An Alternative Sorting Procedure for Interactive Group Decision Support Based on the Pseudo-criterion Concept

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ABSTRACT

An original interactive procedure is proposed, which aims at overcoming some of the major weaknesses of existing pseudo-criterion based methods for group decision analysis. It refers to absolute judgements of feasible alternatives and is focused on complementary activities of opinion elicitation and robustness analysis. As a foundation, four interdependent principles are introduced – problem localization, interactivity on the basis of progressiveness approach, semiautomatic derivation of criteria weights according to selective effects of veto thresholds, and group consensus seeking. The principles are grounded and realized by appropriate methodological solutions.

Keywords: Decision Support, Multi-criteria Decision Analysis, Pseudo-criterion, Alternative Sorting, Group Work, Consensus, Fuzzy Relation.

INTRODUCTION

The application of the pseudo-criterion and the outranking relation concepts represents one of the fundamental approaches to decision analysis [9]. The use of indifference, preference and veto thresholds deals in an effective and practical way with imprecision, indetermination and uncertainty of the data. Real-life applications of outranking methods show that a threshold model is easily accepted by decision-makers, while in contrast, capturing inaccuracies with probability distributions has been found to be somewhat difficult for people to understand [6]. Yet, existing methods for group decision analysis that are based on the concepts of pseudo-criterion and outranking relation have several serious drawbacks: a substantial cognitive load is put on a person because of many required input parameters, a poor insight into the derivation of results from input data is given, a quantitative decision model is insufficiently adaptable, and most important of all, credible and just group agreement is not assured.

A procedure that overcomes the above-mentioned difficulties has to be interactive and iterative, should incorporate strong analytical capabilities, and must enable firm conformity among decision-makers. The method, which is defined in this paper, rests on four concepts – problem localization, semiautomatic weight derivation, progressiveness principle, and group consensus seeking.

In addition to the classical ELECTRÉ (*ELimination Et Choix Tradusiant la RÉalité*) [7, 9] and PROMETHEE (*Preference Ranking Organisation METHod for Enrichment Evaluations*) [1] families, there already exist some interactive methods. An approach that determines parameter values according to sorted alternatives [2] is related to the machine learning theory. It is highly interactive, but it does not support group deciding. A search procedure that has been introduced in [6] is limited to the criteria weight space, does not consider veto thresholds, and introduces nondifferentiable optimization programs. Interactive trichotomy segmentation [5] is founded on the localization principle. Its drawbacks are the following: alternatives are sorted into three categories, criteria importance coefficients cannot be manually set, it is suited solely to an individual decision-maker, and finally, it does not allow for continuous change from indifference to weak and strict preference, which is essential for the sake of inaccuracy treatment.

PROBLEM LOCALIZATION

To enable high adaptability of a quantitative model and high comparability of individual participants' results, the alternative sorting analysis is implemented. Sorting refers to the absolute assignment of a set of alternatives into pre-existing ordinally defined categories or classes [12]. In contrast to the more usual ranking approach, where as much as $m \cdot (m-1)$ relative pairwise comparisons between alternatives have to be considered, only m pieces of information about category memberships are needed. But sorting by itself does not guarantee a fulfilment of both specified conditions. For this reason, the localization principle is introduced. The global problem of assigning variants to $p + 1$ ordered classes is reduced to the two-categorical partition of a set of feasible alternatives; all acceptable choices belong to the positive category C^+, while unsatisfactory ones are members of the negative category C^-. Many advantages appear:

- Since any two categories have to be delimited by a reference vector, which is also termed a profile, and since for each profile and for each criterion four parameters have to be considered, instead of $4 \cdot p \cdot n$ only $4 \cdot n$ input values are necessary. The cognitive load is thus considerably reduced.
- Because of mental and time constraints, a decision-maker is rarely capable of altering many reference vectors at once. It is therefore difficult for him to figure out how different profiles affect the alternative evaluation process. But when he concentrates on only one profile instead, it is easy for him to modify referential values. By doing so he can tighten or loosen demands and see what effect this has on selection. Consequently, learning about a given problem situation, a decision model and advantges or weaknesses of alternatives is greatly improved.
- Dispersion of alternatives across classes is reduced. Vice versa, comparability of individuals' results is increased.
- Because fewer input parameters are required, unification of opinions becomes an easier task.
- The problem localization principle enables semiautomatic derivation of criteria weights according to veto thresholds.

Implementation of the Localization Principle

In order to implement the alternative sorting analysis, some basic notions of the ELECTRÉ TRI method [7] are used. But yet, the concepts of ELECTRÉ TRI must be modified. The set of alternatives is partitioned into two exclusive categories. They are delimited by the profile b, which is defined as a vector of n referential values on criteria domains. Let this vector be denoted as $(g_1(b), \ldots, g_n(b))$. Let similarly $g_j(a_i)$ denote the value of an alternative a_i that is measured with regard to a criterion x_j. Two parameters allow for compensation – the indifference threshold q_j and the preference threshold p_j. These thresholds form the basis for computing the indices $c_j(a_i, b)$ and $c_j(b, a_i)$, which express the degree of concordance with the assertions "the alternative a_i is at least as good as the profile b" and "the profile b is at least as good as the alternative a_i", respectively. Each index considers a single criterion x_j. Its contribution to the aggregation is determined by the weighting coefficient w_j. The discordance concept is also applied to model partial incompensation between criteria. It is grounded on the veto threshold v_j.

The threshold model generally leads to three different types of binary relations: preference, indifference and incomparability. The incomparability relation occurs when there exist at least two conflicting criteria. Then neither the alternative a_i is treated to be at least as good as the profile b nor the opposite. Since the profile represents the delimitation of the categories C^+ and C^-, it cannot be clearly stated whether the alternative should be assigned to C^+ or to C^-. Consequently, the membership of a_i is undetermined.

It must be assured that each alternative is strictly better or worse than the single profile in order to enable the two-categorical sorting. The localization principle thus calls for the prevention of the incomparability relation. To solve the "incomparability problem", veto thresholds are treated asymmetrically. This is justified by the noncompensatory nature of the veto concept and originates from the explicitly regarded primary viewpoint of the logical evaluation of the truthfulness of presupposed alternative assignment to the positive category C^+. This fixed point of view implicitly determines the complementary logical evaluation, which confirms or rejects the truthfulness of assignment to the negative category C^-. The positive semantics is mathematically denoted as:

$$a_i \in C^+ \Rightarrow a_i \notin C^-,$$
$$a_i \notin C^+ \Rightarrow a_i \in C^-.$$

Considering a decision-maker's beliefs, it is only important whether the alternative a_i is good enough to be assigned to the positive category C^+ and not whether a_i is convenient for C^-. This is utterly reasonable since alternatives belonging to C^+ are solely chosen for further analysis or for implementation. In practice, asymmetry means that an alternative a_i with very poor values on some criteria is excluded from the positive class. It is not important though, if the profile b does not reach one or more veto thresholds when compared with a_i, because this information does not confirm that a_i is a member of the C^+ class nor does it prevent the classification of a_i into the C^- category. Yet small weaknesses of an alternative should be compensated. For this reason, indifference and preference thresholds are treated symmetrically. The interpretation of preferential information is thus symmetrically-asymmetrical and leads to the assignment rule. The alternative a_i is good enough to be sorted into the positive category C^+, when all its weaknesses that are measured according to the q_j and p_j thresholds are compensated with advantages and when no difference between $g_j(b)$ and $g_j(a_i)$ exceeds the veto threshold v_j.

Since the incomparability relation no longer exists, another mechanism is introduced to indicate conflicting alternatives and to help a decision-maker express robust values of parameters. It is the below described progressiveness approach.

Aggregation of Partial Indices

To express the degree of concordance with the assertion "the alternative a_i belongs to the C^+ class", the indices $c_j(a_i, b)$ and $c_j(b, a_i)$ are aggregated with a fuzzy averaging operator:

$$\sigma_j(a_i) = \frac{c_j(a_i, b) + (1 - c_j(b, a_i))}{2} = \frac{c_j(a_i, b) + \widetilde{c}_j(a_i, b)}{2},$$

$$\sigma(a_i) = \frac{\sum_{j=1..n} w_j \cdot \sigma_j(a_i)}{\sum_{j=1..n} w_j}.$$

As $\sigma(a_i) = ½$ denotes strict equality among the alternative and the profile, the classical λ-cut may be used to determine the "crisp" membership of the alternative:

$$a_i \in C^+ \Leftrightarrow \sigma(a_i) \geq \lambda, \text{ where } \lambda \in [½, 1].$$

Because of the introduced positive semantics and because the index $\sigma_j(a_i)$ combines the indices $c_j(a_i, b)$ and $c_j(b, a_i)$, there is no need to explicitly verify whether the alternative is a member of the negative category. The fuzzy union operator is used to compute the degree of discordance with the assertion "the alternative a_i belongs to the C^+ class":

$$d(a_i) = \max_{j=1..n} d_j(a_i).$$

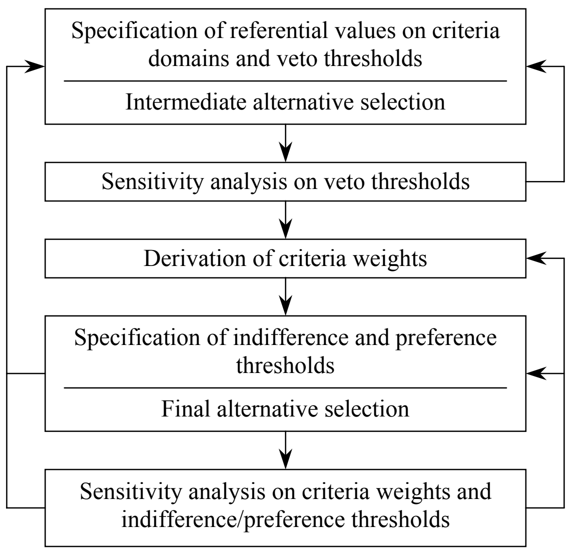
An alternative cannot be excluded from the positive class with greater certainty than it is excluded according to a criterion on which its performance is the poorest. The noncompensatory absolute influence of veto thresholds is the reason why the indices $\sigma(a_i)$ and $d(a_i)$ need not, and should not, be joint together.

PROGRESSIVENESS APPROACH

Because veto thresholds have an asymmetrical noncompensatory effect, they are dealt with independently of criteria weights or compensatory indifference and preference thresholds. Because, on the other hand, criteria weights are dependent on discordance indices, they are determined after veto thresholds have been set. For these reasons, a decision-maker does not need to specify all types of preferential parameters simultaneously. Instead, he concentrates on only a subset of input data at a time, which again gives rise to some benefits in comparison to the existing techniques:

- Information burden is substantially reduced.
- It can be seen how each individual parameter influences the alternative selection.
- It is not required that the decision-maker's preferences are precise at the starting-point of the analysis since the learning process is strongly emphasized.

Progressive and separate consideration of different parameters and their iterative adaptation causes a partial reconstruction of a quantitative model. According to a subset of specified input information, partial results are derived and recurrently mediated to the individual, who can respond. In this way, preferences can be easily controlled – verified, compared to opinions of other participants and, if necessary, modified. Robust solutions are thereby obtained. Figure shows the interchanging elicitation and analysis phases of the problem solving process.



Sensitivity analysis is carried out for the purpose of proving that alternatives are robustly sorted. Three types of distance metrics are applied. They reflect the minimum changes of veto, weight and evaluation vectors that cause the reassignment to another category. The influence of veto thresholds is further analysed by comparing selective strengths of criteria. In addition, indices $\sigma(a_i)$ and $d(a_i)$ are separately examined and can thus point to conflicting alternatives.

SEMIAUTOMATIC WEIGHT DERIVATION

Assessment of criteria importance weights is a basic step in preference elicitation. It is a hard and time consuming task [6, 8]. There exist some structured techniques, such as Simos' procedure [3] or AHP (*Analytic Hierarchy Process*) [10], that help people make more reasonable estimates. However, they are not capable of automatic weight derivation according to a given problem situation, which is reflected through values of input parameters. The presented research tries to bridge this gap. Two mechanisms are proposed that determine the criteria weights by considering the selective influence of veto thresholds.

Although a relationship between the veto threshold v_j and the criterion weight w_j has been discussed in the past [8], it has neither been mathematically founded nor practically applied. As the veto threshold has a noncompensatory nature, it can exclude any alternative from the positive category C^+ regardless of other parameter values. The closer to the preference threshold p_j it is set, the more alternatives it eliminates from C^+, and a stronger selective influence it has. The criterion x_j thus contributes to a higher degree to the evaluation process and consequently to the final decision as well.

It is essential that the criterion importance is not dependent on the indifference and preference thresholds. The effect of these thresholds has to be compensated. Since the weighting factors w_j represent the concept that determines the compensation rates, it is the role of the weights to direct the influence of the q_j and p_j thresholds, and not the opposite.

Two approaches are introduced to obtain cardinal weights. Both construct a fuzzy veto relation by organizing partial discordance indices:

$$V = \{((x_j, a_i), \mu_V(x_j, a_i)) \mid (x_j, a_i) \in X \times A\},$$
$$\mu_V(x_j, a_i) = d_j(a_i) \text{ where } i = 1, \ldots, m,\ j = 1, \ldots, n.$$

Selective Strength Based Approach

It consists of the following five steps:

1. All possible α-cuts of the fuzzy relation V are taken. With a heuristic rule, partial selective strengths of criteria are calculated for each crisp relation.
2. Partial selective strengths are joint by use of an algorithm, which considers veto certainty and similarities between intermediate results.
3. Differences between each two complete selective strengths are transformed with a linear or an exponential function so that ratios of pairs of criteria weights are reflected through a comparison matrix.
4. A decision-maker modifies ratios in the comparison matrix according to his personal beliefs that reflect his cultural, social, psychological and genetic background.
5. Numerical values of weights are computed from the adjusted matrix.

Selective strengths: The selective strength of the j-th criterion is computed according to the number of alternatives that are excluded from the positive category C^+ because of the discordance effect of the veto threshold v_j, and simultaneously according to the number of other criteria from the set $X \setminus \{x_j\}$ that oppose a veto on the assignment of the same alternatives to the C^+ class. The partial selective strength of the criterion x_j considers only the i-th alternative and the single cut-level α_k. It equals to zero when v_j does not contradict the assertion $a_i \in C^+$ or when all criteria oppose a veto on this assertion. It indicates to which degree a criterion outperforms the weakest criterion:

$$\varphi_{ji}^{k} = \begin{cases} \operatorname{card}(x_l \in X \setminus \{x_j\} : d_l(a_i) < \alpha_k)\ , & d_j(a_i) \ge \alpha_k \\ 0 & , d_j(a_i) < \alpha_k . \end{cases}$$

It can be stated that the partial strength of the j-th criterion, which excludes the i-th alternative from the positive category, equals the number of criteria, which do not exclude the same alternative. To aggregate the claculated indices that are obtained for separate alternatives and for different cut-levels $\alpha_k \in L(V)$, an algorithm is defined. It rests on the following principles:

- The higher the cut-level is, the more certain is the crisp veto relation. Selective strengths of criteria, which are bound to cuts with levels $\alpha_k >> 0$, are consequently more substantial for a given problem situation than those, which correspond to low levels $\alpha_k \approx 0$. They are entitled to contribute to a greater extent to the total strengths.
- The criterion x_j gains the highest possible selective strength, measured according to the alternative a_i, at the first cut for which the discordance degree $d_j(a_i)$ exceeds the α_k threshold. At levels $\alpha_{k'} > \alpha_k$, the indices equal to zero and need not be dealt with, while at $\alpha_{k'} < \alpha_k$, additional criteria might oppose a veto and thus $\varphi_{ji}^{k'} \le \varphi_{ji}^{k}$ holds true.
- When $\varphi_{ji}^{k_1} = \ldots = \varphi_{ji}^{k_h}$ for adjacent $\alpha_{k_1} > \ldots > \alpha_{k_h}$, only the cut with the highest level is considered. This originates from the logical maximum concept.
- When the difference $\delta = \varphi_{ji}^{k} - \varphi_{ji}^{k'}$ exceeds 0 for $\alpha_{k'} < \alpha_k$, the total strength of x_j according to a_i falls by $\alpha_{k'} \cdot \delta$. The difference δ has to be lessened by the certainty factor of results. The decrease is a consequence of a weaker – yet positive – veto effect of one or more additional critera.

The total selective strength Φ_j of the criterion x_j is not based on average values of partial results. Instead, it relies on similarities between α-cuts. It therefore represents a more consistent result than would be achieved by applying the weighted sum function.

The Φ_j strengths can be of great help to a decision-maker when specifying the criteria weights, as they reflect the relationships between criteria importances. In addition, when the value of Φ_j is too high or too low, the domination or the discrimination of the j-th criterion is exposed. This is a clear sign for a decision-maker to modify input parameters if he wishes that all criteria are adequately participating in the evaluation process.

Conversion of selective strengths to criteria weights: Selective strengths must, however, not be directly interpreted as criteria weights. The meaning of the "zero strength" has to be considered. If $\Phi_j = 0$, it certainly does not imply that the weight w_j also equals to zero. Similarly, the maximal possible strength $\Phi_{\max} = m \cdot (n - 1)$, which appears when exactly a single criterion eliminates all alternatives form the positive category, has to be dealt with. When the difference $\Delta_{ij} = \Phi_i - \Phi_j$ is small compared to $\Phi_{\max} = \Delta_{\max}$, it is reasonable that the same weight $w_i = w_j$ is assigned to both criteria x_i and x_j. Finally, computed strengths should be modified by an individual in order to properly reflect his personal beliefs.

Two interrelated problems arise:

1. the conversion of indices Φ_j to weights w_j, for $j = 1, \ldots, n$,
2. the representation of indices Φ_j in such a way that criteria importances are correctly and intelligibly expressed, and that a decision-maker is able to easily as well as quickly adjust them.

To solve both problems, the differences Δ_{ij} of selective strengths are transformed and included in a $n \times n$ pairwise comparison matrix. This matrix contains ratios of criteria weights and is consistent with the concepts of AHP [10] – it is reciprocal and it is limited to the scale of 1 to 9. Because of the above illustrated "Φ_{max} problem" and because the ratio Φ_i / Φ_j can be computed only if $\Phi_j \neq 0$ holds true, an original approach is introduced which combines ratios with intervals.

Let the weight ratio be denoted as $r_{ij} \approx w_i / w_j$. The fundamental presumption is that the ratio r_{ij} increases linearly according to the difference $\Delta_{ij} = \Phi_i - \Phi_j$ between the selective strengths of two criteria:

$$r_{ij} = a \cdot \Delta_{ij} + b.$$

The ratio r_{ij} remains the same for all feasible values of Φ_i and Φ_j, if only the difference Δ_{ij} is constant. The interpretation is that the additional strength $d\Phi_j$, which is gained by the criterion x_j, influences the weight increase with equal intensity regardless of the initial value Φ_j. Since Φ_{max} has the maximum possible priority over $\Phi_{min} = 0$, it is evident that $r_{max} = 9$ is assigned to $\Delta_{max} = \Phi_{max} - \Phi_{min}$. Thus:

$$r_{ij} = \frac{8}{\Delta_{max}} \cdot \Delta_{ij} + 1 .$$

Only non-negative differences are considered; if $\Delta_{ij} < 0$ holds, a reciprocal value $r_{ij} = 1 / r_{ji}$ is taken. The constant $b = 1$ ensures that $\Delta_{ij} = 0$ is transformed to the $r_{ij} = 1$ ratio, which indicates total equality of criteria. Since the linear function does not guarantee matrix consistency, the exponential function is also defined:

$$r_{ij} = (r_{max})^E , \; E = \frac{\Delta_{ij}}{\Delta_{max}} .$$

Now:

$r_{ij} = 1$, if $\Delta_{ij} = 0$,
$r_{ij} = r_{max} = 9$, if $\Delta_{ij} = \Delta_{max}$, and
$r_{ik} = r_{ij} \cdot r_{jk}$ for $\Delta_{ik} = \Delta_{ij} + \Delta_{jk}$.

Statistical experiments show that the inconsistency rate of a pairwise matrix, which is constructed with the linear function, is very low. The results were obtained by the power method and are summarized in Table 1. All values are considerably better than required. They do not rise above the worst acceptable level of CR = 0.1 in any of 40000 test cases.

Table 1. Inconsistency rates of pairwise comparison matrices

	6 criteria, 6 alternatives	6 criteria, 8 alternatives	10 criteria, 4 alternatives	10 criteria, 30 alternatives
Average	0.0106	0.0104	0.0112	0.0106
Deviation	0.0050	0.0050	0.0032	0.0030
Maximum	0.0309	0.0268	0.0236	0.0146

Binary Relation Based Approach

It consists of the following steps:

1. A fuzzy binary relation on the criteria set is constructed. Its transitive closure is found.
2. Every α-cut of the transitive closure is analysed to obtain a unique partial order of criteria.
3. Partial orders are combined into a single weak order.

The fuzzy binary relation

$$B = \{((x_i, x_j), \mu_B(x_i, x_j)) \mid (x_i, x_j) \in X \times X\}$$

is interpreted with the assertion "the criterion x_i is at least as selective as the criterion x_j". It is constructed from the fuzzy veto relation V by applying the Bandler and Kohout's triangle superproduct composition:

$$B = V * V^T ,$$
$$\mu_B(x_i, x_j) = \cap_{k=1..m} (\mu_V(x_i, a_k) \leftarrow \mu_V(x_j, a_k)) .$$

For the inner fuzzy operator, the Lukasiewitcz's implication is used. It compares two criteria in regard to their restrictive veto effects on a single alternative:

$$\mu_B^k(x_i, x_j) = \mu_V(x_i, a_k) \leftarrow \mu_V(x_j, a_k)$$
$$= \min(1 - \mu_V(x_j, a_k) + \mu_V(x_i, a_k), 1) .$$

Werners' fuzzy "and" serves as the outer aggregation operator:

$$\mu_B(x_i, x_j) = \gamma \cdot \min_{k=1..m} \mu_B^k(x_i, x_j) + (1-\gamma) \cdot \frac{\sum_{k=1..m} \mu_B^k(x_i, x_j)}{m} , \; 0 \le \gamma << 1 .$$

By setting the value of γ at considerably less than 1, necessary compensation is ensured. If the degree of truth $\mu_B(x_i, x_j)$ would be computed according to the logical minimum, as is usual for the intersection operator in the realm of fuzzy set theory, then the relative selectiveness of the i-th criterion in comparison to x_j would be reduced to only those alternatives that are to a greater extent rejected by x_j. Thereby, the criterion importance would be dependent solely on the weakest discordance index, while alternatives that are actually excluded from the positive class would not participate in the weight determination process.

For the purpose of being analysed, the binary relation has to be at least a fuzzy quasiorder relation, which means that it is reflexive and transitive. Reflexivity is a consequence of the implication operator, while transitivity is generally unachieved. The transitive closure Q is therefore constructed. It is analysed with respect to various levels $\alpha_k \in L(Q)$. From the perspective of a single α-cut, four relations between criteria exist:

$$x_i \succ x_j \Leftrightarrow (x_i\, Q_\alpha\, x_j) \wedge \neg(x_j\, Q_\alpha\, x_i) ,$$
$$x_i \prec x_j \Leftrightarrow \neg(x_i\, Q_\alpha\, x_j) \wedge (x_j\, Q_\alpha\, x_i) ,$$
$$x_i \approx x_j \Leftrightarrow (x_i\, Q_\alpha\, x_j) \wedge (x_j\, Q_\alpha\, x_i) ,$$
$$x_i \,?\, x_j \Leftrightarrow \neg(x_i\, Q_\alpha\, x_j) \wedge \neg(x_j\, Q_\alpha\, x_i) .$$

A different partial order of criteria is derived for each α-cut. These orders can be represented by means of Hasse diagrams. When, however, there are many graphs, determining unique criteria importances might be inconvenient for a decision-maker. This is the reason why partial orders are automatically combined into a single weak order by applying a procedure based on a distance measure π between preference, indifference and incomparability relations [11]. The procedure computes and compares dominance indices. It relies upon the presumption that the criterion x_i is the more influential the more are relations in which it is with the other criteria $x_j \in X \setminus \{x_i\}$ distant from the antiideal considering all cut-levels $\alpha_k \in L(Q)$:

$$\theta_k(x_i) = \sum_{j \neq i} \pi(\prec, R_{ij}^k) , \text{ where } R_{ij}^k \in \{\succ, \prec, \approx, ?\} ,$$
$$\Theta(x_i) = \sum_k \alpha_k \cdot \theta_k(x_i) .$$

The criterion with the highest dominance index Θ_i is the most important one, and so on. Obtained results are not so reliable as selective strengths, because they are bound to an arbitrary real-valued positive constant a:

$$\pi(\succ,\prec) = 2 \cdot a\,,\ \pi(\succ,?) = \frac{5}{3} \cdot a\,,\ \pi(\approx,?) = \frac{4}{3} \cdot a\,,\ \pi(\approx,\succ) = a\ .$$

Comparison of Approaches

First, selective strengths Φ_j are compared to weighted sums of partial strengths as well as to dominance indices Θ_j. For this purpose, criteria are arranged in the descending order. Ratios between the best evaluation and all other evaluations are taken in regard to a separate approach. For example $\Phi_1 : \Phi_2$, $\Phi_1 : \Phi_3$, $\Phi_1 : \Phi_4$ … Here $\Phi_1 = \max \{\Phi_j\}$, $\Phi_2 = \max \{\Phi_j\} \setminus \Phi_1$, and so on. Ratios of these ratios are computed, as is $(\Phi_1 : \Phi_j) : (\Theta_1 : \Theta_j)$. Results that were obtained from 10000 test cases for 8 criteria and 5 alternatives are presented in Table 2.

Table 2. Ratios of selective strengths to weighted sums and dominance indices

	Weighted sums		Dominance indices	
Criteria	Average	Deviation	Average	Deviation
1 : 2	0.9884	0.0996	1.1760	0.2141
1 : 3	0.9756	0.1116	1.3167	0.2860
1 : 4	0.9630	0.1213	1.4630	0.3494
1 : 5	0.9549	0.1368	1.6239	0.4253
1 : 6	0.9267	0.1763	1.8230	0.6092
1 : 7	0.8250	0.2406	2.2152	1.1815
1 : 8	0.5973	0.2765	3.5501	3.0862

Ratios between weighted sums are higher than ratios between selective strengths. They tend to be somewhat unnatural, since the same information is included in the calculation many times. Differences in discordance degrees are consequently intensified and can potentially lead to considerable, immoderate differences in criteria weights. Dominance indices, on the other hand, are rather unstable, which is evident from the measured levels of standard deviation as well as from Table 3. The latter shows how many perturbations occured among weak criteria orders that were derived by individual approaches. A perturbation is interpreted as an event when two criteria swap places.

Table 3. Perturbations in weak criteria orders

	Cases	Average	Maximum
Selective strengths : weighted sums	5764	0.8861	9
Selective strengths : dominance indices	7215	1.2676	9
Weighted sums : dominance indices	7874	1.5512	12

GROUP CONSENSUS SEEKING

In group decision-making, methods belonging to the ELECTRÉ and PROMETHEE families perform a final evaluation of alternatives by compensating values of preferential parameters, which are set by individual group members. A decision thus results from aggregated values. However, these aggregations do not necessariliy represent the opinion of any decision-maker. So, a chosen alternative might not be preferred by the majority of involved people; it could merely be a consequence of considerable disharmony within the group. Moreover, in the case of the ELECTRÉ methods, the credibility of a decision is also hindered by the fact that the coalition is limited to criteria weights, which denote just a subset of input data.

For the above-listed reasons, a procedure is needed that takes into account the following facts:

- All preferential parameters are important in group decision-making.
- A consensus, or at least a compromise, should be reached. An alternative that is chosen according to average parameter values is neither a consensus nor a compromise.
- The level of consensus should be known.
- Equality among the involved people should be guaranteed.

It is a quite reasonable assumption that there exist considerable discrepancies between initial preferential specifications of individual group members. These differences can even increase during the course of discussion and mathematical analysis. Reaching an agreement about a subset of acceptable alternatives is therefore a hard task, which cannot be solved instantly, but rather requires a progressive, iterative, unremitting deepening of problem understanding as well as adapting of personal opinions in order to harmonize with beliefs of other involved decision-makers. To reach uniformity on individuals' views, an active mechanism for convergent group consensus seeking is needed. It should be able to tell a decision-maker how he can modify his preferential parameters so that they will – to as high degree as possible – correspond to the preferences of the whole group [4].

Compromise

The proposed two-categorical sorting ensures a compromise in a very simple way. An acceptable alternative is assigned to the positive class. It thereby receives one vote. As all participants operate on the same alternative set, votes are plainly added. Let o be the number of decision-makers and C_k^+ the subset of alternatives that are approved by the k-th individual. Then the sum of votes for the i-th alternative is:

$$\upsilon_i = \mathrm{card}\,(a_i \in C_k^+,\, k = 1,\ldots,o)\ .$$

Alternatives can now be ranked from the most preferable ones to those that receive the least votes. It is thus clear how many participants in the decision-making process agree upon a given choice and it can never happen that a decision is made, which is not in accordance with the opinion of the majority.

Consensus and Agreement Measures

Since a high level of comparability is attained as a consequence of the applied localization principle, it is an uncomplicated task to define a consensus measure. Let z_i be the consensus degree reached for the i-th alternative. The equality $z_i = 0$ holds true, if exactly half of individuals in the group assign the alternative a_i to the positive category C^+ and the other half to the negative category C^-. In this case, it is totally undetermined whether a_i is an appropriate choice. On the contrary, z_i equals to 1 when all participants classify a_i into the same category. Then the group is perfectly uniform. Let $\nu_i^+ = \upsilon_i$ and $\nu_i^- = o - \upsilon_i$ denote how many participants assign the alternative a_i to the C^+ class and to the C^- class, respectively. Then:

$$z_i = \frac{\nu_i - \rho}{o - \rho}\,,\ \text{where}\ \nu_i = \max\,(\nu_i^+, \nu_i^-)\ \text{and}\ \rho = \left\lfloor \frac{o}{2} \right\rfloor .$$

An operator, which aggregates the partial consensus indices, should not only ensure compensation but has to consider the weakest alternative as well:

$$Z = \gamma \cdot \min_{i=1..m} z_i + (1-\gamma) \cdot \frac{\sum_{i=1..m} z_i}{m}\,,\ \gamma \in [0,1]\ .$$

Another measure is important for the sake of active preference unification in the process of group consensus seeking. It is called the degree of agreement. The more people that assign an alternative to the same category as an individual does, the higher the level of agreement that is reached from the perspective of this person:

$$\zeta_i^k = \begin{cases} \frac{v_i^+ - 1}{o - 1}, a_i \in C_k^+ \\ \frac{v_i^- - 1}{o - 1}, a_i \in C_k^-. \end{cases}$$

Mechanism of Consensus Seeking

The active mechanism of directing group members toward unified opinions is founded on the progressive increasing of the consensus degree Z, that is on the convergence of aggregated values z_i toward the specified threshold ξ. A decision-maker with the lowest degree of agreement is selected. Since this participant is in the strongest opposition to the collective choice, his preferential attitude is the principal reason why the value of Z is not high enough. He has to adjust input parameters to such an extent that someone else becomes the most contradictive group member. Because it is always "the turn" of the participant with the lowest computed agreement level, two important gains arise:

- the values of ζ^k incessantly increase and hence ensure the convergence of Z toward the threshold ξ,
- equality among involved decision-makers is guaranteed, as the only measure of the required conformation to opinions of colleagues is the deviation from the collective choice, which is independent of the person's rank.

It is reasonable that a decision-maker reassigns only alternatives with a low agreement index and with a low robustness level. In the opposite case, either a satisfactory degree of agreement is reached from this person's perspective, or his opinion, which is expressed through the values of input parameters, is so firmly stated that the conformation to the group is not sensible in spite of a considerable contradiction with it. Therefore, the decision support system has to show the k-th group member all partial agreement indices ordered from the lowest to the highest. For each alternative a_i, data on its sensitivity have to be additionally interpreted. The obtained information enables the manual selection of alternatives, which are subject to reassignment. This is essential, because a decision-maker must be able to reject the proposed category changes. When he is convinced that his judgement is right, he may insist on his own choice. Other participants are thereby stimulated to rethink about the decision, enlighten their understanding of the problem situation from another possible point of view, and consider important facts that they have perhaps overlooked.

Suppose a decision-maker specifies which non-robustly sorted alternatives with a low agreement level he is prepared to reassign to the other category. New values of parameters of the decision model – referential values of the profile $g_j(b)$, thresholds q_j, p_j and v_j, and weights w_j – can then be automatically derived for each of n criteria so that the required changes are attained for the chosen alternatives and so that the memberships of all other alternatives are preserved. The adjustment of parameters consists of two phases. At first, the discordance effects are eliminated by loosening the veto thresholds for all criteria according to which the measured performances are intolerably poor. Next, the parameters $g_j(b)$, q_j, p_j and w_j are set. An approach is used which was defined at the Lamsade institute [2]. As the desired categories of all alternatives are known, the decision support system is confronted with the problem of parameter determination on the ground of a sorted alternative set. The problem is solved by an optimization program.

CONCLUSION

In this paper, an interactive alternative sorting procedure, which aims at overcoming some of the major weaknesses of existing pseudo-criterion based methods, was proposed. As its central concept, the localized two-categorical decision analysis was introduced. To enable such analysis, the asymmetrical treatment of veto thresholds was grounded and realized, and appropriate fuzzy aggregation operators for computing the concordance and discordance degrees were defined. The problem localization assured high adaptability of a quantitative model and high comparability of individual group members' results. It hence laid the foundation for three other principles – semiautomatic weight derivation, the progressiveness approach, and group consensus seeking. Two techniques were introduced that are able to determine criteria weights according to the influence of noncompensatory veto thresholds. Both of them define a fuzzy veto relation by organizing partial discordance indices. The first one computes the selective strengths of criteria and transforms them into a pairwise comparison matrix, which can be adjusted by decision-makers in order to correctly reflect their personal beliefs. The second uses the triangle superproduct composition to construct a binary relation on the criteria set. It obtains a weak order by applying a distance measure between preference, indifference and incomparability relations. The techniques were statistically compared. Finally, the mechanism for the iterative unification of decision-makers' opinions was described, being capable of automatic adjustment of preferential parameters. A mathematical optimization program was applied to reach robust conclusions. The consensus and agreement measures were also defined in order to ensure convergence.

Within the scope of further research work, the procedure will be compared to existing methods and evaluated by statistical tests as it is necessary to prove its usefulness, reliability, credibility and convergence toward just group choices. It will also have to be approved in realistic problem situations.

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