is

$$f_{\rm res} = \int_0^t dt' \times \left\{ Q(\mu, J_r, \Phi_r, t') + \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,\ X} (Y - Y_r) \right\}. \qquad (40)$$

The time-dependent growth rate can be evaluated from the expression

$$\dot{U} = 2\gamma U, \qquad (41)$$

where $U = \int u d^3\mathbf{r}$ is the total wave energy and u stands for the wave energy density. The integration is carried out over the entire region of the wave localization. The value $\dot{U}$ can be determined from the energy balance. The total change in wave energy due to interaction with resonant particles is

$$\dot{U} = -\int \mathbf{j}\cdot\delta\mathbf{E} d^3\mathbf{r}, \qquad (42)$$

where $\mathbf{j}$ is the current density of the resonant particles. It is customary to define this by means of a distribution function f normalized as $\int f d^3\mathbf{v} = n$ (where n is the particle number density and the integration is carried out over velocity space). Thus

$$\mathbf{j} = e \int \mathbf{v} f d^3\mathbf{v}, \qquad (43)$$

and therefore

$$\dot{U} = -\, e \int \mathbf{v}\cdot\delta\mathbf{E} f d^3\mathbf{v} d^3\mathbf{r}. \qquad (44)$$

In our case this implies

$$\dot{U} = -\, e \int r\omega_d E_\phi \cos(\omega t - m\phi)\, f_{res} d\Gamma, \qquad (45)$$

where $d\Gamma = d\mu dX d\varphi dY d\psi$ (see (37)). After substitution of (13) and (40) into (45) and keeping in mind the resonant condition (14), we find

$$\dot{U} = -\omega \sum_N \int \alpha_N \cos\lambda(t) \times \int_0^t dt' \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,X} (Y - Y_r)\, d\Gamma. \qquad (46)$$

Using (30), we then obtain

$$\dot{U} = 2\omega \sum_N \int \alpha_N \sin 2\xi(t) \times \int_0^t dt' \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,\ X} \frac{\dot{\xi}(t')}{\Omega'} d\Gamma. \qquad (47)$$

Using $d\Gamma = d\mu dX d\varphi dY d\psi = d\mu dX d\varphi\, (4/N\Omega')\, d\dot{\xi} d\xi$ we obtain

$$\dot{U} = 8\omega \sum_N \int d\mu dX d\varphi \frac{\sin 2\xi(t)}{N} \alpha_N \times \int_0^t dt' \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,\ X} \frac{\dot{\xi}(t')}{(\Omega')^2} d\dot{\xi} d\xi, \qquad (48)$$

or using the relation derived from (31) and (32) for the Jacobian $d\dot{\xi} d\xi = \left(1/\kappa^2 \tau_w\right) dF d\kappa$,

$$\dot{U} = 8\omega \sum_N \int d\mu dX d\varphi \int_0^\infty \frac{d\kappa}{\kappa^2} \int_{-K(\kappa)}^{K(\kappa)} \frac{dF}{\tau_w} \frac{\alpha_N}{(\Omega')^2} \frac{\sin 2\xi(t)}{N} \times \int_0^t dt' \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,\ X} \dot{\xi}(t'), \quad (49)$$

where $K(\kappa)$ is the full elliptic integral of the first kind [*Abramowitz and Stegun*, 1964]. Substituting (33) and (34) in (49), we then obtain

$$\dot{U} = 8\omega \sum_N \int d\mu dX d\varphi \times \int_0^\infty \frac{d\kappa}{\kappa^3} \int_{-K(\kappa)}^{K(\kappa)} \frac{dF}{N} \frac{\alpha_N^2}{\Omega'} \sin 2am[F(\xi_0, \kappa) + t/\kappa\tau_w, \kappa] \times \int_0^t dt' \left[\frac{\partial Q(t')}{\partial Y}\right]_{\mu,\ X} dn\left[F(\xi_0, \kappa) + t'/\kappa\tau_w\right]. \quad (50)$$

Using $\partial\left[dn(F,\kappa)\right]/\partial F = -(1/2)\kappa^2 \sin 2am(F,\kappa)$ to expand the elliptical Jacobian functions in a Fourier series, and treating separately $\kappa < 1$ or $\kappa > 1$, we obtain after straightforward calculations

$$\gamma = \frac{\pi^2\omega}{2U} \sum_N \int \frac{d\mu dX d\varphi}{N} \times \int_0^t dt' \frac{\alpha_N^2}{\Omega'} \left[\frac{\partial Q}{\partial Y}\right]_{\mu,\ X} N\left(\frac{t-t'}{\tau_w}\right). \quad (51)$$

Here $N(z)$ is the O'Neil function [*O'Neil*, 1965],

$$N(z) = \frac{64}{\pi} \sum_{n=0}^\infty \int_0^1 d\kappa \left[\frac{2n\pi^2 q^{2n} \sin\frac{n\pi}{\kappa K} z}{\kappa^5 K^2 (1+q^{2n})^2} + \frac{(2n+1)\pi^2 q^{2n+1} \kappa \sin\frac{(2n+1)\pi}{2K} z}{K^2(1+q^{2n+1})^2}\right], \quad (52)$$

where $K = F(\kappa, \pi/2)$ is the complete elliptical integral of the first kind, $q = \exp\left[-\pi K(\kappa')/K(\kappa)\right]$ and $\kappa' = (1-\kappa^2)^{1/2}$.

4. Nonlinear Drift-Bounce Resonance Growth Rates

The nonlinear growth and saturation of Pgs, driven by drift-bounce resonance, depend critically upon the form and time dependence of the driving resonant particle distribution. In what follows we derive the wave growth rate and amplitude modulation resulting from two limiting cases of sudden and quasi-stationary particle injections.

4.1. Sudden Resonant Particle Injection

In this case we consider the resonant particles to be suddenly injected at the initial moment $t = 0$ into the region of wave localization. After this time the influx of resonance particles is absent, and hence we can approximate the source by $Q_0(t) = f_0\delta(t)$. Thus the expression for the growth rate is

$$\gamma(t) = \frac{\pi^2\omega}{2U} \sum_N \int \frac{d\mu dX d\varphi}{N} \frac{\alpha_N^2}{\Omega'} \left[\frac{\partial f_0}{\partial Y}\right]_{\mu,\ X} N\left(\frac{t}{\tau_w}\right). \quad (53)$$

The expected pattern of the nonlinear growth and evolution of the Pg wave packet for this delta function driver is presented as the solid line in Figure 4. The dashed line refers to the exponential growth derived from purely linear theory. For times less than the linear time t_0, as defined by $t_0 = 1/\gamma_L$, where γ_L is the linear growth rate, the quantitative behavior of both the linear and nonlinear solutions is the same. In the nonlinear case, however, the wave amplitude reaches a maximum value B_m in an additional nonlinear timescale $\sim \tau_w$ at time $t = t_0 + \tau_w$. The wave envelope then oscillates for a few τ_w, before tending to a constant asymptotic value $B_\infty < B_m$.

At $t = 0$, however, the nonlinear growth rate will be identical to that generated by linear theory, and the nonlinear and linear wave growth can clearly be seen to be identical at $t = 0$ in Figure 4. Since at $t = 0$, $N(t/\tau_w) \to 1$, then the growth rate reduces to

$$\gamma_L = \frac{\pi^2\omega}{2U} \sum_N \int \frac{d\mu dX d\varphi}{N} \frac{\alpha_N^2}{\Omega'} \left(\frac{\partial f_0}{\partial Y}\right)_{\mu,\ X} \quad (54)$$

or

$$\gamma_L = \frac{\pi^2\omega}{2U} \sum_N \int d\mu d\Phi dJ d\varphi \times \delta(N\omega_b + m\bar{\omega}_d - \omega) \frac{\alpha_N^2}{N} \left(\frac{\partial f_0}{\partial Y}\right)_{\mu,\ X}. \quad (55)$$

The latter expression for this linear growth rate coincides with that obtained by *Southwood* [1973] if one takes into account that

$$\left(\frac{\partial f_0}{\partial Y}\right)_X = \omega\left[\left(\frac{\partial f_0}{\partial W}\right)_{\mu,\ L} - \frac{m}{eB_{eq}R_E^2 L\omega}\left(\frac{\partial f_0}{\partial L}\right)_{\mu,\ W}\right] = \omega\frac{df_0}{dW}, \quad (56)$$

where $B_{eq} = B_E L^{-3}$ is the geomagnetic field strength in the equatorial plane of the L-valued field line.

4.2. Quasi-stationary Injection

Rather than the delta function type of injection considered above, we can also consider the more realistic

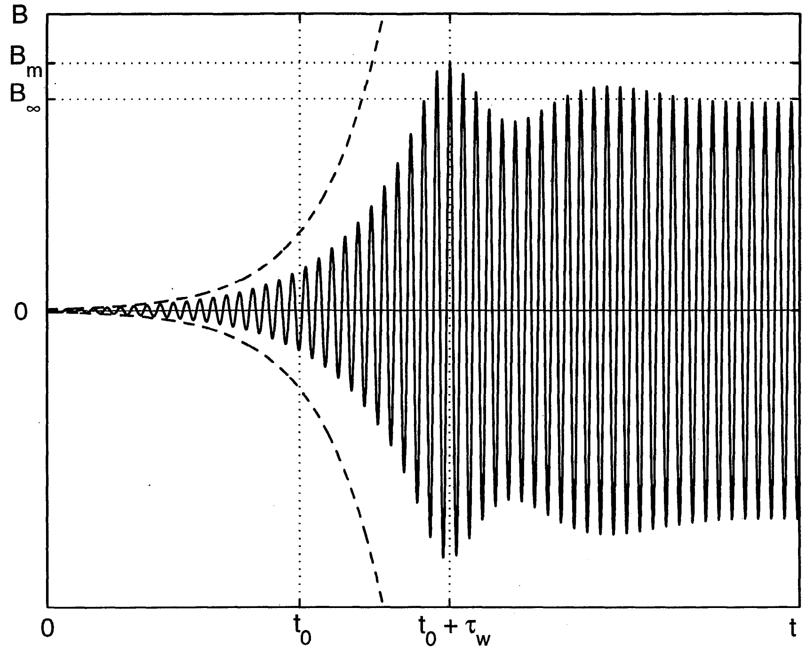

Figure 4. Pulsation evolution expected from nonlinear theory with sudden resonant particle injection (solid line). Initially, the wave grows exponentially over the linear timescale $t_0 = 1/\gamma_L$, where γ_L is the linear growth rate. This is followed by an oscillatory saturation during a characteristic timescale approximately equal to τ_w, i.e., the period of particle phase bouncing around the resonance with the wave. The dashed line shows the envelope of linear growth.

case in which the injection of resonant particles has a finite duration. If we take the case where the variation of the source is so slow that $\partial \ln Q/\partial t \ll \tau_w^{-1}$, then the expression (51) may be further simplified by integrating it by parts. In this case we obtain

$$\dot{U} = \pi^2\omega^2 \int d\mu d\Phi d\varphi \left[\left(\frac{\alpha_N^2 \tau_w}{\Omega'} \frac{dQ}{dW} \right)_t D(0) - \left(\frac{\alpha_N^2 \tau_w}{\Omega'} \frac{dQ}{dW} \right)_0 D(t) \right], \quad (57)$$

where

$$D(t) = 128 \int_0^1 \frac{d\kappa}{\kappa^4 K} \sum_{n=1}^{\infty} \left[\frac{q^{2n}}{(1+q^{2n})^2} \cos \frac{n\pi}{K\kappa} \frac{t}{\tau_w} + \frac{\kappa^5 q^{2n-1}}{(1+q^{2n-1})^2} \cos \frac{2n-1}{2} \frac{\pi}{K} \frac{t}{\tau_w} \right]. \quad (58)$$

The second term in (57), which contains $D(t)$, vanishes within a few τ_w. Therefore one may neglect these oscillations in the course of the wave evolution at $t \geq \tau_w$. Then

$$\dot{U} = \pi^2\omega^2 D(0) \sum_N \int d\mu d\Phi d\varphi \frac{\alpha_N^2 \tau_w}{\Omega'} \frac{dQ}{dW}, \quad (59)$$

where

$$D(0) = 128 \int_0^1 \frac{d\kappa}{\kappa^4 K} \sum_{n=1}^{\infty} \left[\frac{q^{2n}}{(1+q^{2n})^2} + \frac{\kappa^5 q^{2n-1}}{(1+q^{2n-1})^2} \right] \simeq 4. \quad (60)$$

Since $\alpha_N \sim E \sim b$, $\tau_w \sim b^{1/2}$, and $U \sim b^2$, we can rewrite (59) as

$$\dot{U} = \pi^2\omega^2 D(0) \sum_N \int d\mu d\Phi d\varphi \times \left(\frac{\alpha_N^2 \tau_w}{\Omega'} \right)_0 \left[\frac{b(t)}{b(0)} \right]^{3/2} \frac{dQ}{dW}. \quad (61)$$

After integration of (61) we obtain

$$U(t) = U(0) \left[1 + \frac{\pi^2\omega^2 D(0)}{4U(0)} \times \sum_N \int_0^t dt' \int d\mu d\Phi d\varphi \left(\frac{\alpha_N^2 \tau_w}{\Omega'} \right)_0 \frac{dQ(t')}{dW} \right]^4. \quad (62)$$

Suppose further that the injection is not only quasi-stationary but is also constant. In this case the wave amplitude will vary as

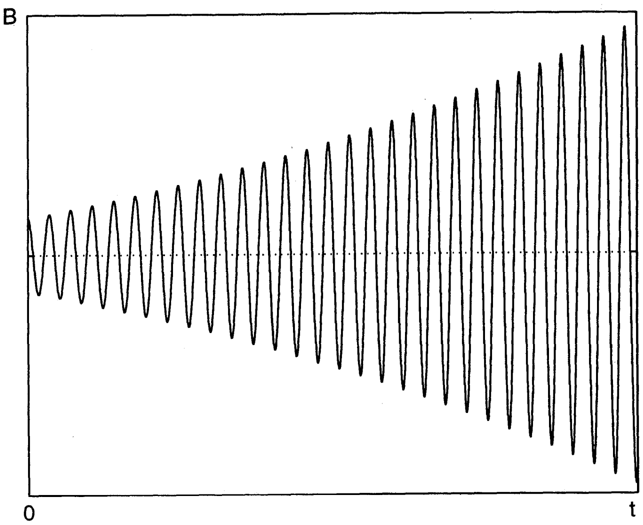

Figure 5. Pulsation evolution expected from nonlinear theory with constant (quasi-stationary) particle injection.

$$b(t) = b(0)\left[1 + \frac{\pi^2\omega^2 D(0)t}{4U(0)} \times \sum_N \int d\mu d\Phi d\varphi \left(\frac{\alpha_N^2 \tau_w}{\Omega'}\right)_0 \frac{dQ}{dW}\right]^2, \tag{63}$$

i.e., asymptotically $b(t) \propto t^2$. Figure 5 shows an example of such evolution.

Note that this case of a slowly changing source of resonant particles seems to be the most relevant to real magnetospheric conditions. It is hard to imagine that a source, which actually appears as a kind of drifting cloud of energetic particles, would have sharp temporal dependence with timescales less than or equal to those of the pulsation period. In this respect the case of sudden injection has only illustrative interest while the quasi-stationary source serves as a reasonable approach to the problem in hand.

5. Discussion

In the previous section we calculated the nonlinear growth and amplitude modulations of Pgs driven by nonlinear drift-bounce resonance. These calculations, of course, neglect the damping effects of ohmic dissipation in the ionosphere. In order for waves to be observed for extended intervals of numerous wave periods, the growth of the instability must overcome ionospheric damping. The linear growth rate of typical waves is given by $\gamma_L \sim \omega\beta$ [e.g., *Southwood*, 1976; *Hughes et al.*, 1978], where $\beta = 2\mu_0 p/B_0^2$ with p as the plasma pressure, and hence for $\beta \sim 0.2$ we have $\gamma_L/\omega \sim 0.2$. Such a growth significantly exceeds the ionospheric damping, even on the nightside. The majority of Pgs are observed in the early morning hours, having foot points in a dark ionosphere. Assuming typical nightside height-integrated Pedersen conductivities $\Sigma_P \sim 0.5-1$ mho, and using the results of *Newton et al.* [1978] at $L = 7$, we find that the ionospheric damping decrement $\gamma_I/\omega \sim 0.075 - 0.04$, much slower than typical wave growth rates.

Observations of Pgs, such as those discussed in section 2, show wave trains that continue for numerous cycles, typically lasting continuously for many hours. If we define the e-folding ionospheric damping time as τ_I, we find that $\tau_I \sim$ 2-4 wave periods for the conditions above [e.g., *Chisham et al.*, 1997]. Hence τ_I is much shorter than the length of typical Pg wave trains, and we infer that Pgs must be continually driven. If Pgs are continually driven over extended periods, then we need to estimate the nonlinear timescale τ_w and assess whether the nonlinear saturation effects are important for Pgs.

Linear theory assumes that the particles, which may be trapped in the wave potential (see Figure 3), interact only once with the wave. If the particles driving the waves are trapped and interact with the waves more than once during the liftime of the wave packet, then a nonlinear theory must be adopted. The nonlinear particle trapping timescale is τ_w, and this can be estimated from $\tau_w/\tau_A \sim (B_0/b)^{1/2}$, where τ_A is the Alfvén wave period. Taking a typical Pg amplitude of ~10 nT in the Earth's magnetosphere [e.g., *Kokubun et al.*, 1989] and taking B_0 to be ~100 nT at the geostationary orbit results in $\tau_w/\tau_A \sim 3$. Consequently, as indicated in Figure 4, the nonlinear timescale τ_w occurs well within the lifetime of typical Pg wave packets.

The estimates of τ_w and τ_I given above suggest that these parameters have similar magnitudes, both being much shorter than typical Pg wave packet lifetimes. Despite the fact that the waves are being continually damped by the ionosphere, the overall Pg amplitude is being maintained by an injection of energy from resonant particles. Since this results in a wave train much longer than τ_w, the resonant particles will interact more than once with the wave fields. The linear theory, which predicts an exponentially growing wave solution, is inadequate and is only valid for times short compared to Pg lifetimes (see Figure 4). This means that if Pg behavior is to be fully understood, a nonlinear theory such as that developed in this paper is required.

It should be noted that similar nonlinear evolution of monochromatic ion-acoustic waves was studied experimentally by *Ikezi et al.* [1978], but the effects of the particle injection in the resonance region were not discussed.

6. Conclusions

The Pg is a unique class of geomagnetic pulsation believed to be associated with injected protons undergoing drift-bounce resonance in the magnetosphere. Previous theoretical explanations of the phenomenon, as discussed in the introduction, are flawed by their assumptions of either small-amplitude (linear theory) or broadband wave packets (random phase approximation) which do not fit the observed properties of these pulsations.

In our paper we have presented a nonlinear description of the drift-bounce resonance in the presence of resonant particle injection. We have demonstrated that nonlinear effects such as amplitude modulation and saturation should occur within the observed lifetime of typical Pg wave packets, hence these nonlinear effects must be included in any model that attempts to explain Pg excitation and evolution.

For clarity we have analyzed two simplified but important cases of sudden and quasi-stationary particle injections. The case of sudden particle injection meets the usual initial problem of the nonlinear wave growth due to instability of an initially given particle distribution. In this case the growth rate starts from its linear value and then begins to oscillate at $t \geq \tau_w$. Finally, at $t \gg \tau_w$ the growth rate vanishes owing to the nonlinear phase mixing and the wave amplitude saturates. In the more realistic case of quasi-stationary particle injection, when the characteristic time of the particle source variation is much longer than τ_w, the evolution of the wave amplitude starts to follow the temporal behavior of the source of newly injected particles. Both cases illustrate the importance of the nonlinear saturation of the excitation mechanism, and, in reality, depending on the form of the source function Q, various regimes for the temporal behavior may be realized.

If Pgs are driven by drift-bounce resonance with injected energetic ions, then the observed modulation of the wave packets may give an indication of temporal variations in the resonant source populations. For example, the particle injection is likely to last for a finite time. As we have shown, the drift-bounce resonance mechanism saturates after a time $\sim \tau_w$, and hence wave amplitude modulations on timescales longer than this are likely to be due to temporal variations in the source. The ionosphere is expected to damp the waves on timescales $\sim \tau_I$, which may be $\sim \tau_w$; however, this damping will generally not be expected to produce any wave amplitude modulations. Moreover, since Pg wave trains last very much longer than τ_I, we can infer that the waves are being driven over an extended period. Hence amplitude modulations during the lifetime of the Pg wave packet could be, at least in part, due to variations in the source populations, and this might provide an important diagnostic for future Pg investigations.

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References

Abramowitz, M., and I. A. Stegun, Handbook of Mathematical Functions, *Appl. Math. Ser.*, vol. 55, Natl. Inst. of Stand. and Technol., Gaithersburg, MD., 1964.

Birkeland, K., Expedition Norvegienne de 1899-1900 pour l'etude des aurores boreales, Resultant des recherches magnetiques, pp. 7-12, *Skr. Nor. Vidensk Akad., Kl. 1, Mat. Naturvidensk Kl.*, *1*, 1901.

Brekke, A., T. Feder, and S. Berger, Pc4 giant pulsations recorded in Tromso, 1929-1985, *J. Atmos. Terr. Phys.*, *49*, 1027, 1987.

Bud'ko, N. I., V. I. Karpman, and O. A. Pokhotelov, Nonlinear theory of circularly polarized VLF and ULF waves in the magnetosphere, *Cosmic Electrodyn.*, *3*, 165, 1972.

Chisham, G., Giant pulsations: An explanation for their rarity and occurrence during geomagnetically quiet times, *J. Geophys. Res.*, *101*, 24,755, 1996.

Chisham, G., and D. Orr, Statistical studies of giant pulsations (Pgs): Harmonic mode, *Planet. Space Sci.*, *39*, 999, 1991.

Chisham, G., and D. Orr, The association between giant pulsations (Pgs) and the auroral oval, *Ann. Geophys.*, *12*, 649, 1994.

Chisham, G., D. Orr, and T. K. Yeoman, Observations of a giant pulsation (Pg) across an extended array of ground magnetometers and on auroral radar, *Planet. Space Sci.*, *40*, 953, 1992.

Chisham, G., I. R. Mann, and D. Orr, A statistical study of giant pulsation latitudinal polarization and amplitude variation, *J. Geophys. Res.*, *102*, 9619, 1997.

Glassmeier, K.-H., Magnetometer array observations of a gaint pulsation event, *J. Geophys.*, *54*, 125, 1980.

Goldstein, H., *Classical Mechanics*, Addison-Wesley, Reading, Mass., 1965.

Green, C. A., Observations of Pg pulsations in the northern auroral zone and at other lower latitude conjugate regions, *Planet. Space Sci.*, *27*, 6, 1979.

Green, C. A., Giant pulsations in the plasmasphere, *Planet. Space Sci.*, *33*, 1155, 1985.

Hillebrand, O., J. Münch, and R. L. McPherron, Ground-satellite correlative study of a giant pulsation event, *J. Geophys.*, *51*, 129, 1982.

Hughes, W. J., and D. J. Southwood, The screening of micropulsations signals by the atmosphere and ionosphere, *J. Geophys. Res.*, *81*, 3234, 1976.

Hughes, W. J., D. J. Southwood, B. Mauk, R. L. McPherron, and J. N. Barfield, Alfvén waves generated by an inverted plasma energy distribution, *Nature*, *275*, 43, 1978.

Ikezi, H., K. Schwartzenegger, A. L. Simons, Y. Iosawa, and T. Kamimura, Nonlinear self-modulation of ion-acoustic waves, *Phys. Fluids*, *21*, 239, 1978.

Karpman, V. I., B. I. Meerson, A. B. Mikhailovskii, and O. A. Pokhotelov, The effects of bounce resonances on wave growth rates in the magnetosphere, *Planet. Space Sci.*, *25*, 573, 1977.

Kokubun, S., K. N. Erickson, T. A. Fritz, and R. L. McPherron, Local time asymmetry of Pc4-5 pulsations and associated particle modulations at synchronous orbit, *J. Geophys. Res.*, *94*, 6607, 1989.

Meirovich, L., *Methods of Analytical Dynamics*, McGraw-Hill, New York, 1970.

Meyerson, B. I., and O. A. Pokhotelov, Self-consistent diffusion of trapped particles on bounce-drift resonance with geomagnetic pulsations, *Geomagn. Aeron.*, *18*, 79, 1978.

Mikhailovskii, A. B., and O. A. Pokhotelov, New mechanism for generation of geomagnetic pulsations by fast particles, *Sov. J. Plasma Phys.*, *1*, 430, 1975.
Nayfeh, A. H., *Perturbation Methods*, John Wiley, New York, 1973.
Newton, R. S., D. J. Southwood, and W. J. Hughes, Damping of geomagnetic pulsations by the ionosphere, *Planet. Space Sci.*, *26*, 201, 1978.
Northrop, T. C., *The Adiabatic Motion of Charged Particles*, Wiley-Interscience, New York, 1963.
O'Neil, T. M., Collisionless damping of nonlinear plasma oscillations, *Phys. Fluids, 8*, 2255, 1965.
Poulter, E. M., W. Allan, E. Nielsen, and K.-H. Glassmeier, STARE radar observations of a Pg pulsation, *J. Geophys. Res.*, *88*, 5668, 1983.
Southwood, D. J. ,The behavior of ULF waves and particles in the magnetosphere, *Planet. Space Sci.*, *21*, 53, 1973.
Southwood, D. J., A general approach to low-frequency instability in the ring current plasma, *J. Geophys. Res.*, *81*, 3340, 1976.
Southwood, D. J., J. W. Dungey, and R. J. Etherington, Bounce resonant interaction between pulsations and trapped particles, *Planet. Space Sci.*, *17*, 349, 1969.
Takahashi, K., N. Sato, J. Warnecke, H. Lühr, H. E. Spence, and Y. Tonegawa, On the standing wave mode of giant pulsations, *J. Geophys. Res, 97*, 10,717, 1992.

O. A. Pokhotelov and Y. G. Khabazin, Institute of Physics of the Earth, 123810 Moscow, Russia (e-mail: pokh@uipe-ras.scgis.ru, khabazin@uipe-ras.scgis.ru)
I. R. Mann and D. K. Milling, Department of Physics, University of York, Heslington, York, YO10 5DD, England (e-mail: ian@aurora.york.ac.uk, dave@samsun.york.,ac.uk)
P. K. Shukla, Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany
L. Stenflo, Department of Plasma Physics, Umeå University, S-90187, Umeå, Sweden

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