

An evaluation of combined geophysical and geotechnical methods to characterise beach thickness

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Abstract

Beaches provide sediment stores and have an important role in the development of the coastline in response to climate change. Quantification of beach thickness and volume is required to assess coastal sediment transport budgets. Therefore, portable, rapid, non-invasive techniques are required to evaluate thickness where environmental sensitivities exclude invasive methods. Site methods and data are described for a toolbox of electrical, electromagnetic, seismic and mechanical based techniques that were evaluated at a coastal site at Easington, Yorkshire. Geophysical and geotechnical properties are shown to be dependent upon moisture content, porosity and lithology of the beach and the morphology of the beach-platform interface. Thickness interpretation, using an inexpensive geographic information system (GIS) to integrate data, allowed these controls and relationships to be understood. Guidelines for efficient site practices based upon this case history including procedures and techniques are presented using a systematic approach. Field results indicated that a mixed sand and gravel beach is highly variable and cannot be represented in models as a homogeneous layer of variable thickness overlying a bedrock half space.

Introduction

Beaches on the open coast represent a dynamic environment, characterised by the dissipation of wave and tidal energy, and the erosion, transport and deposition of sediment. An accurate determination of beach thickness is necessary in order to gain a better understanding of beach sediment transport processes and budgets. Research into beach thickness has been limited, however, and clear quantifiable sediment volumes are not available for many sections of coastline. The stability of the coastline is dependant upon its geological context, sediment regime, energy environment (wave, tide, and wind conditions), and the altitude at which this energy is delivered (sea level), together with any secondary effects resulting from human interference. Sediments stored at the shoreline in the form of beaches perform an important function in protecting the backshore or hinterland from erosion. As a consequence, potential changes in forcing conditions resulting from future climate change are likely to have significant implications for the open coastline (Futurecoast

2002). Sea level rise, storm surge activity and wave climate are of particular importance. The past evolution of open-coast beaches has involved gradual retreat in response to rising sea levels. Those coastlines composed of soft easily erodible materials, such as the Holderness coast of eastern England, have retreated rapidly. In the future, it is anticipated that the rates of recession of undefended shorelines are likely to accelerate due to rising sea levels and possible increases in storminess. Easington is of regional significance, as much data has been gathered at the site in recent years that can provide useful information on the development of beaches on till platforms, such as those found along the Holderness coast as well as other sections of the eastern England coastline.

Shore platforms formed of relatively non-resistant till are sensitive to climate change. It is probable that resulting changes in wave climate will promote an increase in the rate of platform erosion/lowering. A reduction in energy dissipation due to increasing water depths over the platform could arise from an accelerated sea-level rise. This is likely to increase the wave energy reaching the beach, thereby increasing erosion potential. Also, if the direction of wave approach alters, then the pattern of wave refraction over the platform may alter, leading to migration of wave and erosion foci along coast (Futurecoast, 2002).

Coastal sites present difficult surveying conditions, including regular tidal flooding and difficult ground conditions, which limit vehicular access to survey sites. The ground conditions change with each cycle of tidal flooding causing changes in ground surface level and the distribution of seawater within mixed beach sediments. In some cases, site delineation using georeferenced pegs and floats that are positioned on a regular basis is required such that all geophysical and geotechnical datasets can be successfully spatially correlated. Rapid, relatively inexpensive, portable site investigation techniques are essential at sites that are only exposed for a few hours. When using techniques that are sensitive to seawater saturation, such as electrical resistivity and electromagnetic techniques, it is useful to independently map the level of the seawater within the beach and incorporate this information into the interpretation.

Many geophysical and geotechnical properties are directly related to mineralogy and the relative proportions of solid grains, air and moisture within the sediment, (Buchan *et al.* 1972; Baker 1991; Hight *et al.* 1997; Gunn *et al.* 2003). Thus, changes in moisture content and the shape, orientation and packing of the grains all exert significant control on the in situ properties of deposits, and are themselves controlled by the dominant coastal processes at a beach site. Key to a successful investigation is an understanding of the sedimentology of the beach environment such that the contrast in the properties between the base of the beach and the underlying platform can be characterised. For example, this allows some standard geophysical survey parameters, such as

penetration, spatial resolution and contrast in properties to be addressed (Telford *et al.* 1976; Parasnis 1980), and a suitable suite of geophysical surveys planned for beach thickness characterisation. Some physical properties of geotechnical significance can be estimated using geophysical methods (McDowell *et al.* 2002); for example, relationships between elastic moduli and seismic wave velocities have been discussed by McCann *et al.* (1986), Butcher & Powell (1996) and Gordon *et al.* (1996). Shear modulus is controlled by the interaction between neighbouring grains or crystals within the solid framework of the soil or rock (Laughton 1957; Buchan *et al.* 1972; Stoll 1977; Gunn *et al.* 2003). These interactions are particularly influenced by the grain size range and packing density in beach sediments. Thus, survey methods combining portable cone penetration and shear wave velocity based techniques are well suited to investigations at coastal sites.

This paper provides a case history detailing the information gathered and the subsequent geophysical surveys used to assess the thickness of a beach site on the Holderness coast at Easington, East Riding of Yorkshire. The study is used to discuss the scope and limitations of individual techniques. Use of newly emerging geographic information systems (GIS) to aid interpretation is demonstrated, particularly when combining techniques to improve confidence in survey results. Results have shown that this beach environment is highly variable and cannot be simply modelled as a low conductivity, low strength, low stiffness layer overlying a stiffer, higher conductivity bedrock with higher strength.

Coastline at Easington

Easington is situated towards the southern end of the Holderness coastline (Fig. 1). It is a cliffed coastline composed of glacial tills, extending from Bridlington in the north to Kilnsea in the south, a few kilometres to the south of Easington. Sand and gravel-sized sediment generally travels south along the coast as littoral drift, or, in the case of the fine-grained sediments, as suspended sediment in the nearshore zone (Motyka & Brampton, 1993). The physical characteristics of the present-day beach deposits along the Holderness coast vary considerably over fairly short distances. The composition of the beach at Easington is intimately linked with the available sources of sediment, and as such is characteristic of a typical mixed-sediment, medium- to high-energy beach. The beach profile is relatively unstable, as indicated by the frequent marked changes in the position of the high tide mark. This instability is also manifest in the presence of dynamic bars and troughs in the intertidal beach zone.

Geology of the cliffs and sub-beach

The geology of the cliffs on the Holderness coast consists dominantly of relatively weak glacial tills which are subject to failure as the result of small landslides triggered by marine erosion and sub-

aerial terrestrial processes (Futurecoast, 2002). Glacial tills also form the subtidal shoreface along the majority of the coast. The sediment supply from the eroding cliffs is very high and is matched by erosion from the shoreface. The cliffs along with the shoreface provide the main source of sediment within the system; hence the majority of sediment on this frontage is generated locally. Sand and gravel-sized sediment from the coastal recession forms important beach building material along this coastline. Historically, despite the release of beach-building sediment from cliff erosion, throughout this frontage foreshore steepening or beach translation rather than accreting beaches have accompanied the trend of cliff retreat. This has been the case over the last 100 years (Futurecoast 2002). Cliff top recession rates are variable along the Holderness coast both spatially and temporally but there is a general increase in rate from north to south reflecting greater exposure to wave activity. Long-term average rates measured over the last 150 years at Easington are extreme. Individual landslide events may result in recession exceeding 10 metres over periods ranging from days to weeks (Futurecoast, 2002). The cliffs and sub-beach platform within the study area at Easington consists of tills of the Holderness Formation (Bowen, 1999). Examples of sections of the tills along the Holderness coastline are provided by Madgett & Catt (1978), Butcher (1991) and Berridge & Pattison (1994). In boreholes on the foreshore Catt & Digby (1988) recorded the same lithology comprising the Basement Till extending down to the upper surface of the Chalk situated at approximately -27m OD. The Basement Till is composed of a highly consolidated, stiff, very dark grey (Munsell Colour Chart 5Y 3/1) to dark greyish brown (10YR 4/2) matrix-supported diamicton that reaches a maximum observed thickness of 5m within cliff sections. The texture of the matrix is dominantly clayey sandy silt with occasional clasts of local and distant provenance. Some of the clasts, especially the limestones and ironstones, exhibit well-developed striations (including cross-sets) and streamlined keel morphologies, which are strongly diagnostic of abrasion and ploughing beneath a glacier. Further evidence that supports a subglacial origin for the till is the presence of highly deformed masses of attenuated chalk taking the form of attenuated laminae, boudinage, augens and small isoclinal fold noses. These structural features are characteristic of brittle and ductile styles of deformation under moderate-high levels of shear strain within a subglacial deformation till (Hart & Boulton, 1991; Benn & Evans, 1996). A striking feature of the till is that it displays a well-developed joint structure that dips sub-vertically/vertically (81-90°) towards the northwest. This could purely be a function of weathering, or perhaps, more likely, that the Easington area was overridden by ice at a later date and the till compacted and dewatered.

Beach morphology and sediment distribution

The beach at Easington comprises an upper high-tide beach (Komar 1976), composed of mixed sand and gravel, fronted by a predominantly sandy low-tide terrace (tidal flat), both of which are fully

exposed at low tide. The profile of the beach is shown schematically in Fig. 1. The low tide terrace consists of a variable-depth veneer of mobile sand and some accumulated gravel, overlying a cohesive till platform. The thickness of the sand on the terrace varies with changes in incident wave energy, as does the thickness of the high tide beach. The rising tide passes quickly over the low-tide terrace slope and serves to dissipate wave energy due to shallow water depths. The high-tide beach generally consists of coarser-grained sediments than those on the terrace, thereby maintaining a steeper profile. Proximity to the cliff sediment source, and the concentration of wave energy for a longer period of time during high tide, help to determine the difference in the sediment characteristics of the high and low tide beaches. It is predominantly the characteristics of the high tide beach that are of interest to this study.

The grain size of the high-tide beach surface at Easington varies down the profile (Figs. 1 and 2), with fine to medium sand at the top near the base of the cliff around $x=0\text{m}$ on the local survey grid, grading to coarse sand and fine gravel on the lower beach face at around $x=30\text{m}$, and returning to fine to medium sand on the upper low-tide terrace at $x=48\text{m}$. The mean grain size is greatest where energy is focussed at the wave plunge point marking the break of slope between the high tide beach and the low-tide terrace, at approximately the position of the $x=36\text{m}$ line on the survey grid. The composition of the high-tide beach sediments also varies with depth. In shore-normal cross-section, upper beach sediments typically show planar laminated, horizontal to seaward dipping sand laminations, with steeper-gradient, coarser-grained, more discontinuous laminae in the lower foreshore (Short & Hesp, 1999), coincident with higher energy conditions around the plunge point. At Easington, a distinctive layer of medium-sized gravel is exposed at the base of the high-tide beach (Fig. 1 & Plate 1), indicating that such a gravel layer exists between the underlying till platform surface and the overlying mixed sand and gravel. Investigation with a penetrometer at fixed points down the beach profile identified this horizon. It is likely that this represents a gravel lag deposit that has been eroded from the underlying till before burial by the overlying beach sediments. Similar, though discontinuous, lags of gravel also exist on the surface of the low-tide terrace, at locations where the sand cover is thin enough to expose the surface of the platform.

At the time of observation, during the surveys in July 2004, the distribution of high-tide beach sediments generally reflected degree of exposure to the prevailing wave climate, with coarse sands and gravels on the lower foreshore, and finer sands developing a berm on the upper foreshore towards the base of the cliff (Plate 1). The beach profile shows a characteristic berm at the top of the beach at the upper swash limit of Mean High Water Springs, with a further mid-foreshore berm around the position of the Mean High Water strand line, (Fig. 2). The beach profile was lowered by storm activity during the week preceding the field campaign, but overall showed the typical

characteristics of a sediment-filled “swell” profile. In contrast, the veneer of sand covering the low tide terrace was thin, being of the order of only 10-15 centimetres thick at the interface between the high and low tide beaches. Subsequent observation during October 2004, showed a much flatter beach profile, and a more widespread distribution of gravel-sized sediments on the beach surface (Plate 2), with both high-tide beach berms having been removed, and a significant proportion of the sand redistributed to form a nearshore bar, with a corresponding greater thickness of sand across the low tide terrace. This is illustrated by a greater degree of burial of the tank defence blocks at the base of the high-tide beach, and a less marked change in gradient between the high and low tide beaches (shown by comparison of Plates 1 and 2). At the top of the beach below the cliff face, there was a significant change in beach level, of the order of 1-1.2m during this period.

Survey Techniques

The test site was located on the coast to the southwest of Easington village. Access was via the Seaside Road that runs from the village to the beach. A 48m x 24m survey grid was laid out as shown in Fig. 2, with its long axis running shore-normal. Table 1 shows the range of geophysical properties expected above and below the interface for the distribution of geological materials at site. Lithological characteristics of the beach materials at the time of the survey have been discussed above. Testing on samples of exposed till near the test site indicated the matrix to be a composite soil type ranging from clayey sandy SILT to silty sandy CLAY, with a moisture content of 18%w/w and a bulk density of 2.1Mgm^{-3} ; values that fall within the ranges provided by Bell (2002). Seawater saturation has a significant effect on some geophysical properties of the beach deposits, which must be taken into account when interpreting the geophysical survey. Therefore, it is useful to employ an independent means of mapping seawater distribution such as the electrical resistance probe described below. The whole site was only exposed for approximately two hours either side of the low tide time, thus leaving only a four hour window for surveying. Throughout exposure, a spring line was observed to break from below a longshore zone of coarse gravel approximately equivalent to the $x=36\text{m}$ plane, (see Figs 1 and 2). Surveying times had to be short, thus favouring rapid, easily deployable, portable techniques. Where non-portable seismic refraction techniques were undertaken, survey lines had to be laid out rapidly and a portable shear wave source was required to ensure relatively short survey times. The surveys undertaken over this site included electrical resistivity tomography (ERT), electromagnetic (EM), shear wave velocity profiling using continuous surface waves (CSW), positioning of the surface elevation with a global positioning system (GPS), shear wave refraction, cone penetration resistance profiling, radar reflection profiling, and saturation profiling using an electrical resistance probe. Fig. 2 indicates the position of each survey line and profile location.

| Estimates of Geophysical Properties | | | | | | | | |
|--|-----------------------|------------------------|----------------------|------------------------|------------------------|-----------------------|----------------------|------------------------|
| Un-saturated Deposits | | | | | Saturated Deposits | | | |
| | Density | Electrical Resistivity | Shear Wave Velocity | Pressure Wave Velocity | Electrical Resistivity | Density | Shear Wave Velocity | Pressure Wave Velocity |
| Lithology | (Mgm ⁻³) | ($\Omega.m$) | (ms ⁻¹) | (ms ⁻¹) | ($\Omega.m$) | (Mgm ⁻³) | (ms ⁻¹) | (ms ⁻¹) |
| Properties of deposits above the interface – Beach Deposits | | | | | | | | |
| Sand | 1.9 | 10 - 10 000 | 80 - 200 | 400 - 600 | 3 – 10 | 2.2 | 80 - 200 | 1600.0 |
| Mixed Sand & Gravel | 1.9 | 10 – 10 000 | 150 - 250 | 400- 600 | 3 - 10 | 2.2 | 150 - 250 | 1700.0 |
| Properties of deposits below the interface - Bedrock | | | | | | | | |
| Till | 2.3 | 10 - 40 | 300 - 500 | 1700-2000 | 10 - 40 | 2.3 | 300 - 500 | 1700-2000 |

Table 1. Range of geophysical properties anticipated at site.

Shear Wave Refraction

The shear wave refraction survey exploits the critically refracted wave along the boundary between an upper layer of finite thickness and relatively low velocity and a lower layer presumed infinitely thick of substantially higher velocity. Grainger *et al.* (1973), Telford *et al.* (1976), Parasnis (1980), Palmer (1981), Palmer (1986) and Lankston (1990) have provided explanations of field methodologies. Hammer blows forcing a railway sleeper to move along its axis, perpendicular to the seismic spread (Abbis 1981) generated horizontally polarised shear waves that were detected by a line of 24 horizontally polarised geophones, again orientated perpendicular to the seismic spread. Table 2 provides typical arrival times for direct, non-critically refracted waves propagating through the beach and the till platform. A piezoelectric sensor on the sleeper was used to detect acceleration due to movement and provide the trigger for event timing. Coupling was greatly improved by using fins mounted on the underside of the sleeper that protruded into the formation to 0.1m. Examples of arrivals for longshore wave direction are shown in Fig. 3 where the Dry Sand Location was between $x=0m$ and $x=12m$, the Wet Sand Location between $x=24m$ and $x=36m$ and the Till Platform Location seaward of $x=48m$ on the grid in Fig. 2. Different velocities at the two sand locations can be attributed to increased effective stress with depth resulting in faster shear waves at larger source-receiver distances and the effect of water suction or increased packing density resulting in faster waves at the wet location. Refraction survey effectiveness increases with larger ratios between the beach material and the underlying till, e.g. Kassenaar (1992) suggested ratios of at least 1.5, which are only satisfied over shorter source-receiver distances on dry sand locations. In this case, there was insufficient contrast between the shear wave velocities in the beach and the till platform for a successful shear wave refraction survey.

| Easington Beach and Bedrock Shear Wave Velocity | | | |
|--|---|---------------------|---------------------|
| | Velocity for Source to Receiver Distance | | |
| Measured on: | 6m | 12m | 24m |
| Dry Sand Location | 111ms ⁻¹ | 134ms ⁻¹ | 180ms ⁻¹ |
| Wet Sand Location | 166ms ⁻¹ | 189ms ⁻¹ | 194ms ⁻¹ |
| Till Platform Location | 174ms ⁻¹ | 207ms ⁻¹ | 223ms ⁻¹ |
| | Velocity Contrast Sand : Till Ratio | | |
| Dry Sand Location | 1:1.57 (64%) | 1:1.54 (65%) | 1:1.24 (81%) |
| Wet Sand Location | 1:1.05 (95%) | 1:1.10 (91%) | 1:1.15 (87%) |

Table 2. Typical measured velocities for non-critically refracted shear waves through the beach and till platform.

Cone Penetration Resistance

The principle of the lightweight dynamic penetrometer is to drive a cone of known area into the ground with blows from a standard hammer onto the head of a piston attached to the cone by a steel rod. Commercial equipment is instrumented such that speed of impact and the penetration per blow are measured and used to calculate dynamic cone resistance using the Dutch formula (Langton 1999). The lightweight penetrometer is manually operated, which may cause variability in results. However, using this equipment in a sequence of weathered and unweathered tills at a test site at Cowden, Holderness (20km from the survey site) Langton (1999) produced comparable penetration resistance profiles to those described by Butcher *et al.* (1996), who employed motorised equipment and bentonite slurries to reduce friction effects. The lightweight equipment employs a 16mm diameter cone of 2cm² area with 0.5m long, 14mm diameter extension rods for standard use. Where the till at the survey site was expected to be close to the surface, problems with side friction were combated by using sacrificial cones of 22.5mm diameter and 4cm² area, such as along the line of $x=48\text{m}$ in Fig. 2. Where sacrificial cones were not used the effects of side friction can usually be observed as around a five-fold increase in penetration resistance over an interval of 0.5m. Gravel lags at the base of the beach and changes in strength within the till are identified on the penetration resistance-depth profiles, Fig. 4. For example, along $x=36\text{m}$, gravel lags are characterised by a series of peaks of approximately 20-30 MPa above local minima that represent the top of the till. The cone pushing against gravel-sized clasts before either breaking or pushing between them causes these peaks. The top of the till is represented by a value of approximately 5 MPa at (36,0), (36, 12), (48,0) and (48,12) but only about half this value at (36,24) and (48,24) on the grid. [N.B. co-ordinate labelling, for example (36,0), on the figures is equivalent to (36m,0m) but does not include the m dimension for clarity]. An overall increase in undrained shear strength with depth in the till was implied from the cone resistance profiles. This is consistent with increased shear wave velocities as the wave continually refracts more deeply into the formation as source-receiver distance increases.

Continuous Surface Wave

Continuous surface waves (CSW) are produced by a sinusoidal signal generated by an electromagnetic vertical vibrator seated on the ground surface. In practice, this produces a series of finite duration pulses, each at a single frequency over a range of frequencies, for example from 5Hz to 100Hz in increments of 0.5Hz or 1Hz. Field data acquisition at each frequency is synchronized with the control to the vibrator and field dispersion curves are generated from the recorded signals at two or more receivers, aligned shore parallel, using a method based on the steady state Rayleigh method described by Viktorov (1967), Richart *et al.* (1970), Nazarian & Stokoe II (1984), Sánchez-Salineró (1987), Nazarian & Desai (1993), Joh (1996) and Foti (2000). The incoming sea can affect signal to noise levels, but the investigations were undertaken at low tide conditions when the sea was over 75m away, and field data were generally of very good quality. The field data were inverted to produce shear wave velocity profiles with depth using WinSASW 2.2.1 following the procedure described by Joh (1996), with examples shown in Fig. 5. The inversion is non-unique and the ground properties as characterised by the cone penetration resistance profiles were used to aid the procedure. For example, gravel lags within the beach deposits often appear as intervals with high penetration resistance and also intervals of localised higher shear wave velocity as seen above 1m at locations (0, 0) and (12, 24) and above 0.5m at (36, 12) in Fig. 5. The CSW inversions provide shear wave velocity profiles that are very consistent with the velocities measured during the shear wave refraction survey. The upper three graphs (values of $x < 36\text{m}$ in Fig. 5) can be divided into three general zones representing the upper 0.5m of beach of approximately 100ms^{-1} , lower beach from 0.5m to 2.0m of approximate velocity range from 100ms^{-1} to 200ms^{-1} and the underlying till bedrock of approximate velocity range from 200ms^{-1} to 300ms^{-1} . The profile at (24, 24) shows the influence of effective stress causing an increase in the shear wave velocity with depth to 1.5m in the beach sediment with apparently little gravel (on the basis of the cone resistance profile). The top of the till in the lower graphs (values of $x > 24\text{m}$ in Fig. 5) has velocities below 200ms^{-1} and relatively low cone resistances (Fig. 4), which could possibly be related to the till in these locations suffering greater direct sub-aerial and sub-marine exposure time, particularly in winter months. It is also possible that this is representative of the heterogeneity of the till, with soft silty clay layers at the top of the till in these locations. The profile at (36, 12) also demonstrates how the top of the till would be a hidden layer and not be identified on a shear wave refraction survey.

Electrical Resistance Probe

Observations of seawater drained from the beach producing spring lines around $x=36\text{m}$ at low tide indicated that the groundwater levels at the site were controlled by the tidal fluctuations. It is important that the depth of the groundwater table within the beach deposits is known when interpreting electrical and electromagnetic survey data because saltwater saturation significantly affects the in situ electrical properties (see Table 1). It is also important to have knowledge of the depth interval of the partial saturation zone and the change in geophysical properties within this zone. Probes utilising time domain reflectometry (Hook & Livingston 1996) or resistivity measurements (Jackson *et al.* 2002) can be deployed to independently assess seawater levels within the beach. Fig. 6 shows a schematic of the measuring head of a simple resistance probe that used a four electrode technique (Telford *et al.* 1976, Parasnis 1980) to measure the electrical resistance of the localised beach volume adjacent to the measuring head. The current and voltage electrodes are only 1mm in diameter and erroneously large resistivities can occur when large gravel-sized grains contact the electrodes, as exemplified by the measurement shown in Fig. 6 at just below 2m at (0m, 12m). The beach at the site predominantly comprised well-draining, coarse materials giving rise to large magnitude resistance changes within a 0.1m interval from approximately 1000Ω above the partially saturated zone to approximately 30Ω at the seawater table, thus allowing it to be mapped. The groundwater level is dynamic, and the timing of the electrical resistance probings should take account of the time and duration of the EM and ERT surveys with respect to low tide. The example in Fig. 6 shows a suite of measurements made over the period of an hour, beginning at approximately 1hour before low tide, which was comparable to the timing of the EM and the ERT surveys.

Ground Penetrating Radar

Ground penetrating radar (GPR) uses a transmitting antenna to provide a short pulse of high frequency (25 - 1000 MHz) electromagnetic energy into the ground. Variations in the electrical impedance within the ground generate reflections that are detected at the ground surface by the same or another antenna attached to a receiver unit (Davis and Annan, 1989; Reynolds, 1997). Variations in electrical impedance are largely due to variations in the relative permittivity or dielectric constant of the ground, and thus respond well to water distribution and sedimentological structure (Neal 2004). Depth of penetration depends largely on factors such as the variability in the dielectric constant within the ground, attenuation of the waves within materials, and the frequency of operation. Davis & Annan (1989) and Neal (2004) provide tables of relevant properties. For example, water has a dielectric constant of 80, unsaturated sand and gravel around 5 and saturated

sand and gravel around 16, and hence there are high reflection coefficients between dry and wet materials. The electrical conductivity significantly affects the attenuation of radar waves, and on a beach, seawater intrusion is a significant factor in the effectiveness of the GPR technique. Surveys were undertaken around the low tide such that the beach could drain to acceptable water levels.

Common-offset, single-fold reflection profiling was undertaken with a fixed antennae geometry moved along a survey line to map reflections versus position. Unshielded antennae were operated with 2m spacing at a centre frequency of 50MHz resulting in wavelengths of up to 2 m. Given the variability of the materials at the site, generally conductivities were within the range from 0.001 to 0.1Sm^{-1} , equating to attenuations from around 1 to 300dBm^{-1} (Neal 2004). The radar section was built up from successive traces that were recorded with the radar stationary at each point. All radar traces were accumulated by stacking traces in order to improve the signal to noise ratio. Data were collected over the local grid along profiles in the y direction (i.e. x constant profiles parallel to the shoreline). The results are plotted in section form as two way travel time against traverse position. A time to depth conversion has been applied to the data by adopting an estimated electromagnetic propagation velocity of 0.1 m ns^{-1} . Two profiles are shown in Fig. 7, that along $x=0$ was at the top of the beach near the cliffs, whilst that along $x=48$ was on the low tide platform. Along $x=0\text{m}$ a distinct reflection is observed at a depth of approximately 2 m, i.e. the top of the till identified and consistent with cone resistance profiles, (see Fig. 7a). The sea level at the time of the survey (low tide) would have been some 5 m below the top of the beach. Perched water above till was confirmed to be of very low thickness using the electrical resistance probe, discussed above. Above this reflection, the section is characterised by its lack of reflectivity and is interpreted as a unit of mixed sand and gravel of low moisture, which appears to thin towards $y=24$. The survey along $x=48$ (Fig. 7b) is a section over the till as there was no significant sediment cover. The strong reflector at about 0.5 m depth is considered to be the ground wave. Structure within the till may have been imaged, appearing for example, as sub-horizontal reflectors and gently dipping reflectors. Variability in the radargram beneath this reflector appears as either scattered chaotic reflectors from $y=0$ to $y=12$, translucent with weak reflectors from $y=12$ to $y=20$ or with strong reflectors to $y=24\text{m}$. Unshielded antennae were used and it is suspected that side reflections from the concrete blocks (see Fig. 2 for positions) are responsible for these features. If where there are no blocks from $y=16$ to $y=24$, the persistent reflector just beneath the ground wave is due to heterogeneity within the upper part of the till, it is suspected that this would be a near-surface layer with a high proportion of very coarse material, such as the cobble rich layers shown in Plate 3.

EM Conductivity

Ground Conductivity surveys employ electromagnetic coil-coil coupling with different depths of

exploration (McNeill, 1980). The Easington beach survey was conducted across the local survey grid from (-2.5, 0) to (40, 24) using profiles (along the y -direction) spaced 2 m apart. The instrument used had a single transmitter orientated as a vertical dipole and two receivers housed in a boom around 4m long. The nearest receiver was at 4m from the transmitter orientated as a vertical dipole and the second receiver was 4.1m from the transmitter orientated as a horizontal dipole. This receiver configuration provides two different depths of investigation of about 2.5 m (shallow) and 6 m (deep). The boom was operated at slow walking pace, on a wheeled cart, recording coupling ratios every 0.5 seconds. Data were collected at both the shallow and deep depths of investigation. This resulted in an along-profile sampling interval of about 0.3m. The survey of the main site took about one hour to complete. The out-of-phase coupling ratios, for each receiver, can be converted to apparent conductivity using standard procedures (McNeill, 1980). The data for each receiver may then be used to form maps of apparent conductivity for two different depths of investigation (e.g. shallow and deep). The conductivities are apparent since the conductivity is assumed not to vary with depth. The data can, however, be treated in a more rigorous manner using an inversion scheme that attempts to recover the true variation of conductivity of depth at each measurement location. Due to the limited information obtained, it is necessary to regularise the inversion and only smooth conductivity variations with depth are permitted. It is also assumed that the conductivity variation is one-dimensional (1D). The 1D models obtained are stitched together to form a volumetric assessment of the conductivity distribution below the survey region. The conductivity model is shown in Fig. 8 as a 3D perspective view draped below topography. The volume size is 42.5m x 24m x 7.5m and the view is along the local y -direction parallel to the cliff (on the left) and sea (on the right). The conductivity range observed is far greater than that encountered in normal geological circumstances due primarily to saline invasion of the beach where the highest conductivities (> 500 mS/m) are defined in a thin at-surface zone between $x=20$ and 36m. Another zone of elevated conductivities is related to structure within the till, has a strong alignment with the cliff/beach axis and appears either as a lens that dips in a seaward direction, or as two shallow beachward dipping layers of higher conductivity separated by a lower conductivity layer, as discussed in the Thickness Interpretation and Discussion sections below.

Electrical Resistivity Tomography

Electrical resistivity tomography (ERT) is a method by which spatial models (2D or 3D) of subsurface resistivity distributions are generated. In the case of the beach environment it was anticipated that ERT could be used to image resistivity variations associated with changes in the lithology and the water content of beach materials. ERT data were collected from two survey lines (Fig. 2). The first line (ERT-1), with a strike perpendicular to the shoreline, comprised 64 electrode positions at 1 m intervals, and extended from (-5, 12) to (58, 12). The second line (ERT-2), which was oriented parallel to the

shoreline, comprised 32 electrode positions at 1 m intervals, and extended from (21, -3) to (12, 28). The two lines intersected at (12, 12). The resistivity data were collected using an 8-channel resistivity imaging system, and a dipole-dipole array configuration with 'a' spacings of 1, 2, and 3 m and 'n-levels' of 1 to 8. The survey was designed to achieve a maximum depth of investigation of approximately 6 m below ground level. The ERT-1 dataset consisted of 811 measurements that took 18 minutes to collect, whilst the ERT-2 dataset included 299 measurements that were collected within 5 minutes. Data acquisition was carried out shortly before low tide (1437 hours) at between 1230 and 1400 hours on 20th July 2004; throughout the ERT survey the till platform was exposed in drainage gullies on the lower foreshore seaward of the concrete blocks (Fig. 2).

The data were inverted using a 2D smoothness constrained nonlinear least-squares algorithm (Loke and Barker, 1995). The forward problem was solved using a finite element method, which permits topography to be easily incorporated into the inversion process (Tong and Yang, 1990). In this case, good convergence between the observed and model data was achieved, as indicated by RMS errors for models ERT-1 and ERT-2 of 1.05 and 1.54 % respectively. The 2D models resulting from the inversion process are shown in Fig. 9. Model resistivities ranged from less than 1 Ωm to 160 Ωm (equivalent to 1000mS/m to 6.25mS/m), reflecting significant variations in the beach materials. As anticipated, the ERT-2 model, which runs parallel to the shoreline, shows only limited lateral changes in resistivity, whilst ERT-1 displays significant lateral variations reflecting changing beach composition from the base of the cliff to the till platform. Distinct layering can be seen in both the ERT models. In the area of the high-tide beach a resistive surface layer ($>50 \Omega\text{m}$ or $<20\text{mS/m}$ conductivity) overlies a highly conductive zone ($<5 \Omega\text{m}$ or $>200\text{mS/m}$ conductivity), which in turn gives way to more resistive materials at the base of the models (5 to 50 Ωm). Although this sequence is reproduced in both ERT-1 and ERT-2, the base of the conductive zone falls at a slightly greater depth in ERT-2. This small discrepancy is likely to be a function of decreasing model resolution with depth, and off-line 3D effects (e.g. Chambers et al., 2002). The distribution of resistive surface materials, which thin and disappear towards the low-tide beach, corresponds well to that which would be expected from the unsaturated sands and gravel of the high tide beach. The conductive zone extends across the full length of ERT-1, although it thins markedly from several metres at the high-tide beach to tens of centimetres on the till platform. The low resistivities of this material indicate saturation by saline water; this assertion is supported by electrical resistance probe measurements (Fig. 6), which also indicate a change from unsaturated to saturated conditions at the same points. Given the absence of substantial sand cover on the till platform it is likely that this zone represents both saturated sands and gravel and weathered till. The moderate resistivities of the materials comprising the bottom layer of models are consistent with unweathered till bedrock.

Thickness Interpretation

Matching *in situ* geology with consistent data from a number of geophysical and geotechnical datasets improves confidence in the geophysical ground model. Newly emerging IT packages such as Rockworks2004TM or GSI-3D (Hinze et al. 1999; Sobisch 2000) provide very powerful platforms to store, present and aid interpretation of geological and geophysical data. When gathering large quantities of field data it is recommended that survey planning includes sufficient time for the integration of all geological, geotechnical and geophysical data in a well-considered phase of interpretation that includes the use of 3D modelling and display software. The field data gathered at the Easington site has been integrated using Rockworks2004TM to illustrate key features. Surface topography, as provided by a GPS survey, is vital for the sub-surface positioning of all data. It should be noted that beach surface topography is dynamic and should be surveyed regularly. The example in Fig. 9a shows the ERT-1 2D section with topography and the level of groundwater table at the time of survey; the groundwater table was confirmed by the electrical resistance probe profiles, which transects the sharp boundary between the wedge of high resistivity unsaturated beach and the saturated zone below. Note again the spring line at around $x=36\text{m}$, where seaward of this point the beach is saturated and appears as the shallow surface high conductivity zone in Fig. 8b.

The till surface, as shown in Fig 9, is based on an integrated interpretation of all the relevant field data. In the west near the cliff, the top of the till is coincident with the groundwater table level and thus strong reflectors on the radargram and the boundary between high and low resistivity zones. However, seaward, between $x=6\text{m}$ and $x=36\text{m}$ the top of the till transects this resistivity boundary, suggesting that in places the top of the till has similar resistivities to saturated beached deposits. This was confirmed with 0.5m Wenner surveys in the saturated beach and on the till platform. Here the top of the till is interpreted on the basis of the cone penetration and continuous surface wave data, for example where it is coincident with shear wave velocities of 200ms^{-1} or greater and low penetration resistances around 5MPa, which rapidly increase within a 0.5m interval. The resistivity and conductivity data show layering within the till, where a higher resistivity layer within the till underlies a gravel lag at the base of the beach at around 36m. Figure 10 shows the emergence of the till layer at between $x=36$ and $x=40\text{m}$ as higher resistivity or lower conductivity. Note that conductivity colour table has been reversed (i.e. hot to cold) such that low conductivities show as yellows and reds. Figure 10 shows that this higher resistivity layer within the till coincides with higher penetration resistances and high shear wave velocities suggesting that it is stiffer and more compact than the till above. Using the GIS, a model of a mixed sand and gravel beach has been

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developed that includes unsaturated mixed sand and gravel in the upper high tide beach of approximately 2m thick, which thins over a terrace slope around 24m long to a thickness of around 0.4m, and continues to thin to less than 200mm over a low tide beach that forms a veneer over the till platform. The platform is of variable strength and stiffness and appears to be layered with a soft, high conductivity layer overlying a stiffer low conductivity layer.

Discussion of Methodologies at Coastal Sites

Survey activity at coastal sites should be undertaken in a modular fashion to maximize the information gained at each site during successive phases of surveying, with each additional survey building upon the previous phase. A methodology is required that ensures that the decision to use a particular technique is correctly made because supporting field evidence was available. Thus, survey efficiency can be significantly improved if certain key activities are undertaken within a field procedure in the correct order. Prior to choosing techniques with which to survey a site, there are some fundamental factors that need to be established:

Step 1. Establish the lithologies of the deposit (sand/gravel etc) and the bedrock if possible.

Step 2. Establish the degree of water saturation in the deposit and its variability with tidal cycles.

Step 3. Establish a contrast in the geophysical/geotechnical properties of the deposit and the bedrock.

Step 4. Establish that the geophysical/geotechnical contrast is laterally continuous.

Step 1 can be achieved by observation of a suitably qualified geologist. In practical terms this requires a literature review and reconnaissance visits by a field geologist with relevant experience, and thereafter a report of their findings to the survey crews. Step 2 can be achieved using a simple probe that either directly measures moisture content or a moisture content proxy, for example an electrical resistance probe. Step 3 can be achieved using probes to gauge geophysical properties of the beach and bedrock. For example, shear wave probes have been used to verify the shear wave velocities in the sand materials and the Mercia Mudstone bedrock at a sand and gravel quarry in Nottingham (Gunn *et al.* 2005). Such tools would be used in conjunction with the lightweight cone penetrometer tool. Cone penetration profiling should be undertaken at a series of point locations, for example, along the intended line of a seismic refraction survey. The resulting penetration resistance profiles can be examined to identify potential stiffness interfaces that could be interpreted as the top-bedrock boundary. These data would provide evidence to decide whether or not to use follow-up surface wave and seismic refraction surveys. The identification of a step-like increase in cone penetration resistance on a single profile provides sufficient evidence to justify a continuous surface wave profile at that location. Step 4 is achieved with the identification of similar features on a line of penetration profiles, thus providing evidence that the feature is laterally continuous and justifies a

shear wave refraction survey. These steps can be translated into a survey methodology such as the following example. Proposed here is a two-stage methodology that includes a more detailed reconnoitring visit that involves several prescribed activities that can feed into a plan for the final site survey.

Reconnoitring Visit: In addition to standard procedures (tides, access, amenities) the reconnoitring site visit should also include the following activities:

- i. Participation of an experienced field geologist who subsequently reports their findings.
- ii. Cone resistance and moisture content profiling at key locations at the site including the bedrock if exposed.
- iii. Other geophysical property probing (e.g. shear waves) at key site locations.
- iv. The collection of samples for further examination and laboratory tests.

Survey Plan: These primary data can be considered when establishing a survey plan for each site. This will involve a choice from a 'toolbox' of techniques. Such a plan could include:

- i. Use of ground penetrating radar in coarse well-drained deposits, where it is established that seawater saturation will not present problems.
- ii. Continuous surface wave (CSW) profiling where potential stiffness interfaces have been identified (inferred) using the penetrometer.
- iii. Shear wave refraction where potential continuous refractors have been identified using CSW, or inferred along a line of penetrometer profiles.
- iv. More detailed surveys using electromagnetic (Dual EM and EM38) or electrical resistivity imaging, and further exploration of newly developed equipment where appropriate and available.

Finally, it is anticipated that climate change will include rising sea levels and changes in wave direction that will change the pattern of coastal erosion. DEFRA's concern with the future development of the UK coastline in response to climate change is the theme of new projects following on from Futurecoast (DEFRA 2002), where an understanding of the long-term dynamic nature of coastal processes causing coastal erosion is advised. Such strategic programmes should include studies of the processes and rates of current soft cliff recession and the role of the platform in coastline evolution. New methods should include rapid scanning of cliff elevations and beach surfaces using combined surveying and laser techniques as discussed by Hobbs *et al.* (2002), where combined geophysical-geotechnical methods can be used to establish the thickness of beach sediment.

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