Adaptive ‘imperfect’ Decision Feedback Equalizer for a Frequency Selective Communication Channel

Ramkumar Balasubramanyam, Predrag B. Rapajic
School of Engineering, the University of Greenwich
Chatham Maritime, ME4 4TB, UK
{r.balasubramanyam,p.rapajic}@gre.ac.uk

Abstract— **In this paper, we show that, for an uncoded receiver system, the proposed least mean square (LMS) decision aided equalizer (DAE), with, the backward step-size constant greater than forward step-size constant, compared to classical equal-step size design, has a lower mean square error by upto 5 dB, for a frequency selective wireless communication channel. Classical LMS DAE with equal step-size constants, can be considered as perfect decision feedback system, compared to, the proposed unequal step-size, as a imperfect decision feedback system. We provide, Wiener DAE, considering imperfect decision feedback information, during training mode and provide analysis for LMS DAE with unequal step size constants.**

I. Introduction

Interference mitigation in a wireless communication receiver system, results in a signal to interference noise ratio (SINR) versus bit error rate (BER) performance improvement. Inter symbol interference (ISI) and deep fades in the channel, increases with higher frequency selectivity, causing a corresponding increase in interference noise, reducing the BER performance. In a receiver design, with unknown wireless channel state information (CSI), the effects of interference noise, degrades the SINR versus BER performance. A decision aided estimator (DAE), is employed to reduce the input SINR for a particular BER performance, by mitigating interference noise [1], [2]. A classical adaptive DAE receiver [3], ‘learns’ the CSI, using least mean square (LMS) algorithm, to update the forward and backward filter coefficients. The common feature of classical DAE receiver is, 1) to operate in moderate to high SINR and 2) to assume that, perfect decision feedback information is available at the feedback filter. In a classical LMS DAE, the equalizer structure, has always used equal step-size constants to update the filter coefficients [4], [3], which is, implicitly assumes ’perfect’ decision feedback information, to converge towards steady-state mean square error (MSE).

However, unequal step-size constant LMS DAE, provides a potential for ‘imperfect’ decision information to be included to converge to lower MSE steady-state values. The detector in a DAE shown in figure 1, makes perfect decisions which are fed into the feedback filter. But, the weighing of filter coefficients can be influenced by unequal step-size constants, which treats the decision feedback information as ‘imperfect’. In this paper, we show that unequal step-size constants for LMS DAE, have the ability to vary the extent of MSE convergence behavior towards lower steady-state values for frequency selective communication channels. The consequence of this result can be

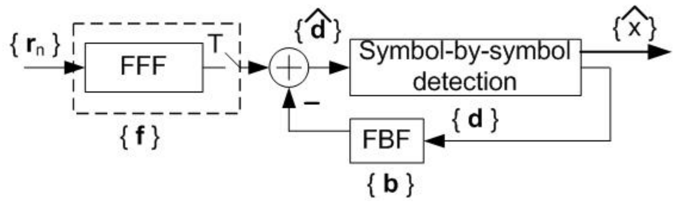

Fig. 1. A classical DAE, **r** is the output information vector of transmitted symbols **x** passed through a frequency selective ISI channel. Training mode, which, performs channel estimation, uses the structure of a DAE, shown in the figure.

applied to coded communication systems [5], [6], which is not discussed in this paper. Hence, the step-size constants, provide additional degree of freedom, per filter coefficient, for LMS DAE. However, the question is, what are the optimal settings of the forward and backward step-size constants (μ_f, μ_b respectively)? In order to understand the influence of the step-size constants, on the MSE performance, a Wiener DAE solution, for imperfect decision feedback information during training mode is derived. We can then infer, the effect of unequal step-size constants on MSE, using LMS DAE, over an uncoded communication system. The contributions of this paper are,

1) An LMS DAE with, the backward filter step-size (μ_b) greater than forward filter step-size (μ_f), provides a reduction in steady-state MSE by upto 5 dB for an uncoded system, in an highly frequency selective wireless communication channel (figure 3). The higher the frequency selectivity, the greater, is the reduction in MSE, such that, $\mu_b > \mu_f$.
2) We derive a Wiener solution, for the forward and backward filter coefficients of a DAE receiver structure, with imperfect channel estimate, during training mode (equation 20 and 22), which is used to explain the reduction of steady-state MSE for an LMS DAE, with $\mu_b > \mu_f$.
3) The BER versus unequal step-size constants performance for an uncoded LMS DAE follows the steady MSE behavior, for high input SINR. The figures 4., and 5., show BER optimization can be achieved by assuming unequal-step size.

The rest of the paper is organized as follows, in section II we discuss the system model for an uncoded wireless communication system. In section III, we discuss the effect of imperfect decision feedback information, in lowering the MSE, using classical DAE structure, during training mode. In addition, we discuss the lowering of steady-state MSE using classical LMS DAE, with unequal step-size constants. In section IV we describe the simulation results obtained in this paper. We summarize the results presented by the authors in section V.

II. Wireless Communication System Model

A communication receiver system is shown in figure 1., where, it is assumed that, there is no channel coding applied, to the binary phase shift keyed (BPSK) digital information. The channel model used in a wireless communication system, presents an observation equation, with the complex baseband signal,

$$\mathbf{r}^l(i) = \mathbf{H}^l\mathbf{x}(i) + \mathbf{n}^l(i) \tag{1}$$

where, $\mathbf{r}(i)^l$=$[r(i)_1, ..., r(i)_N]^T$, i∈{0,L-1}, is the received signal transpose vector, of length L, sampled at chip rate, where N is the filter length, $\mathbf{H}$ is the hermitian matrix of the wireless communication channel impulse response, $\mathbf{x}(i)$is the transmitted symbol vector and $\mathbf{n}(i)$=$[n(i)_1, ..., n(i)_N]^T$, is the additive white gaussian noise (AWGN), with covariance $E\{\mathbf{n}(n)\mathbf{n}^H(n)\} = \sigma^2\mathbf{I}$. The channel response matrix is $\mathbf{H}^l$=$[\mathbf{h}^l, ..., \mathbf{p}^l]$, where, l is the packet index of L_p packets, l∈{0,L_p-1}. The binary information $\mathbf{x}$, is transmitted over a radio-channel of bandwidth B, at a rate R=1/T symbols per second. The M-tap wireless communication channel coefficients $\mathbf{h}_k$=$[h_1, ..., h_M]^T$, with, unequally spaced delays (of interval τ) convolves ($\star$) with the transmitted signatures, to form signatures $\mathbf{p}_k^l$=$\mathbf{g}_k \star \mathbf{h}_k$. As the wireless channel is unknown, the channel response matrix at the receiver is unknown. Since, the channel response matrix $\mathbf{H}^l$, is estimated once for a packet, we drop l in the per-packet analysis, provided in this paper.

III. Classical Decision Aided Estimation

A. Problem statement

In considering the estimation of channel, using training mode, the figure 1., shows two receiver system components for channel estimation. A feed-forward filter (FFF), is fed with received vector $\mathbf{r}$, while, a feedback filter is fed with decision information from a binary detector. The estimation error is given by,

$$\epsilon(i) = \hat{d}(i) - x(i) \tag{2}$$

where, $\epsilon(i)$, is the error in estimating the training sequence correctly, x(i) is the i th training symbol. The principle of MSE, averages over the entire packet, so dropping the index i in the future reference,

$$\begin{aligned} \varepsilon &= E\{\mid \epsilon \mid^2\} \\ &= E\{\mid \hat{\mathbf{d}} - x \mid^2\} \end{aligned} \tag{3}$$

where, $\hat{\mathbf{d}}$ is the estimated symbol at the DAE, given by,

$$\hat{\mathbf{d}} = \mathbf{f}^H\mathbf{r} - \mathbf{b}^H\mathbf{d} \tag{4}$$

Substituting (4) into (3)

$$\begin{aligned} \varepsilon &= E\{\mid \epsilon(i) \mid^2\} \\ &= E\{\mid \hat{\mathbf{d}} - x \mid^2\} \\ &= E\{\mid \mathbf{f}^H\mathbf{r} - \mathbf{b}^H\mathbf{d} - x \mid^2\} \end{aligned} \tag{5}$$

Solving (3), we can arrive at MMSE solution, which, can be implemented adaptively, using the LMS DAE given by,

$$\begin{aligned} \mathbf{f}(i+1) &= \mathbf{f}(i) + \mu_f e(i)\mathbf{r}(i) \\ \mathbf{b}(i+1) &= \mathbf{b}(i) + \mu_b e(i)\mathbf{d}(i) \end{aligned} \tag{6}$$

where, μ_f and μ_b are step-size parameters for updating the feed-forward ($\mathbf{f}$) and feedback filter ($\mathbf{b}$) taps respectively. The classical solution, for LMS DAE however, uses the same step-size constant, $\mu_b = \mu_f$ [3].
However, from the update equation (6), it is not clear as to whether the choice of the step-size ratio r, defined as $r = \frac{\mu_b}{\mu_f}$, has any effect on the final MSE, which leads to the question: what are the optimal settings of μ_f, μ_b? In order to understand the influence of the step-size constants on the MSE performance, we will first need to access the Wiener solution for a DAE under the conditions that, feedback information through the backward filter, has imperfect decision information. Since, the choice of forward, backward filter coefficients, is critically dependent on its adjustment, during training sequence. We have to focus our analysis on filter coefficient design, during training mode, as part of channel estimation.

B. Wiener Decision Aided Estimator

During training mode, the channel response matrix $\mathbf{H}$ and vector $\mathbf{x}$ are not distinguishable, under the assumption that, the wireless communication channel is unknown at the receiver. Therefore, we have chosen a novel approach to finding the expression for Wiener coefficients during training mode. Normalizing and expanding (3), we have,

$$\begin{aligned} \varepsilon = & (\mathbf{f}^H E\{\mathbf{r}\mathbf{r}^H\}\mathbf{f} - \mathbf{f}^H E\{\mathbf{r}\mathbf{d}^H\}\mathbf{b} - \mathbf{b}^H E\{\mathbf{d}\mathbf{r}^H\}\mathbf{f} \\ & -\mathbf{f}^H E\{\mathbf{r}x\} - E\{x\mathbf{r}^H\}\mathbf{f} + \mathbf{b}^H E\{\mathbf{d}\mathbf{d}^H\}\mathbf{b} \\ & +\mathbf{b}^H E\{\mathbf{d}x\} + E\{x\mathbf{d}^H\}\mathbf{b} + 1 \end{aligned} \tag{7}$$

Assuming the following expectations, and co-variances,

$$\mathbf{R} = E(\mathbf{r}.\mathbf{r}^H) \tag{8}$$

$$\mathbf{R}_{rd} = E(\mathbf{r}.\mathbf{d}^H) \tag{9}$$

$$\mathbf{p} = E(\mathbf{r}.x) \tag{10}$$

$$\mathbf{R}_{dd} = E(\mathbf{d}.\mathbf{d}^H) \tag{11}$$

$$\mathbf{R}_{dx} = E(\mathbf{d}.x) \tag{12}$$

Taking partial differential w.r.t $\mathbf{f}$,$\mathbf{b}$ and equation to zero in order to find MMSE filter coefficients, we have, (7), (8)

$$\begin{aligned} \frac{\partial \varepsilon}{\partial \mathbf{f}} &= 2\mathbf{f}E\{\mathbf{r}\mathbf{r}^H\} - 2\mathbf{b}E\{\mathbf{d}\mathbf{r}^H\} - 2E\{\mathbf{r}x\} \\ \frac{\partial \varepsilon}{\partial \mathbf{b}} &= -2\mathbf{f}E\{\mathbf{r}\mathbf{d}^H\} + 2\mathbf{b}E\{\mathbf{d}\mathbf{d}^H\} + 2E\{\mathbf{d}x\} \end{aligned} \tag{13}$$

In keeping with the principle, that we cannot discard any information, without establishing the extent of its usefulness, for an imperfect decision feedback DAE in training mode, we allow the following assumptions,

$$\begin{aligned} \mathbf{R}_{rd} &\neq 0 \\ \mathbf{R}_{dd} &\neq \mathbf{I} \\ E(\mathbf{d}.x) &\neq 0 \\ E(\mathbf{d}) &\neq 0 \end{aligned} \quad (14)$$

Substituting the assumptions, into (13) and equating to zero for MMSE solution, we get

$$\mathbf{fR} - \mathbf{bR}_{rd}^H - \mathbf{p} = 0 \quad (15)$$

$$\mathbf{f}_{opt} = (\mathbf{b}_{opt} \quad (16)$$

$$\mathbf{R}_{rd}^H + \mathbf{p})\mathbf{R}^{-1} \quad (17)$$

$$-\mathbf{fR}_{rd}^H + \mathbf{bR}_{dd}^H + \mathbf{r}_{dx} = 0 \quad (18)$$

substituting, for $\mathbf{f}_{opt}$ into (18), we get,

$$\begin{aligned} -(\mathbf{bR}_{rd} + \mathbf{p})\mathbf{R}^{-1}\mathbf{R}_{rd} + \mathbf{bR}_{dd} + \mathbf{r}_{dx} = 0 \\ -\mathbf{bR}_{rd}\mathbf{R}^{-1}\mathbf{R}_{rd}^H - \mathbf{pR}^{-1}\mathbf{R}_{rd} + \mathbf{bR}_{dd} + \mathbf{r}_{dx} = 0 \\ \mathbf{b}(\mathbf{R}_{dd} - \mathbf{R}_{rd}\mathbf{R}^{-1}\mathbf{R}_{rd}^H) - \mathbf{pR}^{-1}\mathbf{R}_{rd} + \mathbf{r}_{dx} = 0 \end{aligned} \quad (19)$$

$$\mathbf{b}_{opt} = (\frac{\mathbf{pR}^{-1}\mathbf{R}_{rd} - \mathbf{r}_{dx}}{\mathbf{R}_{dd} - \mathbf{R}_{rd}\mathbf{R}^{-1}\mathbf{R}_{rd}^H}) \quad (20)$$

Substituting $\mathbf{b}_{opt}$ into equation (15),

$$\mathbf{f}_{opt} = ((\frac{\mathbf{pR}^{-1}\mathbf{R}_{rd} - \mathbf{r}_{dx}}{\mathbf{R}_{dd} - \mathbf{R}_{rd}\mathbf{R}^{-1}\mathbf{R}_{rd}^H})\mathbf{R}_{rd}^H + \mathbf{p})\mathbf{R}^{-1} \quad (21)$$

simplifying it further assuming $\mathbf{R}_{rd}\mathbf{R}_{rd}^{-1} = \mathbf{I}$, we get,

$$\mathbf{f}_{opt} = \frac{\mathbf{pR}^{-1}\mathbf{R}_{rd} - \mathbf{r}_{dx}}{\mathbf{R}_{dd}\mathbf{R}_{rd}^{-1} - \mathbf{R}_{rd}\mathbf{R}^{-1}} + \mathbf{pR}^{-1} \quad (22)$$

1) Analysis of Wiener DAE for imperfect decision feedback: The forward and backward filter coefficients derived in (22),(20), for the case of imperfect decision information, can be reduced to perfect decision feedback Wiener DAE results [7], by making appropriate assumptions,

$$\begin{aligned} \mid \mathbf{R}_{rd} - \mathbf{I} \mid &\ll 0 \\ \mathbf{R}_{dd} &= \mathbf{I} \\ E(\mathbf{d}.x) &= 0 \\ \mathbf{f}_{opt}^p &= \mathbf{pR}^{-1}\mathbf{R}_{rd} \\ \mathbf{b}_{opt}^p &= \mathbf{fR}^{-1} \end{aligned} \quad (23)$$

where, $\mathbf{f}_{opt}^p$, and $\mathbf{b}_{opt}^p$ are Wiener filter coefficients assuming perfect decision feedback information. So, the vector set, for $\mathbf{f}_{opt}$ given by (22), and the vector set for $\mathbf{b}_{opt}$ given by (20), in a Wiener DAE considering imperfect decision feedback information, can be represented as,

$$\begin{aligned} \mathbf{f}_{opt} &= \{\mathbf{f}_{opt}^p, \mathbf{f}^i\} \\ \mathbf{b}_{opt} &= \{\mathbf{b}_{opt}^p, \mathbf{b}^i\} \end{aligned} \quad (24)$$

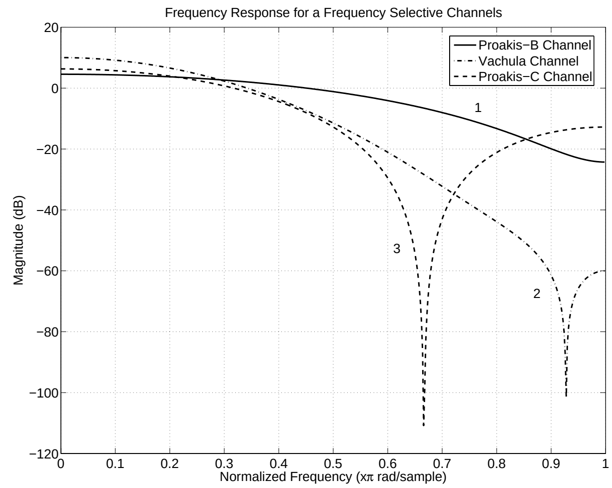

Fig. 2. Frequency responses of different frequency selective channels.

1) Proakis-B channel [8].
2) Vachula channel [9].
3) Proakis-C channel [8].

where, $\mathbf{f}^i$, and $\mathbf{b}^i$ are the filter coefficients adjustments, considering imperfect decision feedback information. Clearly, the forward and backward filter coefficient solutions, are affected by assuming imperfect decision feedback information, which, have to accounted separately, when using adaptive DAE algorithm.

C. LMS Decision Aided Estimation

The MSE $E[\epsilon(i)^2]$ is a convex function of vector $\mathbf{f}$, $\mathbf{b}$, defined by (20) and (22). As stated in section III-A, training sequence is used in practice, to tune the forward and backward filters, using (6), which, converges channel estimation errors using MSE principle. The classical wireless communication channel estimate, using LMS DAE, with the forward and backward filters adjusted using step-size μ_b,μ_f, assumes perfect decision feedback information. In an LMS DAE, this is reflected by setting μ_b=$\mu_f = \mu$. The adaptive coefficients, are updated classically, according to the following principle equations[7],

$$\begin{aligned} \mathbf{f}_k(i+1) &= \mathbf{f}_k(i) + \frac{1}{2}\mu[-\nabla_f \varepsilon(i)] \\ \mathbf{b}_k(i+1) &= \mathbf{b}_k(i) + \frac{1}{2}\mu[-\nabla_b \varepsilon(i)] \end{aligned} \quad (25)$$

where, $\mathbf{f}_k(i+1)$, and $\mathbf{b}_k(i+1)$, are the filter coefficient at $i+1$ th symbol interval. $\nabla_f \varepsilon(i)$ is the gradient (partial-derivative) of an i th symbol, w.r.t $\mathbf{f}$, $\nabla_b \varepsilon(i)$ is the gradient of an i th symbol, w.r.t $\mathbf{b}$. Substituting (13) into (25) and simplifying, we get the following set of update equations for the case, where, decision feedback information into LMS DAE is perfect,

$$\begin{aligned} \mathbf{f}_k(i+1) &= \mathbf{f}_k(i) + \mu[-\mathbf{fR} + \mathbf{p}] \\ \mathbf{b}_k(i+1) &= \mathbf{b}_k(i) + \mu[\mathbf{f}^H\mathbf{R}_{rd} - \mathbf{b}] \end{aligned} \quad (26)$$

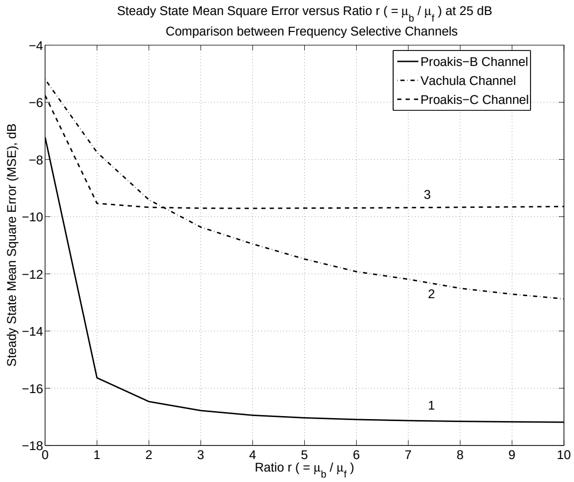

Fig. 3. Convergence analysis against steady-state MSE plot versus ratio r ($= \frac{\mu_b}{\mu_f}$). Results for uncoded system at 25 dB with LMS DFE for different frequency selective channels. The steady-state MSE for optimum r=1 is greater than r=10. The forward and backward filter coefficients are updated constantly in directed mode.

1) Proakis-B steady-state MSE results [8].
2) Vachula steady-state MSE results [9].
3) Proakis-c steady-state MSE results [8].

1) Unequal step-size behavior in LMS DAE: The step-size ratio $r = \frac{\mu_b}{\mu_f}$, provides an additional degree of freedom for every filter coefficient, using LMS DAE, such that, imperfect decision feedback information into feedback filter is weighed by a factor $r = \frac{\mu_b}{\mu_f}$, according to MSE criteria. A different set of update equations for LMS DAE, when imperfect decisions (15),(18) are taken into account and substituted into (13),

$$\begin{aligned} \mathbf{f}_k(i+1) &= \mathbf{f}_k(i) + \mu_f[-\mathbf{f}\mathbf{R} + \mathbf{b}\mathbf{R}_{rd}^H + \mathbf{p}] \\ \mathbf{b}_k(i+1) &= \mathbf{b}_k(i) + \mu_b[\mathbf{f}\mathbf{R}_{rd}^H - \mathbf{b}\mathbf{R}_{dd}^H - \mathbf{r}_{dx}] \end{aligned} \quad (27)$$

Assuming that, $\upsilon_b \rightarrow 0$ and , is the residual MSE for estimated symbol using LMS DAE, which, approaches zero, for large enough training sequence,

$$\begin{aligned} &\mathbf{f}\mathbf{R}_{rd}^H - \mathbf{b}\mathbf{R}_{dd}^H - \mathbf{r}_{dx} = \upsilon_b \\ &\mathbf{b} = \mathbf{f}\mathbf{R}_{dd}^{-1}\mathbf{R}_{rd}^H - \mathbf{R}_{dd}^{-1}\mathbf{r}_{dx} - \upsilon_b.\mathbf{I}.\mathbf{R}_{dd}^{-1} \end{aligned} \quad (28)$$

substituting $\mathbf{b}$ into the update equation for $\mathbf{f}_k(i+1)$,

$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) + \mu_f[-\mathbf{f}(\mathbf{R} - \mathbf{R}_{dd}^{-1}) + \mathbf{p} - \mathbf{R}_{dd}^{-1}(\mathbf{r}_{dx} - \upsilon_b.\mathbf{I})\mathbf{R}_{rd}^H] \quad (29)$$

$$\mathbf{b}_k(i+1) = \mathbf{b}_k(i) + \mu_b[\mathbf{f}\mathbf{R}_{rd}^H - \mathbf{b}\mathbf{R}_{dd}^H - \mathbf{r}_{dx}] \quad (30)$$

The forward filter coefficients of (29), are updated using a step-size constant μ and a factor $\mathbf{R}_{dd}^{-1}(\mathbf{r}_{dx} - \upsilon_b.\mathbf{I})\mathbf{R}_{rd}^H$, compared to (26). Similarly, the backward filter coefficients are updated using step-size μ and a factor $\mathbf{r}_{dx}$. The filter coefficient adjustments in (24), can now be justified using LMS DAE filter coefficient update, using, unequal step-sizes.

The same principle can be extended to a continuously adapting

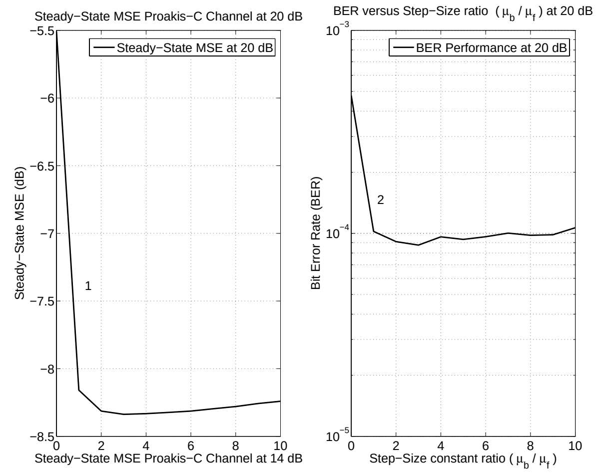

Fig. 4. The two subplots show the correspondence of steady-state MSE and uncoded BER performance versus ratio r ($= \frac{\mu_b}{\mu_f}$), using DAE shown in figure 1., for Proakis-C communication channel [8], at 20 dB (input SINR).

1) Steady-State MSE at 20 dB for Proakis-C channel [8]
2) BER versus step-size ratio r performance measure for Proakis-C channel

LMS DAE, where, the MSE is estimated symbol is compared to the detected symbol **d**, assuming, sufficient training has been provided during channel estimation. But, the analysis for decision directed mode, within 'Turbo' receiver systems is complicated and outside the scope of this paper.

IV. Simulation Results and Analysis

We use a Vachula model used in [9], Proakis-B and Proakis-C in [8] to demonstrate three results, with different channel responses, as shown in figure 2. Although, Proakis-B channel has non-flat fade, the Vachula, Proakis-C channels have highly frequency selective frequency responses. An uncoded BPSK information is transmitted through a frequency selective communication channel. The received samples (1) are fed to feed-forward filter of the DAE, which, consists of 18 taps. The feedback filter consists of 2 taps. All the taps are initialized to zero values. With training data, we run the LMS algorithm for 10,000 iterations, sufficiently long for the algorithm to converge. With the taps thus obtained, we run the algorithm for another 10,000 iterations in the decision directed mode. We compute the average of (3), in the decision directed mode and then find the ensemble average by repeating the experiment over 200 independent trials. The final MSE thus obtained for different values of the ratio r (=$\frac{\mu_b}{\mu_f}$) are shown in figure 3., for an SINR of 25 dB. The step-size μ_f associated with the feed-forward filter is fixed at 0.0025. Clearly, MSE behavior for Proakis-B channel, does not exhibit, the slow exponential drop in MSE compared to Proakis-C or Vachula channel. In a DAE, the simulation results show that, the MSE decreases sharply in the beginning, and later reaches a steady-state region, with increase in μ_b. The results show that, the simple choice of choosing the step-sizes to μ_f=μ_b is not optimal, and a higher

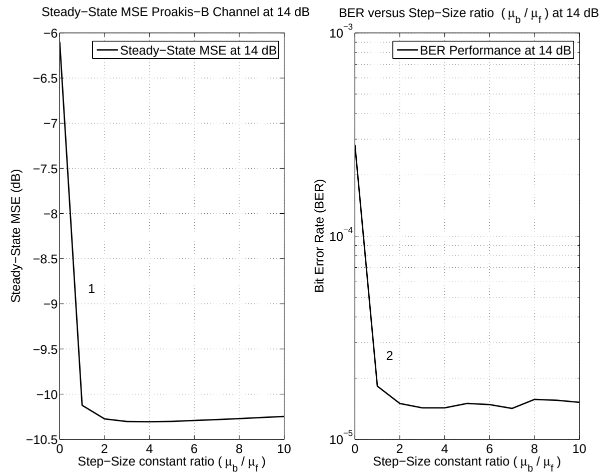

Fig. 5. The two subplots show the correspondence of steady-state MSE and uncoded BER performance versus ratio r ($=\frac{\mu_b}{\mu_f}$), using DAE shown in figure 1., for Proakis-B communication channel [8], at 14 dB (input SINR).

1) Steady-State MSE at 14 dB for Proakis-B channel [8]
2) BER versus step-size ratio r performance measure for Proakis-B channel

value of μ_b compared to μ_f results in more than 1.5 dB gain in terms of MSE for a frequency selective channel. It can be said that, the MSE for a frequency selective channel does not have a correspondence BER results, especially at low SINR.
The reason for a higher steady-state MSE at r=0 is not difficult to understand, as in this case, only the feed-forward filter is active and it is just a linear equalizer. For the channel considered above, linear equalizers produce excessive noise enhancement [8]. As r is increased the FBF starts playing its role and the combination of feed-forward and feedback filter functions as a DAE. Since, DAE has the potential to compensate for amplitude distortion without providing noise enhancement, we get lower steady-state MSE. However, even at $r = 1$, the feed-forward and feedback filter do not combine in an optimal manner, to minimize the steady-state MSE. In our example, a higher value of r pushes the combination of feed-forward and feedback filter towards the optimal, and feed-forward filter tends to be a whitening matched filter [3].
The figures 4., 5., show the BER performance results versus the ratio r ($=\frac{\mu_b}{\mu_f}$) for Proakis-C and Proakis-B frequency selective channel. The figures have been shown separately, to clearly point the correspondence between MSE and BER results. There optimizing the BER results based on steady-state MSE results, at high SINR. Different input SINR's have been used to obtain the results for Proakis-C and Proakis-B channel, due to practical issues in generating BER results at high SINR.

V. Conclusion

The steady-state MSE results follow the BER performance results at high SINR for an LMS DAE. The simulation results have demonstrated the optimization in BER performance results, by using unequal step-size constant values for an adaptive DAE. The analysis provided here shows that, the reduction in steady-state MSE can be explained by assuming 'imperfect' co-variance matrix of the detected symbols.

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