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Producer Choice and Technical Unemployment: John E. Tozer's Mathematical Model (1838)[*]

Abstract

The paper presents Tozer's study of the effects of the mechanization of productive activities on employment as an effort to devise a mathematical model, as an analytical method that would be more general and robust than Ricardo's numerical examples. The contradictory nature of this achievement is emphasized: while with the help of algebra, Tozer made significant progress in model building in economics, it is argued that his contribution to a deeper understanding of the phenomenon under study is much less satisfactory, due to the difficulties he faced in his effort to incorporate consumption and demand into a classical analytical framework.

Keywords

J.E. Tozer, D. Ricardo, machinery, technical unemployment, mathematical modeling.

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0. Introduction

In 1838, John Edward Tozer (1806 – 70), a Fellow at Caius College in Cambridge[1], published an article entitled "Mathematical Investigation of the Effect of Machinery on the Wealth of a Community in Which it is Employed, and on the Fund for the Payment of Wages", in which he discussed the following problem: "A portion of capital, which either has been or would have been employed in the payment of wages, is used in the construction of machinery; to determine the effect on the wealth of the community, and on the fund for the payment of the labourer" (Tozer 1991 [1838]: 164).

The question of the effects that the mechanization of productive activities may have on employment levels was a much debated subject in early 19th century England. David Ricardo addressed this issue in the famous chapter 31 ("On Machinery") that he added to the third edition of his *Principles of Political Economy and Taxation* (1821). His well-known claim that the use of machinery may be detrimental to workers aroused great controversy among his contemporary economists. Tozer was one of those who criticized Ricardo, basically on the grounds that the latter's use of numerical examples as his main device for discovery and proof was inadequate: "The method that has generally been employed has been to take numerical examples, and the results of these have frequently been assumed to lead legitimately to general conclusions". Since such examples involve specific cases in point, no general conclusions can be drawn: "the numbers have been generally assumed without reference to realities, and though it may sometimes have been carefully stated, that the conclusions could not possess a higher degree of truth than the premises, the impressions on the minds of general readers would be favourable to that particular conclusion which the example chosen tended to support" (Tozer 1991 [1838]: 164).

Tozer tried to broach the problem of mechanization from a different angle, devising an analytical apparatus that can be viewed as a sort of "model" before its time, capable of dealing with technical unemployment in a full and coherent fashion. Instead of the numerical examples upon which Ricardo and others had based most of their work, he chose to use algebra, so as to render his deductions "necessary" and "general" (Tozer 1991 [1838]: 164).

The question arises whether Tozer's intention to go beyond the simple arithmetic of numerical examples, and devise a more general and reliable methodology based on algebraic tools, led him to satisfactory results. Did it improve his understanding of the phenomenon of mechanization and of its effects on employment, so that it could beat numerical examples?

[1] Additional information on Tozer's life, scientific production, and connections to other scholars of the time can be found in Gehrke (2000).

Did it pave the way towards relevant progress in the studies being done at the time on mechanization-related issues? A previous article by Christian Gehrke (2000) provides a partial answer to these questions, by claiming that Tozer's application of his own model to the numerical data that Ricardo had used in a well-known example of chapter 31, revealed a surprising inconsistency in the argument put forward by the author of the *Principles*. In this sense, Tozer's attempt to emphasize the inadequacy of numerical examples can be considered a success. In the present paper, I aim at exploring the 1838 model in further detail, in order to evaluate its strengths and weaknesses in representing the phenomenon of mechanization and its social effects. To what extent was it a reliable method for estimating how the mechanization of productive activities affected living conditions in a community? To answer this question, I distinguish the mathematical form and the economic content of the model, and examine each of them separately, in order to assess to what degree the activity of modeling with the help of algebraic tools may have contributed to a better understanding of mechanization.

Although I present a slightly simplified version of Tozer's algebra, it will undoubtedly appear cumbersome, difficult to follow, and obsolete in terms of what we know today. It may thus be believed that the difficulties in the author's argument, if any, are the result of poor mathematical formulation. However, I show that it is not the case: though coarse and at times tiring, Tozer's formulae do not lead him into trouble. In contrast, I claim that the author's insufficient understanding of some key economic linkages is largely responsible for one major shortcoming of his model. My argument basically relies on a thorough examination of the main analytical tool upon which Tozer's theoretical construct is based, i.e. the notion of "community gain". He introduced it in an attempt to assess technical change not from the viewpoint of a producer who will change a production technique if that helps to cut production costs, but from the perspective of society as a whole, including consumers. Surprisingly enough, this notion turns out to be fairly similar to other surplus measurement definitions that were being concocted at the time by French economists and engineers, ranging from Jean-Baptiste Say (1972 [1803]) to Henri Navier (1832). It can also be criticized in much the same way as Jules Dupuit (1934a [1844], 1934b [1849]) attacked these analyses: specifically, Tozer's suggested equation over-estimates the advantage consumers derive from the implementation of more capital-intensive techniques. I suggest that this difficulty can be explained by the fact that Tozer was on the whole unable to model consumers' point of view, in that similarly to most theoreticians of classical inspiration, he was still beholden to a schema focused on evaluating production costs from a capitalist's perspective. The lack of a

sufficiently developed theory of consumption and demand, not inadequate mathematics, is responsible for Tozer's failure to devise a correct measure of the gain of the community. In all fairness, however, it should be said that the problem of including consumption and demand within the classical analytical framework, where a greater weight had traditionally been placed on production and supply conditions, had no trivial solution. Even Dupuit's measure, able as it is to incorporate aspects of consumers' evaluation of goods and services that Tozer's community gain concept fails to capture, would not be entirely satisfactory here, due to the partial analysis framework on which it is based.

By way of conclusion, I stress that the interest of Tozer's article is twofold. First, it is evidence that mathematics did contribute to the progress of economic theory, by making it more rigorous and general –thus in this sense, the author was right in criticizing those who reasoned in terms of numerical examples. Yet his inadequate formula for calculating the gain to the community also hints that the use of mathematics *per se* did not solve all problems, and on the whole, it improved his understanding of the phenomenon under study only marginally –thus suggesting that a solid grasp of socio-economic relationships, independently of the mathematical support, is an indispensable requirement for consistent modeling.

Second, Tozer's paper clearly brings to light the emergence of a tension in the economic thought of the first half of the nineteenth century, insofar as, like other scholars of different nationality and background, he felt the need to incorporate consumption and demand into economic analysis, but found it difficult to do so while preserving at the same time the basic theoretical framework and the main achievements of the classical school. This seems to be one reason why Tozer's model, after all, contributed little to the advancement of a theory of consumer behavior –and perhaps, one reason why other theoreticians of the time moved much further away from the classical tradition.

I begin by presenting Tozer's article, a necessary precondition for what follows (par. 1), while at the same time introducing a few specific remarks on particularly interesting points. I then use this material to discuss the mathematical form and the economic content of the model, by drawing attention to the community gain concept (par. 2). Lastly, I summarize the main achievements and the shortcomings of Tozer's work.

1. Tozer's article

1.1 Machine-building reduces the fund available for paying wages

Tozer distinguished the phase in which the machine is being built from the one in which it is ready and utilized in production. Beginning with the first, he assumed that a capitalist possesses a stock that he uses initially to pay the wages of workers employed to produce some commodity. Imagine that he decides at a certain moment to redirect some of this capital towards a machine-making project. If $n > 1$ years are necessary for the machinery to be built, the capitalist will invest every year a constant portion of his capital, say i. At the beginning of the first year, he can use the difference between the total initial capital stock and the investment i to continue paying workers their wages. The amount available for them will be reduced by a quantity equal to i, and will therefore be lower than it was in the previous period, or than it would have been in the absence of any plans to build a machine. At the beginning of the second year, with a general rate of profit in the economy of r, the investment undertaken the year before will be worth $i(1 + r)$; the new investment will still be equal to i; and the total reduction in the amount available for the payment of wages will be equal to the sum of these two quantities: $i[1 + (1 + r)]$. At the start of the third year, the total value of worker wages will fall by $i[1 + (1 + r) + (1 + r)^2]$, and so on, so that at the beginning of the n^{th} year, the reduction in the fund set aside for paying wages is $i[1 + (1 + r) + (1 + r)^2 + \ldots + (1 + r)^{n-1}]$. This is the sum of a geometric series, equal to $\frac{i[(1+r)^n - 1]}{r}$. This quantity represents the future value of the n-1 initial investments, plus the new investment i. The end result is that the value V of the machine at the end of the last year will be equal to:

$$(1) \qquad V = \frac{i(1+r)[(1+r)^n - 1]}{r}.$$

The capitalist's machine-building investment implies a reduction in the amount of funds available to pay wages. At the end of the n years needed to build the machine, the reduction in this fund will be equal to the value of the investment V. However, the reduction in the stock that all the entrepreneurs in the economy set aside for the payment of wages will be lower than V, because some of the expenses incurred in manufacturing the machine will be directed to pay the wages of the workers producing it. Tozer assumed that this is a constant fraction, equal to m ($m < 1$) every year. As a result, the total reduction in the stock destined for the payment of wages is equal to $I_1 = i(1\text{-}m)$ the first year, with i representing the diminution of this stock in the branch producing the commodity in question, and mi its increase in the branch producing the machine. The second year, the stock that all entrepreneurs set aside for

paying wages falls, compared to the initial period, by $I_2 = i[1 + (1 + r) – m]$, with the element $i[1 + (1 + r)]$ representing the drop in this stock in the branch in question, and mi its rise in the machine-producing sector. The third year, the reduction of the stock destined for the payment of wages is equal to $I_3 = i[1 + (1 + r) + (1 + r)^2 – m]$, and so on. In the n^{th} year, the overall fall in the fund used to pay wages in the economy is:

$$(2) \qquad I_n = i[\frac{(1+r)^n - 1}{r} - m]$$

Despite the compensation symbolized by m, the wage fund throughout the whole of the economy falls every period. Note that its diminution is an increasing function of time.

1.2 On the use of machines and its positive effects for the community

Let us now consider the machine-utilization phase. Tozer's starting point is his calculation of an annuity A representing that part of the cost of producing the commodity in question that can be attributed to the machine's utilization over one year. A is equivalent to gross profits, including both the net profits Vr that the machine yields annually and its depreciation, i.e. an amount δ that has to be set aside every year to replace the machine after n' years. Since δ is equal to $\frac{Vr}{[(1+r)^{n'} - 1]}$, then:

$$(3) \qquad A = Vr + \frac{Vr}{[(1+r)^{n'} - 1]} = \frac{Vr(1+r)^{n'}}{[(1+r)^{n'} - 1]}$$

Let p_1 and y_1 be respectively the price and the produced quantity of the commodity, one year after the new machine is introduced, and let p_0 and y_0 be the price and level of production of the same commodity, before the machine is introduced. Tozer included these variables in his calculation of an index G representing the gain to the community, i.e. the flow of benefits accruing annually to the whole community after the mechanization:

$$(4) \qquad G = V(1+r)\frac{y_1}{y_0} - A\,.$$

At first glance, Tozer's interpretation of equation (4) seems to be relatively opaque: "the gain to the community may be measured by the price that would have been paid for the produce y, minus the price that is actually paid for it, when the profits of the capitalist have reached the average rate" (Tozer 1991 [1838]: 167). Gehrke (2000: 489) has tried to clarify this excerpt, by considering that, before the machine's introduction, a capital equal to V would have allowed for an annual production with a value of $V(1 + r)$. Calling the physical production of the commodity y_0, and its unitary price p_0, we can write $V(1 + r) = p_0 y_0$. As such, the first term

of equation (4) is equal to p_0y_1. After the machine is introduced, the unitary price of the produce p_1 is equal to the annuity A divided by the quantity produced: $p_1 = \frac{A}{y_1}$. The second term of (4) is therefore equal to p_1y_1. By re-writing equation (4) to incorporate these findings, Gehrke obtains:

(5) $G = p_0y_1 - p_1y_1 = (p_0 - p_1)\, y_1$

This interpretation gives an insight into Tozer's second formulation of the gain notion, based on the idea that "this gain may be measured by the saving in expenditure, added to the cost of the additional produce enjoyed, reckoning that cost at the original price" (Tozer 1991 [1838]: 167). Before introducing the machine, a sum equal in value to p_0y_0 had to be spent in order to obtain a physical produce equal to y_0; afterwards, the expenditure corresponding to a produce of y_1 is equal to p_1y_1. The machine's installation makes it possible to lower total spending by an amount equal to $p_0y_0 - p_1y_1$. At the same time, consideration must be given to the additional produce enabled by the machine: reckoning it at the initial price, its value is $p_0(y_1 - y_0)$. By considering that $p_0y_0 = V(1 + r)$ and that $p_1y_1 = A$, and since $p_0\ (y_1 - y_0) = p_0y_0 \frac{(y_1 - y_0)}{y_0}$, Tozer obtained the following equation:

(6) $$G = V(1+r) - A + V(1+r)\frac{y_1 - y_0}{y_0} = V(1+r)\frac{y_1}{y_0} - A$$

thus returning to equation (4).

Tozer explained that the condition $p_0 > p_1$ would always be satisfied when a mechanization occurs since, if p_1 were greater than p_0, the capitalists who have introduced the machine would be forced to face higher costs compared to other capitalists who only use human labor. The net effect would be that the machine could no longer be profitably used in production: "It may be observed that p_1 cannot be $> p_0$; if it were, more than the ordinary rate of profit would arise from employing labour, and the machine would be superseded" (Tozer 1991 [1838]: 168). Tozer showed that the gain G is positive if $p_0 > p_1$. Since $p_0y_0 = V(1 + r)$, and $p_1y_1 = A$, it follows that $y_0 = \frac{V(1+r)}{p_0}$, and $y_1 = \frac{A}{p_1}$. The ratio $\frac{y_1}{y_0}$ is equal to:

(7) $$\frac{y_1}{y_0} = \frac{p_0}{p_1} \cdot \frac{A}{V(1+r)}$$

By replacing (7) in (4), we get:

(8) $$G = V(1+r)\cdot\frac{p_0}{p_1}\cdot\frac{A}{V(1+r)} - A = A(\frac{p_0}{p_1} - 1)$$

This expression is obviously positive when $p_0 > p_1$.

At this stage, two preliminary remarks can already be made. First, the above argument presents *G* as an effort to integrate into a single indicator the viewpoint of both producers and consumers. I have mentioned that producers switch to a new technique if this allows them to reduce production costs; since the resulting lower prices are obviously advantageous for consumers, a positive *G* means that a decision taken by the former benefits the latter, so that "the gains of the capitalist are included in those of the community at large" (Tozer 1991 [1838]: 169). Second, the line of reasoning that has been followed brings to light the far-reaching implications of Tozer's calculation, meaning that any mechanization, decided solely by capitalists on account of their own interests, is good for society as a whole.

1.3 Variations in the rate of profit

Equations (4) – (6) are valid if the mechanization of a productive branch has no consequences for the overall economic system, i.e. if the general rate of profit in the economy remains at its initial level *r* after the technical change. This is why the interpretation proposed in equation (5), which only shows the price and the quantity of the merchandise in question, is legitimate. Tozer viewed this as a specific case in point, and tried to extend his analysis to cover situations in which the mechanization of a productive branch leads to changes in the rate of profit in the economy as a whole. In general, "the motive of the capitalist in supplanting labour by machinery is to procure for his capital more than the ordinary rate of profit" (Tozer 1991 [1838]: 168). The definition of the annuity *A* does not vary as the rate of profit rises – one only has to replace *r* by its new value *r*' in (3). Similarly, the definition of *G* does not change, because it is supposed to take capitalists' interests into account (Tozer 1991 [1838]: 169).

Under what conditions will the community gain *G* be positive if the rate of profit rises? For Tozer, the community gains whenever the price falls from p_0 to $p_1 < p_0$ as the rate of profit rises from *r* to *r'* (Tozer 1991 [1838]: 169). Tozer justified this by considering p_0, the price of the commodity before mechanization, in relation to two post-mechanization possibilities: a new price p'' preserving the previous general rate of profit *r*, and a price p_1 that allows the rate of profit to rise to $r' > r$. The value of the increased produce enabled by the machine's implementation is given by annuity *A*, as defined in equation (3). This can be re-written as:

(9) $$A = \frac{Vr(1+r)^{n'}}{[(1+r)^{n'}-1]} = \frac{Vr}{[1-(1+r)^{-n'}]}.$$

As a result,

(10) $$p''y_1 = A = \frac{Vr}{[1-(1+r)^{-n'}]}, \quad \text{and} \quad p_1y_1 = A' = \frac{Vr'}{[1-(1+r')^{-n'}]}.$$

It follows that:

(11) $$p'' = p_1 \cdot \frac{r}{r'} \cdot \frac{[1-(1+r')^{-n'}]}{[1-(1+r)^{-n'}]}$$

The community gain is positive if the initial price p_0 is greater than p'', i.e.:

(12) $$p_0 > p_1 \cdot \frac{r}{r'} \cdot \frac{[1-(1+r')^{-n'}]}{[1-(1+r)^{-n'}]}$$

Gehrke (2000: 492) notes that when $r' > r$, the term $\frac{r}{r'} \cdot \frac{[1-(1+r')^{-n'}]}{[1-(1+r)^{-n'}]}$ is smaller than 1. Consequently, inequality (12) may be satisfied even in some cases in which $p_0 < p_1$. It follows that the condition $p_0 > p_1$ is sufficient, but not necessary, for $G > 0$.

On this basis, I believe that Tozer's findings can be summarized in the terms of modern classical theory, by saying that the introduction of machinery in the process of production of a non-basic commodity –i.e. a luxury good- has a positive effect on the whole community ($G > 0$) if and only if it lowers the price of the commodity under consideration. On the other hand, the mechanization of a branch producing a basic commodity –i.e. a necessary good- has effects on the economic system as a whole, leading to a modification in the general rate of profit: in this case, not only will a lower price always cause G to be greater than zero, but a positive gain may also arise without any significant decrease in the price of the commodity.

However, Tozer himself did not draw any precise distinction between the two cases, and did not further investigate the conditions under which a variation in the general rate of profit may take place. In the remaining part of his article –arguably, for simplicity's sake- he took into account only cases in which the rate of profit is the same before and after the introduction of machinery. Apparently, he was more interested in highlighting the implications for the political debate of a situation in which an increase in the rate of profit is accompanied by a positive collective gain, because in this case, the capitalist's search for individual advantages proves to be compatible with the interests of the community: "If we assume that a capitalist will employ machinery or labour as one or the other will procure for him the highest rate of profit, then the employment of machinery will always increase the wealth of the community.

Not only is the capitalist unable to secure his own advantage at the expense of any other class, he cannot even prevent a general participation in the benefit" (Tozer 1991 [1838]: 172).

1.4 Increase over time in the fund for the payment of wages

Tozer then tries to study the effects of mechanization on the fund set aside for the payment of wages. His starting point is the observation that, at the beginning of the machine-utilization phase, this fund will fall by an amount equal to V. Why? We have seen that during the machine-building phase, the annual diminution in the wage payment fund set aside by all the entrepreneurs in the economy is less than the annual diminution in the wage fund of the branch producing the commodity under consideration, because a portion m of the annual investment i is being paid to the workers in the mechanical sector producing the machine. However, at the end of the final year of the machine-building phase, the machine-building workers will have completed their task, and therefore they will be made redundant. The reduction in the stock set aside for the payment of wages in the whole of the economy at the end of the machine-building phase -which coincides with the beginning of the machine-utilization phase- is equal to the reduction in the wage fund of the sector producing the commodity in question, which is equivalent to the value of the machine V.

Nonetheless, the actual reduction in the fund set aside to pay wages during the utilization period will be less than V. One initial form of compensation stems from the building of new machines - insofar as the machine that is currently used will have to be replaced at the end of its lifespan, capitalists will set aside a portion m of the annuity A, after having deducted Vr in profits, so as to be able to advance the wages of the workers manufacturing the new machine. Another form of compensation stems from the possible existence of a positive collective gain. The quantity G is an annuity calculated in monetary or other units of measurement, from which a given portion k (where $k < 1$) is destined for immediate consumption, and another portion $(1 - k)$ for additional investment. In all likelihood, a portion m_1 of the spending on additional consumption kG, plus a portion m_2 of the spending on new investments $(1 - k)G$, will increase the fund for the payment of wages.

With given k, m, m_1 and m_2, Tozer calculated the annual variation in the wage fund during the machine-utilization phase. Let D_t be the reduction of these sums in period t. Then:

(13) $$D_1 = V - m(A - Vr) - [m_1k + m_2(1 - k)]G$$

$$D_2 = V - m(A - Vr) - [m_1k + 2m_2(1 - k)]G$$

$$\ldots\ldots$$

$$D_t = V - m(A - Vr) - [m_1k + tm_2(1 - k)]G$$

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As time passes, the reduction in the wage fund becomes increasingly smaller. Tozer thought that this reduction may even turn negative, meaning that from a certain date onwards, the wage fund will start to increase. This date t^* corresponds to the number of years that have to elapse, starting from the beginning of the machine-utilization phase, to satisfy the condition $D_{t^*} = V - m(A - Vr) - [m_1k + t^*m_2(1 - k)]G < 0$. By substituting the value of the variables A and G into (13), Tozer obtained:

$$(14) \quad D_{t^*} = V - m \cdot \{\frac{Vr(1+r)^{n'}}{[(1+r)^{n'}-1]} - Vr\} - [m_1k + t^*m_2(1-k)] \cdot \{V(1+r)\frac{y_1}{y_0} + - \frac{Vr(1+r)^{n'}}{[(1+r)^{n'}-1]}\} < 0,$$

which can be rewritten as:

$$(15) \quad V \cdot \{1 + mr - m\frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]} - [m_1k + t^*m_2(1-k)] \cdot [(1+r)\frac{y_1}{y_0} + - \frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]}]\} < 0.$$

Since $V > 0$, the sign of this inequality depends on the term between parentheses. Hence the need to solve:

$$(16) \ [1 + mr - m\frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]}] - m_1k\cdot[(1+r)\frac{y_1}{y_0} - \frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]}] < t^*m_2(1-k)\cdot[(1 + + r)\frac{y_1}{y_0} + - \frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]}].$$

By isolating t^*, we get:

$$(17) \quad t^* > \frac{1 - m\cdot\frac{r}{[(1+r)^{n'}-1]}}{m_2(1-k)[(1+r)\frac{y_1}{y_0} - \frac{r(1+r)^{n'}}{[(1+r)^{n'}-1]}} - \frac{m_1}{m_2}\cdot\frac{k}{(1-k)}$$

From date t^* onwards, the wage fund increases every year. In summary, the reduction in the wage fund is initially generated by the transformation of circulating capital into fixed capital; then, with increasing use of this fixed capital, unemployment progressively starts to dissipate, until it falls back to its initial level. Afterwards, it may even fall below the initial level, due to the creation of additional jobs: “The operation on the labourer is to abstract a fund which has

been or would have been annually employed in the payment of wages, and annually renewed by the produce due to his exertions, and to supply a new fund, by increasing the wealth of the community, a portion of which will in general be paid as wages; this portion is at first smaller than the fund abstracted, but it increases without any assignable limit, the rapidity of increase depending on the proportion in which the new fund is divided between the labourer and the other classes of society" (Tozer 1991 [1838]: 172).

The implications of Tozer's reasoning for the political debates of his time are noteworthy. The claim that the initial reduction in the wage fund may be more than fully compensated over time, and that as a consequence, the introduction of new machinery turns out to be beneficial for the working classes, is mitigated by the fact that such positive outcomes are only to be expected in the very long run. As shown above, Tozer's formulas imply that the wage fund deteriorates steadily during the entire machine-building phase, and that it remains lower than its initial level after the utilization phase has begun. A mechanism capable of gradually offsetting this reduction is only set in motion after the machine has been put to use; it may take long to increase the wage fund up to its initial level (and possibly above it), depending on the speed of accumulation, as reflected in the values of the parameters m, m_1, m_2, k. Hence, the negative effects of the introduction of machinery on the wage fund are not limited to the transition phase from what we would call today one long-period position (corresponding to the old non-machinery based production technique) to another one (the new technique, using the machine), but may last for some time after the new technique has been adopted: strictly speaking, technological unemployment is not a transitional phenomenon only. In this sense, although Tozer hints that unemployment will eventually be eradicated, his algebraic formulation does not disprove Ricardo's view that the introduction of machinery may be "very injurious to the interests of the class of labourers" (Ricardo, 1951 [1817-21]: 388), and that "the opinion entertained by the labouring class, that the employment of machinery is frequently detrimental to their interests, is not founded on prejudice and error, but is conformable to the correct principles of political economy"(1951 [1817-21]: 392).

Before concluding on this point, it should also be emphasized that Tozer makes no explicit assumptions about the real wage rate. A reduction (rise) in the wage fund may imply falling (rising) employment levels or a falling (rising) real wage rate, or both: without any clearly stated hypotheses on wages, it is difficult to evaluate the overall effects of a reduction followed by a later increase in the wage fund. It may be argued that Tozer implicitly assumed a given real wage rate –e.g. fixed at some "subsistence" level, as in the classical tradition of his time- so that any variation in the wage fund has repercussions on the level of employment

only, as he seems to claim. This, however, is a drastic simplification, because the mechanization is likely to be associated with higher capital growth, implying increasing demand for labor that tends to increase wages above the subsistence level: the adjustment process might be more complicated than Tozer believed, involving changes both in the wage rate and in employment levels.

1.5 Adjusting the notion of gain

It has been shown earlier that the calculation of the gain G as suggested in equations (4) – (6) presupposes a notion of community that includes consumers; a possible criticism is that it neglects their being workers at the same time. Indeed, the definition of G does not account for the fact that the advantage that households, as consumers, derive from reductions in the price of the goods they buy may be undermined by the loss of income from which these very same households may suffer due to the fall in employment levels. Tozer seems to have realized that this leaves open the question of the relationship between the capitalist and working classes, the fundamental problem that Ricardo had tried to deal with in his chapter 31. He attempted to overcome this difficulty at least in part, by considering that, if the introduction of machinery causes unemployment for some of the workers who used to be employed before, the community will have additional costs to pay. Specifically, if the community takes care of the newly redundant workers, whose wages had been paid with a portion lV of the capital ($l < 1$), the notion of gain will assume the following form:

$$(18) \qquad G' = V(1+r)\frac{y_1}{y_0} - A - lV$$

The notion of gain expressed in equation (18) indirectly incorporates workers' living conditions, by including the costs that consumers and producers have to pay due to the presence of a pool of unemployed persons –arguably in the shape of higher taxes to fund additional government-run social assistance programs, or of increased donations to charities.

Two remarks are to be added at this point. On the one hand, this correction is somewhat incomplete by modern standards, because it fails to take into account the demand for consumption goods that comes from spending welfare benefits. On the other hand, it can be shown that in equation (18) a price reduction does not imply a positive gain. By substituting equation (4) into (18), we may rewrite the latter as: $G' = G - lV$. This alternative formulation shows that the gain of the community may be either positive or negative, depending on the comparison between the cost that society is ready to incur, represented by the total amount set aside to help persons suffering from technical unemployment lV, and the advantage G =

$V(1+r)\frac{y_1}{y_0}$ - A that stems from the reduction in the price of the commodity under consideration.

On this basis, the question arises of the consequences of $G' \leq 0$ on the variable D_t -i.e. the diminution in the wage fund resulting from the use of the machine in year t. Consider first the case $G' = 0$: the demand for capital goods and consumption goods remains the same, and the only element that makes the reduction in the wage fund smaller than V every year is the need to build a new machine. Equation (13) thus becomes:

(19) $D_t = V - m(A - Vr), \quad \forall t = 1, 2, \ldots, n'$

The fall in the wage fund is constant, smaller than V, and positive. There is no date t^* when the variable D_t becomes negative. The wage fund never rises due to the use of the machine.

Consider now the case $G' < 0$: demand falls for both capital goods and consumption goods. As a result, income drops in all the sectors producing such goods; in turn, this may generate an additional reduction in the wage fund. Equation (13) becomes:

(20) $D_t = V - m(A - Vr) + [m_1k + tm_2(1 - k)]G'$

The variable D_t is positive, and can even be greater than V. The wage fund falls over time. This means that the previous positive findings are considerably weakened when we include the social cost of unemployment into the community gain calculation: in this case, the wage fund is never brought back to its initial level. Yet Tozer did not emphasize this possibility. In the conclusion to his article, he stressed again the positive effects of mechanization, as well as the compatibility between the capitalist's interests and those of society as a whole.

2. Mathematical form and economic content in Tozer's model

2.1 Is the modern notion of "model" appropriate?

Before any further discussion of the merits and shortcomings of Tozer's work, I should justify the claim made in the introduction that it can be seen as a sort of "model" before its time. According to Giorgio Israel (1996), the trend towards mathematical modeling dates back to the early 20th century, since very few models were built before the year 1900. Can Tozer be thought of as a forerunner of these later developments? To answer this question, let us examine his work in the light of some of the distinctive characteristics of modern modeling. A model can be thought of as a theoretical construct that represents an economic phenomenon with a set of variables and of clearly stated relationships between them, thus providing a consistent framework that is supposed to enable logical reasoning about this phenomenon. It is particularly interesting for our purposes to notice that a model is not supposed to capture

the "true" nature of a phenomenon; nor does it purport to justify it in the context of a general and unitary explanation of reality. It is meaningful insofar as it can be expected to account for the different aspects of the phenomenon under study in a coherent manner. In fact, several models can be built to describe one and the same phenomenon, and inversely, the same model can sometimes be used to describe different phenomena, so as to ascertain analogies amongst them. This aspect seems to correspond to Tozer's intentions, since he was seeking to isolate the question of mechanization and its effects on employment from other problems in political economy (Tozer 1991 [1838]: 163), so that his sole focus would become whether it is possible to trace the various aspects of this phenomenon correctly. As he wrote in the introduction of his 1838 article: "the particular problem under consideration is of very limited extent, and of very easy solution" (Tozer 1991 [1838]: 164).

Tozer was well aware of the advantages of this method. In his view, focusing on one specific problem allows rigorous analysis, provided everything else remains the same: "the science that results […] acquires an almost entirely demonstrative character – becomes a series of propositions which are logical deductions from assumed definitions, and form those properties of the things defined which furnish axiomatic truths" (Tozer 1991 [1838]: 163-64). If the economic reasoning follows an unerringly deductive path, the use of mathematics becomes not only possible, but even indispensable: "[the science that results] is therefore a subject to which mathematical reasoning is not only proper but necessary" (Tozer 1991 [1838]: 164). By "mathematical reasoning", the author is clearly not referring to the arithmetic used in the numerical examples upon which his contemporaries had based most of their work, but to algebra, capable of sustaining economic deductions that are both "necessary" and "general" (Tozer 1991 [1838]: 164).

This argument seems to be confirmed by the appendix to the 1838 article, where Tozer applied his own model to study the numerical examples that John Barton, Sismonde de Sismondi, John R. McCulloch and David Ricardo had used to present their own positions on the problem of the social effects of machines being introduced into production. Tozer did not explicitly state the purpose of this exercise, but in line with the methodological attitudes he expressed in his introduction, I think that this was an attempt by him to offer a concrete portrayal of the generality of his algebraic model: numerical examples can only describe a particular case in point, while a model can be adapted to the study of many cases.

Despite the image of Tozer as an anti-Ricardian inductivist that some commentators have promoted (in particular Henderson 1996: 249-58), the above remarks hint that in a certain, definite sense, Tozer was trying to turn economics into a deductive science based on

mathematics. This objective is undoubtedly highly ambitious, and the approach he adopted, consisting in the construction of a mathematical model representing mechanization, is very modern in nature. In this respect, Tozer's work was ahead of its time, and can be seen as a forerunner of later developments in mathematical economics.

2.2 The mathematical form: is Tozer's algebra inconsistent?

Although I have slightly altered the original notation and presented a "modernized" version of Tozer's equations, more accessible to today's readers, the mathematical arguments presented above still appear rather awkward. Sometimes, they look complicated, as in the case of the formulae (14) – (17) above, or even somewhat obscure, as in the case of the gain formula itself. Despite the novelty of Tozer's approach that anticipates the more modern trend towards mathematical modeling, his algebra does not entirely meet the present standards of elegance and conciseness. This criticism should be nuanced, however, by considering that several authors in mathematical economics shared these same problems during the first decades of the 19^{th} century.

More significantly, it is to be stressed that there are no purely mathematical incongruities in Tozer's paper. Indeed, cumbersome notation does not invalidate formulae (14) – (17), but the author's failure to make appropriate assumptions concerning the real wage rate may well cast doubt on their usefulness. Similarly, difficulties in interpreting the gain formula are mainly due to unclear statements about its meaning, not to poor algebraic formulation. For this reason, I now turn to the economic content of Tozer's model, in order to assess whether the imperfections that have just been mentioned are traceable, not to unsound mathematics, but to a somewhat inconsistent view of economic phenomena.

2.3 The economic content of Tozer's models: producers vs. consumers

To answer this question, I focus on the main analytical tool upon which Tozer's theoretical construct is based –i.e. the notion of "community gain". The above presentation suggests that the author used some existing tools, e.g. annuity formulae commonly used in financial mathematics in order to measure the contribution of machinery or of other fixed capital to value formation. However, these concepts and analytical devices were originally intended to assess technical change exclusively from the viewpoint of a producer, who will change a production technique if that helps to cut production costs. Tozer could not be content with them, because he aimed at evaluating the impact of technical change from the perspective of the whole of society, including consumers. Hence, he tried to go beyond existing concepts

and to find a way of modeling the perspective of consumers. This attempt resulted in the introduction of his notion of gain *G*, which is supposed to summarize the point of view of producers and consumers in a single numerical index.

Nonetheless, his effort was not really successful, due to his lack of a deeper understanding of demand and consumption phenomena. The problem is that when prices fall, a distinction should be made between two types of consequences: a price effect -i.e. the possibility of acquiring the same quantity as before, but at a lower price-, and a quantity effect, that is, the fact that a lower price may make it worthwhile to use the commodity in new ways or to satisfy less urgent needs, which implies that consumers may have good reasons to buy more of it. Tozer was able to incorporate the price effect, since he only had to refer to the criteria that producers habitually apply to calculate unitary production costs, which depend on factors that can be called "objective" because they are supposed to be known to all producers -to wit, currently available production techniques and input prices. Regarding the quantity effect, Tozer did realize that a technical change can alter not only a commodity's unitary production cost but also the quantity that can be sold: he stated explicitly that mechanization causes a shift from (p_0, y_0) to (p_1, y_1), where production levels y_0 and y_1 are not necessarily equal. Nevertheless, he did not have a clear idea of how to calculate the price at which he should be valuing the additional quantity sold owing to the lower price. Eventually, he valued the final equilibrium quantity y_1 at the initial price p_0 and then at the new price p_1, before calculating the difference between the two values p_0y_1 and p_1y_1. But since the quantity y_1 - y_0 was not bought at the initial price p_0, this process is inaccurate, causing an over-assessment of the gain. Equations (4) – (6) are only valid if the quantity purchased remains unchanged when the price falls. Tozer tried but failed to measure the surplus from consumers' viewpoint. His effort was not really successful, and Dupuit's solution a few years later (1844) would leave him far behind.

Figures 1 and 2 illustrate this important point. On a diagram in which the quantity is represented on the X-axis and the price on the Y-axis, the surplus in Dupuit's sense of this term is equal to the area of the trapezoid $p_0q'qp_1$ (figure 1). Dupuit (1934a [1844], Annexe) drew such a diagram himself, including a representation of the demand schedule and of consumers' surplus, while Tozer did not; yet I take the liberty of representing his own suggested measurement on a similar scheme, so as to compare it with Dupuit's. Tozer's measure of gain, equal to (p_0 - p_1) y_1 according to equation (5), could thus be represented by the rectangle $p_0q''qp_1$ (figure 2).

Needless to say, if the gain of the community is to be measured by Dupuit's criterion instead of Tozer's, mechanization appears to be less favorable to society than he claimed. In particular, equation (13) indicates that a lower *G* may imply a larger reduction in the wage fund, and possibly the need to wait longer (i.e. in the terms of the model, a higher *t**) before this unfavorable trend is reversed, and the wage fund starts to rise again.

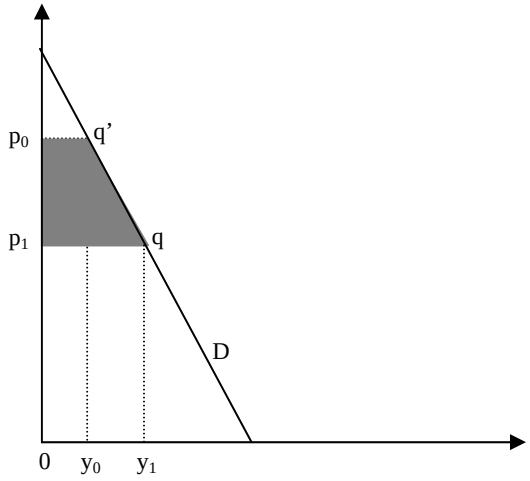

Figure 1

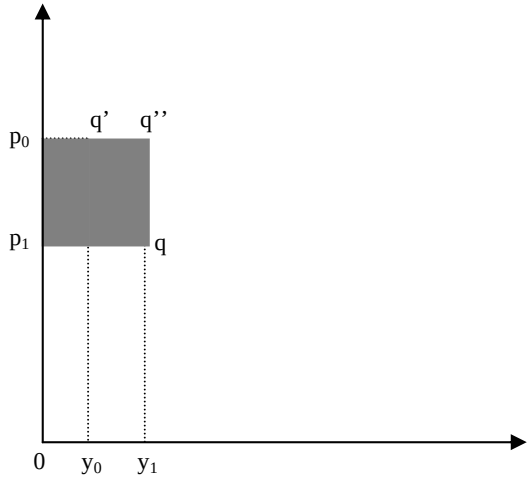

Figure 2

Tozer's inaccuracy can be explained by considering that to correctly formulate the notion of gain, he would have had to overcome the basic asymmetry that exists between producers and consumers, one that stems from the fact that the advantages of a technical change can be calculated relatively easily for producers (since all one needs to know is the variation in production costs per unit sold), while the advantages consumers derive from purchasing a good will depend on subjective factors that are more qualitative than quantitative in nature, and which cannot be expressed numerically right from the start. Tozer gets around this problem without solving it, by neglecting consumers' specificities and therefore the need to introduce subjective factors into his analysis. As a matter of fact, he simply applies a producer evaluation criterion (i.e. measuring variation in production cost terms) to consumers.

It may be argued that Tozer's inability to take into account consumers' point of view in a proper manner is due to his being strongly influenced by classical theorists, who emphasized quantities produced rather than quantities demanded. However, other authors of his time, who did emphasize the role of utility and demand in economic theory, ran into similar problems. For instance Say (1972 [1803]), who had identified consumers' advantage as the production cost of the concerned commodity multiplied by its quantity, was unable to prove that a fall in

this cost is a real advantage for them. By referring to figure 2, his notion of consumers' gain, at a price p_0, can be represented by the surface of the rectangle $0y_0q'p_0$, whereas at a price $p_1 < p_0$, it becomes $0y_1qp_1$. A fall in production costs that translates into a lower price will therefore only become advantageous for consumers if $0y_1qp_1 > 0y_0q'p_0$. According to this criterion, paradoxically, if the quantity sold remains unchanged, the fall in price is considered to be harmful to consumers. Another example of the difficulties that other authors of the time were experiencing when dealing with this problem is Navier (1832), who, in the same way as Tozer, overestimated the surplus, by taking the product of the price variation and the total quantity sold after the decrease in price –in our symbols, $(p_0 - p_1)y_1$: this is why Dupuit strongly criticized him. Apparently all these authors, while more or less influenced by classical theories focusing on production conditions, were trying to develop a deeper understanding of phenomena related to consumption. Tozer's work was much less developed in this respect than Say's, but it is to be emphasized that no one of these authors found an appropriate device to account for consumer behavior.

It was not until Dupuit that a measurement procedure adapted to consumer behavior descriptions was finalized. By advancing the idea that consumer surplus can be measured by the maximum sacrifice s/he is willing to make to obtain a given good, the French engineer was replacing objectively calculable production costs by a subjective notion of benefit. On the one hand, different quantities of an object can offer different advantages to a given individual, depending on the relative importance of the needs that the individual is seeking to satisfy through the use of the object. On the other hand, equal quantities of one and the same object can provide different advantages to different individuals, due to differences in needs and personal tastes. Thus, consumer surplus, which may have appeared initially too vague a notion to be quantifiable, could in the end be expressed numerically, thereby reducing the aforementioned gap between producers and consumers in economic theory. This achievement came at a cost, however, insofar as consumer surplus arguments rely on a partial analysis framework, and thus suffer from the limitations that this inevitably entails.

3. Conclusions

I have interpreted Tozer's analytical construct as a mathematical model, in the modern sense of the term. It is a theoretical device, capable of coping with the specific problem under study in a coherent manner, and general enough to cover a number of particular cases, previously presented by other authors with the help of simple numerical examples. It even allowed criticizing these authors, including Ricardo.

Tozer's main finding is that mechanization, implemented by self-interested capitalists, turns out to be good for society as a whole. However, this is only true in the very long run; indeed, during the phase in which the machine is being introduced, technological unemployment may well arise, and even persist for a while after the machine has been put to use. More complex versions of the model, taking into account the social cost of supporting the workers that have been made redundant by the introduction of machinery, lead to even less optimistic conclusions, in that they make unemployment a more likely long-term outcome. In this sense, Tozer's algebra ultimately confirms the main point that Ricardo had been trying to prove with the help of numerical examples in his Chapter 31.

Despite its merits, I have highlighted some of the imperfections of this model. In particular, the key notion of community gain is unsatisfactory, as it leads to over-estimate the gain that society obtains from mechanization. I have argued that such an inadequacy is not due to the mathematical form of Tozer's article, which is coarse in many respects, but not substantially inexact. Rather, it is the result of the author's insufficient understanding of the underlying economic concepts and linkages. Tozer's idea of community gain, by hinting at the fact that a decrease in price benefits consumers, clearly indicated that the latter's perspective had to be duly evaluated, in order to accurately assess the well-being of the entire society. However, while the author seems to have had a fairly sound understanding of producers' choice criteria, and modeled them suitably, he did not have a solid grasp of consumption and demand behavior. This led him to a biased definition of the gain to the community *G,* which invalidates some of his calculations, further supporting the concern that long-term unemployment may well result from mechanization.

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