Is individual rationality essential to market price formation?
The contribution of Zero-Intelligence agent trading models

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0. Introduction

The experimental and computational literature on “Zero-Intelligence” (ZI) agents that has flourished after Dhananjay K. Gode and Shyam Sunder’s “Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality” (1993a), is presently questioning parts of conventional microeconomics. In an experiment with artificial agents, Gode and Sunder showed that some of the most relevant findings of economic theory are robust to modifications of the behavioral assumptions adopted for economic agents. Specifically, they provided evidence that under some conditions, very weak individual rationality requirements are enough to obtain the equilibrium price and quantity that standard supply-and-demand models would have predicted, and an efficient allocation of resources. Under an appropriate trading mechanism, it is only required that individuals trade at no loss, i.e. sellers never sell below their cost and buyers never pay more than the maximum sacrifice they are prepared to make for an object. The two authors concluded that it is primarily the market institution that ensures this outcome, while “the effect of human motivations and cognitive abilities has a second-order magnitude at best” (1993a, p. 133).

This finding has momentous implications for economic theory: in conjunction with the Sonnenschein-Mantel-Debreu theorem, it hints that individual rationality is neither a necessary nor a sufficient condition to obtain regularity properties at the macro level (Sunder 2007). Unsurprisingly, then, Gode and Sunder’s article has had a strong, if controversial impact on the profession. Sunder (2006a, 2006b) and Philip Mirowski (2002) came to criticize the detailed study of micro behavior that characterizes much of today’s economics, and suggested a new direction of research, concentrating instead on institutions and structures. Others continued to focus on individual economic behavior, trying to incorporate the lessons learned from the ZI simulations in different ways: some ventured into testing whether Zero-Intelligence can be a good predictor of observed behavior, especially in financial markets (e.g. Farmer, Patelli and Zovko 2004), while critics were eager to draw attention to situations in which human rational agents outperform ZI traders (Brewer, Huang, Nelson, and Plott 2002).

Nonetheless, the admittedly provocative “Zero-Intelligence” terminology fails to properly clarify what intelligence and rationality are. Did the ZI simulations really eliminate any form of individual rationality, and focus on the structure of interactions only, as Gode and Sunder claimed –or was there a minimum level of rationality that could not be dispensed with? Are individuals who are able to keep away from unprofitable exchanges unquestionably deprived of any cognitive capabilities, and can they truly be said to be mechanically responding to externally imposed constraints only? What is at stake is not much the “Zero-Intelligence” label itself, but rather the more general issue of how important are the cognitive limitations of human beings for aggregate properties of socio-economic systems.

Answering these questions is difficult, all the more so as the partial equilibrium setting of Gode and Sunder’s experiment, with induced, step-level market supply and demand schedules, does not facilitate integration of their results into theoretical models of resource allocation, typically formulated in terms of utility functions, preferences, and endowments. To be sure, this methodological choice had deep historical roots, as the two authors were inspired by a previous, seminal article of Vernon L. Smith (1962) –who in turn, borrowed the basic analytical framework of Edward Chamberlin, a known Marshallian, who performed a pioneering experiment on the organization of markets in 1948. Science is incremental, and by building on preceding work, market experiments have kept alive the Marshallian tradition, which has meanwhile lost ground in most other parts of economics. Ironically, the novelty of the laboratory experimental methodology was combined with an older analytical framework.

That two-good supply-and-demand schemes date back to an earlier stage of development of the economics discipline may yet shed light on the notion of individual rationality that is incorporated in the ZI methodology and on its implications for economic theory. In this article, I look at Gode and Sunder’s contribution through the lens of the works that laid the foundations of the partial equilibrium framework of analysis. The earlier literature contains explicit discussions of the assumptions, features, and limitations of this approach, and may help to establish its role in yielding Gode and Sunder’s strong result. Founding texts, dating back to a time in which individual optimizing behavior could not be taken for granted, also offer a variety of potentially enlightening reflections on the cognitive skills that are involved in trading.

I refer to Alfred Marshall (*Principles of Economics*, 1890), but also to the lesser-known Jules Dupuit[1] (“De la mesure de l’utilité des travaux publics”, 1844, and “De l’influence des péages sur l’utilité des voies de communication”, 1849[2]). Not only did the latter anticipate many of the basic concepts to be used in partial-equilibrium supply-and-demand models, including consumer surplus arguments, long before Marshall; but also, most significantly for the problem at hand, his analyses involve individuals who avoid losses, but are content with

[1] Jules Dupuit (1804 – 1866) served as a civil engineer for the French government. He was in charge of roads and waterways in various regions, and then worked for the municipal water system in Paris. He was involved in economic policy debates, and published widely on economic topics.

[2] An English translation of Dupuit’s 1844 article, entitled “On the Measurement of the Utility of Public Works”, came out in 1952, while only a section of the 1849 article has been translated, under the title “On Tolls and Transport Charges” (1962).

this relatively modest result, and do not go as far as to try to maximize their gains. Thus, despite the obvious diversity of Dupuit's and Gode and Sunder's contributions, not least because of the time and space distance that separates them, they exhibit a striking similarity, in that they basically portray the same kind of individual behavior, within the same analytical framework.

Reference to older writers does not imply that this article engages in historical research *stricto sensu*; in fact, I avail myself of historical material as a tool to guide my questioning on, and to structure my interpretation of, ZI models. Specifically, I draw on Marshall and Dupuit to elaborate an alternative interpretation of ZI behavior; to clarify the meaning and implications of a particular assumption of Gode and Sunder's model; and finally, to identify the limits of applicability of the ZI methodology.

With this approach, I show that ZI agents are in fact endowed with more-than-zero-intelligence, despite the label that was attributed to them. Because individual choices must be utility-improving in order for Gode and Sunder's result to obtain (although they need not be utility-maximizing), they involve some degree of rationality, somewhat in between optimization and total lack of intelligence. It follows that rationality in economics is more complex than commonly believed, including not only utility maximization, but a larger set of behavioral rules. Although acknowledged since the very origins of the discipline, this wider notion of rationality is still poorly understood, and needs further exploration.

I also provide evidence that in the original experiment performed by Gode and Sunder, ZI behavior was in fact "augmented" with an additional assumption, which is of a behavioral nature and presupposes some (positive) degree of individual rationality. Finally, I emphasize that the partial equilibrium setting and the assumption of a competitive environment allowed a particularly simple representation of agents' behavior (i.e. random choice from a given set), so as to make it appear "unintelligent"; however, a more complex model of individual choice-making would be necessary if one, or both, of these assumptions were removed.

The rest of the paper is organized as follows. Section 1 reviews the salient points of ZI trading models and spells out the questions to be addressed later. Sections 2, 3, and 4 contain the main arguments, and the last section concludes.

1. "Zero-Intelligence Agents": constrained behavior and its effects

The original motivation for the ZI project came from Gary Becker's claim (1962) that essential features of economics such as downward-sloping demand functions can be thought of as macro-level consequences of individual behavioral rules less restrictive than those implied by utility maximization. A second source of inspiration was Smith's suggestion (1962) that a variety of trading mechanisms, not just Walrasian tâtonnement, might yield very similar equilibrium price and quantity predictions[3].

In an effort to better understand the reasons underlying these findings, Gode and Sunder performed an experiment in which human traders, whose motivations, expectations, and attitudes towards risk would be difficult to control for, were replaced by programmed robots that submitted random bids and asks –hence the "Zero-Intelligence" label. The two authors adopted a partial equilibrium framework, and divided traders between buyers and sellers. Following Smith's (1976) induced value theory, they endowed each buyer with valuations (i.e. the price below which she could profitably buy) for a certain number of units of a good, and each seller with costs (i.e. the price above which she could profitably sell), again for some number of units; each trader's valuations or costs were supposed to be private information. Valuations of units j, $j + 1$ of buyer i satisfied $v_{i,j} \geq v_{i, j+1}$, while costs for units j, $j + 1$ of seller i satisfied $c_{i, j} \leq c_{i, j+1}$; traders had to buy/sell their j^{th} unit before buying/selling their $j + 1^{th}$. Aggregate demand and supply curves could be obtained on this basis, by sorting, respectively, individual valuations from highest to lowest, and individual costs from lowest to highest. The intersection of these two curves corresponded to the competitive equilibrium price and quantity that standard partial analysis would predict.

Consider, for instance, an experimental market in which buyers are willing to buy five units (A, B, C, D, E) at a maximum price of 10 for A, 8 for B, 6 for C, 4 for D and 2 for E; in turn, sellers would accept to sell five units (V, W, X, Y, Z) at a minimum price of 9 for V, 7 for W, 5 for X, 3 for Y and 1 for Z. In competitive equilibrium, price is in the range of 5 to 6; buyers' units A, B and C, and sellers' X, Y and Z, are traded; total gains from trade (i.e. the sum of consumer's and producer's surplus) add up to 15[4]. The question then is what types of individual behavioral rules and what trading institutions enable agents to reach this solution.

Gode and Sunder considered two cases, namely ZI-U (U = unconstrained) agents that simply chose bids and asks at random over some predetermined range, and ZI-C (C = constrained) agents that also chose at random, but were prevented from selling below their costs or buying above their values. Following an established tradition in experimental markets, exchanges followed double auction rules: buyers could post any bid order and sellers

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[3] In addition to the introduction to the 1993 article, Sunder (2004, 2006b) provides a detailed account of how the project came about.

[4] I thank Robert Sugden for suggesting this example.

could post any ask order at any time. Buyers were free to accept posted asks and sellers were free to accept bids at any time; if a bid and a ask matched or crossed, a transaction occurred.

Under these conditions it turned out that, with ZI-U traders, transaction prices were extremely volatile, with no convergence to the competitive (partial) equilibrium; but with ZI-C agents, volatility was significantly reduced and prices exhibited a tendency to approach the theoretical equilibrium prediction, though with some noise[5]. Besides, taking Smith's definition of the allocative efficiency of a market as the ratio of the actual to the potential gains from trade (i.e. the sum of consumer's and producer's surplus), Gode and Sunder found that efficiency was low with ZI-U agents, but close to 100% with ZI-C agents.

What are the reasons for this result, astonishing at first sight? A crucial point is the imposition of the "C" condition, which in line with Becker (1962), was interpreted as a budget constraint. Gode and Sunder saw it as a form of "market discipline", considering that the trading institution itself forbade traders to buy or sell at a loss because they then would not have been able to settle their accounts (1993a, p. 123). The constraint was thought to eliminate one source of inefficiency, arising whenever traders engage in unprofitable trades.

But this is not enough. In the case of the above example, for instance, the "C" condition holds if unit A is traded with V, B with W, C with X, D with Y, and E with Z, but this produces only 5 in total surplus, i.e. 33% efficiency. It is thus essential to find some way of ensuring that units for which buyers' willingness-to-pay is highest are paired with units for which sellers' willingness-to-accept is lowest, and conversely, that buyers with low willingness-to-pay and sellers with high willingness-to-accept are prevented from trading. Ideally in our example, first A should be traded with Z, then B with Y, and finally C with X, while the remaining units should not be exchanged at all.

The assumption that agents act at random, subject to the constraint that they never sell below their costs or buy above their values, contributes to generate this effect, because it implies that buyers with the highest values tend to post the highest bids, and are willing to accept a wider range of asks posted by sellers (and conversely for sellers). This is still insufficient to guarantee the result, though; there should also be some mechanism of pairing buyers and sellers that reduces the magnitude and probability of the efficiency losses arising when traders fail to negotiate profitable trades (buyers cannot find suitable sellers), and when

[5] Gode and Sunder reported that the root mean squared deviation of transaction prices from equilibrium prices tended toward zero as the number of transactions increased, and that regression of the root mean squared deviation on the transaction sequence number yielded significantly negative coefficients (1993a, pp. 129-31).

extramarginal traders (i.e. those whose values or costs lie to the right of the intersection between supply and demand) displace intramarginal ones (those to the left of the intersection).

The double auction trading mechanism provides the key condition for appropriately pairing buyers and sellers. It is a standard double auction rule, also included in Gode and Sunder's experiment, that when two or more bids (resp. asks) are posted, only the highest (resp. lowest) is operative. This rule gives priority to higher bids over lower bids, and to lower asks over higher asks; together with the constrained-random behavior of agents, it favors intramarginal traders over extramarginal ones, and implies that buyers with high values (hence, with high expected bids) and sellers with low costs (thus low expected asks) are likely to find trading partners first. In addition, the double auction allows multiple rounds (thus increasing the probability that buyers find suitable sellers), and allows traders to observe market data (so that intramarginal buyers can more quickly outbid, and intramarginal sellers can more quickly undercut, extramarginal buyers and sellers).

A further feature of this path is that it leads trading prices toward the predicted equilibrium level, by progressively dampening price adjustments over time. Although individual units are not necessarily exchanged in the order in which they appear in supply and demand curves (so that actual transaction prices sometimes are above the demand curve or below the supply curve), price priority rules make this possibility less likely. As highest-value buyers and lowest-cost sellers complete their transactions and leave the market, the set of possible bids and asks narrows, and the range of feasible transaction prices correspondingly shrinks; as even more units are traded, the set of possible bids and asks narrows further, and so on, so that the last transaction price is likely to be at or near the equilibrium level[6].

Gode and Sunder pointed out that the double auction mechanism differs from Walrasian tâtonnement (1993, p. 120); in fact, it draws on a tradition of thought that can be traced back to Marshall. While the Marshallian cross has often been understood as an equilibrium condition only, one line of interpretation (Chamberlin 1948, Tricou 1994, Leijonhufvud 2006) emphasizes that Marshall's conception of the market allows sequential, bilateral transactions in which each pair of traders negotiates a price, and supply and demand conditions change after each transaction –as in double auction. Bilateral contracts, disequilibrium exchanges and

[6] Chamberlin (1948, p. 102) and Smith (1962, p. 115) observed that demand and supply schedules continually alter as trading takes place. When a contract is made, a unit is withdrawn from the market, and the demand and supply schedules are shifted to the left in a way that depends on this unit's position on the schedules. Thus if lower cost/higher value units are traded first and if all trades must take place in the area bounded by the supply and demand curves (i.e. the C condition holds), prices have less and less room to vary as the trading process occurs, and eventually tend toward the level that corresponds to the intersection of the (initial) demand and supply curves.

multiple prices distinguish this approach to price formation from the Walrasian, characterized by a multilateral, simultaneous transaction at the (unique) equilibrium price (Tricou 1994, p. 26). Indeed, the very favorable properties of the exchange dynamics generated by double auction rules –such that in our example, first A is traded with Z, then B with Y, and finally C with X– are sometimes referred to as "Marshallian dynamics" or "Marshallian path" (Cason and Friedman, 1993; Easley and Ledyard, 1993; Brewer et al., 2002).

In sum, the orderly outcomes that can be observed at the system level appear to depend mainly on market institutions, i.e. "market discipline" that imposes the budget constraint, and the trading rules under which a Marshallian path emerges. Hence, strong individual rationality assumptions seem unnecessary. Gode and Sunder interpreted these results as reinforcing the idea put forward by Becker, that "households may be irrational and yet markets quite rational" (1962, p. 8). In their own wording,

> "Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality" (1993a, p. 119).

The question that I address in this paper is whether this interpretation is correct: is it right to argue that the "C" constraint is a form of market discipline, imposed from trading institutions on individual behavior? If not, to what extent, and under which conditions, does enforcement of the C restriction depend on agents' rationality? Do any additional behavioral assumptions play a role in yielding efficiency and/or price convergence? Finally, under which conditions is it possible to model trade at no loss simply by assuming that agents randomly choose bids or asks from a predetermined set? An attempt to answer these questions follows.

2. The constraint: market discipline or individual rationality?

To establish whether Gode and Sunder's interpretation of the "C" condition as market discipline is warranted, I first examine whether it truly represents a budget constraint, as the two authors suggested. On this basis, I try to assess what degree of individual rationality may be needed to meet the constraint.

2.1 Budget-constrained or utility-improving behavior?

The C condition is defined on the basis of the concepts of "valuation" and "cost", widely used in market experiments but not much in microeconomic arguments, typically framed in terms of preferences, utility functions, and endowments. In a pure exchange economy with two goods, a valuation denotes the maximum amount of good 2 that an agent would give up to obtain one unit of good 1, and represents this agent's willingness-to-pay for good 1. Within this framework, a cost can be symmetrically defined as the reciprocal of a valuation, and denotes willingness-to-accept good 2 against good 1. Cost denotes here a subjective notion independent of technological conditions –this is why, like a valuation, it is supposed to be private information in most market experiments (Smith 1982, p. 935).

These concepts derive from Dupuit and Marshall, who introduced them as indicators of the "strength of individual desire" (Dupuit 1844, p. 94), i.e. of the utility an individual attaches to a good (expressed in terms of another good). Depending on heterogeneities in preferences, different people may attach different utilities to the same good; further, valuations and costs need not be constant for the same individual, whose willingness-to-pay for the first units may be higher than for the following ones (and conversely, willingness-to-accept may increase with the number of units). People are not supposed to know others' valuations or costs; at most, they may attempt to "guess" them (Dupuit 1849b, pp. 12, 16).

It follows that valuations and costs of one good in terms of the other are subjective notions, depending not only on an individual's endowment (or income), but also on preferences. In a sense, these concepts can be thought of as partial equilibrium equivalents of the Marginal Rate of Substitution. A valuation exhibits the properties of a MRS, as it is nonincreasing in the number of units assigned to a subject; nondecreasing induced costs are the counterpart of noninecreasing buyers' valuations. Things do not significantly change in experimental environments, where values and costs are induced by the experimenter, because inducing values and costs ultimately means creating a structure of incentives (through a system of monetary rewards) that drives subjects toward behaving as if their preferences were really reflected in these values/costs, under a non-satiation assumption (Smith 1976, p. 275). Hence values and costs always represent individual preferences and utility in some sense.

Therefore, the C constraint is not a *budget* constraint, contrary to Gode and Sunder's own interpretation. The prohibition to trade in excess of values means that a buyer is not allowed to buy good 1 if its price in terms of good 2 exceeds the quantity of good 2 she is willing to give up in exchange of one extra unit of good 1. In fact, buyers were prevented from engaging in transactions that would result in lower levels of utility for them; it would be entirely different to impose that buyers should make choices from their feasible sets of trades.

This ambiguity may have arisen out of the assumption that a buyer's endowment of good 2 equals the value she attaches to a unit of good 1. But Gode and Sunder did not say it explicitly in their paper; what's more, it would be an unnecessarily restrictive assumption. Of course under bankruptcy laws and other regulations, an agent can at most give up her entire endowment of good 2 in order to get an additional unit of good 1: the importance of settlement of transactions for the functioning of markets, strongly emphasized by Gode and Sunder, is reflected in the fact that personal wealth usually places a limit on what one can bid. But to the extent that both goods yield a positive utility, an individual may be willing to give up *less* than her entire endowment of good 2 to get more of good 1. Ruling out this possibility is a restriction on individual preferences that does not derive from the standard assumption that agents meet their budget constraints[7]. As a matter of fact, other experimental studies on markets explicitly rule it out. For instance Kagel, Harstad, and Levin (1987) performed an experiment with no formal injunction against bidding in excess of one's value. Anticipating that some subjects would do so, they endowed them with an initial "capital balance", to which gains were added and losses subtracted; subjects whose capital balance dropped to zero or less were no longer allowed to bid. It is clear then that capital balance, or endowment, and valuation are distinct concepts.

A similar argument holds for sellers. In a pure exchange design with two goods, costs are not really incurred by sellers to produce the good; as noticed above, the constraint only concerns sellers' willingness-to-accept, which is a subjective notion depending on preferences. Thus, the condition that must be met is that they do not engage in utility-diminishing sales.

In sum, the constraint in the ZI-C experiment does not push agents toward meeting their budget constraints, but toward engaging in *utility-improving* transactions. The trade-at-no-loss condition that Gode and Sunder imposed should rather be renamed trade-at-no-*utility*-loss. Subsequent efforts to extend the model to general equilibrium settings (Crockett 2008; Gjerstad and Shachat 2007) are clearer in this respect, as they explicitly interpret the constraint as imposing (weak) utility improvement. Gode and Sunder's insistence on the budget constraint probably arose out of their reliance on Becker's 1962 article, showing that individual irrational behavior may be consistent with regularities at the aggregate level, provided agents choose from their opportunity set. But Becker used a general equilibrium framework, in which the distinction between endowment and preferences appears more clearly; the shift to a partial equilibrium setting seems to have added confusion, so that in effect, ZI-C behavior does not take up Becker's suggestion.

[7] A similar point has been made by Gjerstad and Shachat (2007).

The question, then, is what brings utility-improving behavior about. The subjective character of the C condition and the analogy with the MRS indicate that it results to some extent from individual intelligence or rationality or certain cognitive capacity; its origins in Dupuit and Marshall, outlined above, support this interpretation. One may even go as far as Gjerstad and Shachat (2007), who interpret the ZI-C condition as a sort of *rationality constraint*, following Luce and Raiffa's interpretation of rationality as an attribute of all agents who attempt to take part in trades that do not diminish their utility (1957, pp. 192-193).

Gode and Sunder had rather interpreted the C condition as an effect of market discipline, because their artificial agents were equipped with no learning or optimizing algorithm, and above all because their experimental design allowed them to enforce the restriction from above by inducing agents' values and costs, and by programming the experiment so as to automatically disable bids above values and asks below costs; in fact, they acted as a centralized, perfectly informed authority capable of controlling agents' behavior. But these conditions are stronger than what is implied by double auction rules alone, and can hardly be interpreted outside the controlled environment of the lab: a central authority could only enforce the C condition if it knew subjects' values and costs (which is unrealistic), and if it interfered with individual choices (which is at odds with free market principles). In the absence of such strict controls, institutions can still impinge on individual behavior by creating incentives to adopt utility-improving behavior; but this process presupposes, again, some cognitive capacity of the individual, who should be able to respond to such incentives and possibly to learn –which would require a more sophisticated modeling framework.

2.2 Utility improvement or utility maximization?

To assess the degree of rationality that the C condition presupposes, it is instructive to have a closer look at Gjerstad and Shachat's (2007) claim that a "rationality constraint" is at work in the ZI-C simulations. Their argument is based on a representation of Gode and Sunder's bargaining situation in an Edgeworth box, aiming to show that the restriction that buyers do not bid in excess of their values and sellers do not sell below their costs is in fact equivalent to imposing that bids and asks be restricted to the upper contour set of buyers and sellers.

Gjerstad and Shachat tried to reformulate experimental environments with induced supply and demand as Edgeworth exchange economies (with two agents and two goods), in which economic fundamentals such as preferences, utility functions, and endowments, are more easily understood. More precisely, their idea is that given an induced demand function (i.e. buyers' valuations, assigned by the experimenter), it is possible to construct a quasi-linear utility function for buyers, such that the induced demand can be derived from constrained maximization of this quasi-linear utility function; likewise, given an induced supply function (i.e. sellers' costs, also exogenously assigned), it is possible to construct a quasi-linear utility function for sellers, such that the given induced supply can be derived from constrained maximization of this quasi-linear utility function[8].

This method of converting one theoretical environment into another, however, presupposes maximization as a necessary precondition, as individuals must maximize their quasi-linear utility functions for their demand and supply schedules to be well-defined. This is equivalent to reintroducing an assumption of perfectly rational behavior from the very beginning, thus ruling out the very question of whether, or to what extent, ZI-C agents behave (ir)rationally.

An alternative interpretation of the degree of rationality that is involved in ZI-C behavior can be found in Dupuit and Marshall, who did not conceive demand and supply schedules that indicate the quantities consumers would buy or sell at each possible market price, so as to maximize their utility. They did not envisage individuals who take prices as given and use this information to determine optimal quantities; instead, these earlier writers mapped quantities into prices, with a conceptual experiment in which an individual determines, for each unit of the good in question, the maximum price she is willing to pay to acquire it, or the minimum price she is willing to accept to give it up. In other words, buyers and sellers determine, respectively, their valuations and costs as functions of quantities. Complex calculations may be needed to do so, as valuations and costs depend on several factors (notably the individual's income and preferences), and may differ from one unit to another, but always exist (Dupuit 1849a, p. 110).

Notice that an individual's valuation is an upper limit that he will under no circumstances voluntarily exceed. Dupuit wrote that "the purchaser never pays more for the product than the value he places on its utility" (1844, p. 89). Consumers choose the alternative they value most between an object and the monetary equivalent of its price. They avoid utility losses, and their gains from an exchange are necessarily non-negative. Yet buyers' ability never to pay more than what they think is the value of an object does not imply that they maximize their utility. Dupuit's consumers would accept to pay any price lower than the utility they attach to an object, but this price may be well above the cost of production –hence

[8] In passing, this argument calls to mind the standard result that if utility functions are quasi-linear and individuals maximize their utility, then the area to the left of the demand curve, i.e. consumer's surplus, is an exact measure of a welfare variation, not simply an approximation. Under this condition, microeconomics has preserved the consumer surplus arguments first introduced by Dupuit and Marshall.

even a small price decrease would raise their satisfaction, without being incompatible with producers' need to cover their costs. For each unit of the good, consumers would gain (i.e. consumer's surplus would be positive) if the actual transaction price turned out to be lower than the maximum amount they are willing to pay. Thus the highest price a consumer is ready to pay for an object does not correspond to an optimum, but rather indicates, for each unit of the good, the limits of this individual's willingness to pay. Although their gains are always non-negative, consumers do not systematically strive to make them as large as possible, in Dupuit's view; they never exceed their valuations, thus always obtaining at least a "minimum" level of utility, so to speak, but do not maximize utility either. In this perspective, Dupuit's demand schedule represents the relationship between quantity and willingness-to-pay for a number of individuals: it is the upper boundary of the set of utility-improving price-quantity combinations, not a *locus* of optimal points. Axel Leijonhufvud (2006) comes to a similar conclusion with respect to Marshall's demand and supply schedules, arguing that they represent valuations/costs as functions of quantities, not most preferred quantities as functions of prices. They define the set within which agents trade at no loss, and are not derived from individual optimization calculations[9].

This interpretation of the partial equilibrium supply-and-demand framework fits with the conditions of the ZI experiment, in which for each unit of the good, a buyer gained the difference between her valuation and the actual transaction price (symmetrically, a seller gained the difference between transaction price and cost), and efficiency was defined as a function of these gains; as in Dupuit's and Marshall's original schemes, the area delimited by demand and supply curves corresponded to the set of price-quantity combinations that would improve agents' utility. This interpretation clearly appears in Smith's seminal 1962 article, laying the theoretical foundations for experimental studies on markets:

> "Each buyer receives a card containing a number, known only to that buyer, which represents the *maximum* price he is willing to pay for one unit of the fictitious commodity. It is explained that the buyers are not to buy a unit of the commodity at a

[9] In support of his argument, Leijonhufvud emphasizes that Marshall drew his supply-and-demand diagrams with quantity on the horizontal and price on the vertical axis, while Walras had price on the horizontal, quantity on the vertical axis. It is the substantial difference in their approaches that accounts for this dissimilarity, he claims, not mathematical convention. Admittedly, Dupuit drew his demand curve with price on the *x*-axis and quantity on the *y*-axis, as Walras. This does not imply any behavioral assumption, though, but conveys Dupuit's intuition that valuations are private information and cannot be observed by third parties; instead, demand behavior can in principle be observed, with a thought exercise in which the observer gradually increases the price of a good, and keeps track of the total quantity that is withdrawn from consumption at each step. The diagram is constructed from an observer's viewpoint, and does not presuppose individuals who (optimally) map prices into quantities.

price exceeding that appearing on their buyer's card; they would be *quite happy to purchase a unit at any price below this number* […]. It is further explained that each buyer should think of himself as making a *pure profit equal to the difference between his actual contract price and the maximum reservation price* on his card. These reservation prices generate a *demand* curve" (1962, p. 112, my emphasis).

A symmetric argument holds for sellers, so that the ensuing demand and supply schedules "do nothing beyond setting extreme limits to the observable price-quantity behavior in this market" (Smith 1962, p. 114).

It follows that Gjerstad and Shachat's representation of induced supply and demand in a general equilibrium setting can be misleading, due to the maximization assumption that it necessarily implies, but that the original Dupuit/Marshall approach leaves out. The fact that enforcement of the "C" condition requires some individual cognitive effort does not make ZI-C traders equivalent to utility-maximizing agents: their behavior is simply utility-improving.

3. The "Marshallian Path": an effect of the trading institution only?

Recall that in the ZI simulations, it was assumed that in accordance with the principle of nonincreasing marginal rate of substitution, $v_{i,j} \geq v_{i,j+1}$ for each buyer *i* and unit *j*, and symmetrically $c_{i,j} \leq c_{i,j+1}$ for each seller *i* and unit *j*. It was further assumed that traders had to buy/sell their j^{th} unit before buying/selling their $j + 1^{th}$ unit. This sequencing restriction is common in market experiments, so that Gode and Sunder did not feel the need to justify it. Yet Duffy (2006, p. 958) noticed that strictly speaking, this assumption is not directly implied by double auction trading rules, and suggested that it may have played some role in yielding price convergence and efficiency in the ZI-C case. Indeed, it may have made it easier for low-cost and high-value units to be traded earlier, thus facilitating the emergence of Marshallian dynamics. In this perspective, it is important to characterize the nature and implications of this sequencing restriction.

To begin with, the meaning of this assumption is made clear by a leading textbook in experimental economics: "providing buyers with multiple units but restricting them to purchase the highest-valued unit first implements an assumption that individual demand is downward-sloping", and that similarly, "requiring sellers to sell the lower-cost unit first induces upward-sloping individual supply functions" (Davis and Holt 1993, p. 10). Sequencing is equivalent to imposing nice individual supplies and demands; then, horizontally summing across individual demands and supplies generates well-behaved market demand and supply schedules.

Do such restrictions on individual demands and supplies require individual rationality? Looking at Dupuit's work suggests that agents who attribute monotonically nonincreasing values to the units they wish to buy (resp. nondecreasing costs to the units they wish to sell), and trade their j^{th} unit before their $j + 1^{th}$, do exhibit some degree of rationality. They are able to prioritize their needs, and to evaluate different units of a good in different ways, according to the importance of the needs they are meant to satisfy; most significantly, agents are capable of acting consistently, by choosing to acquire first the units that have the highest value to them (or conversely, to cede first the units that are least worth to them). In this respect, it is instructive to look at one of Dupuit's illustrative examples. Consider, he said, that people need their first two hectoliters of water for strictly necessary "personal purposes", attach a high value to them (say, 50 francs each, at least), and buy them first; additional hectoliters of water, meant "for less urgent and less essential needs", have lower value, and are last to be purchased. For instance, after the first two hectoliters, an individual might destine the following two "to scrub his house every day", and attach a value of 20 to them; the next six hectoliters, at a value of 10, to water the garden; ten hectoliters, at a value of 5, "to keep up the level of his pond"; the last 80 hectoliters, worth just 1, to keep a fountain going, and so on (Dupuit 1844, p. 86). That this consumer's actions conform to this ranking of needs could be proven if it were possible to levy a tax on water, to increase it progressively, and to observe the quantity withdrawn from consumption at each step: one would notice that the individual in question first gives up the less valuable units (the 80 fountain hectoliters), then the more valuable (those for garden, housekeeping, and so on):

> "A tax of 4 francs per hectoliter, when the price is 1 franc, will immediately reduce consumption from 100 hectoliters to 20; a tax of 9 francs, from 20 hectoliters to 10; a tax of 19 francs, from 10 hectoliters to four; and so on" (Dupuit, 1844, p. 86).

It would obviously be irrational to give up the various units of the good in reverse order.

It becomes then clear that the sequencing restriction has behavioral content. It leads Gode and Sunder's non-utility-maximizing agents to act in a way that still fulfills the key predictions of the theory of choice based on utility maximization, notably downward-sloping demand and upward-sloping supply schedules.

This assumption rules out a broad class of irrational behavior, including all cases in which individual supply and demand schedules are not nice. This would have no appreciable consequences if it was true that, as argued by Becker (1962), a shift in agents' opportunity sets is conducive to negatively inclined demands or positively inclined supplies even with irrational individual decision rules. For instance, an increase in the price of a good shifts consumption opportunities toward other goods, and leaves less chance of consuming this one, even for an impulsive individual who chooses at random from her opportunity set. But the important point of Becker's paper is that this is true at aggregate level, not necessarily at individual level –irrational decision units are "often", but not "always", forced by a change in opportunities to respond rationally. The expected demand schedule of each individual will be negatively inclined, although many actual demand schedules will not be; the actual market demand schedule, instead, will always be negatively inclined, all the more so as the number of individuals is large in a society, because averaging over many decision units cancels out much erratic behavior (Becker 1962, p. 6). This is why "a group of irrational units would […] respond more smoothly and rationally than a single unit would" (Becker 1962, p. 13) [10].

The sequencing restriction that was at work in the ZI simulations is equivalent to imposing that the actual, not only the expected, supply or demand schedule of each individual is nicely behaved; the conditions that were imposed on individual behavior were thus stronger than those considered by Becker. The mechanism leading from individual irrational decision rules to possible rational responses (as reflected in negatively sloped demand schedules and positively sloped supply schedules) was assumed to exist and to be so strong as to operate for all decision units, confirming that they have more-than-zero intelligence.

This result supports Duffy's claim that sequencing is an additional assumption, not directly and exclusively derived from double auction trading rules, and that as such, it contributes to explain Gode and Sunder's results. Stressing the importance of individual cognitive capacities in bringing about Marshallian dynamics, however, does not necessarily mean that institutional features play no role. There may well be circumstances in which price priority rules have a bearing on the behavior of economic agents, by providing incentives for them to trade their high-value and low-cost units first; but modeling the process of endogenous emergence of the sequencing property presupposes some capacity of individuals

[10] Interestingly, this idea was anticipated by Marshall: "There are many classes of things the need for which on the part of any individual is inconstant, fitful, and irregular. […] But […] the variety and the fickleness of individual action are merged in the comparatively regular aggregate of the action of many. In large markets, then —where […] persons of all varieties of tastes, temperaments and occupations are mingled together— the peculiarities in the wants of individuals will compensate one another in a comparatively regular gradation of total demand. Every fall, however slight in the price of a commodity in general use, will, other things being equal, increase the total sales of it; just as an unhealthy autumn increases the mortality of a large town, though many persons are uninjured by it" (Marshall 1890, III.III.19, III.III.20).

to respond to incentives in their best interest, and possibly to learn, and would require a more sophisticated model of how non-zero-intelligence agents react to their environment[11].

4. ZI-C behavior as random search

One reason why ZI-C behaviour was interpreted as unintelligent may be that it was modeled in a very simple way, with a high degree of randomness –it is as if agents' choices were determined by the throw of a multi-sided die that would pick one option from an appropriate, predefined set. This is also a strength of the ZI approach, which has been seen as a "bottom of bottom-up processes" (Duffy, 2006), i.e. a benchmark for assessing the contribution of specific institutional features, or of human cognitive abilities, in experimental settings:

> "the value of this approach lies in building the minimal, necessary restrictions on directed random search that achieve the desired outcome. The ZI approach aids in formulating these restrictions, by greatly simplifying agent behavior, allowing the researcher to concentrate on the institutional restrictions" (Duffy 2006, p. 967).

It is then important to carefully spell out the conditions that make this particular approach suitable to model utility-improving choices, giving rise to nicely behaved supply and demand functions. Once again, comparison with earlier literature is of help.

First, Gode and Sunder's adoption of a partial equilibrium framework ruled out interdependencies among markets for different goods, and entitled them to rely on induced value theory to assign values and costs to agents. If interdependencies were taken into account, valuations and costs could no longer be taken as exogenously given; they would result from a choice process in which each agent would consider several factors to determine her willingness to give up one good in exchange of another. In this case, agents' behavior could not be modeled as random search from a given set, because agents would need to modify the set itself if circumstances (including those prevailing on the markets for other goods) changed. This was clear in Dupuit, who criticized Jean-Baptiste Say's example of

[11] Whether institutional features suffice to bring about Marshallian dynamics in the absence of any sequencing issue is less clear. One version of the ZI experiment, implemented by Gode and Sunder themselves, eliminates the sequencing assumption by assigning each trader the right to buy or sell one unit only; the authors report that they did obtain price convergence and efficiency also in this case (1993, p. 122). But another version, by Brewer et al. (2002), leads to opposite results. Brewer and co-authors endowed agents with multiple units, and eliminated the sequencing assumption by supposing that supply and demand are continuously refreshed: as agents trade and units are removed from the market, new units (with higher or lower value/cost) are attributed to them, in a way that keeps market supply and demand schedules constant over time. They found that under these conditions, no tendency to equilibrium appears in ZI-C simulations.

"a working-class family whose income allows, after satisfying its most basic needs, to spend 30 sous each week to buy meat; if meat costs 10 sous per pound, this family will demand three pounds a week; if it costs 15 sous, it will only demand two pounds" (Dupuit 1849a, p. 113, my translation).

Dupuit's objection consists in observing that

"the consumption figures of this example are perhaps badly chosen, because they presume that whatever the price of meat, the amount destined to buy this good will be the same, which is hardly admissible, because I do not know what this family will do to substitute for the pound of meat it does not buy when it costs 15 sous" (Dupuit 1849a, p. 113, my translation)[12].

In sum, agents determine their values and costs by taking into account their endowment, current prices, and the availability of substitutes and complements, and modify them in case of relative price changes. Clearly, such behavior can hardly be modeled as simply singling out an option at random from a given set: utility-improving choices do not appear unintelligent outside the partial equilibrium framework. Behavior rules have to be suitably modified to extend the ZI approach to many-good cases (see e.g. Crockett et al. 2007).

Second, the ZI approach has been conceived for a competitive environment, and may be difficult to extend to other market structures. Admittedly, the notion of perfect competition is harder to define in a double auction market in which there is no unique price known to everybody, and agents essentially behave as price-makers[13]. In a sense, however, the framework chosen by Gode and Sunder can be thought of as *quasi*-competitive, insofar as one single agent's action is unlikely to exert a strong impact on the system as a whole. The question is, then, which conditions allow generalizing the ZI approach to environments in which one agent's behavior may affect market outcomes.

Dupuit's work may be instructive in this respect, because it mainly deals with monopoly cases. In his description, a monopolist does not know buyers' willingness-to-pay and tries to "guess" it, sometimes by "just going ahead at random" (Dupuit 1849b, p. 16). Businesspeople in this position "have already made good progress" in this way, added Dupuit, thus hinting

[12] In Marshall, "the list of demand prices for tea is drawn out on the assumption that the price of coffee is known; but a failure of the coffee harvest would raise the prices for tea. The demand for gas is liable to be reduced by an improvement in electric lighting; and in the same way a fall in the price of a particular kind of tea may cause it to be substituted for an inferior but cheaper variety" (Marshall 1890, III.III.23).

[13] Nor are competition conditions characterized by a high number of traders in this case, because experimental markets typically involve a small number of participants. Indeed, one of the main results of experimental economics is that the equilibrium predictions of conventional models, founded on perfect competition assumptions, are fulfilled even with as few as six to eight agents (see e.g. Smith 1982, p. 945).

that also with this market structure, simple behavioral rules still yield good results. Nevertheless, a monopolist cannot fail to notice that the demand schedule is downward-sloping. Based on this awareness, the monopolist may tend to mobilize higher cognitive skills in order to increase gains; in particular, Dupuit insisted that attempts to guess consumers' willingness-to-pay may trigger efforts to devise pricing policies conducive to make them pay as much as possible, by putting in place price discrimination schemes that sometimes take highly sophisticated forms[14]. In sum, such a market structure leads individuals to form beliefs and expectations about others' behavior and to act accordingly –thus suggesting that random choice from a given set is unsuitable to model utility-improving behavior in this case.

5. Summary and discussion

The ZI methodology yielded interesting results, but relied on an analytical framework that is little used today, thus giving rise to potential difficulties in interpretation. My study of the foundations of this approach in the work of its forefathers hoped to shed light on its presuppositions, meaning, and implications, so as to better assess Gode and Sunder's findings.

Of course, Dupuit and Marshall's writings are unsophisticated by modern standards, but they contain reflections on founding issues that are often left implicit or taken for granted now. Reading them in light of the questions that the ZI simulations raise has helped to show that the very construction of the two-good supply-and-demand model requires agents to have some cognitive skills. In fact, the ZI project did not entirely remove individual rationality, and did not study the effects of the market institution only.

One question then is whether economics needs individual rationality at all. While the discipline is widely perceived to impose strong, unrealistic individual rationality requirements, some of its main results do not depend on them. Proofs of existence of general equilibrium for a competitive economy hardly rely on optimization assumptions, as they merely assume that Walras' law holds, i.e. that budget constraints are binding for all individuals –which according to Becker's (and then Gode and Sunder's) interpretation is tantamount to assuming nil rationality. To be sure, the proofs also require excess demand functions to be homogeneous of degree zero and continuous, but this does not necessarily

[14] For instance, railway passengers, unlike merchandises for which a classification is possible, "must be left to classify themselves" (Dupuit 1849b, p. 20). This is often done with a host of measures, differentiating as much as possible the comfort provided for passengers, so as to discourage the better-off from traveling in a lower class: the poor conditions in which third-class passengers travel are not intended to hurt them, said Dupuit, but are a sort of insurance "against the avarice of the rich" (Dupuit 1849b, p. 24).

presuppose stronger restrictions on individual behaviour. Homogeneity of degree zero only means that purely nominal changes have no effect, and continuity of the aggregate demand function may not only result from all consumers having strictly convex preferences, but also from individual demands that exhibit discontinuities, provided the economy is large[15].

By contrast, the fundamental theorems of welfare economics do rely on strong individual rationality requirements: the assumption that agents maximize their objective functions is needed to prove that a competitive equilibrium leads to an efficient allocation of resources, and *viceversa*, that any efficient allocation of resources can be sustained by a competitive equilibrium. Finally, and somewhat oddly, stability analyses incorporate an irrationality assumption, first highlighted by Franklin Fisher (1983, p. 85). Since little is known about agents' behavior out of equilibrium, it is assumed that agents always behave in the same way, whether they are in equilibrium or not: they are supposed to make consumption or production plans without taking into account the possibility that they may be unable to complete their desired transactions, and that prices may change –which is obviously unreasonable out of equilibrium. In this perspective, the ZI methodology only challenges the association between efficiency and strong individual rationality assumptions in welfare economics; and it does so only to some degree, because as shown, it does not fully remove individual rationality.

It is unclear whether it would be desirable, and feasible, to lower the degree of individual rationality that is assumed in economic models. A line of thought that goes from Becker (1962) to Gode and Sunder (1993) suggests an answer in the affirmative, meaning that the "invisible hand" is more powerful than commonly thought (Gode and Sunder 1993, p. 119). But Fisher (1983, p. 9, p. 85) suggests instead that no algorithm of market adjustment is satisfactory if founded on irrational individual behavior; sensible assumptions on agents' rational behavior are essential, in his view, to build a reliable theory of market price formation. All in all, the forms of individual rationality that are relevant for economics seem to be more complex than commonly believed, including not only utility maximization, but a variety of behavioral rules, in between optimization and total lack of intelligence.

Another question concerns Gode and Sunder's suggestion that rationality can be found at two levels in an economic system –the individual and the market. Trading rules incorporated in a market institution such as the double auction are "a consequence of

[15] Presentations of the definitive existence results can be found in Debreu (1959) and Arrow and Hahn (1971, Chapters 2-5). Benetti and Cartelier (1994) examine the rationality requirements of existence proofs.

individual rationality because they evolve out of individual choices over time" (Gode and Sunder 1997, p. 606); a set of rules can be seen as a kind of "institutional codification" of human thinking, which arises out of "successive refinements of individual rationality" over a large interval of time, and "trains and protects less rational traders –if the trading rules are 'smart', the traders need not be" (1997, p. 623).

But, if market rules are the ultimate product of agents' rationality, Gode and Sunder's experimental design presupposes that, once the trading mechanism has been built and put to use, there is no need for agents to be rational anymore: at this stage, some or all of them can be replaced by robots acting at random.

This paper's insistence on how individual rationality contributes with institutional features to achieve price convergence and efficiency, suggests a refinement of this interpretation. The market rules that discipline individual behavior are the product of previous individual behaviors, but they have not been created once and for all. They continuously require individuals' consensus and even active cooperation in order to function properly; Gode and Sunder themselves noted that "Rules are meaningful only if individuals follow them" (1997, p. 606, fn. 2). Instead of seeing individuals as passive subjects and institutions as fixed entities with a given, unalterable role, it may be more appropriate to emphasize agents' active participation in bringing about the results that institutions are meant to achieve. The present paper suggests that individual and "aggregate" (i.e. codified into existing institutions) rationality are at work simultaneously, and co-evolve.

So far, there has been limited work in economics on how this process may take place. Rediscovering the older literature as a tool to shed light on a recent contribution on this issue suggests the need for further research in this area.

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