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Data Envelopment Analysis - Basic Models and their Utilization

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Data Envelopment Analysis (DEA) is a decision making tool based on linear programming for measuring the relative efficiency of a set of comparable units. Besides the identification of relatively efficient and inefficient units, DEA identifies the sources and level of inefficiency for each of the inputs and outputs. This paper is a survey of the basic DEA models. A comparison of DEA models is given. The effect of model orientation (input or output) on the efficiency frontier and the effect of the convexity requirements on returns to scale are examined. The paper also explains how DEA models can be used to assess efficiency.

Keywords: Efficiency, DEA models, Efficiency frontier.

1 Introduction

One of the most important principles in any business is the principle of efficiency; where the best possible economic effects (outputs) are attained with as little economic sacrifices as possible (inputs). Efficiency can be defined as the demand that the desired goals are achieved with the minimum use of the available resources. In order to assess the relative efficiency of a business unit, it is necessary to consider the conditions and operation results of other units of the same kind and to determine the real standing of the results of such a comparison.

In a simple case where units have a single output and a single input, efficiency is defined as their ratio. However, typical organizational units have multiple and incommensurate inputs and outputs. Data Envelopment Analysis was introduced by Charnes, Cooper and Rhodes (1978) to assess the relative efficiency of organizational units with multiple inputs to produce multiple outputs. The authors of DEA defined the efficiency of the unit under evaluation as the ratio of the sum of its weight outputs to the sum of its weight inputs.

Recently, the Data Envelopment Analysis method is becoming popular for assessing the relative efficiency of business entities. DEA is a technique of mathematical programming that enables the determination of a unit's efficiency based on its inputs and outputs, and compares it to other units involved in the analysis. The DEA can be described as data-oriented as it effects performance evaluations and other inferences directly from the observed data and with minimal assumptions. The efficiency of a Decision Making Unit (DMU) is measured relative to all other DMUs with the simple restriction that all DMUs lay on or below the extreme frontier. DEA is a non-parametric method as it does not require any assumption about functional form (e.g. a regression equation, a production function, etc.). It is a methodology directed at the frontier rather than at central tendencies. While statistical procedures are based on central tendencies, DEA is a process of extremities. DEA analyzes each DMU separately and calculates a maximum performance measure for each unit. DEA has become one of the most popular fields in operations research, with applications involving a wide range of context (Thanassoulis, 2001).

DEA is one of the most popular fields in operations research (Emrouznejad et al., to appear; Seiford, 1997). Since the seminal work of Charnes, Cooper and Rhodes (1978) and since 1995, there was literally "exponential" growth in the number of publications. Between 1995 and 2003, the number of relevant publications stabilized at about 225 per year. However, in the last four years (2004-2007), the number increased to approximately 360 per year. Journals such as the *European Journal of Operational Research, Journal of Productivity Analysis,* and the *Journal of the Operational Research Society* are the most utilized.

The papers presented show ample possibilities for using the DEA for the evaluation of the performance of bank branches, schools, university departments, farming estates, hospitals and social institutions, military services, entire economic systems (regions) and other things. DEA is a methodology of several different interactive approaches and models used for the assessment of the relative efficiency of DMU and for the assessment of the efficiency frontier. It supplies important information for managing the operations of efficient and inefficient units. This paper is a survey of the basic DEA models. Some ways in which these models can be used are also given.

2 DEA Models

DEA methodology, originally proposed in (Charnes et al., 1978), is used to assess the relative efficiency of a number of entities using a common set of incommensurate inputs to generate a common set of incommensurate outputs. The original motivation for DEA was to compare the productive efficiency of similar organizations, referred to as DMUs. The problem of assessing efficiency is formulated as a task of fractional programming, but the application procedure for DEA consists of solving linear programming (LP) tasks for each of the units under evaluation.

Let x_{ij} - denote the observed magnitude of i - type input for entity j ($x_{ij} > 0,\ i = 1, 2, ..., m, j = 1, 2, ..., n$) and y_{rj} - the observed magnitude of r-type output for entity j ($y_{rj} > 0, r = 1, 2, ..., s, j = 1, 2, ..., n$). Then, the Charnes-Cooper-Rhodes (CCR) model is formulated in the following form for the selected entity k:

MODEL (M1)

$$Maximize \qquad h_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \tag{1}$$

Subject to

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad j = 1,2,\ldots,j_k,\ldots,n \tag{2}$$

$$u_r \ge \varepsilon, \quad r = 1,2,\ldots,s \tag{3}$$

$$v_i \ge \varepsilon, \quad i = 1,2,\ldots,m \tag{4}$$

Where:

- v_i is the weights to be determined for input i;
- m is the number of inputs;
- u_r is the weights to be determined for output r;
- s is the number of outputs;
- h_k is the relative efficiency of DMU_k;
- n is the number of entities;
- ε is a small positive value.

The relative efficiency h_k, of one decision-making unit k, is defined as a ratio of the weighted sums of their outputs (virtual output) and the weighted sums of their inputs (virtual input). As for the decision-making unit k, for which a maximum in objective function (1) is sought, the condition (2) is true, meaning that it is obviously $0 < h_k \le 1$, for each DMU_k. The weights v_i and u_r show the importance of each input and output and are determined in the model so that each DMU is efficient as much as possible. Given that the condition (2) is true for every DMU, it means that each of them lies on the efficiency frontier or beyond it. If Max $h_k = h_k^* = 1$, it means that efficiency is being achieved, so we can tell that DMU_k is efficient. Efficiency is not achieved for $h_k^* < 1$ and DMU_k is not efficient in that case. DMU_k is to be considered relatively inefficient, if it is possible to expand any of its outputs without reducing any of its inputs, and without reducing any other output (output orientation), or if it is possible to reduce any of its inputs without reducing any output and without expanding some other inputs (input orientation).

Problem (1) - (4) is nonlinear, nonconvex, with a linear and fractional objective function and linear and fractional constraints. Using a simple transformation developed by Charnes and Cooper (1962), the above CCR ratio model can be reduced to the LP form (the Primal CCR model) so that the LP methods can be applied. In this model, the denominator has been set equal to 1 and the numerator is being maximized. The input oriented CCR primal model is:

MODEL (M2)

$$Max \quad h_k = \sum_{r=1}^{s} u_r y_{rk} \tag{5}$$

subject to

$$\sum_{i=1}^{m} v_i x_{ik} = 1 \tag{6}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad (j = 1,\ldots, n) \tag{7}$$

$$u_r \ge \varepsilon, \quad (r = 1,.., s) \tag{8}$$

$$v_i \ge \varepsilon, \quad (i = 1,.., m) \tag{9}$$

The mathematical model given above is linear and can be solved using any of the familiar programs packages for LP. However, in practice, it is often solved dual task for problem (5) - (9), which is:

MODEL (M3)

$$Min \quad Z_k - \varepsilon\,(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-) \tag{10}$$

subject to

$$\sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = y_{rk}, \quad (r = 1,2,\ldots, s) \tag{11}$$

$$Z_k \cdot x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0, \quad (i = 1,2,\ldots, m) \tag{12}$$

$$\lambda_j, s_r^+, s_i^- \ge 0; \quad Z_k - sign\ unbound. \tag{13}$$

The basic idea behind DEA is best conveyed in the dual CCR model (M3), which is easy to solve because of its calculating size. The dual model for a given unit using input and output values of other units tries to construct a hypothetical composite unit out of the existing units. If it is possible, the given unit is inefficient, otherwise it is efficient and lies at the efficiency frontier. The efficiency frontier is a set of segments interconnecting all the efficient DMUs and it acts as an envelope for inefficient units. An inefficient unit can be enveloped below (input-oriented model) or above (output-oriented model).

Because the problems described by models (M2) and (M3) are associated and because of the duality theorem in linear programming, DMU_k is efficient if and only if conditions for optimal solution (λ^*, s^{+*}, s^{-*}, Z_k^*) are accomplished for the problem (10)-(13):

$$Z_k^* = 1 \tag{14}$$

$$s^{+*} = s^{-*} = 0 \text{ in all alternate optima} \tag{15}$$

Using the optimal solution (λ^*, s^{+*}, s^{-*}, Z_k^*) of the problem (10) - (13), it can be determined:

$$X''_k = Z_k^* X_k - s^{-*} \tag{16}$$

$$Y''_k = Y_k + s^{+*} \tag{17}$$

It can be shown that after CCR projection (16), (17), DMU_k with altered inputs X''_k and outputs Y''_k becomes efficient. The difference $\mathbf{\Delta Xk = Xk - X''k}$ and $\mathbf{\Delta Yk = Y''k - Yk}$ respectively shows the estimated amount of input and output inefficiency. Thus it can be seen for inefficient DMU_k, how to change its inputs and outputs, so it would become efficient. We should emphasize that, for each DMU_j (j = 1, 2, ..., n) taken as DMU_k, an appropriate linear programming problem is solved (10) - (13). Hence, we should solve *n* linear programming tasks with the form (10) - (13), with (*s+m+1*) variables and (*s+m*) constraints per task.

The CCR models (dual and primal) with input orientation are still the most widely known and used DEA models despite the numerous modified models that have appeared. The CCR models assume constant returns to scale. DMU operates under constant returns to scale if an increase in the inputs results in a proportionate increase in the output levels. These models calculate an overall efficiency in which both its pure technical efficiency and its scale efficiency are aggregated into a single value. The envelopment surface obtained from the CCR model has the shape of a convex cone. The efficient DMUs would lie on top of the structure, while the inefficient ones would be covered under the cone. In a single input and output case, the efficiency frontier is reduced to a straight line. The CCR model yields the same efficiencies regardless of whether it is input- or output-oriented.

The most important extension of the original CCR models is given in Banker *et al.* (1984) where an additional constraint was introduced in model (M3):

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{18}$$

This constraint enables variable returns to scale and provides that the reference set is formed as a convex combination of DMUs, which are in the set (those that have positive value for λ in the optimal solution). The DMU operates under variable returns to scale if it is suspected that an increase in inputs does not result in a proportional change in the outputs. The convexity constraint ensures that the composite unit is of similar scale size as the unit being measured. The BCC model yields a measure of pure technical efficiency that ignores the impact of the scale size by only comparing a DMU to a unit of similar scale. Often, small units are qualitatively different from large units and a comparison between the two may distort measurements of comparative efficiency. The measured efficiency is always at least equal to the one given by the CCR model. The envelopment surface obtained from the BCC model results in a convex hull.

The DEA model can be input or output oriented. The input oriented model contracts the inputs as far as possible while controlling the outputs. In an input oriented model, an inefficient unit is made efficient through the proportional reduction of its inputs, while its outputs proportions are held constant. The output oriented model expands the outputs as far as possible while controlling the inputs. In an output oriented model, an inefficient unit is made efficient through the proportional increase of its outputs, while the inputs' proportions remain unchanged. An inefficient DMU can be made more efficient by projection onto the frontier. Model orientation determines the direction of the projection for inefficient DMUs. In an input orientation, one improves efficiency through the proportional reduction of inputs, whereas an output orientation requires proportional augmentation of the outputs.

The input and output measurements are always the same in the CCR model, but frequently differ in the BCC model. Thus, if we are using the CCR model, we can solve one model and give either interpretation. If we solve the BCC input model, we can only give an input interpretation and we must solve the BCC output model for an output interpretation. Another difference between the BCC and CCR models lies in the scalar transformations of all data for a given DMU. The efficiency measure in the CCR model is unchanged by scalar transformations, since the efficiency ratio of the scaled DMU is unchanged. On the other hand, the scalar transformations of a given DMU change the scalar size and could easily affect the efficiency measurements from the BCC model.

Numerous extensions of the basic DEA models are presented in the literature (Charnes et al., 1995; Cooper et al., 2005; Thanassoulis, 2001). Some of the extensions are:

- constraints are placed on the weights for particular inputs and outputs (Thanassoulis, 2001),
- constraints are placed on the amount of particular virtual inputs and outputs (Thanassoulis, 2001),
- inputs and outputs that cannot be controlled are brought into analysis (Banker and Morey, 1986),
- categorical variables are brought into the model (Banker and Morey, 1986),
- models for ranking relatively efficient DMUs are developed (Andersen and Petersen, 1993).

3 DEA Model Utilization

In the first part of this section, we discuss how DEA models can be used to assess DMUs. A key stage in a DEA assessment is the identification of the input/output variables pertaining to the units being assessed, see (Boussofiane et al., 1991). Since DEA is used to evaluate performances by directly considering input and output data, the results will depend on the input/output choice for the analysis and the number and homogeneity of the DMUs to be evaluated. In this stage it is important to consult the people working in the units that are to be evaluated, so major inputs and outputs can be identified properly. It is important to envelop all the important inputs in the analysis, namely all the resources used, and all the important outputs (the products and services produced). A large number of inputs and outputs compared to the number of units to be evaluated may reduce the discriminating tendency of the method. The larger the number of inputs and outputs compared to the number of units to be evaluated, the greater the chances that the units will allocate appropriate weights to a single subset of inputs and outputs that will make them appear efficient. In order to preserve the discriminating power of the method, the number of the units to be evaluated should be much larger than the number of inputs and outputs. Some authors suggest from experience that the number of units in the DMU should exceed the number of inputs and outputs by at least twice. Boussofiane et al., (1991) propose testing the correlation between the inputs and outputs, as one possible way to reduce their number. If a pair of inputs is positively correlated then one may be omitted without any implications on the efficiency to be rated. The same applies to outputs. The availability of data may also affect the choice of inputs and outputs in practice. If the data on an input or output is not available then the possibility of using a substitute for which such data is either available or can be obtained relatively easily should be checked.

DEA is a methodology with several different interactive approaches and models used for the assessment of the relative efficiency of a DMU and for the assessment of the efficiency frontier. It supplies important information for managing the operations of both efficient and inefficient units. For each inefficient unit, DEA identifies a set of relatively efficient units, thus making a peer group for the inefficient unit. The peer set for an inefficient unit constitutes the units with the same optimum weights as the inefficient unit, but with a relative efficiency rating of 1. Such peer units are identified fairly easily by the fact that they all have a positive value for λ in the optimum solution to (M3) for an inefficient unit. The identification of peer groups should be very useful in practice. Peer units can be used to highlight the weak aspects of the performance of the corresponding inefficient unit. The input/output levels of a peer unit can also sometimes prove useful target levels for the inefficient unit.

From the solution of any DEA model, we can get information on how much a relatively inefficient unit should reduce their inputs or increase their outputs in order to become relatively efficient. For each inefficient DMU (one that lies below the frontier), DEA identifies the sources and level of inefficiency for each input and output. The level of inefficiency is determined by comparison with a single referent DMU or a convex combination of other relevant DMUs located on the efficient frontier that utilize the same level of inputs and produce the same or higher level of outputs. In the previous section, we have seen that we can reach the level of inefficiency using the optimal solution of model (M3) and relations (16) and (17). We can arrive at similar information through sensitivity analysis of the optimal solution in model (M1). These results are very important to managers, because they indicate the sources of inefficiency for relatively inefficient DMUs.

Improvement of efficiency in not only inefficient but also the efficient units can be attained by identifying an efficient operating practice. It can usually be found in the relatively efficient units. However, among the relatively efficient units, some are better than others at setting a good example. A need to distinguish the relatively efficient units and find a good operating practice, emerges from the essence of a DEA model that allows a unit to select the weights that will show it as having maximum efficiency. In this way, the units may appear efficient because all very small input subsets will be ignored within their choice of weights. Moreover, the inputs and outputs assigned larger weights could be given a secondary importance while those that are ignored could be associated with the units' main functions.

To distinguish the relatively efficient DMUs, Boussofiane *et al.* (1991) suggested the following methods (or a combination of these):

- cross efficiency matrix,
- the distribution of virtual inputs and outputs,
- weight restriction,
- the frequency by which an efficient unit appears in the peer groups.

Basic DEA models evaluate the relative efficiency of DMUs but do not allow any ranking of the efficient units themselves. This is the main weakness of the basic DEA models. One way to rank efficient DMUs is to modify basic DEA models. One of these has been formulated by Andersen and Petersen (1993). The basic idea is to compare the unit under evaluation with a linear combination of all the other units in the sample, i.e., the observed DMU is excluded from the peer group. Efficient units may proportionally increase the value of the input vector while preserving efficiency. The units obtain an efficiency score above 1. This score reflects the radial distance of the DMU under evaluation from the production frontier, estimated with that DMU excluded from the sample. This approach provides an efficiency rating for efficient units that is similar to the rating of the inefficient units above.

The second part of this section relates to two of the most important applications of DEA in Serbia. DEA is used very successfully for the comparative analysis and ranking of 30 regions in Serbia (Martić and Savić, 2001) and in assessing the relative efficiency of 20 investment programs in agriculture (Martić et al., 1996).

In Martić and Savić (2001), DEA is used to estimate how well regions in Serbia utilize their resources. Based on the data for 4 inputs (Arable area, Active fixed assets, Consumption of electricity and Population) and 4 outputs (Gross domestic product, Total number of physicians, Total number of pupils in primary school and Total number employed in the social sector), an output-oriented CCR DEA model is applied and it appears that 17 out of the 30 regions are efficient. 5 out of 7 regions from Vojvodina and 12 out of 18 regions from central Serbia are efficient, while all the regions from Kosovo and Metohia are inefficient. Linear Discrimination Analysis (LDA) is also applied to determine the regions. The comparison between the DEA and the LDA results indicated that LDA could be a useful tool for checking the DEA results.

The authors analyzed which changes in inputs and outputs inefficient regions should make in order to become efficient. However, it was shown that goals obtained using the basic CCR models and DEA models with exogenously fixed inputs are not quite realistic. It is difficult to explain that some regions have goals to reduce the size of the population or arable area. For inefficient regions, we experimentally determined realistic goals without any reduction of its inputs. These goals could be achieved more easily but are not in the production possibility set.

In order to rank the 17 efficient regions, an output-oriented version of Andersen-Petersen's DEA model and a cross efficiency matrix are used. The same or similar rank was obtained for 12 regions. A comparison of the obtained ranks shows that the regions were ranked more realistically with the cross efficiency matrix. Nevertheless, the same 8 regions are ranked in the top ten in both approaches. The ranks obtained showed that the region of Nisava, the City of Belgrade and the region of Jablanica utilize their resources most efficiently.

In Martić et al. (1996), it is shown how DEA could be used to provide information concerning efficient and inefficient investments, and also to rank the efficient investments and indicate how to improve the efficiency of these inefficient investments. An example illustrates the application of the approach proposed to assess the relative efficiency of 20 investment programs in agriculture and their ranking. The agriculture bank management often faces a problem in measuring the efficiency of new investments. Normally, the government is effectuating its investments in the agricultural sector through the banking system, by nominating one or several banks to handle the investment loans to various agricultural firms competing for funds. The banks are facing the tremendous problem of deciding, under the constraints of limited funds and the wish to maximize the economic return, which firms to select for their investment portfolio. Further, banks have to take care of the uniform development of all regions, unemployment levels, ecological factors, etc.

The first step in assessing the relative efficiency of a set of investment programs in the agricultural sector is the definition of the sets of common inputs and common outputs. By reviewing the standard practice of evaluating the investment programs that is used by the investment banks, 4 inputs (**Required loan amount**, **Labour costs**, **Production costs** and **Energy consumption**) and 3 outputs (**Expected value of domestic sales**, **Expected value of exports**, **Social justifiability and environmental acceptability**) have been investigated. The definitions and corresponding units of measure are obvious for all inputs and the first two outputs listed. However, social justifiability and environ-

Table 1: Input and output values for 20 investment programs competing for loans

	X1	X2	X3	X4	Y1	Y2	Y3	Z_k	Peer group
P1	250	60	50	30	200	100	90	0.88	P10,P13,P14
P2	1500	400	150	125	600	250	60	0.59	P7,P10,P17
P3	800	350	300	85	600	450	40	1.00	-
P4	500	150	200	75	500	360	60	1.00	-
P5	200	100	120	60	330	250	50	1.00	-
P6	600	100	50	35	180	75	80	0.60	P7,P10,P14,P17
P7	1500	500	90	40	500	200	100	1.00	-
P8	1000	360	300	90	750	500	65	1.00	-
P9	500	120	100	60	350	180	50	0.64	P7,P10,P18
P10	300	45	80	50	440	230	80	1.00	-
P11	700	160	60	30	300	130	100	0.96	P7,P10,P14,P18
P12	500	200	50	20	200	80	85	0.90	P10,P18
P13	200	30	50	40	160	90	100	1.00	-
P14	100	50	20	15	125	50	80	1.00	-
P15	800	180	200	100	700	400	90	0.83	P8,P10,P18
P16	1200	300	250	115	750	400	55	0.71	P8,P10,P18
P17	250	100	20	25	180	70	100	1.00	-
P18	400	80	30	10	130	60	90	1.00	-
P19	1000	250	130	100	600	270	95	0.72	P7,P10,P17
P20	300	75	60	45	225	100	40	0.62	P10,P14,P17

mental acceptability deserve further explanation. Namely, any investment program must be socially justifiable and environmentally acceptable. The term socially justifiable encompasses a number of factors such as the unemployment level, regional level of development and similar. Each of the investment programs proposed has been assigned a social justifiability and environmental acceptability level using a scale from 0 to 100. Thus, an investment program that is fully socially justifiable and environmentally acceptable has an assigned level of 100. Conversely, an investment program that cannot be socially and/or environmentally justified at all, has a level of 0.

An illustrative example, based on the data for 20 investment programs competing for loans, is analyzed here. Their input and output values are presented in Table 1. The 20 linear programs of type (10) - (13) are formulated, consisting of 28 variables $(m+n+s+1=28)$ and 7 constraints $(m + s = 7)$. The results were obtained using the E-DEA programming package (see Martić & Savić, 2001)), developed at the Faculty of Organizational Sciences from Belgrade and are also presented in Table 1.

The solution obtained allows the classification of the investment programs into a set of efficient programs with a relative efficiency of 1 and a set of inefficient programs with a relative efficiency less than 1. The number of relatively efficient investment programs is 10. For each inefficient program, the list of peer programs is given.

Boussofiane *et al.*, 1991, proposes that a simple count of the frequency that an efficient unit appears in different peer groups could be used as an alternative indicator of good operating practice. This count indicates the extent that a relatively efficient unit is a self evaluator or an evaluator of other units. According to the frequency count, the results obtained indicate that the investment program P10 appears in all the peer groups. A satisfactory rating is given to the investment programs P18 and P7, appearing in 5 peer groups, as well as P14 and P17 with 4 appearances.

One of the ways to compare relatively efficient units is the distribution of virtual inputs and outputs. Virtual inputs and/or outputs are obtained by multiplying their magnitudes with the corresponding optimal weights v_i and u_r. Thus, $u_r^* * y_{rj}$ is a virtual output r for unit j, where u_r^* denotes the optimal value for u_r in (M1). The magnitudes of the virtual inputs and outputs show the contribution of each input and output to the relative efficiency of the subject unit in comparison with the other units. The optimal weights for each of the 4 inputs and 3 outputs of the efficient units are obtained as the values of dual variables in (M2). Based on that, the values of the virtual inputs and outputs are computed indicating how each of the efficient units attained their maximum efficiency. The investment programs P3 and P4 have the maximum efficiency due to the satisfactory value of mix y_2/x_4, because the largest proportion in the virtual output comes from y_2 and from x_4 in the virtual input. The investment program P5 is relatively efficient, mostly due to output y_2 because of its best mix y_2/x_1 and the satisfactory value of mix y_2/x_4. A similar reasoning explains the relative efficiency of the other efficient units.

Further, it is possible to analyze the possible proportional change of inputs and/or outputs needed to make an inefficient investment program into an efficient one, establishing the sensitivity of the efficiency to changes in inputs and/or outputs. By using the optimal solution of model (M2) and relation (16) and (17), set targets can be determined for a relatively inefficient unit to guide them towards improved performance. This set of targets is input-oriented as the main changes are to the input levels. The necessary value of the inputs and outputs that make each inefficient investment program become efficient are given in Table 2.

The results obtained, show that DEA can be successfully used in supporting the decision making process of investment banks. The results of the application of the DEA method to assess the relative efficiency of investments in agriculture provide:

- a measure of the greatest relative efficiency that each investment program can achieve according to the factors and fields included in the analysis,
- information about the most important inputs and outputs for each efficient investment,
- the list of efficient investments that form a peer group for each inefficient investment and
- a report on the excess of inputs and the lack of outputs when the relative efficiency of an investment is less than 1.

In Martić, Krčevinac & Petrić, it is also shown how the DEA method could be used for ranking a set of effi-

Table 2: The value of the inputs and outputs that make each inefficient investment program become efficient

	X1	X2	X3	X4	Y1	Y2	Y3
P1	164	53	38	26.5	214	190	90
P2	637	236	89	74	600	288	100
P6	264	60	30	21	180	80	80
P9	315	56	64.5	39	350	180	72
P11	580	153	57	29	300	139	100
P12	450	88	43	18	200	97	98
P15	662	143	165	82	769	400	94
P16	952	175	164	81	750	500	84
P19	622	181	94	72.5	600	362	98
P20	187	47	37	27	225	110	63

cient investment programs in agriculture using Andersen-Petersen's DEA model and a cross efficiency matrix. The results obtained are analyzed and compared.

4 Conclusions

DEA is a non-parameter methodology for evaluating the efficiency of non-profit DMUs. It contains solutions for several mutually connected linear programming mathematical models for each of the DMUs. While each of these models addresses managerial and economic issues and provides useful results, their orientations are different and, more important, they generalize and provide contact with these disciplines and concepts. Thus, the models may focus on increasing, decreasing or constant returns to scale as found in economics, which are here generalized into the form of multiple outputs.

The extensive but probably incomplete bibliography (Emrouznejad et al., to appear) is intended to document the diffusion and growth of DEA usage. The bibliography shows DEA applications in a wide range of contexts, such as education (public schools and universities), health care (hospitals, clinics and physicians), banking, the armed forces (recruiting and aircraft maintenance), auditing, sports, market research, mining, agriculture, retail outlets, organization effectiveness, transportation (ferries and highway maintenance), public housing, index number construction, benchmarking, etc.

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