HANDBOOK 2

Design of Timber Structures according to EC 5

Education and Culture

Leonardo da Vinci

TEMTIS

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Educational Materials for Designing and
Testing of Timber Structures - TEMTIS

HANDBOOK 2 - DESIGN OF TIMBER STRUCTURES ACCORDING TO EC 5

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Preface

This handbook makes specific reference to design of timber structures to European Standards and using products available in Europe.

The handbook is closely linked to Eurocode 5 (EC5), the European code for the design of timber structures.

For better understanding of the Eurocode 5 design rules the worked examples are presented.

The purpose of this handbook is to introduce readers to the design of timber structures. It is designed to serve either as a text for a course in timber structures or as a reference for systematic self-study of the subject.

Finally, the leader of working package WP4 – Handbook 2 – wishes to express his thanks to all contributors (see included list of contributors). Thanks also goes to Mr. Kolbein BELL, Norwegian University of Science and Technology for vetting the language of this handbook.

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Contents

1 Introduction

From the earliest years of recorded history, trees have provided mankind with food and materials for shelter, fuel and tools. Timber is one of the earliest building materials used by our predecessors, and most of us experience a strong affinity with the beauty and intrinsic characteristics of this natural material when it is used in the places we work and live.

Timber is the oldest known building material capable of transferring both tension and compression forces - making it naturally suited as a beam element. It has a very high strength to weight ratio, it is relatively easy to fabricate and to join, and it often out-performs alternative materials in hazardous environments and extremes of temperature (including fire). Timber does not corrode and many species, if detailed correctly, can be very durable. The unique properties of timber have made it a cornerstone contributor to the advance of civilisation and development of society as we know it today.

Timber has been used in the construction of buildings, bridges, machinery, war engines, civil engineering works and boats since mankind first learnt to fashion tools.

Timber is a truly remarkable material. Whilst most of the structural materials we use are processed from finite resources, requiring large amounts of energy and producing significant green house emissions, timber is grown using solar energy, in natural soil which is fertilised by its own compost, fuelled by carbon dioxide and watered by rain. Because it literally grows on trees, timber is the only structural engineering material which can be totally renewed - provided that trees are replanted (plantations) or naturally regenerated (native forests) after felling!

At the same time forests provide a number of unique and varied benefits that include protection of our climate, water and soil and a great range of recreational functions enjoyed by the general public.

Forests and forest based industries and the goods and products they provide affect directly the daily life of all of Europe's 450 M citizens. Within the EU countries, the forests cover 140 millions hectares which accounts for 36 % of the total land area, ranging from 1 % in Cyprus to 71 % in Finland. Europe's forests are extending in area, increasing in growth rate, and expanding in standing volume.

From an engineering point of view, timber is different from wood. Wood is the substance of which the trunks and branches of trees are made. It is cut and used for various purposes. Timber is wood for building.

In the hands of skilled professionals who have an appreciation and understanding of its natural characteristics, timber has significant advantages over alternative structural materials, enhancing the best designs with a sense of appropriateness, unity, serenity and warmth in achieving the marriage of form and function, which is simply not possible with concrete and steel.

2 Design of timber structures

Before starting formal calculations it is necessary to analyse the structure and set up an appropriate design model. In doing so there may be a conflict between simple, but often conservative, models which make the calculations easy, and more complicated models which better reflect the behaviour but with a higher risk of making errors and overlooking failure modes.

The geometrical model must be compatible with the expected workmanship. For structures sensitive to geometrical variations it is especially important to ensure that the structure is produced as assumed during design. The influence of unavoidable deviations from the assumed geometry and of displacements and deformations during loading should be estimated.

Connections often require large areas of contact and this may give rise to local excentricities which may have an important influence. Often there is a certain freedom as regards the modelling as long as a consistent set of assumptions is used.

The Eurocodes are limit state design codes. This means that the requirements concerning structural reliability are linked to clearly defined states beyond which the structure no longer satisfies specified performance criteria. In the Eurocode system only two types of limit states are considered: ultimate limit states and serviceability limit states.

Ultimate limit states are those associated with collapse or with other forms of structural failure. Ultimate limit states include: loss of equilibrium; failure through excessive deformations; transformation of the structure into a mechanism; rupture; loss of stability.

Serviceability limit states include: deformations which affect the appearance or the effective use of the structure; vibrations which cause discomfort to people or damage to the structure; damage (including cracking) which is likely to have an adverse effect on the durability of the structure.

In the Eurocodes the safety verification is based on the partial factor method described below.

2.1 Principles of limit state design
The design models for the different limit states shall, as appropriate, take into account the following:
− different material properties (e.g. strength and stiffness);
− different time-dependent behaviour of the materials (duration of load, creep);
− different climatic conditions (temperature, moisture variations);
− different design situations (stages of construction, change of support conditions).

2.1.1 Ultimate limit states
The analysis of structures shall be carried out using the following values for stiffness properties:

− for a first order linear elastic analysis of a structure, whose distribution of internal forces is not affected by the stiffness distribution within the structure (e.g. all members have the

same time-dependent properties), mean values shall be used;

− for a first order linear elastic analysis of a structure, whose distribution of internal forces is affected by the stiffness distribution within the structure (e.g. composite members containing materials having different time-dependent properties), final mean values adjusted to the load component causing the largest stress in relation to strength shall be used;

− for a second order linear elastic analysis of a structure, design values, not adjusted for duration of load, shall be used.

The slip modulus of a connection for the ultimate limit state, K_u, should be taken as:

$$K_u = \frac{2}{3} K_{ser} \tag{2.1}$$

where K_{ser} is the slip modulus.

2.1.2 Serviceability limit states

The deformation of a structure which results from the effects of actions (such as axial and shear forces, bending moments and joint slip) and from moisture shall remain within appropriate limits, having regard to the possibility of damage to surfacing materials, ceilings, floors, partitions and finishes, and to the functional needs as well as any appearance requirements.

The instantaneous deformation, u_{inst}, see Chapter 7, should be calculated for the characteristic combination of actions using mean values of the appropriate moduli of elasticity, shear moduli and slip moduli.

The final deformation, u_{fin}, see Chapter 7, should be calculated for the quasi-permanent combination of actions.

If the structure consists of members or components having different creep behaviour, the final deformation should be calculated using final mean values of the appropriate moduli of elasticity, shear moduli and slip moduli.

For structures consisting of members, components and connections with the same creep behaviour and under the assumption of a linear relationship between the actions and the corresponding deformations the final deformation, u_{fin}, may be taken as:

$$u_{fin} = u_{fin,G} + u_{fin,Q_1} + u_{fin,Q_i} \tag{2.2}$$

where:

$$u_{fin,G} = u_{inst,G} \left(1 + k_{def}\right) \qquad \text{for a permanent action, } G \tag{2.3}$$

$$u_{fin,Q,1} = u_{inst,Q,1} \left(1 + \psi_{2,1} k_{def}\right) \quad \text{for the leading variable action, } Q_1 \tag{2.4}$$

$$u_{fin,Q,i} = u_{inst,Q,i} \left(\psi_{0,i} + \psi_{2,i} k_{def}\right) \text{ for accompanying variable actions, } Q_i \ (i > 1) \tag{2.5}$$

$u_{\text{inst,G}}$, $u_{\text{inst,Q,1}}$, $u_{\text{inst,Q,i}}$ are the instantaneous deformations for action *G, Q*$_1$, *Q*$_\text{i}$ respectively;

$\psi_{2,1}$, $\psi_{2,\text{i}}$ are the factors for the quasi-permanent value of variable actions;

$\psi_{0,\text{i}}$ are the factors for the combination value of variable actions;

k_{def} is given in Chapter 3 for timber and wood-based materials, and in Chapter 2 for connections.

For serviceability limit states with respect to vibrations, mean values of the appropriate stiffness moduli should be used.

2.2 Basic variables
The main variables are the actions, the material properties and the geometrical data.

2.2.1 Actions and environmental influences
Actions to be used in design may be obtained from the relevant parts of EN 1991.

Note 1: The relevant parts of EN 1991 for use in design include:
EN 1991-1-1   Densities, self-weight and imposed loads
EN 1991-1-3   Snow loads
EN 1991-1-4   Wind actions
EN 1991-1-5   Thermal actions
EN 1991-1-6   Actions during execution
EN 1991-1-7   Accidental actions

Duration of load and moisture content affect the strength and stiffness properties of timber and wood-based elements and shall be taken into account in the design for mechanical resistance and serviceability. Actions caused by the effects of moisture content changes in the timber shall be taken into account.

Load-duration classes
The load-duration classes are characterised by the effect of a constant load acting for a certain period of time in the life of the structure. For a variable action the appropriate class shall be determined on the basis of an estimate of the typical variation of the load with time.

Actions shall be assigned to one of the load-duration classes given in Table 2.1 for strength and stiffness calculations.

Table 2.1 Load-duration classes

Load-duration class	Order of accumulated duration of characteristic load
Permanent	more than 10 years
Long-term	6 months – 10 years
Medium-term	1 week – 6 months

Short-term	less than one week
Instantaneous	

NOTE: Examples of load-duration assignment are given in Table 2.2

Table 2.2 Examples of load-duration assignment

Load-duration class	Examples of loading
Permanent	self-weight
Long-term	storage
Medium-term	imposed floor load, snow
Short-term	snow, wind
Instantaneous	wind, accidental load

Service classes
Structures shall be assigned to one of the service classes given below:

NOTE: The service class system is mainly aimed at assigning strength values and for calculating deformations under defined environmental conditions.

Service class 1 is characterised by a moisture content in the materials corresponding to a temperature of 20 °C and the relative humidity of the surrounding air only exceeding 65 % for a few weeks per year.

NOTE: In service class 1 the average moisture content in most softwoods will not exceed 12 %.

Service class 2 is characterised by a moisture content in the materials corresponding to a temperature of 20 °C and the relative humidity of the surrounding air only exceeding 85 % for a few weeks per year.

NOTE: In service class 2 the average moisture content in most softwoods will not exceed 20 %.

Service class 3 is characterised by climatic conditions leading to higher moisture contents than in service class 2.

2.2.2 Materials and product properties

Load-duration and moisture influences on strength
Modification factors for the influence of load-duration and moisture content on strength are given in Chapter 3.

Where a connection is constituted of two timber elements having different time-dependent behaviour, the calculation of the design load-carrying capacity should be made with the following modification factor k_{mod}:

$$k_{mod} = \sqrt{k_{mod,1} \, k_{mod,2}} \qquad (2.6)$$

where $k_{mod,1}$ and $k_{mod,2}$ are the modification factors for the two timber elements.

Load-duration and moisture influences on deformations

For serviceability limit states, if the structure consists of members or components having different time-dependent properties, the final mean value of modulus of elasticity, $E_{mean,fin}$, shear modulus, $G_{mean,fin}$, and slip modulus, $K_{ser,fin}$, which are used to calculate the final deformation should be taken from the following expressions:

$$E_{mean,fin} = \frac{E_{mean}}{\left(1+k_{def}\right)} \qquad (2.7)$$

$$G_{mean,fin} = \frac{G_{mean}}{\left(1+k_{def}\right)} \qquad (2.8)$$

$$K_{ser,fin} = \frac{K_{ser}}{\left(1+k_{def}\right)} \qquad (2.9)$$

For ultimate limit states, where the distribution of member forces and moments is affected by the stiffness distribution in the structure, the final mean value of modulus of elasticity, $E_{mean,fin}$, shear modulus ,$G_{mean,fin}$, and slip modulus, $K_{ser,fin}$, should be calculated from the following expressions :

$$E_{mean,fin} = \frac{E_{mean}}{\left(1+\psi_2 k_{def}\right)} \qquad (2.10)$$

$$G_{mean,fin} = \frac{G_{mean}}{\left(1+\psi_2 k_{def}\right)} \qquad (2.11)$$

$$K_{ser,fin} = \frac{K_{ser}}{\left(1+\psi_2 k_{def}\right)} \qquad (2.12)$$

where:

E_{mean} is the mean value of modulus of elasticity;

G_{mean} is the mean value of shear modulus;

K_{ser} is the slip modulus;

k_{def} is a factor for the evaluation of creep deformation taking into account the relevant service class;

ψ_2 is the factor for the quasi-permanent value of the action causing the largest stress in relation to the strength (if this action is a permanent action, ψ_2 should be replaced by 1).

NOTE 1: Values of k_{def} are given in Chapter 3.

NOTE 2: Values of ψ_2 are given in EN 1990:2002.

Where a connection is constituted of timber elements with the same time-dependent behaviour, the value of k_{def} should be doubled.

Where a connection is constituted of two wood-based elements having different time-dependent behaviour, the calculation of the final deformation should be made with the following deformation factor k_{def}:

$$k_{\text{def}} = 2 \sqrt{k_{\text{def},1} \; k_{\text{def},2}} \tag{2.13}$$

where $k_{\text{def},1}$ and $k_{\text{def},2}$ are the deformation factors for the two timber elements.

2.3 Verification by the partial factor method

A low probability of getting action values higher than the resistances, in the partial factor method, is achieved by using design values found by multiplying the characteristic actions and dividing the characteristic strength parameters, by partial safety factors.

2.3.1 Design value of material property

The design value X_d of a strength property shall be calculated as:

$$X_d = k_{\text{mod}} \frac{X_k}{\gamma_M} \tag{2.14}$$

where:

X_k is the characteristic value of a strength property;

γ_M is the partial factor for a material property;

k_{mod} is a modification factor taking into account the effect of the duration of load and moisture content.

NOTE 1: Values of k_{mod} are given in Chapter 3, Table 3.1.

NOTE 2: The recommended partial factors for material properties (γ_M) are given in Table 2.3. Information on the National choice may be found in the National annex of each country.

Table 2.3 Recommended partial factors γ_M for material properties and resistances

Fundamental combinations:	
Solid timber	1,3
Glued laminated timber	1,25
LVL, plywood, OSB,	1,2
Particleboards	1,3
Fibreboards, hard	1,3
Fibreboards, medium	1,3
Fibreboards, MDF	1,3
Fibreboards, soft	1,3
Connections	1,3
Punched metal plate fasteners	1,25
Accidental combinations	1,0

The design member stiffness property E_d or G_d shall be calculated as:

$$E_d = \frac{E_{mean}}{\gamma_M} \tag{2.15}$$

$$G_d = \frac{G_{mean}}{\gamma_M} \tag{2.16}$$

where:

E_{mean} is the mean value of modulus of elasticity;

G_{mean} is the mean value of shear modulus.

2.3.2 Design value of geometrical data
Geometrical data for cross-sections and systems may be taken as nominal values from product standards hEN or drawings for the execution.

Design values of geometrical imperfections specified by the Eurocode 5 comprise the effects of

− geometrical imperfections of members;

− the effects of structural imperfections from fabrication and erection;

− inhomogeneity of materials (e.g. due to knots).

2.3.3 Design resistances
The design value R_d of a resistance (load-carrying capacity) shall be calculated as:

$$R_d = k_{mod} \frac{R_k}{\gamma_M}$$

(2.17)

where:

R_k is the characteristic value of load-carrying capacity;

γ_M is the partial factor for a material property,

k_{mod} is a modification factor taking into account the effect of the duration of load and moisture content.

NOTE 1: Values of k_{mod} are given in Chapter 3, Table 3.1.

NOTE 2: For partial factors, see Table 2.3.

3 Design values of material properties

Eurocode 5 in common with the other Eurocodes provides no data on strength and stiffness properties for structural materials. It merely states the rules appropriate to the determination of these values to achieve compatibility with the safety format and the design rules of EC5.

3.1 Introduction

Strength and stiffness parameters
Strength and stiffness parameters shall be determined on the basis of tests for the types of action effects to which the material will be subjected in the structure, or on the basis of comparisons with similar timber species and grades or wood-based materials, or on well-established relations between the different properties.

Stress-strain relations
Since the characteristic values are determined on the assumption of a linear relation between stress and strain until failure, the strength verification of individual members shall also be based on such a linear relation.

For members or parts of members subjected to compression, a non-linear relationship (elastic-plastic) may be used.

Strength modification factors for service classes and load-duration classes
The values of the modification factor k_{mod} given in Table 3.1 should be used.

If a load combination consists of actions belonging to different load-duration classes a value of k_{mod} should be chosen which corresponds to the action with the shortest duration, e.g. for a combination of dead load and a short-term load, a value of k_{mod} corresponding to the short-term load should be used.

Table 3.1 Values of k_{mod}

Material	Standard	Service class	Load-duration class				
			Permanent action	Long term action	Medium term action	Short term action	Instanta-neous action
Solid timber	EN 14081-1	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
Glued laminated timber	EN 14080	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
LVL	EN 14374, EN 14279	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
Plywood	EN 636						
	Part 1, Part 2, Part 3	1	0,60	0,70	0,80	0,90	1,10
	Part 2, Part 3	2	0,60	0,70	0,80	0,90	1,10

	Part 3	3	0,50	0,55	0,65	0,70	0,90
OSB	EN 300						
	OSB/2	1	0,30	0,45	0,65	0,85	1,10
	OSB/3, OSB/4	1	0,40	0,50	0,70	0,90	1,10
	OSB/3, OSB/4	2	0,30	0,40	0,55	0,70	0,90
Particle-	EN 312						
board	Part 4, Part 5	1	0,30	0,45	0,65	0,85	1,10
	Part 5	2	0,20	0,30	0,45	0,60	0,80
	Part 6, Part 7	1	0,40	0,50	0,70	0,90	1,10
	Part 7	2	0,30	0,40	0,55	0,70	0,90
Fibreboard, hard	EN 622-2						
	HB.LA, HB.HLA 1 or 2	1	0,30	0,45	0,65	0,85	1,10
	HB.HLA1 or 2	2	0,20	0,30	0,45	0,60	0,80
Fibreboard, medium	EN 622-3						
	MBH.LA1 or 2	1	0,20	0,40	0,60	0,80	1,10
	MBH.HLS1 or 2	1	0,20	0,40	0,60	0,80	1,10
	MBH.HLS1 or 2	2	–	–	–	0,45	0,80
Fibreboard, MDF	EN 622-5						
	MDF.LA, MDF.HLS	1	0,20	0,40	0,60	0,80	1,10
	MDF.HLS	2	–	–	–	0,45	0,80

Deformation modification factors for service classes
The values of the deformation factor k_{def} given in Table 3.2 should be used.

3.2 Solid timber
Timber members shall comply with EN 14081-1. Timber members with round cross-section shall comply with EN 14544.

NOTE: Values of strength and stiffness properties (see Table 3.4) are given for structural timber allocated to strength classes in EN 338.

The establishment of strength classes and related strength and stiffness profiles is possible because, independently, nearly all softwoods and hardwoods commercially available exhibit a similar relationship between strength and stiffness properties.

Experimental data shows that all important characteristic strength and stiffness properties can be calculated from either bending strength, modulus of elasticity (E) or density. However, further research is required to establish the effect of timber quality on these relationships and to decide whether accuracy could be improved by modifying these retationships for different strength classes.

Deciduous species (hardwoods) have a different anatomical structure from coniferous species (softwoods). They generally have higher densities but not correspondingly higher strength and stiffness properties. This is why EN 338 provides separate strength classes for coniferous and deciduous species. Poplar, increasingly used for structural purposes, shows a density/strength

relationship closer to that of coniferous species and was therefore assigned to coniferous strength classes.

Due to the relationships between strength, stiffness and density a species /source/ grade combination can be assigned to a specific strength class based on the characteristic values of bending strength, modulus of elasticity and density.

According to EN 338 a timber population can thus be assigned to a strength class provided

- the timber has been visually or machine strength graded according to the specifications of EN 518 or EN 519;

- the characteristic strength, stiffness and density values have been determined according to EN 384 "Determination of characteristic values of mechanical properties and density";

- the characteristic values of bending strength, modulus of elasticity and density of the population are equal to or greater than the corresponding values of the related strength class.

The effect of member size on strength may be taken into account.

Table 3.2 Values of k_{def} for timber and wood-based materials

Material	Standard	Service class		
		1	**2**	**3**
Solid timber	EN 14081-1	0,60	0,80	2,00
Glued Laminated timber	EN 14080	0,60	0,80	2,00
LVL	EN 14374, EN 14279	0,60	0,80	2,00
Plywood	EN 636			
	Part 1	0,80	–	–
	Part 2	0,80	1,00	–
	Part 3	0,80	1,00	2,50
OSB	EN 300			

	OSB/2	2,25	–	–
	OSB/3, OSB/4	1,50	2,25	–
Particleboard	EN 312			
	Part 4	2,25	–	–
	Part 5	2,25	3,00	–
	Part 6	1,50	–	–
	Part 7	1,50	2,25	–
Fibreboard, hard	EN 622-2			
	HB.LA	2,25	–	–
	HB.HLA1, HB.HLA2	2,25	3,00	–
Fibreboard, medium	EN 622-3			
	MBH.LA1, MBH.LA2	3,00	–	–
	MBH.HLS1, MBH.HLS2	3,00	4,00	–
Fibreboard, MDF	EN 622-5			
	MDF.LA	2,25	–	–
	MDF.HLS	2,25	3,00	–

For rectangular solid timber with a characteristic timber density $\rho_k \leq 700$ kg/m^3, the reference depth in bending or width (maximum cross-sectional dimension) in tension is 150 mm. For depths in bending or widths in tension of solid timber less than 150 mm the characteristic values for $f_{m,k}$ and $f_{t,0,k}$ may be increased by the factor k_h, given by:

$$k_h = \min \begin{cases} \left(\dfrac{150}{h}\right)^{0,2} \\ 1,3 \end{cases}$$

(3.1)

where h is the depth for bending members or width for tension members, in mm.

For timber which is installed at or near its fibre saturation point, and which is likely to dry out under load, the values of k_{def}, given in Table 3.2, should be increased by 1,0.

Finger joints shall comply with EN 385.

3.3 Glued laminated timber
Glued laminated timber members shall comply with EN 14080.

NOTE: Values of strength and stiffness properties are given for glued laminated timber allocated to strength classes in EN 1194.

Formulae for calculating the mechanical properties of glulam from the lamination properties are given in Table 3.3.

The basic requirements for the laminations which are used in the formulae of Table 3.3 are the tension characteristic strength and the mean modulus of elasticity. The density of the

laminations is an indicative property. These properties shall be either the tabulated values given in EN 338 or derived according to the principles given in EN 1194.

The requirements for glue line integrity are based on the testing of the glue line in a full cross-sectional specimen, cut from a manufactured member. Depending on the service class, delamination tests (according to EN 391 "Glued laminated timber - delamination test of glue lines") or block shear tests (according to EN 392 "Glued laminated timber - glue line shear test") must be performed.

Table 3.3 Mechanical properties of glued laminated timber (in N/mm^2)

Property		
Bending	$f_{m,g,k}$	$= 7 + 1,15\, f_{t,0,l,k}$
Tension	$f_{t,0,g,k}$	$= 5 + 0,8\, f_{t,0,l,k}$
	$f_{t,90,g,k}$	$= 0,2 + 0,015\, f_{t,0,l,k}$
Compresion	$f_{c,0,g,k}$	$= 7,2\, f_{t,0,l,k}^{0,45}$
	$f_{c,90,g,k}$	$= 0,7\, f_{t,0,l,k}^{0,5}$
Shear	$f_{v,g,k}$	$= 0,32\, f_{t,0,l,k}^{0.8}$
Modulus of elasticity	$E_{0,g,mean}$	$= 1,05\, E_{0,l,mean}$
	$E_{0,g,05}$	$= 0,85\, E_{0,l,mean}$
	$E_{90,g,mean}$	$= 0,035\, E_{0,l,mean}$
Shear modulus	$G_{g,mean}$	$= 0,065\, E_{0,l,mean}$
Density	$\rho_{g,k}$	$= 1,10\, \rho_{l,k}$
NOTE: For combined glued laminated timber the formulae apply to the properties of the individual parts of the cross-section. It is assumed that zones of different lamination grades amount to at least 1/6 of the beam depth or two laminations, whichever is the greater.		

The effect of member size on strength may be taken into account.

For rectangular glued laminated timber, the reference depth in bending or width in tension is 600 mm. For depths in bending or widths in tension of glued laminated timber less than 600 mm the characteristic values for $f_{m,k}$ and $f_{t,0,k}$ may be increased by the factor k_h, given by

$$k_h = \min \begin{cases} \left(\dfrac{600}{h} \right)^{0,1} \\ 1,1 \end{cases} \tag{3.2}$$

where h is the depth for bending members or width for tensile members, in mm.

Large finger joints complying with the requirements of ENV 387 shall not be used for products to be installed in service class 3, where the direction of grain changes at the joint.

The effect of member size on the tensile strength perpendicular to the grain shall be taken into account.

3.4 Laminated veneer lumber (LVL)
LVL structural members shall comply with EN 14374.

For rectangular LVL with the grain of all veneers running essentially in one direction, the effect of member size on bending and tensile strength shall be taken into account.

The reference depth in bending is 300 mm. For depths in bending not equal to 300 mm the characteristic value for $f_{m,k}$ should be multiplied by the factor k_h ,given by

$$k_h = \min \begin{cases} \left(\dfrac{300}{h}\right)^s \\ 1,2 \end{cases} \tag{3.3}$$

where:

h is the depth of the member, in mm;

s is the size effect exponent, see below.

The reference length in tension is 3000 mm. For lengths in tension not equal to 3000 mm the characteristic value for $f_{t,0,k}$ should be multiplied by the factor k_ℓ given by

$$k_\ell = \min \begin{cases} \left(\dfrac{3000}{\ell}\right)^{s/2} \\ 1,1 \end{cases} \tag{3.4}$$

where ℓ is the length, in mm.

The size effect exponent s for LVL shall be taken as declared in accordance with EN 14374.

Large finger joints complying with the requirements of ENV 387 shall not be used for products to be installed in service class 3, where the direction of grain changes at the joint.

For LVL with the grain of all veneers running essentially in one direction, the effect of member size on the tensile strength perpendicular to the grain shall be taken into account.

3.5 Wood-based panels

Wood-based panels shall comply with EN 13986 and LVL used as panels shall comply with EN 14279.

The use of softboards according to EN 622-4 should be restricted to wind bracing and should be designed by testing.

3.6 Adhesives

Adhesives for structural purposes shall produce joints of such strength and durability that the integrity of the bond is maintained in the assigned service class throughout the expected life of the structure.

Adhesives which comply with Type I specification as defined in EN 301 may be used in all service classes.

Adhesives which comply with Type II specification as defined in EN 301 should only be used in service classes 1 or 2 and not under prolonged exposure to temperatures in excess of 50 °C. More about adhesives in Chapter 4.

3.7 Metal fasteners

Metal fasteners shall comply with EN 14592 and metal connectors shall comply with EN 14545.

Table 3.4 Strength classes and characteristic values according to EN 338

		Coniferous species and Poplar												Deciduous species					
		C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50	D30	D35	D40	D50	D60	D70
Strength properties in N/mm^2																			
Bending	$f_{m,k}$	14	16	18	20	22	24	27	30	35	40	45	50	30	35	40	50	60	70
Tension parallel to grain	$f_{t,0,k}$	8	10	11	12	13	14	16	18	21	24	27	30	18	21	24	30	36	42
Tension perpendicular to grain	$f_{t,90,k}$	0,4	0,5	0,5	0,5	0,5	0,5	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Compression parallel to grain	$f_{c,0,k}$	16	17	18	19	20	21	22	23	25	26	27	29	23	25	26	29	32	34
Compression perpendicular to grain	$f_{c,90,k}$	2,0	2,2	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	3,1	3,2	8,0	8,4	8,8	9,7	10,5	13,5
Shear	$f_{v,k}$	1,7	1,8	2,0	2,2	2,4	2,5	2,8	3,0	3,4	3,8	3,8	3,8	3,0	3,4	3,8	4,6	5,3	6,0
Stiffness properties in kN/mm^2																			
Mean value of modulus of elasticity parallel to grain	$E_{0,mean}$	7	8	9	9,5	10	11	11,5	12	13	14	15	16	10	10	11	14	17	20
5% value of modulus of elasticity parallel to grain	$E_{0,05}$	4,7	5,4	6,0	6,4	6,7	7,4	7,7	8,0	8,7	9,4	10,0	10,7	8,0	8,7	9,4	11,8	14,3	16,8
Mean value of modulus of elasticity pependicular to grain	$E_{90,mean}$	0,23	0,27	0,30	0,32	0,33	0,37	0,38	0,40	0,43	0,47	0,50	0,53	0,64	0,69	0,75	0,93	1,13	1,33
Mean value of shear modulus	G_{mean}	0,44	0,5	0,56	0,59	0,63	0,69	0,72	0,75	0,81	0,88	0,94	1,00	0,60	0,65	0,70	0,88	1,06	1,25
Density in kg/m^3																			
Density	ρ_k	290	310	320	330	340	350	370	380	400	420	440	460	530	560	590	650	700	900
Mean value of density	ρ_{mean}	350	370	380	390	410	420	450	460	480	500	520	550	640	670	700	780	840	1080

4 Wood adhesives

At present there is one established EN-standard for classification of structural wood adhesives, namely EN 301, "Adhesives, phenolic and aminoplastic, for load bearing timber structures: Classification and performance requirements". The corresponding test standard is EN 302, "Adhesives for load-bearing timber structures - Test methods. The standards apply to phenolic and aminoplastic adhesives only. These adhesives are classified as:

- type I-adhesives, which will stand full outdoor exposure, and temperatures above 50 °C;

- type II-adhesives, which may be used in heated and ventilated buildings, and exterior protected from the weather. They will stand short exposure to the weather, but not prolonged exposure to weather or to temperatures above 50 °C.

According to EC5 only adhesives complying with EN 301 may be approved at the moment. Current types of structural wood adhesives are listed below.

Resorcinol formaldehyde (RF) and Phenol-resorcinol formaldehyde (PRF) adhesives
RF's and PRF's are type I adhesives according to EN 301. They are used in laminated beams, fingerjointing of structural members, I-beams, box beams etc., both indoors and outdoors.

Phenol-formaldehyde adhesives (PF), hot-setting
Hot-setting PF's cannot be classified according to EN 301.

Phenol-formaldehyde adhesives (PF), cold-setting
Cold-setting PF's are classified according to EN 301, but the current types are likely to be eliminated by the "acid damage test" given in EN 302-3.

Urea-formaldehyde adhesives (UF)
Only special cold-setting UF's are suitable for structural purposes. In a fire they will tend to delaminate. UF's for structural purposes are classified according to EN 301 as type II-adhesives.

Melamine-urea formaldehyde adhesives (MUF)
The cold set ones are classified according to EN 301. They are, however, less resistant than the resorcinols, and not suitable for marine purposes. However, MUF's are often preferred for economic reasons, and because of their lighter colour.

Casein adhesives
Caseins are probably the oldest type of structural adhesive and have been used for industrial glulam production since before 1920. Caseins do not meet the requirements of EN 301.

Epoxy adhesives
Epoxy adhesives have very good gapfilling properties. Epoxies have very good strength and durability properties, and the weather resistance for the best ones lies between MUF's and PRF's.

Two-part polyurethanes
These adhesives have good strength and durability, but experience seems to indicate that they are not weather-resistant, at least not all of them.

5 Durability

Timber is susceptible to biological attack whereas metal components may corrode.
Under ideal conditions timber structures can be in use for centuries without significant biological deterioration. However, if conditions are not ideal, many widely used wood species need a preservative treatment to be protected from the biological agencies responsible for timber degradation, mainly fungi and insects.

5.1 Resistance to biological organisms and corrosion

Timber and wood-based materials shall either have adequate natural durability in accordance with EN 350-2 for the particular hazard class (defined in EN 335-1, EN 335-2 and EN 335-3), or be given a preservative treatment selected in accordance with EN 351-1 and EN 460.

Preservative treatment may affect the strength and stiffness properties.

Rules for specification of preservation treatments are given in EN 350-2 and EN 335.

Metal fasteners and other structural connections shall, where necessary, either be inherently corrosion-resistant or be protected against corrosion.

Examples of minimum corrosion protection or material specifications for different service classes are given in Table 5.1.

Table 5.1 Examples of minimum specifications for material protection against corrosion for fasteners (related to ISO 2081)

Fastener	Service Class[b]		
	1	**2**	**3**
Nails and screws with $d \leq 4$ mm	None	Fe/Zn 12c[a]	Fe/Zn 25c[a]
Bolts, dowels, nails and screws with $d > 4$ mm	None	None	Fe/Zn 25c[a]
Staples	Fe/Zn 12c[a]	Fe/Zn 12c[a]	Stainless steel
Punched metal plate fasteners and steel plates up to 3 mm thickness	Fe/Zn 12c[a]	Fe/Zn 12c[a]	Stainless steel
Steel plates from 3 mm up to 5 mm in thickness	None	Fe/Zn 12c[a]	Fe/Zn 25c[a]
Steel plates over 5 mm thickness	None	None	Fe/Zn 25c[a]
[a] If hot dip zinc coating is used, Fe/Zn 12c should be replaced by Z275 and Fe/Zn 25c by Z350 in accordance with EN 10147			
[b] For especially corrosive conditions consideration should be given to heavier hot dip coatings or stainless steel.			

5.2 Biological attack

The two main biological agencies responsible for timber degradation are fungi and insects although in specific situations, timber can also be attacked by marine borers.

Fungal attack
This occurs in timber which has a high moisture content, generally between 20 % and 30 %.

Insect attack
Insect attack is encouraged by warm conditions which favour their development and reproduction.

5.3 Classification of hazard conditions

The levels of exposure to moisture are defined differently in EC5 and EN 335-I "Durability of wood and wood-based products - Definition of hazard (use) classes of biological attack - Part 1: General". EC5 provides for three service classes relating to the variation of timber performance with moisture content, see Chapter 2.

In EN 335-1, five hazard (use) classes are defined with respect to the risk of biological attacks:

Hazard (use) class 1, situation in which timber or wood-based product is under cover, fully protected from the weather and not exposed to wetting;

Hazard (use) class 2, situation in which timber or wood-based product is under cover and fully protected from the weather but where high environmental humidity can lead to occasional but not persistent wetting;

Hazard (use) class 3, situation in which timber or wood-based product is not covered and not in contact with the ground. It is either continually exposed to the weather or is protected from the weather but subject to frequent wetting;

Hazard (use) class 4, situation in which timber or wood-based product is in contact with the ground or fresh water and thus is permanently exposed to wetting;

Hazard (use) class 5, situation in which timber or wood-based product is permanently exposed to salt water.

5.4 Prevention of fungal attack

It is possible to lower the risk by reducing the timber moisture content through careful construction details.

5.5 Prevention of insect attack

Initially, the natural durability of the selected timber species should be established with respect to the particular insect species to which it may be exposed. It is also necessary to establish whether the particular insect is present in the region in which the timber is to be used.

6 Ultimate limit states

Timber structures are generally analysed using elastic (linear/nonlinear) structural analysis techniques. This is quite appropriate for the serviceability limit state (which is fairly representative of the performance of the structure from year to year). Even the ultimate limit state (which models the failure of structural element under an extreme loading condition) can be adequately handled by a linear elastic analysis.

6.1 Design of cross-sections subjected to stress in one principal direction
This section deals with the design of simple members in a single action.

6.1.1 Assumptions
Section 6.1 applies to straight solid timber, glued laminated timber or wood-based structural products of constant cross-section, whose grain runs essentially parallel to the length of the member. The member is assumed to be subjected to stresses in the direction of only one of its principal axes (see Figure 6.1).
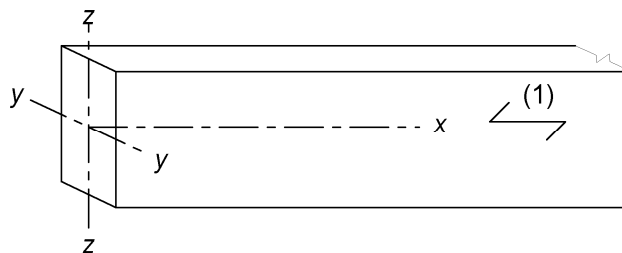
Key:
(1) direction of grain
Figure 6.1 Member Axes

6.1.2 Tension parallel to the grain
Tension members generally have a uniform tension field throughout the length of the member, and the entire cross section, which means that any corner at any point on the member has the potential to be a critical location. However a bending member under uniformly distributed loading will have a bending moment diagram that varies from zero at each end to the maximum at the centre. The critical locations for tension are near to the centre, and only one half of the beam cross section will have tension, so the volume of the member that is critical for flaws is much less than that for tension members.

The inhomogeneities and other deviations from an ideal orthotropic material, which are typical for structural timber, are often called defects. As just mentioned, these defects will cause a fairly large strength reduction in tension parallel to the grain. For softwood (spruce, fir) typical average value are in the range of $f_{t,0} = 10$ to 35 N/mm^2.

In EC5 the characteristic tensile strength parallel to the grain of solid timber is related to a width of 150 mm. For widths of solid timber less than 150 mm the characteristic values may be increased by a factor k_h, see Equation 3.1.

For glulam the reference width is 600 mm and, analogously, for widths smaller than 600 mm a factor k_h, defined by Equation 3.2, should be applied.

The following expression shall be satisfied:

$$\sigma_{t,0,d} \leq f_{t,0,d} \tag{6.1}$$

where

$\sigma_{t,0,d}$ is the design tensile stress along the grain;

$f_{t,0,d}$ is the design tensile strength along the grain.

6.1.3 Tension perpendicular to the grain
The lowest strength for timber is in tension perpendicular to the grain.
In timber members tensile stresses perpendicular to the grain should be avoided or kept as low as possible.
The effect of member size shall be taken into account (see Section 6.4).

6.1.4 Compression parallel to the grain
At the ultimate limit state, the compression member will have achieved its compressive capacity whether limited by material crushing (see Figure 6.2) or buckling. In contrast to the brittle, explosive failure of tension members, the compression failure is quiet and gradual. Buckling is quite silent as it is not associated with material failure at all, and crushing is accompanied by a "crunching or crackling" sound. However, in spite of the silence of failure, any structural failure can lead to loss or at least partial loss of the structural system and place a risk on human life. Both modes of failure are just as serious as the more dramatic tensile and bending failures.
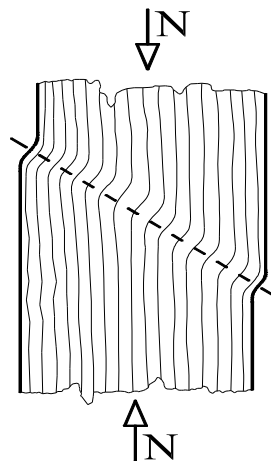
Figure 6.2 Failure mechanisms in compression

The strength in compression parallel to the grain is also somewhat reduced by the growth defects. Characteristic values are in the range 25 to 40 N/mm^2. The reduction in strength depends on the testing method.

The following expression shall be satisfied:

$$\sigma_{c,0,d} \leq f_{c,0,d} \tag{6.2}$$

where:

$\sigma_{c,0,d}$ is the design compressive stress along the grain;

$f_{c,0,d}$ is the design compressive strength along the grain.

NOTE: Rules for the instability of members are given in 6.3.

6.1.5 Compression perpendicular to the grain

Bearing capacity either over a support or under a load plate is a function of the crushing strength of the wood fibre. Where the bearing capacity is exceeded, local crushing occurs. This type of failure is quite ductile, but in some cases, fibre damage in the region of a support may cause flexural failure in that location.

The bearing capacity is a complex function of the bearing area. Where the bearing does not completely cover the area of timber, testing has shown a considerable increase in bearing capacity.
This is known as an "edge effect". Figure 6.3 shows bearing failure under heavily loaded beams. The influence of growth defects on the strength perpendicular to the grain is small.
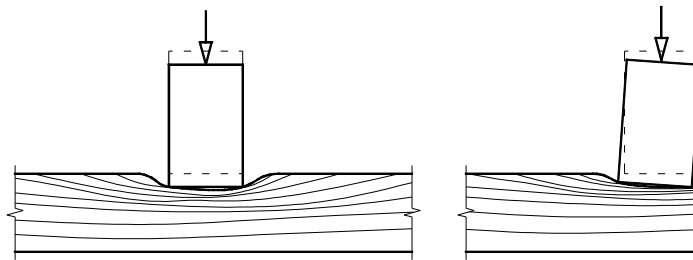
Figure 6.3 Bearing effects at supports and points of concentrated load application

The following expression shall be satisfied:

$$\sigma_{c,90,d} \leq k_{c,90} f_{c,90,d} \tag{6.3}$$

where:

$\sigma_{c,90,d}$ is the design compressive stress in the contact area perpendicular to the grain;

$f_{c,90,d}$ is the design compressive strength perpendicular to the grain;

$k_{c,90}$ is a factor taking into account the load configuration, possibility of splitting and degree of compressive deformation.

The value of $k_{c,90}$ should be taken as 1,0, unless the member arrangements in the following paragraphs apply. In these cases the higher value of $k_{c,90}$ specified may be taken, up to a limiting value of $k_{c,90} = 4,0$.

NOTE: When a higher value of $k_{c,90}$ is used, and contact extends over the full member width b, the resulting compressive deformation at the ultimate limit state will be approximately 10 % of the member depth.

For a beam member resting on supports (see Figure 6.4), the factor $k_{c,90}$ should be calculated from the following expressions:

−   When the distance from the edge of a support to the end of a beam $a, \leq h/3$:

$$k_{c,90} = \left(2,38 - \frac{\ell}{250} \right)\left(1 + \frac{h}{12\ell} \right) \tag{6.4}$$

−   At internal supports:

$$k_{c,90} = \left(2,38 - \frac{\ell}{250} \right)\left(1 + \frac{h}{6\ell} \right) \tag{6.5}$$

where:

ℓ is the contact length in mm;

h is member depth in mm.
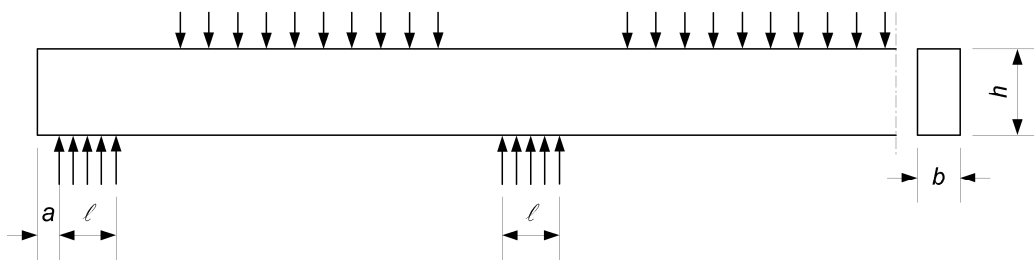
Figure 6.4 Beam on supports

For a member with a depth $h \leq 2,5b$ where a concentrated force with contact over the full width b of the member is applied to one face directly over a continuous or discrete support on the opposite face, see Figure 6.5, the factor $k_{c,90}$ is given by:

$$k_{c,90} = \left(2,38 - \frac{\ell}{250} \right)\left(\frac{\ell_{ef}}{\ell} \right)^{0,5} \tag{6.6}$$

where:

ℓ_{ef} is the effective length of distribution, in mm;

ℓ is the contact length, see Figure 6.5, in mm.

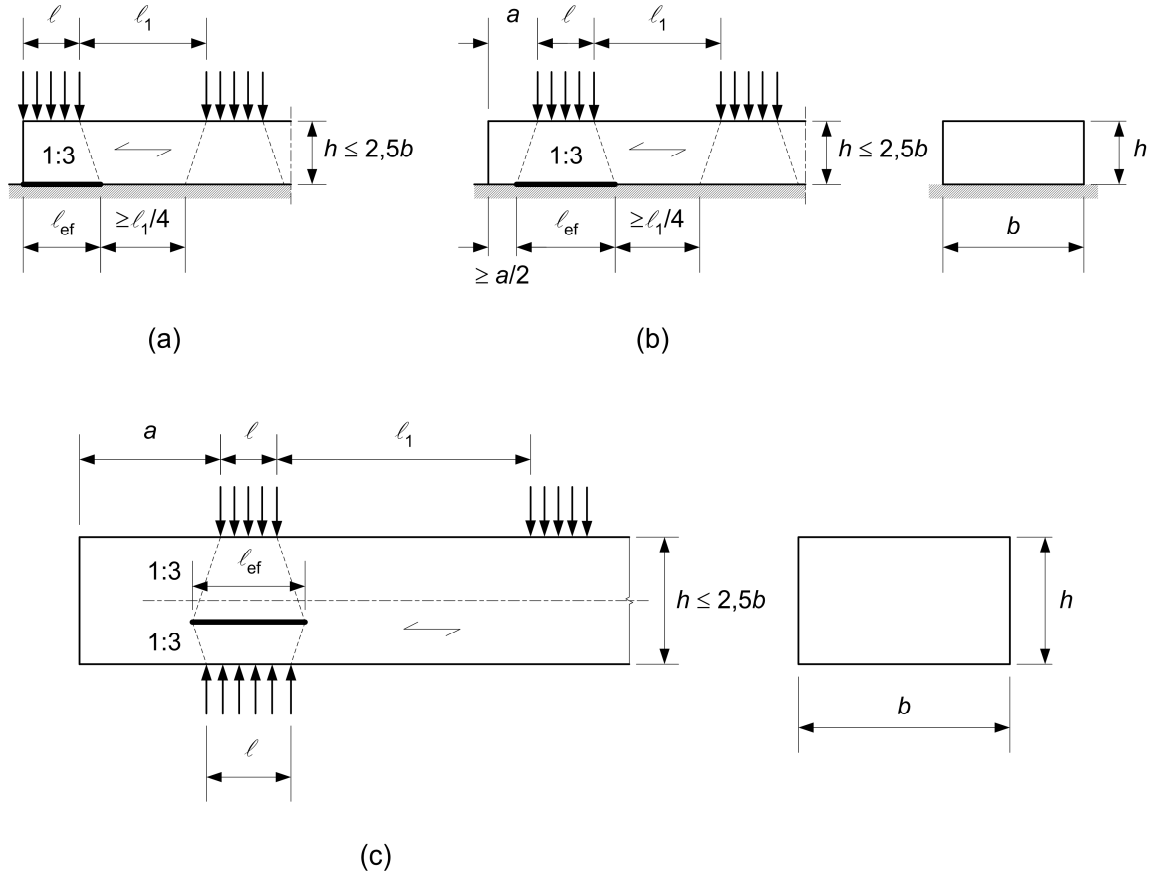(a)   (b)

(c)

**Figure 6.5 Determination of effective lengths for a member with h/b ≤ 2,5,
(a) and (b) continuous support, (c) discrete supports**

The effective length of distribution ℓ_{ef} should be determined from a stress dispersal line with a vertical inclination of 1:3 over the depth h, but curtailed by a distance of $a/2$ from any end, or a distance of $\ell_1/4$ from any adjacent compressed area, see Figure 6.5a and b.
For the particular positions of forces below, the effective length is given by:
- for loads adjacent to the end of the member, see Figure 6.5a

$$\ell_{ef} = \ell + \frac{h}{3}$$ (6.7)

- when the distance from the edge of a concentrated load to the end of the member a,

$\geq \dfrac{2}{3}h$,see Figure 6.5b

$$\ell_{ef} = \ell + \frac{2h}{3}$$ (6.8)

where h is the depth of the member or 40 mm, whichever is the largest.

For members on discrete supports, provided that $a \geq h$ and $\ell_1 \geq 2h$, see Figure 6.5c, the effective length should be calculated as:

$$\ell_{ef} = 0,5 \left(\ell + \ell_s + \frac{2h}{3} \right)$$
(6.9)

where h is the depth of the member or 40 mm, whichever is the largest.

For a member with a depth $h > 2,5b$ loaded with a concentrated compressive force on two opposite sides as shown in Figure 6.6b, or with a concentrated compressive force on one side and a continuous support on the other, see Figure 6.6a, the factor $k_{c,90}$ should be calculated according to expression (6.10), provided that the following conditions are fulfilled:

− the applied compressive force occurs over the full member width b;

− the contact length ℓ is less than the greater of h or 100 mm:

$$k_{c,90} = \frac{\ell_{ef}}{\ell}$$
(6.10)

where:

ℓ \quad is the contact length according to Figure 6.6;

ℓ_{ef} \quad is the effective length of distribution according to Figure 6.6

The effective length of distribution should not extend by more than ℓ beyond either edge of the contact length.

For members whose depth varies linearly over the support (e.g. bottom chords of trusses at the heel joint), the depth h should be taken as the member depth at the centreline of the support, and the effective length ℓ_{ef} should be taken as equal to the contact length ℓ.

IMPORTANT: In amendment EC5/A1 simpler rules are given, see Chapter 13.
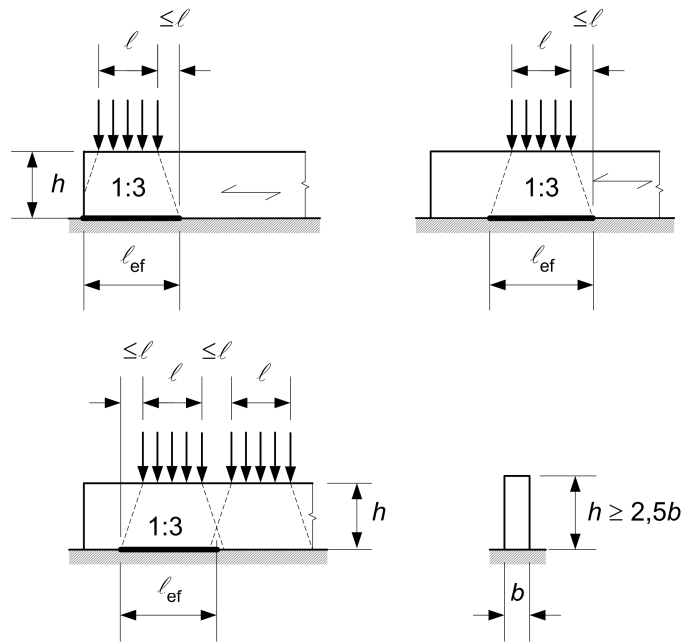
(a)
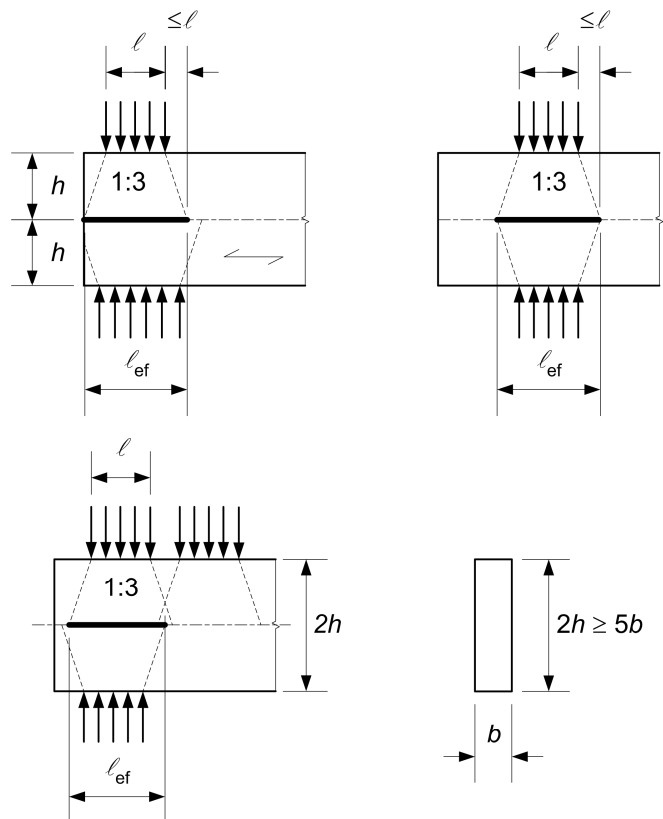
(b)

Figure 6.6 Determination of effective lengths for a member with $h/b > 2,5$ on (a) a continuous support, (b) discrete supports

6.1.6 Bending

The most common use of a beam is to resist loads by bending about its major principal axis. However, the introduction of forces which are not in the plane of principal bending results in bi-axial bending (i.e. bending about both the major and the minor principal axes). Additionally, the introduction of axial loads in tension or compression results in a further combined stress effect. For beams which are subjected to bi-axial bending, the following two conditions both need to be satisfied:

$$\frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.11}$$

$$k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.12}$$

where:

$\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the design bending stresses about the principal axes as shown in

Figure 6.1;

$f_{m,y,d}$ and $f_{m,z,d}$ are the corresponding design bending strengths.

NOTE: The factor k_m makes allowance for re-distribution of stresses and the effect of inhomogeneities of the material in a cross-section.

The value of the factor k_m should be taken as follows:

For solid timber, glued laminated timber and LVL:

for rectangular sections: $k_m = 0{,}7$

for other cross-sections: $k_m = 1{,}0$

For other wood-based structural products, for all cross-sections: $k_m = 1{,}0$.

A check shall also be made of the instability condition (see 6.3).

6.1.7 Shear

When bending is produced by transverse loading, shear stresses will be present according to the theory of elasticity. Shear stresses transverse to the beam axis will always be accompanied by equal shear stresses parallel to the beam axis.

For shear with a stress component parallel to the grain, see Figure 6.7(a), as well as for shear with both stress components perpendicular to the grain, see Figure 6.7(b), the following expression shall be satisfied:

$$\tau_d \leq f_{v,d} \tag{6.13}$$

where:

τ_d is the design shear stress;

$f_{v,d}$ is the design shear strength for the actual condition.

NOTE: The shear strength for rolling shear is approximately equal to twice the tension strength perpendicular to grain.
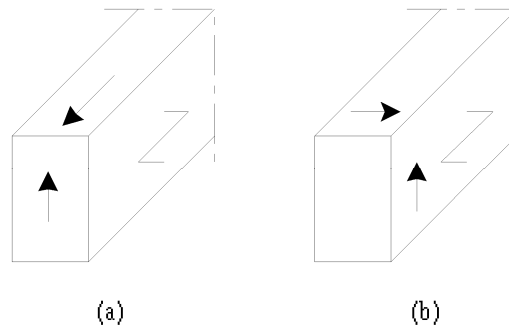

(a)   (b)

Figure 6.7(a) Member with a shear stress component parallel to the grain (b) Member with both stress components perpendicular to the grain (rolling shear)

At supports, the contribution to the total shear force of a concentrated load *F* acting on the top side of the beam and within a distance h or h_{ef} from the edge of the support may be disregarded (see Figure 6.8). For beams with a notch at the support this reduction in the shear force applies only when the notch is on the opposite side to the support.

Figure 6.8 Conditions at a support, for which the concentrated force *F* may be disregarded in the calculation of the shear force

IMPORTANT: In amendment EC5/A1 new rules are given, see Chapter 13.

6.1.8 Torsion
Torsional stresses are introduced when the applied load tends to twist a member. This will occur when a beam supports a load which is applied eccentrically to the principal cross sectional axis. A transmission mast may, for instance, be subjected to an eccentric horizontal load, resulting in a combination of shear and torsion.

The following expression shall be satisfied:

$$\tau_{\text{tor,d}} \le k_{\text{shape}} \, f_{\text{v,d}} \tag{6.14}$$

with

$$k_{\text{shape}} = \begin{cases} 1{,}2 & \text{for a circular cross section} \\ \min \begin{cases} 1+0{,}15 \, \dfrac{h}{b} \\ 2{,}0 \end{cases} & \text{for a rectangular cross section} \end{cases} \tag{6.15}$$

where:

$\tau_{\text{tor,d}}$ is the design torsional stress;

$f_{\text{v,d}}$ is the design shear strength;

k_{shape} is a factor depending on the shape of the cross-section;

h is the larger cross-sectional dimension;

b is the smaller cross-sectional dimension.

6.2 Design of cross-sections subjected to combined stresses

While the design of many members is to resist a single action such as bending, tension or compression, there are many cases in which members are subjected to two of these actions simultaneously.

6.2.1 Assumptions

Section 6.2 applies to straight solid timber, glued laminated timber or wood-based structural products of constant cross-section, whose grain runs essentially parallel to the length of the member. The member is assumed to be subjected to stresses from combined actions or to stresses acting in two or three of its principal axes.

6.2.2 Compression stresses at an angle to the grain

Interaction of compressive stresses in two or more directions shall be taken into account.

The compressive stresses at an angle α to the grain, (see Figure 6.9), should satisfy the following expression:

$$\sigma_{\text{c},\alpha,\text{d}} \le \frac{f_{\text{c,0,d}}}{\dfrac{f_{\text{c,0,d}}}{k_{\text{c,90}} \, f_{\text{c,90,d}}} \sin^2 \alpha + \cos^2 \alpha} \tag{6.16}$$

where:

$\sigma_{\text{c},\alpha,\text{d}}$ is the compressive stress at an angle α to the grain;

$k_{\text{c,90}}$ is a factor given in 6.1.5 taking into account the effect of any of stresses perpendicular to the grain.
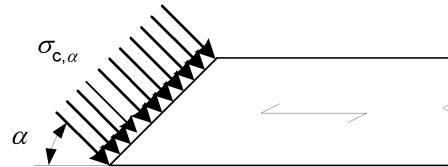
Figure 6.9 Compressive stresses at an angle to the grain

6.2.3 Combined bending and axial tension

The following expressions shall be satisfied:

$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.17}$$

$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.18}$$

The values of k_m given in 6.1.6 apply.

6.2.4 Combined bending and axial compression

The following expressions shall be satisfied:

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.19}$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,0,d}}\right)^2 + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \tag{6.20}$$

The values of k_m given in 6.1.6 apply.

NOTE: To check the instability condition, a method is given in 6.3.

6.3 Stability of members

When a slender column is loaded axially, there exists a tendency for it to deflect sideways (see Figure 6.10). This type of instability is called flexural buckling. The strength of slender members depends not only on the strength of the material but also on the stiffness, in the case of timber columns mainly on the bending stiffness. Therefore, apart from the compression and bending strength, the modulus of elasticity is an important material property influencing the load-bearing capacity of slender columns. The additional bending stresses caused by lateral deflections must be taken into account in a stability design. If a (geometric) nonlinear analysis, incorporating geometric imperfections (shape errors), is the basis for the design, the additional bending stresses are determined explicitly. In the case of linear static analyses, the higher order effects are accounted for approximately through modification (k) factors.

When designing beams, the prime concern is to provide adequate load carrying capacity and stiffness against bending about its major principal axis, usually in the vertical plane. This leads to a cross-sectional shape in which the stiffness in the vertical plane is often much greater than that in the horizontal plane.Whenever a slender structural element is loaded in its stiff plane (axially in the case of the column) there is a tendency for it to fail by buckling in a more flexible plane (by deflecting sideways in the case of the column). The response of a slender simply supported beam, subjected to bending moments in the vertical plane; is termed lateral-torsional buckling as it involves both lateral deflection and twisting (see Figure 6.11).
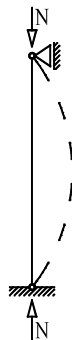
Figure 6.10 Two-hinged column buckling in compression
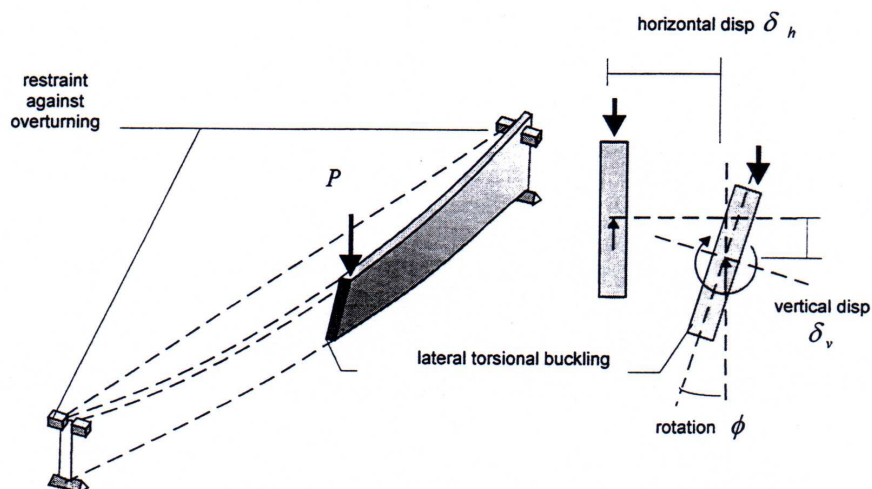
Figure 6.11 Lateral-torsional buckling of simply supported beam

6.3.1 Assumptions
The bending stresses due to initial curvature, eccentricities and induced deflection shall be taken into account, in addition to those due to any lateral load.

Column stability and lateral torsional stability shall be verified using the characteristic properties, e.g. $E_{0,05}$

The stability of columns subjected to either compression or combined compression and bending should be verified in accordance with 6.3.2.

The lateral torsional stability of beams subjected to either bending or combined bending and compression should be verified in accordance with 6.3.3.

The deviation from straightness measured midway between the supports should for columns and beams where lateral instability can occur, or members in frames, be limited to 1/500 times the length of glued laminated timber or LVL members and to 1/300 times the length of solid timber.

6.3.2 Columns subjected to either compression or combined compression and bending

The relative slenderness ratios should be taken as:

$$\lambda_{rel,y} = \frac{\lambda_y}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} \qquad (6.21)$$

and

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi} \sqrt{\frac{f_{c,0,k}}{E_{0,05}}} \qquad (6.22)$$

where:

λ_y and $\lambda_{rel,y}$ are slenderness ratios corresponding to bending about the *y*-axis (deflection in the *z*-direction);

λ_z and $\lambda_{rel,z}$ are slenderness ratios corresponding to bending about the *z*-axis;

$E_{0,05}$ is the fifth percentile value of the modulus of elasticity parallel to the grain.

Where both $\lambda_{rel,z} \leq 0,3$ and $\lambda_{rel,y} \leq 0,3$ the stresses should satisfy the expressions (6.19) and (6.20) in 6.2.4.

In all other cases the stresses, which will be increased due to deflection, should satisfy the following expressions:

$$\frac{\sigma_{c,0,d}}{k_{c,y} f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (6.23)$$

$$\frac{\sigma_{c,0,d}}{k_{c,z} f_{c,0,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1 \qquad (6.24)$$

where the symbols are defined as follows:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} \qquad (6.25)$$

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} \qquad (6.26)$$

$$k_y = 0,5 \left(1 + \beta_c \left(\lambda_{rel,y} - 0,3\right) + \lambda_{rel,y}^2\right) \qquad (6.27)$$

$$k_z = 0,5 \left(1 + \beta_c \left(\lambda_{rel,z} - 0,3\right) + \lambda_{rel,z}^2\right) \tag{6.28}$$

where:

β_c is a factor for members within the straightness limits:

$$\beta_c = \begin{cases} 0,2 & \text{for solid timber} \\ 0,1 & \text{for glued laminated timber and LVL} \end{cases} \tag{6.29}$$

k_m as given in 6.1.6.

6.3.3 Beams subjected to either bending or combined bending and compression

Lateral torsional stability shall be verified both in the case where only a moment M_y exists about the strong axis y and where a combination of moment M_y and compressive force N_c exists.

The relative slenderness for bending should be taken as:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} \tag{6.30}$$

where $\sigma_{m,crit}$ is the critical bending stress calculated according to the classical theory of stability, using 5-percentile stiffness values.

The critical bending stress should be taken as:

$$\sigma_{m,crit} = \frac{M_{y,crit}}{W_y} = \frac{\pi \sqrt{E_{0,05} I_z G_{0,05} I_{tor}}}{\ell_{ef} W_y} \tag{6.31}$$

where:

$E_{0,05}$ is the fifth percentile value of modulus of elasticity parallel to grain;

$G_{0,05}$ is the fifth percentile value of shear modulus parallel to grain;

I_z is the second moment of area about the weak axis z.

I_{tor} is the torsional moment of inertia;

ℓ_{ef} is the effective length of the beam, depending on the support conditions and the load configuration, acccording to Table 6.1;

W_y is the section modulus about the strong axis y.

For softwood with solid rectangular cross-section, $\sigma_{m,crit}$ should be taken as:

$$\sigma_{m,crit} = \frac{0,78 b^2}{h \, \ell_{ef}} E_{0,05} \tag{6.32}$$

where:

b is the width of the beam;

h is the depth of the beam.

In the case where only a moment M_y exists about the strong axis y, the stresses should satisfy the following expression:

$$\sigma_{m,d} \le k_{crit} \, f_{m,d} \tag{6.33}$$

where:

$\sigma_{m,d}$ is the design bending stress;

$f_{m,d}$ is the design bending strength;

k_{crit} is a factor which takes into account the reduced bending strength due to lateral buckling.

Table 6.1 Effective length as a ratio of the span

Beam type	Loading type	ℓ_{ef}/ℓ[a]
Simply supported	Constant moment Uniformly distributed load Concentrated force at the middle of the span	1,0 0,9 0,8
Cantilever	Uniformly distributed load Concentrated force at the free end	0,5 0,8
[a] The ratio between the effective length ℓ_{ef} and the span ℓ is valid for a beam with torsionally restrained supports and loaded at the centre of gravity. If the load is applied at the compression edge of the beam, ℓ_{ef} should be increased by $2h$ and may be decreased by $0,5h$ for a load at the tension edge of the beam.		

For beams with an initial lateral deviation from straightness within the limits, k_{crit} may be determined from expression (6.34)

$$k_{crit} = \begin{cases} 1 & \text{for } \lambda_{rel,m} \le 0,75 \\[2mm] 1,56 - 0,75\lambda_{rel,m} & \text{for } 0,75 < \lambda_{rel,m} \le 1,4 \\[2mm] \dfrac{1}{\lambda_{rel,m}^2} & \text{for } 1,4 < \lambda_{rel,m} \end{cases} \tag{6.34}$$

The factor k_{crit} may be taken as 1,0 for a beam where lateral displacement of its compressive edge is prevented throughout its length and where torsional rotation is prevented at its supports.

In the case where a combination of moment M_y about the strong axis y and compressive force N_c exists, the stresses should satisfy the following expression:

$$\left(\frac{\sigma_{m,d}}{k_{crit}f_{m,d}}\right)^2 + \frac{\sigma_{c,d}}{k_{c,z}f_{c,0,d}} \leq 1 \qquad (6.35)$$

where:

$\sigma_{m,d}$ is the design bending stress;

$\sigma_{c,d}$ is the design compressive stress;

$f_{c,0,d}$ is the design compressive strength parallel to grain;

$k_{c,z}$ is given by expression (6.26).

6.4 Design of cross-sections in members with varying cross-section or curved shape

Due to the range of sizes, lengths and shapes available, glulam is frequently used for different types of beams and arches. It is rare for sawn timber to be used as tapered or curved beams because of the difficulty of obtaining large sized cross section material and difficulties in bending it about its major axis to give a curved longitudinal profile.

6.4.1 Assumptions

The effects of combined axial force and bending moment shall be taken into account.

The relevant parts of 6.2 and 6.3 should be verified.

The stress at a cross-section from an axial force may be calculated from
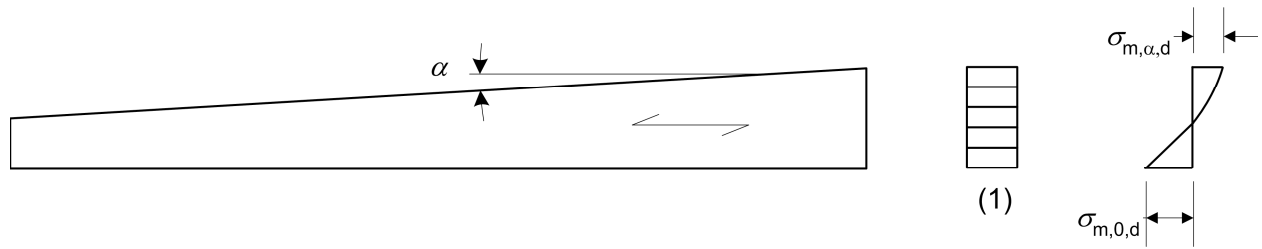
$$\sigma_N = \frac{N}{A} \qquad (6.36)$$

where:

σ_N is the axial stress;

N is the axial force;

A is the area of the cross-section.

6.4.2 Single tapered beams

The influence of the taper on the bending stresses parallel to the surface shall be taken into account.

Key:
(1) cross-section

Figure 6.12 Single tapered beam

The design bending stresses, $\sigma_{m,\alpha,d}$ and $\sigma_{m,0,d}$ (see Figure 6.12) may be taken as:

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 M_d}{b h^2} \qquad (6.37)$$

At the outermost fibre of the tapered edge, the stresses should satisfy the following expression:

$$\sigma_{m,\alpha,d} \leq k_{m,\alpha} f_{m,d} \qquad (6.38)$$

where:

$\sigma_{m,\alpha,d}$ is the design bending stress at an angle to grain;

$f_{m,d}$ is the design bending strength;

$k_{m,\alpha}$ should be calculated as:

For tensile stresses parallel to the tapered edge:

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{m,d}}{0,75 f_{v,d}} \tan \alpha \right)^2 + \left(\dfrac{f_{m,d}}{f_{t,90,d}} \tan^2 \alpha \right)^2}} \qquad (6.39)$$

For compressive stresses parallel to the tapered edge:

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{m,d}}{1,5 f_{v,d}} \tan \alpha \right)^2 + \left(\dfrac{f_{m,d}}{f_{c,90,d}} \tan^2 \alpha \right)^2}} \qquad (6.40)$$

6.4.3 Double tapered, curved and pitched cambered beams
This section applies only to glued laminated timber and LVL.

The requirements of 6.4.2 apply to the parts of the beam which have a single taper.

In the apex zone (see Figure 6.13), the bending stresses should satisfy the following expression:

$$\sigma_{m,d} \leq k_r \, f_{m,d} \tag{6.41}$$

where k_r takes into account the strength reduction due to bending of the laminates during production.

NOTE: In curved and pitched cambered beams the apex zone extends over the curved part of the beam.

The apex bending stress should be calculated as follows:

$$\sigma_{m,d} = k_\ell \frac{6 M_{ap,d}}{b \, h_{ap}^2} \tag{6.42}$$

with:

$$k_\ell = k_1 + k_2 \left(\frac{h_{ap}}{r}\right) + k_3 \left(\frac{h_{ap}}{r}\right)^2 + k_4 \left(\frac{h_{ap}}{r}\right)^3 \tag{6.43}$$

$$k_1 = 1 + 1{,}4 \, \tan \alpha_{ap} + 5{,}4 \, \tan^2 \alpha_{ap} \tag{6.44}$$

$$k_2 = 0{,}35 - 8 \, \tan \alpha_{ap} \tag{6.45}$$

$$k_3 = 0{,}6 + 8{,}3 \, \tan \alpha_{ap} - 7{,}8 \, \tan^2 \alpha_{ap} \tag{6.46}$$

$$k_4 = 6 \, \tan^2 \alpha_{ap} \tag{6.47}$$

$$r = r_{in} + 0{,}5 \, h_{ap} \tag{6.48}$$

where:

$M_{ap,d}$ is the design moment at the apex;

h_{ap} is the depth of the beam at the apex, see Figure 6.13;

b is the width of the beam;

r_{in} is the inner radius, see Figure 6.13;

α_{ap} is the angle of the taper in the middle of the apex zone, see Figure 6.13.
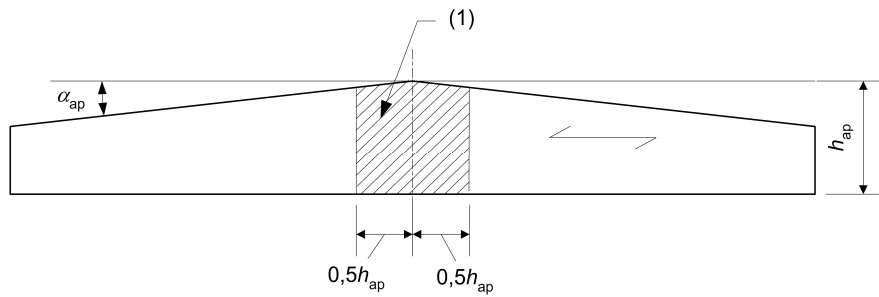
For double tapered beams $k_r = 1{,}0$. For curved and pitched cambered beams k_r should be taken as:

$$k_r = \begin{cases} 1 & \text{for } \dfrac{r_{in}}{t} \geq 240 \\[2ex] 0{,}76 + 0{,}001 \dfrac{r_{in}}{t} & \text{for } \dfrac{r_{in}}{t} < 240 \end{cases} \tag{6.49}$$
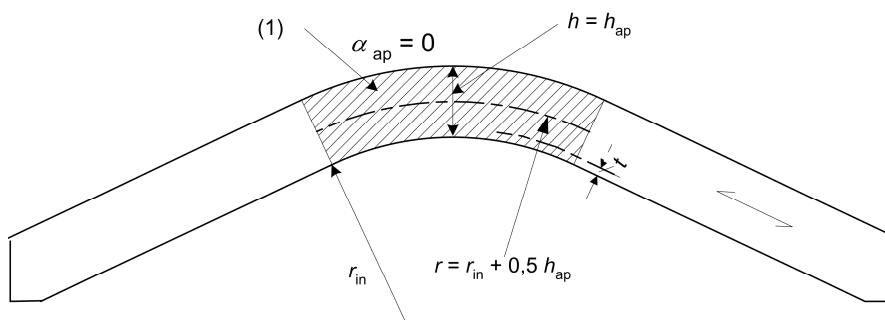
where

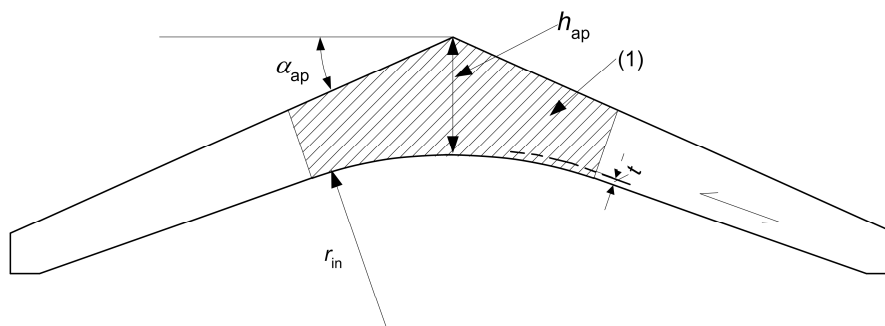r_{in} is the inner radius, see Figure 6.13;

t is the lamination thickness.

(a)

(b)

(c)

Key:
(1) Apex Zone

NOTE: In curved and pitched cambered beams the apex zone extends over the curved parts of the beam.

Figure 6.13 Double tapered (a), curved (b) and pitched cambered (c) beams with the fibre direction parallel to the lower edge of the beam

In the apex zone the greatest tensile stress perpendicular to the grain, $\sigma_{t,90,d}$, should satisfy the following expression:

$$\sigma_{t,90,d} \leq k_{dis} \, k_{vol} \, f_{t,90,d} \tag{6.50}$$

with

$$k_{\text{vol}} = \begin{cases} 1,0 & \text{for solid timber} \\ \left(\dfrac{V_0}{V}\right)^{0,2} & \text{for glued laminated timber and LVL with} \\ & \text{all veneers parallel to the beam axis} \end{cases} \tag{6.51}$$

$$k_{\text{dis}} = \begin{cases} 1,4 & \text{for double tapered and curved beams} \\ 1,7 & \text{for pitched cambered beams} \end{cases} \tag{6.52}$$

where:

k_{dis} is a factor which takes into account the effect of the stress distribution in the apex zone;

k_{vol} is a volume factor;

$f_{\text{t,90,d}}$ is the design tensile strength perpendicular to the grain;

V_0 is the reference volume of 0,01m³;

V is the stressed volume of the apex zone, in m³, (see Figure 6.13) and should not be taken greater than $2V_b/3$, where V_b is the total volume of the beam.

For combined tension perpendicular to grain and shear the following expression shall be satisfied:

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{\text{dis}}\, k_{\text{vol}}\, f_{t,90,d}} \leq 1 \tag{6.53}$$

where:

τ_d is the design shear stress;

$f_{v,d}$ is the design shear strength;

$\sigma_{t,90,d}$ is the design tensile stress perpendicular to grain;

k_{dis} and k_{vol} are given in expressions (6.51) and (6.52).

The greatest tensile stress perpendicular to the grain due to the bending moment should be calculated as follows:

$$\sigma_{t,90,d} = k_p \frac{6 M_{\text{ap,d}}}{b\, h_{\text{ap}}^2} \tag{6.54}$$

or, as an alternative to expression (6.54), as

$$\sigma_{t,90,d} = k_p \frac{6 M_{\text{ap,d}}}{b\, h_{\text{ap}}^2} - 0,6 \frac{p_d}{b} \tag{6.55}$$

where:

p_d is the uniformly distributed load acting on the top of the beam over the apex area;

b   is the width of the beam;

$M_{ap,d}$ is the design moment at apex resulting in tensile stresses parallel to the inner curved edge;

with:

$$k_p = k_5 + k_6\left(\frac{h_{ap}}{r}\right) + k_7\left(\frac{h_{ap}}{r}\right)^2 \qquad (6.56)$$

$$k_5 = 0,2 \ \tan\alpha_{ap} \qquad (6.57)$$

$$k_6 = 0,25 - 1,5 \ \tan\alpha_{ap} + 2,6 \ \tan^2\alpha_{ap} \qquad (6.58)$$

$$k_7 = 2,1 \ \tan\alpha_{ap} - 4 \ \tan^2\alpha_{ap} \qquad (6.59)$$

6.5. Notched members

It is not uncommon for the ends of beams to be notched at the bottom, to increase clearance or to bring the top surface of a particular beam level with other beams or girdes. Notches usually create stress concentrations in the region of the re-entrant corners.

6.5.1 Assumptions

The effects of stress concentrations at the notch shall be taken into account in the strength verification of members.

The effect of stress concentrations may be disregarded in the following cases:

− tension or compression parallel to the grain;

− bending with tensile stresses at the notch if the taper is not steeper than 1:*i* = 1:10, that is *i* ≥ 10, see Figure 6.14a;

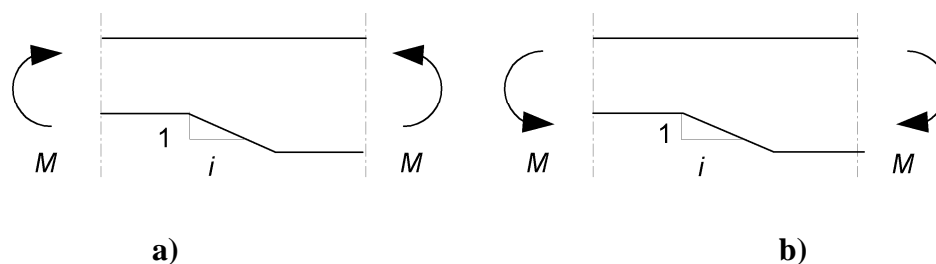− bending with compressive stresses at the notch, see Figure 6.14b.

**Figure 6.14 Bending at a notch: a) with tensile stresses at the notch,
b) with compressive stresses at the notch**

6.5.2 Beams with a notch at the support

For beams with rectangular cross-sections and where grain runs essentially parallel to the length of the member, the shear stresses at the notched support should be calculated using the effective (reduced) depth h_{ef} (see Figure 6.15).

It should be verified that

$$\tau_{\mathrm{d}} = \frac{1,5V}{b\,h_{\mathrm{ef}}} \le k_{\mathrm{v}} f_{\mathrm{v,d}} \tag{6.60}$$

where k_{v} is a reduction factor defined as follows:

− For beams notched at the opposite side to the support (see Figure 6.15b)

$$k_{\mathrm{v}} = 1,0 \tag{6.61}$$

− For beams notched on the same side as the support (see Figure 6.15a)

$$k_{\mathrm{v}} = \min \left\{ \begin{array}{l} 1 \\[2em] \dfrac{k_{\mathrm{n}}\left(1 + \dfrac{1,1\,i^{1,5}}{\sqrt{h}}\right)}{\sqrt{h}\left(\sqrt{\alpha(1-\alpha)} + 0,8\dfrac{x}{h}\sqrt{\dfrac{1}{\alpha} - \alpha^2}\right)} \end{array} \right. \tag{6.62}$$

where:

i is the notch inclination (see Figure 6.15a);

h is the beam depth in mm;

x is the distance from line of action of the support reaction to the corner of the notch;

$$\alpha = \frac{h_{\mathrm{ef}}}{h}$$

$$k_{\mathrm{n}} = \begin{cases} 4,5 & \text{for LVL} \\ 5 & \text{for solid timber} \\ 6,5 & \text{for glued laminated timber} \end{cases} \tag{6.63}$$

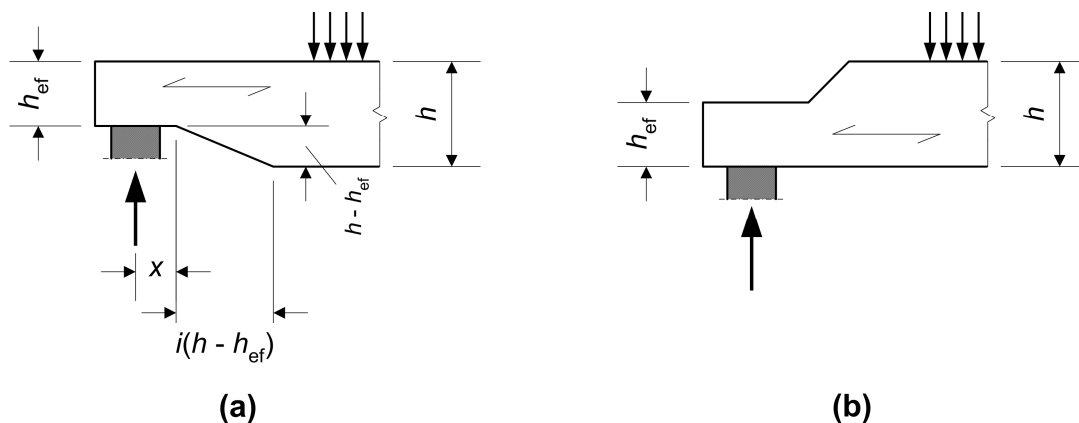**(a)**   **(b)**

Figure 6.15 End-notched beams

6.6 System strength

When several equally spaced similar members, components or assemblies are laterally connected by a continuous load distribution system, the member strength properties may be multiplied by a system strength factor k_{sys}.

Provided the continuous load-distribution system is capable of transfering the loads from one member to the neighbouring members, the factor k_{sys} should be 1,1.

The strength verification of the load distribution system should be carried out assuming the loads are of short-term duration.

NOTE: For roof trusses with a maximum centre to centre distance of 1,2 m it may be assumed that tiling battens, purlins or panels can transfer the load to the neighbouring trusses provided that these load-distribution members are continuous over at least two spans, and any joints are staggered.

For laminated timber decks or floors the values of k_{sys} given in Figure 6.16 should be used.
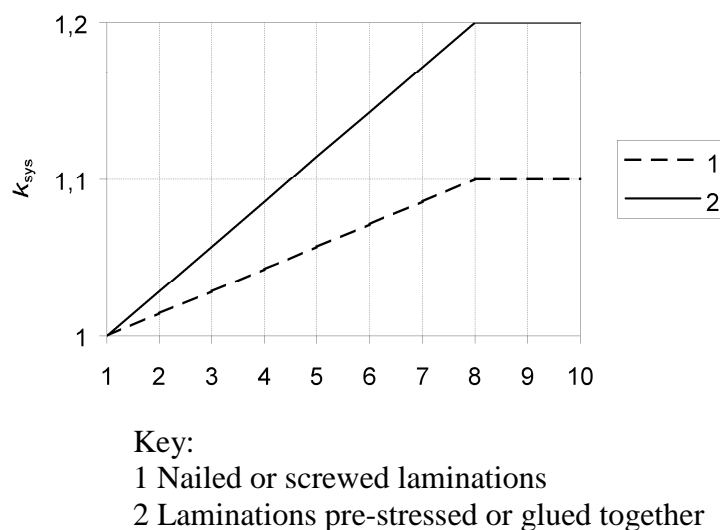

Key:
1 Nailed or screwed laminations
2 Laminations pre-stressed or glued together

Figure 6.16 System strength factor k_{sys} for laminated deck plates of solid timber or glued laminated members

7 Serviceability limit states

The overall performance of structures should satisfy two basic requirements. The first is safety, usually expressed in terms of load bearing capacity, and the second is serviceability, which refers to the ability of the structural system and its elements to perform satisfactorily in normal use.

7.1 Joint slip

For joints made with mechanical fasteners the slip modulus K_{ser} per shear plane per fastener under service load should be taken from Table 7.1 with ρ_m (mean density) in kg/m³ and d (diameter) or d_c (connector diameter) in mm.

Table 7.1 Values of K_{ser} for dowel-type fasteners and connectors in N/mm

in timber-to-timber and wood-based panel-to-timber connections

Fastener type	K_{ser}
Dowels Bolts with or without clearance[a] Screws Nails (with pre-drilling)	$\rho_m^{1,5} d/23$
Nails (without pre-drilling)	$\rho_m^{1,5} d^{0,8}/30$
Staples	$\rho_m^{1,5} d^{0,8}/80$
Split-ring connectors type A according to EN 912 Shear-plate connectors type B according to EN 912	$\rho_m d_c/2$

Toothed-plate connectors:
  – Connectors types C1 to C9 according to EN 912
  – Connectors type C10 and C11 according to EN 912
	$1,5\rho_m d_c/4$
	$\rho_m d_c/2$

[a] The clearance should be added separately to the deformation.	

If the mean densities $\rho_{m,1}$ and $\rho_{m,2}$ of the two jointed wood-based members are different then ρ_m in the above expressions should be taken as

$$\rho_m = \sqrt{\rho_{m,1}\rho_{m,2}}$$

(7.1)

For steel-to-timber or concrete-to-timber connections, K_{ser} should be based on ρ_m for the timber member and may be multiplied by 2,0.

7.2 Limiting values for deflections of beams

The fact that variable loads (such as imposed loads on floors and snow loads on roofs) often dominate in timber structures means that the deflection will vary considerably during the lifetime of the structure. This has to be considered in a rational serviceability design.

The components of deflection resulting from a combination of actions are shown in Figure 7.1, where the symbols are defined as follows:

− w_c is the precamber (if applied);

− w_{inst} is the instantaneous deflection;

− w_{creep} is the creep deflection;

− w_{fin} is the final deflection;
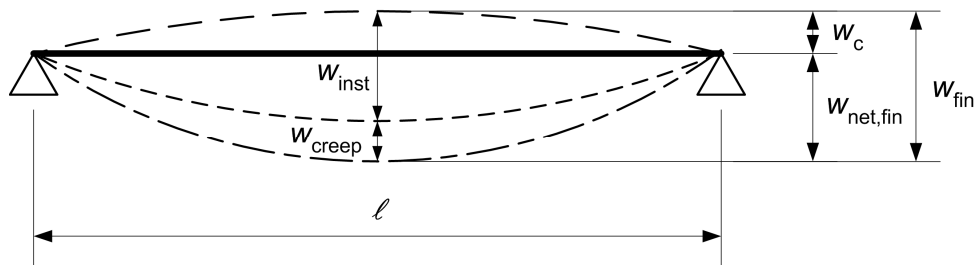
− $w_{net,fin}$ is the net final deflection.

Figure 7.1 Components of deflection

The net deflection below a straight line between the supports, $w_{net,fin}$, should be taken as:

$$w_{net,fin} = w_{inst} + w_{creep} - w_c = w_{fin} - w_c \tag{7.2}$$

NOTE: The recommended range of limiting values of deflections for beams with span ℓ is given in Table 7.2 depending upon the level of deformation deemed to be acceptable.

Table 7.2 Examples of limiting values for deflections of beams

	w_{inst}	$w_{net,fin}$	w_{fin}
Beam on two supports	$\ell/300$ to $\ell/500$	$\ell/250$ to $\ell/350$	$\ell/150$ to $\ell/300$
Cantilevering beams	$\ell/150$ to $\ell/250$	$\ell/125$ to $\ell/175$	$\ell/75$ to $\ell/150$

7.3 Vibrations

In general there are many load-response cases where structural vibrations may effect the structure's serviceability. The main concern, however, is with regard to human discomfort. People are in most cases the critical sensor of vibration. Among different dynamic actions, human activity and installed machinery are regarded as the two most important sources of vibration in timber-framed buildings. Human activity not only includes footfall from normal

walking, but also children's jumping, etc. Two critical load response cases are finally identified:

- Human discomfort from footfall-induced vibrations.

- Human discomfort from machine-induced vibrations.

7.3.1 Assumptions

It shall be ensured that the actions which can be reasonably anticipated on a member, component or structure, do not cause vibrations that can impair the function of the structure or cause unacceptable discomfort to the users.

The vibration level should be estimated by measurements or by calculation taking into account the expected stiffness of the member, component or structure and the modal damping ratio.

For floors, unless other values are proven to be more appropriate, a modal damping ratio of $\zeta = 0,01$ (i.e. 1 %) should be assumed.

7.3.2 Vibrations from machinery

Vibrations caused by rotating machinery and other operational equipment shall be limited for the unfavourable combinations of permanent load and variable loads that can be expected.

For floors, acceptable levels for continuous vibration should be taken from Figure 5a in Appendix A of ISO 2631-2 with a multiplying factor of 1,0.

Residential floors

For residential floors with a fundamental frequency less than 8 Hz ($f_1 \leq 8 Hz$) a special investigation should be made.

For residential floors with a fundamental frequency greater than 8 Hz ($f_1 > 8$ Hz) the following requirements should be satisfied:

$$\frac{w}{F} \leq a \quad \text{mm/kN} \tag{7.3}$$

and

$$v \leq b^{(f_1\zeta-1)} \quad \text{m/(Ns}^2) \tag{7.4}$$

where:

w is the maximum instantaneous vertical deflection caused by a vertical concentrated static force F applied at any point on the floor, taking account of load distribution;

v is the unit impulse velocity response, i.e. the maximum initial value of the vertical floor vibration velocity (in m/s) caused by an ideal unit impulse (1 Ns) applied at the point of the floor giving maximum response. Components above 40 Hz may be disregarded;

ζ is the modal damping ratio.

NOTE: The recommended range of limiting values of *a* and *b* and the recommended relationship between *a* and *b* is given in Figure 7.2.
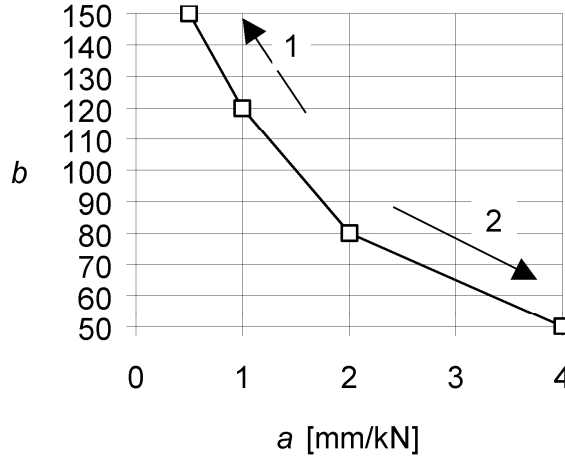
Key:
1   Better performance
2   Poorer performance

Figure 7.2 Recommended range of and relationship between *a* and *b*

The calculations for floors in 7.3.2 should be made under the assumption that the floor is unloaded, i.e., only the mass corresponding to the self-weight of the floor and other permanent actions.

For a rectangular floor with overall dimensions $\ell \times b$, simply supported along all four edges and with timber beams having a span ℓ, the fundamental frequency f_1 may approximately be calculated as

$$f_1 = \frac{\pi}{2\,\ell^2} \sqrt{\frac{(EI)_\ell}{m}} \qquad\qquad (7.5)$$

where:

m is the mass per unit area in kg/m²;

ℓ is the floor span, in m;

$(EI)_\ell$ is the equivalent plate bending stiffness of the floor about an axis perpendicular to the beam direction, in Nm²/m.

For a rectangular floor with overall dimensions $b \times \ell$, simply supported along all four edges, the value *v* may, as an approximation, be taken as:

$$v = \frac{4\,(0,4\ +\ 0,6\,n_{40})}{m\,b\,\ell\ +\ 200} \tag{7.6}$$

where:

v is the unit impulse velocity response, in m/(Ns^2);

n_{40} is the number of first-order modes with natural frequencies up to 40 Hz;

b is the floor width, in m;

m is the mass, in kg/m^2;

ℓ is the floor span, in m.

The value of n_{40} may be calculated from:

$$n_{40} = \left\{ \left[\left(\frac{40}{f_1} \right)^2 - 1 \right] \left(\frac{b}{\ell} \right)^4 \frac{(EI)_\ell}{(EI)_b} \right\}^{0,25} \tag{7.7}$$

where $(EI)_b$ is the equivalent plate bending stiffness, in Nm^2/m, of the floor about an axis parallel to the beams, where $(EI)_b < (EI)_\ell$.

8 Connections with metal fasteners

For timber structures, the serviceability and the durability of the structure depend mainly on the design of the joints between the elements. For commonly used connections, a distinction is made between carpentry joints and mechanical joints that can be made from several types of fastener.

For a given structure, the selection of fasteners is not only controlled by the loading and the load-carrying capacity conditions. It also involves considerations such as aesthetics, cost-efficiency and the fabrication process. The erection method and the preference of the designer and/or the architect may also play a role. It is impossible to specify a set of rules from which the best connection can be designed for any given structure. The main idea is that the simpler the joint and the fewer the fasteners, the better is the structural result.

The traditional mechanical fasteners are divided into two groups depending on how they transfer the forces between the connected members.

The main group corresponds to the dowel type fasteners. Here, the load transfer involves both the bending behaviour of the dowel and the bearing and shear stresses in the timber along the shank of the dowel. Staples, nails, screws, bolts and dowels belong to this group. The second type includes fasteners such as split-rings, shear-plates, and punched metal plates for which the load transmission is primarily achieved by a large bearing area at the surface of the members. This handbook deals only with dowel type fasteners.
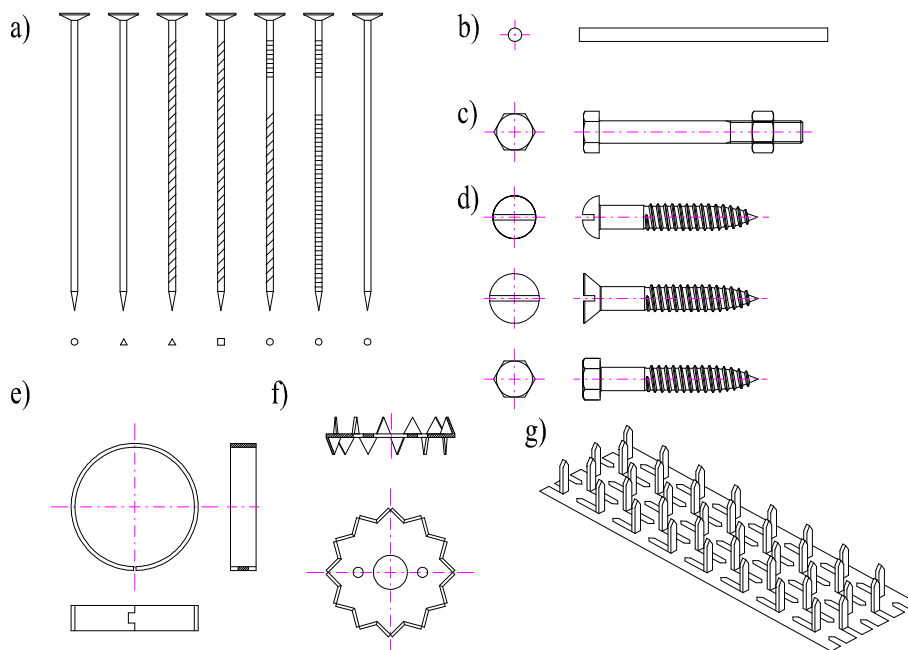
Figure 8.0 Metal fasteners
a) nails, b) dowel, c) bolt, d) srews, e) split ring connector, f) toothed-plate connector

g) punched metal plate fastener

8.1   Basic assumptions
There is a huge variety of configurations and design loadings of connections.

8.1.1 Fastener requirements
Unless rules are given in this chapter, the characteristic load-carrying capacity, and the stiffness of the connections shall be determined from tests according to EN 1075, EN 1380, EN 1381, EN 26891 and EN 28970. If the relevant standards describe tension and compression tests, the tests for the determination of the characteristic load-carrying capacity shall be performed in tension.

8.1.2 Multiple fastener connections
The arrangement and sizes of the fasteners in a connection, and the fastener spacings, edge and end distances shall be chosen so that the expected strength and stiffness can be obtained.

It shall be taken into account that the load-carrying capacity of a multiple fastener connection, consisting of fasteners of the same type and dimension, may be lower than the summation of the individual load-carrying capacities for each fastener.

When a connection comprises different types of fasteners, or when the stiffness of the connections in respective shear planes of a multiple shear plane connection is different, their compatibility should be verified.

For one row of fasteners parallel to the grain direction, the effective characteristic load-carrying capacity parallel to the row, $F_{v,ef,Rk}$, should be taken as:

$$F_{v,ef,Rk} = n_{ef} F_{v,Rk}$$ (8.1)

where:

$F_{v,ef,Rk}$ is the effective characteristic load-carrying capacity of one row of fasteners parallel to the grain;

n_{ef} is the effective number of fasteners in line parallel to the grain;

$F_{v,Rk}$ is the characteristic load-carrying capacity of each fastener parallel to the grain.

NOTE: Values of n_{ef} for rows parallel to grain are given in 8.3.1.1 and 8.5.1.1.

For a force acting at an angle to the direction of the row, it should be verified that the force component parallel to the row is less than or equal to the load-carrying capacity calculated according to expression (8.1).

8.1.3 Multiple shear plane connections
In multiple shear plane connections the resistance of each shear plane should be determined by assuming that each shear plane is part of a series of three-member connections.

To be able to combine the resistance from individual shear planes in a multiple shear plane connection, the governing failure mode of the fasteners in the respective shear planes should be compatible with each other and should not consist of a combination of failure modes (a), (b), (g) and (h) from Figure 8.2 or modes (c), (f) and (j/l) from Figure 8.3 with the other failure modes.

8.1.4 Connection forces at an angle to the grain

When a force in a connection acts at an angle to the grain, (see Figure 8.1), the possibility of splitting caused by the tension force component $F_{Ed} \sin \alpha$, perpendicular to the grain, shall be taken into account.

To take account of the possibility of splitting caused by the tension force component, $F_{Ed} \sin \alpha$, perpendicular to the grain, the following shall be satisfied:

$$F_{v,Ed} \leq F_{90,Rd} \tag{8.2}$$

with

$$F_{v,Ed} = \max \begin{cases} F_{v,Ed,1} \\ F_{v,Ed,2} \end{cases} \tag{8.3}$$

where:

$F_{90,Rd}$ is the design splitting capacity, calculated from the characteristic splitting capacity $F_{90,Rk}$ according to 2.3.3;

$F_{v,Ed,1}$, $F_{v,Ed,2}$ are the design shear forces on either side of the connection (see Figure 8.1).

For softwoods, the characteristic splitting capacity for the arrangement shown in Figure 8.1 should be taken as:

$$F_{90,Rk} = 14 \, b \, w \sqrt{\frac{h_e}{\left(1 - \dfrac{h_e}{h}\right)}} \tag{8.4}$$

where:

$$w = \begin{cases} \max \left\{ \begin{array}{l} \left(\dfrac{w_{pl}}{100}\right)^{0,35} \quad \text{for punched metal plate fasteners} \\ 1 \end{array} \right. \\ 1 \qquad\qquad\quad \text{for all other fasteners} \end{cases} \tag{8.5}$$

and:

$F_{90,Rk}$ is the characteristic splitting capacity, in N;

w is a modification factor;

h_e is the loaded edge distance to the centre of the most distant fastener or to the edge of the punched metal plate fastener, in mm;

h is the timber member height, in mm;

b   is the member thickness, in mm;

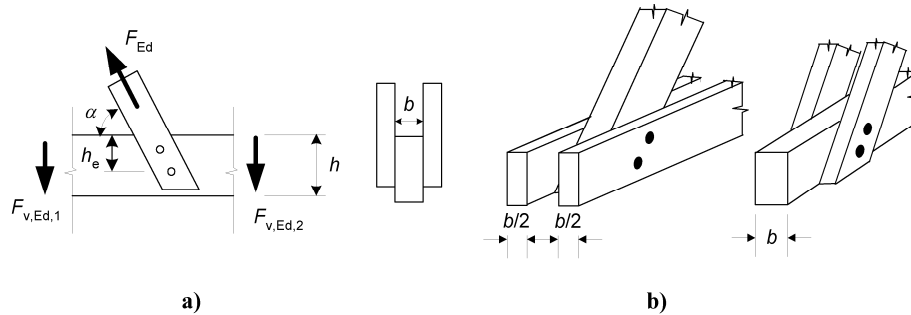w_{pl} is the width of the punched metal plate fastener parallel to the grain, in mm.

Figure 8.1 Inclined force transmitted by a connection

8.1.5 Alternating connection forces
The characteristic load-carrying capacity of a connection shall be reduced if the connection is subject to alternating internal forces due to long-term or medium-term actions.

The effect on connection strength of long-term or medium-term actions alternating between a tensile design force $F_{t,Ed}$ and a compressive design force $F_{c,Ed}$ should be taken into account by designing the connection for $(F_{t,Ed} + 0{,}5F_{c,Ed})$ and $(F_{c,Ed} + 0{,}5F_{t,Ed})$.

8.2 Lateral load-carrying capacity of metal dowel-type fasteners
The failure of laterally loaded fasteners include both crushing of the timber and bending of the fastener.

8.2.1 Asumptions
For the determination of the characteristic load-carrying capacity of connections with metal dowel-type fasteners the contributions of the yield strength, the embedment strength, and the withdrawal strength of the fastener shall be considered.

8.2.2 Timber-to-timber and panel-to-timber connections
The characteristic load-carrying capacity for nails, staples, bolts, dowels and screws per shear plane per fastener, should be taken as the minimum value found from the following expressions:

– For fasteners in single shear

$$F_{v,Rk} = \min \begin{cases} f_{h,1,k}t_1d & \text{(a)} \\[4pt] f_{h,2,k}t_2d & \text{(b)} \\[4pt] \dfrac{f_{h,1,k}t_1d}{1+\beta}\left[\sqrt{\beta+2\beta^2\left[1+\dfrac{t_2}{t_1}+\left(\dfrac{t_2}{t_1}\right)^2\right]+\beta^3\left(\dfrac{t_2}{t_1}\right)^2}-\beta\left(1+\dfrac{t_2}{t_1}\right)\right]+\dfrac{F_{ax,Rk}}{4} & \text{(c)} \\[4pt] 1{,}05\dfrac{f_{h,1,k}t_1d}{2+\beta}\left[\sqrt{2\beta(1+\beta)+\dfrac{4\beta(2+\beta)M_{y,Rk}}{f_{h,1,k}d\ t_1^2}}-\beta\right]+\dfrac{F_{ax,Rk}}{4} & \text{(d)} \\[4pt] 1{,}05\dfrac{f_{h,1,k}t_2d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta)+\dfrac{4\beta(1+2\beta)M_{y,Rk}}{f_{h,1,k}d\ t_2^2}}-\beta\right]+\dfrac{F_{ax,Rk}}{4} & \text{(e)} \\[4pt] 1{,}15\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,Rk}f_{h,1,k}d}+\dfrac{F_{ax,Rk}}{4} & \text{(f)} \end{cases} \qquad (8.6)$$

− For fasteners in double shear:

$$F_{v,Rk} = \min \begin{cases} f_{h,1,k}t_1d & \text{(g)} \\[4pt] 0{,}5f_{h,2,k}t_2d & \text{(h)} \\[4pt] 1{,}05\dfrac{f_{h,1,k}t_1d}{2+\beta}\left[\sqrt{2\beta(1+\beta)+\dfrac{4\beta(2+\beta)M_{y,Rk}}{f_{h,1,k}d\ t_1^2}}-\beta\right]+\dfrac{F_{ax,Rk}}{4} & \text{(j)} \\[4pt] 1{,}15\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,Rk}f_{h,1,k}d}+\dfrac{F_{ax,Rk}}{4} & \text{(k)} \end{cases} \qquad (8.7)$$

with

$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}} \qquad (8.8)$$

where:

$F_{v,Rk}$ is the characteristic load-carrying capacity per shear plane per fastener;

t_i is the timber or board thickness or penetration depth, with i either 1 or 2, see also 8.3 to 8.7 ;

$f_{h,i,k}$ is the characteristic embedment strength in timber member i;

d is the fastener diameter;

$M_{y,Rk}$ is the characteristic fastener yield moment;

β is the ratio between the embedment strength of the members;

$F_{ax,Rk}$ is the characteristic axial withdrawal capacity of the fastener.

NOTE: Plasticity of joints can be assured when relatively slender fasteners are used. In that case, failure modes (f) and (k) are governing.

In the expressions (8.6) and (8.7), the first term on the right hand side is the load-carrying capacity according to the Johansen yield theory, whilst the second term $F_{ax,Rk}/4$ is the contribution from the rope effect. The contribution to the load-carrying capacity due to the rope effect should be limited to following percentages of the Johansen part:

− Round nails   15 %

− Square nails   25 %

− Other nails   50 %

− Screws   100%

− Bolts   25 %

− Dowels   0 %

If $F_{ax,Rk}$ is not known then the contribution from the rope effect should be taken as zero.

For single shear fasteners the characteristic withdrawal capacity, $F_{ax,Rk}$, is taken as the lower of the capacities in the two members. The different modes of failure are illustrated in Figure 8.2. For the withdrawal capacity, $F_{ax,Rk}$, of bolts the resistance provided by the washers may be taken into account, see 8.5.2.

If no design rules are given below, the characteristic embedment strength $f_{h,k}$ should be determined according to EN 383 and EN 14358.

If no design rules are given below, the characteristic yield moment $M_{y,k}$ should be determined according to EN 409 and EN 14358.
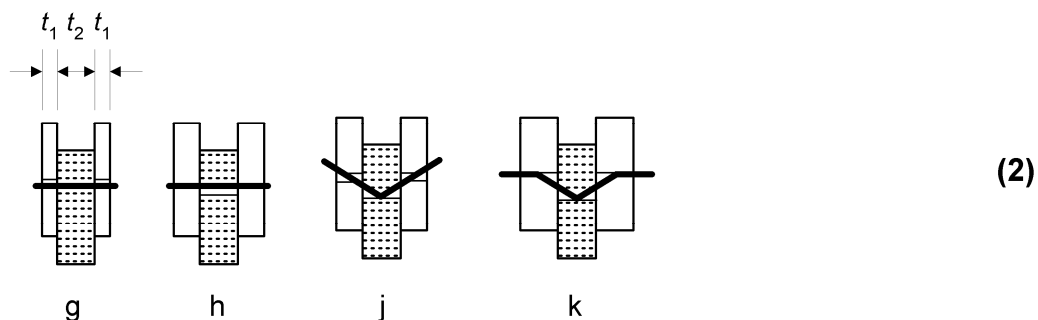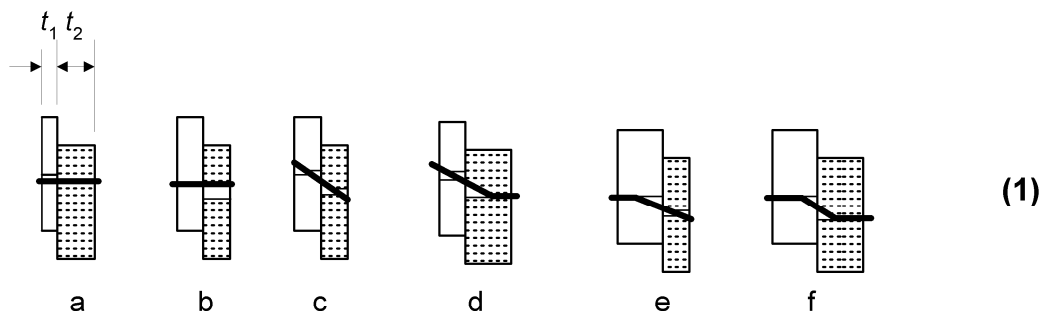
Key:
(1) Single shear
(2) Double shear

NOTE: The letters correspond to the references of the expressions (8.6) and (8.7).

Figure 8.2 Failure modes for timber and panel connections.

8.2.3 Steel-to-timber connections

The characteristic load-carrying capacity of a steel-to-timber connection depends on the thickness of the steel plates. Steel plates of thickness less than or equal to $0,5d$ are classified as thin plates and steel plates of thickness greater than or equal to d with the tolerance on hole diameters being less than $0,1d$ are classified as thick plates. The characteristic load-carrying capacity of connections with steel plate thickness between a thin and a thick plate should be calculated by linear interpolation between the limiting thin and thick plate values.

The strength of the steel plate shall be checked.

The characteristic load-carrying capacity for nails, bolts, dowels and screws per shear plane per fastener should be taken as the minimum value found from the following expressions:

− For a thin steel plate in single shear:

$$F_{v,Rk} = \min \begin{cases} 0,4\, f_{h,k}\, t_1\, d & \text{(a)} \\[2mm] 1,15\sqrt{2\, M_{y,Rk}\, f_{h,k}\, d} + \dfrac{F_{ax,Rk}}{4} & \text{(b)} \end{cases} \qquad (8.9)$$

− For a thick steel plate in single shear:

$$F_{v,Rk} = \min \begin{cases} f_{h,k}\, t_1\, d\left[\sqrt{2 + \dfrac{4 M_{y,Rk}}{f_{h,k}\, d\, t_1^2}} - 1\right] + \dfrac{F_{ax,Rk}}{4} & \text{(c)} \\[4mm] 2,3\sqrt{M_{y,Rk}\, f_{h,k}\, d} + \dfrac{F_{ax,Rk}}{4} & \text{(d)} \\[4mm] f_{h,k}\, t_1\, d & \text{(e)} \end{cases} \qquad (8.10)$$

− For a steel plate of any thickness as the central member of a double shear connection:

$$F_{v,Rk} = \min \begin{cases} f_{h,1,k}\, t_1\, d & \text{(f)} \\[2mm] f_{h,1,k}\, t_1\, d\left[\sqrt{2 + \dfrac{4 M_{y,Rk}}{f_{h,1,k}\, d\, t_1^2}} - 1\right] + \dfrac{F_{ax,Rk}}{4} & \text{(g)} \\[4mm] 2,3\sqrt{M_{y,Rk}\, f_{h,1,k}\, d} + \dfrac{F_{ax,Rk}}{4} & \text{(h)} \end{cases} \qquad (8.11)$$

− For thin steel plates as the outer members of a double shear connection:

$$F_{v,Rk} = \min \begin{cases} 0,5\, f_{h,2,k}\, t_2\, d & \text{(j)} \\ 1,15\sqrt{2\, M_{y,Rk}\, f_{h,2,k}\, d} + \dfrac{F_{ax,Rk}}{4} & \text{(k)} \end{cases}$$

(8.12)

− For thick steel plates as the outer members of a double shear connection:

$$F_{v,Rk} = \min \begin{cases} 0,5\, f_{h,2,k}\, t_2\, d & \text{(l)} \\ 2,3\sqrt{M_{y,Rk}\, f_{h,2,k}\, d} + \dfrac{F_{ax,Rk}}{4} & \text{(m)} \end{cases}$$

(8.13)

where:

$F_{v,Rk}$ is the characteristic load-carrying capacity per shear plane per fastener;

$f_{h,k}$ is the characteristic embedment strength in the timber member;

t_1 is the smaller of the thickness of the timber side member or the penetration depth;

t_2 is the thickness of the timber middle member;

d is the fastener diameter;

$M_{y,Rk}$ is the characteristic fastener yield moment;

$F_{ax,Rk}$ is the characteristic withdrawal capacity of the fastener.

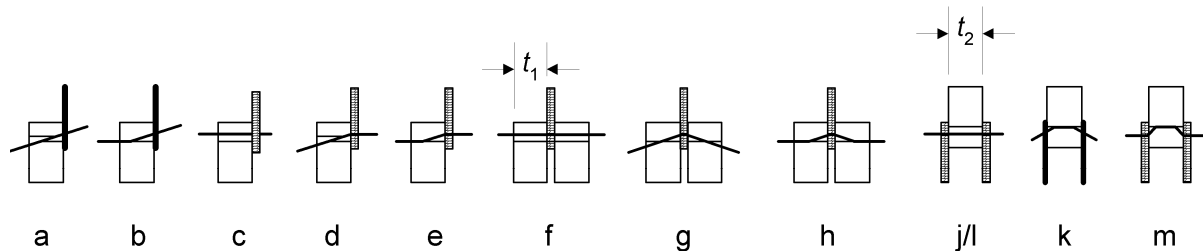NOTE 1: The different failure modes are illustrated in Figure 8.3.

Figure 8.3 Failure modes for steel-to-timber connections

For the limitation of the rope effect $F_{ax,Rk}$ 8.2.2 applies.

It shall be taken into account that the load-carrying capacity of steel-to-timber connections with a loaded end may be reduced by failure along the perimeter of the fastener group.

8.3 Nailed connections
Nails are the most commonly used fasteners in timber construction.

8.3.1 Laterally loaded nails
The failure of laterally loaded nails include both crushing of the timber and bending of the nail.

8.3.1.1 Asumptions

The symbols for the thicknesses in single and double shear connections (see Figure 8.4) are defined as follows:

t_1 is:

the headside thickness in a single shear connection;

the minimum of the head side timber thickness and the pointside penetration in a double shear connection;

t_2 is:

the pointside penetration in a single shear connection;

the central member thickness in a double shear connection.

Timber should be pre-drilled when:

− the characteristic density of the timber is greater than 500 kg/m^3;

− the diameter d of the nail exceeds 8 mm.

IMPORTANT: In amendment EC5/A1 new rules are given, see Chapter 13.

For square and grooved nails, the nail diameter d should be taken as the side dimension.

For smooth nails produced from wire with a minimum tensile strength of 600 N/mm^2, the following characteristic values for yield moment should be used:

$$M_{y,Rk} = \begin{cases} 0,3 \, f_u \, d^{2,6} & \text{for round nails} \\ 0,45 \, f_u \, d^{2,6} & \text{for square nails} \end{cases} \tag{8.14}$$

where:

$M_{y,Rk}$ is the characteristic value for the yield moment, in Nmm;

d is the nail diameter as defined in EN 14592, in mm;

f_u is the tensile strength of the wire, in N/mm^2.

For nails with diameters up to 8 mm, the following characteristic embedment strengths in timber and LVL apply:

− without predrilled holes

$$f_{h,k} = 0,082 \, \rho_k \, d^{-0,3} \qquad \text{N/mm}^2 \tag{8.15}$$

− with predrilled holes

$$f_{h,k} = 0,082 \, (1 - 0,01 \, d) \, \rho_k \quad \text{N/mm}^2 \tag{8.16}$$

where:

ρ_k is the characteristic timber density, in kg/m³;
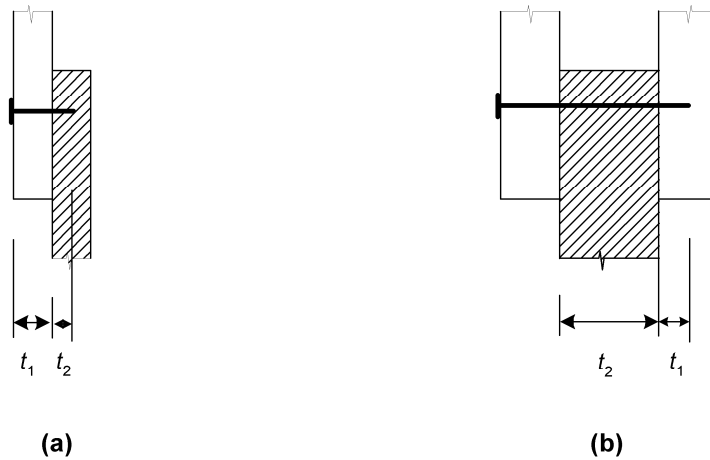
d is the nail diameter, in mm.

Figure 8.4 Definitions of t_1 and t_2 (a) single shear connection, (b) double shear connection

For nails with diameters greater than 8 mm the characteristic embedment strength values for bolts according to 8.5.1 apply.

In a three-member connection, nails may overlap in the central member provided (t - t_2) is greater than $4d$ (see Figure 8.5).
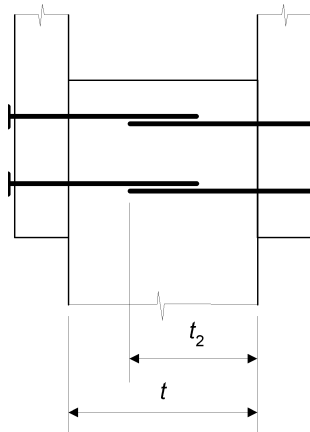

Figure 8.5 Overlapping nails

For one row of n nails parallel to the grain, unless the nails of that row are staggered perpendicular to grain by at least $1d$ (see Figure 8.6), the load-carrying capacity parallel to the grain (see 8.1.2) should be calculated using the effective number of fasteners n_{ef}, where:

$$n_{ef} = n^{k_{ef}} \tag{8.17}$$

where:

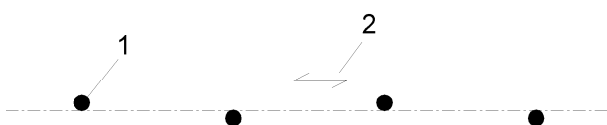n_{ef} is the effective number of nails in the row;

n is the number of nails in a row;

k_{ef} is given in Table 8.1.

Table 8.1 – Values of k_{ef}

Spacing[a]	k_{ef}	
	Not predrilled	Predrilled
$a_1 \geq 14d$	1,0	1,0
$a_1 = 10d$	0,85	0,85
$a_1 = 7d$	0,7	0,7
$a_1 = 4d$	-	0,5
[a] For intermediate spacings, linear interpolation of k_{ef} is permitted		

Key:
1   Nail
2   Grain direction

Figure 8.6 Nails in a row parallel to grain staggered perpendicular to grain by *d*

There should be at least two nails in a connection.

8.3.1.2 Nailed timber-to-timber connections

For smooth nails the pointside penetration length should be at least 8*d*.

For nails other than smooth nails, as defined in EN 14592, the pointside penetration length should be at least 6*d*.

Smooth nails in end grain should not be considered capable of transmitting lateral forces.

As an alternative to 8.3.1.2, for nails in end grain the following rules apply:

− In secondary structures smooth nails may be used. The design values of the load-carrying capacity should be taken as 1/3 of the values for nails installed at right angles to the grain;

− Nails other than smooth nails, as defined in EN 14592, may be used in structures other than secondary structures. The design values of the load-carrying capacity should be taken as 1/3 of the values for smooth nails of equivalent diameter installed at right angles to the grain, provided that:

  − the nails are only laterally loaded;

  − there are at least three nails per connection;

  − the pointside penetration is at least 10*d*;

  − the connection is not exposed to service class 3 conditions;

− the prescribed spacings and edge distances given in Table 8.2 are satisfied.

Note: An example of a secondary structure is a fascia board nailed to rafters.

Minimum spacings and edge and end distances are given in Table 8.2, where (see Figure 8.7):

a_1 is the spacing of nails within one row parallel to grain;

a_2 is the spacing of rows of nails perpendicular to grain;

$a_{3,c}$ is the distance between nail and unloaded end;

$a_{3,t}$ is the distance between nail and loaded end;

$a_{4,c}$ is the distance between nail and unloaded edge;

$a_{4,t}$ is the distance between nail and loaded edge;

α is the angle between the force and the grain direction.

Table 8.2 Minimum spacings and edge and end distances for nails

Spacing or distance (see Figure 8.7)	Angle α	Minimum spacing or end/edge distance		
		without predrilled holes		with predrilled holes
		$\rho_k \leq 420 \text{kg/m}^3$	$420 \text{ kg/m}^3 < \rho_k \leq 500 \text{ kg/m}^3$	
Spacing a_1 (parallel to grain)	$0° \leq \alpha \leq 360°$	$d < 5$ mm: $(5+5\lvert\cos\alpha\rvert)d$ $d \geq 5$ mm: $(5+7\lvert\cos\alpha\rvert)d$	$(7+8\lvert\cos\alpha\rvert)\,d$	$(4+\lvert\cos\alpha\rvert)\,d$
Spacing a_2 (perpendicular to grain)	$0° \leq \alpha \leq 360°$	$5d$	$7d$	$(3+\lvert\sin\alpha\rvert)\,d$
Distance $a_{3,t}$ (loaded end)	$-90° \leq \alpha \leq 90°$	$(10+5\cos\alpha)d$	$(15 + 5\cos\alpha)\,d$	$(7+5\cos\alpha)\,d$
Distance $a_{3,c}$ (unloaded end)	$90° \leq \alpha \leq 270°$	$10d$	$15d$	$7d$
Distance $a_{4,t}$ (loaded edge)	$0° \leq \alpha \leq 180°$	$d < 5$ mm: $(5+2\sin\alpha)\,d$ $d \geq 5$ mm: $(5+5\sin\alpha)\,d$	$d < 5$ mm: $(7+2\sin\alpha)\,d$ $d \geq 5$ mm: $(7 + 5\sin\alpha)\,d$	$d < 5$ mm: $(3 + 2\sin\alpha)\,d$ $d \geq 5$ mm: $(3 + 4\sin\alpha)\,d$
Distance $a_{4,c}$ (unloaded edge)	$180° \leq \alpha \leq 360°$	$5d$	$7d$	$3d$

Timber should be pre-drilled when the thickness of the timber members is smaller than

$$t = \max \begin{cases} 7d \\ (13d - 30)\dfrac{\rho_k}{400} \end{cases}$$ (8.18)

where:

t is the minimum thickness of timber member to avoid pre-drilling, in mm;

ρ_k is the characteristic timber density in kg/m³;

d is the nail diameter, in mm.

Timber of species especially sensitive to splitting should be pre-drilled when the thickness of the timber members is smaller than

$$t = \max \begin{cases} 14d \\ (13d - 30)\dfrac{\rho_k}{200} \end{cases}$$ (8.19)

Expression (8.19) may be replaced by expression (8.18) for edge distances given by:

$a_4 \geq 10\, d$ for $\rho_k \leq 420$ kg/m³

$a_4 \geq 14\, d$ for 420 kg/m³ $\leq \rho_k \leq 500$ kg/ m³.

Note: Examples of species sensitive to splitting are fir (abies alba), Douglas fir (pseudotsuga menziesii) and spruce (picea abies).

Key:
(1) Loaded end
(2) Unloaded end
(3) Loaded edge
(4) Unloaded edge
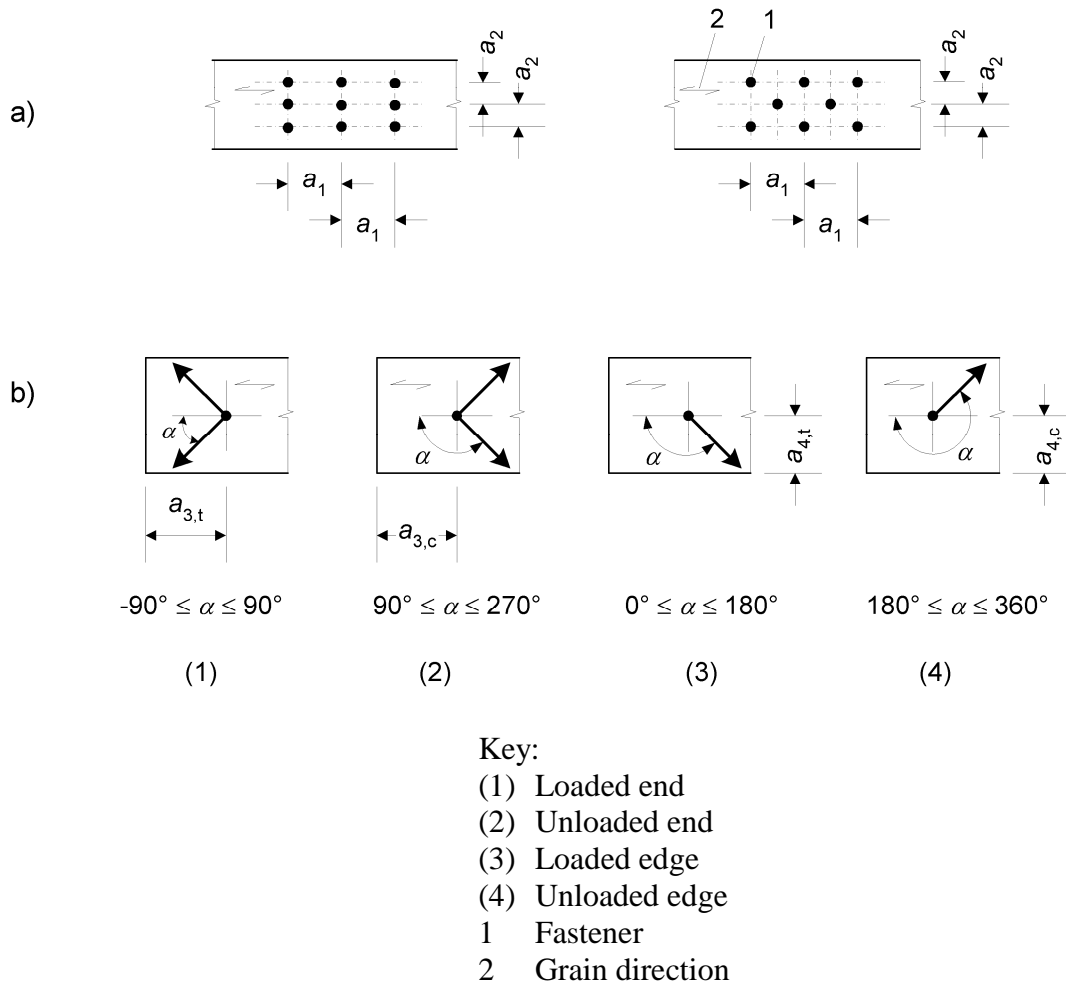1   Fastener
2   Grain direction

Figure 8.7 – Spacings and end and edge distances
(a) Spacing parallel to grain in a row and perpendicular to grain between rows, (b) Edge and end distances

8.3.1.3 Nailed panel-to-timber connections

Minimum nail spacings for all nailed panel-to-timber connections are those given in Table 8.2, multiplied by a factor of 0,85. The end/edge distances for nails remain unchanged unless otherwise stated below.

Minimum edge and end distances in plywood members should be taken as $3d$ for an unloaded edge (or end) and $(3 + 4 \sin \alpha)d$ for a loaded edge (or end), where α is the angle between the direction of the load and the loaded edge (or end).

For nails with a head diameter of at least $2d$, the characteristic embedment strengths are as follows:

− for plywood:

$$f_{h,k} = 0{,}11\,\rho_k\,d^{-0,3} \tag{8.20}$$

where:

$f_{h,k}$ is the characteristic embedment strength, in N/mm²;

ρ_k is the characteristic plywood density in kg/m³;

d is the nail diameter, in mm;

− for hardboard in accordance with EN 622-2:

$$f_{h,k} = 30\,d^{-0,3}t^{0,6} \tag{8.21}$$

where:

$f_{h,k}$ is the characteristic embedment strength, in N/mm²;

d is the nail diameter, in mm;

t is the panel thickness, in mm.

− for particleboard and OSB:

$$f_{h,k} = 65\,d^{-0,7}\,t^{0,1} \tag{8.22}$$

where:

$f_{h,k}$ is the characteristic embedment strength, in N/mm²;

d is the nail diameter, in mm;

t is the panel thickness, in mm.

8.3.1.4 Nailed steel-to-timber connections

The minimum edge and end distances for nails given in Table 8.2 apply. Minimum nail spacings are those given in Table 8.2, multiplied by a factor of 0,7.

8.3.2 Axially loaded nails

Smooth nails shall not be used to resist permanent or long-term axial loading.
IMPORTANT: In amendment EC5/A1 new rules are given, see Chapter 13.

For threaded nails, only the threaded part should be considered capable of transmitting axial load.

Nails in end grain should be considered incapable of transmitting axial load.

The characteristic withdrawal capacity of nails, $F_{ax,Rk}$, for nailing perpendicular to the grain (Figure 8.8 (a)) and for slant nailing (Figure 8.8 (b)), should be taken as the smaller of the values found from the following expressions:

− For nails other than smooth nails, as defined in EN 14592:

$$F_{ax,Rk} = \begin{cases} f_{ax,k}\, d\, t_{pen} & \text{(a)} \\ f_{head,k}\, d_h^2 & \text{(b)} \end{cases}$$ (8.23)

- For smooth nails:

$$F_{ax,Rk} = \begin{cases} f_{ax,k}\, d\, t_{pen} & \text{(a)} \\ f_{ax,k}\, d\, t + f_{head,k}\, d_h^2 & \text{(b)} \end{cases}$$ (8.24)

where:

$f_{ax,k}$ is the characteristic pointside withdrawal strength;

$f_{head,k}$ is the characteristic headside pull-through strength;

d is the nail diameter according to 8.3.1.1;

t_{pen} is the pointside penetration length or the length of the threaded part in the pointside member;

t is the thickness of the headside member;

d_h is the nail head diameter.

The characteristic strengths $f_{ax,k}$ and $f_{head,k}$ should be determined by tests in accordance with EN 1382, EN 1383 and EN 14358 unless specified in the following.

For smooth nails with a pointside penetration of at least $12d$, the characteristic values of the withdrawal and pull-through strengths should be found from the following expressions:

$$f_{ax,k} = 20 \times 10^{-6}\, \rho_k^2$$ (8.25)

$$f_{head,k} = 70 \times 10^{-6}\, \rho_k^2$$ (8.26)

where:

ρ_k is the characteristic timber density in kg/m³;

For smooth nails, the pointside penetration t_{pen} should be at least $8d$. For nails with a pointside penetration smaller than $12d$ the withdrawal capacity should be multiplied by $(t_{pen}/4d - 2)$. For threaded nails, the pointside penetration should be at least $6d$. For nails with a pointside penetration smaller than $8d$ the withdrawal capacity should be multiplied by $(t_{pen}/2d - 3)$.

For structural timber which is installed at or near fibre saturation point, and which is likely to dry out under load, the values of $f_{ax,k}$ and $f_{head,k}$ should be multiplied by 2/3.

The spacings, end and edge distances for laterally loaded nails apply to axially loaded nails.

For slant nailing the distance to the loaded edge should be at least 10d (see Figure 8.8 (b)). There should be at least two slant nails in a connection.
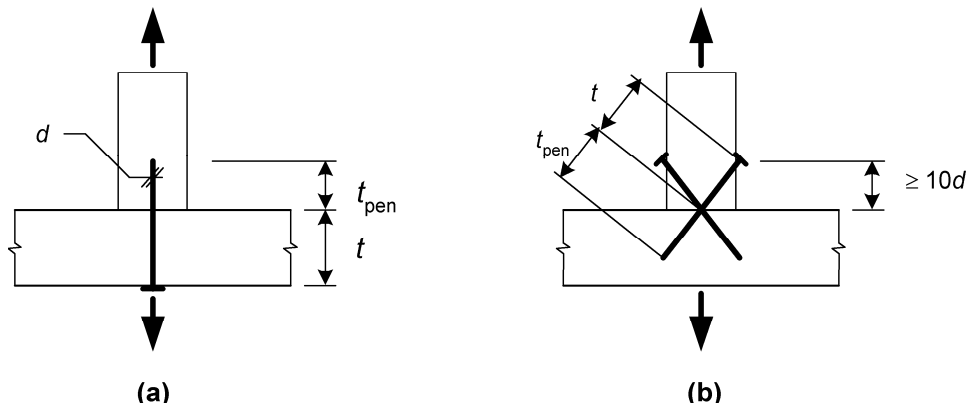

Figure 8.8 (a) Nailing perpendicular to grain and (b) slant nailing

8.3.3 Combined laterally and axially loaded nails

For connections subjected to a combination of axial load ($F_{ax,Ed}$)and lateral load ($F_{v,Ed}$)the following expressions should be satisfied:

− for smooth nails:

$$\frac{F_{ax,Ed}}{F_{ax,Rd}} + \frac{F_{v,Ed}}{F_{v,Rd}} \leq 1 \tag{8.27}$$

− for nails other than smooth nails, as defined in EN 14592:

$$\left(\frac{F_{ax,Ed}}{F_{ax,Rd}}\right)^2 + \left(\frac{F_{v,Ed}}{F_{v,Rd}}\right)^2 \leq 1 \tag{8.28}$$

where:

$F_{ax,Rd}$ and $F_{v,Rd}$ are the design load-carrying capacities of the connection loaded with axial load or lateral load respectively.

8.4 Stapled connections

The rules given in 8.3, except for expressions (8.15), (8.16) and (8.19), apply for round or nearly round or rectangular staples with bevelled or symmetrical pointed legs.

For staples with rectangular cross-sections the diameter d should be taken as the square root of the product of both dimensions.

The width b of the staple crown should be at least 6d, and the pointside penetration length t_2 should be at least 14d, see Figure 8.9.

There should be at least two staples in a connection.

The lateral design load-carrying capacity per staple per shear plane should be considered as equivalent to that of two nails with the staple diameter, provided that the angle between the crown and the direction of the grain of the timber under the crown is greater than 30°, see Figure 8.10. If the angle between the crown and the direction of the grain under the crown is equal to or less than 30°, then the lateral design load-carrying capacity should be multiplied by a factor of 0,7.

For staples produced from wire with a minimum tensile strength of 800 N/mm², the following characteristic yield moment per leg should be used:

$$M_{y,Rk} = 240\, d^{2,6} \tag{8.29}$$

where:

$M_{y,Rk}$ is the characteristic yield moment, in Nmm;

d is the staple leg diameter, in mm.

For a row of n staples parallel to the grain, the load-carrying capacity in that direction should be calculated using the effective number of fasteners n_{ef} according to 8.3.1.1- expression (8.17).

Minimum staple spacings, edge and end distances are given in Table 8.3, and illustrated in Figure 8.10 where $\varTheta$ is the angle between the staple crown and the grain direction.
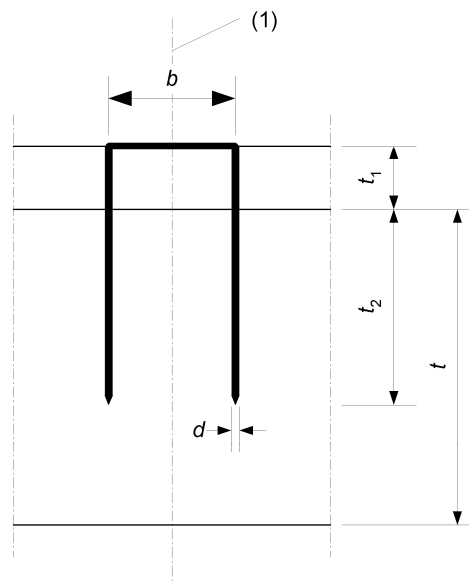
Key:
(1) staple centre
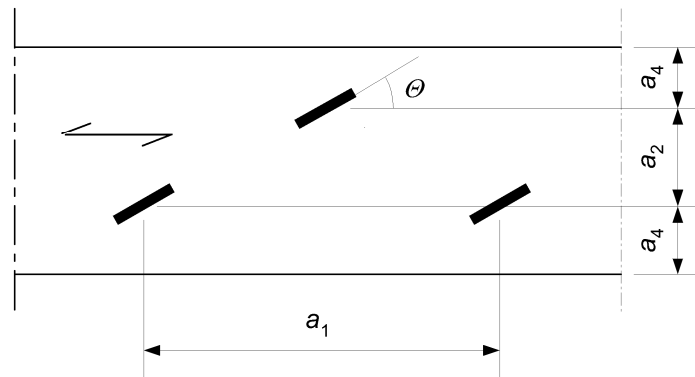
Figure 8.9 Staple dimensions

Figure 8.10 Definition of spacing for staples

Table 8.3 Minimum spacings and edge and end distances for staples

Spacing and edge/end distances (see Figure 8.7)	Angle	Minimum spacing or edge/end distance
a_1 (parallel to grain) for $\Theta \geq 30°$ for $\Theta < 30°$	$0° \leq \alpha \leq 360°$	$(10 + 5 \lvert \cos \alpha \rvert)\, d$ $(15 + 5 \lvert \cos \alpha \rvert)\, d$
a_2 (perpendicular to grain)	$0° \leq \alpha \leq 360^{0°}$	$15\, d$
$a_{3,t}$ (loaded end)	$-90° \leq \alpha \leq 90°$	$(15 + 5 \lvert \cos \alpha \rvert)\, d$
$a_{3,c}$ (unloaded end)	$90° \leq \alpha \leq 270°$	$15\, d$
$a_{4,t}$ (loaded edge)	$0° \leq \alpha \leq 180°$	$(15 + 5 \lvert \sin \alpha \rvert)\, d$
$a_{4,c}$ (unloaded edge)	$180° \leq \alpha \leq 360°$	$10\, d$

8.5 Bolted connections

Bolts are installed into pre-drilled holes in the timber.

8.5.1 Laterally loaded bolts

The failure of laterally loaded bolts include both crushing of the timber and bending of the bolt.

8.5.1.1 General and bolted timber-to-timber connections

For bolts the following characteristic value for the yield moment should be used:

$$M_{y,Rk} = 0,3\, f_{u,k}\, d^{2,6} \tag{8.30}$$

where:

$M_{y,Rk}$ is the characteristic value for the yield moment, in Nmm;

$f_{u,k}$ is the characteristic tensile strength, in N/mm²;

d is the bolt diameter, in mm.

For bolts up to 30 mm diameter, the following characteristic embedment strength values in timber and LVL should be used, at an angle α□ to the grain:

$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90} \sin^2\alpha + \cos^2\alpha} \tag{8.31}$$

$$f_{h,0,k} = 0,082 \, (1 - 0,01 \, d) \, \rho_k \tag{8.32}$$

where:

$$k_{90} = \begin{cases} 1,35 + 0,015 \, d & \text{for softwoods} \\ 1,30 + 0,015 \, d & \text{for LVL} \\ 0,90 + 0,015 \, d & \text{for hardwoods} \end{cases} \tag{8.33}$$

and:

$f_{h,0,k}$ is the characteristc embedment strength parallel to grain, in N/mm^2;

ρ_k is the characteristic timber density, in kg/m³;

α is the angle of the load to the grain;

d is the bolt diameter, in mm.

Minimum spacings and edge and end distances should be taken from Table 8.4, with symbols illustrated in Figure 8.7.

Table 8.4 Minimum values of spacing and edge and end distances for bolts

Spacing and end/edge distances (see Figure 8.7)	Angle	Minimum spacing or distance
a_1 (parallel to grain)	$0° \leq \alpha \leq 360°$	$(4 + \lvert \cos \alpha \rvert) \, d$
a_2 (perpendicular to grain)	$0° \leq \alpha \leq 360°$	$4 \, d$
$a_{3,t}$ (loaded end)	$-90° \leq \alpha \leq 90°$	max (7 d; 80 mm)
$a_{3,c}$ (unloaded end)	$90° \leq \alpha < 150°$	max [(1 + 6 sin α) d; 4d]
	$150° \leq \alpha < 210°$	$4 \, d$
	$210° \leq \alpha \leq 270°$	max [(1 + 6 sin α) d; 4d]
$a_{4,t}$ (loaded edge)	$0° \leq \alpha \leq 180°$	max [(2 + 2 sin α) d; 3d]
$a_{4,c}$ (unloaded edge)	$180° \leq \alpha \leq 360°$	$3 \, d$

For one row of n bolts parallel to the grain direction, the load-carrying capacity parallel to grain, see 8.1.2, should be calculated using the effective number of bolts n_{ef} where:

$$n_{ef} = \min \begin{cases} n \\ n^{0,9} \sqrt[4]{\dfrac{a_1}{13d}} \end{cases} \tag{8.34}$$

where:

a_1 is the spacing between bolts in the grain direction;

d is the bolt diameter

n is the number of bolts in the row.

For loads perpendicular to grain, the effective number of fasteners should be taken as

$$n_{ef} = n \qquad (8.35)$$

For angles $0° < \alpha < 90°$ between load and grain direction, n_{ef} may be determined by linear interpolation between expressions (8.34) and (8.35).

Bolt holes in timber should have a diameter not more than 1 mm larger than the bolt. Bolt holes in steel plates should have a diameter not more than 2 mm or $0,1d$ (whichever is the greater) lager than the bolt diameter d. Washers with a side length or a diameter of at least $3d$ and thickness of at least $0,3d$ should be used under the head and nut. Washers should have a full bearing area.

8.5.1.2 Bolted panel-to-timber connections

For plywood the following embedment strength, in N/mm^2, should be used at all angles to the face grain:

$$f_{h,k} = 0,11 \, (1 - 0,01 \, d) \, \rho_k \qquad (8.36)$$

where:

ρ_k is the characteristic plywood density, in kg/m³;

d is the bolt diameter, in mm.

For particleboard and OSB the following embedment strength value, in N/mm^2, should be used at all angles to the face grain:

$$f_{h,k} = 50 \, d^{-0,6} \, t^{0,2} \qquad (8.37)$$

where:

d is the bolt diameter, in mm;

t is the panel thickness, in mm.

8.5.1.3 Bolted steel-to-timber connections

The rules given in 8.2.3 apply.

8.5.2 Axially loaded bolts

The axial load-bearing capacity and withdrawal capacity of a bolt should be taken as the lower value of:

− the bolt tensile capacity;

- the load-bearing capacity of either the washer or (for steel-to-timber connections) the steel plate.

The bearing capacity of a washer should be calculated assuming a characteristic compressive strength on the contact area of $3,0 f_{c,90,k}$.

The bearing capacity per bolt of a steel plate should not exceed that of a circular washer with a diameter which is the minimum of:

- $12t$, where t is the plate thickness;

- $4d$, where d is the bolt diameter.

8.6 Dowelled connections

The rules given in 8.5.1 except minimum spacing and edge and end distances apply.

The dowel diameter should be greater than 6 mm and less than 30 mm.

Minimum spacing and edge and end distances are given in Table 8.5, with symbols illustrated in Figure 8.7.

Table 8.5 Minimum spacings and edge and end distances for dowels

Spacing and edge/end distances (see Figure 8.7)	Angle	Minimum spacing or edge/end distance
a_1 (parallel to grain)	$0° \leq \alpha \leq 360°$	$(3 + 2 \left| \cos \alpha \right|) \, d$
a_2 (perpendicular to grain)	$0° \leq \alpha \leq 360°$	$3 \, d$
$a_{3,t}$ (loaded end)	$-90° \leq \alpha \leq 90°$	max ($7 \, d$; 80 mm)
$a_{3,c}$ (unloaded end)	$90° \leq \alpha < 150°$	$\max(a_{3,t} \left| \sin \alpha \right|; 3d)$
	$150° \leq \alpha < 210°$	$3 \, d$
	$210° \leq \alpha \leq 270°$	$\max(a_{3,t} \left| \sin \alpha \right|; 3d)$
$a_{4,t}$ (loaded edge)	$0° \leq \alpha \leq 180°$	$\max((2 + 2 \sin \alpha) \, d; 3d)$
$a_{4,c}$ (unloaded edge)	$180° \leq \alpha \leq 360°$	$3 \, d$

The minimum dowel diameter should be 6 mm. The tolerances on the dowel diameter should be -0/+0,1 mm. Pre-bored holes in the timber members should have a diameter not greater than the dowel.

8.7 Screwed connections

Screws are installed into a drilled hole, by turning the screw and allowing the flutes on the thread of the screw to draw it in.

8.7.1 Laterally loaded screws

The effect of the threaded part of the screw shall be taken into account in determining the load-carrying capacity, by using an effective diameter d_{ef}

For smooth shank screws, where the outer thread diameter is equal to the shank diameter, the rules given in 8.2 apply, provided that:
− The effective diameter d_{ef} is taken as the smooth shank diameter;

− The smooth shank penetrates into the member containing the point of the screw by not less than $4d$.

Where the conditions are not satisfied, the screw load-carrying capacity should be calculated using an effective diameter d_{ef} taken as 1,1 times the thread root diameter.

For smooth shank screws with a diameter $d > 6$ mm, the rules in 8.5.1 apply.

For smooth shank screws with a diameter of 6 mm or less, the rules of 8.3.1 apply.

For screws in softwoods with a smooth shank diameter $d \leq 6$ mm, pre-drilling is not required. For all screws in hardwoods and for screws in softwoods with a diameter $d > 6$ mm, pre-drilling is required, with following requiremets:
− The lead hole for the shank should have the same diameter as the shank and the same depth as the length of the shank;

− The lead hole for the threaded portion should have a diameter of approximately 70% of the shank diameter.

8.7.2 Axially loaded screws

The following failure modes should be verified when assessing the load-carrying capacity of connections with axially loaded screws:

− the withdrawal capacity of the threaded part of the screw;

− for screws used in combination with steel plates, the tear-off capacity of the screw head should be greater than the tensile strength of the screw;

− the pull-through strength of the screw head;

− the tension strength of the screw;

− for screws used in conjunction with steel plates, failure along the circumference of a group of screws (block shear or plug shear);

Minimum spacing and edge distances for axially loaded screws should be taken from Table 8.6.

Table 8.6 Minimum spacings and edge distances for axially loaded screws

Screws driven	Minimum spacing	Minimum edge distance
At right angle to the grain	4*d*	4*d*
In end grain	4*d*	2,5*d*

The minimum pointside penetration length of the threaded part should be 6*d*.

The characteristic withdrawal capacity of connections with axially loaded screws should be taken as:

$$F_{ax,\alpha,Rk} = n_{ef} \, (\pi \, d \, l_{ef})^{0,8} \, f_{ax,\alpha,k} \tag{8.38}$$

where:

$F_{ax,\alpha,Rk}$ is the characteristic withdrawal capacity of the connection at an angle α to the grain;

n_{ef} is the effective number of screws;

d is the outer diameter measured on the threaded part;

l_{ef} is the pointside penetration length of the threaded part minus one screw diameter;

$f_{ax,\alpha,k}$ is the characteristic withdrawal strength at an angle α to the grain.

The characteristic withdrawal strength at an angle α to the grain should be taken as:

$$f_{ax,\alpha,k} = \frac{f_{ax,k}}{\sin^2\alpha + 1,5\cos^2\alpha} \tag{8.39}$$

with:

$$f_{ax,k} = 3,6 \times 10^{-3} \, \rho_k^{1,5} \tag{8.40}$$

where:

$f_{ax,\alpha,k}$ is the characteristic withdrawal strength at an angle α to the grain;

$f_{ax,k}$ is the characteristic withdrawal strength perpendicular to the grain;

ρ_k is the characteristic density, in kg/m^3.

NOTE: Failure modes in the steel or in the timber around the screw are brittle, i.e. with small ultimate deformation and therefore have a limited possibility for stress redistribution.

The pull-through capacity of the head shall be determined by tests, in accordance with EN 1383.
For a connection with a group of screws loaded by a force component parallel to the shank, the effective number of screws is given by:

$$n_{ef} = n^{0,9} \tag{8.41}$$

where:

n_{ef} is the effective number of screws;

n is the number of screws acting together in a connection.

IMPORTANT: In amendment EC5/A1 new rules are given, see Chapter 13.

8.7.3 Combined laterally and axially loaded screws

For screwed connections subjected to a combination of axial load and lateral load, expression (8.28) should be satisfied.

9 Components and wall diaphragms
Components and wall diaphragms have a high load capacity and stiffness compared with their weight.

9.1 Components
Component usually comprises minimum two parts.

9.1.1 Glued thin-webbed beams
If a linear variation of strain over the depth of the beam is assumed, the axial stresses in the wood-based flanges should satisfy the following expressions:

$$\sigma_{f,c,max,d} \leq f_{m,d} \tag{9.1}$$

$$\sigma_{f,t,max,d} \leq f_{m,d} \tag{9.2}$$

$$\sigma_{f,c,d} \leq k_c f_{c,0,d} \tag{9.3}$$

$$\sigma_{f,t,d} \leq f_{t,0,d} \tag{9.4}$$
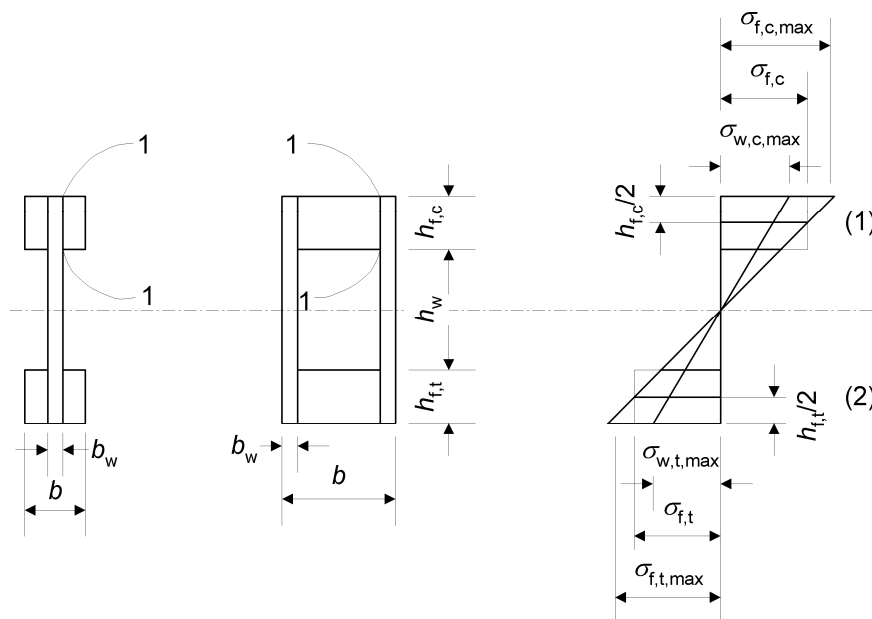
where:

$\sigma_{f,c,max,d}$ is the extreme fibre flange design compressive stress;

$\sigma_{f,t,max,d}$ is the extreme fibre flange design tensile stress;

$\sigma_{f,c,d}$ is the mean flange design compressive stress;

$\sigma_{f,t,d}$ is the mean flange design tensile stress;

k_c is a factor which takes into account lateral instability.

Key:

(1) compression
(2) tension

Figure 9.1 Thin-webbed beams

The factor k_c may be determined (conservatively, especially for box beams) according to 6.3.2 with

$$\lambda_z = \sqrt{12}\left(\frac{\ell_c}{b}\right) \tag{9.5}$$

where:

ℓ_c is the distance between the sections where lateral deflection of the compressive flange is prevented;

b is given in Figure 9.1.

If a special investigation is made with respect to the lateral instability of the beam as a whole, it may be assumed that $k_c = 1,0$.

The axial stresses in the webs should satisfy the following expressions:

$$\sigma_{w,c,d} \le f_{c,w,d} \tag{9.6}$$

$$\sigma_{w,t,d} \le f_{t,w,d} \tag{9.7}$$

where:

$\sigma_{w,c,d}$ and $\sigma_{w,t,d}$ are the design compressive and tensile stresses in the webs;

$f_{c,w,d}$ and $f_{t,w,d}$ are the design compressive and tensile bending strengths of the webs.

Unless other values are given, the design in-plane bending strength of the webs should be taken as the design tensile or compressive strength.

It shall be verified that any glued splices have sufficient strength.

Unless a detailed buckling analysis is made it should be verified that:

$$h_w \le 70b_w \tag{9.8}$$

and

$$F_{v,w,Ed} \le \begin{cases} b_w h_w \left(1 + \dfrac{0,5(h_{f,t} + h_{f,c})}{h_w}\right) f_{v,0,d} & \text{for } h_w \le 35b_w \\[3mm] 35\, b_w^2 \left(1 + \dfrac{0,5(h_{f,t} + h_{f,c})}{h_w}\right) f_{v,0,d} & \text{for } 35b_w \le h_w \le 70b_w \end{cases} \tag{9.9}$$

where:

$F_{v,w,Ed}$ is the design shear force acting on each web;

h_w is the clear distance between flanges;

$h_{f,c}$ is the compressive flange depth;

$h_{f,t}$ is the tensile flange depth;

b_w is the width of each web;

$f_{v,0,d}$ is the design panel shear strength.

For webs of wood-based panels, it should, for sections 1-1 in Figure 9.1, be verified that:

$$\tau_{mean,d} \leq \begin{cases} f_{v,90,d} & \text{for } h_f \leq 4\, b_{ef} \\[2em] f_{v,90,d}\left(\dfrac{4b_{ef}}{h_f}\right)^{0,8} & \text{for } h_f > 4\, b_{ef} \end{cases} \tag{9.10}$$

where:

$\tau_{mean,d}$ is the design shear stress at the sections 1-1, assuming a uniform stress distribution;

$f_{v,90,d}$ is the design planar (rolling) shear strength of the web;

h_f is either $h_{f,c}$ or $h_{f,t}$.

$$b_{ef} = \begin{cases} b_w & \text{for boxed beams} \\ b_w/2 & \text{for I-beams} \end{cases} \tag{9.11}$$

9.1.2 Glued thin-flanged beams

This section assumes a linear variation of strain over the depth of the beam.

In the strength verification of glued thin-flanged beams, account shall be taken of the non-uniform distribution of stresses in the flanges due to shear lag and buckling.

Unless a more detailed calculation is made, the assembly should be considered as a number of I-beams or U-beams (see Figure 9.2) with effective flange widths b_{ef}, as follows:

− For I-beams

$$b_{ef} = b_{c,ef} + b_w \qquad (\text{or } b_{t,ef} + b_w) \tag{9.12}$$

− For U-beams

$$b_{ef} = 0{,}5b_{c,ef} + b_w \qquad (\text{or } 0{,}5b_{t,ef} + b_w) \tag{9.13}$$

The values of $b_{c,ef}$ and $b_{t,ef}$ should not be greater than the maximum value calculated for shear lag from Table 9.1. In addition the value of $b_{c,ef}$ should not be greater than the maximum value calculated for plate buckling from Table 9.1.

Maximum effective flange widths due to the effects of shear lag and plate buckling should be taken from Table 9.1, where ℓ is the span of the beam.

Table 9.1 Maximum effective flange widths due to the effects of shear lag and plate buckling

Flange material	Shear lag	Plate buckling
Plywood, with grain direction in the outer plies:		
− Parallel to the webs	$0,1\ell$	$20h_f$
− Perpendicular to the webs	$0,1\ell$	$25h_f$
Oriented strand board	$0,15\ell$	$25h_f$
Particleboard or fibreboard with random fibre orientation	$0,2\ell$	$30h_f$

Unless a detailed buckling investigation is made, the unrestrained flange width should not be greater than twice the effective flange width due to plate buckling, from Table 9.1.

For webs of wood-based panels, it should, for sections 1-1 of an I-shaped cross-section in Figure 9.2, be verified that:

$$\tau_{mean,d} \leq \begin{cases} f_{v,90,d} & \text{for } b_w \leq 8h_f \\ f_{v,90,d}\left(\dfrac{8h_f}{b_w}\right)^{0,8} & \text{for } b_w > 8h_f \end{cases} \tag{9.14}$$

where:

$\tau_{mean,d}$ is the design shear stress at the sections 1-1, assuming a uniform stress distribution;

$f_{v,90,d}$ is the design planar (rolling) shear strength of the flange.

For section 1-1 of a U-shaped cross-section, the same expressions should be verified, but with $8h_f$ substituted by $4h_f$.

The axial stresses in the flanges, based on the relevant effective flange width, should satisfy the following expressions:

$$\sigma_{f,c,d} \leq f_{f,c,d} \tag{9.15}$$

$$\sigma_{f,t,d} \leq f_{f,t,d} \tag{9.16}$$

where:

$\sigma_{f,c,d}$ is the mean flange design compressive stress;

$\sigma_{f,t,d}$ is the mean flange design tensile stress;

$f_{f,c,d}$ is the flange design compressive strength;

$f_{f,t,d}$ is the flange design tensile strength.

It shall be verified that any glued splices have sufficient strength.

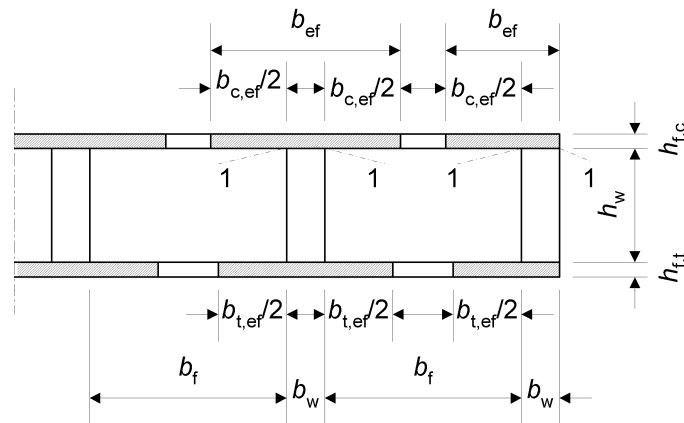The axial stresses in the wood-based webs should satisfy the expressions (9.6) to (9.7) defined in 9.1.1

Figure 9.2 Thin-flanged beam

9.1.3 Mechanically jointed beams
If the cross-section of a structural member is composed of several parts connected by mechanical fasteners, consideration shall be given to the influence of the slip occurring in the joints.

Calculations should be carried out assuming a linear relationship between force and slip.

If the spacing of the fasteners varies in the longitudinal direction according to the shear force between s_{min} and s_{max} ($\leq 4s_{min}$), an effective spacing s_{ef} may be used as follows:

$$s_{ef} = 0,75\, s_{min} + 0,25\, s_{max} \tag{9.17}$$

A method for the calculation of the load-carrying capacity of mechanically jointed beams is given in Chapter 10.

9.1.4 Mechanically jointed and glued columns
Deformations due to slip in joints, to shear and bending in packs, gussets, shafts and flanges, and to axial forces in the lattice shall be taken into account in the strength verification.

A method for the calculation of the load-carrying capacity of I- and box-columns, spaced columns and lattice columns is given in Chapter 11.

9.2. Wall diaphragms
In order to stabilize timber frame buildings against lateral loads, the diaphragm action of walls is often used.

9.2.1 *General*

Wall diaphragms shall be designed to resist both horizontal and vertical actions imposed upon them.

The wall shall be adequately restrained to avoid overturning and sliding.

Wall diaphragms deemed to provide resistance to racking shall be stiffened in-plane by board materials, diagonal bracing or moment connections.

The racking resistance of a wall shall be determined either by test according to EN 594 or by calculations, employing appropriate analytical methods or design models.

The design of wall diaphragms shall take account of both the material construction and geometric make-up of the wall under consideration.

The response of wall diaphragms to actions shall be assessed to ensure the construction remains within appropriate serviceability limits.

9.2.2 *Simplified analysis of wall diaphragms – Method A*

The simplified method given in this section should only be applied to wall diaphragms with a tie-down at their end, that is the vertical member at the end is directly connected to the construction below.

The design load-carrying capacity $F_{v,Rd}$ (the design racking resistance) under a force $F_{v,Ed}$ acting at the top of a cantilevered panel secured against uplift (by vertical actions or by anchoring) should be determined using the following simplified method of analysis for walls made up of one or more panels, where each wall panel consists of a sheet fixed to one side of a timber frame, provided that:

− the spacing of fasteners is constant along the perimeter of every sheet;

− the width of each sheet is at least $h/4$.

For a wall made up of several wall panels, the design racking load-carrying capacity of a wall should be calculated from

$$F_{v,Rd} = \sum F_{i,v,Rd} \tag{9.18}$$

where $F_{i,v,Rd}$ is the design racking load-carrying capacity of the wall panel.

The design racking load-carrying capacity of each wall panel, $F_{i,v,Rd}$, against a force $F_{i,v,Ed}$ according to Figure 9.3 should be calculated from

$$F_{i,v,Rd} = \frac{F_{f,Rd}\, b_i\, c_i}{s} \tag{9.19}$$

where:

$F_{f,Rd}$ is the lateral design capacity of an individual fastener;

b_i is the wall panel width;

s is the fastener spacing.

and

$$c_i = \begin{cases} 1 & \text{for} \quad b_i \geq b_0 \\ \dfrac{b_i}{b_0} & \text{for} \quad b_i < b_0 \end{cases} \qquad (9.20)$$

where:

$b_0 = h/2$

h is the height of the wall.

For fasteners along the edges of an individual sheet, the design lateral load-carrying capacity should be increased by a factor of 1,2 over the corresponding values given in Chapter 8. In determining the fastener spacing in accordance with the requirements of Chapter 8, the edges should be assumed to be unloaded.
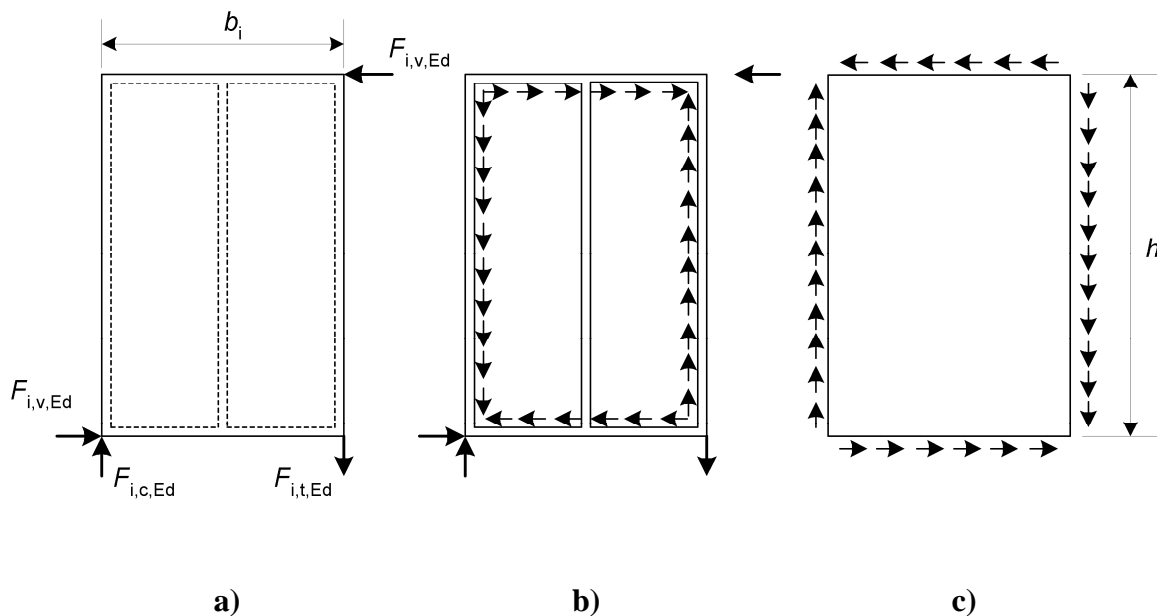

Figure 9.3 Forces acting on:
a) wall panel;
b) framing;
c) sheet

Wall panels which contain a door or window opening should not be considered to contribute to the racking load-carrying capacity.

For wall panels with sheets on both sides the following rules apply:

− if the sheets and fasteners are of the same type and dimension then the total racking load-carrying capacity of the wall should be taken as the sum of the racking load-carrying capacities of the individual sides

− if different types of sheets are used, 75 % of the racking load-carrying capacity of the weaker side may, unless some other value is shown to be valid, be taken into consideration if fasteners with similar slip moduli are used. In other cases not more than 50 % should be taken into consideration.

The external forces $F_{i,c,Ed}$ and $F_{i,t,Ed}$ according to Figure 9.3 should be determined from

$$F_{i,c,Ed} \;=\; F_{i,t,Ed} \;=\; \frac{F_{i,v,Ed}\, h}{b_i} \qquad\qquad (9.21)$$

where h is the height of the wall.
These forces can either be transmitted to the sheets in the adjacent wall panel or transmitted to the construction situated above or below. When tensile forces are transmitted to the construction situated below, the panel should be anchored by stiff fasteners. Buckling of wall studs should be checked in accordance with 6.3.2. Where the ends of vertical members bear on horizontal framing members, the compression perpendicular to the grain stresses in the horizontal members should be assessed according to 6.1.5.

The external forces which arise in wall panels containing door or window openings and in wall panels of smaller width, see Figure 9.4, can similarly be transmitted to the construction situated above or below.
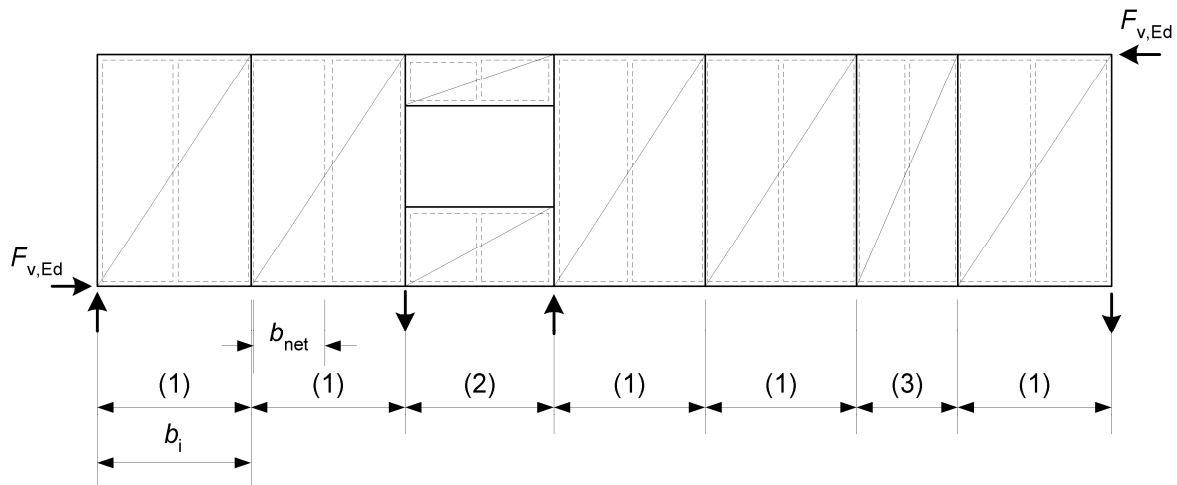
Key:
(1) Wall panel (normal width)
(2) Wall panel with window
(3) Wall panel (smaller width)

Figure 9.4 Example of the assembly of wall panels containing a wall panel with a window opening and a wall panel of smaller width

Shear buckling of the sheet may be disregarded, provided that $\dfrac{b_{net}}{t} \leq 100$

where:

b_{net} is the clear distance between studs;

t is the thickness of the sheet.

In order that the centre stud may be considered to constitute a support for a sheet, the spacing of fasteners in the centre stud should not be greater than twice the spacing of the fasteners along the edges of the sheet.

Where each panel consists of a prefabricated wall element, the transfer of shear forces between the separate wall elements should be verified.

In contact areas between vertical studs and horizontal timber members, compression stresses perpendicular to grain should be verified in the timber members.

10 Mechanically jointed beams

Cross-section of beams may be composed of several parts connected by mechanical joints. The mechanical joints mainly carry shear force.

10.1 Simplified analysis

The solution require that for every part simple bending-theory is valid and shear displacement is omitted.

10.1.1 Cross-sections

The cross-sections shown in Figure 10.1 are considered.

10.1.2 Assumptions

The design method is based on the theory of linear elasticity and the following assumptions:

− the beams are simply supported with a span ℓ. For continuous beams the expressions may be used with ℓ equal to 0,8 of the relevant span and for cantilevered beams with ℓ equal to twice the cantilever length

− the individual parts (of wood, wood-based panels) are either full length or made with glued end joints

− the individual parts are connected to each other by mechanical fasteners with a slip modulus K

− the spacing s between the fasteners is constant or varies uniformly according to the shear force between s_{min} and s_{max}, with $s_{max} \leq 4\, s_{min}$

− the load is acting in the z-direction giving a moment $M = M(x)$ varying sinusoidally or parabolically and a shear force $V = V(x)$.

10.1.3 Spacings

Where a flange consists of two parts jointed to a web or where a web consists of two parts (as in a box beam), the spacing s_i is determined by the sum of the fasteners per unit length in the two jointing planes.

10.1.4 Deflections resulting from bending moments

Deflections are calculated by using an effective bending stiffness $(EI)_{ef}$, determined in accordance with 10.2.

Figure 10.1 Cross-section (left) and distribution of bending stresses (right).
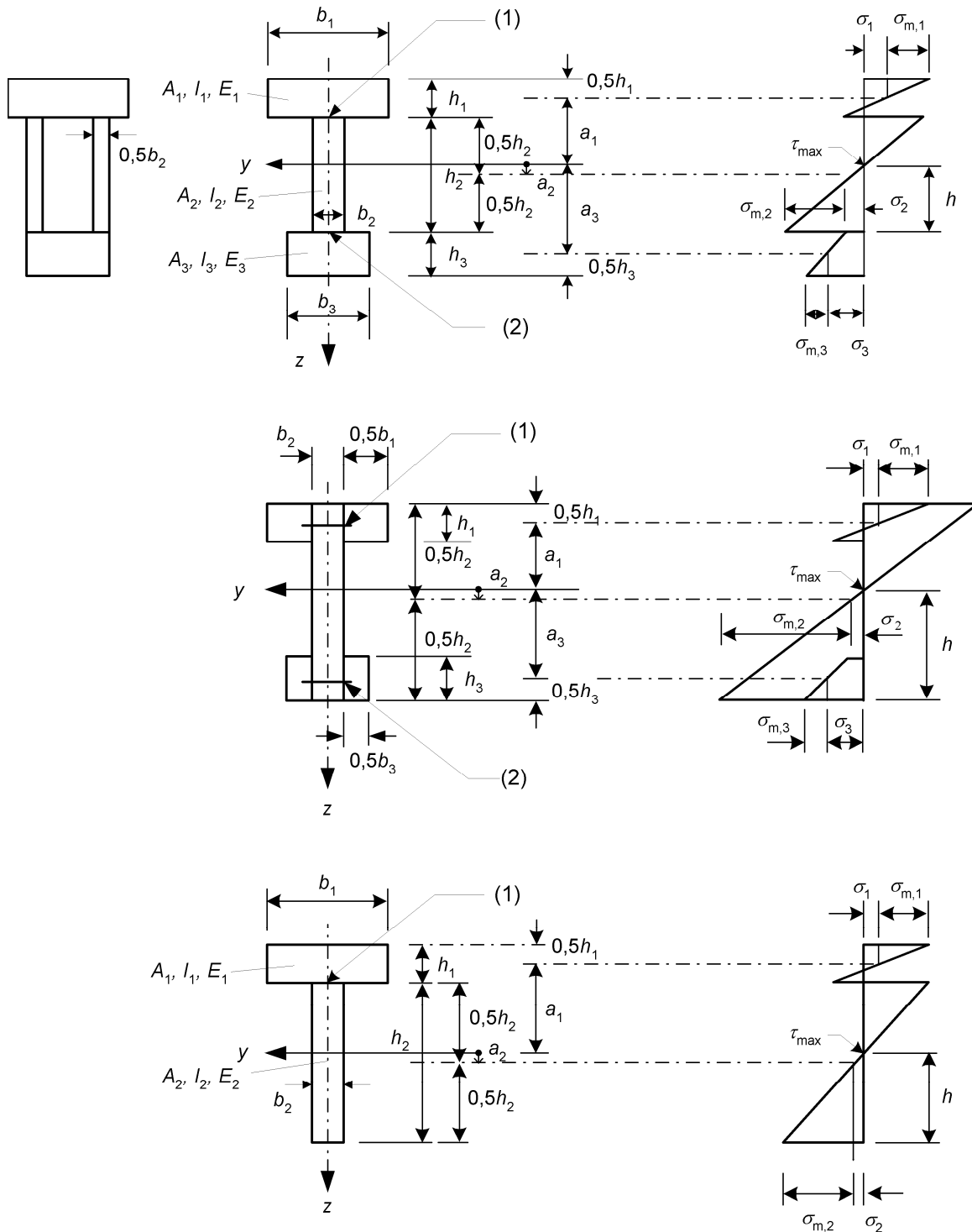
Key:
(1) spacing: s_1 slip modulus: K_1 load: F_1
(2) spacing: s_3 slip modulus: K_3 load: F_3

All measurements are positive except for a_2 which is taken as positive as shown.

10.2 Effective bending stiffness
The effective bending stiffness should be taken as:

$$(EI)_{ef} = \sum_{i=1}^{3} (E_i I_i + \gamma_i E_i A_i a_i^2) \tag{10.1}$$

using mean values of E and where:

$$A_i = b_i h_i \tag{10.2}$$

$$I_i = \frac{b_i h_i^3}{12} \tag{10.3}$$

$$\gamma_2 = 1 \tag{10.4}$$

$$\gamma_i = \left[1 + \pi^2 E_i A_i s_i / (K_i l^2) \right]^{-1} \quad \text{for } i = 1 \text{ and } i = 3 \tag{10.5}$$

$$a_2 = \frac{\gamma_1 E_1 A_1 (h_1 + h_2) - \gamma_3 E_3 A_3 (h_2 + h_3)}{2 \sum_{i=1}^{3} \gamma_i E_i A_i} \tag{10.6}$$

where the symbols are defined in Figure 10.1.

$K_i = K_{ser,i}$ for the serviceability limit state calculations;

$K_i = K_{u,i}$ for the ultimate limit state calculations.
For T-sections $h_3 = 0$

10.3 Normal stresses
The normal stresses should be taken as:

$$\sigma_i = \frac{\gamma_i E_i a_i M}{(EI)_{ef}} \tag{10.7}$$

$$\sigma_{m,i} = \frac{0,5 E_i h_i M}{(EI)_{ef}} \tag{10.8}$$

10.4 Maximum shear stress
The maximum shear stresses occur where the normal stresses are zero. The maximum shear stresses in the web member (part 2 in Figure 10.1) should be taken as:

$$\tau_{2,max} = \frac{\gamma_3 E_3 A_3 a_3 + 0,5 E_2 b_2 h_2^2}{b_2 (EI)_{ef}} V \tag{10.9}$$

10.5 Fastener load
The load on a fastener should be taken as:

$$F_i = \frac{\gamma_i \, E_i \, A_i \, a_i \, s_i}{(EI)_{ef}} V$$

(10.10)

where:

$i = 1$ and 3, respectively;

$s_i = s_i(x)$ is the spacing of the fasteners as defined in 10.1.3.

11 Built-up columns

For slender columns, the design capacity is limited by buckling. Buckling is related to slenderness which means that in order to increase the member capacity, the slenderness must be decreased. This can be achieved in two ways: reduction of effective length and increase in radius of gyration about the critical buckling axis.

11.1 General

The effective length of column can be reduced by installing some restraints to movement in the critical buckling direction. The increase in radius of gyration can be achieved by increasing the size of the member or by using a member made from multiple compression elements.

11.1.1 Assumptions

The following assumptions apply:

− the columns are simply supported with a length ℓ;

− the individual parts are full length;

− the load is an axial force F_c acting at the geometric centre of gravity, (see 11.2.3).

11.1.2 Load-carrying capacity

For column deflection in the y-direction (see Figure 11.1) the load-carrying capacity should be taken as the sum of the load-carrying capacities of the individual members.

For column deflection in the z-direction (see Figure 11.1) it should be verified that:

$$\sigma_{c,0,d} \ \le \ k_c f_{c,0,d} \tag{11.1}$$

where:

$$\sigma_{c,0,d} \ = \ \frac{F_{c,d}}{A_{tot}} \tag{11.2}$$

where:

A_{tot} is the total cross-sectional area;

k_c is determined in accordance with 6.3.2 but with an effective slenderness ratio λ_{ef}

determined in accordance with sections 11.2 - 11.4.

11.2 Mechanically jointed columns

11.2.1 Effective slenderness ratio

The effective slenderness ratio should be taken as:

$$\lambda_{ef} = \ell \sqrt{\frac{A_{tot}}{I_{ef}}} \tag{11.3}$$

with

$$I_{\text{ef}} = \frac{(EI)_{\text{ef}}}{E_{\text{mean}}} \qquad\qquad (11.4)$$

where $(EI)_{\text{ef}}$ is determined in accordance with Chapter 10..

11.2.2 Load on fasteners
The load on a fastener should be determined in accordance with Chapter 10, where

$$V_{\text{d}} = \begin{cases} \dfrac{F_{\text{c,d}}}{120\,k_{\text{c}}} & \text{for } \lambda_{\text{ef}} < 30 \\[3mm] \dfrac{F_{\text{c,d}}\lambda_{\text{ef}}}{3600\,k_{\text{c}}} & \text{for } 30 \leq \lambda_{\text{ef}} < 60 \\[3mm] \dfrac{F_{\text{c,d}}}{60\,k_{\text{c}}} & \text{for } 60 \leq \lambda_{\text{ef}} \end{cases} \qquad\qquad (11.5)$$

11.2.3 Combined loads
In cases where small moments (e.g. from self weight) are acting in adition to axial load, 6.3.2 applies.

11.3 Spaced columns with packs or gussets

11.3.1 Assumptions
Columns as shown in Figure 11.1 are considered, i.e. columns comprising shafts spaced by packs or gussets. The joints may be either nailed or glued or bolted with suitable connectors.

The following assumptions apply:

− the cross-section is composed of two, three or four identical shafts;

− the cross-sections are symmetrical about both axes;

− the number of unrestrained bays is at least three, i.e. the shafts are at least connected at the ends and at the third points;

− the free distance a between the shafts is not greater than three times the shaft thickness h for columns with packs and not greater than 6 times the shaft thickness for columns with gussets;

− the joints, packs and gussets are designed in accordance with 11.2.2;

− the pack length ℓ_2 satisfies the condition: $\ell_2/a \geq 1{,}5$;

− there are at least four nails or two bolts with connectors in each shear plane. For nailed joints there are at least four nails in a row at each end in the longitudinal direction of the column;

− the gussets satisfies the condition: $\ell_2/a \geq 2$;

− the columns are subjected to concentric axial loads.

For columns with two shafts A_{tot} and I_{tot} should be calculated as

$$A_{tot} = 2A \tag{11.6}$$

$$I_{tot} = \frac{b\left[(2h+a)^3 - a^3\right]}{12} \tag{11.7}$$

For columns with three shafts A_{tot} and I_{tot} should be calculated as

$$A_{tot} = 3A \tag{11.8}$$

$$I_{tot} = \frac{b\left[(3h+2a)^3 - (h+2a)^3 + h^3\right]}{12} \tag{11.9}$$
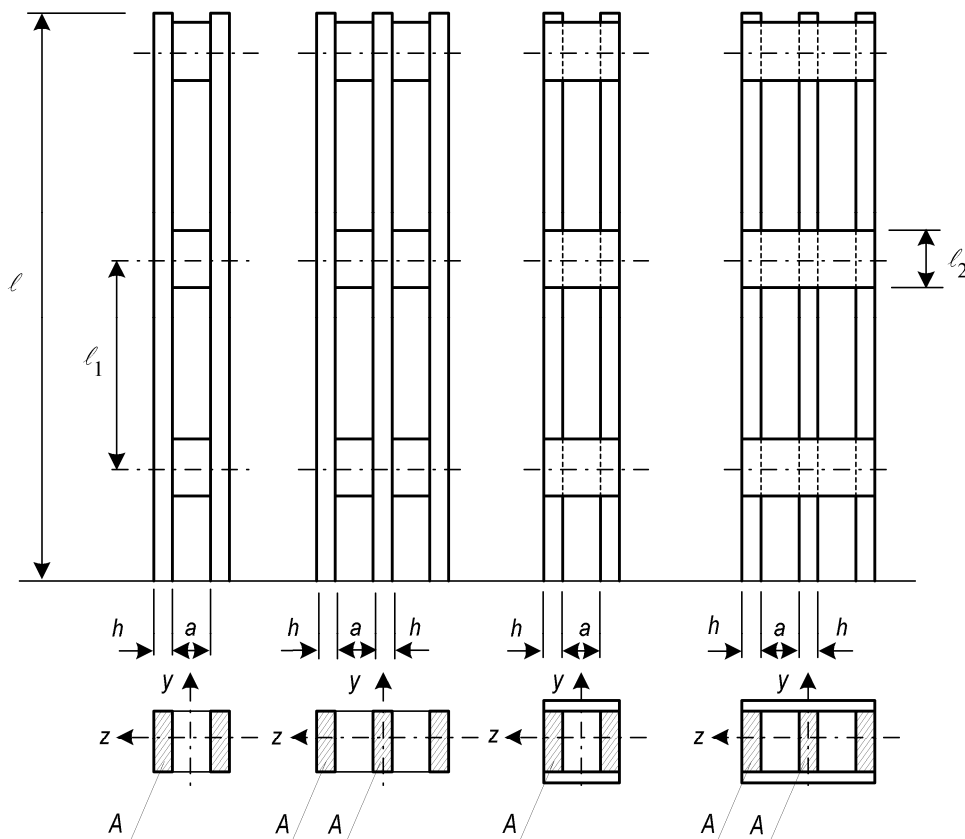

Figure 11.1 – Spaced columns

11.3.2 Axial load-carrying capacity
For column deflection in the y-direction (see Figure 11.1) the load-carrying capacity should

be taken as the sum of the load-carrying capacities of the individual members.

For column deflection in the z-direction 11.1.2 applies with

$$\lambda_{ef} = \sqrt{\lambda^2 + \eta\frac{n}{2}\lambda_1^2} \qquad\qquad (11.10)$$

where:

λ is the slenderness ratio for a solid column with the same length, the same area (A_{tot}) and the same second moment of area (I_{tot}), i.e.,

$$\lambda = \ell\sqrt{A_{tot}/I_{tot}} \qquad\qquad (11.11)$$

λ_1 is the slenderness ratio for the shafts and has to be set into expression (11.10) with a minimum value of at least 30, i.e.

$$\lambda_1 = \sqrt{12}\frac{\ell_1}{h} \qquad\qquad (11.12)$$

n is the number of shafts;

η is a factor given in Table 11.1.

Table 11.1 – The factor η

	Packs			Gussets	
	Glued	Nailed	Bolted[a]	Glued	Nailed
Permanent/long-term loading	1	4	3,5	3	6
Medium/short-term loading	1	3	2,5	2	4,5
[a] with connectors					

11.3.3 Load on fasteners, gussets or packs

The load on the fasteners and the gussets or packs are as shown in Figure 11.2 with V_d according to section 11.2.2.

The shear forces on the gussets or packs, see Figure 11.2, should be calculated from:

$$T_d = \frac{V_d l_1}{a_1} \qquad\qquad (11.13)$$
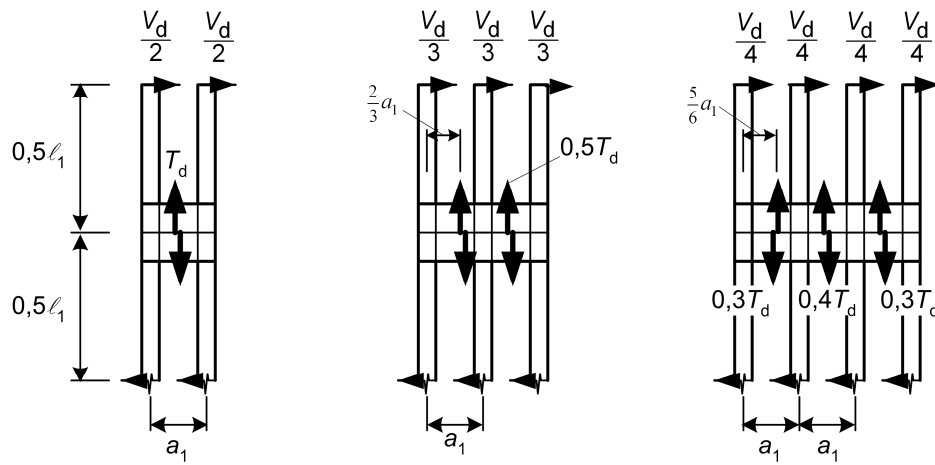
Figure 11.2 Shear force distribution and loads on gussets or packs

12 Worked examples

12.1 Column with solid cross-section

Column with cross-section 100 x 100 mm, buckling length $l = 3\,000$ mm.

Timber of strength class C22 according to EN 338 ($f_{c,0,k} = 20$ MPa and $E_{0,05} = 6\,700$ MPa).

Design compressive force $N_d = 30$ kN (medium-term). Service class 1.

Design compressive strength

$$f_{c,0,d} = k_{mod}\frac{f_{c,0,k}}{\gamma_M} = 0,8\frac{20}{1,3} = 12,3\,\text{MPa}$$

Design compressive stress

$$\sigma_{c,0,d} = \frac{N_d}{A} = \frac{30\cdot10^3}{10\cdot10^3} = 3,0\,\text{MPa}$$

Slenderness ratio

$$\lambda = \frac{\ell_{ef}}{i} = \frac{3\,000}{0,289\cdot100} = 103,8$$

Buckling resistance

$$\sigma_{c,crit} = \pi^2\frac{E_{0,05}}{\lambda^2} = 3,14^2\frac{6\,700}{103,8^2} = 6,1\,\text{MPa}$$

$$\lambda_{rel} = \sqrt{\frac{f_{c,0,k}}{\sigma_{c,crit}}} = \sqrt{\frac{20}{6,1}} = 1,8$$

$$k = 0,5\left[1 + \beta_c\left(\lambda_{rel} - 0,3\right) + \lambda_{rel}^2\right] = 0,5\left[1 + 0,2\left(1,8 - 0,3\right) + 1,8^2\right] = 2,27$$

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}} = \frac{1}{2,27 + \sqrt{2,27^2 - 1,8^2}} = 0,29$$

Verification of failure condition

$$\frac{\sigma_{c,0,d}}{k_c\,f_{c,0,d}} \le 1$$

$$\frac{3,0}{0,29\cdot12,4} = 0,83 < 1$$

12.2 Beam with solid cross-section

Simply supported timber beam with cross-section 50 x 200 mm, clear span ℓ = 3 500 mm.

Timber of strength class C22 according to EN 338 ($f_{m,k}$ = 22 MPa, $f_{v,k}$ = 2,4 MPa,

$E_{0,05}$ = 6 700 MPa).

Design uniformly distributed load of 2 kNm^{-1} (medium-term). Service class 1.

Design bending and shear strength

$$f_{m,d} = k_{mod} \frac{f_{m,k}}{\gamma_M} = 0,8 \frac{22,0}{1,3} = 13,5 \text{ MPa}$$

$$f_{v,d} = k_{mod} \frac{f_{v,k}}{\gamma_M} = 0,8 \frac{2,4}{1,3} = 1,48 \text{ MPa}$$

a) Bending (beam is assumed to be laterally restrained throughout the length of its compression edge)

Verification of failure condition

$$\sigma_{m,d} \le f_{m,d}$$

$$\sigma_{m,d} = \frac{M_d}{W} = \frac{1}{8} \frac{q_d \ell^2}{W} = \frac{1}{8} \frac{2 \cdot 3500^2 \cdot 6}{50 \cdot 200^2} = 9,2 \text{ MPa} < 13,5 \text{ MPa}$$

b) Bending (beam is not assumed to be laterally restrained throughout the length of its compression edge)

Buckling resistance

$$\sigma_{m,crit} = \frac{0,78 \, b^2 \, E_{0,05}}{h \, \ell_{ef}} = \frac{0,78 \cdot 50^2 \cdot 6700}{200 \cdot (0,9 \cdot 3500 + 400)} = 18,4 \text{ MPa}$$

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{m,crit}}} = \sqrt{\frac{22}{18,4}} = 1,06$$

$$k_{crit} = 1,56 - 0,75 \, \lambda_{rel,m} = 1,56 - 0,75 \cdot 1,06 = 0,76$$

$$k_{crit} \cdot f_{m,d} = 0,76 \cdot 13,5 = 10,3 \text{ MPa}$$

Verification of failure condition

$$\sigma_{m,d} \le k_{crit} \cdot f_{m,d}$$

$$\sigma_{m,d} = \frac{M_d}{W} = \frac{1}{8} \frac{q_d \ell^2}{W} = \frac{2 \cdot 3500^2 \cdot 6}{8 \cdot 50 \cdot 200^2} = 9,2 \text{ MPa} < 10,3 \text{ MPa}$$

c) Shear

$\tau_{v,d} \le f_{v,d}$

Effective width

$b_{ef} = k_{cr}\, b$

$k_{cr} = 0,67$ is taking into account cracks caused by too rapid drying

Verification of failure condition

$$\tau_{v,d} = \frac{3\,V_d}{2\,A} = \frac{3 \cdot 1 \cdot 2 \cdot 3500}{2 \cdot 2 \cdot 0,67 \cdot 50 \cdot 200} = 0,78\,\text{MPa} < 1,48\,\text{MPa}$$

12.3 Step joint

Joint of a compression member with cross-section 140 x 140 mm, see Figure below (cutting depth is 45 mm, shear length in chord 250 mm and $\beta = 45°$).

Design values of timber properties are $f_{c,0,d} = 11,03$ MPa, $f_{c,90,d} = 2,21$ MPa, $f_{v,d} = 1,32$ MPa.

Design compressive force $N_d = 55$ kN.

Design compressive strength at an angle to the grain

$$f_{c,\alpha,d} = \frac{f_{c,0,d}}{\dfrac{f_{c,0,d}}{k_{c,90}\, f_{c,90,d}} \sin^2 \alpha + \cos^2 \alpha} = \frac{11,03}{\dfrac{11,03}{2,81} \sin^2 22,5° + \cos^2 22,5°} = 7,72\,\text{MPa}$$

Verification of failure conditions

$$\sigma_{c,\alpha,d} = \frac{N_d \cos^2 \alpha}{b\, t_z} = \frac{55 \cdot 10^3 \cos^2 22,5°}{140 \cdot 45} = 7,45 \text{ MPa} < 7,72 \text{ MPa}$$

$$\tau_{v,d} = \frac{N_d \cos \beta}{b\ell_z} = \frac{55 \cdot 10^3 \cos 45°}{140 \cdot 250} = 1,11 \text{ MPa} < 1,32 \text{ MPa}$$

12.4 Timber-framed wall

The walls assembly presented in Figure 1 is subjected to the total design horizontal force $F_{H,d,tot} = 25$ kN (short-term) acting at the top of the wall assembly.
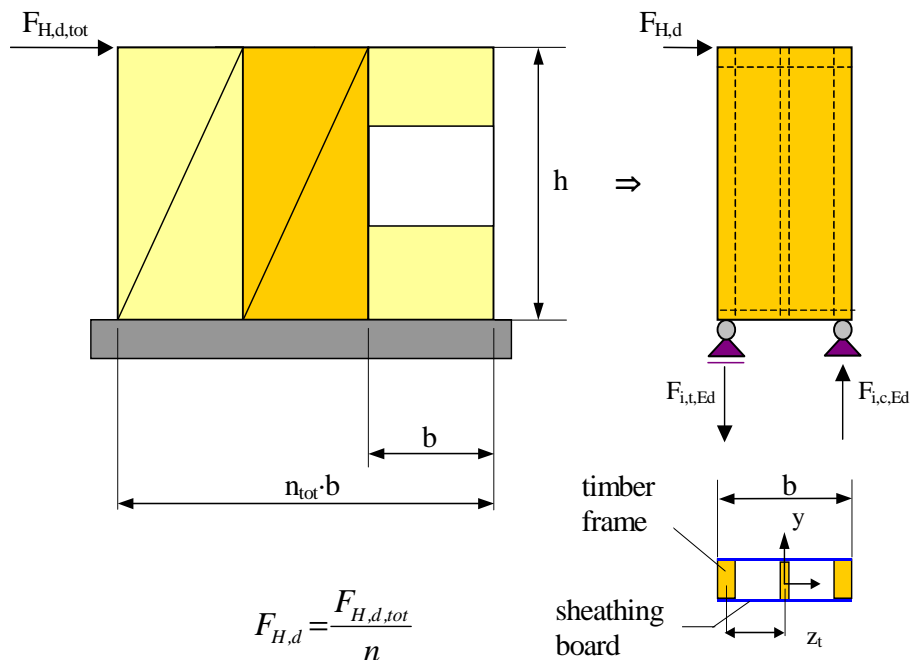

$$F_{H,d} = \frac{F_{H,d,tot}}{n}$$

Figure 1: Example of the wall assembly

The single panel wall element of actual dimensions $h = 2\ 635$ mm and $b = 1\ 250$ mm is composed of timber studs (2 x 90 x 90 mm and 1 x 44 x 90 mm) and timber girders (2 x 80 x 90 mm). The plywood sheathing boards of the thickness $t = 15$ mm are fixed to the timber frame using staples of $\varnothing$ 1,53 mm and length $l = 35$ mm at an average spacing of $s = 75$ mm (Figure 2).
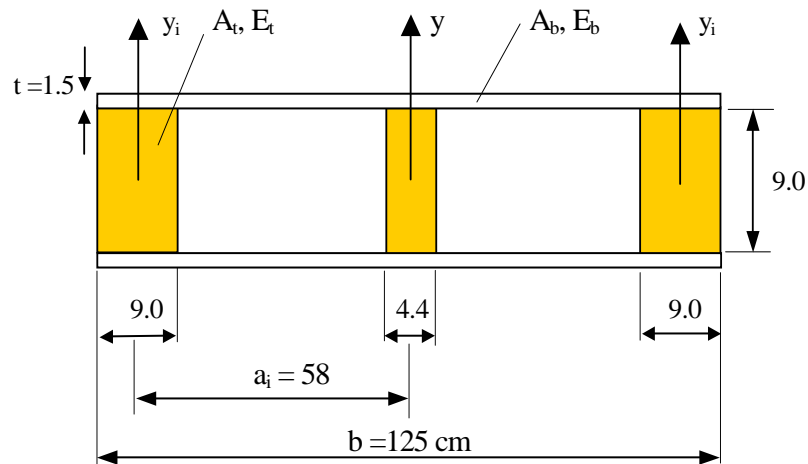
Figure 2: Cross-section of the single wall element

Material properties for the timber of quality C22 are taken from EN338 and for the Swedian plywood boards (S-plywood) from Steck »*Holzwerkstoffe – Sperrholz, Holzbauwerke: Bemessung und Baustoffe nach Eurocode 5, Step 1*«, 1995. All material properties are listed in *Table 1*.

Table 1: Properties of used materials

	$E_{0,m}$ [N/mm^2]	$f_{m,k}$ [N/mm^2]	$f_{t,0,k}$ [N/mm^2]	$f_{c,0,k}$ [N/mm^2]	ρ_k [kg/m^3]	ρ_m [kg/m^3]
C22	10 000	22,0	13,0	20,0	340,0	410,0
S – plywood[*]	9 200	23,0	15,0	15,0	410,0	410,0

[*] *The values are given for 12 mm typical thickness of the board.*

---

a) Characteristic fastener yield moment

$$M_{y,Rk} = 240 \cdot d^{2.6} = 240 \cdot 1,53^{2.6} = 725,12 \, \text{Nmm}$$

b) Characteristic embedment strength

in plywood: $f_{h,1,k} = 0.11 \cdot \rho_k \cdot d^{-0.3} = 0,11 \cdot 410 \cdot 1,53^{-0.3} = 39,70 \, \text{N/mm}^2$

in timber: $f_{h,2,k} = 0.082 \cdot \rho_k \cdot d^{-0.3} = 0,082 \cdot 340 \cdot 1,53^{-0.3} = 24,54 \, \text{N/mm}^2$

c) Lateral characteristic capacity of an individual fastener (t_1 = 15 mm, t_2 = 20 mm).

Lateral characteristic load-carrying capacity per staple per shear plane should be considered as equivalent to that of two nails with the staple diameter:

$$F_{f,Rk} = 2 \cdot f_{h,1,k} \cdot t_1 \cdot d = 1\,822,14\ \text{N}$$

$$F_{f,Rk} = 2 \cdot f_{h,2,k} \cdot t_2 \cdot d = 1\,501,88\ \text{N}$$

$$F_{v,Rk} = 2 \cdot \frac{f_{h,1,k} \cdot t_1 \cdot d}{1+\beta} \cdot \left[\sqrt{\beta + 2\beta^2 \cdot \left[1 + \frac{t_2}{t_1} + \left(\frac{t_2}{t_1}\right)^2\right] + \beta^3 \cdot \left(\frac{t_2}{t_1}\right)^2} - \beta \cdot \left(1 + \frac{t_2}{t_1}\right) \right] + \frac{F_{ax,Rk}}{4} = 678,04\ \text{N}$$

$$F_{f,Rk} = 2 \cdot 1,05 \cdot \frac{f_{h,1,k} \cdot t_1 \cdot d}{2+\beta} \cdot \left[\sqrt{2\beta \cdot (1+\beta) + \frac{4\beta \cdot (2+\beta) \cdot M_{y,Rk}}{f_{h,1,k} \cdot d \cdot t_1^2}} - \beta \right] + \frac{F_{ax,Rk}}{4} = 667,10\ \text{N}$$

$$F_{f,Rk} = 2 \cdot 1,05 \cdot \frac{f_{h,1,k} \cdot t_2 \cdot d}{2+\beta} \cdot \left[\sqrt{2\beta^2 \cdot (1+\beta) + \frac{4\beta \cdot (2+\beta) \cdot M_{y,Rk}}{f_{h,1,k} \cdot d \cdot t_2^2}} - \beta \right] + \frac{F_{ax,Rk}}{4} = 705,88\ \text{N}$$

$$F_{f,Rk} = 2 \cdot 1,15 \cdot \sqrt{\frac{2\beta}{1+\beta}} \cdot \sqrt{2M_{y,Rk} \cdot f_{h,1,k} \cdot d} + \frac{F_{ax,Rk}}{4} = 596,67\ \text{N}$$

$$F_{f,Rk} = 596,67\ \text{N}$$

d) Characteristic racking load-carrying capacity of one wall panel (Eurocode 5-1-1; Method A)

$$F_{i,v,Rk} = 2 \cdot \frac{F_{f,Rk} \cdot b_i \cdot c_i}{s} = 2 \cdot \frac{596,67 \cdot 125,0 \cdot 0,949}{7,5} = 18\,874,66\ \text{N} = 18,87\ \text{kN}$$

$$c_i = \frac{b_i}{b_0} = \frac{125 \cdot 2}{263,5} = 0,949;\quad b_0 = \frac{h}{2}$$

e) Characteristic racking load-carrying capacity of the wall assembly (the wall element with the opening is not considered)

$$F_{v,Rk} = \sum F_{i,v,Rk} = 2 \cdot 18,87\ \text{kN} = 37,74\ \text{kN}$$

f) Design racking load-carrying capacity of the wall assembly ($k_{mod} = 0,9$)

$$F_{v,Rd} = k_{mod} \cdot \frac{F_{v,Rk}}{\gamma_M} = 0,9 \cdot \frac{37,74}{1,30} = 26,13\ \text{kN}$$

g) Ultimate limit state criteria

$$F_{v,Rd} > F_{H,d,tot}$$
$$26,13\ \text{kN} > 25,0\ \text{kN}$$

h) Design external forces in the supports (Figure 1)

$$F_{i,c,Ed} = F_{i,t,Ed} = \frac{F_{H,d} \cdot h}{b} = \frac{25,0 \cdot 263,5}{2 \cdot 125} = 26,35 \, \text{kN}$$

12.5a Single tapered beam – span 12 m

Assessment of a single tapered beam (Figure 10.1). Material: glue laminated timber (GL 24h), service class 1. Characteristic values: Dead load $g_k = 4,5 \, \text{kNm}^{-1}$, snow $s_k = 4,5 \, \text{kNm}^{-1}$.
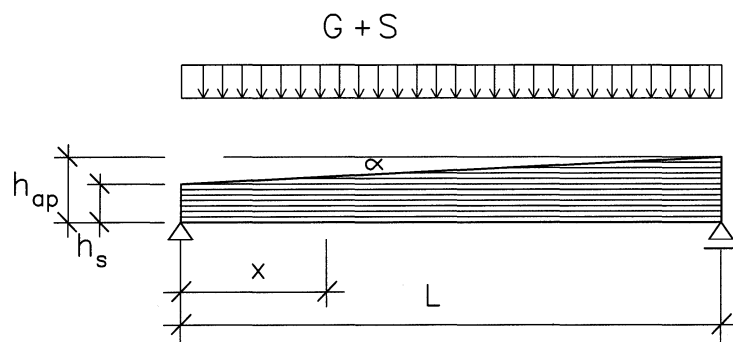Materials and geometrical characteristics of the beam:

Figure 10.1 Scheme of the single tapered beam

Span: $L = 12\,000 \, \text{mm}$

Depth of the beam at the apex: $h_{ap} = 1\,200 \, \text{mm}$

Angle of the taper: $\alpha = 3°$

Width of the beam: $b = 140 \, \text{mm}$

Precamber of the beam: $w_c = 30 \, \text{mm}$

$f_{m,g,k} = 24 \, \text{MPa}$

$f_{v,g,k} = 2,7 \, \text{MPa}$

$f_{c,90,g,k} = 2,7 \, \text{MPa}$

$f_{t,90,g,k} = 0,4 \, \text{MPa}$

$E_{0,mean,g} = 11\,600 \, \text{MPa}$

The beam is prevented against lateral-torsional buckling.

Design bending strength

$$f_{m,g,d} = k_{mod} \frac{f_{m,g,k}}{\gamma_M} = 0,9 \frac{24}{1,25} = 17,28 \, \text{MPa}$$

Design shear strength

$$f_{v,g,d} = k_{mod} \frac{f_{v,g,k}}{\gamma_M} = 0,9 \frac{2,7}{1,25} = 1,94 \text{ MPa}$$

Design compressive strength perpendicular to the grain

$$f_{c,90,g,d} = k_{mod} \frac{f_{c,90,g,k}}{\gamma_M} = 0,9 \frac{2,7}{1,25} = 1,94 \text{ MPa}$$

Basic combination of the load

$$q_d = 1,35 g_k + 1,5 p_k = 1,35 \cdot 4,5 + 1,5 \cdot 4,5 = 12,825 \text{ kNm}^{-1}$$

Shear force at a support

$$V_d = q_d \frac{L}{2} = 12,825 \frac{12}{2} = 76,95 \text{ kN}$$

Depth of the beam at the support

$$h_S = h_{ap} - tg\alpha \cdot L = 1,2 - tg\ 3^0 \cdot 12 = 0,571 \text{ m}$$

Verification of failure conditions

a) Shear at support

$$\tau_{v,d} = \frac{3V_d}{2bh_0} = \frac{3 \cdot 76,95 \cdot 10^3}{2 \cdot 140 \cdot 571} = 1,44 \text{ MPa} < 1,94 \text{ MPa}$$

b) Bending at critical cross-section

Critical cross-section position

$$x = \frac{L}{1 + \frac{h_{ap}}{h_s}} = \frac{12}{1 + \frac{1,2}{0,571}} = 3,87 \text{ m}$$

Depth of the beam at critical cross-section

$$h_x = \frac{2 \cdot h_{ap}}{1 + \frac{h_{ap}}{h_s}} = \frac{2 \cdot 1,2}{1 + \frac{1,2}{0,571}} = 0,774 \text{ m}$$

Bending moment at critical cross-section

$$M_d = V_d x - \frac{q_d x^2}{2} = 76,95 \cdot 3,87 - \frac{12,825 \cdot 3,87^2}{2} = 201,76 \text{ kNm}$$

Stress at critical cross-section

$$\sigma_{m,0,d} = \sigma_{m,\alpha,d} = \frac{6M_d}{bh_x^2}$$

$$\sigma_{m,0,d} \le f_{m,g,d}$$

$$\sigma_{m,0,d} = \frac{6 \cdot 201,76 \cdot 10^6}{140 \cdot 774^2} = 14,43 \text{ MPa} < 17,28 \text{ MPa} \quad \Rightarrow \text{allowed}$$

$$\sigma_{m,\alpha,d} \le k_{m,\alpha} \cdot f_{m,g,d}$$

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{m,g,d}}{1,5 f_{v,g,d}} tg\alpha\right)^2 + \left(\dfrac{f_{m,g,d}}{f_{c,90,g,d}} tg^2\alpha\right)^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{17,28}{1,5 \cdot 1,94} tg 3^0\right)^2 + \left(\dfrac{17,28}{1,94} tg^2 3^0\right)^2}} = 0,9112$$

$$\sigma_{m,\alpha,d} = 14,43 \text{ } MPa < 0,9112 \cdot 17,28 = 15,74 \text{ MPa} \quad \Rightarrow \text{allowed}$$

c) Deflection

$$w_m = k_u \cdot w_0$$

Coefficient k_u – see *Figure 10.2*.

$$h_0 = \frac{h_s + h_{ap}}{2} = \frac{0,571 + 1,2}{2} = 0,886 \text{ m}$$

$$\frac{h_{ap}}{h_s} = \frac{1,2}{0,571} = 2,10 \Rightarrow k_u = 1,1166$$

$$k_{def} = 0,6$$

c1) Instantaneous deflection

$$w_{inst,g} = k_u \cdot \frac{5 \cdot g \cdot L^4}{384 \cdot E \cdot I_y} = 1,1166 \cdot \frac{5 \cdot 4,5 \cdot 12^4 \cdot 10^{12} \cdot 12}{384 \cdot 11600 \cdot 140 \cdot 886^3} = 14,42 \text{ mm}$$

$$w_{inst,s} = k_u \cdot \frac{5 \cdot s \cdot L^4}{384 \cdot E \cdot I_y} = 1,1166 \cdot \frac{5 \cdot 4,5 \cdot 12^4 \cdot 10^{12} \cdot 12}{384 \cdot 11600 \cdot 140 \cdot 886^3} = 14,42 \text{ mm}$$

$$w_{inst} = w_{inst,g} + w_{inst,s} = 14,42 + 14,42 = 28,84 \text{ } mm = \frac{L}{416} < \frac{L}{400} \Rightarrow \text{allowed}$$
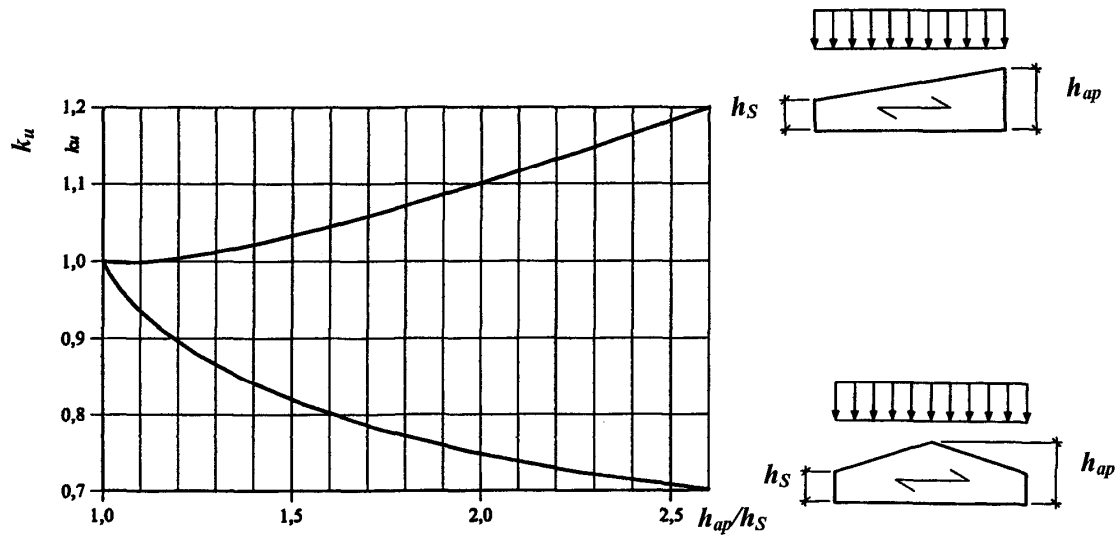
Figure 10.2 Coefficient k_u

c2) Final deflection

$$w_{fin,g} = w_{inst,g} \cdot \left(1 + k_{def}\right) = 14,42 \cdot \left(1 + 0,6\right) = 23,07 \text{ mm}$$

$$w_{fin,s} = w_{inst,s} \cdot \left(1 + \psi_2 \cdot k_{def}\right) = 14,42 \cdot \left(1 + 0 \cdot 0,6\right) = 14,42 \text{ mm}$$

$$w_{fin} = w_{fin,g} + w_{fin,s} = 23,07 + 14,42 = 37,49 \text{ mm} = \frac{L}{320} < \frac{L}{250} \Rightarrow \text{allowed}$$

c3) Net final deflection

$$w_{net,fin} = w_{fin} - w_c = 37,49 - 30 = 7,49 \text{ mm} = \frac{L}{1602} < \frac{L}{300} \Rightarrow \text{allowed}$$

12.5b Single tapered beam – span 15 m
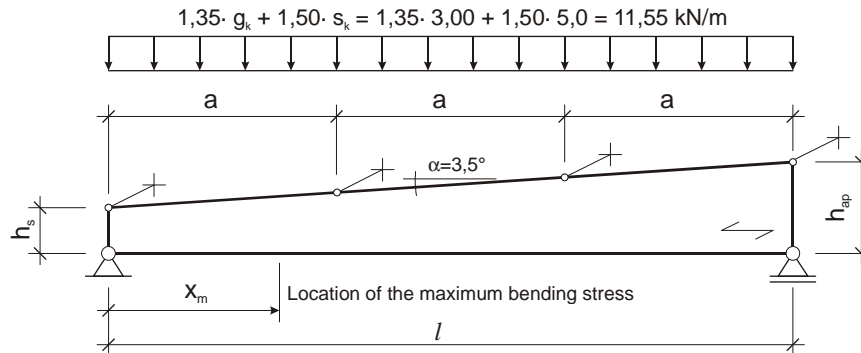
Material:

Glulam GL28h (according to EN 14080), $\gamma_M = 1,25$;

Service class 1, Load duration class 'short-term' (snow): $k_{mod} = 0,90$

Dimensions:

$b = 180$ mm, $h_s = 450$ mm, $h_{ap} = 1\,370$ mm, $l = 15\,000$ mm,

$a = 5\,000$ mm (Distance of lateral-buckling supports)

Design values – Material:

$$f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_M} = 0,90 \cdot \frac{28,0}{1,25} = 20,16 \text{ N/mm}^2$$

$$f_{v,d} = k_{mod} \cdot \frac{f_{v,k}}{\gamma_M} = 0,90 \cdot \frac{3,2}{1,25} = 2,30 \text{ N/mm}^2$$

$$f_{c,90,d} = k_{mod} \cdot \frac{f_{c,90,k}}{\gamma_M} = 0,90 \cdot \frac{3,0}{1,25} = 2,16 \text{ N/mm}^2$$

Design values – Internal forces:

Distance of design location:

$$x_m = \frac{l}{(1 + \frac{h_{ap}}{h_s})} = \frac{15\,000}{(1 + \frac{1\,370}{450})} = 3\,709 \text{ mm}$$

Height of the beam at the location of maximum stresses:

$$h_{x_m} = h_s + \frac{(h_{ap} - h_s)}{l} \cdot x_m = 450 + \frac{(1\,370 - 450)}{15\,000} \cdot 3\,709 = 677 \text{ mm}$$

$$V_d = \frac{(g_d + s_d) \cdot l}{2} = \frac{11,55 \cdot 15,00}{2} = 86,63 \text{ kN}$$

$$M_{x_m,d} = V_d \cdot x_m - (g_d + s_d) \cdot \frac{x_m^2}{2} = 86,33 \cdot 3,71 - 11,55 \cdot \frac{3,71^2}{2} = 240,80 \text{ kNm}$$

Verification:

Bending (at the location of maximum stresses)

- at the outermost fibre of the beam in the tension zone (uncutted fibres)

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_d}{b \cdot h_{x_m}^2} = \frac{6 \cdot 240,80 \cdot 10^6}{180 \cdot 677^2} = 17,51 \text{ N/mm}^2$$

With: $\alpha = 0 \rightarrow k_{m,\alpha} = 1,0$

$$\frac{\sigma_{m,0,d}}{k_{m,\alpha} \cdot f_{m,d}} = \frac{17,51}{1,0 \cdot 20,16} = 0,87 < 1,0$$

- at the outermost fibre of the beam in the compression zone (cutted fibres)

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_d}{b \cdot h_{x_m}^2} = \frac{6 \cdot 240,80 \cdot 10^6}{180 \cdot 677^2} = 17,51 \text{ N/mm}^2$$

With $\alpha = 3,5°$:

$$k_{m,\alpha} = \cfrac{1}{\sqrt{1 + \left(\cfrac{f_{m,d}}{1,5 \cdot f_{v,d}} \cdot \tan\alpha\right)^2 + \left(\cfrac{f_{m,d}}{f_{c,90,d}} \cdot \tan^2\alpha\right)^2}} =$$

$$= \cfrac{1}{\sqrt{1 + \left(\cfrac{20,16}{1,5 \cdot 2,16} \cdot \tan 3,5\right)^2 + \left(\cfrac{20,16}{2,30} \cdot \tan^2 3,5\right)^2}} = 0,93$$
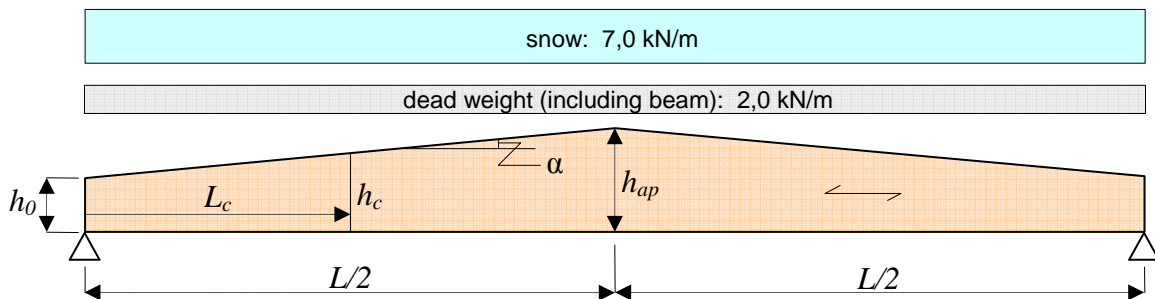
$$\frac{\sigma_{m,0,d}}{k_{m,\alpha} \cdot f_{m,d}} = \frac{17,51}{0,93 \cdot 20,16} = 0,93 < 1,0$$

Shear stresses (at the supports)

$$\tau_{\mathrm{d}} = 1,5 \cdot \frac{V_d}{b \cdot h} = 1,5 \cdot \frac{86,63 \cdot 10^3}{180 \cdot 450} = 1,60 \text{ N/mm}^2$$

$$\frac{\tau_{\mathrm{d}}}{f_{\mathrm{v,d}}} = \frac{1,60}{2,30} = 0,70 < 1,0$$

12.6a Double tapered beam – span 20 m

snow: 7,0 kN/m

dead weight (including beam): 2,0 kN/m

$h_0 = 600$ mm, $h_{\mathrm{ap}} = 1\ 100$ mm, $L = 20\ 000$ mm, width of beam: $b = 190$ mm

Glulam: GL36c $\rightarrow$ $f_{\mathrm{m,k}} = 36$ MPa $\quad f_{\mathrm{v,k}} = 3,8$ MPa $\quad f_{\mathrm{c,90,k}} = 3,3$ MPa $\quad f_{\mathrm{t,90,k}} = 0,5$ MPa

Load duration: short term $\quad p = 1,5 \cdot 7,0 + 1,2 \cdot 2,0 = 12,9$ kN/m

Service class: 2 $\quad \rightarrow \quad k_{\mathrm{mod}} = 0,9$

Assumption: Lateral torsional buckling is prevented by sufficient transverse bracing ($k_{\mathrm{crit}} = 1$)

Ultimate limit state

Design strength: $f_{\mathrm{d}} = k_{\mathrm{mod}} \dfrac{f_{\mathrm{k}}}{\gamma_{\mathrm{M}}} = 0,9 \dfrac{f_{\mathrm{k}}}{1,25} = 0,72 \cdot f_{\mathrm{k}} \quad \rightarrow \quad f_{\mathrm{m,d}} = 0,72 \cdot 36 = 25,9$ MPa

$f_{\mathrm{v,d}} = 0,72 \cdot 3,8 = 2,74$ MPa; $\quad f_{\mathrm{c,90,d}} = 0,72 \cdot 3,3 = 2,38$ MPa; $\quad f_{\mathrm{t,90,d}} = 0,72 \cdot 0,5 = 0,36$ MPa

Critical section with respect to *bending*, for a uniformly distributed load, is at distance

$L_{\mathrm{c}} = L(h_0/2h_{\mathrm{ap}}) = 20\ 000\ (600/2 \cdot 1100) = 5\ 450$ mm from the support, where

$h_{\mathrm{c}} = h_0 + (h_{\mathrm{ap}} - h_0) \cdot 2L_{\mathrm{c}}/L = 600 + 273 = 873$ mm

Also: $\tan \alpha = (h_{\mathrm{ap}} - h_0)/(0,5 \cdot L) = (1100 - 600)/10\ 000 = 0,05$

Nominal bending stress at critical section:

$$\sigma_{m,\alpha,d} = \frac{M_c}{W_c} = \frac{0,5 \cdot pL_c(L - L_c)}{bh_c^2 / 6} = \frac{3 \cdot 12,9 \cdot 5,45 \, (20 - 5,45) \cdot 10^6}{190 \cdot 873^2} = 21,2 \text{ MPa}$$

Verification of failure condition, (6.38),

$$\sigma_{m,\alpha,d} \leq k_{m,\alpha} f_{m,d}$$

where the stress modification factor due to compression at the tapered egde is defined by (6.40):

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{m,d}}{1,5 f_{v,d}} \tan \alpha\right)^2 + \left(\dfrac{f_{m,d}}{f_{c,90,d}} \tan^2 \alpha\right)^2}} = 0,953$$

Hence $k_{m,\alpha} \, f_{m,d} = 0,953 \cdot 25,9 = 24,7 \text{ MPa} > \sigma_{m,\alpha,d} = 21,2 \text{ MPa}$

At the apex, the bending stress is defined by (6.42):

$$\sigma_{m,d} = k_\ell \frac{6 M_{ap,d}}{b \, h_{ap}^2} \quad \text{where} \quad k_\ell = 1 + 1,4 \tan \alpha + 5,4 \tan^2 \alpha = 1,084$$

$$\sigma_{m,d} = 1,084 \frac{6 \cdot 12,9 \cdot 20^2 / 8}{190 \cdot 1100^2} 10^6 = 18,2 \text{ MPa}$$

The requirement, (6.41), is: $\sigma_{m,d} \leq k_r \, f_{m,d}$

Since $k_r = 1,0$ (see 6.49) the bending stress at apex is well below the limit.

Largest *tensile stress perpendicular to grain* is defined by (6.54):

$$\sigma_{t,90,d} = k_p \frac{6 M_{ap,d}}{b \, h_{ap}^2} \quad \text{where} \quad k_p = k_5 = 0,2 \tan \alpha = 0,01 \text{ (see 6.56)}$$

$$\sigma_{t,90,d} = 0,01 \frac{6 \cdot 12,9 \cdot 20^2 / 8}{190 \cdot 1100^2} 10^6 = 0,17 \text{ MPa}$$

The design requirement is, (6.53),

$$\frac{\tau_d}{f_{v,d}} + \frac{\sigma_{t,90,d}}{k_{dis} \, k_{vol} \, f_{t,90,d}} = \frac{0}{2,74} + \frac{0,17}{1,4 \cdot 0,53 \cdot 0,36} = 0,63 < 1$$

The volume factor, k_{vol}, has been determined by (6.51) with $V = 0,19 \cdot 1,1 \cdot 1,1 = 0,23 \, \text{m}^3$.

The *shear stresses* should, according to the current version of the code, not exceed the shear strength τ_d. However, a modification to the code, reducing the width of the section by a

"cracking" factor k_{cr}, will most likely be made in the near future. The value for glulam is

$k_{cr} = 0,67$. For a rectangular section this means that the shear stress should not exceed

$k_{cr} \, \tau_d = 0,67 \cdot 2,74 = 1,83$ MPa

Maximum shear stress, at the support,

$$\tau_d = \frac{3}{2} \frac{V}{A} = \frac{3 \cdot 129\,000}{190 \cdot 600} = 1,70 \text{ MPa} \quad < \quad 1,83 \text{ MPa}$$

Conclusion: All strength requirements are satisfied.

Serviceability limit state

Maximum displacement for this beam, due to a uniformly distributed load, is (by a computer analysis) found to be 1,63 times that of a corresponding beam with uniform height equal to $h_{ap} = 1100$ mm. For GL36c: $E_0 = 14\,700$ MPa. From Section 2.1.2:

$$w_{inst} = 1,63 \frac{5 \cdot p \cdot 20\,000^4}{384 \cdot 14\,700 \cdot 190 \cdot 1100^3 / 12} = 10,96 \cdot p$$

$$w_{net,fin} = w_{inst,G} \, (1 + k_{def}) + w_{inst,Q} \, (1 + \psi_{2,1} \, k_{def})$$
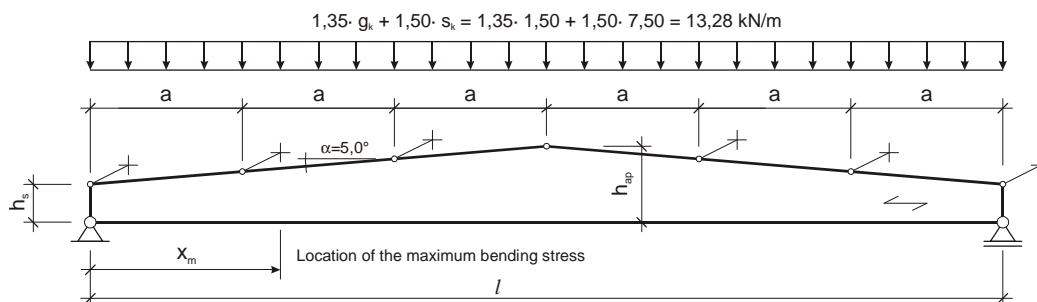
$$w_{net,fin} = 10,96 \cdot 2 \, (1 + 0,8) + 10,96 \cdot 7 \, (1 + 0,2 \cdot 0,8) = 39,5 + 89,0 = 128,5 \text{ mm}$$

In other words, $w_{net,fin} = L/155$, which is well above the recommended value of table 7.2.

Conclusion: The displacement may, depending on the type of building, be too large.

It may be considered to produce the beam with a precamber of, say 100 mm.

12.6b Double tapered beam – span 24 m

$1,35 \cdot g_k + 1,50 \cdot s_k = 1,35 \cdot 1,50 + 1,50 \cdot 7,50 = 13,28$ kN/m

$\alpha = 5,0°$

h_s h_{ap}

x_m Location of the maximum bending stress

l

Material:

Glulam GL24h (according to EN 14080), $\gamma_M = 1,25$;

Service class 2, Load duration class 'short-term' (snow): $k_{mod} = 0,90$

Dimensions:

$b = 180$ mm, $h_s = 750$ mm, $h_{ap} = 1\,800$ mm, $l = 24\,000$ mm,

$a = 4\,000$ mm (Distance of lateral-buckling supports)

Design values – Material:

$$f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_M} = 0,90 \cdot \frac{24,0}{1,25} = 17,28 \text{ N/mm}^2$$

$$f_{v,d} = k_{mod} \cdot \frac{f_{v,k}}{\gamma_M} = 0,90 \cdot \frac{2,7}{1,25} = 1,94 \text{ N/mm}^2$$

$$f_{c,90,d} = k_{mod} \cdot \frac{f_{c,90,k}}{\gamma_M} = 0,90 \cdot \frac{2,7}{1,25} = 1,94 \text{ N/mm}^2$$

$$f_{t,90,d} = k_{mod} \cdot \frac{f_{t,90,k}}{\gamma_M} = 0,90 \cdot \frac{0,4}{1,25} = 0,29 \text{ N/mm}^2$$

Design values – Internal forces:

Distance of design location:

$$x_m = \frac{l \cdot h_s}{2 \cdot h_{ap}} = \frac{24\,000 \cdot 750}{2 \cdot 1\,800} = 5\,000 \text{ mm}$$

Height of the beam at the location of maximum stresses:

$$h_{x_m} = h_s + \frac{(h_{ap} - h_s)}{\dfrac{l}{2}} \cdot x_m = 750 + \frac{(1\,800 - 750)}{\dfrac{24\,000}{2}} \cdot 5\,000 = 1188 \text{ mm}$$

$$V_d = \frac{(g_d + s_d) \cdot l}{2} = \frac{13,28 \cdot 24,00}{2} = 159,36 \text{ kN}$$

$$M_{x_m,d} = V_d \cdot x_m - (g_d + s_d) \cdot \frac{x_m^2}{2} = 159,36 \cdot 5,00 - 13,28 \cdot \frac{5,00^2}{2} = 630,80 \text{ kNm}$$

$$M_{ap,d} = \frac{(g_d + s_d) \cdot l^2}{8} = \frac{13,28 \cdot 24,00^2}{8} = 956,16 \text{ kNm}$$

Verification at the location of maximum bending stresses

- at the outermost fibre of the beam in the tension zone (no cutted fibres)

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_d}{b \cdot h_{x_m}{}^2} = \frac{6 \cdot 630,80 \cdot 10^6}{180 \cdot 1188^2} = 14,90 \text{ N/mm}^2$$

With: $\alpha = 0 \rightarrow k_{m,\alpha} = 1,0$

$$\frac{\sigma_{m,0,d}}{k_{m,\alpha} \cdot f_{m,d}} = \frac{14,90}{1,0 \cdot 17,28} = 0,86 < 1,0$$

- at the outermost fibre of the beam in the compression zone (cutted fibres)

$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \cdot M_d}{b \cdot h_{x_m}{}^2} = \frac{6 \cdot 630,80 \cdot 10^6}{180 \cdot 1188^2} = 14,90 \text{ N/mm}^2$$

With $\alpha = 5°$:

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\dfrac{f_{m,d}}{1,5 \cdot f_{v,d}} \cdot \tan\alpha\right)^2 + \left(\dfrac{f_{m,d}}{f_{c,90,d}} \cdot \tan^2\alpha\right)^2}} =$$

$$= \frac{1}{\sqrt{1 + \left(\dfrac{17,28}{1,5 \cdot 1,94} \cdot \tan 5\right)^2 + \left(\dfrac{17,28}{1,94} \cdot \tan^2 5\right)^2}} = 0,89$$

$$\frac{\sigma_{m,0,d}}{k_{m,\alpha} \cdot f_{m,d}} = \frac{14,90}{0,89 \cdot 17,28} = 0,97 < 1,0$$

Verification of the shear stresses (at the supports)

$$\tau_d = 1,5 \cdot \frac{V_d}{b \cdot h} = 1,5 \cdot \frac{159,36 \cdot 10^3}{180 \cdot 750} = 1,77 \text{ N/mm}^2$$

$$\frac{\tau_d}{f_{v,d}} = \frac{1,77}{1,94} = 0,91 < 1,0$$

Verifications in the apex zone

Bending stresses

$$\sigma_{m,ap,d} = k_\ell \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2}$$

With: $k_\ell = k_1 + k_2 \cdot \left(\dfrac{h_{ap}}{r}\right) + k_3 \cdot \left(\dfrac{h_{ap}}{r}\right)^2 + k_4 \cdot \left(\dfrac{h_{ap}}{r}\right)^3$, $\alpha_{ap} = 5°$ and $r = \infty \rightarrow \left(\dfrac{h_{ap}}{r}\right) = 0$

$$k_\ell = k_1 = 1 + 1,4 \cdot \tan\alpha_{ap} + 5,4 \cdot \tan^2\alpha_{ap} = 1 + 1,4 \cdot \tan 5 + 5,4 \cdot \tan^2 5 = 1,16$$

$$\sigma_{m,ap,d} = k_\ell \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 1,16 \cdot \frac{6 \cdot 956,16 \cdot 10^6}{180 \cdot 1\,800^2} = 11,41 \text{ N/mm}^2$$

With $k_r = 1,0$ for double tapered beams:

$$\frac{\sigma_{m,ap,d}}{k_r \cdot f_{m,d}} = \frac{11,41}{1,0 \cdot 17,28} = 0,66 < 1,0$$

Tension stresses perpendicular to grain

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2}$$

With: $k_p = k_5 + k_6 \cdot \left(\dfrac{h_{ap}}{r}\right) + k_7 \cdot \left(\dfrac{h_{ap}}{r}\right)^2$, $\alpha_{ap} = 5°$ and $r = \infty \rightarrow \left(\dfrac{h_{ap}}{r}\right) = 0$

$$k_p = k_5 = 0,2 \cdot \tan\alpha_{ap} = 0,2 \cdot \tan 5 = 0,0175$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0,0175 \cdot \frac{6 \cdot 956,16 \cdot 10^6}{180 \cdot 1800^2} = 0,17 \text{ N/mm}^2$$

With the reference volume $V_0 = 0,01 \text{ m}^3$ and

$$V = b \cdot h_{ap}^2 \cdot \left(1 - \frac{\tan\alpha_{ap}}{4}\right) = 0,18 \cdot 1,80^2 \cdot \left(1 - \frac{\tan 5,0}{4}\right) = 0,57 \text{ m}^3 \text{ the volume factor is:}$$

$$k_{vol} = \left(\frac{V_0}{V}\right)^{0,2} = \left(\frac{0,01}{0,57}\right)^{0,2} = 0,45 \text{ and}$$
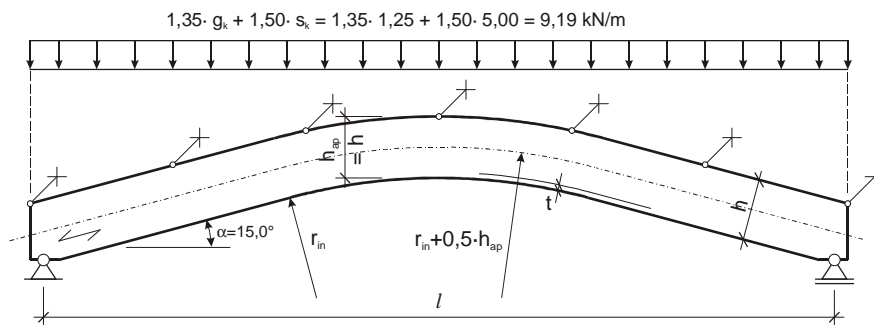
$k_{dis} = 1,4$ for double tapered beams

$$\frac{\sigma_{t,90,ap,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0,17}{1,4 \cdot 0,45 \cdot 0,29} = 0,93 < 1,0$$

Remark:

Since no shear force is acting in the middle of the beam a combined verification for tension stresses perpendicular to grain and shear forces is not necessary.

12.7 Curved beam

Material:

Glulam GL28h (according to EN 14080), $\gamma_M = 1,25$;

Service class 2, Load duration class 'short-term' (snow): $k_{mod} = 0,90$

Dimensions:

$b = 180 \text{ mm}$, $h = h_{ap} = 1\,600 \text{ mm}$, $l = 20\,000 \text{ mm}$, $r_{in} = 15\,000 \text{ mm}$, $t = 40 \text{ mm}$

Design values – Material:

$$f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_M} = 0,90 \cdot \frac{28,0}{1,25} = 20,16 \text{ N/mm}^2$$

$$f_{t,90,d} = k_{mod} \cdot \frac{f_{t,90,k}}{\gamma_M} = 0,90 \cdot \frac{0,45}{1,25} = 0,32 \text{ N/mm}^2$$

Design values – Internal forces:

$$V_d = \frac{(g_d + s_d) \cdot l}{2} = \frac{9,19 \cdot 20,00}{2} = 91,90 \text{ kN}$$

$$M_{max,d} = M_{ap,d} = \frac{(g_d + s_d) \cdot l^2}{8} = \frac{9,19 \cdot 20,00^2}{8} = 459,50 \text{ kNm}$$

Verification

Bending stresses

$$\sigma_{m,max,d} = \sigma_{m,ap,d} = k_\ell \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2}$$

With: $k_\ell = k_1 + k_2 \cdot \left(\frac{h_{ap}}{r} \right) + k_3 \cdot \left(\frac{h_{ap}}{r} \right)^2 + k_4 \cdot \left(\frac{h_{ap}}{r} \right)^3$ and $\alpha_{ap} = 0°$

$k_1 = 1 + 1,4 \cdot \tan \alpha_{ap} + 5,4 \cdot \tan^2 \alpha_{ap} = 1 + 1,4 \cdot \tan 0 + 5,4 \cdot \tan^2 0 = 1,00$

$k_2 = 0,35 - 8 \cdot \tan \alpha_{ap} = 0,35 - 8 \cdot \tan 0 = 0,35$

$k_3 = 0,6 + 8,3 \cdot \tan \alpha_{ap} - 7,8 \cdot \tan^2 \alpha_{ap} = 0,6 + 8,3 \cdot \tan 0 - 7,8 \cdot \tan^2 0 = 0,60$

$k_4 = 6 \cdot \tan^2 \alpha_{ap} = 6 \cdot \tan^2 0 = 0$

$r = r_{in} + 0,5 \cdot h_{ap} = 15\,000 + 0,5 \cdot 1\,600 = 15\,800$ mm

$$k_\ell = 1,00 + 0,35 \cdot \left(\frac{1\,600}{15\,800} \right) + 0,60 \cdot \left(\frac{1\,600}{15\,800} \right)^2 + 0 \cdot \left(\frac{1\,600}{15\,800} \right)^3 = 1,04$$

$$\sigma_{m,ap,d} = k_l \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 1,04 \cdot \frac{6 \cdot 459,50 \cdot 10^6}{180 \cdot 1\,600^2} = 6,22 \text{ N/mm}^2$$

With $\frac{r_{in}}{t} = \frac{15\,000}{40} = 375 > 240$: $k_r = 1,00$ for curved beams

$$\frac{\sigma_{m,ap,d}}{k_r \cdot f_{m,d}} = \frac{6,22}{1,00 \cdot 20,16} = 0,31 < 1,0$$

Tension stresses perpendicular to grain

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2}$$

With: $k_p = k_5 + k_6 \cdot \left(\frac{h_{ap}}{r} \right) + k_7 \cdot \left(\frac{h_{ap}}{r} \right)^2$ and $\alpha_{ap} = 0°$:

$k_5 = 0,2 \cdot \tan \alpha_{ap} = 0,2 \cdot \tan 0 = 0$

$k_6 = 0,25 - 1,5 \cdot \tan \alpha_{ap} + 2,6 \cdot \tan^2 \alpha_{ap} = 0,25 - 1,50 \cdot \tan 0 + 2,6 \cdot \tan^2 0 = 0,25$

$k_7 = 2,1 \cdot \tan \alpha_{ap} - 4 \cdot \tan^2 \alpha_{ap} = 2,1 \cdot \tan 0 - 4 \cdot \tan^2 0 = 0$

$$k_p = 0 + 0,25 \cdot \left(\frac{1600}{15800} \right) + 0 \cdot \left(\frac{1600}{15800} \right)^2 = 0,0253$$

$$\sigma_{t,90,ap,d} = k_p \cdot \frac{6 \cdot M_{ap,d}}{b \cdot h_{ap}^2} = 0,0253 \cdot \frac{6 \cdot 459,50 \cdot 10^6}{180 \cdot 1\,600^2} = 0,15 \text{ N/mm}^2$$

With the reference volume $V_0 = 0,01 \text{ m}^3$ and

$$V = \frac{\beta \cdot \pi}{180} \cdot b \cdot \left(h_{ap}^2 + 2 \cdot r_{in} \cdot h_{ap} \right) = \frac{15 \cdot \pi}{180} \cdot 0,18 \cdot \left(1,60^2 + 2 \cdot 15,00 \cdot 1,60 \right) = 2,38 \text{ m}^3$$

the volume factor is:

$$k_{vol} = \left(\frac{V_0}{V} \right)^{0,2} = \left(\frac{0,01}{2,38} \right)^{0,2} = 0,335 \text{ and}$$
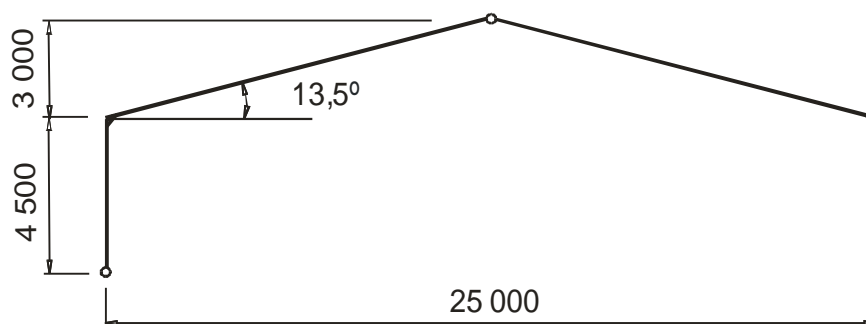
$k_{dis} = 1,4$ for curved beams

$$\frac{\sigma_{t,90,ap,d}}{k_{dis} \cdot k_{vol} \cdot f_{t,90,d}} = \frac{0,15}{1,4 \cdot 0,335 \cdot 0,32} = 1,00 = 1,0$$

12.8 Moment resisting joint

Design and assessment of moment resisting joint in the corner of the three-hinged plane frame. Material: glued laminated timber (GL 24h), service class 1.

Geometrical characteristics of the frame:

Span:	L = 25 m
Depth of the rafter:	$h_R = 1\,480$ mm
Width of the rafter:	$b_R = 200$ mm
Depth of the column:	$h_C = 1\,480$ mm

Width of the column: $b_C = 2 \times 120$ mm

Angle of the rafter: $\alpha = 13,5^0$

Material properties (characteristic values):

$f_{m,g,k} = 24$ MPa

$f_{v,g,k} = 2,7$ MPa

$\rho_k = 380$ kg/m^3

Design bending strength

$$f_{m,g,d} = k_{mod} \frac{f_{m,g,k}}{\gamma_M} = 0,9 \frac{24}{1,25} = 17,28 \text{ MPa}$$

Design shear strength

$$f_{v,g,d} = k_{mod} \frac{f_{v,g,k}}{\gamma_M} = 0,9 \frac{2,7}{1,25} = 1,94 \text{ MPa}$$

Dowels:

Steel grade S235 ∅ 24 mm (4.6): $f_{u,k} = 400$ MPa

Internal forces at the corner:

Column: $M_d = 676,8 \cdot 10^6$ Nmm, $V_{d,C} = 150,4 \cdot 10^3$ N, $N_{d,C} = 178,1 \cdot 10^3$ N

Rafter: $M_d = 676,8 \cdot 10^6$ Nmm, $V_{d,R} = 138,1 \cdot 10^3$ N, $N_{d,R} = 187,8 \cdot 10^3$ N
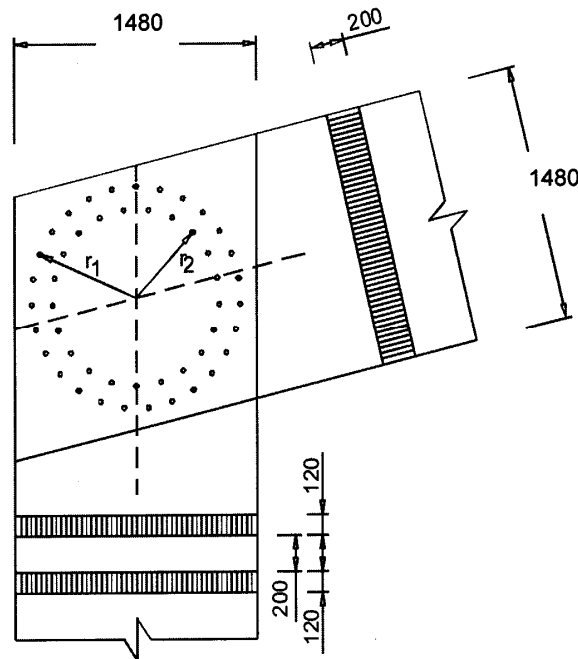
Design of dowel joints:

Outer circle: $r_1 \leq 0,5\,h - 4\,d = 0,5 \cdot 1480 - 4 \cdot 24 = 644$ mm $\Rightarrow r_1 = 644$ mm

Inside circle: $r_2 \leq r_1 - 5\,d = 644 - 5 \cdot 24 = 524$ mm $\Rightarrow r_2 = 524$ mm

Number of dowels in circles:

$$n_1 \leq \frac{2\,\pi\,r_1}{6\,d} = \frac{2 \cdot \pi \cdot 644}{6 \cdot 24} = 28,1 \text{ ks} \qquad \Rightarrow n_1 = 28$$

$$n_2 \leq \frac{2\,\pi\,r_2}{6\,d} = \frac{2 \cdot \pi \cdot 524}{6 \cdot 24} = 22,8 \text{ ks} \qquad \Rightarrow n_2 = 22$$

Load of dowels:

Load of dowel in column and rafter of the frame due to bending moment:

$$F_{\mathrm{M}} = M_{\mathrm{d}} \frac{r_1}{n_1\, r_1^2 + n_2\, r_2^2} = 676,8 \cdot 10^6 \frac{644}{28 \cdot 644^2 + 22 \cdot 524^2} = 24,69 \cdot 10^3 \text{ N}$$

Load of dowel in column of the frame due to shear and normal force:

$$F_{\mathrm{V,C}} = \frac{V_{\mathrm{d,C}}}{n_1 + n_2} = \frac{150,4 \cdot 10^3}{28 + 22} = 3,00 \cdot 10^3 \text{ N}$$

$$F_{\mathrm{N,C}} = \frac{N_{\mathrm{d,C}}}{n_1 + n_2} = \frac{178,1 \cdot 10^3}{28 + 22} = 3,56 \cdot 10^3 \text{ N}$$

Load of dowel in rafter of the frame due to shear and normal force:

$$F_{\mathrm{V,R}} = \frac{V_{\mathrm{d,R}}}{n_1 + n_2} = \frac{138,1 \cdot 10^3}{28 + 22} = 2,76 \cdot 10^3 \text{ N}$$

$$F_{\mathrm{N,R}} = \frac{N_{\mathrm{d,R}}}{n_1 + n_2} = \frac{187,8 \cdot 10^3}{28 + 22} = 3,76 \cdot 10^3 \text{ N}$$

Total load of dowel in the axis of the rafter and column of the frame:

$$F_{\mathrm{d,C}} = \sqrt{\left(F_{\mathrm{M}} + F_{\mathrm{V,C}}\right)^2 + F_{\mathrm{N,C}}^2} = \sqrt{\left(24,69 \cdot 10^3 + 3,00 \cdot 10^3\right)^2 + \left(3,56 \cdot 10^3\right)^2} = 27,92 \cdot 10^3 \text{ N}$$

$$F_{d,R} = \sqrt{\left(F_M + F_{V,R}\right)^2 + F_{N,R}^2} = \sqrt{\left(24,69 \cdot 10^3 + 2,76 \cdot 10^3\right)^2 + \left(3,76 \cdot 10^3\right)^2} = 27,71 \cdot 10^3 \text{ N}$$

Shear force in column and rafter in joint:

$$V_M = \left(\frac{M_d}{\pi} \frac{n_1\, r_1 + n_2\, r_2}{n_1\, r_1^2 + n_2\, r_2^2}\right) = \left(\frac{676,8 \cdot 10^6}{\pi} \frac{28 \cdot 644 + 22 \cdot 524}{28 \cdot 644^2 + 22 \cdot 524^2}\right) = 360,74 \cdot 10^3 \text{ N}$$

$$F_{V,d,C} = V_M - \frac{V_{d,C}}{2} = 360,74 \cdot 10^3 - \frac{150,4 \cdot 10^3}{2} = 285,5 \cdot 10^3 \text{ N}$$

$$F_{V,d,R} = V_M - \frac{V_{d,R}}{2} = 360,74 \cdot 10^3 - \frac{138,1 \cdot 10^3}{2} = 291,7 \cdot 10^3 \text{ N}$$

The mechanical properties of dowels:

Embedding strength in fibres direction (characteristic value):

$$f_{h,0,k} = 0,082\left(1 - 0,01\, d\right) \rho_k = 0,082 \cdot \left(1 - 0,01 \cdot 24\right) \cdot 380 = 23,68 \text{ MPa}$$

a) Carrying capacity of dowel in column axis:

Angle between load and timber fibres:

$$\alpha_1 = \arctan\left(\frac{F_M + F_{V,C}}{F_{N,C}}\right) = \arctan\left(\frac{24,69 \cdot 10^3 + 3,0 \cdot 10^3}{3,56 \cdot 10^3}\right) = 82,7°$$

$$\alpha_2 = \alpha - \left(\frac{\pi}{2} - \alpha_1\right) = 13,5 - \left(90 - 82,7\right) = 6,2°$$

Embedding strength (characteristic value):

$$k_{90} = 1,35 + 0,015\, d = 1,35 + 0,015 \cdot 24 = 1,71$$

$$f_{h,1,k} = \frac{f_{h,0,k}}{k_{90} \cdot \sin^2 \alpha_1 + \cos^2 \alpha_1} = \frac{23,68}{1,71 \cdot \sin^2 82,7 + \cos^2 82,7} = 13,94 \text{ MPa}$$

$$f_{h,2,k} = \frac{f_{h,0,k}}{k_{90} \cdot \sin^2 \alpha_2 + \cos^2 \alpha_2} = \frac{23,68}{1,71 \cdot \sin^2 6,2 + \cos^2 6,2} = 23,49 \text{ MPa}$$

$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}} = \frac{23,49}{13,94} = 1,685$$

Yield moment (characteristic value):

$$M_{y,Rk} = 0,3 \, f_{u,k} \, d^{2,6} = 0,3 \cdot 400 \cdot 24^{2,6} = 465,3 \cdot 10^3 \text{ Nmm}$$

$$t_1 = 120 \text{ mm} , \; t_2 = 200 \text{ mm}$$

$$F_{v,Rk,C} = \min \begin{cases} f_{h,1,k} \cdot t_1 \cdot d = 13,94 \cdot 120 \cdot 24 = 40,1 \cdot 10^3 \text{ N} \\[2mm] 0,5 \cdot f_{h,2,k} \cdot t_2 \cdot d = 0,5 \cdot 23,49 \cdot 200 \cdot 24 = 56,4 \cdot 10^3 \text{ N} \\[2mm] 1,05 \dfrac{f_{h,1,k} \cdot t_1 \cdot d}{2+\beta} \left[\sqrt{2\,\beta\,(1+\beta) + \dfrac{4\,\beta\,(2+\beta)\,M_{y,Rk}}{f_{h,1,k} \cdot d \cdot t_1^2}} - \beta \right] + \left[\dfrac{F_{ax,Rk}}{4} \right]^* = \\[2mm] = 1,05 \dfrac{13,94 \cdot 120 \cdot 24}{2+1,685} \cdot \\[2mm] \quad \cdot \left[\sqrt{2 \cdot 1,685 \cdot (1+1,685) + \dfrac{4 \cdot 1,685 \cdot (2+1,685) \cdot 465,3 \cdot 10^3}{13,94 \cdot 120^2 \cdot 24}} - 1,685 \right] = 19,4 \cdot 10^3 \text{ N} \\[2mm] 1,15 \sqrt{\dfrac{2\,\beta}{1+\beta}} \sqrt{2\,M_{y,Rk}\,f_{h,1,k}\,d} + \left[\dfrac{F_{ax,Rk}}{4} \right]^* = \\[2mm] = 1,15 \sqrt{\dfrac{2 \cdot 1,685}{1+1,685}} \sqrt{2 \cdot 465,3 \cdot 10^3 \cdot 13,94 \cdot 24} = 22,7 \cdot 10^3 \text{ N} \\[2mm] {}^* F_{ax,Rk} = 0 \end{cases}$$

$$F_{v,Rd,C} = \frac{k_{mod} \cdot F_{v,Rk}}{\gamma_M} = \frac{0.9 \cdot 19.4 \cdot 10^3}{1.25} = 13.97 \cdot 10^3 \; N$$

b) Carrying capacity of dowel in rafter axis:

Angle between load and timber fibres:

$$\alpha_2 = arctg \left(\frac{F_M + F_{V,R}}{F_{N,R}} \right) = arctg \left(\frac{24.69 \cdot 10^3 + 2.76 \cdot 10^3}{3.76 \cdot 10^3} \right) = 82.2°$$

$$\alpha_1 = \frac{\pi}{2} + \alpha - \alpha_1 = 90 + 13.5 - 82.2 = 21.3°$$

Embedding strength (characteristic value):

$$f_{h,1,k} = \frac{f_{h,0,k}}{k_{90} \cdot \sin^2 \alpha_1 + \cos^2 \alpha_1} = \frac{23.68}{1.71 \cdot \sin^2 21.3 + \cos^2 21.3} = 21.65 \, MPa$$

$$f_{h,2,k} = \frac{f_{h,0,k}}{k_{90} \cdot \sin^2 \alpha_2 + \cos^2 \alpha_2} = \frac{23.68}{1.71 \cdot \sin^2 82.2 + \cos^2 82.2} = 13.95 \, MPa$$

$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}} = \frac{13.95}{21.65} = 0.644$$

$t_1 = 120mm$, $t_2 = 200mm$

$$F_{v,Rk,R} = \min \begin{cases} f_{h,1,k} \cdot t_1 \cdot d = 21.65 \cdot 120 \cdot 24 = 62.4 \cdot 10^3 \, N \\[2mm] 0,5 \cdot f_{h,2,k} \cdot t_2 \cdot d = 0.5 \cdot 13.95 \cdot 200 \cdot 24 = 33.5 \cdot 10^3 \, N \\[2mm] 1.05 \dfrac{f_{h,1,k} \cdot t_1 \cdot d}{2+\beta} \left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,Rk}}{f_{h,1,k} \cdot d \cdot t_1^2}} - \beta \right] + \left[\dfrac{F_{ax,Rk}}{4} \right]^* = \\[2mm] = 1.05 \dfrac{21.65 \cdot 120 \cdot 24}{2+0.644} \cdot \\[2mm] \quad \cdot \left[\sqrt{2 \cdot 0.644 \cdot (1+0.644) + \dfrac{4 \cdot 0.644 \cdot (2+0.644) \cdot 465.3 \cdot 10^3}{21.65 \cdot 120^2 \cdot 24}} - 0.644 \right] = 23.5 \cdot 10^3 \, N \\[2mm] 1.15 \sqrt{\dfrac{2\beta}{1+\beta}} \sqrt{2M_{y,Rk} f_{h,1,k} d} + \left[\dfrac{F_{ax,Rk}}{4} \right]^* = \\[2mm] = 1.15 \sqrt{\dfrac{2 \cdot 0.644}{1+0.644}} \sqrt{2 \cdot 465.3 \cdot 10^3 \cdot 21.65 \cdot 24} = 22.4 \cdot 10^3 \, N \\[2mm] {}^* F_{ax,Rk} = 0 \end{cases}$$

$$F_{v,Rd,R} = \frac{k_{mod} \cdot F_{v,Rk}}{\gamma_M} = \frac{0,9 \cdot 22,4 \cdot 10^3}{1,25} = 16,13 \cdot 10^3 \, N$$

Verification of failure conditions:

a) Carrying capacity of the joint of frame column and rafter assessment:

- Column:

$$F_{d,C} = 27,92 \cdot 10^3 \, N \leq 2 \cdot F_{v,Rd,C} = 2 \cdot 13,97 \cdot 10^3 = 27,94 \cdot 10^3 \, N \qquad \Rightarrow \text{allowed}$$

- Rafter:

$$F_{d,R} = 27{,}71 \cdot 10^3 \text{ N} \leq 2 \cdot F_{v,Rd,R} = 2 \cdot 16{,}13 \cdot 10^3 = 32{,}26 \cdot 10^3 \text{ N} \qquad \Rightarrow \text{allowed}$$
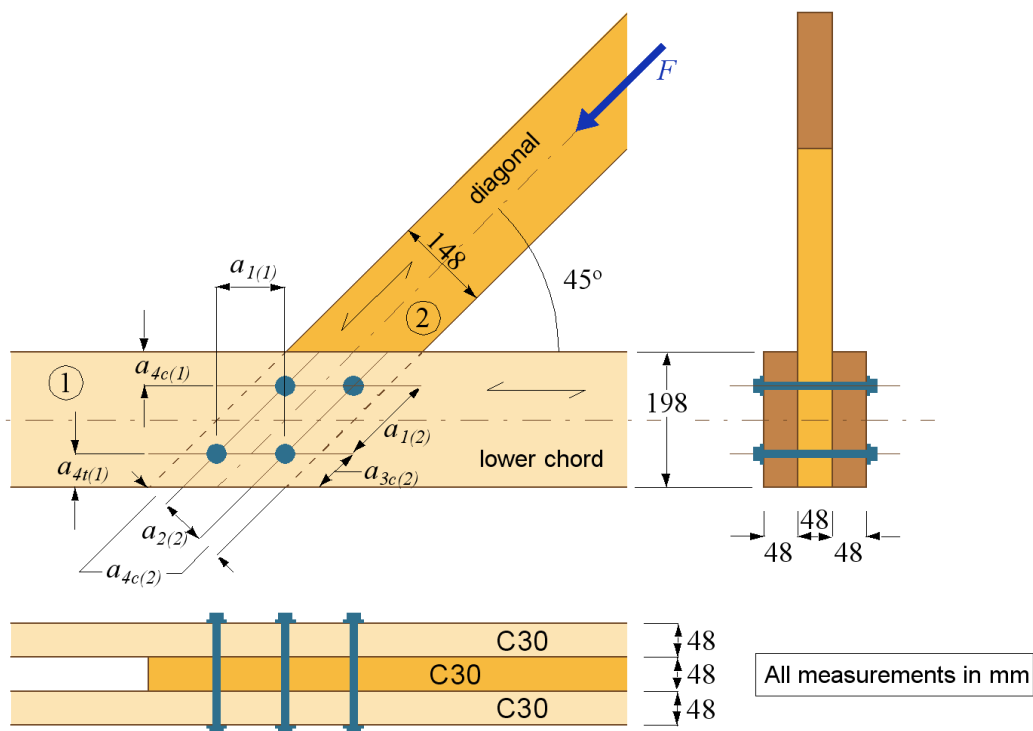
b) Shear stress in frame column and rafter assessment:

- Column:

$$\tau_{v,C} = \frac{3 \cdot F_{V,d,C}}{2 \cdot b \cdot h} = \frac{3 \cdot 285{,}5 \cdot 10^3}{2 \cdot 2 \cdot 120 \cdot 1\,480} = 1{,}21 \text{ MPa} \leq f_{v,g,d} = 1{,}94 \; MPa \quad \Rightarrow \text{allowed}$$

- Rafter:

$$\tau_{v,R} = \frac{3 \cdot F_{V,d,R}}{2 \cdot b \cdot h} = \frac{3 \cdot 291{,}7 \cdot 10^3}{2 \cdot 200 \cdot 1\,480} = 1{,}48 \text{ MPa} \leq f_{v,g,d} = 1{,}94 \text{ MPa} \qquad \Rightarrow \text{allowed}$$

12.9 Joint transmitting inclined forces

Determine the largest *design* force *F* that can be transmitted by means of *bolts* with a characteristic tensile strength of $f_{u,k} = 800$ MPa.

Other problem characteristics are:

- Timber quality: C30 (all members): $\rho_k = 380$ kg/m^3

- Loading is *short term*, and service class is 2

Minimum spacing as well as edge and end distances suggest 4 bolts, and with respect to the diagonal, in which the force is parallel to grain, we need a total width of at least $3d + 4d + 3d = 10d$, where d is the bolt diameter. Hence $\underline{d = 14}$ mm is the largest bolt diameter possible.

For fasteners in double shear in timber-to-timber connections the characteristic load-carrying capacity per shear plane is determined by the failure modes **g**, **h**, **j** and **k** of Eq. (8.7).

The yield moment for one bolt is: $M_{y,Rk} = 0,3\, f_{u,k} d^{2,6} = 0,3 \cdot 800 \cdot 14^{2,6} = 229\,160$ Nmm

We first consider the force F which is parallel to grain in the diagonal, but forms an angle of 45 degrees with the grain of the chord.

With $k_{90} = 1,35 + 0,015d = 1,35 + 0,21 = 1,56$ we find the following characteristic embedment strengths (se Eqs (8.32) and (8.31)):

$$f_{h,2,k} = 0,082\,(1 - 0,01\,d)\,\rho_k = 0,082 \cdot 0,86 \cdot 380 = 26,8 \text{ MPa (diagonal)}$$

$$f_{h,1,k} = \frac{f_{h,2,k}}{k_{90} \sin^2 \alpha + \cos^2 \alpha} = \frac{26,8}{1,56 \cdot 0,5 + 0,5} = 20,9 \text{ MPa (chord)} \quad \rightarrow \quad \beta = \frac{f_{h,2,k}}{f_{h,1,k}} = 1,28$$

Disregarding the *rope* effect, the formulas of (8.7) give the following characteristic capacities per bolt and shear plane:

g: 14 070 N   **h: 9 005 N**   **j:** 9 530 N   **k:** 14 125 N

The capacity is governed by failure mode h, and since this mode is independent of the axial withdrawal capacity, the rope effect does not come into play.

In order to determine the *effective* number of bolts we need to know the distance a_1 (see figure). With reference to the figure we choose the following distances:

$a_{4t(1)} = 55$ mm $> (2 + 2\sin 45)d = 48$ mm $\quad \rightarrow \quad a_{3c(2)} = 78$ mm $> 4d = 56$ mm

$a_{4c(1)} = 50$ mm $> 3d = 42$ mm $\quad \rightarrow \quad a_{1(2)} = 131$ mm $> 5d = 70$ mm

$a_{2(2)} = 60$ mm $> 4d = 56$ mm $\quad \rightarrow \quad a_{4c(2)} = 44$ mm $> 3d = 42$ mm

Hence, with $n = 2$: $\quad n_{ef(2)} = \min\left\{ n^{0,9} \sqrt[4]{\dfrac{a_{1(2)}}{13d}}\, ,\, n \right\} = \min\{1,72\,,\,2\} = 1,72$, and the characteristic

capacity of the entire connection is: $F_{k(2)} = (1,72 \cdot 2) \cdot 9\,005 \cdot 2 = 61\,955 = \underline{62,0 \text{ kN}}$

According to 8.1.2 (5) we also need to check the load-carrying capacity of the horizontal component of the force F. This problem is defined by a force 0,71 F in the chord (parallel to grain) being transmitted to the diagonal:

With $n = 2$ and $a_{1(1)} = \sqrt{2a_{2(2)}^2} = 85$ mm, we find: $n_{ef(1)} = 1,54$

We also need to compute new capacities per bolt and shear plane, since the force is now parallel to the chord grain, but acts at an angle of 45 degrees in the diagonal.

Hence: $f_{h,1,k} = 26,8$ MPa and $f_{h,2,k} = 20,9$ MPa $\rightarrow$ $\beta = \dfrac{f_{h,2,k}}{f_{h,1,k}} = 0,78$

Again, disregarding the *rope* effect, the formulas of (8.7) now give the following characteristic capacities per bolt and shear plane:

g: 18 010 N $\quad$ **h: 7 035 N** $\quad$ **j:** 10 023 N $\quad$ **k:** 14 125 N

Again, failure mode h governs, and we now find the capacity of the entire connection to be:

$F_{k(1)} = (1,54 \cdot 2) \cdot 7035 \cdot 2 / 0,71 = 61035 = \underline{61,0 \text{ kN}}$

Although there is little in it, it is the horizontal component of F that governs the capacity.

With $k_{mod} = 0,9$ and $\gamma_M = 1,3$ we find the design capacity of the connection to be

$$F_d = F_k \frac{k_{mod}}{\gamma_M} = 61,0 \cdot 0,9 / 1,3 = \underline{\textbf{42,2 kN}}$$

The characteristic *splitting* capacity of the connection is, according to (8.4),

$$F_{90,Rk} = 14\, b\, w \sqrt{\frac{h_e}{\left(1 - \dfrac{h_e}{h}\right)}} = 14 \cdot (2 \cdot 48) \cdot 1 \sqrt{\frac{198 - 55}{\left(1 - \dfrac{198 - 55}{198}\right)}} = 30\,495 = 30,5 \text{ kN}$$

If we assume that the vertical component of F, that is $0,71 \cdot 61,0 = 43,3$ kN, is divided into two *equal* shear forces on each side of the connection, splitting is no problem, but we do not have sufficient information about the problem to make this claim.

13 Annex: Amendment A1

This amendment A1 modify the Eurocode 5.

6.1.5 Compression perpendicular to the grain

The following expression shall be satisfied:

$$\sigma_{c,90,d} \leq k_{c,90} f_{c,90,d} \qquad (6.3)$$

with:

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}}$$

where:

$\sigma_{c,90,d}$ is the design compressive stress in the effective contact area perpendicular to the grain;

$F_{c,90,d}$ is the design compressive load perpendicular to the grain;

A_{ef} is the effective contact area in compression perpendicular to the grain;

$f_{c,90,d}$ is the design compressive strength perpendicular to the grain;

$k_{c,90}$ is a factor taking into account the load configuration, the possibility of splitting and the degree of compressive deformation.

The effective contact area perpendicular to the grain, A_{ef}, should be determined taking into account an effective contact length parallel to the grain, where the actual contact length, ℓ, at each side is increased by 30 mm, but not more than a, ℓ or $\ell_1/2$, see Figure 6.4.

The value of $k_{c,90}$ should be taken as 1,0 unless the conditions in the following paragraphs apply. In these cases the higher value of $k_{c,90}$ specified may be taken, with a limiting value of $k_{c,90} = 1,75$.

For members on continuous supports, provided that $\ell_1 \geq 2h$, see Figure 6.4a, the value of $k_{c,90}$ should be taken as:

− $k_{c,90} = 1,25$ for solid softwood timber

− $k_{c,90} = 1,5$ for glued laminated softwood timber

where h is the depth of the member and ℓ is the contact length.

For members on discrete supports, provided that $\ell_1 \geq 2h$, see Figure 6.4b, the value of $k_{c,90}$ should be taken as:

− $k_{c,90} = 1,5$ for solid softwood timber

− $k_{c,90} = 1,75$ for glued laminated softwood timber provided that $\ell \leq 400$ mm

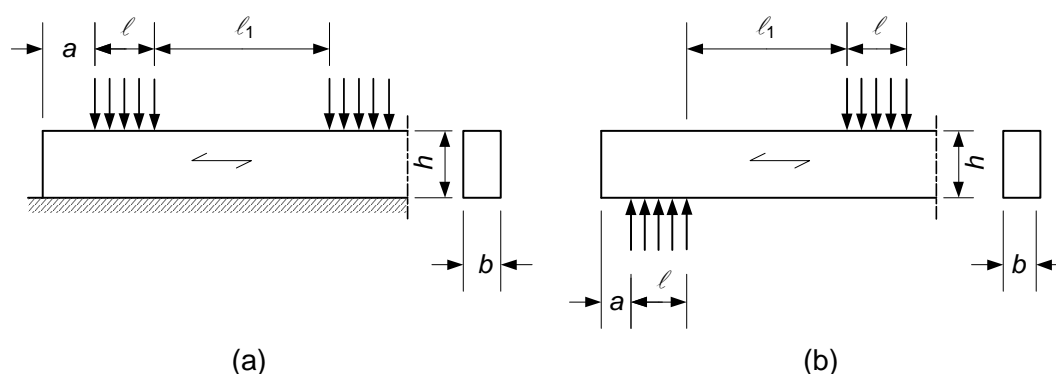where h is the depth of the member and ℓ is the contact length.

Figure 6.4 Member on (a) continuous and (b) discrete supports

NOTE: Figures 6.5 and 6.6 and Expressions (6.4) to (6.10) are void.

6.1.7 Shear

For shear with a stress component parallel to the grain, see Figure 6.7(a), as well as for shear with both stress components perpendicular to the grain, see Figure 6.7(b), the following expression shall be satisfied:

$$\tau_d \leq f_{v,d} \tag{6.13}$$

where:

τ_d is the design shear stress;

$f_{v,d}$ is the design shear strength for the actual condition.

NOTE: The shear strength for rolling shear is approximately equal to twice the tensile strength perpendicular to grain.

For the verification of shear resistance of members in bending, the influence of cracks should be taken into account using an effective width of the member given as:
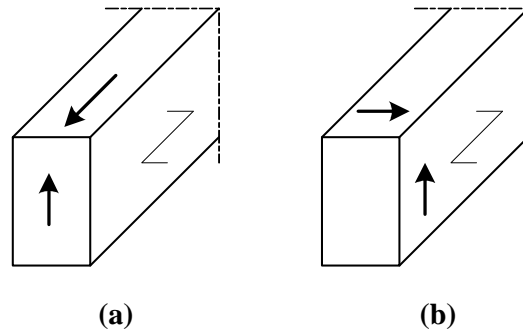
$$b_{ef} = k_{cr}\, b \tag{6.13a}$$

where b is the width of the relevant section of the member.

NOTE: The recommended value for k_{cr} is given as

$k_{cr} = 0,67$ for solid timber

$k_{cr} = 0,67$ for glued laminated timber

$k_{cr} = 1,0$ for other wood-based products in accordance with EN 13986 and EN 14374.

(a)   (b)

Figure 6.7 (a) Member with a shear stress component parallel to the grain
(b) Member with both stress components perpendicular to the grain (rolling shear)

At supports, the contribution to the total shear force of a concentrated load F acting on the top side of the beam and within a distance h or h_{ef} from the edge of the support may be disregarded (see Figure 6.8). For beams with a notch at the support this reduction in the shear force applies only when the notch is on the opposite side to the support.
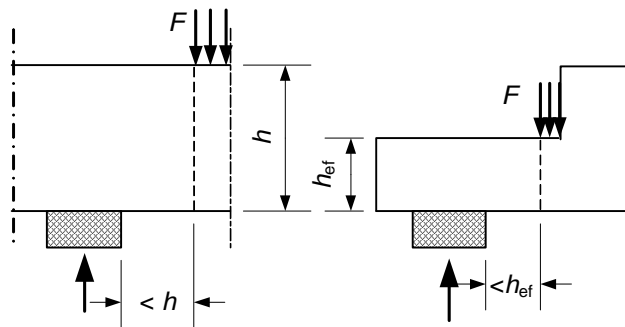

Figure 6.8 Conditions at a support, for which the concentrated force F
may be disregarded in the calculation of the shear force

8.3.1   Laterally loaded nails

8.3.1.1 Asumptions

Timber should be pre-drilled when:
− the characteristic density of the timber is greater than 500 kg/m³;

− the diameter d of the nail exceeds 6 mm.

8.3.2 Axially loaded nails

Nails used to resist permanent or long-term axial loading shall be threaded.

NOTE: The following definition of threaded nails is given in EN 14592: Nail that has its shank profiled or deformed over a part of its length of minimum 4,5 d (4,5 times the nominal diameter) and that has a characteristic withdrawal parameter $f_{ax,k}$ greater than or equal to 6 N/mm2 when measured on timber with a characteristic density of 350 kg/m3 when conditioned to constant mass at 20 ºC and 65 % relative humidity, conditioned to constant mass at 20°C and 65 % relative humidity.
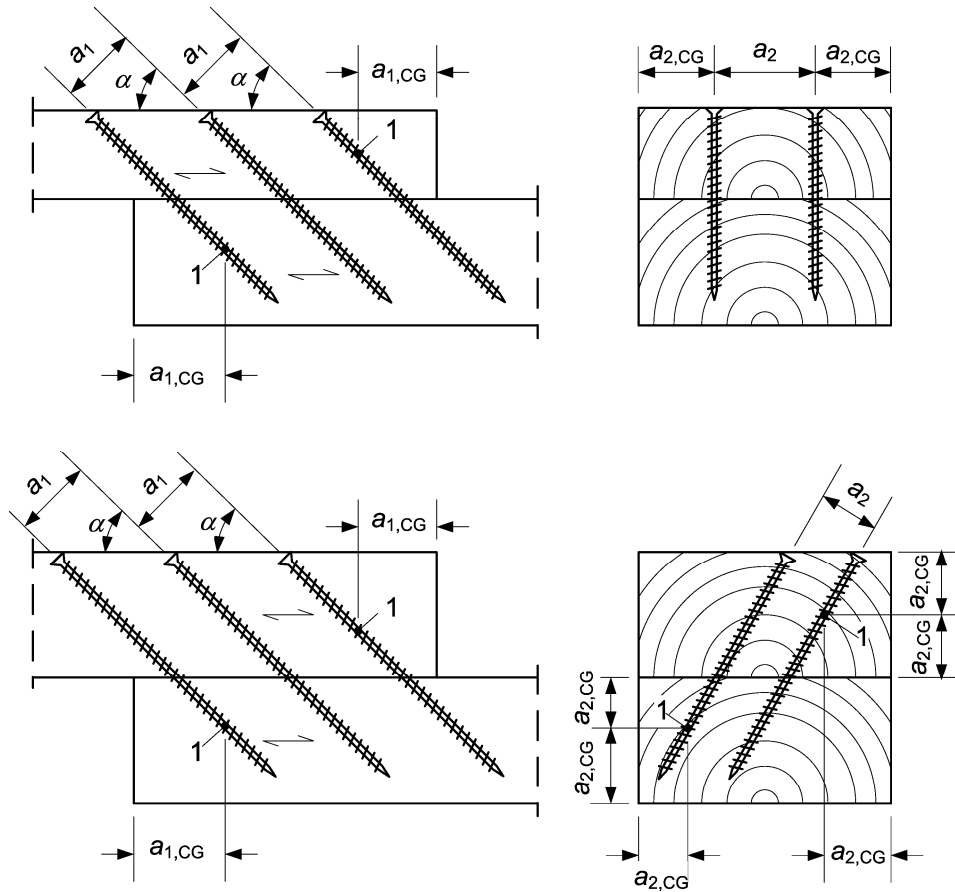
8.7.2 Axially loaded screws

For the verification of resistance of axially loaded screws, the following failure modes shall be taken into account:

- the withdrawal failure of the threaded part of the screw;

- the tear-off failure of the screw head of screws used in combination with steel plates, the tear-off resistance of the screw head should be greater than the tensile strength of the screw;

- the pull-through failure of the screw head;

- the tensile failure of the screw;

- the buckling failure of the screw when loaded in compression;

- failure along the circumference of a group of screws used in conjunction with steel plates (block shear or plug shear);

Minimum spacings and end and edge distances for axially loaded screws, see figure 8.11a, should be taken from Table 8.6, provided the timber thickness $t \geq 12d$.

Table 8.6 – Minimum spacings and end and edge distances for axially loaded screws

Minimum screw spacing in a plane parallel to the grain	Minimum screw spacing perpendicular to a plane parallel to the grain	Minimum end distance of the centre of gravity of the threaded part of the screw in the member	Minimum edge distance of the centre of gravity of the threaded part of the screw in the member
a_1	a_2	$a_{1,CG}$	$a_{2,CG}$
7d	5d	10d	4d

Key: 1   Centre of gravity of the threaded part of the screw in the member

Figure 8.11.a Spacings and end and edge distances

The minimum point side penetration length of the threaded part should be 6d.

For connections with screws in accordance with EN 14592 with

- 6 mm $\leq d \leq$ 12 mm

- 0,6 $\leq d_1/d \leq$ 0,75

where

d is the outer thread diameter;

d_1 is the inner thread diameter

the characteristic withdrawal capacity should be taken as:

$$F_{\text{ax,k,Rk}} = \frac{n_{\text{ef}} \, f_{\text{ax,k}} \, d \, \ell_{\text{ef}} \, k_{\text{d}}}{1,2\cos^2\alpha + \sin^2\alpha} \tag{8.38}$$

where:

$$f_{\text{ax,k}} = 0,52 \, d^{-0,5} \, \ell_{\text{ef}}^{-0,1} \, \rho_{\text{k}}^{0,8} \tag{8.39}$$

$$k_{\text{d}} = \min \begin{cases} \dfrac{d}{8} \\ 1 \end{cases} \tag{8.40}$$

$F_{ax,\alpha,Rk}$ is the characteristic withdrawal capacity of the connection at an angle α to the grain, in N;

$f_{ax,k}$ is the characteristic withdrawal strength perpendicular to the grain, in N/mm^2;

n_{ef} is the effective number of screws, see 8.7.2(8);

ℓ_{ef} is the penetration length of the threaded part, in mm;

ρ_k is the characteristic density, in kg/m^3;

α is the angle between the screw axis and the grain direction, with $\alpha \geq 30°$.

NOTE: Failure modes in the steel or in the timber around the screw are brittle, i.e. with small ultimate deformation and therefore have a limited possibility for stress redistribution.

Where the requirements with respect to the outer and inner thread diameter given in (4) are not satisfied, the characteristic withdrawal capacity, $F_{ax,\alpha,Rk}$, should be taken as:

$$F_{ax,\alpha,Rk} = \frac{n_{ef}\, f_{ax,k}\, d\, \ell_{ef}}{1,2\cos^2\alpha + \sin^2\alpha}\left(\frac{\rho_k}{\rho_a}\right)^{0,8} \qquad (8,40a)$$

where

$f_{ax,k}$ is the characteristic withdrawal parameter perpendicular to the grain determined in accordance with EN 14592 for the associated density ρ_a;

ρ_a is the associated density for $f_{ax,k}$, in kg/m^3

and the other symbols are explained in (4).

The characteristic pull-through resistance of connections with axially loaded screws should be taken as:

$$F_{ax,\alpha,Rk} = n_{ef}\, f_{head,k}\, d_h^2\, \left(\frac{\rho_k}{\rho_a}\right)^{0,8} \qquad (8.40b)$$

where:

$F_{ax,\alpha,Rk}$ is the characteristic pull-through capacity of the connection at an angle α to the grain in N, with $\alpha \geq 30°$;

$f_{head,k}$ is the characteristic pull-through parameter of the screw determined in accordance with EN 14592 for the associated density ρ_a;

d_h is the diameter of the screw head in mm

and the other symbols are explained in (4).

The characteristic tensile resistance of the connection (head tear-off or tensile capacity of shank), $F_{t,Rk}$, should be taken as:

$$F_{t,Rk} = n_{ef}\, f_{tens,k} \qquad (8.40c)$$

where

$f_{tens,k}$ is the characteristic tensile capacity of the screw determined in accordance with EN 14592;

n_{ef} is the effective number of screws, see 8.7.2(8).

For a connection with a group of screws loaded by a force component parallel to the shank, the effective number of screws is given by:

$$n_{ef} = n^{0,9} \tag{8.41}$$

where:

n_{ef} is the effective number of screws;

n is the number of screws acting together in a connection.

For pre-drilling screws in softwoods with a smooth shank diameter $d \leq 6$ mm, pre-drilling is not required. For all screws in hardwoods and for screws in softwoods with a diameter $d > 6$ mm, pre-drilling is required, with the following requirements:

− The lead hole for the shank should have the same diameter as the shank and the same depth as the length of the shank
− The lead hole for the threaded portion should have a diameter of approximately 70 % of the shank diameter.

For timber densities greater than 500 kg/m^3, the pre-drilling diameter should be determined by tests.

Where pre-drilling is applied to selfdrilling screws, the lead hole diameter shall not be greater than the inner thread diameter d_1.

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