

Cost models for lot streaming in a multistage flow shop

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Received 7 January 2002; accepted 27 May 2004

Available online 27 August 2004

Abstract

Lot streaming is a technique used to split a processing batch into several transfer batches. In this way, overlapping operations can be performed in different manufacturing stages, and production can be accelerated. This paper proposes two cost models for solving lot streaming problems in a multistage flow shop. The purpose is to determine the optimal processing batch size and the optimal number of transfer batches that minimize the total annual cost in each model. In the first model, a more complete and accurate method is developed to compute the costs of raw materials, work-in-process, and finished-product inventories. The total cost includes the setup cost, the transfer batch movement cost, the three-type inventory holding cost, and the finished-product shipment cost. The second model contains not only the four costs in the first model, but also the imputed cost associated with the makespan time. The total annual cost functions in both models are shown to be convex, and two solution approaches are suggested. An experiment consisting of three phases was conducted to explore the effect on the optimal solution when changing the value of one parameter at a time. The results indicate that three parameters have significant effects on the optimal solution. © 2004 Elsevier Ltd. All rights reserved.

Keywords: Lot streaming; Processing batch; Transfer batch; Multistage flow shop; Cost model

1. Introduction

In recent years, the philosophy of time-based competition has been widely used in a variety of businesses. Reduction in manufacturing lead time is a very important means of gaining competitive advantages when this philosophy is adopted by an enterprise. Lot streaming is a procedure in which a processing batch (i.e., a lot) is split into several smaller transfer batches (i.e., sublots). Each subplot is processed serially by a given number of workstations (i.e., stages), and precedence relationships are determined by a manufacturing or assembly structure. In this way, several operations in different stages can be performed simultaneously, thereby accelerating production. Lot streaming was first introduced by Reiter [1], and it has gradually received more attention in academic and industrial fields. This procedure for splitting a processing batch and overlapping operations in different stages is one of the effective methods used to shorten manufacturing lead times and reduce inventories. For this reason, several researchers (e.g., [2,3]) studied lot streaming and found that it is one of the major optimized production technology (OPT) techniques that can be used in general manufacturing systems.

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The literature on lot streaming can be divided into two parts. One part focuses on time models. The purpose of some time models is only to determine the optimal allocation of sublots for the single-product case. However, some time models aim to obtain both the optimal allocation of sublots and the optimal production sequence for the multiple-product case. To achieve aforementioned goals, the objective functions of these time models based on a time-related performance measurement criterion (e.g., the makespan time or the mean flow time) are minimized. In this area, Kropp and Smunt [2] developed linear and quadratic programming models that minimize the makespan time and the mean flow time, respectively. They investigated the optimal lot splitting policies in a multistage flow shop for the single-product problem. Baker and Pyke [4] proposed a computationally efficient algorithm for finding the optimal allocation of two sublots for the purpose of minimizing the makespan time. They also developed several heuristic approaches to handle more than two sublots in flow shops using the technique of network analysis. Trietsch and Baker [5] presented an overview of basic time models and their solution procedures. Chen and Steiner [6,7] explored the structural properties of schedules (i.e., the allocation of sublots) for the single-product case. Their models minimize the makespan time with detached and attached setup times in three-machine flow shops. Glass and Potts [8] developed a two-phase method to find the optimal allocation of sublots. In the first phase, a powerful relaxation algorithm that uses the machine dominance property is derived to reduce the number of machines so that only dominant machines should be considered. In the second phase, the critical path structure of an optimal solution that in fact is an optimal allocation of sublots is characterized by applying the network representation. Kalir and Sarin [9] used an optimization method to determine the optimal number of sublots in which the makespan time is affected by the subplot movement time. In addition, they developed an algorithm to determine the optimal number of sublots, where the setup time has an impact on the makespan time. Multiple-product lot streaming problems are more complex than classical scheduling problems which do not consider splitting or overlapping. The reason is that the former problems concurrently consider two decisions on the optimal production sequence and the optimal allocation of sublots, provided that the number of sublots for each product must be known. So far as we know, the previous studies on optimizing lot streaming models for multiple-product problems were based on no more than three machines in a flow shop. Hence, all of the previous studies simplified the problem to one that could be sequenced by Johnson's algorithm [10] (see, for example, [11–13]).

Another part of the literature deals with cost models that only consider the single-product case in multistage flow shops due to the fact that multiple-product problems are extremely intractable. The objective of solving a cost model is to determine the optimal processing lot size and/or the optimal number of sublots that minimize the total cost. In this part of the literature, Szendrovits [14] first introduced a cost model to solve the problem. Although the total cost in Szendrovits's model is a function of the processing lot size and the number of sublots, only the optimal processing lot size is obtained since the number of sublots is assumed to be constant. Goyal [15] extended Szendrovits's model to a more realistic model in two ways. First, he considered the effect of the number of sublots on the processing lot size by including the subplot movement cost in the total cost function. He also used a method to accurately measure work-in-process (WIP) inventories. Second, the solution procedure in Goyal's model is similar to that in Szendrovits's model. However, a major difference is that Goyal obtained a processing lot size for each predetermined number of sublots, and then chose one with the lowest total cost. Graves and Kostreva [16] applied the concept of Szendrovits's model to overlapping operations in a material requirements planning (MRP) framework. They studied only one manufacturing segment consisting of two workstations and constructed a more complicated cost model in which the setup cost, the subplot movement cost, and the holding cost of the input, WIP, and output inventories are included. Ranga et al. [17] proposed a cost model in which the setup time, the wait time, and the subplot movement time that constitute a part of the makespan time are also considered. Conversely, Szendrovits [14] used an arbitrary factor to multiply the technological lead time based only on the processing time. They used the same method used by Szendrovits to measure WIP inventories. Because Ranga et al.'s model is much complex and has no closed-form solution, the optimal solution is obtained for two cases: (1) completely balanced flow shops, where the unit processing times for all stages were identical; and (2) unbalanced flow shops where the unit processing times for all stages need not be the same, but the number of sublots is pre-specified.

The above literature review reveals that time models have attracted much more attention than cost models. For this reason, this paper presents two cost models for solving lot streaming problems in a multistage flow shop. The three main contributions in our paper are as follows: (1) A more complete and accurate method compared to those of Goyal [15] and Graves and Kostreva [16] is proposed to measure the costs of raw materials, WIP, and finished-product inventories. More specifically, Goyal excluded consideration of raw materials, and Graves and Kostreva only considered a two-stage manufacturing system rather than a general multistage flow shop. (2) Two cost functions have two decision variables (i.e., the processing lot size and the number of sublots), as opposed to one decision variable and one fixed-value variable used in the papers by Szendrovits [14], Goyal [15], and Ranga et al. [17]. (3) We recognize the importance of reducing the makespan time. Hence, this paper first incorporates the imputed cost associated with the makespan time into the second cost model. In particular, the imputed makespan cost makes the second cost model more general than the existing cost models.

2. Notations and assumptions

2.1. Notation

The notations used in this paper are defined as follows:

- m number of stages;
- n number of sublots (decision variable);
- j order of stage, $j = 1, \dots, m$, where stage m represents the last stage to complete production;
- D demand for the finished-product per year (units/year);
- Q processing lot size (units)(decision variable);
- t_j processing time per unit for stage j (unit time/unit);
- S_j setup cost per cycle for stage j (\$/cycle), where a cycle is the time required to produce a processing lot;
- G_1 subplot movement cost per movement (\$/movement);
- G_2 finished-product shipment cost per shipment (\$/shipment);
- C_0 value of raw materials per unit (\$/unit);
- C_j value of WIP inventories per unit for stage j , $j = 1, \dots, m - 1$ (\$/unit);
- C_m value of finished-product inventories per unit, where $C_0 < C_1 < \dots < C_{m-1} < C_m$ (\$/unit);
- h inventory holding cost rate per unit time for stage j (1/unit time);
- r cost per unit time (\$/unit time).

2.2. Assumptions

The following assumptions are made in this paper:

1. A flow shop contains m stages and produces only one product, and each stage has only one machine.
2. The annual demand for the finished-product, D , is deterministic and known over the infinite planning horizon.
3. All sublots are equal sizes in different stages. There are no production interruption times between any two adjacent sublots in the same stage.
4. The number of transporters used to move sublots, and the capacity of each transporter are unconstrained.
5. The buffer area between two stages is sufficient to store sublots of any size.
6. The processing time per unit for stage j , t_j , is known and fixed, and the subplot movement time is ignored.
7. For model simplicity, the setup time for each stage is neglected. However, the setup cost for stage j , S_j , is independent of the setup time.
8. The subplot movement cost, G_1 , and the finished-product shipment cost, G_2 , do not depend upon the subplot size.
9. No shortages are allowed.
10. Raw materials are procured from outside sources, their replenishment rates are infinite, and the value of raw materials per unit, C_0 , is known and constant (i.e., no quantity discounts). In stage 1, the point of time of raw material replenishment is the start time of each subplot's production. In addition, each replenishment quantity is equal to the subplot size.
11. When a subplot is finished in the last stage (i.e., stage m), it should be shipped to the customer immediately.

3. Computations of makespan and inventories

Based on Assumption (3), the makespan time for producing a processing lot is

$$M(Q, n) = \frac{Q}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1}) \delta_j \right], \quad (1)$$

where

$$\delta_j = \begin{cases} 1 & \text{if } t_j > t_{j-1} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad t_0 = 0, \quad j = 1, \dots, m.$$

Eq. (1) was called the no-idling makespan time by Baker and Jia [18]. Theoretically, increasing the number of sublots will reduce the makespan time. However, its marginal proportion of reduction in the makespan time will gradually decrease with the increase of the number of sublots. In other words, as Baker and Pyke [4] pointed out, the benefits of lot streaming show diminishing marginal returns as the number of sublots increases. The number of sublots cannot be infinite since time and cost are required when moving a subplot from one stage to the next stage. There exists an optimal number of sublots in the range of one unit (i.e., $n = 1$) and a processing lot (i.e., $n = Q$).

Fig. 1 depicts three types of inventories (as described in Section 1) and the makespan time for producing a processing lot in a m -stage flow shop. In Fig. 1, the value of raw materials in stage 1 can be computed by summing the dotted-line triangular areas. The result is

$$\frac{Q^2}{2n} t_1 C_0. \quad (2)$$

Similarly, the value of finished-product inventories in stage m is the sum of the bold-line triangular areas. It is given by

$$\frac{Q^2}{2n} t_m C_m. \quad (3)$$

Furthermore, the value of WIP inventories has two parts. In one part, each subplot waiting to be produced in each stage except for stage 1 can be represented by a shaded rectangular area. Hence, the waiting time for producing a processing lot (i.e., n sublots) for stage j is

$$\sum_{k=2}^n (n-k+1) |t_{j-1} - t_j| \frac{Q}{n} = \frac{Q(n-1)}{2} |t_{j-1} - t_j| \quad \text{for } j = 2, \dots, m. \quad (4)$$

Then, the value of WIP inventories for stage j can be obtained by multiplying $Q/n C_{j-1}$ by the resulting value in Eq. (4). That is,

$$\frac{Q^2}{2} \left(1 - \frac{1}{n}\right) C_{j-1} |t_{j-1} - t_j| \quad \text{for } j = 2, \dots, m. \quad (5)$$

Finally, the total waiting time for producing a processing lot for all stages (in fact, no waiting time exists in stage 1) is

$$\frac{Q(n-1)}{2} \sum_{j=2}^m |t_{j-1} - t_j|. \quad (6)$$

The total value of WIP inventories in this part becomes

$$\frac{Q^2}{2} \left(1 - \frac{1}{n}\right) \sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|. \quad (7)$$

In the other part, the WIP of each subplot in each stage except for stages 1 and m can be represented by dotted-line and bold-line triangular areas, while the WIP of each subplot in stage 1 can be represented by a bold-line triangular area, and that in stage m can be represented by a dotted-line triangular area. Therefore, the value of each subplot's WIP in each stage except for stages 1 and m is

$$\frac{Q^2}{2n} t_j (C_{j-1} + C_j) \quad \text{for } j = 2, \dots, m-1. \quad (8)$$

Hence, the total value of WIP inventories in the second part is

$$\frac{Q^2}{2n} \left[t_1 C_1 + \sum_{j=2}^{m-1} t_j (C_{j-1} + C_j) + t_m C_{m-1} \right]. \quad (9)$$

As a result, the total value of raw materials, WIP and finished-product inventories is the sum of the resulting values in Eqs. (2), (3), (7), and (9). That is,

$$\frac{Q^2}{2n} t_1 C_0 + \frac{Q^2}{2n} t_m C_m + \frac{Q^2}{2} \left(1 - \frac{1}{n}\right) \sum_{j=2}^m C_{j-1} |t_{j-1} - t_j| + \frac{Q^2}{2n} \left[t_1 C_1 + \sum_{j=2}^{m-1} t_j (C_{j-1} + C_j) + t_m C_{m-1} \right]. \quad (10)$$

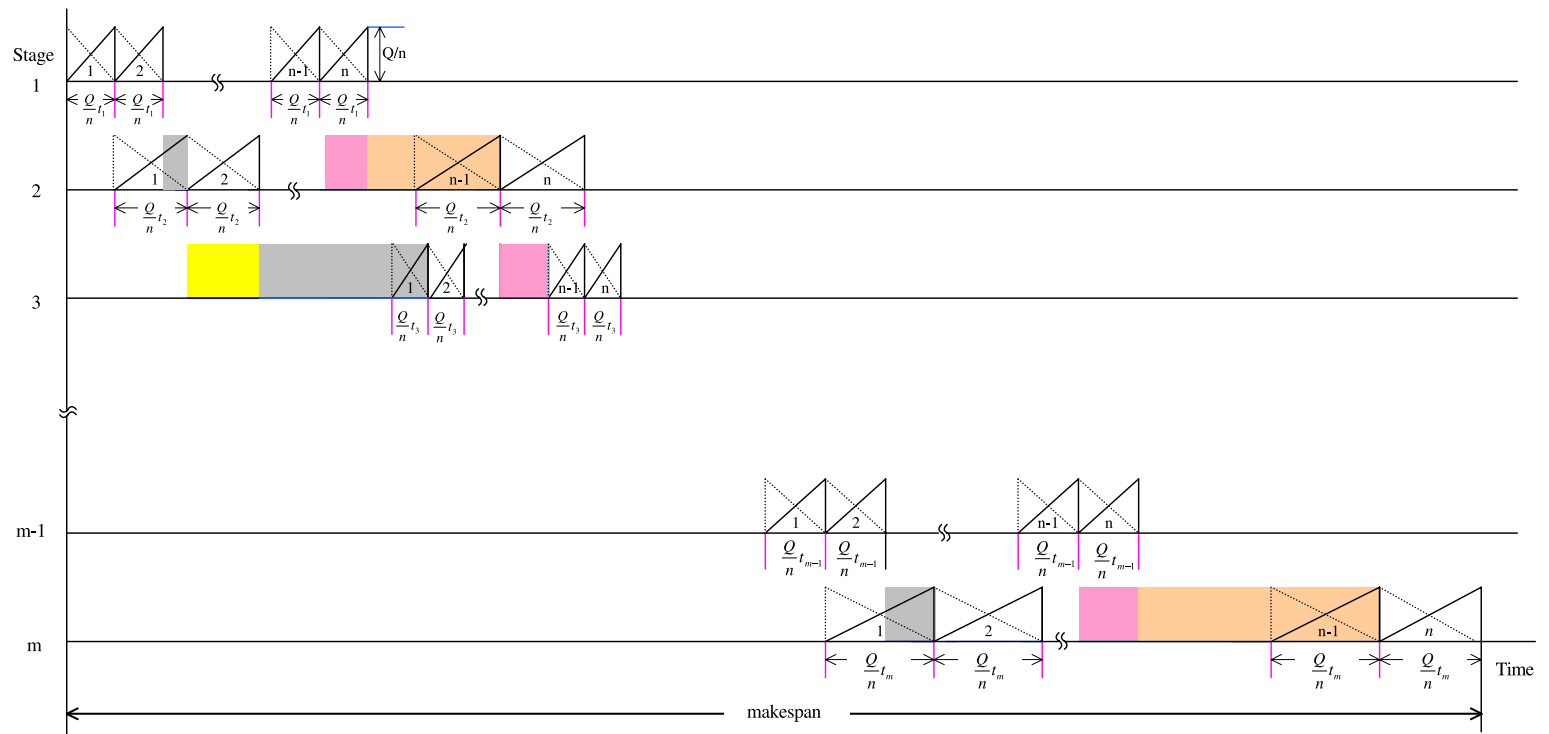


Fig. 1. Three types of inventories and the makespan time.

However, a more concise form obtained by combining the first, second, and last terms in Eq. (10) is given by

$$\frac{Q^2}{2} \left(1 - \frac{1}{n}\right) \sum_{j=2}^m C_{j-1} |t_{j-1} - t_j| + \frac{Q^2}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j) \right]. \quad (11)$$

4. Cost model formulation and optimization

4.1. The first cost model

The first cost model (Model I) consists of the setup cost, the subplot movement cost, the three-type inventory holding cost, and the finished-product shipment cost. Accordingly, the total cost per cycle can be written as

$$\begin{aligned} CTC_1(Q, n) = & \sum_{j=1}^m S_j + G_1 n(m-1) + \frac{Q^2}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j| \right) h \\ & + \frac{Q^2}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j) \right] h + G_2 n. \end{aligned} \quad (12)$$

Multiplying $CTC_1(Q, n)$ in Eq. (12) by D/Q , the total cost per year becomes

$$\begin{aligned} YTC_1(Q, n) = & \frac{D}{Q} \sum_{j=1}^m S_j + \frac{D}{Q} G_1 n(m-1) + \frac{QD}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j| \right) h \\ & + \frac{QD}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j) \right] h + \frac{D}{Q} G_2 n. \end{aligned} \quad (13)$$

Because $YTC_1(Q, n)$ in Eq. (13) is a convex function (the detailed proof is given in Appendix A), an optimal solution exists. In order to find it, we take the partial derivatives of $YTC_1(Q, n)$ in Eq. (13) with respect to Q and n , respectively, and set each of them to zero. Consequently, we have

$$Q^0 = \sqrt{\frac{2n^0(\sum_{j=1}^m S_j + G_1 n^0(m-1) + n^0 G_2)}{(n^0 - 1)(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|)h + [\sum_{j=1}^m t_j (C_{j-1} + C_j)]h}} \quad (14)$$

and

$$\begin{aligned} n^0 = & Q^0 \sqrt{\frac{[\sum_{j=1}^m t_j (C_{j-1} + C_j)]h - (\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|)h}{2(G_1(m-1) + G_2)}} \\ = & \sqrt{\frac{(\sum_{j=1}^m S_j)(\sum_{j=1}^m t_j (C_{j-1} + C_j)) - \sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|}{(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|)(G_1(m-1) + G_2)}}. \end{aligned} \quad (15)$$

It should be emphasized here that the value of n^0 in Eq. (15) becomes infinite when the unit processing times for all stages are identical (i.e., $t_1 = t_2 = \dots = t_m$ and the flow shop is completely balanced). Obviously, the value of Q^0 in Eq. (14) is also infinite.

In general, the value of n^0 obtained by using Eq. (15) is not guaranteed to be an integer. We can find that an optimal solution with an integer number of sublots, denoted by (Q^{0*}, n^{0*}) , is definitely close to (Q^0, n^0) since $YTC_1(Q, n)$ is convex. In this case, two approaches can be used to determine the optimal values of Q^{0*} and n^{0*} for an unbalanced flow shop. The first one simply uses a computer package, such as LINGO, under the condition that the value of n in Eq. (13) is set to be a positive integer. If such a computer package is not available, the second approach based on Eqs. (14) and (15) can be used to search for the optimal values of Q^{0*} and n^{0*} . Two steps of this search method are as follows:

Step 1: Input relevant parameters into Eq. (15), use it to obtain the value of n^0 , and find the values of $\lfloor n^0 \rfloor$ and $\lceil n^0 \rceil$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x , and $\lceil x \rceil$ denotes the smallest integer larger than or equal to x . Substitute the values of $\lfloor n^0 \rfloor$ and $\lceil n^0 \rceil$ into n^0 in Eq. (14), respectively, and obtain the values of $Q_{\lfloor n^0 \rfloor}^0$ and $Q_{\lceil n^0 \rceil}^0$.

Step 2: Obtain the values of $YTC_1(Q_{\lfloor n^0 \rfloor}, \lfloor n^0 \rfloor)$ and $YTC_1(Q_{\lceil n^0 \rceil}, \lceil n^0 \rceil)$, and select the one with the lowest total cost. The optimal solution is found.

4.2. The second cost model

The second cost model (Model II) contains the four costs included in Model I and the imputed cost associated with the makespan time. Consequently, the total cost per cycle can be written as

$$CTC_2(Q, n) = \sum_{j=1}^m S_j + G_1 n(m-1) + \frac{Q^2}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|\right) h + \frac{Q^2}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j)\right] h + G_2 n + \frac{Q}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1}) \delta_j\right] r. \quad (16)$$

Multiplying $CTC_2(Q, n)$ in Eq. (16) by D/Q , the total cost per year becomes

$$YTC_2(Q, n) = \frac{D}{Q} \sum_{j=1}^m S_j + \frac{D}{Q} G_1 n(m-1) + \frac{QD}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|\right) h + \frac{QD}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j)\right] h + \frac{D}{Q} G_2 n + \frac{D}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1}) \delta_j\right] r. \quad (17)$$

Since $YTC_2(Q, n)$ in Eq. (17) is also convex (the detailed proof is shown in Appendix B), an optimal solution can be determined by means of partial differentiation of Eq. (17) with respect to Q and n , respectively. By setting each of them to zero, we get

$$Q' = \sqrt{\frac{2n'(\sum_{j=1}^m S_j + G_1 n'(m-1) + n'G_2)}{(n' - 1)(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|)h + [\sum_{j=1}^m t_j (C_{j-1} + C_j)]h}} \quad (18)$$

and

$$n' = \sqrt{\frac{Q'^2 h [\sum_{j=1}^m t_j (C_{j-1} + C_j) - (\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|)] + 2Q' r (\sum_{j=1}^m t_j - \sum_{j=1}^m (t_j - t_{j-1}) \delta_j)}{2(G_1(m-1) + G_2)}}. \quad (19)$$

We can also use the two approaches mentioned above to obtain the optimal solution with an integer number of sublots, denoted by (Q^*, n^*) , for an unbalanced flow shop. The first approach uses LINGO. The second approach differs from that presented in Model I. The main reason is that n' is not independent of Q' (as shown in Eqs. (18) and (19)). An iterative procedure based on Eqs. (18) and (19) is developed to search for the optimal values of Q^* and n^* . Three steps of this procedure are as follows:

Step 1: Set $k = 1$, $Q'_k = L$, which is a reasonable positive integer, and the tolerance, ε , a minimal positive number. Substituting the initial guess value L into Q' in Eq. (19), we can obtain the initial value of n'_k .

Step 2: Set $k = k + 1$. Substituting the value of n'_{k-1} into n' in Eq. (18), we can obtain the value of Q'_k . Then, substituting the value of Q'_k into Q' in Eq. (19), we can obtain the value of n'_k . This step is repeated until $|n'_k - n'_{k-1}| < \varepsilon$.

Step 3: Find an integer n^* that closely approaches the value of n'_k . Substituting this integer into n' in Eq. (18), we can obtain the optimal value of Q^* .

Also note that the result obtained by implementing Steps 1 and 2 in the above iterative procedure will converge to (Q'_k, n'_k) , which is the same as solving directly using Eq. (17), in which Q and n are assumed to be real numbers, when the unit processing times for all stages are not equal. On the other hand, the values of Q'_k and n'_k will gradually become extremely large as the stopping condition $|n'_k - n'_{k-1}| < \varepsilon$ can never be satisfied, and a large number of iterations will be performed in Step 2 when the unit processing times for all stages are identical.

Table 1
Related time and cost data

Stage j	t_j (min)	S_j (\$)	C_j (\$)
1	2	150	0.6
2	4	40	0.7
3	3	100	0.8
4	5	90	0.9
5	2	100	1
Total	16	480	

Table 2
The results obtained by implementing Steps 1 and 2 in the illustration of Model II

L	k	1	2	3	4	5	6	7	8
Underestimated	Q'_k	50	260.4027	400.7651	448.1164	463.4031	468.1140	469.4932	469.9840
	n'_k	2.9510	7.3193	9.5334	10.2375	10.4616	10.5304	10.5505	10.5576
Overestimated	Q'_k	800	557.5799	495.6600	477.8704	472.5472	470.8477	470.3182	
	n'_k	15.1438	11.8137	10.9299	10.6724	10.5950	10.5702	10.5625	

5. Numerical illustration and experimental design and analysis

5.1. Numerical illustration

A numerical example is used to illustrate the solution procedures of Models I and II. Suppose that there are five stages in a flow shop. Relevant input parameters are: $D = 600,000$ units per year, $G_1 = \$5$ per movement, $G_2 = \$100$ per shipment, $C_0 = \$0.5$ per unit, $h = 0.002$ per year, and $r = \$2$ per minute. In addition, the unit processing time, the cost per setup, and the value per unit WIP of each stage are given in Table 1.

5.1.1. Illustration of Model I

From Eq. (13), the total annual cost is

$$YTC_1(Q, n) = \frac{28,800,000}{Q} + 1,200,000 \frac{n}{Q} + 372Q \left(1 - \frac{1}{n}\right) + 1,452 \frac{Q}{n} + 6,000,000 \frac{n}{Q}.$$

The search method mentioned previously is used:

Step 1: Use Eq. (15) to obtain n^0 (i.e., $n^0 = 3.41$). As a result, $\lfloor n^0 \rfloor = 3$ and $\lceil n^0 \rceil = 4$. Then, we can obtain $Q_3^0 = 262.398$ and $Q_4^0 = 299.532$ using Eq. (14).

Step 2: Compute $YTC_1(262.398, 3)$ and $YTC_1(299.532, 4)$. Finally, the optimal solution is $(262.398, 3)$ with a total annual cost of \$384,150 (which is identical to that obtained using LINGO to solve Eq. (13) directly).

5.1.2. Illustration of Model II

According to Eq. (17), the total annual cost is

$$YTC_2(Q, n) = \frac{28,800,000}{Q} + 1,200,000 \frac{n}{Q} + 372Q \left(1 - \frac{1}{n}\right) + 1,452 \frac{Q}{n} + 6,000,000 \frac{n}{Q} + \frac{1,920,000}{n} + 720,000 \left(1 - \frac{1}{n}\right).$$

The iterative procedure is used:

Step 1: Set $k = 1$, $Q'_1 = L = 50$, and $\varepsilon = 0.01$. The initial value of n'_1 is 2.9510 after solving Eq. (19).

Step 2: Set $k = 2$. Using Eq. (18), the value of Q'_2 is 260.4027. It is clear that $Q'_1 = L = 50$ is an underestimated value. Then, using Eq. (19), the value of n'_2 is 7.3193. The results of the other iterations in this step are given in Table 2. This step is terminated at $k = 8$ (i.e., $|n'_8 - n'_7| = 0.0071 < 0.01$). Hence, the value of n'_8 is 10.5576.

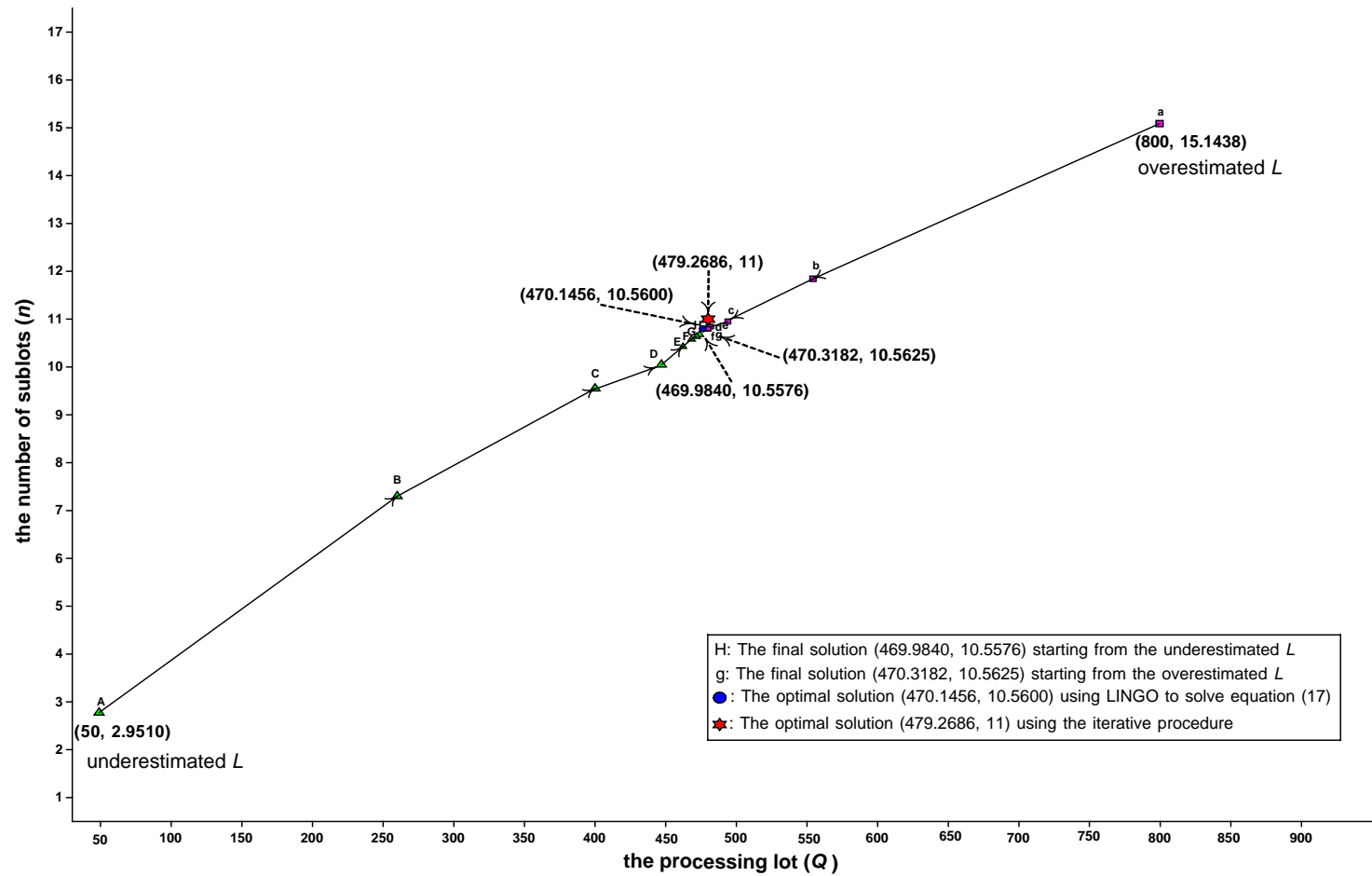


Fig. 2. A schematic representation of the illustration in Model II with the underestimated and overestimated values of L .

Table 3

The results of the first phase of the experiment

r	Model II			
	Q^{0*}	n'^*	YTC_2	$M(Q'^*, n'^*)$
0.0625	299.532	4	416474.5	2546.022
0.125	299.532	4	448349.5	2546.022
0.25	331.970	5	510396.7	2655.76
0.5	361.158	6	628717.9	2768.878
1	412.813	8	853591.9	2992.894
2	479.269	11	1279778.0	3311.313
4	555.075	15	2092906.0	3700.500
8	681.183	23	3659468.0	4383.265
16	834.878	35	6706959.0	5247.805
32	1032.137	54	12684750.0	6383.958
64	1283.092	84	24484760.0	7851.301

Table 4

The results of the second phase of the experiment

$(t_1, t_2, t_3, t_4, t_5)$	Model I				Model II			
	Q^{0*}	n^{0*}	YTC_1	$M(Q^{0*}, n^{0*})$	Q'^*	n'^*	YTC_2	$M(Q'^*, n'^*)$
(2, 3, 5, 7, 10)	278.343	5	465612.4	3729.796	507.665	15	1874938.0	5652.004
(2, 3, 4, 5, 6)	391.397	6	367913.0	3261.642	756.657	18	1232017.0	5128.453
(2, 2, 2, 2, 2)	∞	∞	—	∞	∞	∞	—	∞
(6, 5, 4, 3, 2)	409.197	6	351909.1	3409.975	750.789	17	1221600.0	5123.031
(10, 7, 5, 3, 2)	301.765	5	429472.6	4043.651	542.280	15	1840537.0	6037.384
(1, 3, 5, 3, 1)	282.843	3	356381.8	2168.463	456.070	9	1117130.0	2685.746
(5, 3, 1, 3, 5)	297.113	4	387731.9	3268.243	459.696	10	1614550.0	4505.021
(1, 2, 4, 3, 2)	359.573	4	320379.8	2157.438	596.754	11	929231.0	2821.019
(2, 5, 3, 6, 1)	235.339	3	428317.6	2588.729	346.410	8	1593831.0	3160.991
(1, 5, 1, 2, 7)	199.172	2	433796.3	2688.822	304.017	6	1893658.0	3597.535

Step 3: Let $n' = 11$. Use Eq. (18) to obtain the value of Q'^* (which is 479.2628). Finally, the optimal solution is (479.2628, 11) with a total annual cost of \$1,279,778.

To more explicitly demonstrate the computational process of this illustration presented in Table 2, Fig. 2 shows a schematic representation of cases with underestimated and overestimated values of L . In either the underestimated or overestimated case, the magnitude of improvement at the start of the iterative procedure is significant, but it gradually decreases with the increase of k . When the iterative procedure is terminated because the condition that $|n'_k - n'_{k-1}| < \varepsilon$ is satisfied, the final solution (i.e., $n'_8 = 10.5576$ in the underestimated case and $n'_7 = 10.5625$ in the overestimated case) is very close to the integer optimal solution (i.e., $n'^* = 11$).

5.2. Experimental design and analysis

The experiment was conducted in three phases. In the first phase of the experiment, the purpose was to explore the effect of changing the value of the cost per unit time, r , on the optimal solution in Model II. The levels of the cost per unit time were 0.0625, 0.125, 0.25, 0.5, 1, 2, 4, 8, 16, 32, and 64. The values of other parameters were identical to those given in the above numerical illustration. The results of the first phase of the experiment are listed in Table 3. In the second phase, the objective was to investigate whether the unit processing time for stage j , t_j , would have a significant impact on the optimal solutions in Models I and II. The values of t_1, t_2, \dots, t_5 were chosen from 1, 2, \dots , 10 according to several unit-processing-time configurations in a flow shop, including increasing, decreasing, completely balanced, and other configurations. Table 4 summarizes the results of the second phase. In the third phase, the aim was to deal with the impact of changing the values of G_1, G_2, h , and $\sum_{j=1}^m S_j$, on the optimal solutions in Models I and II. Note that only one of the above four values was changed at a time. Table 5 shows the results of the third phase.

Some important conclusions drawn from Tables 3, 4, and 5 are as follows:

1. From Tables 3, 4, and 5, the optimal value of n'^* obtained by solving Model II is larger than that of n^{0*} obtained by solving Model I. The reason is that the imputed makespan cost is not included in Model I.
2. From Table 3, the optimal values of Q'^* and n'^* in Model II increase as the value of r increases. Obviously, the optimal values of Q'^* and n'^* gradually become close to those of Q^{0*} and n^{0*} in Model I when the value of r decreases. In addition, according to Eqs. (1) and (17), although $YTC_2(Q^{0*}, n^{0*}) > YTC_2(Q'^*, n'^*)$, it can be found that $M(Q^{0*}, n^{0*}) < M(Q'^*, n'^*)$. The reason is that the purpose of this paper is to minimize the total annual cost rather than to minimize the total annual cost and the makespan time simultaneously. In other words, the minimum of the total annual cost does not guarantee the minimum of the makespan time.
3. From Table 4, the optimal values of (Q^{0*}, n^{0*}) and (Q'^*, n'^*) obtained by solving Models I and II become extremely large as the flow shop approaches complete balance, as expected from Eqs. (14), (15), (18), and (19). Conversely, those of (Q^{0*}, n^{0*}) and (Q'^*, n'^*) are small when the flow shop is unbalanced. In addition, the optimal values of Q'^* and n'^* in Model II are inevitably larger than those of Q^{0*} and n^{0*} in Model I.
4. It also can be seen from Table 5 that the optimal value of n^{0*} obtained by solving Model I is negatively correlated with the value of G_1 or G_2 . The same situation can be applied to the optimal value of n'^* in Model II. On the other hand, each optimal value of Q^{0*} and Q'^* exhibits positive correlation with the value of G_1 or G_2 under the condition that the optimal value of n^{0*} or n'^* is fixed. Furthermore, the optimal values of Q^{0*} , Q'^* , and n'^* , except for that of n^{0*} , are negatively correlated with the value of h . The above four optimal values, however, exhibit positive correlation with the value of $\sum_{j=1}^m S_j$.

6. Conclusions and directions for future research

This paper has developed two cost models to solve lot streaming problems in a multistage flow shop. The contribution of Model I is a more complete and accurate method for taking into account the costs of raw materials, WIP, and finished-product inventories. Model II further includes the imputed cost (as seen from the last term in Eq. (16)), which is transformed from the makespan time (as presented in Eq. (1)). This makes Model II more general than Model I and other existing cost models. It can be proven that the total annual cost functions of Models I and II (i.e., Eqs. (13) and (17)) are convex. In addition, we have suggested the use of LINGO or the proposed approach in each cost model, depending on the availability of the computer package, to find the optimal solutions of the lot streaming problems. The results obtained by conducting an experiment indicate that the inventory holding cost rate (h), the setup cost ($\sum_{j=1}^m S_j$), and the unit-processing-time configuration (t_1, t_2, \dots, t_m) each has a much greater impact on the optimal solution than does the subplot movement cost (G_1) or the finished-product shipment cost (G_2).

A more detailed factorial design can be used in the future to analyse the effect on the optimal solution in each cost model when concurrently changing the values of two or more related parameters. Other interesting research topics will be the use of the imputed mean-flow-time cost rather than the imputed makespan cost to formulate a cost model and simultaneously minimizing the time and cost objective functions in dual-objective lot streaming problems.

Acknowledgements

The authors wish to thank the anonymous referees for their helpful comments. Additionally, we dedicate this paper to Andrew Z. Szendrovits.

Appendix A. Proof of the convexity of $YTC_1(Q, n)$ in Model I

The total annual cost function is

$$YTC_1(Q, n) = \frac{D}{Q} \sum_{j=1}^m S_j + \frac{D}{Q} G_1 n(m-1) + \frac{QD}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1} |t_{j-1} - t_j|\right) h \\ + \frac{QD}{2n} \left[\sum_{j=1}^m t_j (C_{j-1} + C_j) \right] h + \frac{D}{Q} G_2 n.$$

Table 5
The results of the third phase of the experiment

Parameter				Model I				Model II			
G_1	G_2	h	$\sum_{j=1}^m S_j$	Q^{0*}	n^{0*}	YTC_1	$M(Q^{0*}, n^{0*})$	Q'^*	n'^*	YTC_2	$M(Q'^*, n'^*)$
0.3125	100	0.002	480	287.594	4	369270.6	2444.549	450.975	11	1253172.0	3115.827
0.625	100	0.002	480	288.405	4	370312.3	2451.443	452.917	11	1254997.0	3129.245
1.25	100	0.002	480	290.121	4	372386.9	2466.029	456.774	11	1258625.0	3155.893
2.5	100	0.002	480	293.226	4	376501.8	2492.421	464.393	11	1265790.0	3208.533
5	100	0.002	480	262.398	3	384150.0	2449.048	479.269	11	1279778.0	3311.313
10	100	0.002	480	271.607	3	397633.0	2534.999	484.768	10	1305377.0	3393.376
20	100	0.002	480	289.148	3	423312.7	2698.715	506.061	9	1351297.0	3598.656
40	100	0.002	480	256.495	2	467846.1	2821.445	550.417	8	1428123.0	3990.523
80	100	0.002	480	294.690	2	537514.3	3241.590	624.413	7	1548677.0	4638.497
160	100	0.002	480	224.529	1	652031.9	3592.464	731.288	6	1727342.0	5606.541
320	100	0.002	480	277.236	1	805091.8	4435.776	867.791	5	1980525.0	6942.328
640	100	0.002	480	360.211	1	1046053	5763.376	1019.437	4	2328957.0	8665.215
5	6.25	0.002	480	275.085	7	289546.7	2043.489	370.052	19	1100546.0	2415.076
5	12.5	0.002	480	284.006	7	298937.0	2109.759	384.599	18	1118961.0	2521.260
5	25	0.002	480	285.520	6	315214.2	2188.987	404.750	16	1150775.0	2681.469
5	50	0.002	480	266.511	4	342199.9	2265.344	428.095	13	1201940.0	2897.874
5	100	0.002	480	262.398	3	384150.0	2449.048	479.269	11	1279778.0	3311.313
5	200	0.002	480	305.634	3	447521.2	2852.584	514.868	8	1392076.0	3732.793
5	400	0.002	480	294.690	2	537514.3	3241.590	624.422	7	1548677.0	4638.563
5	600	0.002	480	336.390	2	613574.4	3700.290	675.664	6	1665933.0	5180.091
5	800	0.002	480	231.774	1	673070.6	3708.384	683.628	5	1763946.0	5469.024
5	1200	0.002	480	265.043	1	769685.7	4240.688	819.407	5	1923623.0	6555.256
5	1600	0.002	480	294.580	1	855457.8	4713.280	806.516	4	2055566.0	6855.386
5	2400	0.002	480	346.172	1	1005282	5538.752	974.440	4	2271181.0	8282.740

5	100	0.0005	480	524.795	3	192075.0	4898.087	1110.150	15	1046453.0	7401.000
5	100	0.001	480	371.086	3	271635.0	3463.469	733.438	13	1146078.0	4964.811
5	100	0.002	480	262.398	3	384150.0	2449.048	479.269	11	1279778.0	3311.313
5	100	0.004	480	185.534	3	543270.1	1731.651	308.419	9	1460301.0	2193.202
5	100	0.008	480	131.199	3	768299.9	1224.524	206.406	8	1707184.0	1496.444
5	100	0.016	480	92.772	3	1086540	856.872	137.154	7	2046339.0	1018.858
5	100	0.032	480	65.599	3	1536600	612.257	90.289	6	2514872.0	692.216
5	100	0.064	480	46.386	3	2173080	432.936	58.685	5	3168417.0	469.480
5	100	0.128	480	32.800	3	3073200	306.133	41.500	5	4083174.0	332.000
5	100	0.256	480	23.193	3	4346161	216.468	26.475	4	5371247.0	225.038
5	100	0.512	480	16.400	3	6146400	153.067	18.721	4	7173593.0	159.129
5	100	1.024	480	11.596	3	8692322	108.229	13.238	4	9722494.0	112.523
5	100	0.002	8	72.727	1	211200.0	1163.632	364.257	9	1211762.0	2590.272
5	100	0.002	15	74.689	1	216898.1	1195.024	365.427	9	1212913.0	2598.592
5	100	0.002	30	78.730	1	228630.7	1259.680	367.921	9	1215368.0	2616.327
5	100	0.002	60	86.244	1	250452.4	1379.904	372.860	9	1220227.0	2651.449
5	100	0.002	120	153.897	2	280707.7	1692.867	382.546	9	1229759.0	2720.327
5	100	0.002	240	177.705	2	324133.3	1954.755	424.264	10	1247294.0	2969.848
5	100	0.002	480	262.398	3	384150.0	2449.048	479.269	11	1279778.0	3311.313
5	100	0.002	960	398.978	5	469198.5	3191.824	558.291	12	1335860.0	3814.989
5	100	0.002	1920	560.944	7	590433.4	4167.013	693.481	14	1428658.0	4656.230
5	100	0.002	3840	793.725	10	761976.4	5556.075	886.763	16	1574464.0	5874.805
5	100	0.002	7680	1118.205	14	1004467	7507.948	1191.519	20	1795174.0	7744.874
5	100	0.002	15360	1571.000	19	1347422	10252.842	1637.452	26	2120453.0	10454.501

For convenience, let A_1 , A_2 , A_3 , and A_4 be $\sum_{j=1}^m S_j$, $G_1(m-1)$, $(\sum_{j=2}^m C_{j-1}|t_{j-1}-t_j|)h$, and $[\sum_{j=1}^m t_j(C_{j-1}+C_j)]h$, respectively. The cost function becomes

$$YTC_1(Q, n) = \frac{D}{Q}A_1 + \frac{nD}{Q}A_2 + \frac{QD}{2}\left(1 - \frac{1}{n}\right)A_3 + \frac{QD}{2n}A_4 + \frac{G_2nD}{Q}.$$

$$\frac{\partial^2 YTC_1}{\partial Q^2} = \frac{2D}{Q^3}A_1 + \frac{2nD}{Q^3}A_2 + \frac{2G_2nD}{Q^3} > 0.$$

Since $\sum_{j=2}^m C_{j-1}|t_{j-1}-t_j| < \sum_{j=2}^m C_{j-1}(t_{j-1}+t_j)$, it follows that $\sum_{j=1}^m t_j(C_{j-1}+C_j) - \sum_{j=2}^m C_{j-1}(t_{j-1}+t_j) = t_1C_0 + t_mC_m > 0$. As a result, $A_4 - A_3 > 0$. Thus,

$$\frac{\partial^2 YTC_1}{\partial n^2} = \frac{-QD}{n^3}A_3 + \frac{QD}{n^3}A_4 = \frac{QD}{n^3}(A_4 - A_3) > 0$$

and

$$\begin{aligned} & \left(\frac{\partial^2 YTC_1}{\partial Q^2}\right)\left(\frac{\partial^2 YTC_1}{\partial n^2}\right) - \left(\frac{\partial^2 YTC_1}{\partial Q \partial n}\right)\left(\frac{\partial^2 YTC_1}{\partial n \partial Q}\right) = \left(\frac{2D(A_1 + nA_2 + nG_2)}{Q^3}\right)\left(\frac{QD(A_4 - A_3)}{n^3}\right) \\ & - \left[\left(-\frac{D(A_2 + G_2)}{Q^2}\right) + \left(-\frac{D(A_4 - A_3)}{2n^2}\right)\right]^2 = \frac{2D^2A_1(A_4 - A_3)}{Q^2n^3} > 0. \end{aligned}$$

$YTC_1(Q, n)$ is a convex function and the proof is completed.

Appendix B. Proof of the convexity of $YTC_2(Q, n)$ in Model II

The total annual cost function is

$$\begin{aligned} YTC_2(Q, n) = & \frac{D}{Q} \sum_{j=1}^m S_j + \frac{D}{Q} G_1 n(m-1) + \frac{QD}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1}|t_{j-1}-t_j|\right) h \\ & + \frac{QD}{2n} \left[\sum_{j=1}^m t_j(C_{j-1}+C_j)\right] h + \frac{D}{Q} G_2 n + \frac{D}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1})\delta_j\right] r. \end{aligned}$$

Since the sum of two convex functions is still a convex function, we can divide $YTC_2(Q, n)$ into two parts. That is,

$$\begin{aligned} YTC_{21}(Q, n) = & \frac{D}{Q} \sum_{j=1}^m S_j + \frac{D}{Q} G_1 n(m-1) + \frac{QD}{2} \left(1 - \frac{1}{n}\right) \left(\sum_{j=2}^m C_{j-1}|t_{j-1}-t_j|\right) h \\ & + \frac{QD}{2n} \left[\sum_{j=1}^m t_j(C_{j-1}+C_j)\right] h \end{aligned}$$

and

$$YTC_{22}(Q, n) = \frac{D}{Q} G_2 n + \frac{D}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1})\delta_j\right] r.$$

(1) Proof of the convexity of $YTC_{21}(Q, n)$:

Let A_1 , A_2 , A_3 , and A_4 be $\sum_{j=1}^m S_j$, $G_1(m-1)$, $(\sum_{j=2}^m C_{j-1}|t_{j-1}-t_j|)h$, and $[\sum_{j=1}^m t_j(C_{j-1}+C_j)]h$, respectively. The cost function becomes

$$YTC_{21}(Q, n) = \frac{D}{Q}A_1 + \frac{nD}{Q}A_2 + \frac{QD}{2}\left(1 - \frac{1}{n}\right)A_3 + \frac{QD}{2n}A_4,$$

$$\frac{\partial^2 YTC_{21}}{\partial Q^2} = \frac{2D}{Q^3}A_1 + \frac{2nD}{Q^3}A_2 > 0.$$

Since $A_4 - A_3 > 0$, as proven in Appendix A, it follows that

$$\frac{\partial^2 YTC_{21}}{\partial n^2} = \frac{-QD}{n^3} A_3 + \frac{QD}{n^3} A_4 = \frac{QD}{n^3} (A_4 - A_3) > 0$$

and

$$\begin{aligned} & \left(\frac{\partial^2 YTC_{21}}{\partial Q^2} \right) \left(\frac{\partial^2 YTC_{21}}{\partial n^2} \right) - \left(\frac{\partial^2 YTC_{21}}{\partial Q \partial n} \right) \left(\frac{\partial^2 YTC_{21}}{\partial n \partial Q} \right) = \left(\frac{2DA_1 + 2nDA_2}{Q^3} \right) \left(\frac{QD(A_4 - A_3)}{n^3} \right) \\ & - \left[\left(-\frac{DA_2}{Q^2} \right) + \left(-\frac{D(A_4 - A_3)}{2n^2} \right) \right]^2 = \frac{2D^2 A_1 (A_4 - A_3)}{Q^2 n^3} > 0. \end{aligned}$$

Hence, $YTC_{21}(Q, n)$ is a convex function.

(2) Proof of the convexity of $YTC_{22}(Q, n)$:

$$\begin{aligned} YTC_{22}(Q, n) &= \frac{D}{Q} G_2 n + \frac{D}{n} \left[\sum_{j=1}^m t_j + (n-1) \sum_{j=1}^m (t_j - t_{j-1}) \delta_j \right] r \\ &= \frac{D}{Q} G_2 n + \frac{rD}{n} \sum_{j=1}^m t_j + rD \left(1 - \frac{1}{n} \right) \sum_{j=1}^m (t_j - t_{j-1}) \delta_j, \end{aligned}$$

$$\frac{\partial^2 YTC_{22}}{\partial Q^2} = \frac{2G_2 n D}{Q^3} > 0.$$

When $\delta_j = 1$, $\sum_{j=1}^m t_j - \sum_{j=1}^m (t_j - t_{j-1}) = \sum_{j=1}^m t_{j-1} > 0$. When $\delta_j = 0$, $\sum_{j=1}^m t_j > 0$. Consequently,

$$\frac{\partial^2 YTC_{22}}{\partial n^2} = \frac{2rD}{n^3} \left(\sum_{j=1}^m t_j - \sum_{j=1}^m (t_j - t_{j-1}) \delta_j \right) > 0$$

and

$$\begin{aligned} & \left(\frac{\partial^2 YTC_{22}}{\partial Q^2} \right) \left(\frac{\partial^2 YTC_{22}}{\partial n^2} \right) - \left(\frac{\partial^2 YTC_{22}}{\partial Q \partial n} \right) \left(\frac{\partial^2 YTC_{22}}{\partial n \partial Q} \right) \\ &= \left(\frac{2G_2 n D}{Q^3} \right) \left\{ \frac{2rD}{n^3} \left[\sum_{j=1}^m t_j - \sum_{j=1}^m (t_j - t_{j-1}) \delta_j \right] \right\} - \left(\frac{G_2 D}{Q^2} \right)^2 = \frac{3G_2^2 D^2}{Q^4} > 0. \end{aligned}$$

Hence, $YTC_{22}(Q, n)$ is a convex function.

From parts (1) and (2), $YTC_2(Q, n)$ is a convex function, and the proof is completed.

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